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The Impact of Cognitively Guided Instruction on Students' Mathematical Mindsets

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By

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Abstract

The purpose of this research was to analyze the impact of an inquiry-based word problem-solving framework, known as Cognitively Guided Instruction (CGI), on students' mathematical mindsets in an early elementary Montessori classroom. Students received one word problem-solving lesson per week over a six-week period. Students completed a pre-intervention and post-intervention mathematical mindset rubric, as well as CGI assessment. Both qualitative and quantitative results show that students had an increase in their variety of word problem-solving strategies, were able to solve word problems more accurately, and showed increased levels in self-efficacy, perception, and affinity towards math. Based on these results, CGI could be utilized as a supplementary instructional method to build students' mathematical mindsets and word problem-solving skills in Montessori classrooms. Further research is needed to know if these results hold true for other student populations.

Keywords: mathematics, social constructivism, mathematical mindset, word problem solving, Montessori, lower elementary

The Impact of Cognitively Guided Instruction on Students' Mathematical Mindsets

Students in the United States consistently perform below average on mathematical assessments compared to their peers in other developed countries. According to the Programme for International Student Assessment (PISA), only 8% of 15 years old students in the United States in 2018 could model complex mathematical situations and apply appropriate problem-solving strategies to them, compared to an international average of 11% (Organisation for Economic Co-operation and Development, 2019). The ability to model mathematical situations and apply problem-solving strategies correctly is a skill that will be essential in the job market of tomorrow, according to the World Economic Forum (2020). The National Council of Teachers of Mathematics states that problem solving needs to be one of the core processes that students focus on in a mathematics classroom (National Council of Teachers of Mathematics, 2000).

Given how crucial mathematical problem-solving skills are for students, it is necessary that Montessori educators today utilize modern educational frameworks for teaching children word problem solving. Montessori education traditionally has not included methods for guiding children in solving word problems, but rather has focused on children deriving mathematical understandings using concrete materials. This can lead to difficulties when students are asked to solve mathematical problems without the use of materials. When my previous classes of students took the end-of-year state standardized test, they would often perform poorly on the word problems that appeared on the test. It appeared they could not transfer what they learned with concrete materials to a word problem using pencil and paper.

Therefore, the aim of this study was to assist students in beginning to model mathematical situations by applying appropriate problem-solving skills. Additionally, the intervention investigated how lessons on solving word problems impacted students' attitudes and mindsets about mathematics. I chose a constructivist problem-solving framework known as Cognitively Guided Instruction (CGI) because it could allow for students to construct strategies for solving word problems with teacher and peer guidance in a way that aligns with the Montessori approach to math instruction.

This action research project took place in an independent private school in a metropolitan city. The school offers a Montessori-based curriculum for primary and elementary students and International Baccalaureate curriculum for students in middle school and high school. While many elements of Montessori pedagogy exist in the elementary school, certain elements are unique to the school, such as the pairing of grade levels in classrooms or the responsibility of guides. This research took place in a second and third grade “early elementary” classroom with twenty-two students and two co-guides.

Theoretical Framework

This study uses the lens of social constructivism to understand learning. Students do not learn passively, but rather construct knowledge through social interactions and their interpretation of these interactions (Adams, 2006). Learning is inseparably intertwined with the social context where it occurs. Through social constructivism, “the nature of the learning environment is one of experimentation and dialogue, where knowledge is seen within the context of problems to be discussed and solved” (Adams, 2006, p. 245). Students do not learn in silos, but rather within a community of practice. While Vygotsky is seen as the originator of the theory of social constructivism, many other constructivist theorists, such as Piaget, Dewey, Bruner, and Montessori, influenced this theory and will be incorporated into the theoretical framework for this study.

Learning occurs through social processes, causing language to play a fundamental role in learning. Piaget (1926) called the language that allows children to learn from one another *socialized speech*. Socialized speech allows the child to exchange thoughts with others, ask questions, and relay answers. Vygotsky (1978) emphasized the importance of this type of speech, stating “speech and action are part of one and the same psychological function, directed toward the solution of the problem at hand” (p. 86). While learners can construct knowledge through concrete materials, as per Montessori education (Montessori, 1948/2023), the language children use to construct this knowledge is equally important. Vygotsky (1978) stated the more difficult a task is, the more necessary it is for children to use language to help solve that task or problem. Speech is crucial to learning. Therefore, it is necessary that the classroom environment includes opportunities for discussion between students, as well as between teachers and students.

The relationship between a teacher and students plays a crucial role in learning. As Bruner (1966) stated, “The relations between one who instructs and the one who is instructed is never indifferent in its effect upon learning” (p. 42). According to social constructivism, students advance in their learning when they are learning within their Zone of Proximal Development, that is they are given challenging work that is scaffolded by interactions with an adult or capable peers within a community of practice (Vygotsky, 1978). Teachers are responsible for designing problems that push students into the next stage of development (Bruner, 1960). Rather than directly instruct students, teachers should guide and facilitate discussions (Adams, 2006). The view of the teacher as an active guide scaffolding learning through discussion differs from Montessori’s and Dewey’s view of guides as observers who wait to present new lessons until children are independently ready (Dewey, 1938; Lillard, 2005).

Social constructivism closely aligns with Cognitively Guided Instruction (CGI). Within a lesson based on the CGI framework, students discuss their mathematical thinking one-on-one with their teacher, as well as in small group and whole group discussions. Students also listen to and explain the benefits and drawbacks of others' strategies. Learning happens through students "constantly describing, explaining, and justifying the strategies they use to solve a problem" (Carpenter et al., 2015, p. 194). Moreover, the teacher is not seen as the source of information, but rather a learner as well. Teachers are part of the social process of learning, using what their students discuss to design challenging word problems or scaffold future discussions.

Thinking of learning as a socially based process also aligns with many of the attributes that are part of a mathematical mindset, which is defined as a growth mindset that includes attitudes and habits that are specific to math (Boaler, 2016). When students have a strong mathematical mindset, they see math as a "connected subject and a form of communication" (Boaler, 2016, p. 172). They can explain their thinking and how it relates to the ideas of others. They are also able to ask questions to better understand the thinking of others. Moreover, the focus is on learning within a heterogeneous community where all learners have access to the same open-ended tasks with a low floor and high ceiling (Boaler, 2016). As students work on the same task with peers of differing abilities, they can use their social interactions with peers to help them work within their Zone of Proximal Development. Students co-construct their identities as mathematical learners when they work on tasks together. The teacher is not the disseminator of knowledge, with students passively receiving that information. Rather students can build strong mathematical mindsets when they are part of a strong mathematical community where learning is centered on dialogue and constructing knowledge on how to solve mathematical problems together.

Literature Review

This action research project aims to analyze the impact of Cognitively Guided Instruction on students' mathematical mindsets in a Montessori early elementary classroom. This section reviews research on mathematical mindset, factors that impact word problem solving, and instructional methods for word problem solving. It is organized under the headings: mathematical mindset, conditions affecting mathematical problem solving, and explicit versus inquiry-based instructional methods.

Mathematical Mindset

To understand the aim of this study, it is important to understand the research around mathematical mindsets. According to Boaler (2016), students have a mathematical mindset when they have a growth mindset about their ability to learn new subject matter and a growth mindset about mathematics and their role as mathematicians. Students with a growth mindset believe their intelligence is malleable and their abilities in any subject area can grow over time (Dweck, 2008). A growth mindset is juxtaposed to a fixed mindset, wherein students believe that their intelligence in any given area is fixed and cannot change over time (Dweck, 2008). In addition to believing that they can learn anything, students with a mathematical mindset also believe mathematics is a creative, open subject of exploration and inquiry rather than a fixed set of procedures (Boaler, 2016). Their role in mathematics is to ask questions and discover connections rather than learn procedural methods for solving problems correctly (Boaler, 2016).

For students to form a strong mathematical mindset, teachers need to change how they present mathematical problems. Students should work on mathematical tasks that encourage "curiosity, connection making, challenge, creativity" and that involve "collaboration" (Boaler, 2016, p. 57). Boaler defined these as the "5 C's of mathematics engagement" (2016, p. 57). Such tasks should include multiple ways for students to access, visually represent, and solve them. Mathematical tasks should require that students develop ideas rather than use straightforward procedures. They should encourage children to explore and think for themselves before being taught a specific method, and they should lend themselves to mathematical visualization. Mathematical tasks should also be “low floor, high ceiling” (Boaler, 2016, p. 84), meaning that heterogeneous groups of students can access the task and challenge themselves. Finally, students need to be able to justify and reason what they are doing mathematically with every task they solve. When students work on rich mathematical tasks, they see math as more open and discovery-based, building stronger mathematical mindsets and identities (Allen & Schnell, 2016).

Teachers must also change how they interact with students about their mathematical thinking. In many mathematics classrooms, there is a *didactic contract* (Brousseau, 1984) between teachers and students. When a student asks for help with a problem, the teacher either asks the student leading questions (Sun, 2018) or breaks the problem down into parts, making it easier and less cognitively demanding for the student to solve. As a result, the student comes to rely on the teacher for help whenever they feel challenged, and the teacher cannot fully see what the students can do mathematically. An alternative form of interaction that allows students to build a mathematical mindset is for teachers to ask more open-ended, conceptual questions, such as how the student is making sense of the problem or seeing the problem visually (Boaler, 2016; Sun, 2018). If teachers elicit students’ mathematical thinking and support them in “productive struggle” (Townsend et al., 2018, p. 216) rather than provide students with mathematical solutions, students will build stronger growth mindsets (Park et al., 2016).

