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Abstract

Modal parameter estimates from ambient vibration testing are turning into the preferred technique when one is interested in systems under actual loadings and operational conditions. Moreover, with this approach, expensive devices to excite the structure are not needed, since it can be adequately excited by human activities, wind, gust, etc. In this paper, the comparison between two different vibration testing techniques is presented. The first approach takes advantage of the frequency domain decomposition, *FDD*, of the response cross power spectral densities to estimate both the natural frequencies and the *"unscaled"* mode shapes, whereas the second one, starting from the Hilbert transform of auto power spectral densities and taking account of the cross power spectral densities, allows one to get the frequency response functions. Both the techniques have been used to estimate the modal parameters of two simple structures (a beam and a plate), excited by an acoustical random signal.

1 Introduction

Some years ago, instead of deriving the mode shapes from modal surveys (carried out by using the input-output data) *i.e.*, from the frequency response functions (*FRF*), a technique which permits to get the operating deflection shapes has been introduced, see for instance [1, 2, 3]. Recently, the practice to derive the modal parameters - natural frequencies, damping ratios and mode shapes - only from the output data, under suitable conditions for the input signals, has become common, and so these methods have been applied, not only to operating machines, but also in the aerospace field, for instance on airplanes [4] excited either by turbulence or by wind.

Obviously some assumptions, as mentioned before, must be made on the input signals, in particular they are considered random, uncorrelated, with zero mean and unknown standard deviation. The obtained results were completely acceptable, and the modal scale factors (modal masses) could be estimated by a technique, similar to the one introduced in [5], presented in [6, 7, 8, 9].

The paper presents a comparison between two different approaches in order to estimate the modal parameters from ambient vibration tests. The first method is based on the frequency domain decomposition (*FDD*), [10], which, through the singular value decomposition (*SVD*) of the power spectral density matrix (*PSD*) of the output signals, determines both the natural frequencies and the mode shapes. Whereas the damping factors are estimated from the response in the time domain, [11, 12]. The second approach starts from the output auto spectral densities, and by applying the Hilbert transform, allows one to get the frequency response functions in the driving points, [13]. Then, by using the cross spectral densities and the above-mentioned *FRFs*, the whole *FRFs* matrix could be estimated. Obviously, since the random input is unknown, under suitable conditions, all the *FRFs* are biased by an unknown constant, as besides it happens for the previous method. In this last case the modal parameters could be obtained by a least square fitting of the *FRFs*, also by a commercial estimating code. Nevertheless, both techniques allows one only to estimate the unscaled

mode shapes.

If several tests have to be carried out on a single structure, the unknown biasing constant varies from test to test, and then it is necessary to have a reference measurement point, equal for each test, so as to normalize the modal displacements of the whole structure.

Experimental tests were carried out on an aluminum cantilever beam and on a free aluminum plate both excited by an acoustic random signal, irradiated from a loudspeaker driven by the analyzer signal generator.

2 Theoretical basis

Hereafter the two approaches, both the one based on the frequency domain decomposition and the one founded on the Hilbert transform, will be briefly outlined.

2.1 Frequency Domain Decomposition

Modal parameter estimates by using the Frequency Domain Decomposition (*FDD*) technique, core of the commercial code *ARTeMIS* from SVS, [10], are based on the evaluation of the power spectral density matrix of the responses, under the hypothesis that a broad banded input loading is acting on a lightly damped structure. The relation between the *PSD* of both the input and the output signals, *i.e.*, $[G(\omega)]_{xx}$ and $[G(\omega)]_{ff}$ respectively, and the frequency response function matrix, $[H(\omega)]$, could be expressed as (superscripts " $*$ " and " T " denote complex conjugate and transpose respectively):

$$[G(\omega)]_{xx} = [H(\omega)]^* [G(\omega)]_{ff} [H(\omega)]^T \qquad (1)$$

On the other hand, if the first M modes are considered in the frequency band of interest, it is possible to demonstrate that Eq. (1) is equivalent to, [7]:

$$[G(\omega)]_{xx} = \sum_{k=1}^{M} \frac{d_k \{\psi\}_k \{\psi\}_k^H}{j\omega - \lambda_k} + \frac{d_k^* \{\psi\}_k^* \{\psi\}_k^{*H}}{j\omega - \lambda_k^*} \qquad (2)$$

