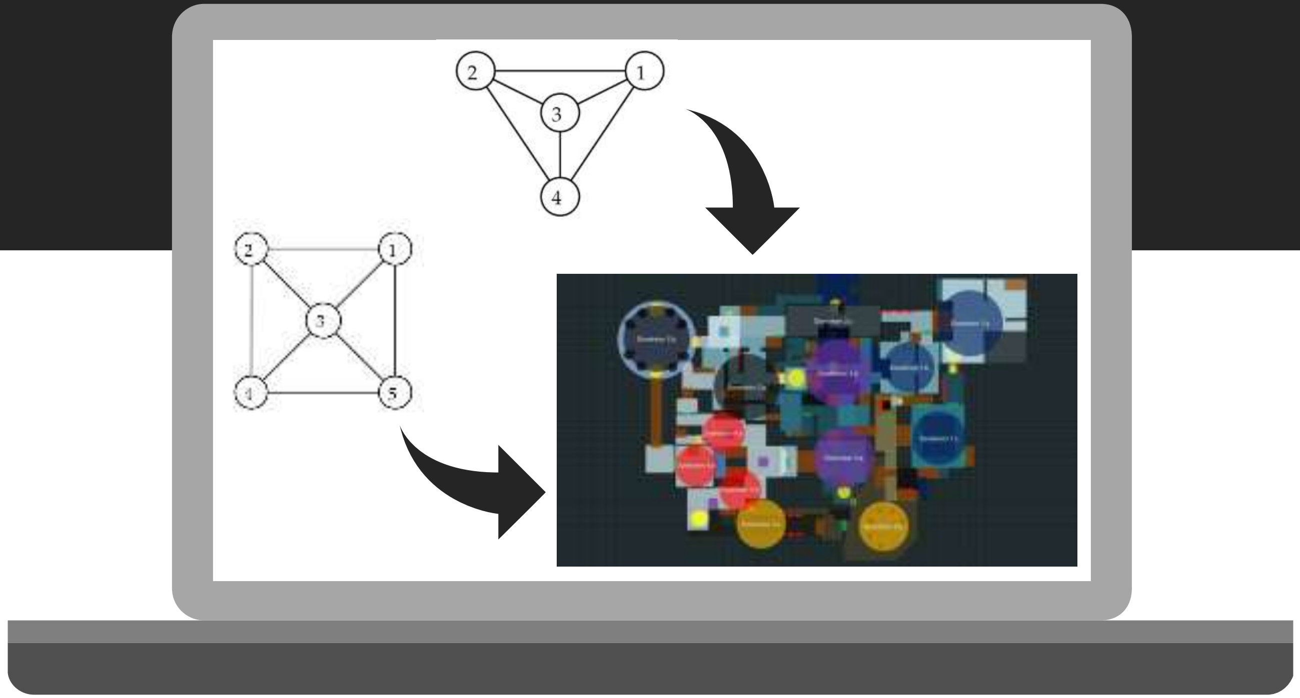
Using Graph Theory to Create a 3D Miniscaped Non-linear Level

Donghua "Elish" Li



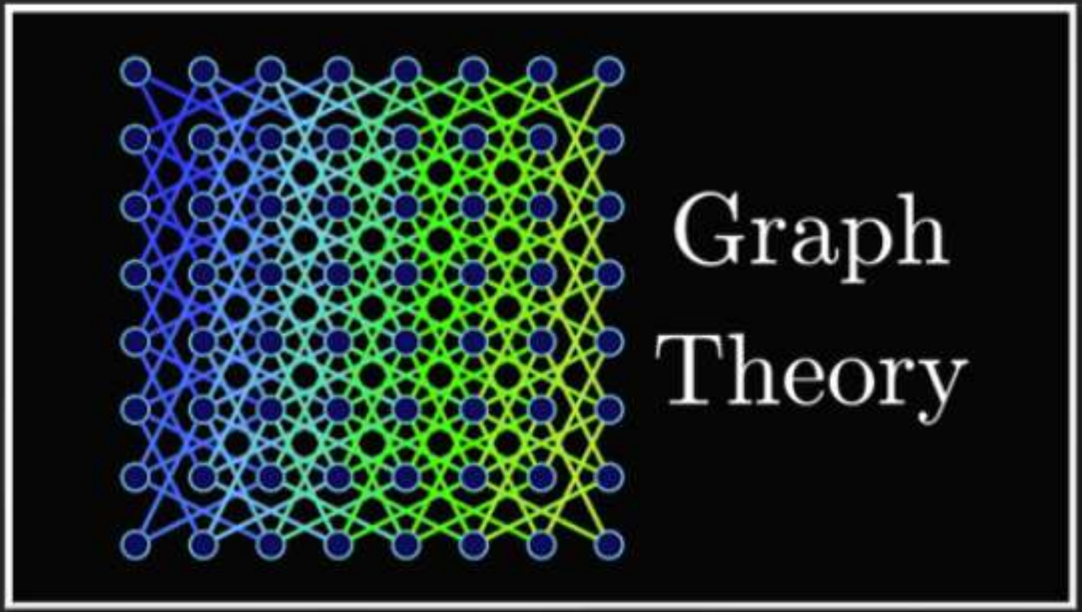
Agenda

1. Thesis Goal
2. Theories & Research
3. Methodology
4. Artifact Description & Map
6. Survey Process & Results
7. Conclusions

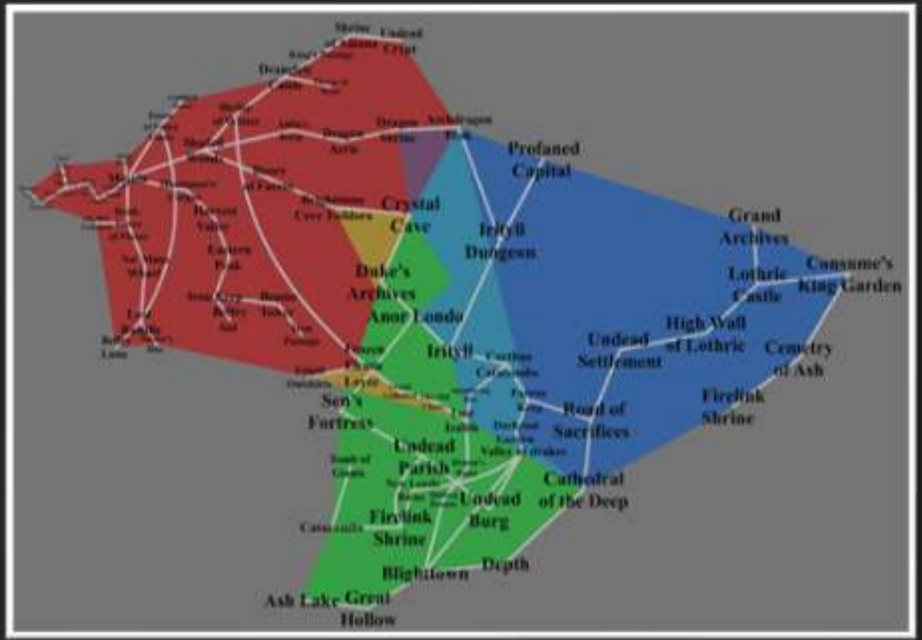
1. Thesis Goal & Hypothesis

Thesis Goal

- Build a methodology to create **3D**, **nonlinear**, **miniscaped** level layouts
- **Methodology Basis**
  1. Graph Theory
  2. Dominion Theory



[1]



[2]

Hypothesis

By using **Graph Theory** and **Dominion Theory**, level designers can create a **3D, non-linear**, and **miniscaped** level where players can navigate without losing their sense of direction or losing track of their objectives

2. Theories & Research

Non-linear Level

- A level that is designed to encourage **unpredictable player movement** and **exploration of the space** [13]


[14]

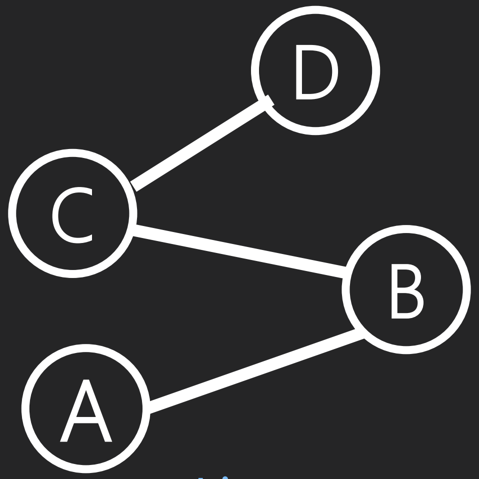
Elden Ring (2022)


[15]

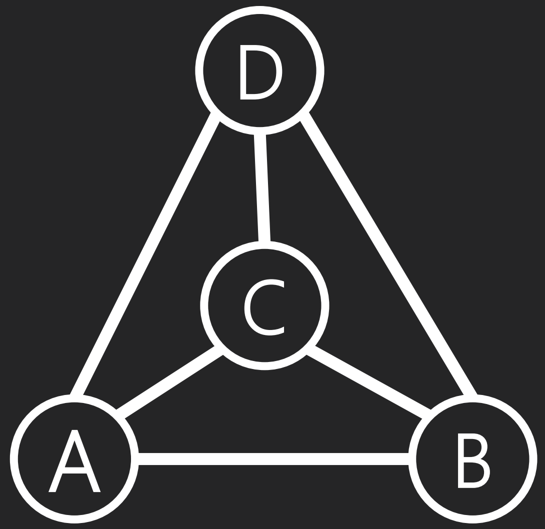
Granblue Fantasy Relink (2024)

Why Non-linearity?

- ***Cons*** **of Non-linear level**
  - Difficult to navigate
  - Frequently travel back and forth
  - Hard to design and implement
- ***Pros*** **of Non-linearity**
  - Sense of player freedom
  - Player feels in control of self
  - More variety in exploration



Linear



Non-linear

Miniscape

- Japanese – “**Hakoniwa**”
- A **dish garden** with plant materials that do not require water (literal)

[16]

Miniscapes in Video Games

- *"In level design, miniscapes are elaborately decorated areas with distinctive themes that are totally different from each other"*
  - Shigeru Miyamoto, Nintendo Tree House Live

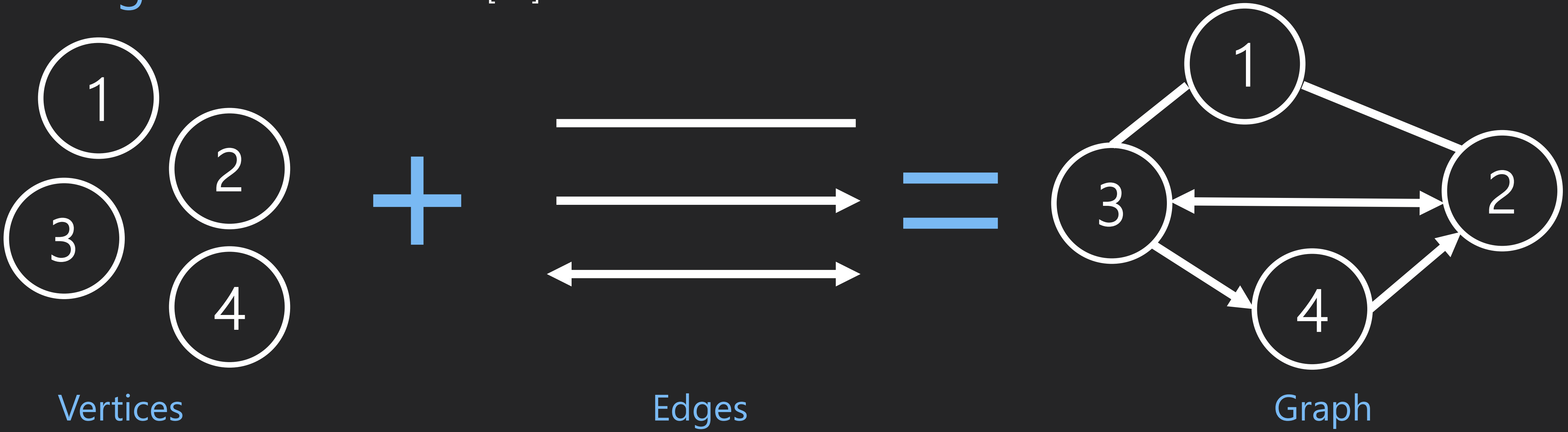
- Each area is distinct visually
- Miniscapes allow for exploration and contain fun

Super Mario 3D World (2013)

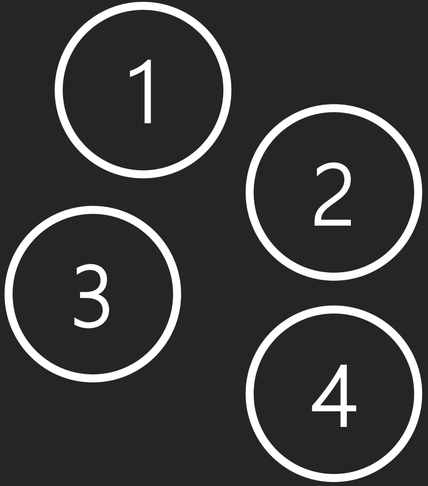
[18]

Graph Theory

- Graph Theory focuses on studying graphs connected by **vertices** and **edges**
- Vertices represent objects or entities
- Edges connect vertices to represent the **interrelationship** among those vertices [22]