Several criteria can measure students' mathematical mindsets. These criteria come from the positive norms that should be present in a mathematical classroom that encourages a growth mindset (Boaler, 2016). Such norms include:

- Everyone can learn high levels of mathematics.
- Mistakes are opportunities for learning and growth.
- Questions drive learning.
- Mathematics should make sense and be creative.
- Communication and collaboration are vital to learning mathematics.
- When solving problems, depth is more valuable than speed.
- Mathematics takes time and effort to learn.
- Mathematics is about the process of learning, not performance on a test.

Im & Park (2022) used these positive classroom norms to create a Likert scale for measuring adolescent students' mathematical mindsets. While other scales exist to measure students' growth mindsets in general, a scale specifically made to measure mathematical mindsets is important. Im & Park found that students can have high levels of growth mindset in other subject areas but still have a low mathematical mindset. Given how prevalent fixed mindset beliefs are in mathematics and science (Boaler, 2016; Dweck, 2008), developing ways to measure students' beliefs in this subject area is crucial.

The academic implications for students with strong mathematical mindsets are vast and varied. Burnette et al. (2013) found that students with an incremental theory of intelligence (growth mindset) were better able to set goals, monitor their progress towards goals, and use mastery-oriented strategies to achieve goals than their peers with an entity theory of intelligence (fixed mindset). Several studies found that students with a growth mindset achieve higher grades overall than students with a fixed mindset (Blackwell et al., 2007; Dar-Nimrod & Heine, 2007; Grant & Dweck, 2003). However, students with a lower socioeconomic status do not experience the same academic results from having a growth mindset as higher socioeconomic peers (King & Trinidad, 2021). Domain-specific interventions for growth mindset, particularly in mathematics, are also more effective overall than domain-general interventions (Bui et al., 2023). Students show improvements in math when their attitudes and mindsets towards math are specifically addressed rather than their attitudes or mindsets towards learning in general. Daly et al. (2019) found that university students were more motivated to solve open-ended mathematical mindset-based word problems than traditional close-ended word problems.

Conditions Affecting Mathematical Problem Solving

Just as there is debate around the role of a growth mindset on students' academic achievement, scholars also debate what conditions are best for creating and presenting word problems to students. Several conditions that affect students' mathematical mindsets in classroom settings also affect their abilities to solve word problems. Differentiation, question design, and the use of modeling and diagrams all impact how students process and solve word problems.

Approaches to differentiating word problems can take a variety of forms in classrooms. The most common form of differentiation is to track students and present them with word problems in small groups or different classes based on their perceived level according to standardized math assessments (Boaler, 2016). This form of tracking can lead students to hold a fixed mindset about their abilities as they work in either low or high-level math groups. An alternative form of differentiating problem solving is the Balanced Mathematics approach used in the Madison Metropolitan School District (Christenson & Wager, 2012). In the Balance

Mathematics approach, students work in flexible problem-solving groups that shift and change throughout the year. According to Christenson and Wager, this approach allows “the teacher to select appropriate number sizes, types of problems, and contexts to expand each child’s understanding while allowing every student to develop, share, and defend his or her solution strategies” (2012, p. 200). While this approach allows for more flexible differentiation than fixed tracking, it still creates a context where children may work on lower levels of math than their peers. What Boaler (2016) recommended through the mathematical mindsets approach is to instead differentiate through the methods that students use to solve word problems. Teachers do not need to create different types or levels of word problems if they give open-ended word problems that can be solved using multiple pathways or methods.

The way that educators design word problems have a significant impact on students’ understanding. Traditionally, word problems can often be written in a way that asks children to ignore common sense or real-life situations and instead requires students to focus on just the numbers and keywords (Boaler, 2016; Carpenter et al., 1999). Due to this, students often provide answers that do not make sense mathematically in a real-world context (Kirkland & McNeil, 2021). When word problems are taken directly from textbooks, teachers miss an opportunity to explicitly design and link problem solving to students’ experiences and cultures. When teachers create word problems for students, they can design problems that make sense contextually for students and target specific learning goals and objectives based on their students’ needs (Barlow, 2010). They can also create word problems that require students to make sense of and justify their answers, such as ones that require students to give a yes/no answer and explain why (Kirkland & McNeil, 2021). Allowing teachers to design or select word problems for their students leads to higher mathematical thinking and problem-solving levels.

Visualization and modeling also play a significant role in helping students make sense of word problems. When students draw models of word problems, such as strip or tape diagrams (Ding, 2018), they can better understand the word problem mathematically (Boaler, 2016; Englard, 2010). Older students often do not independently choose to draw diagrams when given a word problem (Uesaka & Manalo, 2012) and instead will go straight into using algorithms. They are less likely to use diagrams when the cognitive cost of translating a word problem into a model is high, whereas they will be more likely to draw a model when the word problem they are solving is easy to visualize. However, for younger students, drawing a model or using a manipulative for a word problem comes more naturally and should be fostered so that students can keep using that strategy long term. Singapore math is one framework that supports mathematical visualization (Englard, 2010). Rather than spending time in class teaching children mathematical algorithms, teachers using Singapore math spend time with students discussing strategies for visualizing word problems, usually through using bar models, and then deriving their algorithms from those models. This method of concrete to pictorial to abstract supports learners who struggle to make sense of word problems. Visual modeling also helps learners see and make sense of each other's work.

Explicit Versus Inquiry-Based Instructional Methods

While there is a consensus that students need to receive word problems that make sense, are challenging for them, and allow for modeling and visual thinking, how students receive instruction around word problems is widely debated. The research around word problem-solving instruction falls into two main categories. The first category is explicit instruction, also known as closed mathematics instruction or traditional math instruction (Boaler, 1998; Kirschner et al., 2006; Kroesbergen, 2004). The other category is inquiry-based instruction, also called constructivist, discovery, experiential, or open mathematical instruction (Boaler, 1998; Kirschner et al., 2006; Kroesbergen, 2004).

Studies of both forms of instruction provide mixed results. In a study of open versus closed mathematics instruction in England, Boaler (1998) found that high school students who learned an open project-based method of mathematics performed better on tests and had a deeper understanding of mathematics than students who were taught mathematics in a traditional manner with lectures and daily textbook problem sets. Students in the project-based mathematics classroom were better able to solve unfamiliar problems on tests because they were used to transferring mathematics they had learned to novel situations. The students in the traditional mathematics classroom struggled to solve unfamiliar problems on tests because they were used to solving problems familiar to the ones their teacher had shown them how to solve and needed more practice transferring their knowledge to new mathematical situations.

However, other research studies on these forms of instruction do not consistently show similar findings. Explicit instruction can benefit specific groups of students, particularly those with mathematical difficulties. Students with verbal problem-solving difficulties were better able to solve problems when given direct instruction on problem-solving strategies (Babakhani, 2011). Students with diagnosed mathematical difficulties performed better when they received direction instruction around using bar models and other visual strategies (Morin et al., 2017). The effectiveness of explicit instruction depends on the quality and quantity of teacher-student interaction (Doabler et al., 2015). When students receive frequent, high-quality feedback from teachers, they have increased progress in academics.

The importance of feedback aligns with research on the limitations of minimally guided constructivist instruction. When students receive minimal guidance around mathematical learning, they do not make the same academic progress as peers who receive more direct guidance from teachers (Kirschner et al., 2006). Students need feedback from teachers to support the “cognitive processing necessary for learning” (Kirschner et al., 2006, p. 76). When students work on unfamiliar problems with minimal or no scaffolding or guidance, they rely primarily on their working memory to process information, which has a limited capacity to process new material. The cognitive load of solving problems becomes much higher than if students were to receive direct support and guidance.

While minimal guidance or unassisted discovery-based learning is ineffective in helping children progress academically compared to explicit instruction, there is a middle ground instructionally wherein children construct mathematical knowledge, but the teacher plays a more active role in providing scaffolding and feedback. This form of instruction is referred to as guided constructivist instruction, as well as enhanced or assisted discovery (Alfieri et al., 2011). Researchers have found that students who receive this form of instruction perform on par or better academically than students who receive explicit instruction (Alfieri et al., 2011; Kroesbergen, 2004; Lazonder & Harmsen, 2016). Students also showed increased levels of motivation and lessened traditional beliefs around math when receiving guided constructivist instruction. These findings align with Vygotsky’s theory that students learn when given work within their Zone of Proximal Development and that learning happens through social interactions and guidance.

Cognitively Guided Instruction is one problem-solving framework that utilizes a model of guided constructivist instruction. CGI provides scaffolded learning opportunities for students through teachers thoughtfully designing word problems within their Zone of Proximal Development (Carpenter et al., 2015). Children’s thinking informs teacher instruction and guidance (Carpenter et al., 1996; Chambers & Lacampagne, 1994; Jacobs & Phillip, 2010; Kazemi et al., 2016). Teachers provide inquiry-based guidance by conferencing with students as they solve word problems, asking students about their thinking, how they visualize the word problem, and why they are using certain approaches or strategies. At the end of the lesson, the teacher leads a discussion where students show each other their strategies, and the teacher helps provide visual models as needed to show students' mathematical thinking. The focus of this discussion is for the teacher to ask questions and for the students to justify their problem-solving strategies. Children learn through scaffolded communication with one another.