where $\{\psi\}_k$ and λ_k represent the k-th mode and pole, superscript H the hermitian operation, whereas d_k is an unknown scalar quantity that depends on the unknown excitation level if the input loading is uncorrelated, stationary, zero mean, and broad banded. Moreover, recalling that the Fourier transform of an even signal is real even, the *SVD* decomposition of the *PSD* matrix of the responses leads to:

$$[G(\omega)]_{xx} = [U(\omega)] [\Sigma(\omega)] [U(\omega)]^T \qquad (3)$$

where $[U(\omega)]$ and $[\Sigma(\omega)]$ are the matrices of the singular vectors and the singular values respectively. When the above decomposition is evaluated for $\omega = \omega_{n_k}$, then only one singular value, $\sigma_k^{(p)}$, is not zero, so the natural frequency is obtained from this peak (*peak picking method*). If the mode shapes are well separated (corresponding to a structure that vibrates as *SDOF* system near the k-th natural frequency). Therefore, under these conditions, the corresponding singular vector $\{u\}_k^{(p)}$ is an estimate of the k-th mode.

Damping ratios could be finally achieved in the time domain, by means of the logarithmic decrement method, considering the auto correlation functions as results of the Inverse Fast Fourier Transform of the *PSD* matrix, [8, 14].

2.2 Hilbert Transform Identification

Modal parameters could be also estimated by considering output only data starting from the *Hilbert* transform of the auto power spectral densities of the measured responses, as presented in [13]. With this approach, the auto power spectral densities of the input load, $G_{f_i f_i}(\omega)$, together with the corresponding spectrum of the response signal, $G_{x_i x_i}(\omega)$ - both evaluated at the experimental degree of freedom i - could be used to estimate a biased driving point *FRF*. In fact, as shown in [15], one can write:

$$G_{x_i x_i}(\omega) = G_{f_i f_i}(\omega) \; |H_{ii}(\omega)|^2 \tag{4}$$

where $|H_{ii}(\omega)|$ is the magnitude of the driving point *FRF*, that could be, in turn, expressed as:

$$|H_{ii}(\omega)| = \frac{\sqrt{G_{x_i x_i}(\omega)}}{\sqrt{G_{f_i f_i}(\omega)}} \tag{5}$$

Since the power spectral density of the loading, $G_{f_i f_i}(\omega)$, could be considered constant in the frequency band of interest when the excitation is assumed as white noise, performing the *Hilbert* transform of Eq. (4), and recalling that the Hilbert transform of a constant is zero, [16], the following relationship is achieved:

$$\mathcal{H}\left[\ln G_{x_i x_i}(\omega)\right] = 2\,\mathcal{H}\left[\ln |H_{ii}(\omega)|\right] \tag{6}$$

Moreover, the responses are from an actual vibrating structure, therefore they must be causal, so the phase, $\Phi_{ii}(\omega)$, can be estimated by the following relationship, [17]:

$$\Phi_{ii}(\omega) = -\,\mathcal{H}\left[\ln |H_{ii}(\omega)|\right] \tag{7}$$

Substituting Eq. (7) into Eq. (6), one gets:

$$\Phi_{ii}(\omega) = -\frac{1}{2}\mathcal{H}\left[\ln G_{x_i x_i}(\omega)\right] \tag{8}$$

Finally, the biased frequency response functions, evaluated at the driving points, can be obtained only from the knowledge of the output data:

$$H_{ii}(\omega) = |H_{ii}(\omega)| \; e^{j\,\Phi_{ii}(\omega)} = \frac{\sqrt{G_{x_i x_i}(\omega)}}{\sqrt{G_{f_i f_i}(\omega)}} \; e^{-\frac{1}{2}j\,\mathcal{H}\{\ln G_{x_i x_i}(\omega)\}} \tag{9}$$

Indeed, it has been demonstrated that the biasing constant, $\sqrt{G_{f_i f_i}}$, does not change the values of the natural frequencies and damping ratios, [13], so the modal parameters could be evaluated by a least square fitting of the $FRFs$, also with commercial codes.

