Finite Element Physics Informed Neural Network Optimization

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PURPOSE

This research enhances a novel finite element physics-informed neural network (FE-PINN) framework in order to optimize efficiency and results. The enhancements include tuning hyperparameters and considering new methodology in constructing the model architecture. This study achieved near convergence of model prediction to actual data and successfully incorporates finite element discretization into a neural network model.

INTRODUCTION

- PINNs leverage a universal approximation theorem to accurately predict system response and estimate parameters (Raissi 2019)
- PINNs have a limited implementation in civil engineering due to the requirement for previously known physics equations which can be very complex to determine and depend on many factors
- This study proposes a hybrid approach which enhances PINNs with the finite element method to efficiently and accurately estimate parameters and model physical systems.
- The novelty of this study is the implementation of finite element time stepping methodology into the PINN framework and employing a weak form of equilibrium in the forcing function
- The framework is more versatile and can be applied to various civil engineering challenges including inverse and forward problems
- This study will build off of previous work to enhance the architecture of the FE-PINN model to achieve optimal results

BACKGROUND

Artificial Neural Networks (ANN): A type of machine learning model inspired by the structure of biological neural pathways. They are composed of layers of nodes (aka artificial neurons) connected by vectors that can be described by weights and biases. The nodes contain activation functions that help to capture nonlinear relationships between data. During training, data passes through the system and a loss between actual and predicted value is calculated. The loss is backpropagated through the model and weights and biases are updated to minimize loss.

Deep learning: Refers to when an artificial neural network has many layers, allowing it to better construct relationships between inputs and outputs from large and diverse datasets .

Physics-informed neural networks (PINNS): Implement governing physical principles into neural network losses during the training process, thus enforcing adherence to the physics equations. PINNs are effective in solving inverse problems and estimating a model's parameters from limited data. (Raissi 2019).

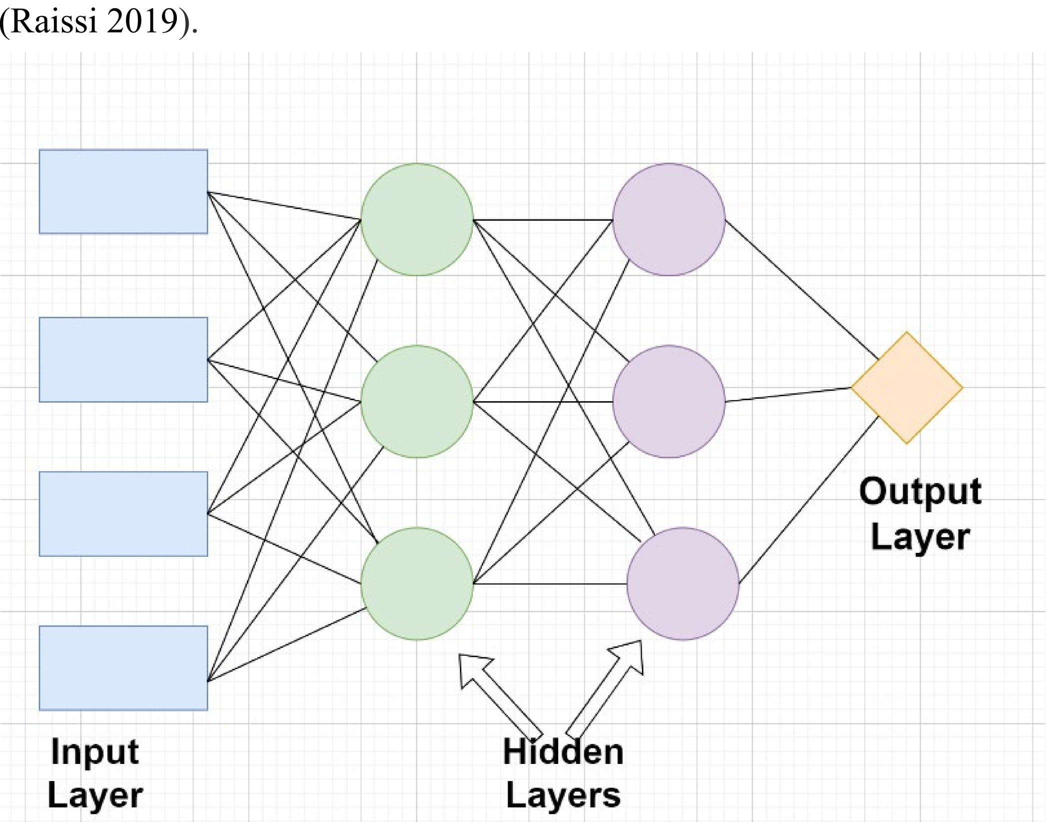


Fig. 1. ANN model.