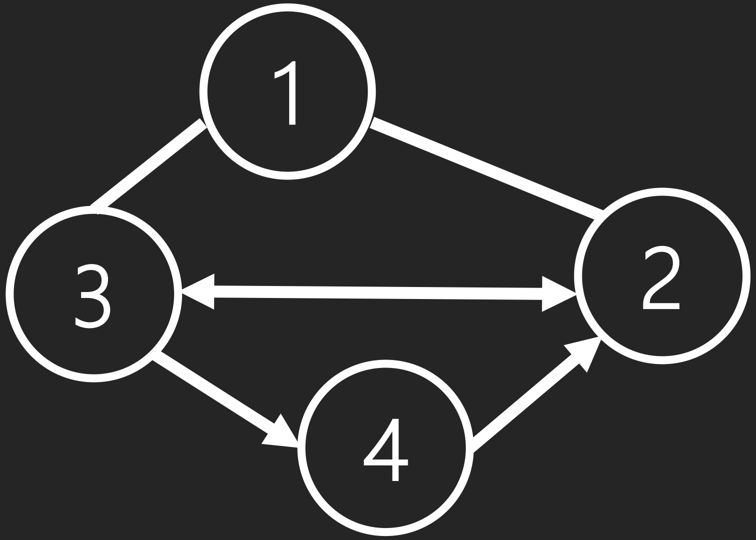


Graph's Elements

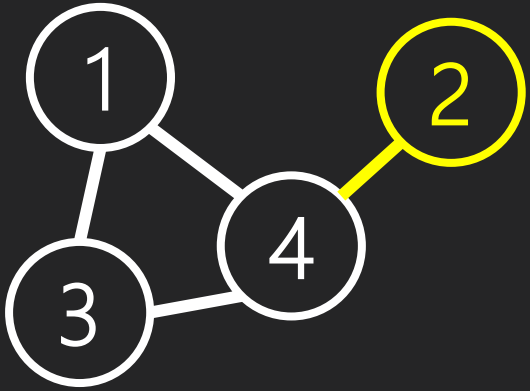


Vertices

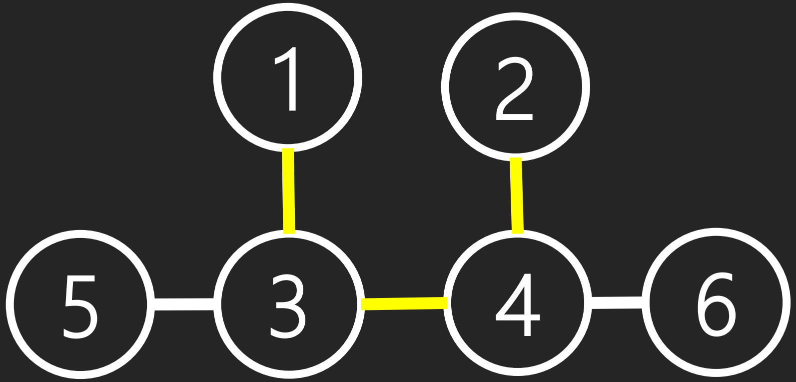
Edges



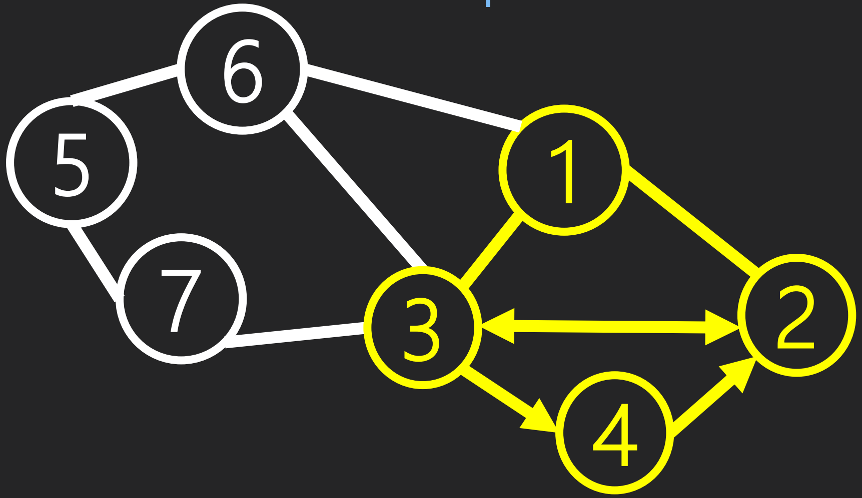
Graph



Leaf

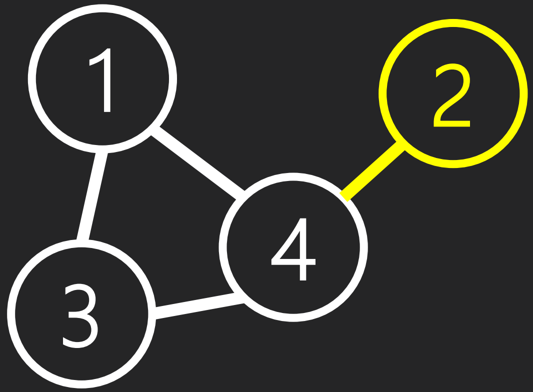


Chain



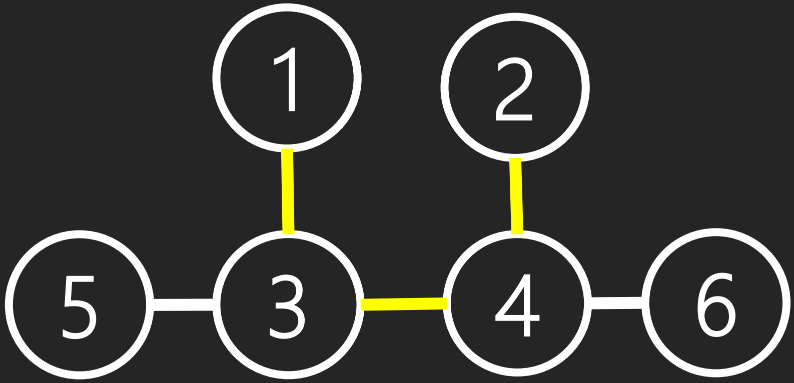
Subgraph

Graph's Elements - Leaf

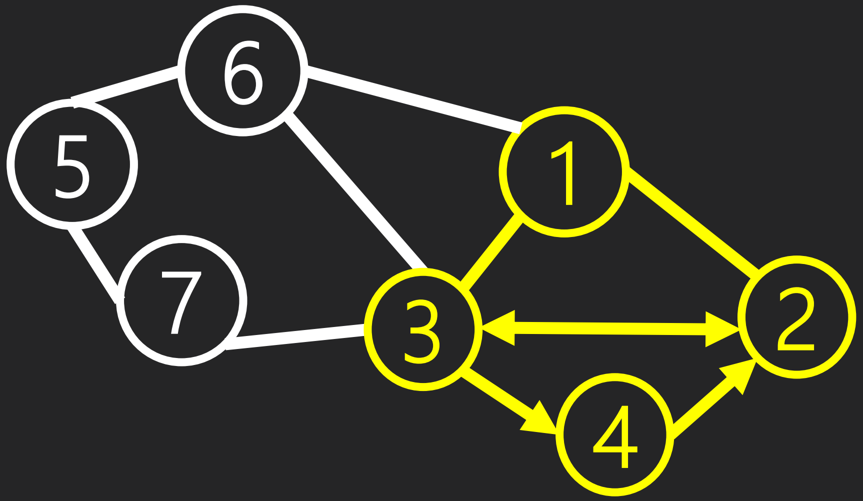


Leaf

- A leaf is a vertex having **only one edge** connecting to its **single** neighbor

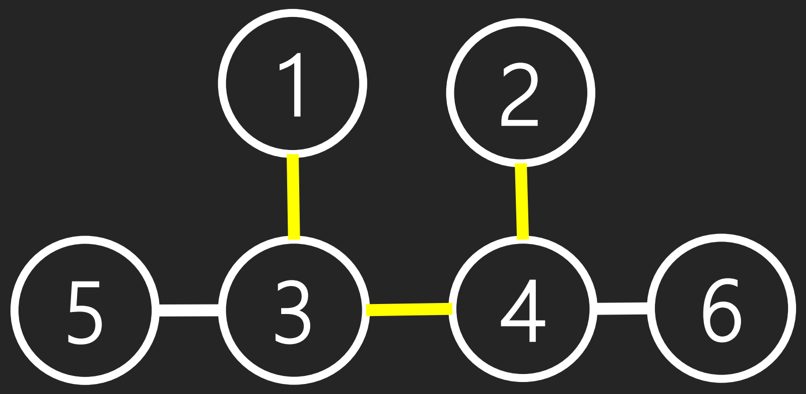


Chain



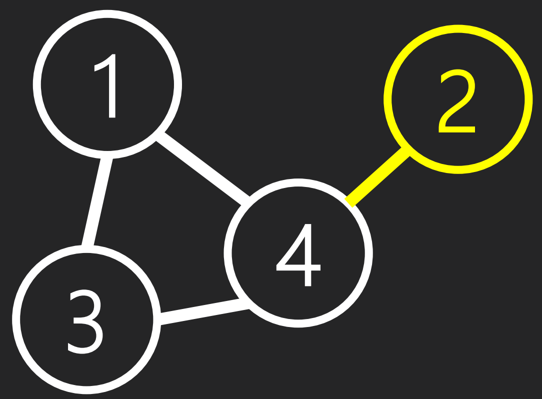
Subgraph

Graph's Elements - Chain

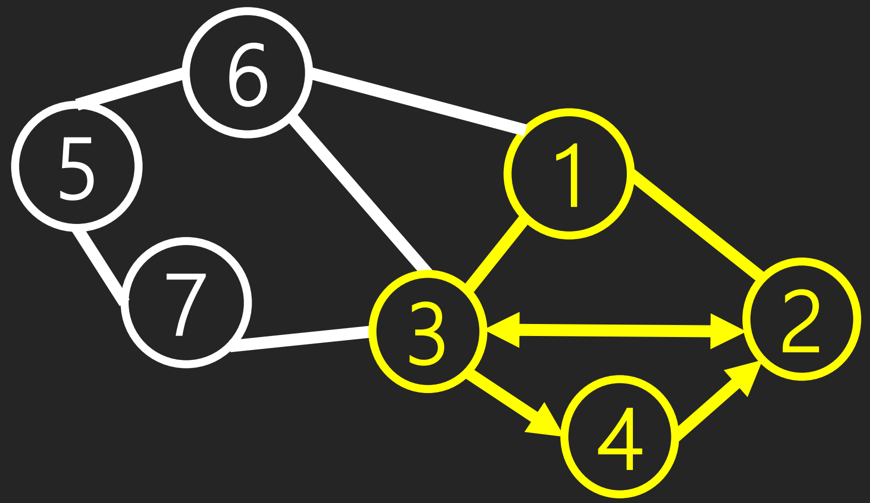


Chain

- A chain is a path formed by **a series of** vertices and edges



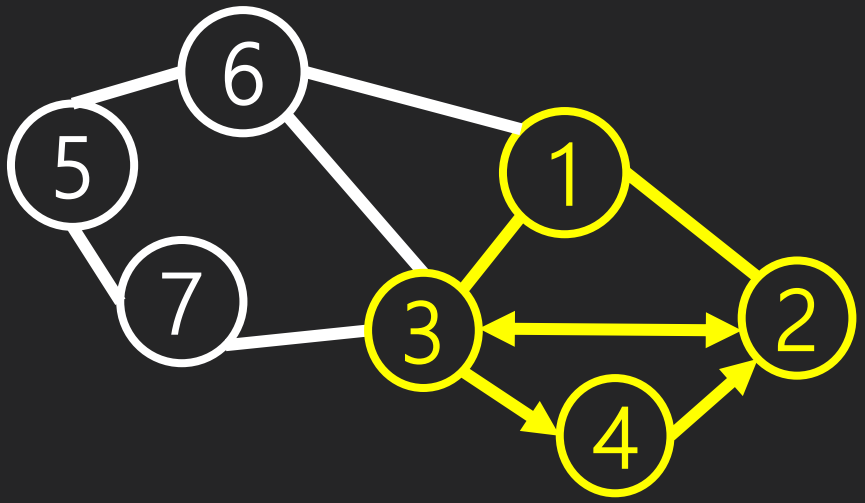
Leaf



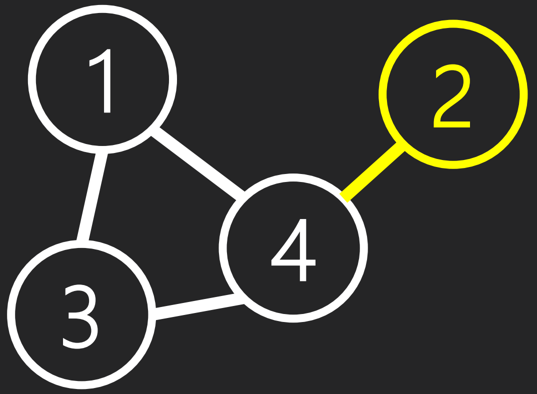
Subgraph

Graph's Elements - Subgraph

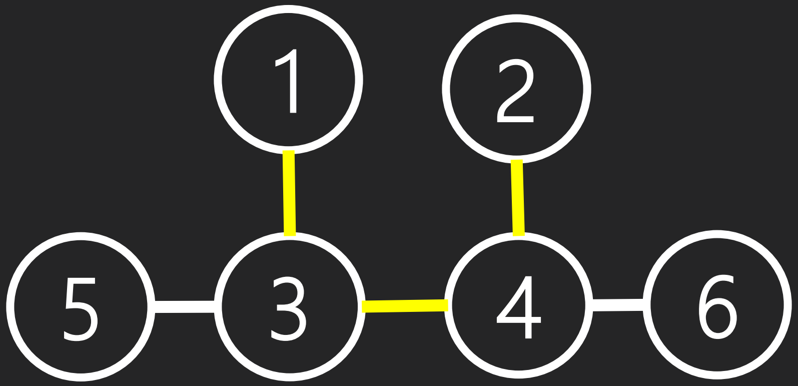
- In simple words a graph is said to be a subgraph if it is **a part of** another graph



Subgraph



Leaf

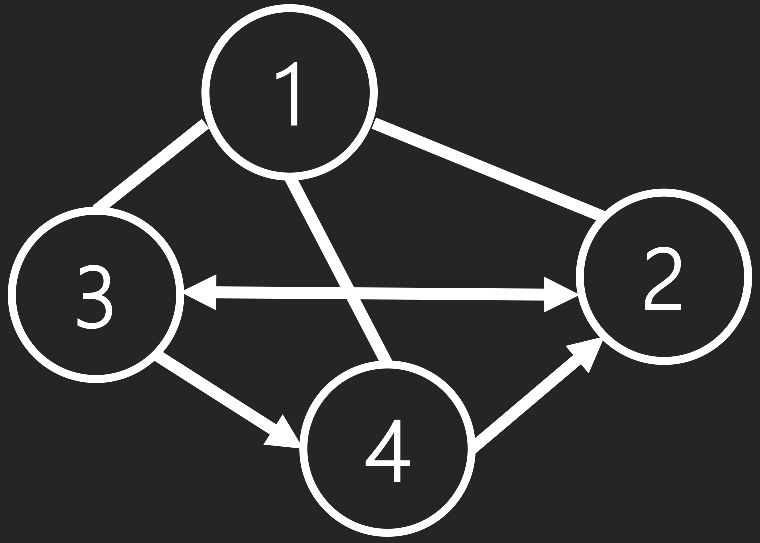


Chain

Connected Graph & Connectivity

Connected Graph:

- A graph that is connected in the sense of a topological space, i.e., there is a **path** from **any point to any other** point in the graph [20]



Connected Graph

Connected Graph

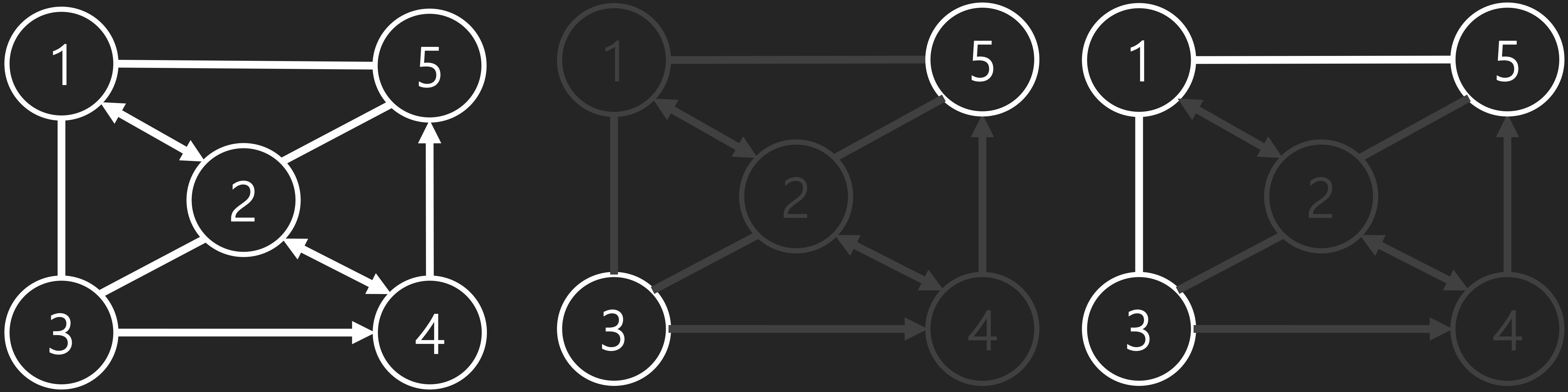
Connected Graph

Not a Connected Graph

Connected & Connectivity

Connectivity:

If there exists one way to remove **k** vertices in a given graph **G,** so that the resulting graph is no longer a connected graph while removing **k-1** vertices will not, **k** is the connectivity of this graph [20]



A Connected Graph **G**

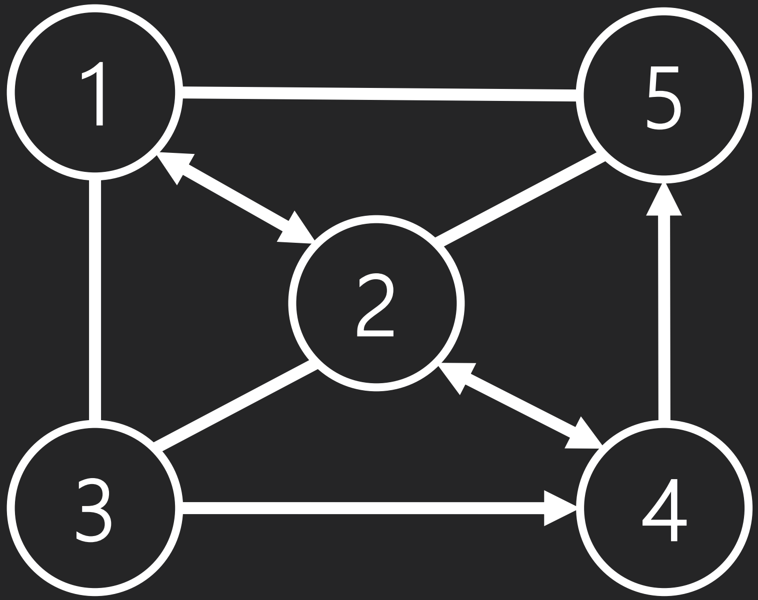
No longer connected if 3 vertices are removed

Removing 2 vertices does not disconnect the graph

Connected & Connectivity

<u>Connectivity</u>:

If there exists one way to remove **k** vertices in a given graph **G,** so that the resulting graph is no longer a connected graph while removing **k-1** vertices will not, **k** is the connectivity of this graph [20]



A Connected Graph **G**

$$\begin{cases} k = 3, \textit{Graph } \boldsymbol{G} \textit{ is disconnected} \\ k - 1 = 2, \textit{Graph } \boldsymbol{G} \textit{ is still connected} \end{cases}$$

Hence, the connectivity on this graph **G** is **3**

Dominion Theory

- **Nodes (Dominions)** have an area of effect
  - Affect player's behavior
  - The gameplay is heavy and concentrated in these areas

- **Ranged-based** instead of Time-based
  - Opt-in next area whenever you want
  - There is time and space between high intensity moments
    - Transition areas among dominions

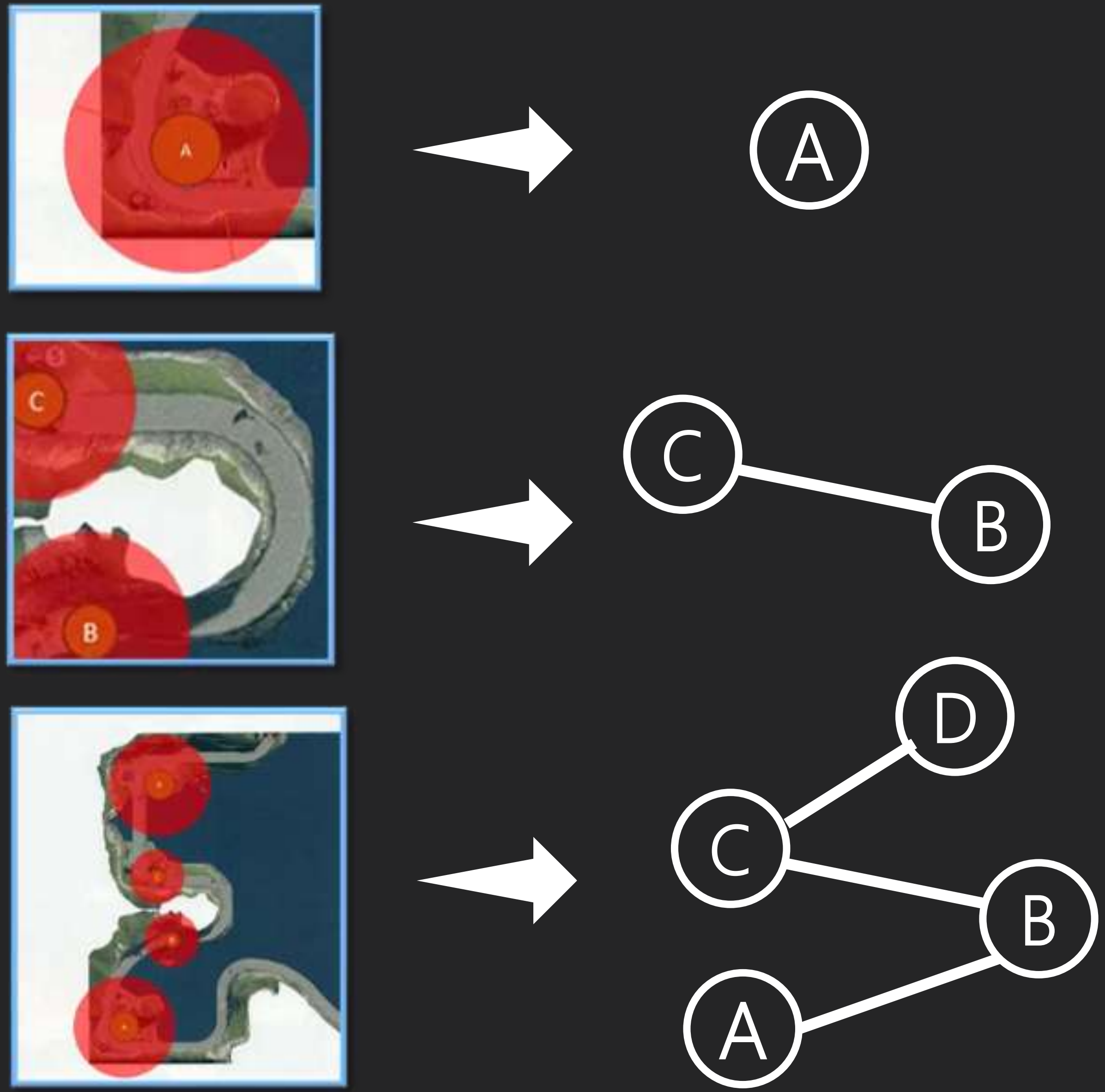
[3]

Half-Life 2 (2004)

Why Dominion Theory and Graph Theory?