The off-diagonal frequency response functions, could be also estimated once the driving point $FRFs$, $H_{ii}(\omega)$, are disposable from Eq. (9). In fact, the FRF relative to the experimental points i and j, could be expressed as follows, [13]:

$$H_{ij}(\omega) = \frac{G_{x_i x_j}(\omega)}{H_{ii}^*(\omega)} \tag{10}$$

Therefore, all the FRF matrix could be estimated evaluating only the auto and cross power densities of the output signals.
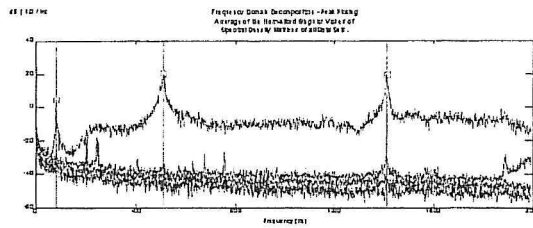
Figure 1: Peak Picking of the beam with FDD.

3   Experimental investigation

Two different structures have been considered in order to test the previous approaches. Both the structures were excited by random acoustic waves generated by a loudspeaker. The resulting signal is therefore random in time, but deterministic in space, being the position of the loudspeaker fixed. The used experimental instrumentation is presented in the following Table 1.

Spectrum Analyzer	LMS SCADAS III
Accelerometers	PCB (352A10)
AC Power Supply	B&K

Table 1: Experimental setup.

3.1   Cantilever Beam

The considered cantilever beam, whose dimensions were $0.16 \times 0.015 \times 0.0295$ m, had four accelerometers glued on it, equally spaced along the beam span and numbered in increasing order from grid point 2, because the grid point 1 corresponds to the clamped end. Since the loudspeaker was positioned apart from the beam in the neighborhood of the free end, it could excite well the last two grid points, and so this type of excitation - from a practical point of view - could be regarded as an almost *MIMO* system.

The natural frequencies estimated by the two approaches, along with the ones theoretically evaluated, are reported in Tab. 2, where subscripts *FDD* and *HTM* stand for Frequency Domain Decomposition and Hilbert Transform Method respectively. The values presented have been obtained by the peak picking method when the *FDD* approach is adopted, as shown in Fig. 1, where the singular values of the *PSD* matrix are depicted as frequency function. On the contrary, the natural frequencies evaluated by the *HTM* method have been estimated by a least square fitting of the biased Frequency Response Functions.

Mode #	f_{Theory} Hz	f_{FDD} Hz	f_{HTM} Hz
1	80.82	80.90	80.64
2	498.3	509	508.5
3	1385	1413	1411

Table 2: Comparison between the natural frequencies for the cantilever beam

Moreover, Tab. 3 presents the comparison between the damping factors obtained by applying the previous methodologies.

As one can see, the errors on the natural frequency estimates, derived from the two methods and the ones achieved from the analytical predictions, are immaterial. On the contrary, although the damping factor estimates are not so coincident, as the natural frequencies, they are completely acceptable.

Mode #	ζ_{FDD} %	ζ_{HTM} %
1	0.94	0.82
2	0.90	0.80
3	0.20	0.26

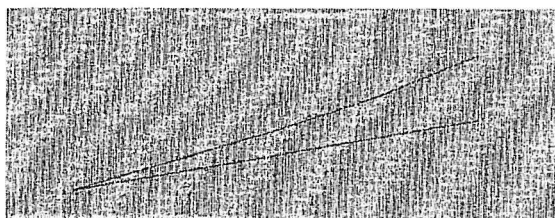Table 3: Comparison between the damping ratios for the cantilever beam

Figure 2: Estimate of Mode #1 of the beam with FDD.

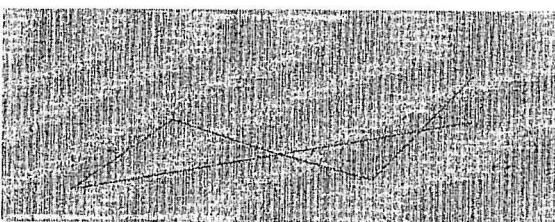Figure 3: Estimate of Mode #2 of the beam with FDD.

Figure 4: Estimate of Mode #3 of the beam with FDD.

The first three mode shapes are shown in the Figs. from 2 to 7. The first three ones have been derived by applying the *FDD*, whereas the last from the *HTM*. As one could expect, because of the good agreement on the natural frequencies and damping ratios, the mode shapes are practically coincident.

3.2 Free-Free Plate

The second structure considered was an aluminum square plate, $0.25 \times 0.25 \times 0.0025$ m, excited by a loud-speaker positioned in front of the middle of the plate, at a distance of 10 cm, while vertical responses have been measured at 9 experimental *DOFs* uniformly distributed on the structure, using 3 *rowing* accelerometers.