Since CGI can be considered a form of guided constructivist learning or assisted discovery, it can benefit students more compared to a traditional model of instruction. At the same time, though, it has limitations, especially when considering the benefit of explicit instruction for students with mathematical learning differences. More studies need to be done on students' academic success using CGI as current research shows mixed results (Schoen et al., 2020). Outside of academic progress, CGI can benefit students in how teachers can use the framework to create a more inclusive, culturally responsive classroom environment (Hankes, 1998; Ladson-Billings, 2000; Moscardini, 2014). Teachers can group students heterogeneously, as students are welcome to use various strategies to solve word problems. Since word problems are teacher-created, they can reflect students' interests, cultural knowledge, and social environments. The emphasis on student-led discussion can help mitigate social and cultural barriers between students as they share their mathematical thinking. One critique of CGI that teachers need to consider is its heavy focus on students' mathematical identities and thinking (Rodriguez et al., 2022). According to Rodriguez et al., teachers need to center the multiple identities of children in every aspect of instruction and continually reflect on making sure they are centering the voices of minoritized students in the classroom, such as emergent bilingual learners. While CGI is a framework that centers student thinking, it still operates within existing structures of power and hierarchy in schools.

Conclusion

There is a wide range of research on how different instructional methods for word problem solving affect student outcomes. Researchers have found that minimally guided instruction does not benefit students academically as it requires too high of a cognitive load. Explicit instruction can benefit learners who have mathematical difficulties but can be challenging to transfer to novel situations. There is an emerging consensus that guided constructivist or inquiry-based instruction may benefit students equally or more than explicit instruction. Cognitively Guided Instruction, a form of guided constructivist instruction, has received mixed results in how it impacts students' academic outcomes but aligns well with research on instructional practices that help build students' mathematical mindsets.

Cognitively Guided Instruction incorporates many of the same positive norms present in mathematical mindset classrooms (Boaler, 2016; Carpenter et al., 2015). Questions drive instruction in CGI. Learning happens through connections and communication between students. Mistakes are valuable learning opportunities as the focus is on the process of solving problems, and not just finding the right answer. Students are encouraged to spend time on one word problem, find multiple ways to solve it, and prioritize depth over speed. Students are also encouraged to be creative and make sense of the word problems they are solving. Finally, students work on higher level word problems than often are required by state curriculum, working on problems involving multiplication and division as early as kindergarten.

Despite the substantial overlap between CGI's framework and the positive norms of mathematical mindset instruction, there needs to be more research on how CGI impacts students' mathematical mindsets. This study aims to fill that gap and see how a structured inquiry-based problem-solving framework can impact students' growth beliefs around mathematics. More research needs to be done on how other problem-solving frameworks, such as Singapore Math or explicit models of instruction, impact students' mathematical mindsets as well.

Methodology

I collected data for this study using a pre-and post-intervention word problem-solving assessment and mathematical mindset rubric, a tally chart of observed behaviors during math lessons, observational notes taken during math lessons, and artifacts of students' work. I triangulated this data to show the effects of Cognitively Guided Instruction on students' mathematical mindsets. I collected data from 20 of the 22 students in my class, of whom nine were second graders and eleven were third graders. Two students were excluded from the data analysis due to incomplete pre- and post-intervention data.

Entering as a new teacher in this classroom, I chose to use a CGI assessment (Appendix A) provided through the CGI Math Teacher Learning Center, which is the professional development agency associated with the researchers who developed CGI. I used the *CGI Math TLC Assessments of Math Understanding* (Levi, 2023), to collect written work about my students mathematical problem-solving skills, but also to observe how my students interacted with word problems, particularly ones they did not know how to solve right away. The assessments for both second and third grade consisted of one subtraction word problem, one missing addend word problem, one multiplication word problem, a subtraction equation, and a missing addend equation with a letter representing the missing addend. The assessment also included a short math survey, which asked the students if they were good at math, if they could figure out math problems by themselves, and if they wanted to have a job that uses math when they grow up. The assessment was completed in small groups so that I could observe students' behavior around the assessment more closely. I provided support in reading questions to students who needed additional help but provided no support in solving the questions.

In addition to this pre-intervention assessment and short survey, students completed a mindset rubric (Appendix B) where they circled always, almost always, sometimes, rarely, or never for a list of attitudes and behaviors associated with a strong mathematical mindset. I modified the list of attitudes and behaviors to be more developmentally appropriate for second and third graders based on a rubric created by Boaler (2022). This rubric was completed in small groups before the first CGI lesson. I read the prompts aloud to the students and provided examples of what each item on the list meant so as to make sure students understood what they were answering.

Once students completed the pre-intervention assessment and mindset rubric, I used a tally chart to collect data about their behavior during both CGI and Montessori math lessons (Appendix C). I tallied how many students promptly came to math lessons versus delaying or refusing to come to a lesson. I tallied how many positive versus negative comments about math were made during the lessons. Finally, I tallied how many students were on task completing work from the lesson. I chose these categories based on the idea that students with a strong mathematical mindset will want to come to math lessons right away, make positive comments about math, and stay on task solving math problems. I also included a space for notes on this template where I could include additional information. Examples of information I included were notes on the positive and negative comments students made, why a student was delayed coming to a lesson, how students were approaching the math work in the lesson, and strategies they were trying to use.

During the CGI lessons, students were provided with a blank sheet of paper with the word problem written at the top of it. I asked students to write down their mathematical thinking as much as possible either through pictures or words. If a student had a difficult time capturing their thinking on paper, I would write down what they were explaining out loud to me on their paper. All students were asked to write a sentence at the end of the lesson explaining how they solved the word problem. These papers were then collected so that I could analyze how students' word problem-solving strategies changed over time, as well as how students progressed in being able to explain their mathematical thinking.

Each CGI lesson consisted of the same structure week to week. I divided my students into heterogeneous groups of 5-6 students at the beginning of the six-week period. These groups remained relatively the same week to week, with a few noted changes when students had to switch to another group or miss the lesson because of a scheduling conflict. I presented a CGI lesson to each group of students for 25-30 minutes one morning a week. At the beginning of each lesson, I showed students a different mathematical mindset poster from Boaler's website YouCubed (2023) and led a discussion with students about that particular mathematical mindset attribute (Appendix D). Examples of topics included: mistakes are opportunities for learning and growth, discussions lead to deeper mathematical thinking, and visualizing mathematics strengthens your brain.

After the short mathematical mindset discussion, I presented the word problem for the lesson (Appendix E). I initially decided to write word problems with two-digit subtraction (such as 54-28) based on the pre-assessment students took. I noticed on the pre-assessment that almost all students had difficulty answering two-digit subtraction problems accurately and showing their work. I also wrote the word problems based on true aspects of my life, such as my pets and plants, so as to provide a relevant and accessible context for students that would further help them connect with me. Students read the word problem along with me out loud and were then given 10 minutes to solve the problem on their paper using as many strategies as possible. Students had access to Montessori materials like the Bead Bars and Colored Counting Bars (Appendix F) to use in helping them solve the word problem.

While students solved the word problem, I conferenced with each student about the approach they were taking and offered inquiry-based guidance if they reached a point where they were stuck or did not know where to begin. I made notes about each student's strategy on my observational tally chart. If students solved the problem in their head, I asked them to show their work on paper as well. I also asked students who finished early to find another way to solve the problem. As students worked on the word problem, I also acknowledged verbally when they were using a mathematical mindset strategy. For example, if a student made a mistake, I would point out how their brain was becoming stronger through that mistake.

Once I noticed that all students had solved the problem and shown work on their paper, I led a discussion where each student shared one of their strategies for solving the math problem. I provided support as needed in helping students explain their strategies. I also built on the strategies they used. For example, if a student showed how they drew circles that they then crossed off to solve the problem, I showed the students how to organize the circles into rows of 10 to make them easier to count. At times, I asked students to write down the alternative methods I was showing them so they could have a better understanding of how to use that strategy in the future. After each student had a chance to explain how they solved the word problem, I asked them to write a short sentence on their paper explaining what strategy or strategies they used. Once students wrote their sentences, I collected their work and asked them to invite the next small group of students to join me.

After six weeks of CGI lessons, students completed the same CGI assessment they completed at the beginning of the assessment, including the math survey. Students also completed the mathematical mindset rubric. Using the assessment and rubric pre- and post-intervention allowed me to see changes over time in students' problem-solving abilities, behaviors around problem solving, and overall mathematical mindset.

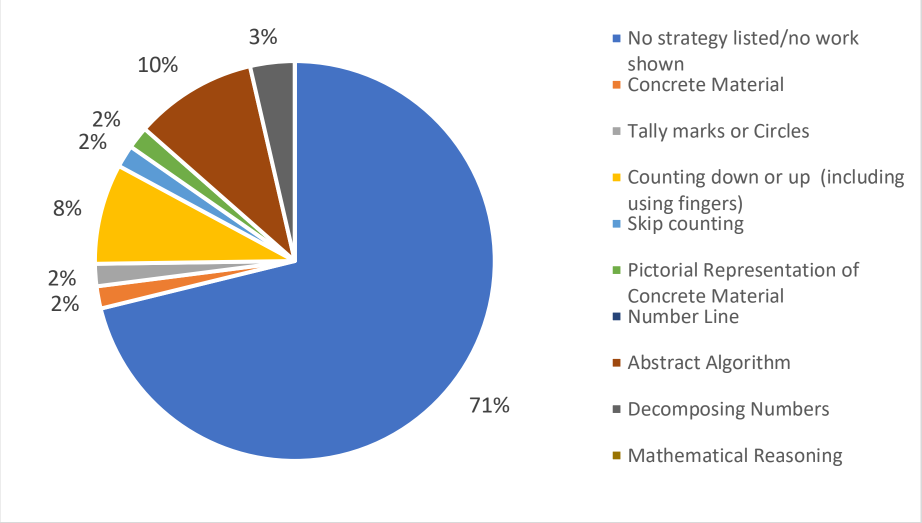
Analysis of Data

Mathematical Strategies

In the pre-intervention CGI assessment, students were not able to explain their work or show strategies for 71% of the problems. The strategies they used on the pre-intervention CGI assessment mostly included mathematical reasoning (10%) and counting up or down from a given number (8%).