- **What is similar?**
  - **Vertices = Gameplay Areas**
  - **Edges = Transitions**
  - **Graphs = Logical Relationships**

  - Use the theories as design tools for creating a level layout



3. Methodology

Graph Theory – Calculating Stability Factor

- From the article –

  **“How to design a ‘*Dark-Souls*-like’ level:
  On topological structures of ‘*Dark-Souls*-like’ game levels”**

- Stability Factor [4]
  - A parameter measuring the logical interrelationship of a level
  - Determines if a level is “**healthy**” enough to be **easily memorized**
  - Ideally, the factor is **greater than 0.94**

γ

Graph Theory – Calculating Stability Factor

1. Simplify the level map to a simplest form
2. Calculate the ***Cheeger*** number according to the following definition:
   1. For a graph ***G*** with m vertices, if there exist ***n*** ways to remove ***k*** vertices such that all nodes in the resulting subgraph are not connected, then the $\boldsymbol{k^{th}}$ order ***Cheeger*** number $\boldsymbol{\lambda_k}$ of graph ***G*** is defined as: $\boldsymbol{\lambda_k = 1 - \frac{n}{\binom{m}{k}}}$, where $\boldsymbol{\binom{m}{k}}$ represents the binomial coefficient that $\binom{m}{k} = \frac{m!}{k!(m-k)!}$
3. Calculate γ by the following formula:
   1. $\gamma_\infty = \lim_{m \to 3} \frac{\sum_{i=1}^{3} \frac{1}{i!} \lambda_i}{\sum_{i=1}^{3} \frac{1}{i!}} = \frac{1}{e} \lim_{m \to 3} \frac{1}{i!} \lambda_i$

Stability Factor Calculation — Example

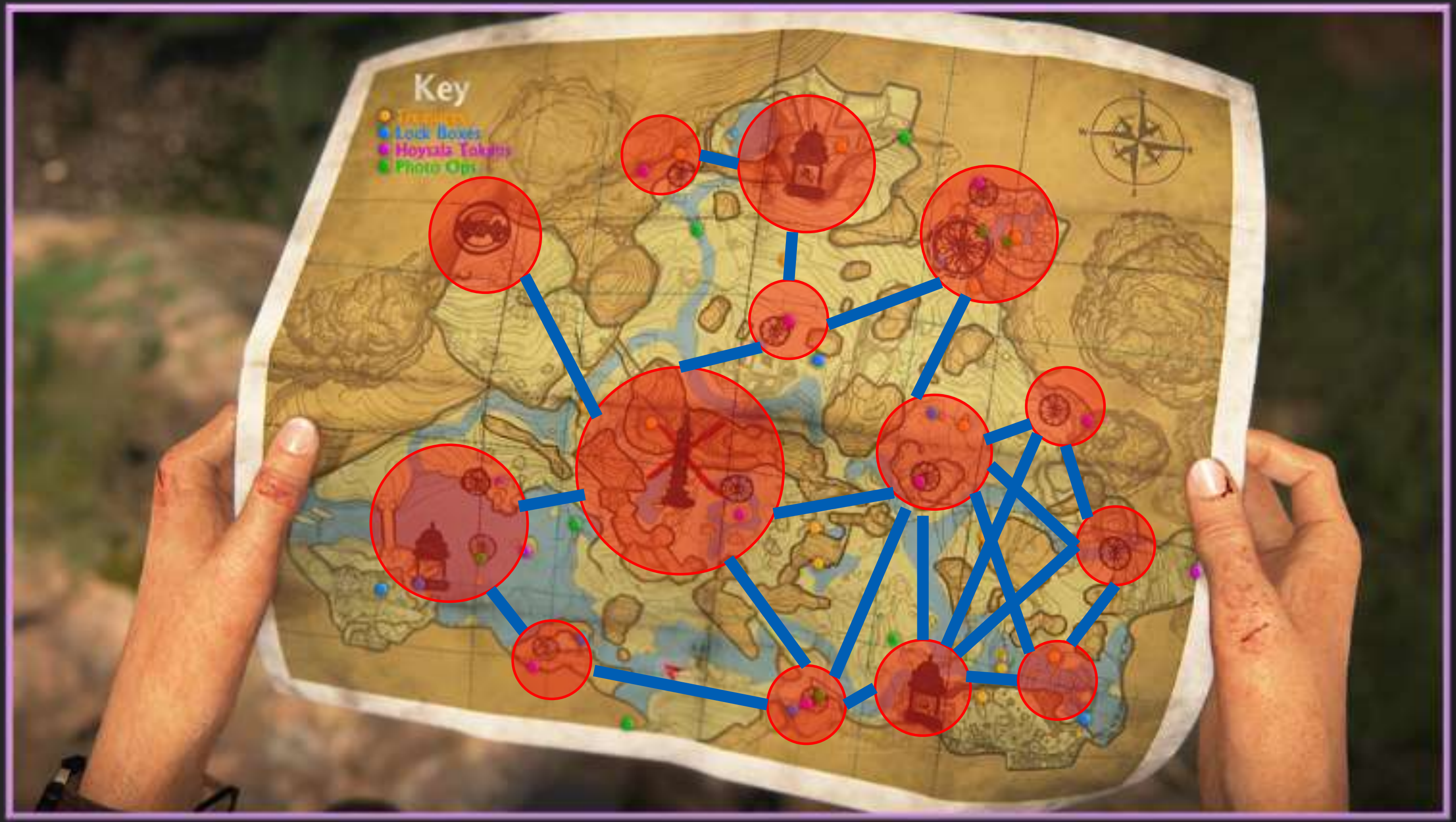
A non-linear level in *Uncharted 4* Chapter 4

[10]

Stability Factor Calculation — Example

1. Simplify the level map to a simplest form

   1.a. Identify **Dominions (Vertices)** and **Transitions (Edges)**

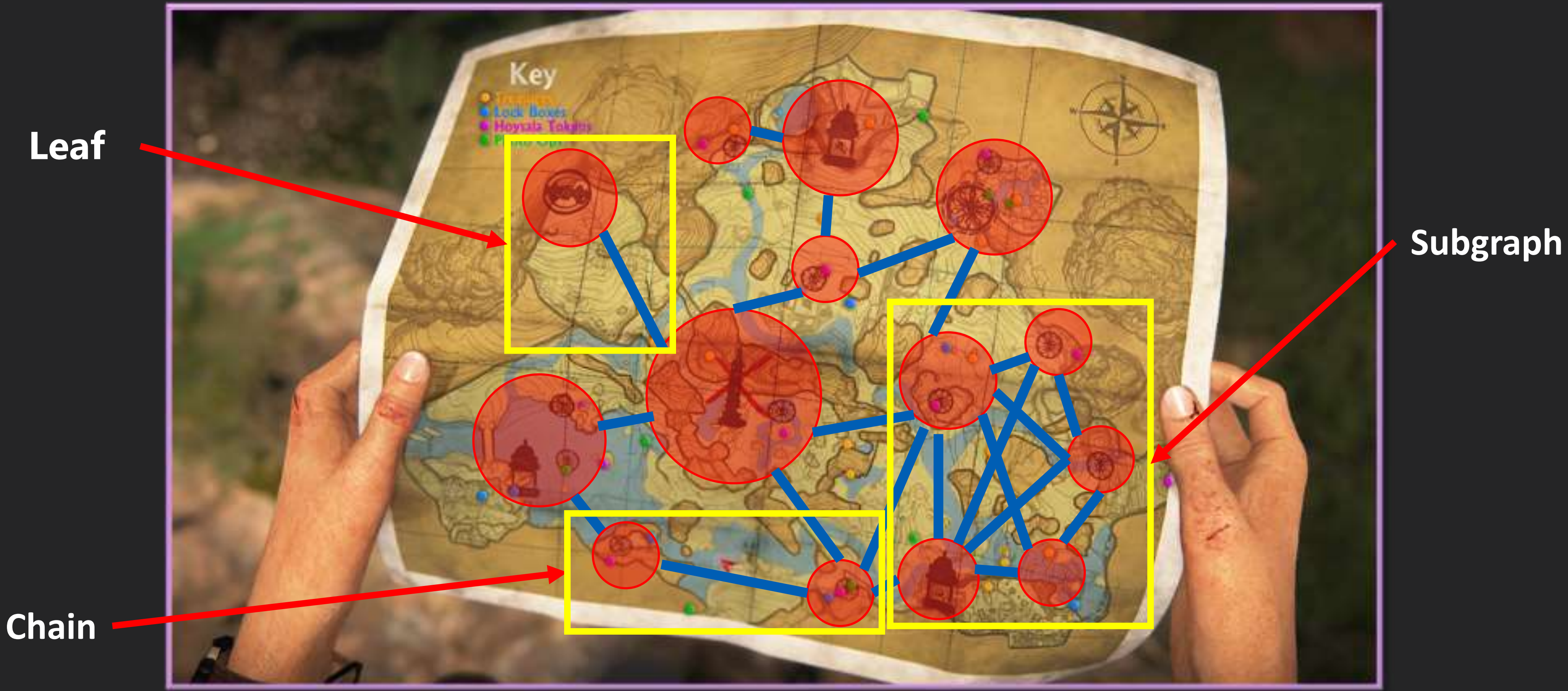


[10]

Stability Factor Calculation — Example

1. Simplify the level map to a simplest form

    1.b. Remove **leaves**, combine **chains,** and generalize **subgraphs**



[10]

Stability Factor Calculation — Example

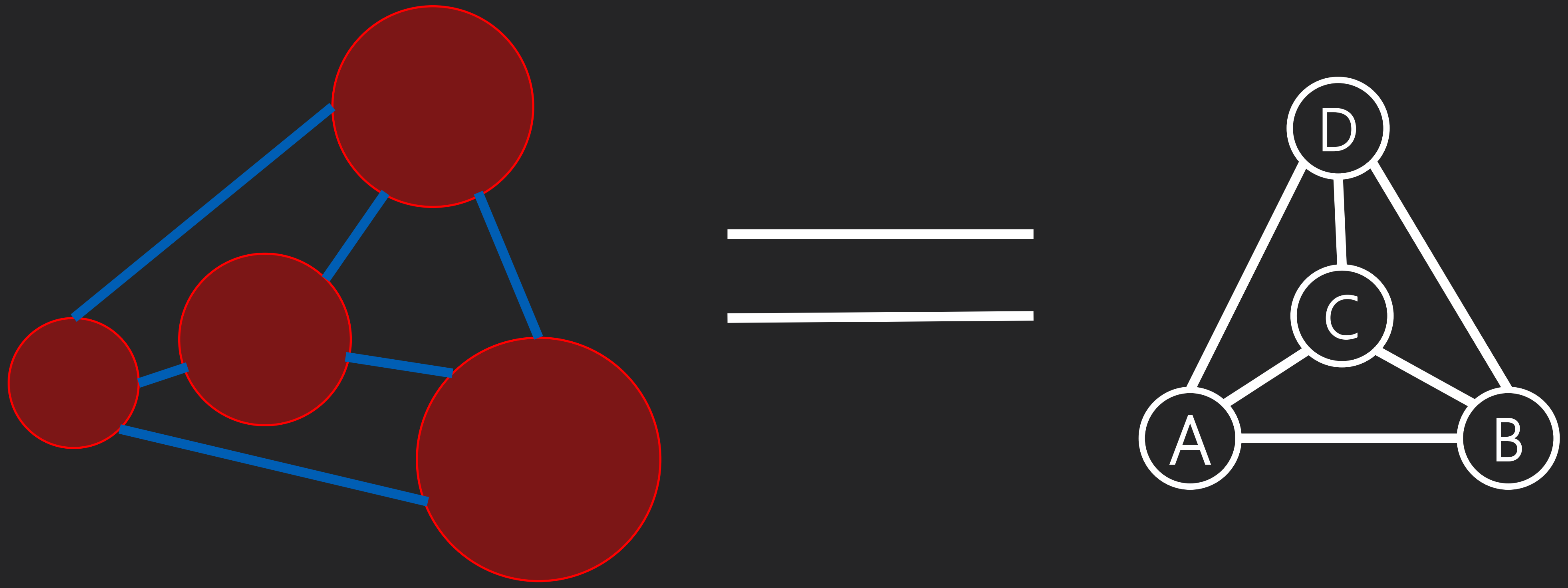
1. Simplify the level map to a simplest form

   Note - **Preserve** vertices and edges containing **important level elements** such as checkpoints, starting points, boss rooms, one-way doors, etc., **as much as possible**

[10]

Stability Factor Calculation — Example

1. Simplify the level map to its simplest form



Redraw the level to a graph

Graph Theory – Calculating Stability Factor

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Graph Theory – Calculating Stability Factor

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Stability Factor Calculation — Example

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Stability Factor Calculation — Example

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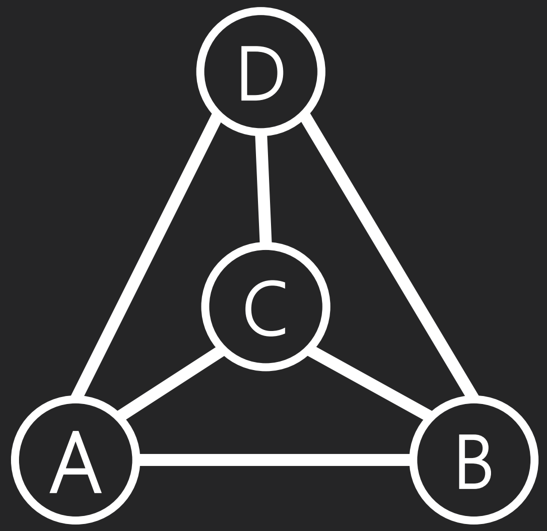

Graph *G* *with* ***m=4***

Stability Factor Calculation — Example

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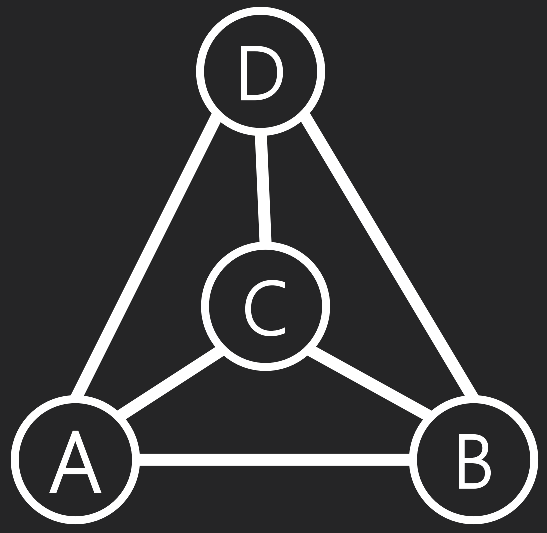

Graph *G* *with* ***m=4***

- If k=1, it means that we are trying to remove **1** vertex to break the connectedness of the graph **G**

Stability Factor Calculation — Example

2. Calculate the ***Cheeger*** number according to the following definition:
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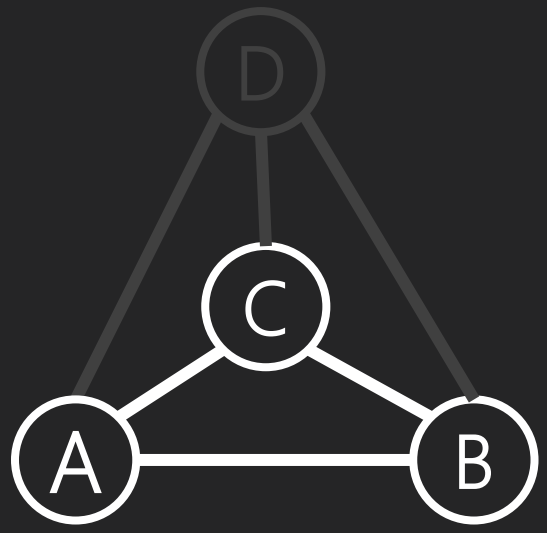

Graph *G* *with* ***m=4***

- If k=1, it means that we are trying to remove **1** vertex to break the connectedness of the graph **G**

If we remove vertex **D**, we have...