METHODS

A novel type of PINN, (finite-element physics informed neural network) or FE-PINN, was created in previous work to utilize the weak form of equilibrium alongside finite element time stepping methodology to determine future displacement of an idealized spring system a single degree of freedom problem. The ODE presenting the weak form of equilibrium which is enforced via the physics loss is as follows:

$$R = M\ddot{U} + C\dot{U} + KU$$

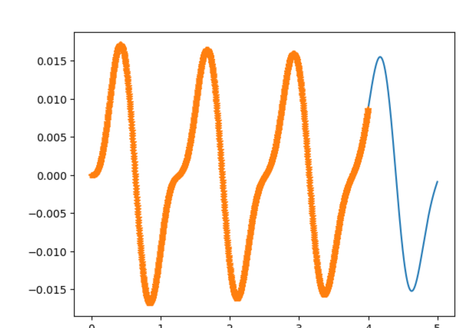
Hyperparameter Tuning :

- The aim of this study is to find the optimal architecture for this FE-PINN solving a simple spring-mass system displacement with the incorporated time-stepping methodology
- Several configurations are to be tested:
  - Varying the number of neurons per layer and number of layers
  - Giving different weights to data and physics losses
  - Implementing new strategies in applying the weak form of equilibrium
  - Decreasing the timesteps

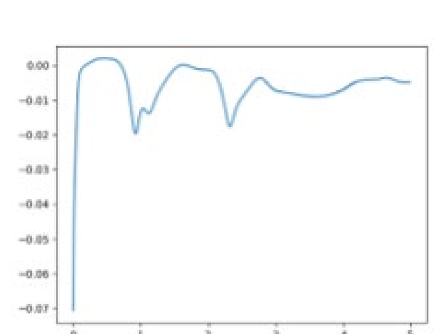
RESULTS

Part 1: Varying Neurons with 5 layers

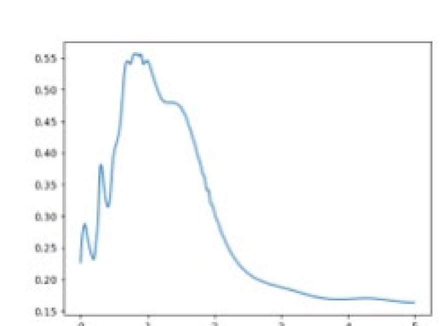
Actual Data:



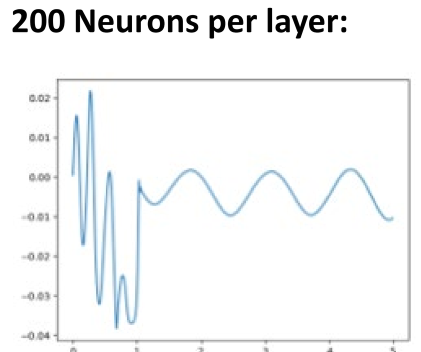
50 Neurons per layer:



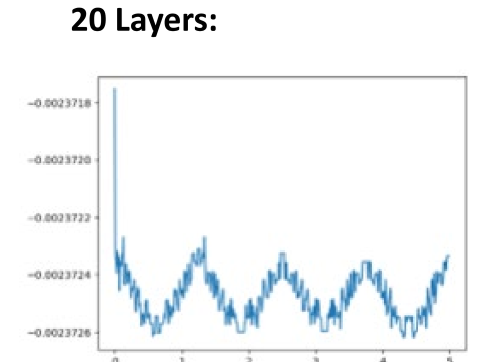
10 Layers:



200 Neurons per layer:

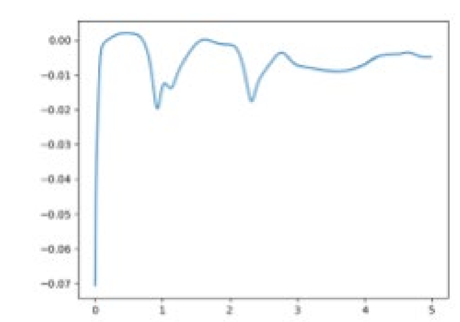


20 Layers:

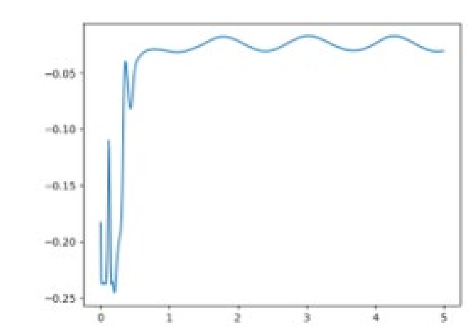


Part 2: Timestep Variation

0.1 Seconds



0.01 Seconds



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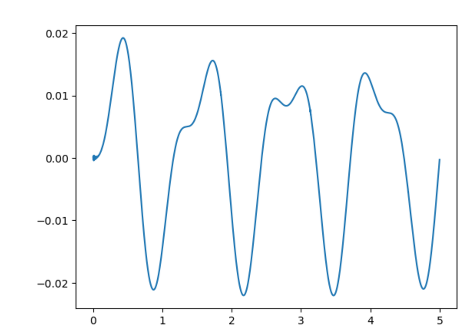

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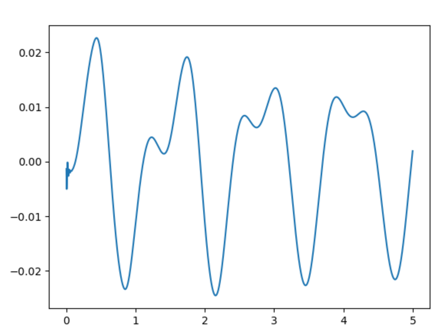

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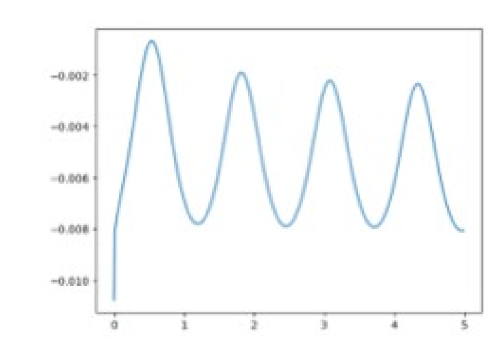

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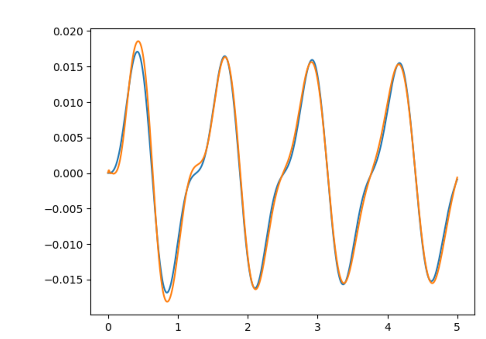

ANALYSIS

- From the hyperparameter tuning results, increasing the number of layers and neurons per layer improves the results
  - Interpretation of the initial hyperparameter experiments is difficult as they do not follow a set pattern
  - It is desirable to balance good results with computational efficiency
- The largest improvement in results was derived from decreasing the timesteps between training datapoints
  - One downside to decreasing the timesteps is the increase in computation power required to train the model and increase in the training duration
- Improvements were also observed by decreasing the model's learning rate, utilizing a reLU activation function as opposed to a hyperbolic tangent, and incorporating an additional physics loss from the velocity estimation as opposed to the solely equilibrium-based loss

NEXT STEPS

1. Create a more robust FE-PINN model that applies to 2 and 3 dimensional problems
2. Analyze new structures and approaches to incorporating time-stepping into the PINN architecture
3. Train the model on noisy data
4. Use nongenerated datasets
5. Incorporate a stopping clause to end training when model predictions are adequate

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