The natural frequencies obtained by applying the two methods are compared with the ones given by the theoretical analysis in Tab. 4. As happened for the cantilever beam, an almost good correlation has been obtained. The peak picking estimate has been shown in Fig. 8.
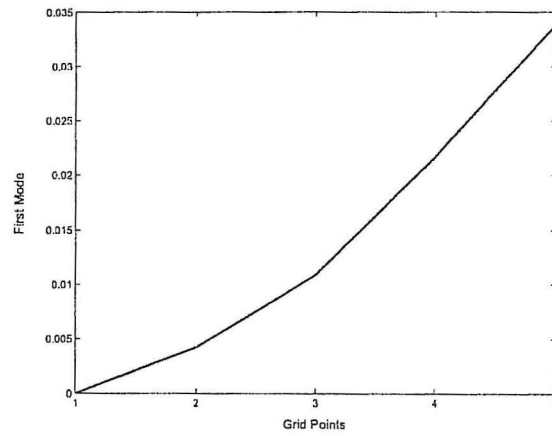
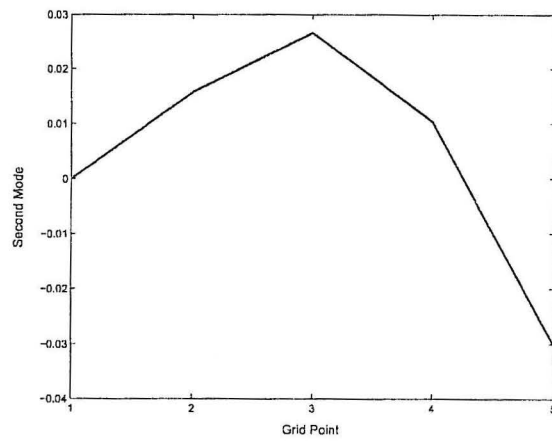Figure 5: Estimate of Mode #1 of the beam with Hilbert Transform.

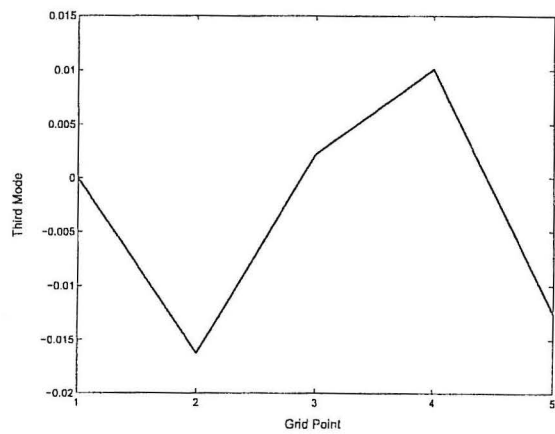Figure 6: Estimate of Mode #2 of the beam with Hilbert Transform.

Figure 7: Estimate of Mode #3 of the beam with Hilbert Transform.

Mode #	f_{Theory} Hz	f_{FDD} Hz	f_{HTM} Hz
1	75.59	69.40	70.90
2	109.9	101.7	101.6
3	136.5	148.7	148.2
4	194.0	187.5	190.8

Table 4: Comparison between the natural frequencies for the free-free plate
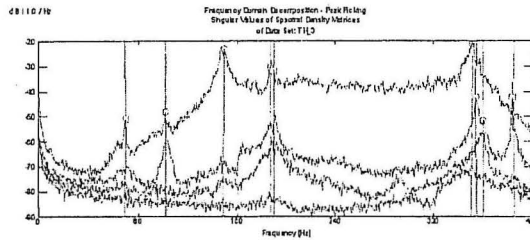

Figure 8: Peak Picking of the plate with FDD.

Figure 9: Estimate of Mode #1 of the plate with FDD

The relative damping ratios have been reported in Tab. 5.

Mode #	ζ_n^{FDD}	ζ_n^{HTM}
1	0.45	0.55
2	0.23	0.48
3	0.81	0.96
4	0.68	0.59

Table 5: Comparison between the damping ratios for the free-free plate

Although with much more scattering, with respect to the values gained for the cantilever beam, from a practical point of view, the estimates are acceptable. Finally, the first three mode shapes of the plate are shown from Figs. 9 to 14. The first three figures are relative to the *FDD* method, on the contrary the last three ones have been obtained by the *HTM* approach.