Vertices **A**, **B**, and **C** still form a connected graph

Stability Factor Calculation — Example

2. Calculate the ***Cheeger*** number according to the following definition:
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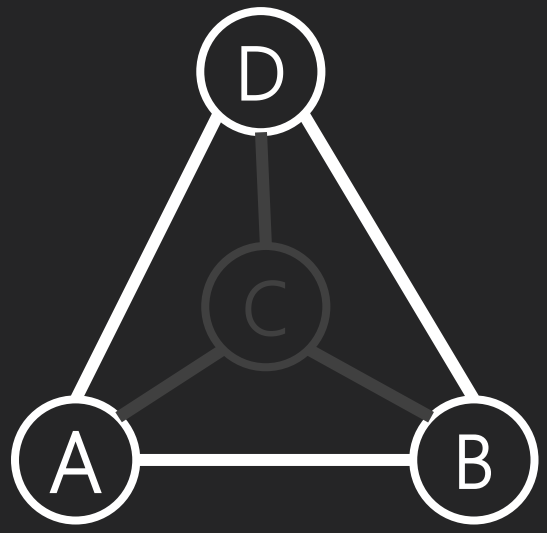

Graph *G* *with* ***m=4***

- If k=1, it means that we are trying to remove **1** vertex to break the connectedness of the graph **G**

If we remove vertex **C**, we have...

Vertices **A**, **B**, and **D** still form a connected graph

Stability Factor Calculation — Example

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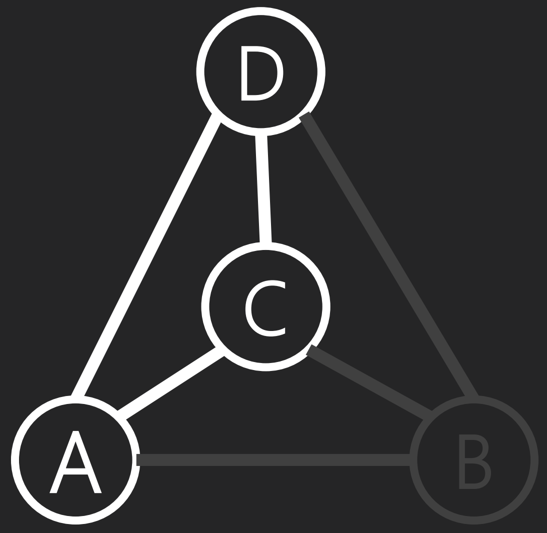

Graph *G* *with* ***m=4***

- If k=1, it means that we are trying to remove **1** vertex to break the connectedness of the graph **G**

If we remove vertex **B**, we have...

Vertices **A**, **C**, and **D** still form a connected graph

Stability Factor Calculation — Example

2. Calculate the ***Cheeger*** number according to the following definition:
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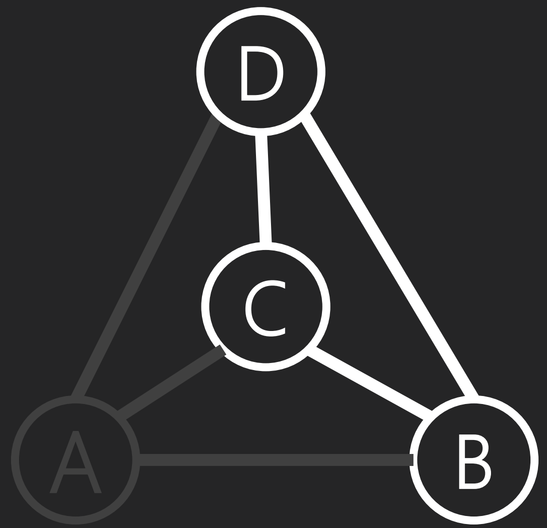

Graph *G* *with* ***m=4***

- If k=1, it means that we are trying to remove **1** vertex to break the connectedness of the graph **G**

If we remove vertex **A**, we have…

Vertices **B**, **C**, and **D** still form a connected graph

Stability Factor Calculation — Example

2. Calculate the ***Cheeger*** number according to the following definition:
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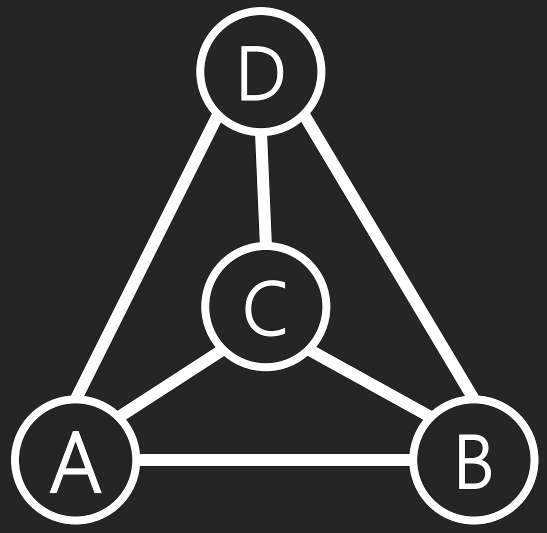


Graph *G* *with* ***m=4***

- If k=1, no matter how we remove a vertex, all vertices in the resulting graph are still connected **(n=0)**

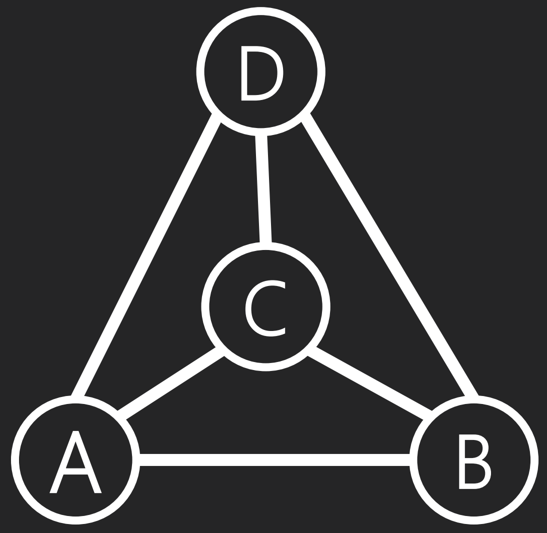
Stability Factor Calculation — Example

2. Calculate the ***Cheeger*** number according to the following definition:

   i. For a graph ***G*** with m vertices, if there exist ***n*** ways to remove ***k*** vertices such that all vertices in the resulting subgraph are not connected, then the k^{th} order ***Cheeger*** number λ_k of graph ***G*** is defined as: $\lambda_k = 1 - \frac{n}{\binom{m}{k}}$, where $\binom{m}{k}$ represents the binomial coefficient that $\binom{m}{k} = \frac{m!}{k!(m-k)!}$

- If k=1, no matter how we remove a vertex, all vertices in the resulting graph are still connected (n=0)
- If k=2, same as what we did before, but **2** vertices will be removed at once

  (Note that a chain is still a connected graph)



Graph *G* *with* ***m=4***

Stability Factor Calculation — Example

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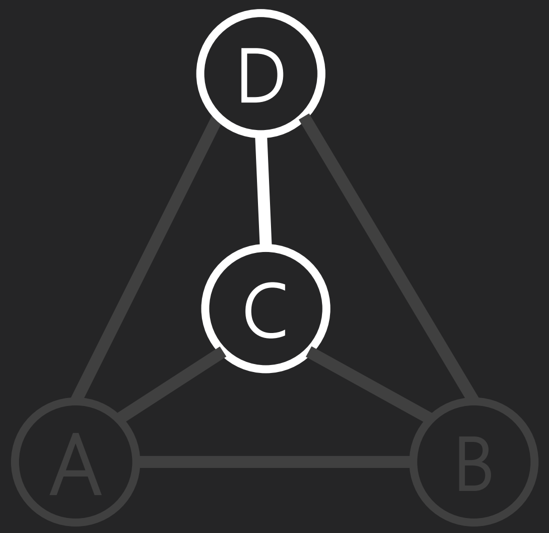

Graph *G* *with* ***m=4***

- If k=1, no matter how we remove a vertex, all vertices in the resulting graph are still connected (n=0)
- If k=2, same as what we did before, but **2** vertices will be removed at once

(Note that a chain is still a connected graph)

If we remove vertices **A and B**, we have…

Vertices **C** and **D** still form a connected graph

Stability Factor Calculation — Example

2. Calculate the ***Cheeger*** number according to the following definition:
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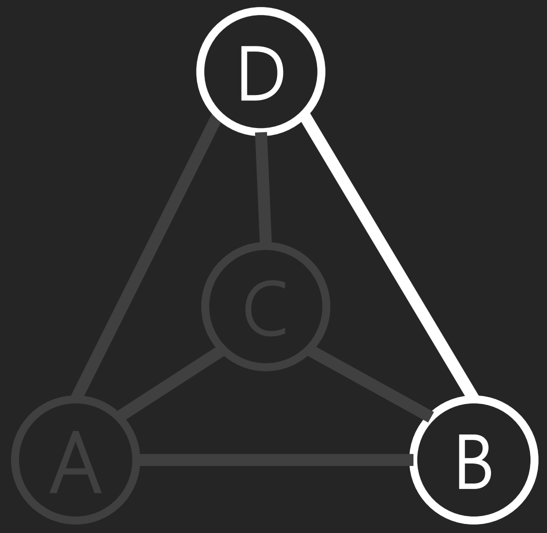

Graph *G* *with* ***m=4***

- If k=1, no matter how we remove a vertex, all vertices in the resulting graph are still connected (n=0)
- If k=2, same as what we did before, but **2** vertices will be removed at once

(Note that a chain is still a connected graph)

If we remove vertices **A and C**, we have…

Vertices **B** and **D** still form a connected graph

Stability Factor Calculation — Example

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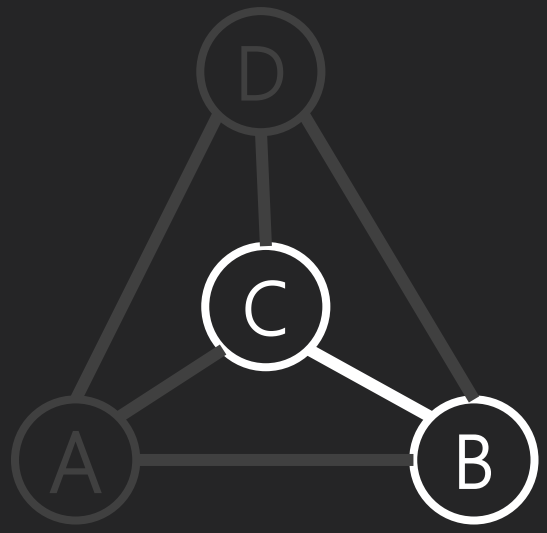

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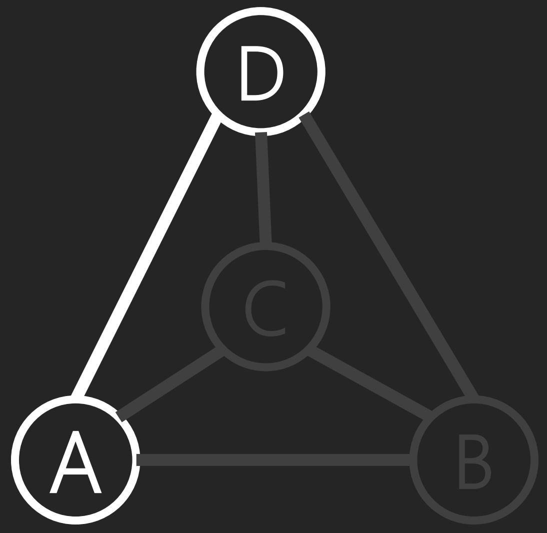

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Stability Factor Calculation — Example

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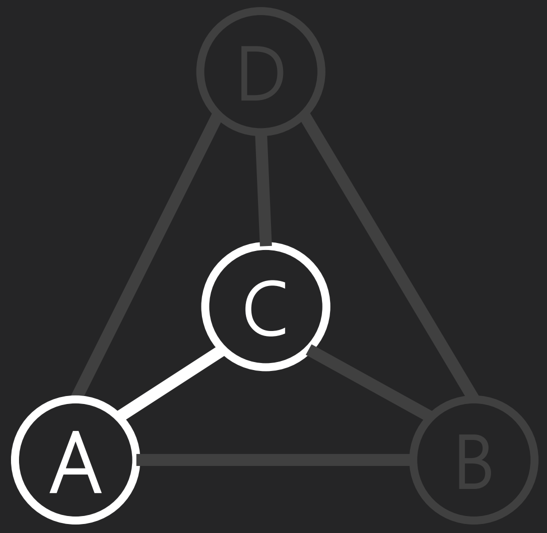

Graph *G* *with* ***m=4***

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(Note that a chain is still a connected graph)

If we remove vertices **B and D**, we have…

Vertices **A** and **C** still form a connected graph

Stability Factor Calculation — Example

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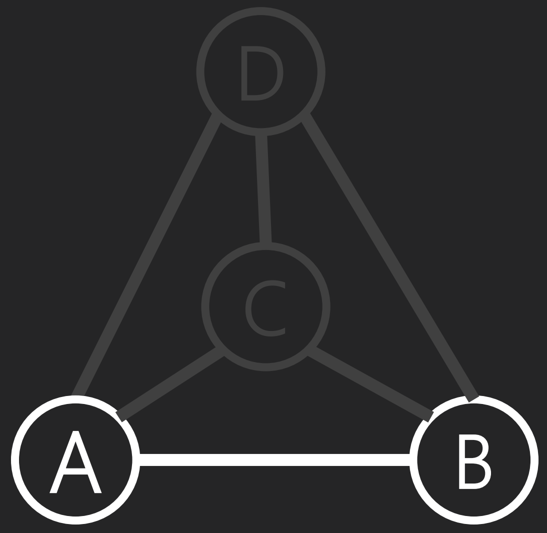

Graph *G* *with* ***m=4***

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  (Note that a chain is still a connected graph)

  If we remove vertices **C and D**, we have…

  Vertices **A** and **B** still form a connected graph

Stability Factor Calculation — Example

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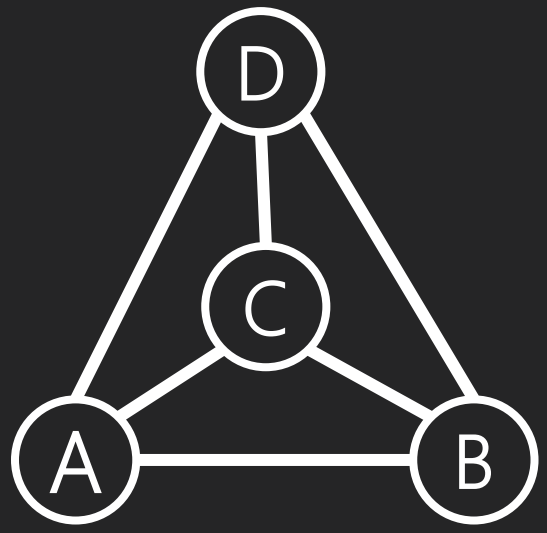

Graph *G* *with* ***m=4***

- If k=1, no matter how we remove a vertex, all vertices in the resulting graph are still connected (n=0)
- If k=2, same as if k=1, all remaining vertices in the resulting graph are still connected **(n=0)**

Stability Factor Calculation — Example

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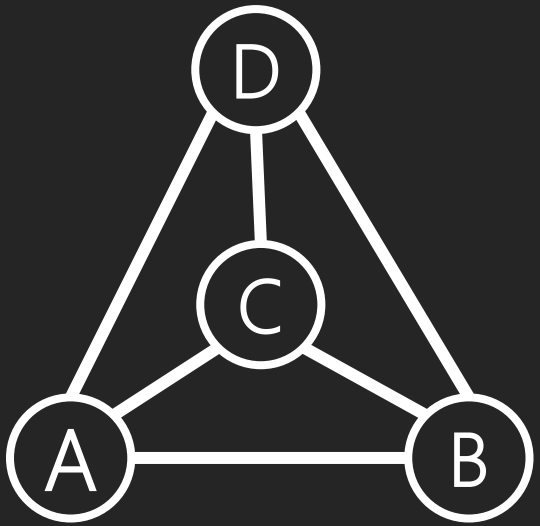

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- If k=3, we need to remove **3** vertices at once to break the connectedness

Stability Factor Calculation — Example

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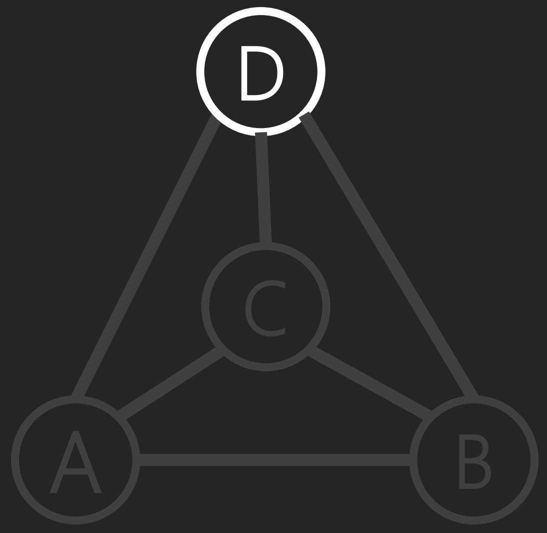

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- If k=3, we need to remove **3** vertices at once to break the connectedness