Figure 1

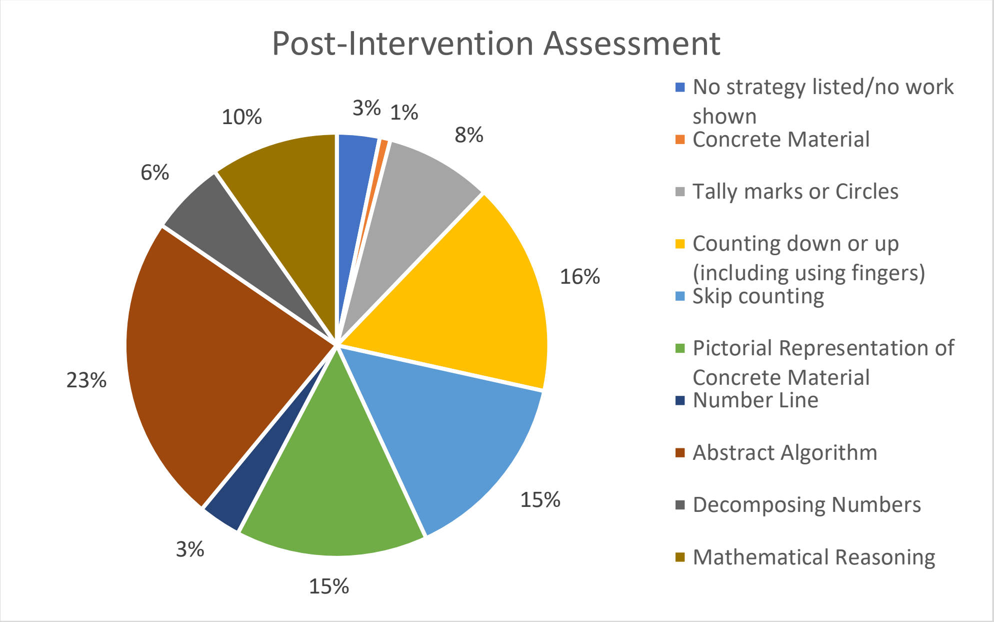
Strategies Used in Pre-Intervention CGI Assessment



In the post-intervention CGI assessment, students used a much wider variety of strategies. Only 4% of problems included no written work or strategy shown. The most frequently used strategies included abstract algorithms (23%), counting up or down from a given number (16%), skip counting (15%), and drawing a representation of a concrete material, such as Base 10 blocks (15%).

Figure 2

Strategies Used in Post-Intervention CGI Assessment



This wider use of strategies in the post-intervention CGI assessment as compared to the pre-intervention CGI assessment is reflected in the students' written problem-solving work each week. In the first week of the intervention, students primarily used concrete materials to solve the subtraction word problem. In the second week, many students transitioned to drawing tally marks or circles to represent each unit in the subtraction problem. In the third week, the majority of students chose to count up or down from a given number. In the fourth week, students used a mixture of strategies. Strategies included using a concrete material, drawing tally marks or circles, using the abstract algorithm, and decomposing numbers. In the fifth and sixth weeks, students primarily drew pictures representing concrete materials (such as the Stamp Game or Base Ten Blocks) or decomposing numbers to solve the problem.

Figure 3

Examples of Pictorial Representations of a Concrete Material

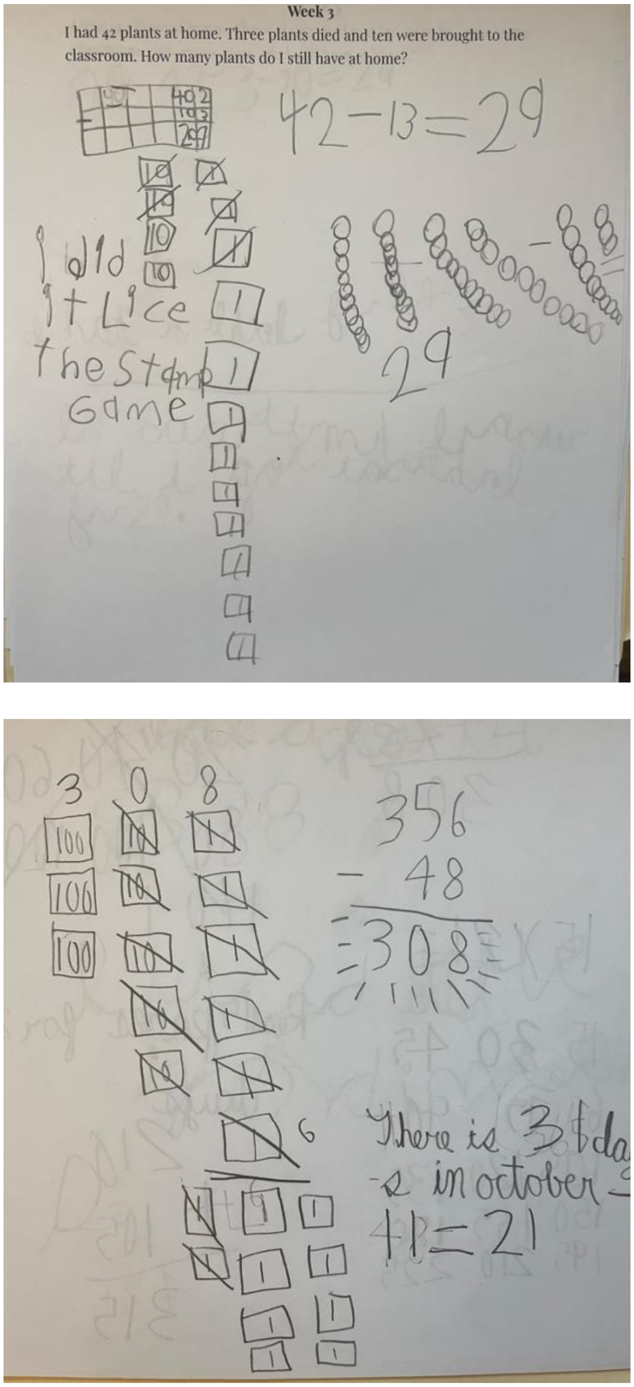
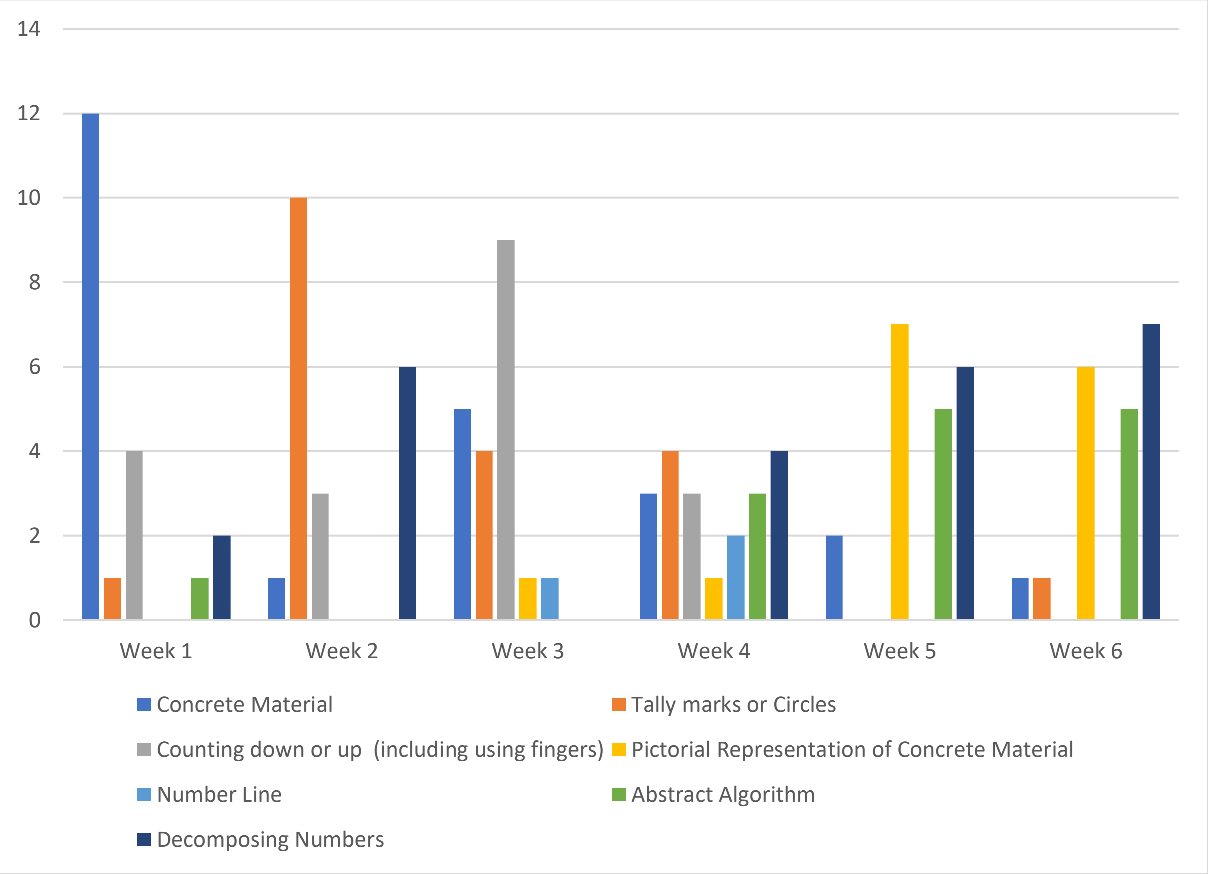