It is worth noting that a good correlation has been achieved for the presented modes, except for the second one. Probably the correlation between the two approaches, especially for the damping ratio and amplitude estimates, could be enhanced if a longer time sequence were recorded. That allows one to perform a higher number of averages, which reduces the standard deviation of the noise, without losing in frequency definition.

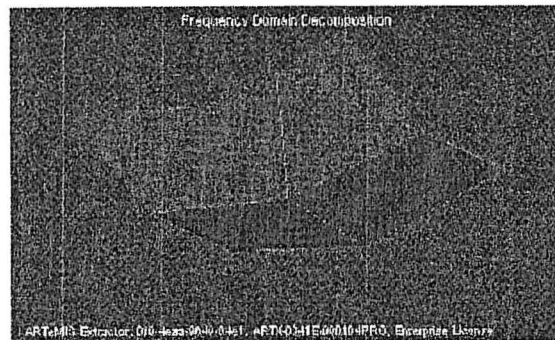Figure 10: Estimate of Mode #2 of the plate with FDD

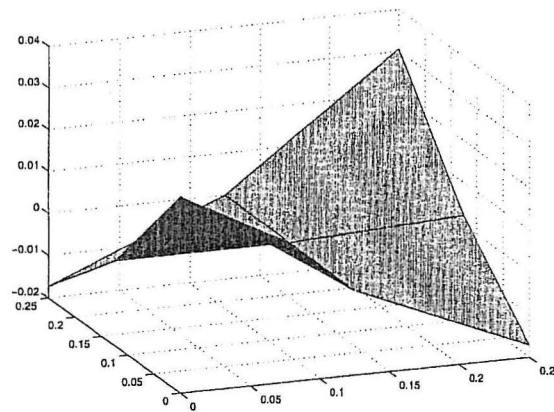Figure 11: Estimate of Mode #3 of the plate with FDD

Figure 12: Estimate of Mode #1 of the plate with Hilbert Transform.

4 Concluding remarks

In this paper, two different approaches in order to get the modal parameters from structures excited by ambient vibrations have been presented. Although the input is unknown, if it is possible to consider it as a white random noise, the auto and cross spectra allow one to get the natural frequencies and the mode shapes. For the damping ratios, the *FDD* method estimates them in the time domain (for instance by the logarithm decrement), whereas the *HTM* approach, evaluating biased Frequency Response Functions, permits to get
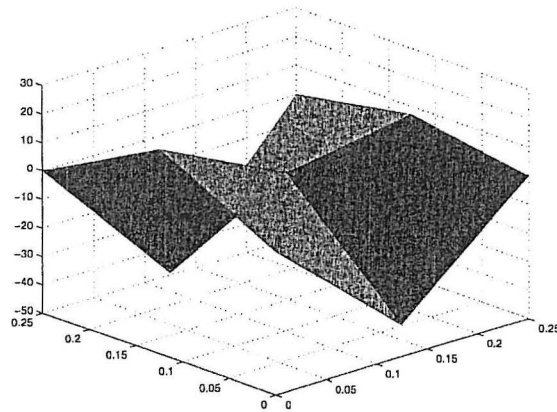
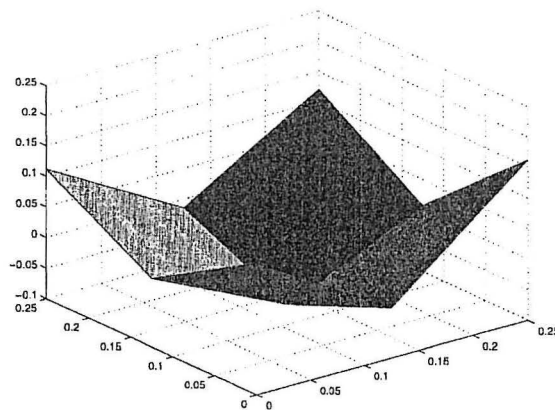Figure 13: Estimate of Mode #2 of the plate with Hilbert Transform.

Figure 14: Estimate of Mode #3 of the plate with Hilbert Transform.

all the modal parameters in the frequency domain. The comparison between the two approaches has shown a generally good agreement among the results for both a cantilever beam and a free-free plate, excited by acoustic random signals. Therefore, although the *FDD* technique has been already used and commercialized in the *ARTeMIS* code, even the *HTM* could represent a valid tool for these purposes.

Acknowledgements

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