If we remove vertices **A**, **B** and **C**, we have…

Vertex **D** itself is still a connected graph

Stability Factor Calculation — Example

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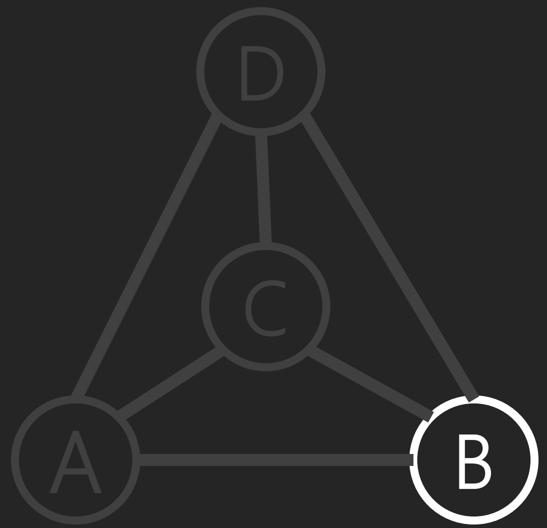

Graph *G* *with* ***m=4***

- If k=1, no matter how we remove a vertex, all vertices in the resulting graph are still connected (n=0)
- If k=2, same as if k=1, all remaining vertices in the resulting graph are still connected (n=0)
- If k=3, we need to remove **3** vertices at once to break the connectedness

Similarly, removing vertices **(A, C, D)** or **(B, C, D)** at once won't break the graph's connectedness

Stability Factor Calculation — Example

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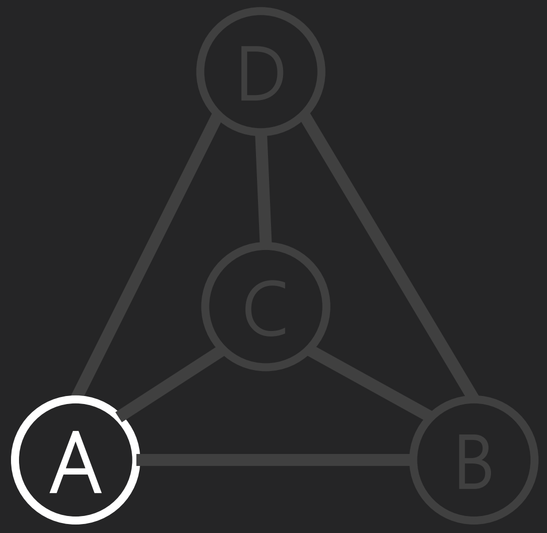

Graph *G* *with* ***m=4***

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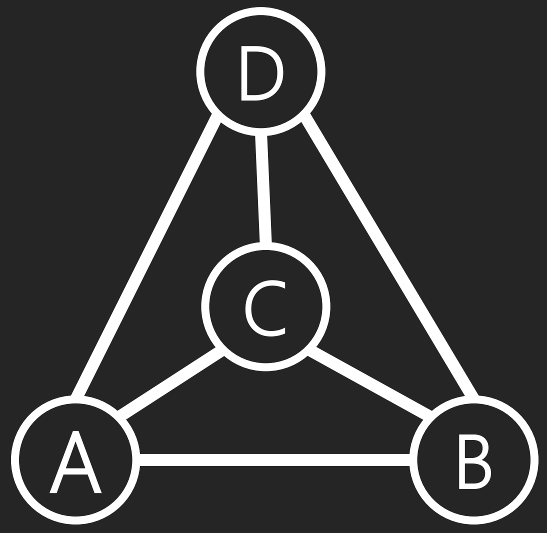
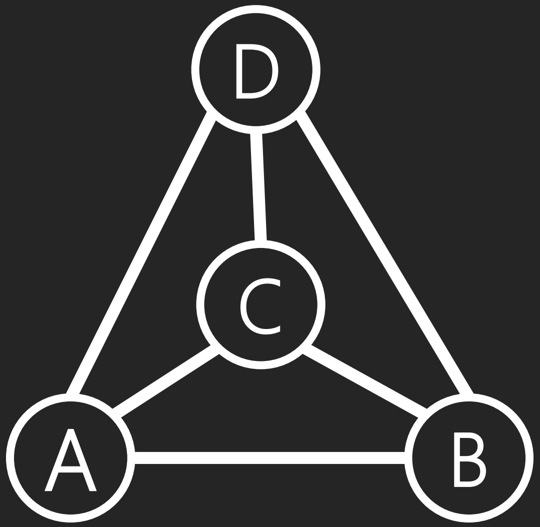


Graph *G* *with* ***m=4***

- If k=1, no matter how we remove a vertex, all vertices in the resulting graph are **still connected (n=0)**
- If k=2, same as if k=1, all remaining vertices in the resulting graph are **still connected (n=0)**
- If k=3, the remaining single vertex is **still connected. (n=0)**
- In this step, we **only** calculate to **k=3**

Stability Factor Calculation — Example

2. Calculate the ***Cheeger*** number according to the following definition:
   i. For a graph ***G*** with m vertices, if there exist ***n*** ways to remove ***k*** vertices such that all vertices in the resulting subgraph are not connected, then the $\boldsymbol{k^{th}}$ order ***Cheeger*** number $\boldsymbol{\lambda_k}$ of graph ***G*** is defined as: $\boldsymbol{\lambda_k = 1 - \frac{n}{\binom{m}{k}}}$, where $\boldsymbol{\binom{m}{k}}$ represents the binomial coefficient that $\binom{m}{k} = \frac{m!}{k!(m-k)!}$



Graph G *with* ***m=4***

$\begin{cases} k = 1 \\ k = 2 \\ k = 3 \end{cases} \rightarrow$ No matter how we remove a vertex, all vertices in the resulting graph are still connected

- Hence, we have:
  - $\lambda_1 = 1 - \frac{0}{\binom{4}{1}} = 1$ *(k=1, n=0)*
  - $\lambda_2 = 1 - \frac{0}{\binom{4}{2}} = 1$ *(k=2, n=0)*
  - $\lambda_3 = 1 - \frac{0}{\binom{4}{3}} = 1$ *(k=3, n=0)*

Calculate each of the *Cheeger* number

Graph Theory – Calculating Stability Factor

1. Simplify the level map to a simplest form
2. Calculate the ***Cheeger*** number according to the following definition:
   1. For a graph ***G*** with m vertices, if there exist ***n*** ways to remove ***k*** vertices such that all nodes in the resulting subgraph are not connected, then the $\boldsymbol{k^{th}}$ order ***Cheeger*** number $\boldsymbol{\lambda_k}$ of graph ***G*** is defined as: $\boldsymbol{\lambda_k = 1 - \frac{n}{\binom{m}{k}}}$, where $\boldsymbol{\binom{m}{k}}$ represents the binomial coefficient that $\binom{m}{k} = \frac{m!}{k!(m-k)!}$
3. Calculate γ by the following formula:
   1. $\gamma_\infty = \lim_{m \to 3} \frac{\sum_{i=1}^{3} \frac{1}{i!} \lambda_i}{\sum_{i=1}^{3} \frac{1}{i!}} = \frac{1}{e} \lim_{m \to 3} \frac{1}{i!} \lambda_i$

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Stability Factor Calculation — Example

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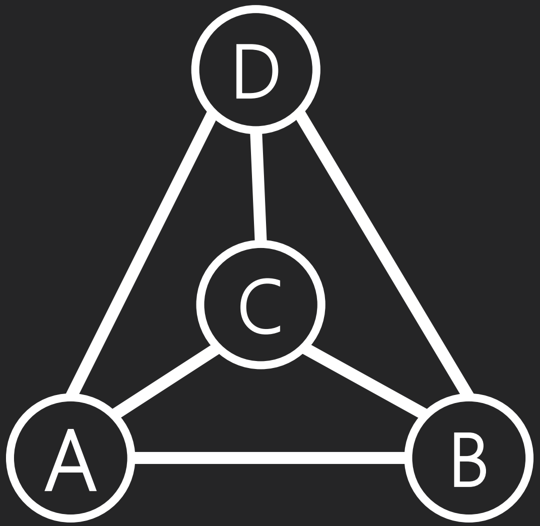

Graph *G* *with* ***m=4***

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- $\lambda_3 = 1 - \frac{0}{\binom{4}{3}} = 1$ *(k=3,n=0)*
- $\gamma^{(3)} = \frac{\sum_{i=1}^{3}\frac{1}{i!}\lambda_i}{\sum_{i=1}^{3}\frac{1}{i!}} = \frac{3}{5}\left(\lambda_1 + \frac{1}{2}\lambda_2 + \frac{1}{6}\lambda_3\right) = \mathbf{1 > 0.94}$

Factor of 1 indicates that this map's layout has good connectedness for exploration

4. Artifact Description & Map

Artifact Description

- *"Lunaric Parchments"*
- *The Elder Scrolls V: Skyrim*
  - Creation Kit: Skyrim

[5]

- A fetch quest - gather certain objects
  - Story:
    - Help investigate a castle under the influence of a dangerous magicka chaos
    - Collect **7** magical parchments to resolve the magicka chaos

Finalize Artifact Outline

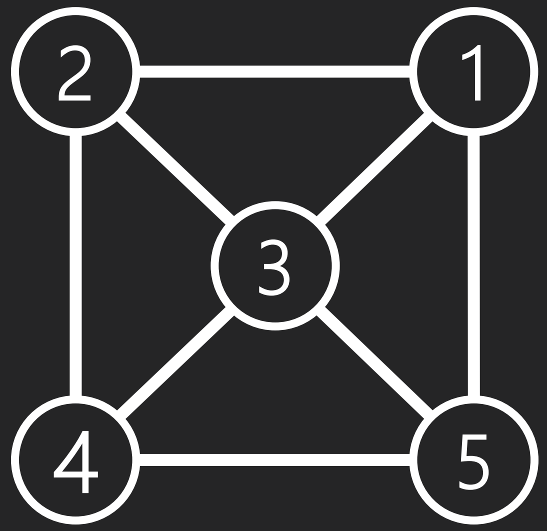
- Level Top-down Snapshot

Finalize Artifact Outline

- Level Top-down Snapshot
- Dominions are decorated with different thematic assets in *The Elder Scrolls V: Skyrim*
  – Miniscaped Definition

Applying the Methodology

1. Designed a graph with a good stability factor

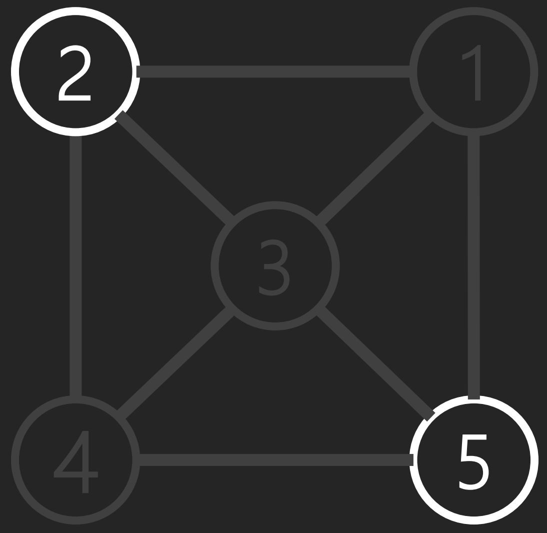


Graph *G* *with **m=5***

- If **k=1**, no matter how we remove a vertex, the resulting graph is still connected
- If **k=2**, no matter how we remove vertices, the resulting graph is still connected

Applying the Methodology

1. Designed a graph with a good stability factor

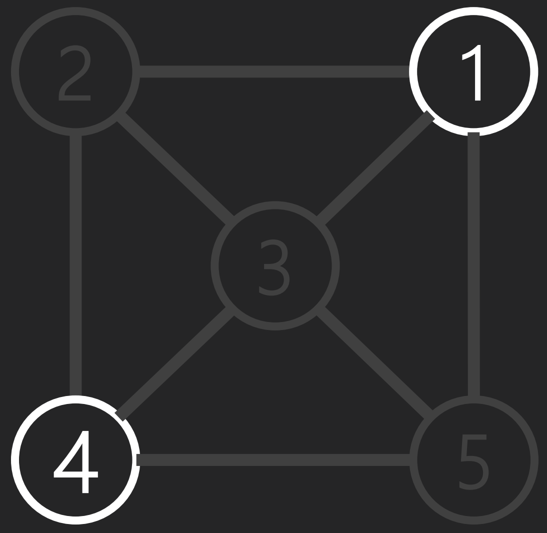


Graph G *with* ***m*=5**

- If **k=1**, no matter how we remove a vertex, the resulting graph is still connected
- If **k=2**, no matter how we remove vertices, the resulting graph is still connected
- If **k=3**, if we remove vertices **(1, 3, 4)** and **(2, 3, 5)**, the resulting graph will not be a connected graph

Applying the Methodology

1. Designed a graph with a good stability factor



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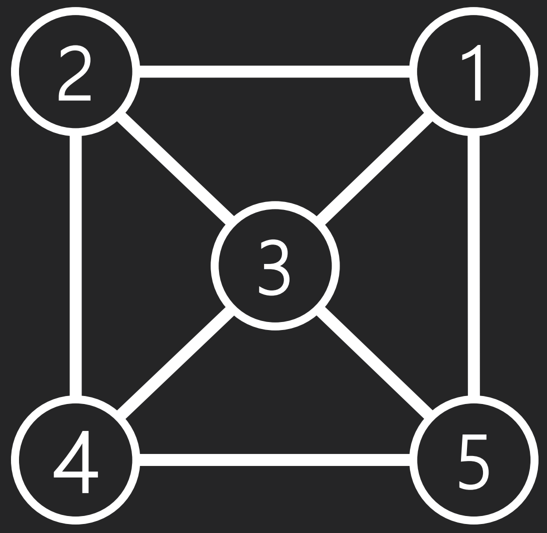
- If **k=2**, no matter how we remove vertices, the resulting graph is still connected
- If **k=3**, if we remove vertices **(1, 3, 4)** and **(2, 3, 5)**, the resulting graph will not be a connected graph

Applying the Methodology

1. Designed a graph with a good stability factor
   a. Verified its Stability Factor



Graph *G* *with* ***m*=5**

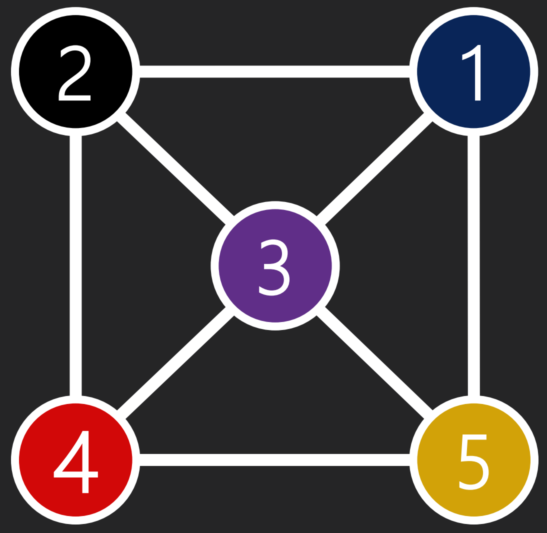
Calculate $\boldsymbol{\gamma}$:

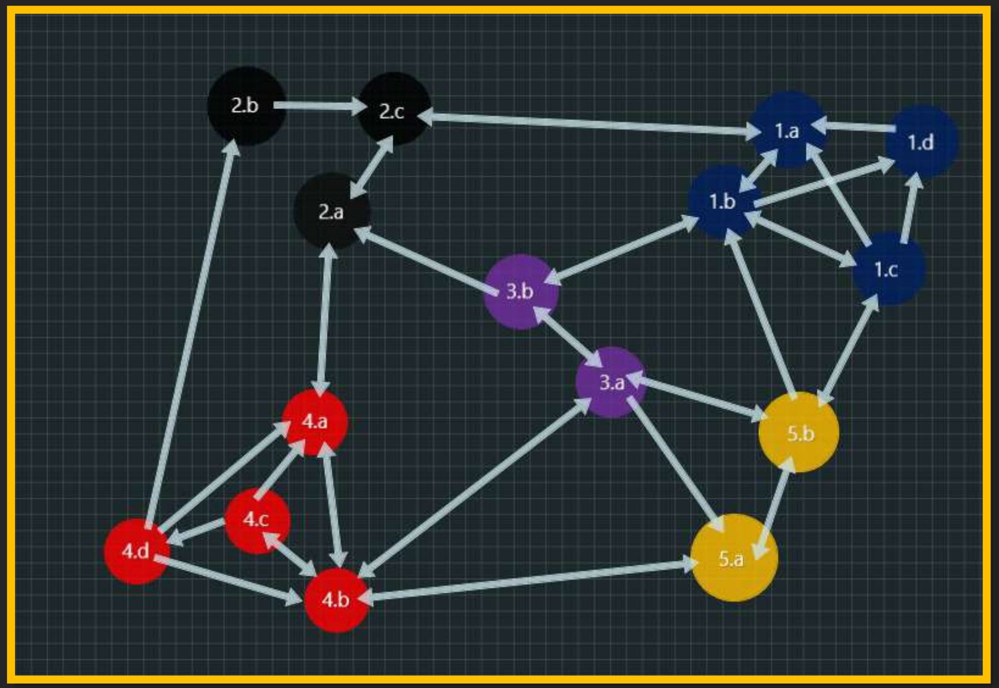
$$\gamma_\infty = \lim_{m\to 3} \frac{\sum_{i=1}^{3} \frac{1}{i!}\lambda_i}{\sum_{i=1}^{3} \frac{1}{i!}} = \frac{1}{e} \lim_{m\to 3} \frac{1}{i!}\lambda_i$$

- $\lambda_1 = 1 - \frac{0}{\binom{5}{1}} = 1$ *(k=1, n=0)*
- $\lambda_2 = 1 - \frac{0}{\binom{5}{2}} = 1$ *(k=2, n=0)*
- $\lambda_3 = 1 - \frac{2}{\binom{5}{2}} = \frac{4}{5}$ *(k=3, n=2)*
- $\gamma^{(3)} = \frac{\sum_{i=1}^{3} \frac{1}{i!}\lambda_i}{\sum_{i=1}^{3} \frac{1}{i!}} = \frac{3}{5}\left(\lambda_1 + \frac{1}{2}\lambda_2 + \frac{1}{6}\lambda_3\right) = 0.9636$
- $\mathbf{0.9636 > 0.94}$