Figure 4

Strategies Recorded During CGI Lessons Each Week



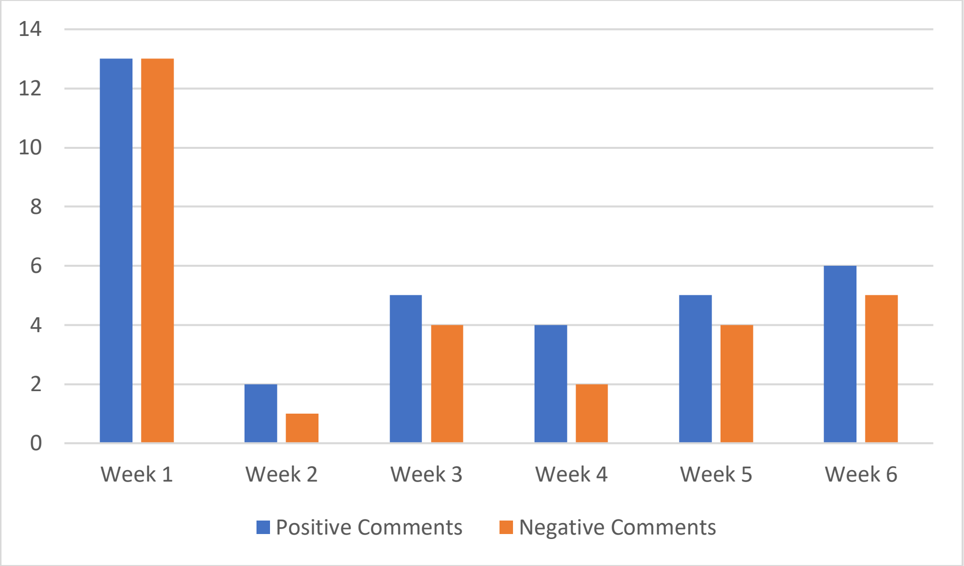
This sequence of strategies used reflects students moving from concrete materials initially to more complex pictorial representations with a few students moving to abstraction as well towards the end. The diversity of strategies used reflects certain aspects of a mathematical mindset, including students' creativity and sense-making around mathematics, as well as the desire for depth over speed. Students often drew very detailed mathematical pictures when problem solving, rather than rushing to find the answer quickly.

Behavior During Math Lessons

The data collected over the six weeks using the tally chart for observed mathematical behaviors shows stability in how students behaved during math lessons week to week. Students started the year with strong behaviors around math lessons, coming to lessons on time, staying focused, and overall remaining positive during lessons. This behavior stayed consistent throughout the six weeks.

Figure 5

Positive and Negative Comments Made During Math Lessons Each Week

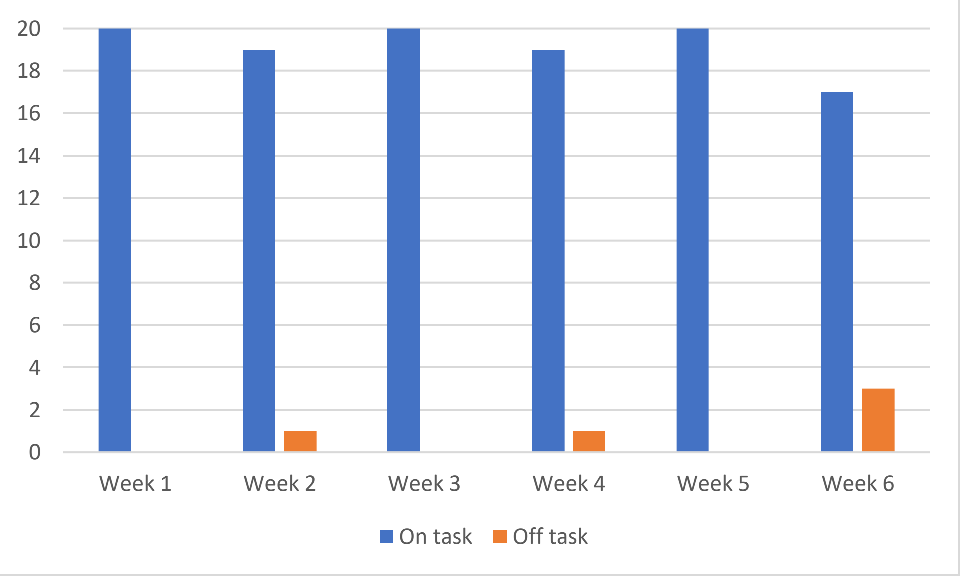


Students began the first six weeks with strong emotions, both positive and negative, around the math lessons that week. However, as the lessons continued, students' comments decreased overall, and slightly more positive comments were made each week than negative comments. Most positive comments centered around the work or lesson being fun or enjoyable. Students made comments like, "I love this lesson!" or "I want to do this every day!" Most negative comments centered around the difficulty of the math work or that particular student feeling tired that day. Comments included, "This is stressful!" when asked to write a sentence explaining their work, and "I don't want to do the work right now." In the first week, two students stated they hated math because they hated mistakes. However, this was not repeated in any of the other weeks.

In addition to making mostly positive comments about math, students stayed consistently focused and on task during math lessons. On task behavior looked like students discussing the problem together, using pencil and paper or materials to solve the problem, or pausing to think. Off task behavior looked like students playing with materials rather than using them for math or talking with each other about topics that did not involve solving the math problem. Slightly more students were off task during the last week of the intervention than in the previous weeks. This was mostly due to the subject of the last word problem. Students were very interested in talking about the topic, rather than the math needed to solve the problem.

Figure 6

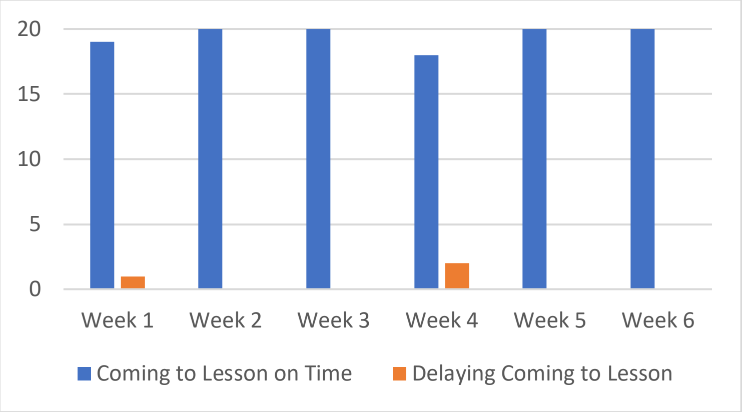
Students On Task and Off Task During CGI Lessons



Students consistently arrived to CGI lessons on time throughout the course of the six weeks. Times when students were delayed mostly involved when they were finishing a work or snack or needed to use the restroom. Students did not delay coming to the lesson simply because they did not want to join. As the weeks progressed, certain students even began to arrive at the lesson early or ask frequently when their group would be because they were excited to join.

Figure 7

Students' Timeliness at CGI Lessons



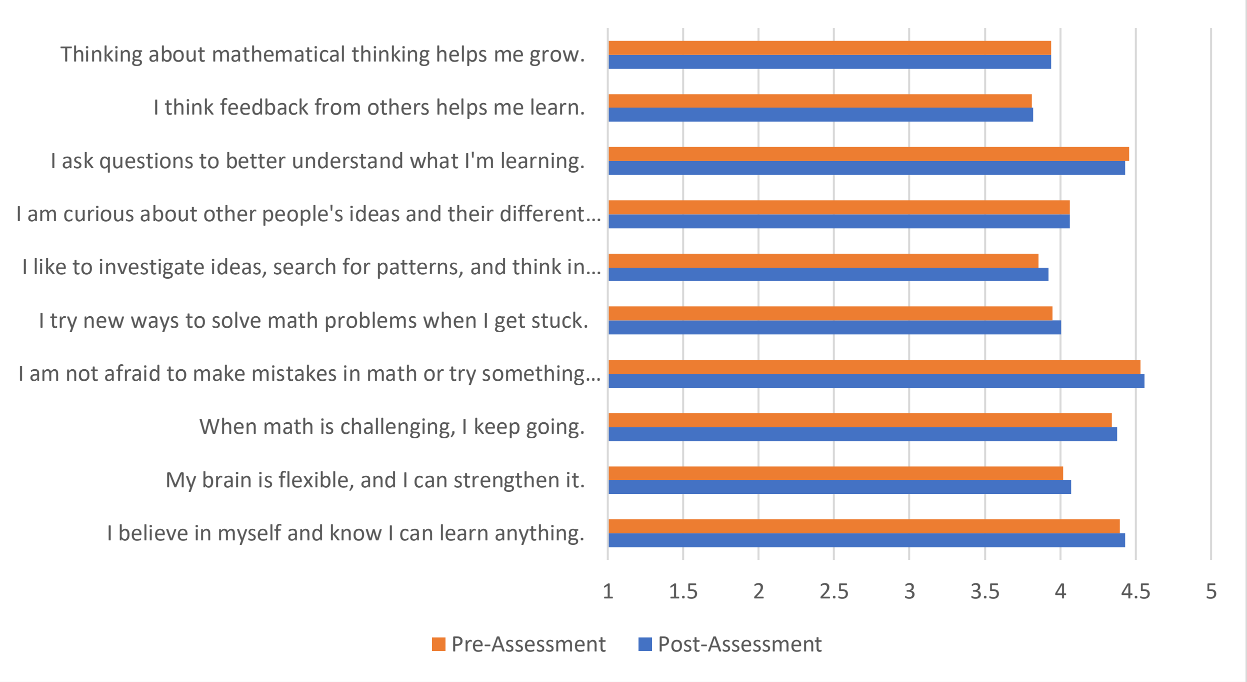
Overall, students showed a consistently positive, focused, and respectful attitude around CGI lessons. This continued readiness to join CGI lessons, as well as other math lessons, is an indicator of students' positive attitude towards math and willingness to complete challenging math work.