Applying the Methodology

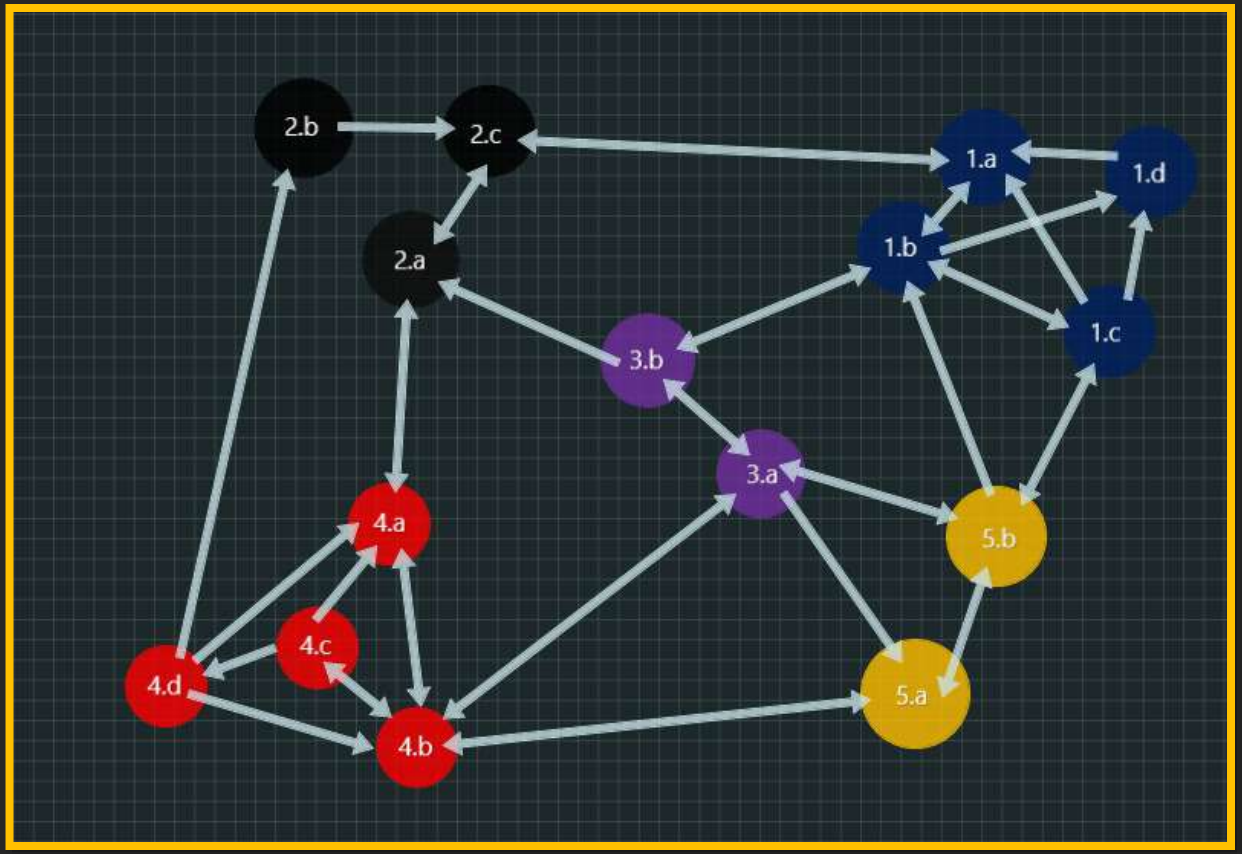
2. Expanded the Graph





Applying the Methodology

3. Detailed the graph to become a level map

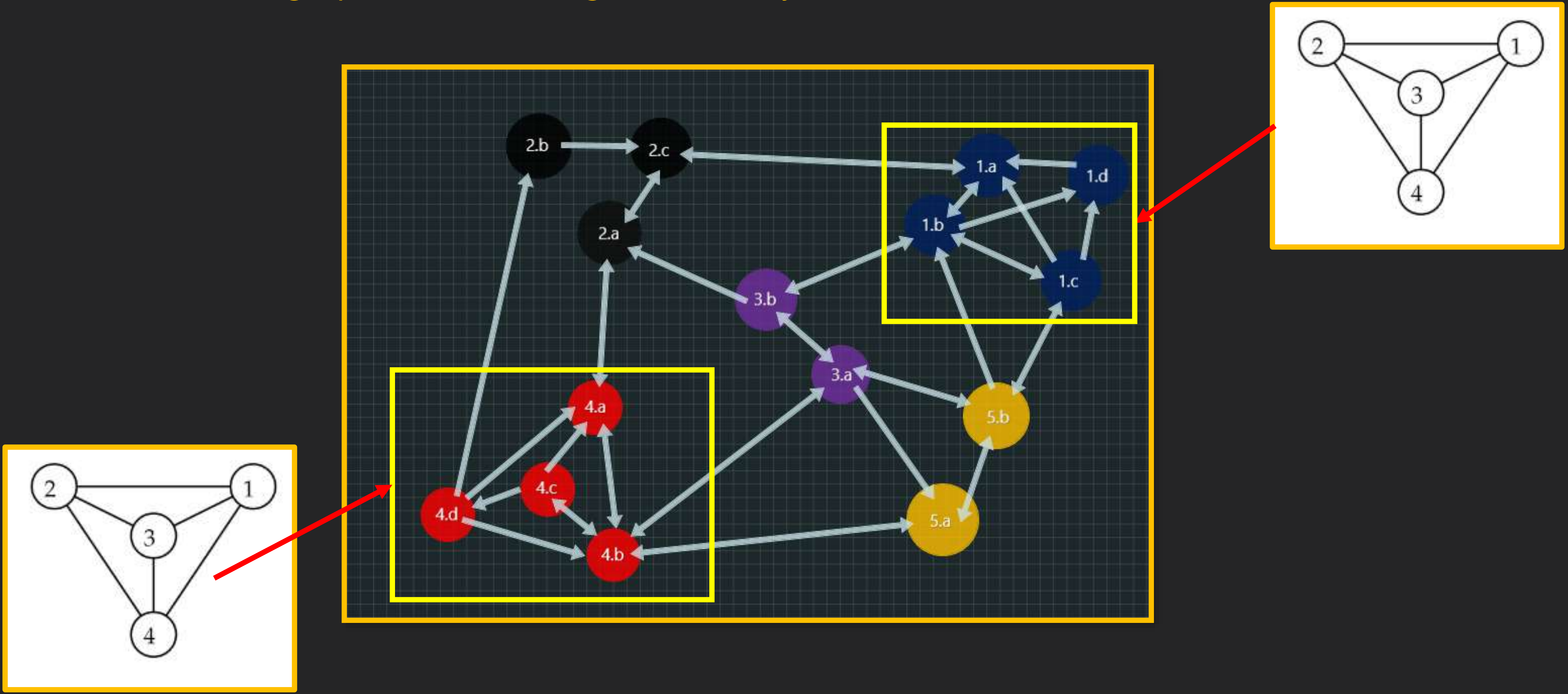


- Removed from the *Uncharted 4* example?
  - **Leaves**
  - **Chains**
  - **Subgraphs**
- They're structures ensuring the level has **enough space** for **gameplay experiences!**

[6]

Applying the Methodology

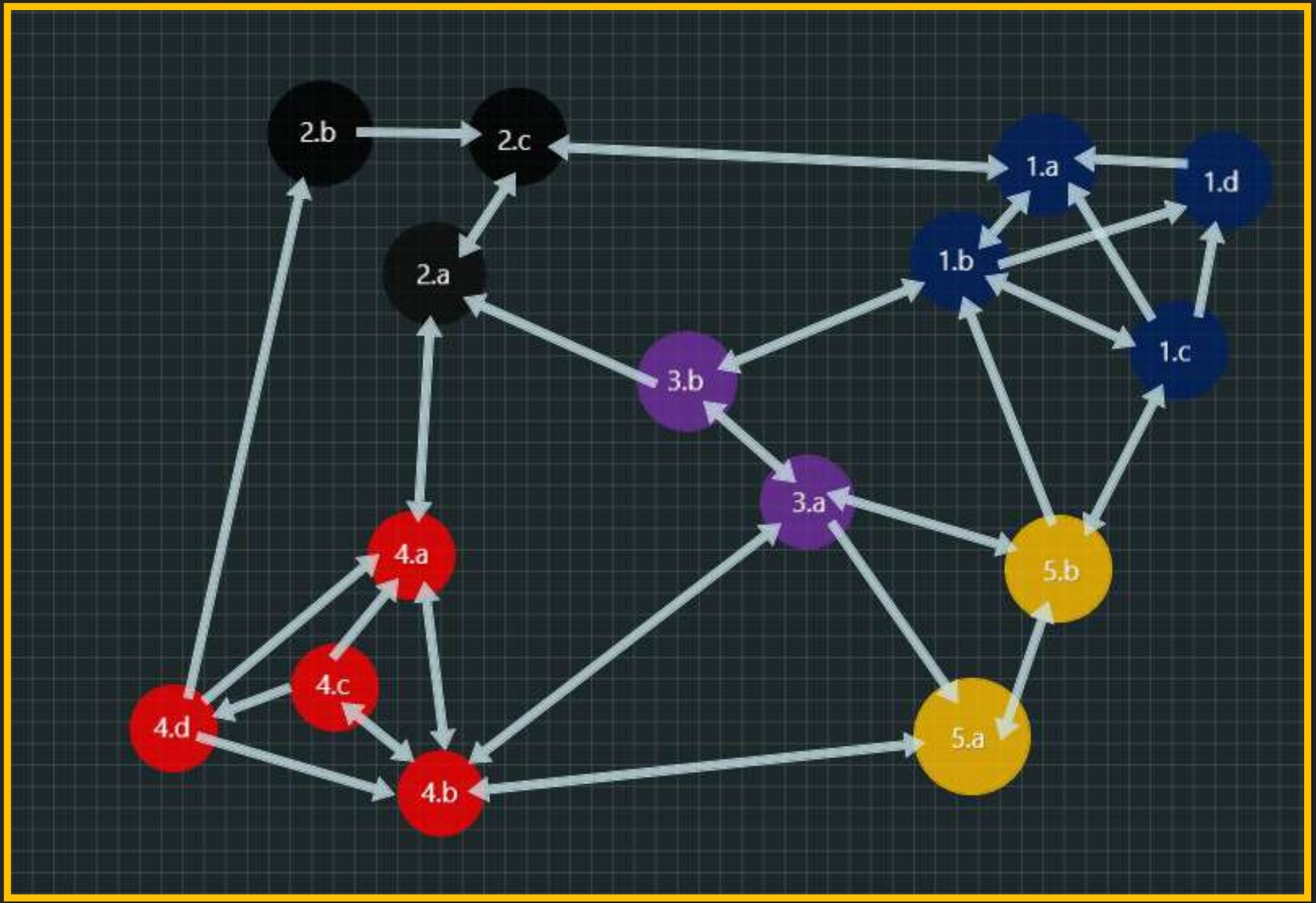
4. Embedded subgraphs that have a good Stability Factor



[6]

Applying the Methodology

5. Detailed the graph to become a level map



[6]

Applying the Methodology

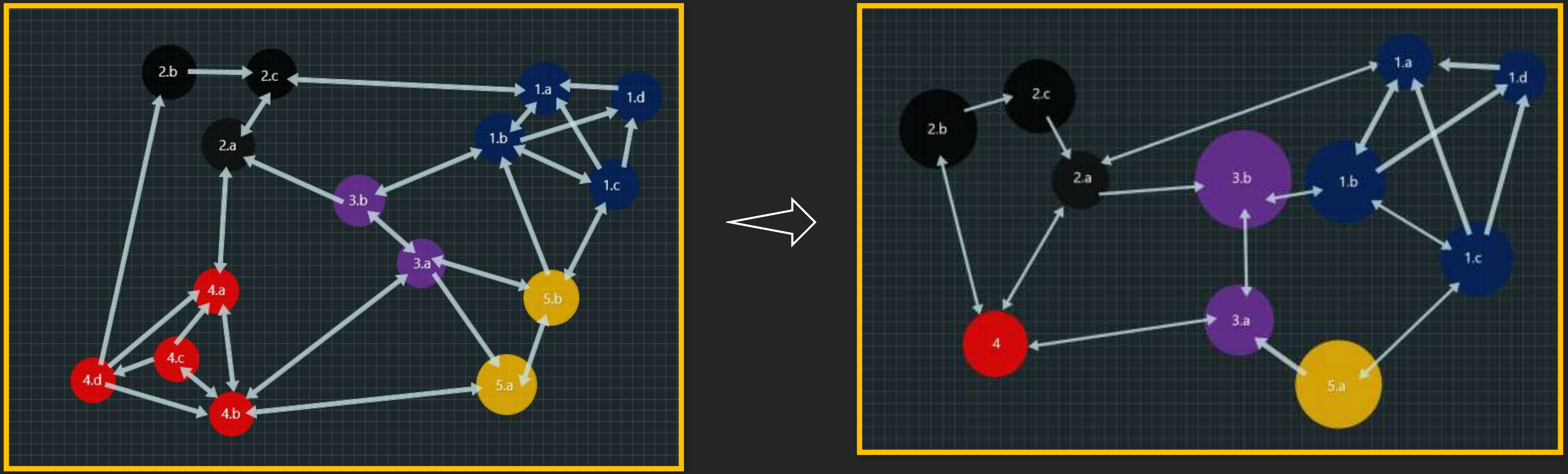
6. Detailed the graph to become a level map

Applying the Methodology

7. Blocked out the level Whitebox

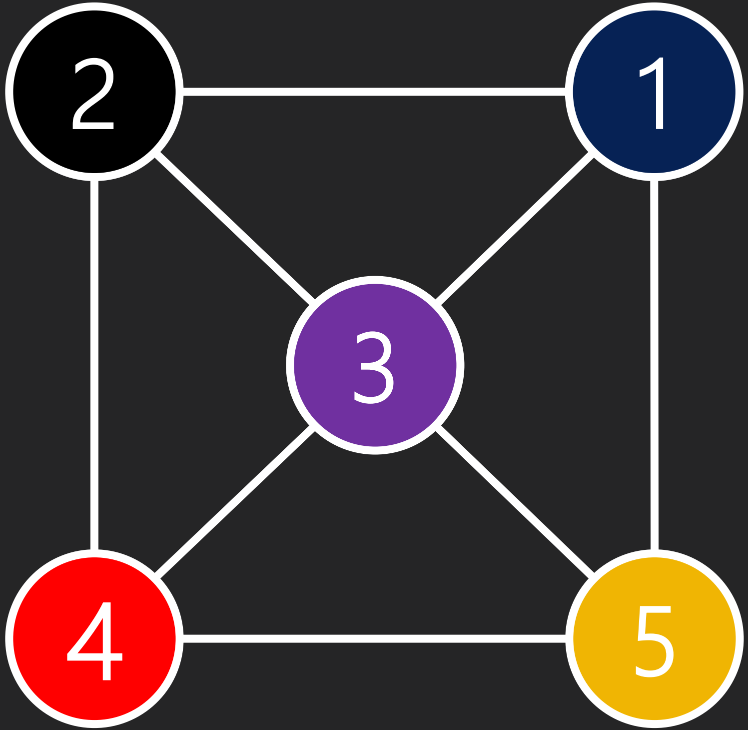
Applying the Methodology

8. Iterated on the structure
   - The player cannot see any landmark from interior spaces (4.c ,4.d & 5.b)
     - Messes with player's sense of space and navigation



[7]

Finalize Artifact Outline



Dominion 2: Castle wall tower in Imperial exterior theme with snow overlay and contaminating magic crystals

Dominion 2: In-game player perspective

2

Dominion 1: Castle wall tower in Markarth exterior theme with snow overlay

Dominion 1: Whiterun City decorative theme

1

4

Dominion 4: Solitude city exterior theme with magic tower

Dominion 4: In-game player perspective

[7]

5

Dominion 5: Sunken garden in Dwemer ruins exterior theme

Dominion 5: In-game player perspective

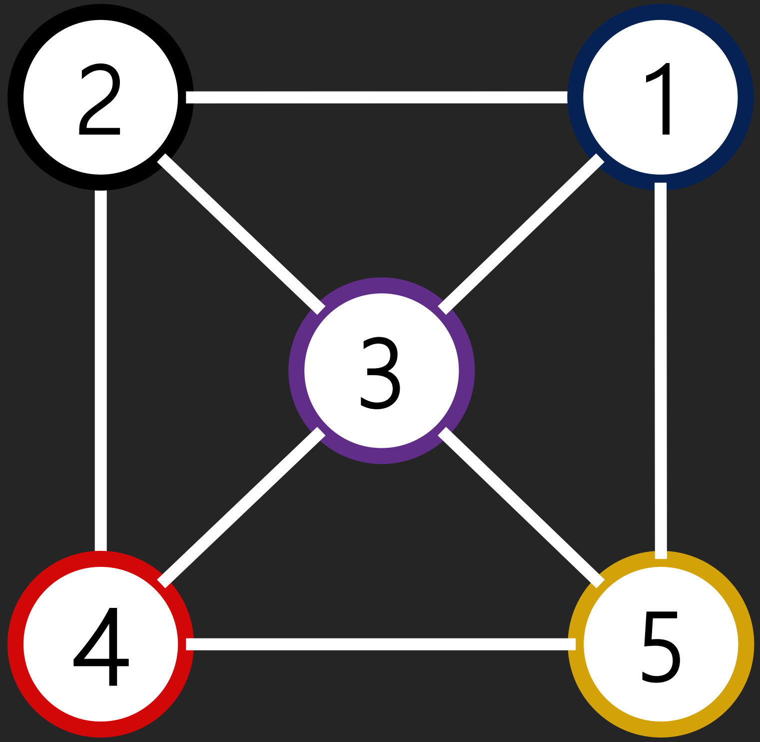
3

Dominion 3: Central courtyard in Labyrinthian exterior theme

Dominion 3: In-game player perspective

Guiding Players - Quest Objectives

Light Beams: Highlighting quest items



Guiding Players - Dominion 1

- Dominion 1 leads the player to observation spots looking for other quest items

Guiding Players - D1

- Used lighting and pickup items to pull the player forward

- Overview of the quest item positions

Guiding Players - D1

- Used lighting and pickup items to pull the player forward
- Overview of the quest item positions