Attitudes Towards Math

The data tools used to assess students' attitudes towards math provided mixed results. The mathematical mindset rubric that assessed the frequency with which students held certain mathematical beliefs and practiced certain behaviors did not show a significant change pre-intervention and post-intervention. The students scored themselves in ten categories using the criteria always (5), almost always (4), sometimes (3), rarely (2), and never (1). The students scored themselves relatively high both pre-intervention and post-intervention, with the lowest average score being 3.8 and the highest average score being 4.5. Students overall thought they almost always showed most mathematical behaviors listed in the rubric.

Figure 8

Students' Average Ratings on the Mathematical Mindset Rubric

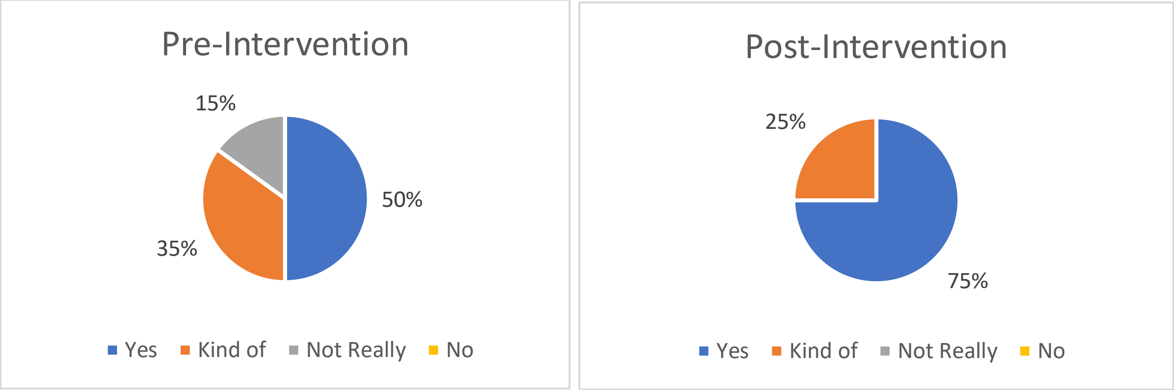


It is unclear from the data if the mathematical mindset rubric used with students was developmentally appropriate for their age-level. Upon further reflection, it is challenging for students at that age-level to assess the frequency with which they do or think something, rather than how they feel about that given topic. Considering that this rubric was based on one written for secondary students, it would make sense if the questions felt difficult for second and third grade students to answer.

What could be considered a more developmentally appropriate way to assess students' attitudes towards math would be the short survey included in the CGI assessment. Students were asked to respond to "I am good at math," with the option to choose yes, kind of, not really, and no. In the pre-intervention assessment, 50% of students said they were good at math, 35% said kind of, and 15% said not really. No student picked no. In the post-intervention assessment, there was a significant change, with 75% of students saying they are good at math, and 25% saying they are kind of good at math.

Figure 9

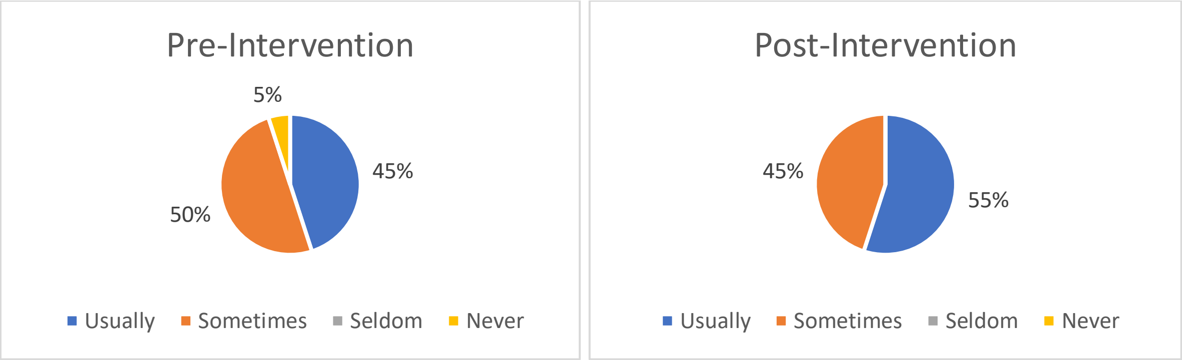
Students' Responses to "I am good at math."



This increased feeling of self-efficacy around math is reflected as well in students' responses to the prompt, "I can figure out how to solve math problems by myself." Pre-intervention, 45% of students said they could figure out math problems by themselves, whereas post-intervention, 55% of students said they could.

Figure 10

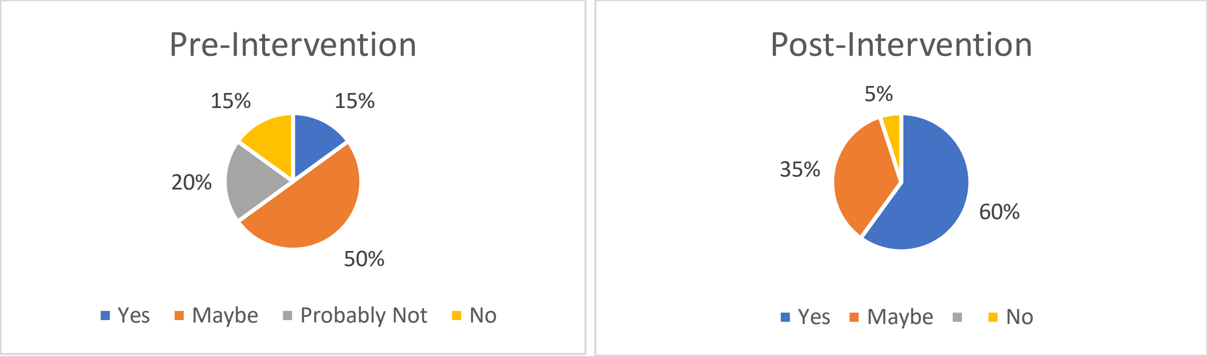
Students' Responses to "I can figure out how to solve math problems by myself."



Perhaps the most indicative of a strong mathematical mindset was the change in students' responses to the prompt, "When I grow up, I want to have a job where I use math." Pre-intervention, 15% of students said they would like to have a job that involves math. Post-intervention, 60% of students said they would like to have a job that involves math. This change does not necessarily indicate that students changed what jobs they want to have when they grow up, but rather that they are beginning to see more how math applies to all aspects of life. As one student said while filling out the post-intervention assessment, “I can’t think of a single job now that doesn’t involve math!”

Figure 11

Students’ Responses to “I want to have a job where I use math.”



Important to note as well is that the student who selected no in the post-intervention assessment was very tired while taking the assessment that day, and their response reflects their attitude in general on that day.

Conclusion

Overall, students showed a significant increase in their ability to use a wider variety of strategies to solve word problems. They also showed an increased accuracy in solving word problems as well, with an average score of 63% on the pre-intervention CGI assessment and 81% on the post-intervention CGI assessment. While students continued to show consistently strong behavior around math lessons, they also showed a significant change in their attitudes towards math, with the majority of students selecting that they are good at math, they can solve math problems on their own, and they would like to have a job that uses math when they grow up. These behaviors and attitudes are reflective of a strong mathematical mindset amongst students that can be further built upon in the future.

Discussion

Results

The aim of this study was to analyze the impact of an inquiry-based word problem-solving framework on students' mathematical mindsets. Cognitively Guided Instruction was selected as the word problem-solving framework as it is based on social constructivism, as students build knowledge about how to solve word problems together. Based on the findings from the study, it can be concluded that Cognitively Guided Instruction had a positive impact on students' mathematical mindsets. Students were able to use a wider variety of strategies over the course of the study to solve word problems. Students showed growth in being able to solve word problems accurately. They were also able to consistently come to lessons on time, stay engaged in the lesson, and keep a positive attitude around math. Students showed increased levels of self-efficacy around math, as well as an increased interest in using math when they grow up.

Limitations

This study was conducted at the beginning of the school year. Students may have scored lower on the pre-intervention CGI assessment due to the "summer slide" than they would have later in the school year. Results on the post-intervention CGI assessment may have been higher as students remembered mathematical concepts that they had learned in previous school years. It is unclear if students would have the same pace of growth continued throughout the school year, or if they would stagnate and become comfortable using certain strategies rather than others. Moreover, students may have scored higher on the post-intervention assessment because the questions on the post-intervention assessment were the same as the questions on the pre-intervention assessment. More data is needed from more standardized forms of assessment to see if students can transfer what they have learned in CGI lessons to different presentations of word problems.

Another limitation of this study was my own knowledge of the students as a researcher. Being new to the classroom and school, I was unfamiliar with what students had or had not been taught the previous school year, as well as their overall mathematical understanding. Because of this, I based the word problems I wrote for them only on the pre-intervention CGI assessment rather than my own instructional or observational notes about what they had learned. I also did not differentiate the problems based on students' levels of mathematical understanding, but rather gave all students the same word problem week to week. It is unclear if certain students could have grown more in their strategy usage if they had been given word problems tailored more to their current level of mathematical understanding.