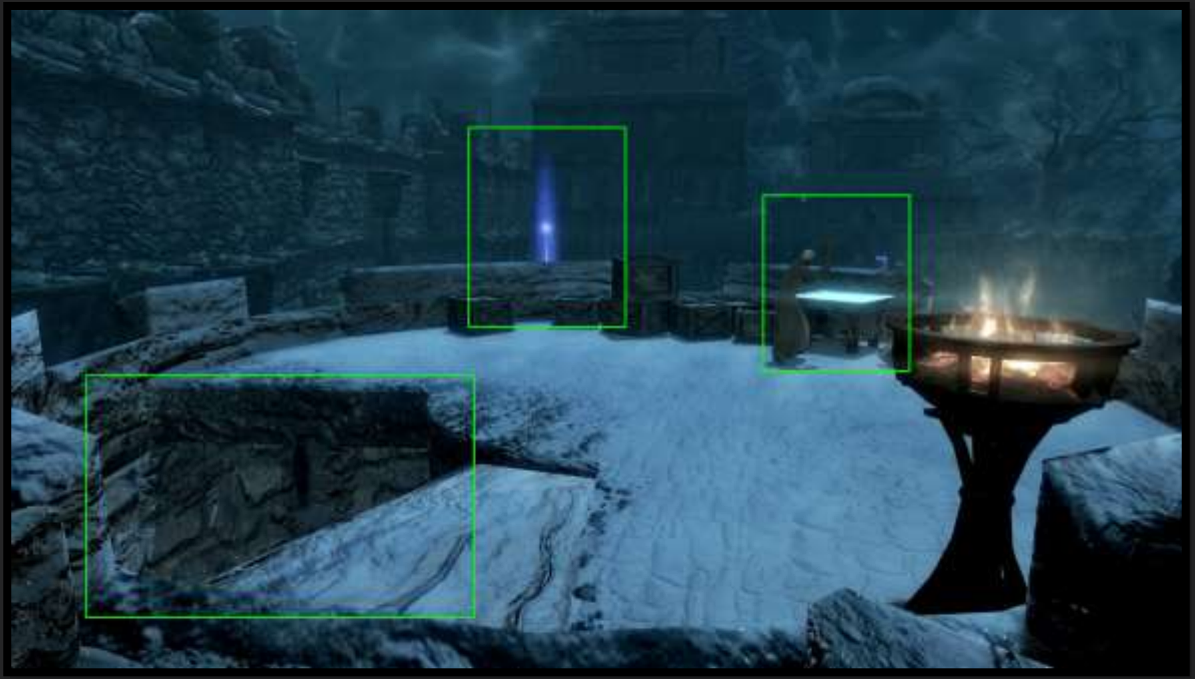
- Overview of the quest item positions
- Pinched objects toward the correct direction

Guiding Players - Dominion 2

Dominion 2 provides good spots to look for quest items

Guiding Players - D2

- Added several spots to observe light beams

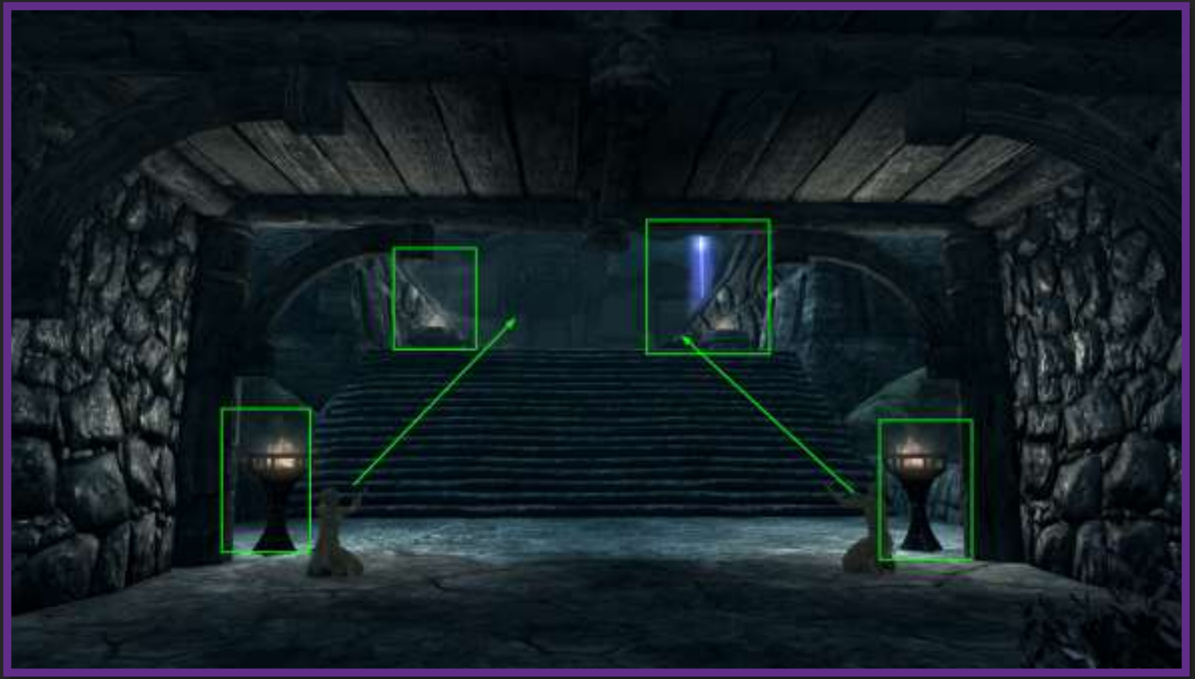


- Used light contrast to hint at the flow
- Used stone NPCs to catch players' attention

Guiding Players - Dominion 3

- Dominion 3 works as a connection point to all the other dominions

Guiding Players - D3

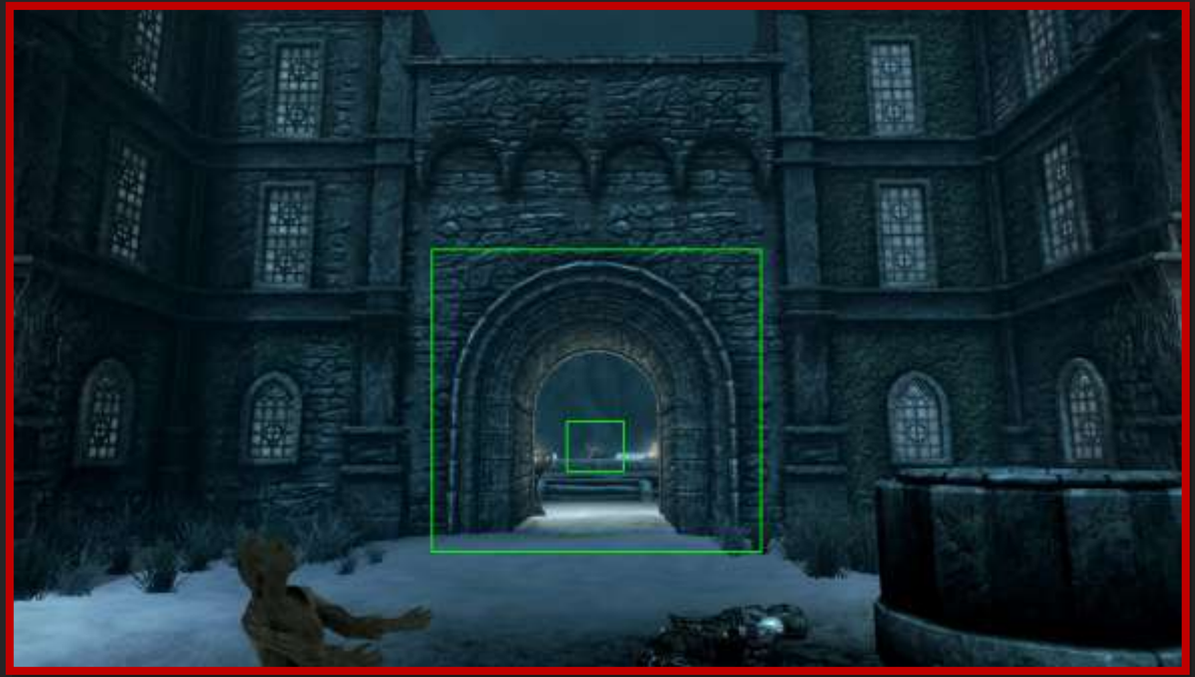


- NPC statues lead the player
- Light contrast highlights the next quest item

- Strategically placed items and lights leading the player to find a hidden stairway to another dominion

Guiding Players - Dominion 4

Dominion 4 is one of the most eye-catching landmarks that help player navigate



Guiding Players - D4

- NPC statues lead the player
- Great stairs with glowing runes lead the player's way

- Framed the handle activating an elevator to the next quest item
- Light contrast catches the player's attention

Guiding Players - Dominion 5

Dominion 5 is an impressive spot for environmental storytelling

Guiding Players - D5

- Light contrast highlights a path to a nearby dominion and the next quest item

- Strategically placed loot items to pull player to a good observation spot
- Motion objects catch the player's attention and point to where to go
- Statues look at the quest items, which guides the player

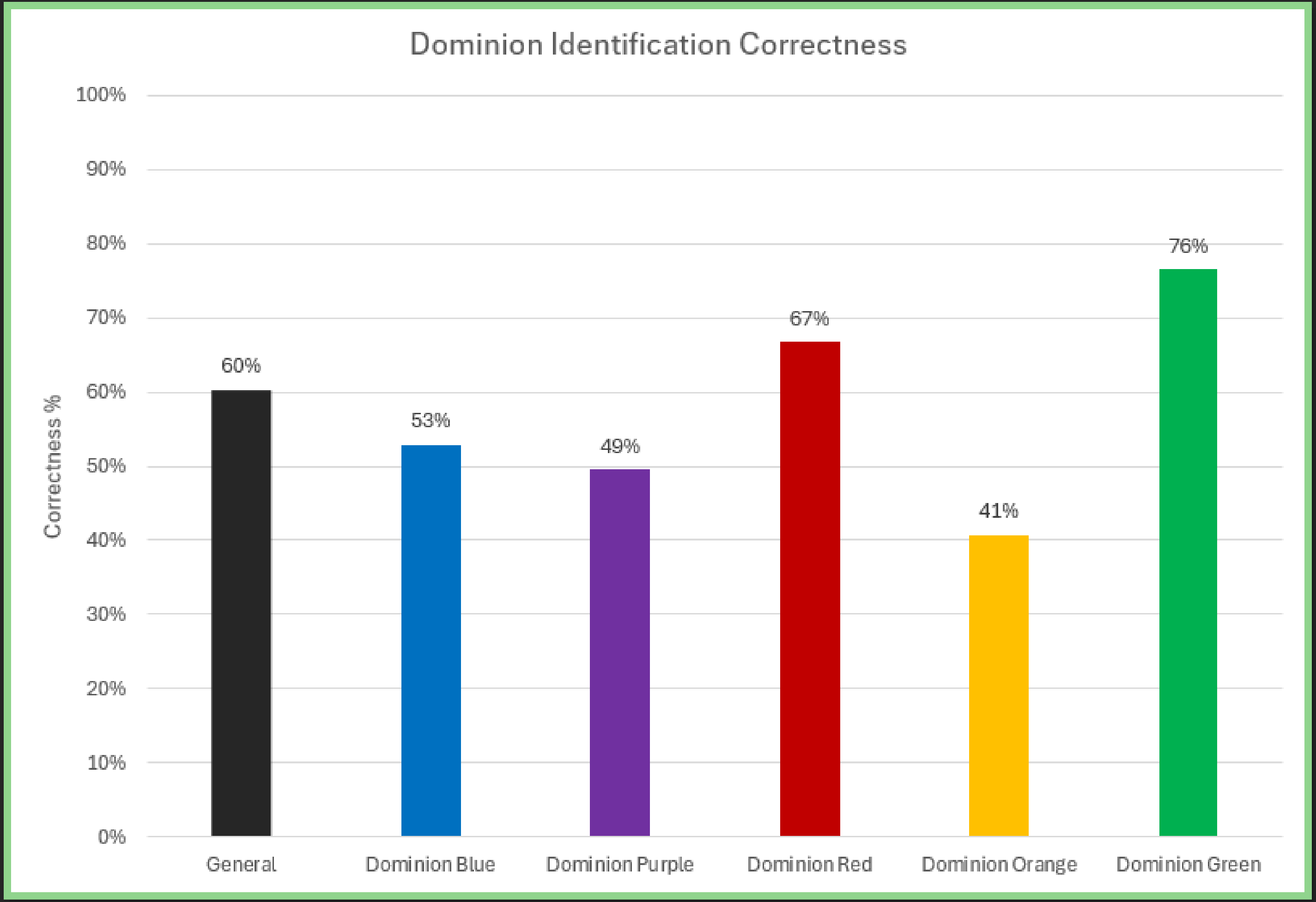
5. Survey Process & Results

Survey Process

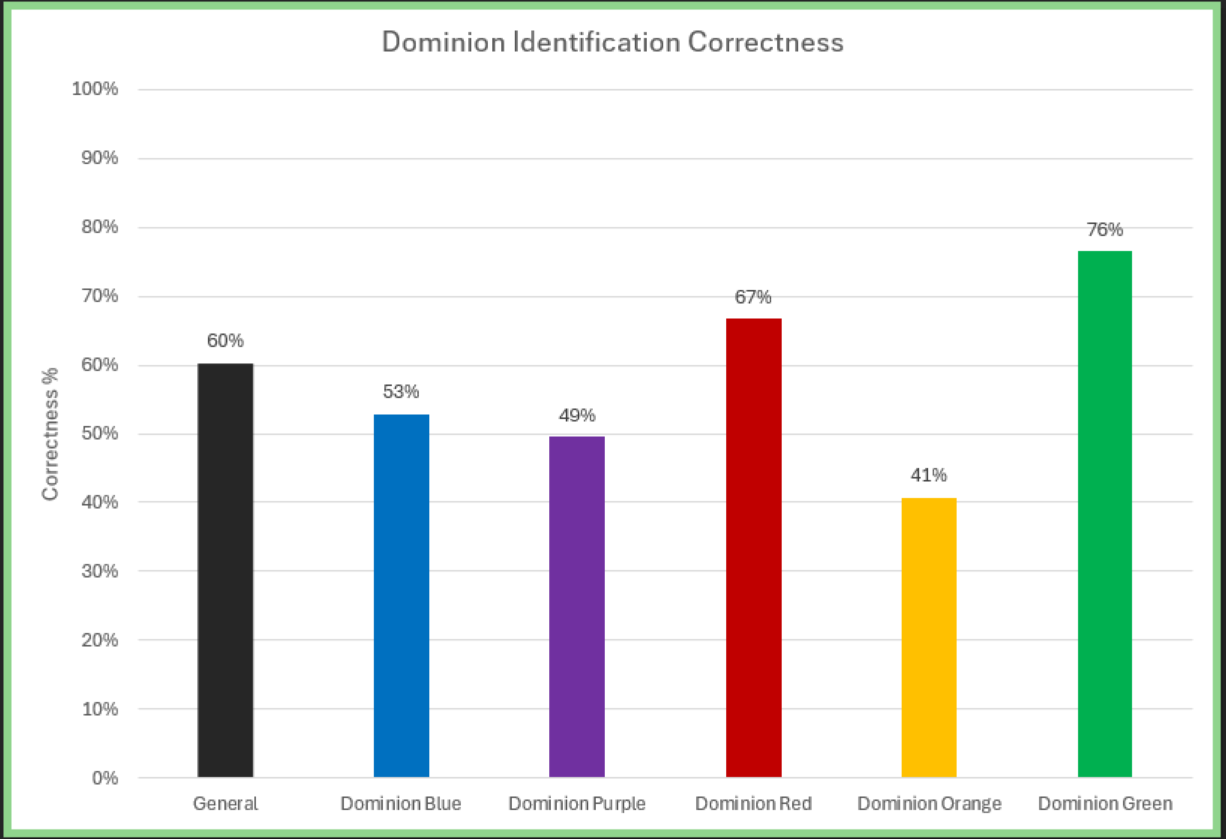
- **17** participants
- Pre-survey = **Quantic Foundry Player Motivation Profile quiz**
- Post-survey = **15 quiz questions** verifying players' mental mapping abilities
  - Mental mapping: ability to recall where a specific place is in a given map

Survey Results

- Players' correctness in identifying individual dominions
- Players' correctness in identifying dominions overall