Finally, being in a private school setting, the population of students taught was not reflective of the overall United States population. Most students started the intervention with strong pro-social behaviors around school: coming to lessons on time, staying focused, and keeping a positive attitude toward learning. It is difficult to draw conclusions from this study about whether the same results would occur with a different population of students, particularly a population with more types of neurodiversity or more varied attitudes and behaviors towards school. To know the extent of how Cognitively Guided Instruction impacts students' mathematical mindsets, this study would need to be conducted in other school settings. This is further proven by the mixed results that currently exist in the overall research around Cognitively Guided Instruction (Schoen et al., 2021).

Recommendations

Based on the pre-intervention assessment data, there is a gap in the Montessori method between students' learning concepts using concrete materials, and then applying those concepts to math word problems. This gap impacts students' mathematical mindsets. They have difficulty seeing how math applies to the real world outside of the Montessori materials and feel unsure of how to solve word problems on their own using pencil and paper. For certain students, this impacts their attitude towards math, causing them to dislike math or dislike challenging or novel math work. There is a strong need for Montessori educators to consider implementing word problem-solving frameworks that align with the constructivist approach of the Montessori method. There is a strong need as well for Montessori educators to consider the mathematical mindsets of their students, and to not assume that the Montessori materials alone are enough to lead students to having positive attitudes about math.

Based on this research, I plan to continue to utilize CGI as a word problem-solving framework in my classroom, teaching a CGI lesson once every other week. I am also planning to incorporate more discussions and social extensions of learning into other Montessori math lessons. For example, I have started having students estimate what the answer will be for a math problem before solving it together using a material, both to build a stronger number sense and to build their comfort around not knowing the exact answer right away. I could also have students talk or write about how they solved a math problem using a material after a lesson or come up with a word problem together that aligns with the math problem they solved.

Finally, I plan to continue to discuss aspects of having a mathematical mindset with my students. More work is needed with certain students around being okay with making mistakes in math, and not knowing the exact right answer to a problem. More work is also needed with certain students in not asking for help right away with unfamiliar math problems but trying strategies and reasoning on their own first. In a culture where it is prevalent to think that some students are good at math and some are bad at math, it is necessary to continue the work of helping students see math through a growth mindset and to help them believe that all people can be capable in math. For students to succeed in math, they first need to believe that they can.

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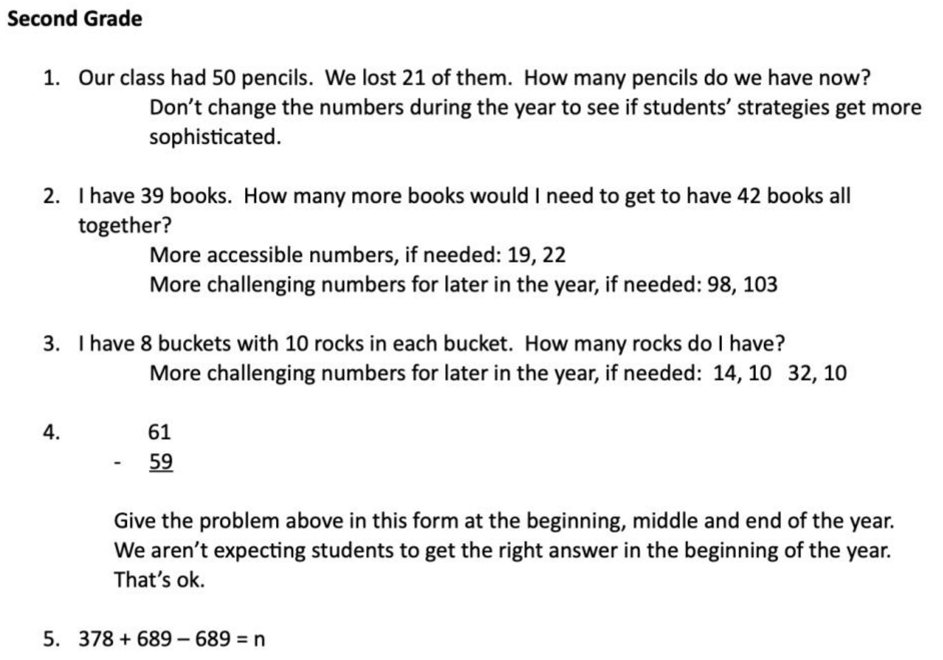
Appendix A

Questions from CGI Assessment



Second Grade

1. Our class had 50 pencils. We lost 21 of them. How many pencils do we have now?
   Don't change the numbers during the year to see if students' strategies get more sophisticated.

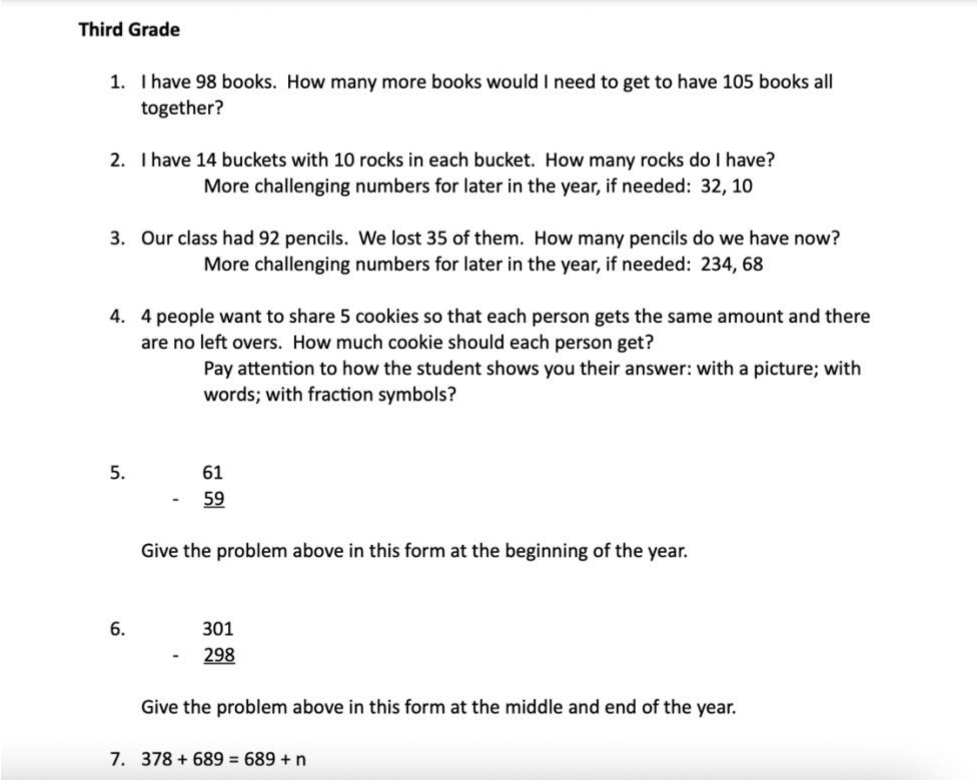
2. I have 39 books. How many more books would I need to get to have 42 books all together?
   More accessible numbers, if needed: 19, 22
   More challenging numbers for later in the year, if needed: 98, 103

3. I have 8 buckets with 10 rocks in each bucket. How many rocks do I have?
   More challenging numbers for later in the year, if needed: 14, 10 32, 10

4.
$$\begin{array}{r} 61 \\ -\ \underline{59} \end{array}$$

   Give the problem above in this form at the beginning, middle and end of the year. We aren't expecting students to get the right answer in the beginning of the year. That's ok.

5. 378 + 689 – 689 = n



Third Grade

1. I have 98 books. How many more books would I need to get to have 105 books all together?

2. I have 14 buckets with 10 rocks in each bucket. How many rocks do I have?
   More challenging numbers for later in the year, if needed: 32, 10

3. Our class had 92 pencils. We lost 35 of them. How many pencils do we have now?
   More challenging numbers for later in the year, if needed: 234, 68

4. 4 people want to share 5 cookies so that each person gets the same amount and there are no left overs. How much cookie should each person get?
   Pay attention to how the student shows you their answer: with a picture; with words; with fraction symbols?

5.
$$\begin{array}{r} 61 \\ -\ \underline{59} \end{array}$$

   Give the problem above in this form at the beginning of the year.

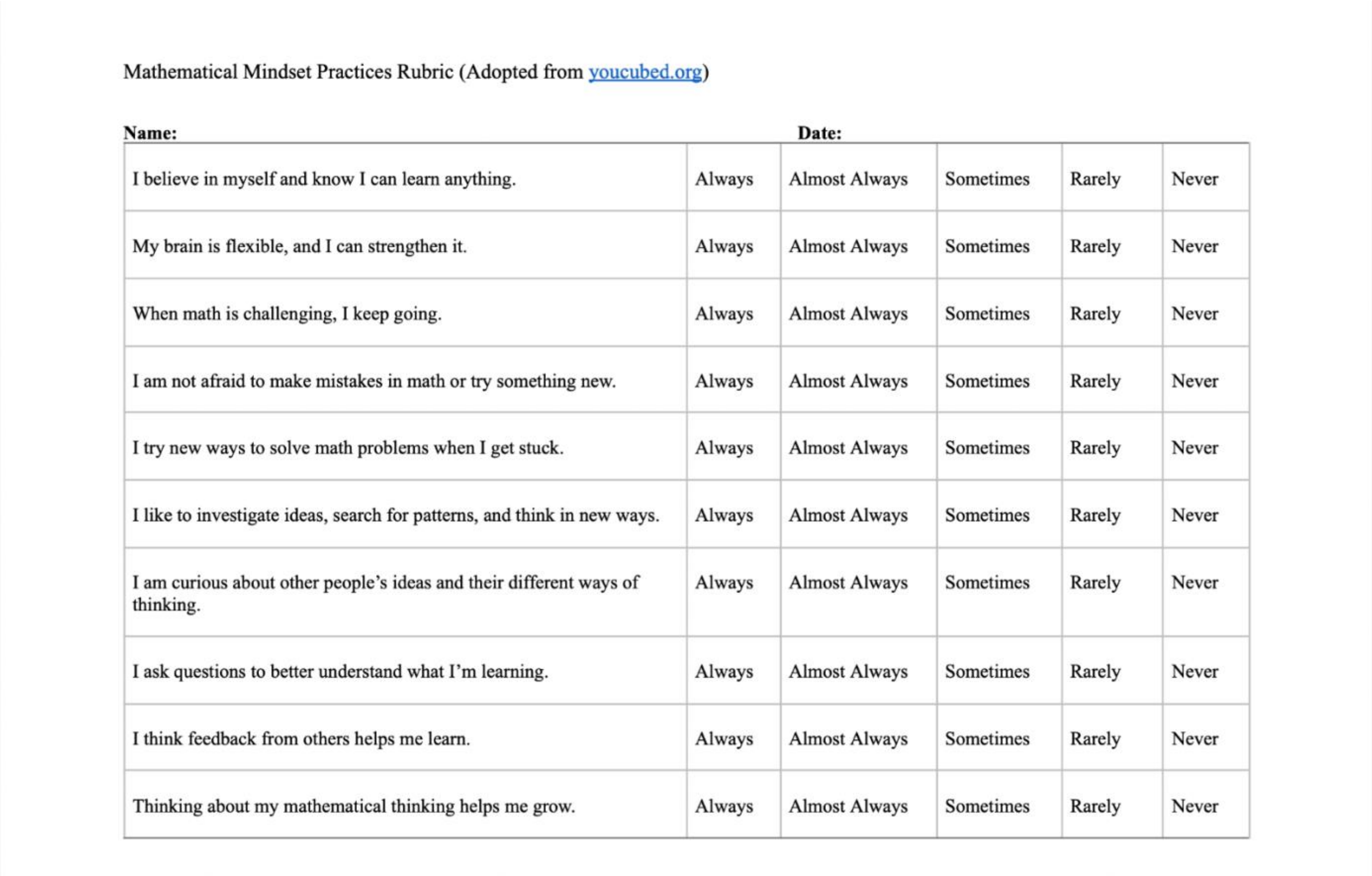
6.
$$\begin{array}{r} 301 \\ -\ \underline{298} \end{array}$$

   Give the problem above in this form at the middle and end of the year.

7. 378 + 689 = 689 + n

Appendix B

Mindset Rubric



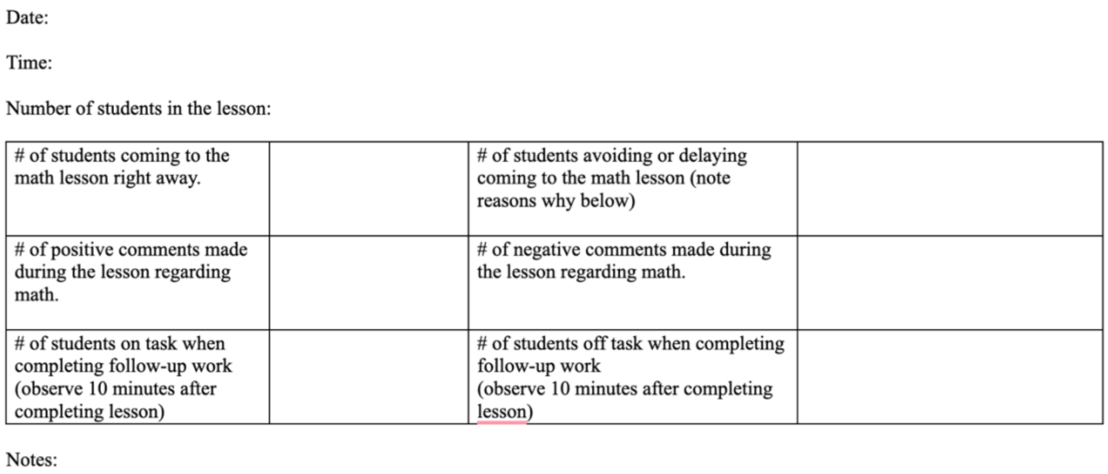
Mathematical Mindset Practices Rubric (Adopted from youcubed.org)

Name: **Date:**

I believe in myself and know I can learn anything.	Always	Almost Always	Sometimes	Rarely	Never
My brain is flexible, and I can strengthen it.	Always	Almost Always	Sometimes	Rarely	Never
When math is challenging, I keep going.	Always	Almost Always	Sometimes	Rarely	Never
I am not afraid to make mistakes in math or try something new.	Always	Almost Always	Sometimes	Rarely	Never
I try new ways to solve math problems when I get stuck.	Always	Almost Always	Sometimes	Rarely	Never
I like to investigate ideas, search for patterns, and think in new ways.	Always	Almost Always	Sometimes	Rarely	Never
I am curious about other people's ideas and their different ways of thinking.	Always	Almost Always	Sometimes	Rarely	Never
I ask questions to better understand what I'm learning.	Always	Almost Always	Sometimes	Rarely	Never
I think feedback from others helps me learn.	Always	Almost Always	Sometimes	Rarely	Never
Thinking about my mathematical thinking helps me grow.	Always	Almost Always	Sometimes	Rarely	Never

Appendix C

Tally Chart and Observational Notes Template



Date:

Time:

Number of students in the lesson:

# of students coming to the math lesson right away.		# of students avoiding or delaying coming to the math lesson (note reasons why below)	
# of positive comments made during the lesson regarding math.		# of negative comments made during the lesson regarding math.	
# of students on task when completing follow-up work (observe 10 minutes after completing lesson)		# of students off task when completing follow-up work (observe 10 minutes after completing lesson)	

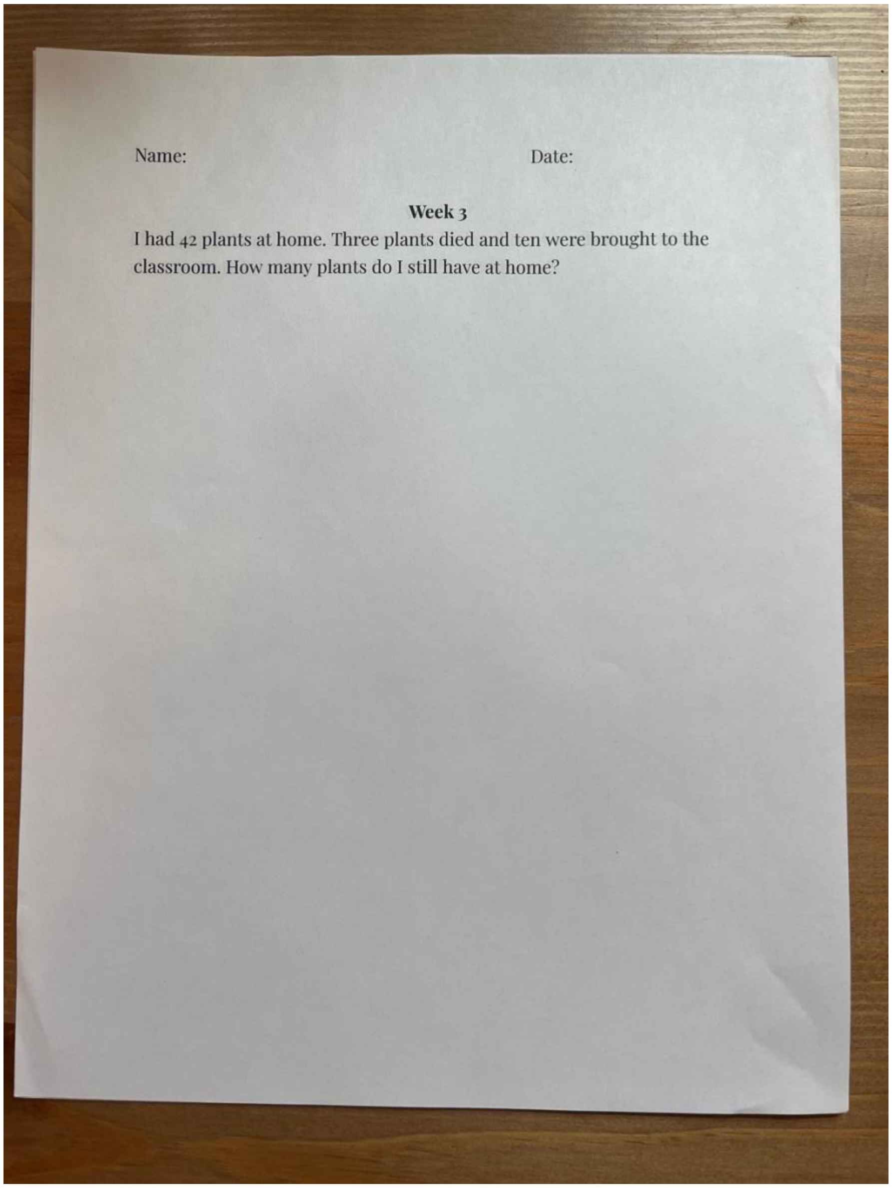
Notes:

Appendix D

Mathematical Mindset Posters from *https://www.youcubed.org/resource/posters/*

Appendix E

Example Word Problem-Solving Paper



Name: Date:

Week 3

I had 42 plants at home. Three plants died and ten were brought to the classroom. How many plants do I still have at home?

Appendix F

Montessori Bead Bars

Montessori Colored Counting Bars