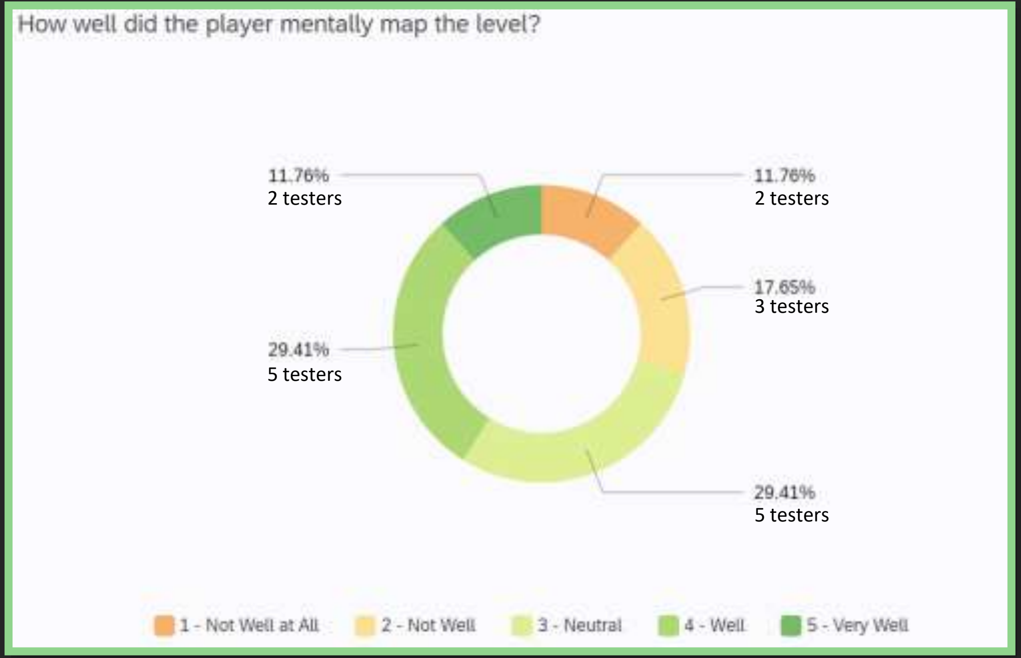
Survey Results



- **Data aligns with assumptions**
- "Memorable": given a description, an objective, or an image, you can picture how to reach an area
  - **Green and Red** dominions are the **most memorable**
  - **Orange dominion** is the **least memorable**
    - Reason for issue – **altitude, low elevation**

Survey Results

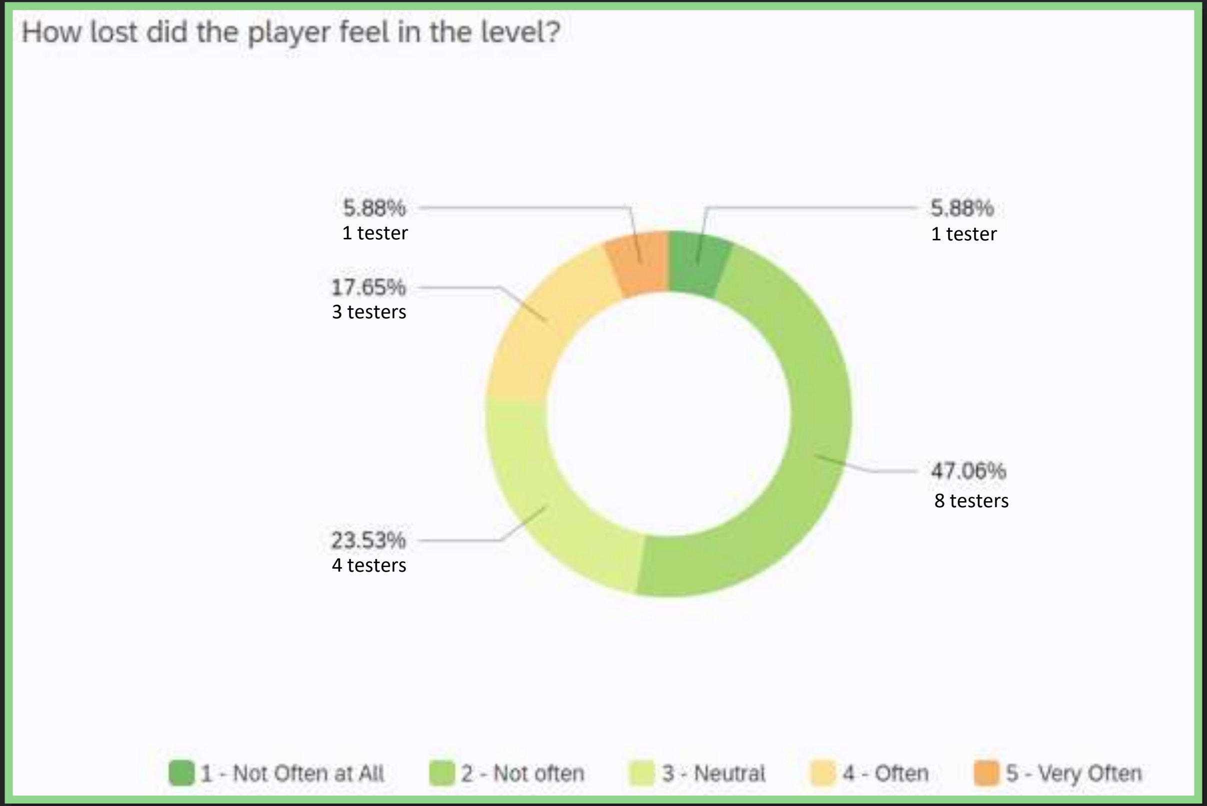
- How well did the player feel about mentally mapping the space?



- **70.58% (12 out of 17 players)** felt they did well when mentally mapping the space
- **29.41% (5 out of 17 players)** felt they did not mentally map the level well

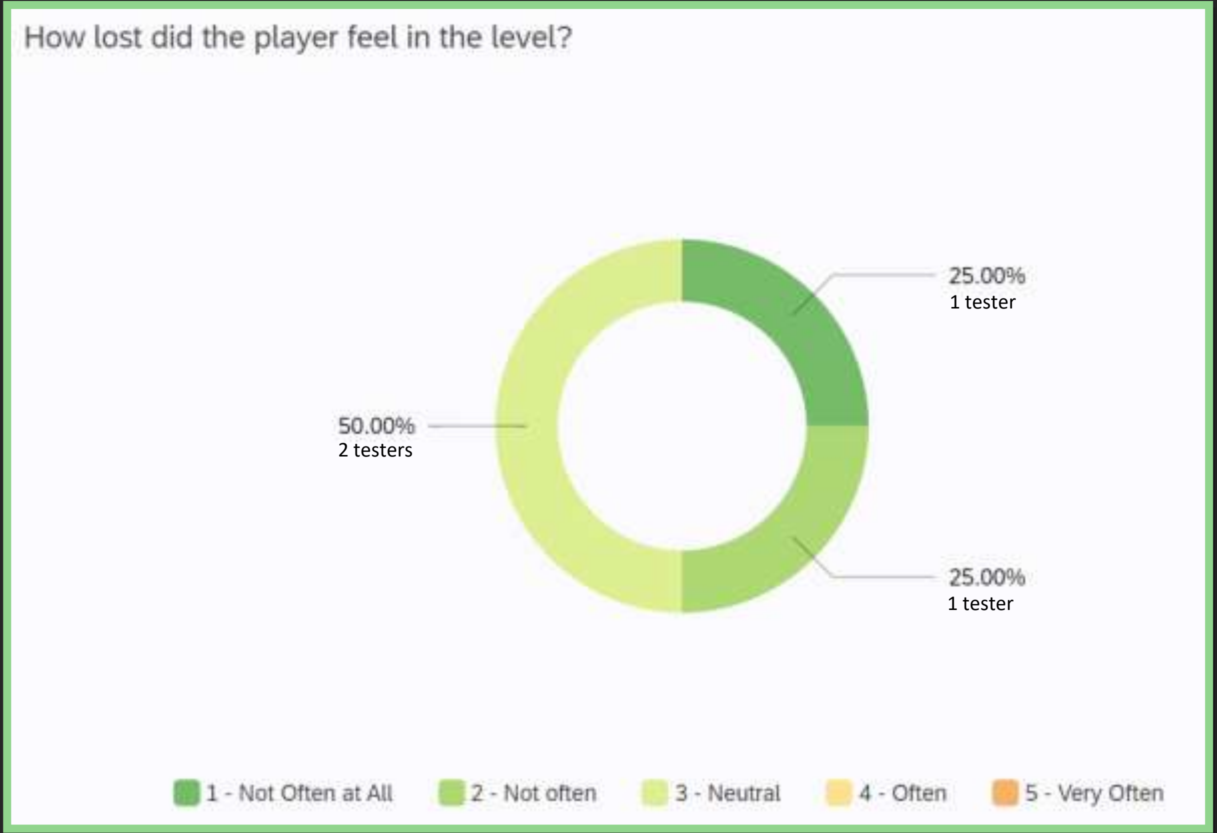
Survey Results

- How lost did the player feel in the level **overall**?



- **76.47% (13 out of 17 players)** didn't feel lost in the level
- **23.53% (4 out of 17 players)** felt lost in the level

Survey Results

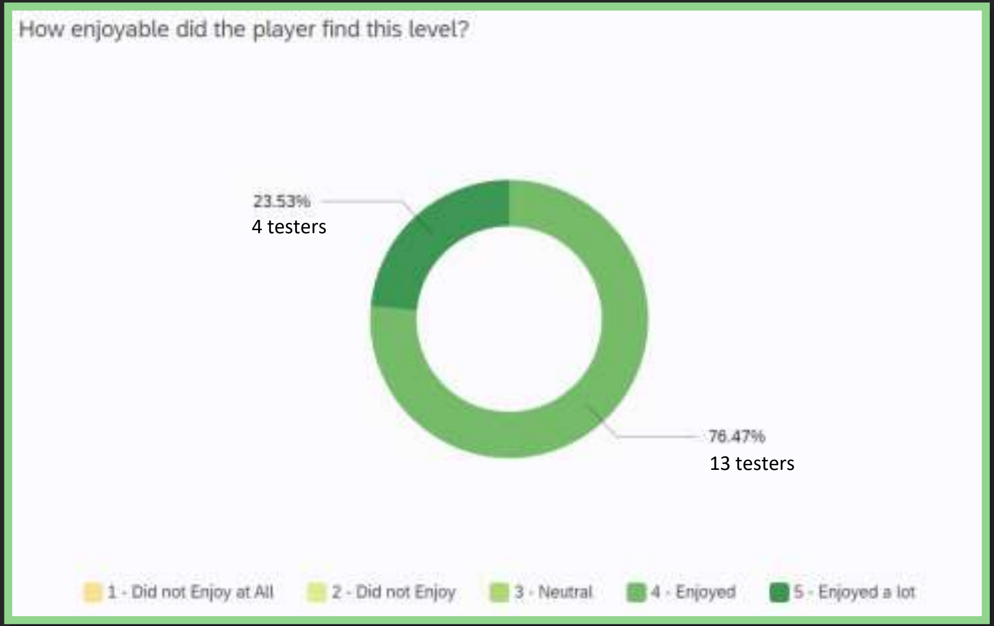


Feeling of being lost for those who had not played *Skyrim* before

- **4 out of 17 players had not played Skyrim before**
- **The players, who had not played *Skyrim* before**, did not feel lost in the level

Survey Results

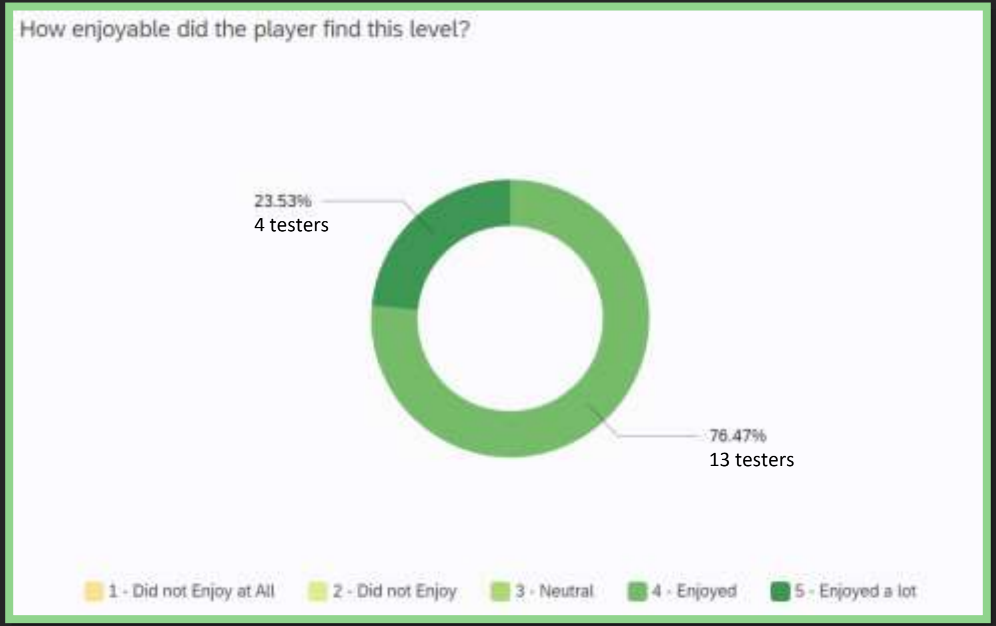
- How enjoyable did the player find this level?



- **All 17** players enjoyed the fun of exploration

Survey Results

- How enjoyable did the player find this level?



- **All 17** players enjoyed the fun of exploration
  - Including players who felt lost in the level

Survey Results

- Quantic Foundry Player Motivation Profile – Pre-test Survey
  - Discovery Type – Acrobat, Gladiator, Bounty Hunter, Architect, and Bard

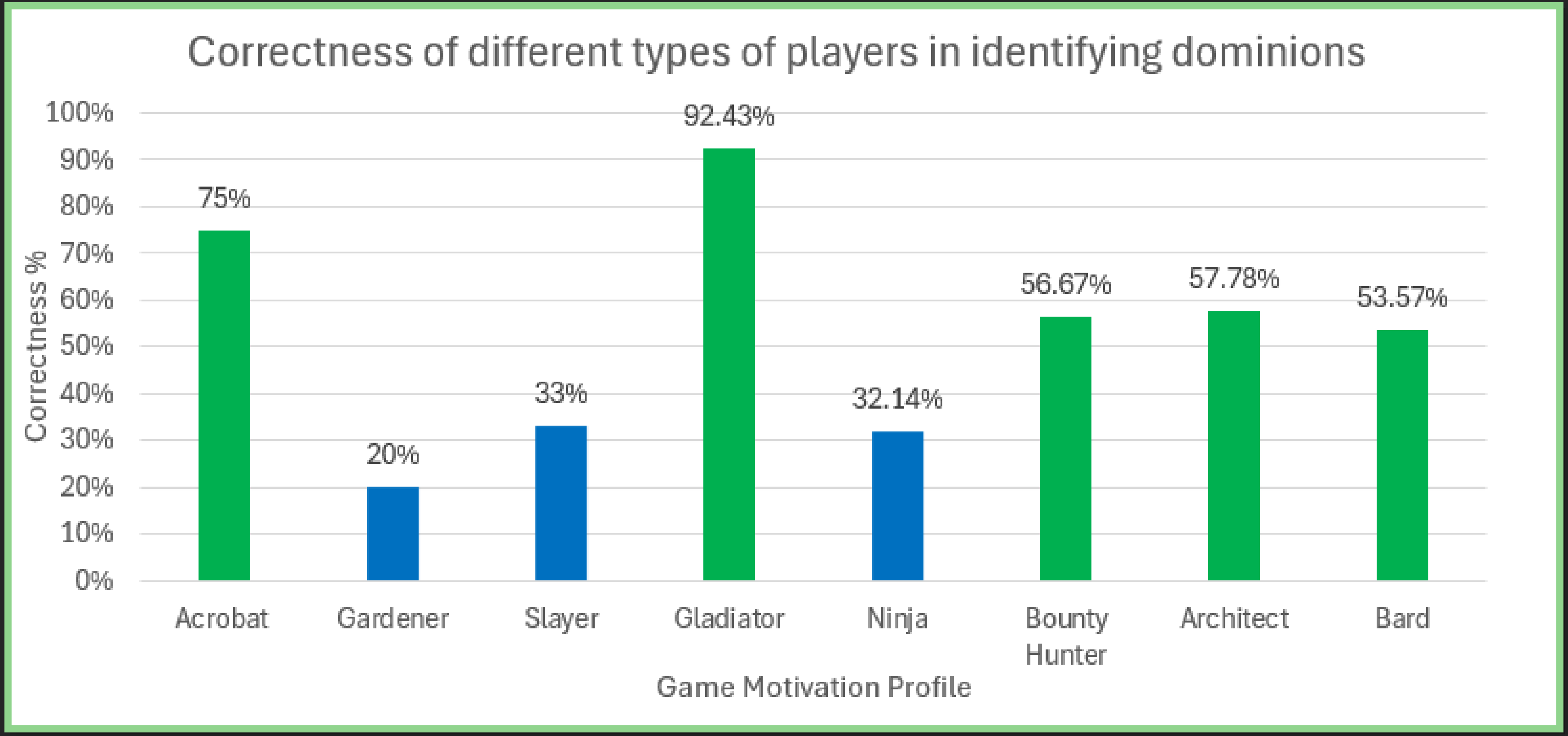
PLAYER SEGMENTS SUMMARY QUANTIC FOUNDRY

	Acrobat	Gardener	Slayer	Skirmisher	Gladiator
Motto	*"Flexing My Reflexes."*	*"Quiet, Relaxing Task Completion."*	*"Cinematic Mayhem With a Purpose."*	*"Jumping Into The Fray of Battle."*	*"Dedicated, hardcore gaming."*
Top Mot.	Challenge + Discovery	Completion	Fantasy + Story + Destruction	Destruction + Competition	Challenge + Completion + Comm.
Pop Games	Spelunky, Celeste, Super Metroid, Tetris	Candy Crush, Solitaire, Animal Crossing	Firewatch, Uncharted, Tomb Raider	Rust, Call of Duty, Battlefield	Mobile Legends, Destiny, Gears of War

	Ninja	Bounty Hunter	Architect	Bard
Motto	*"A Duel of Speed and Skill."*	*"High-Octane Solo World Exploration."*	*"My Empire Begins With This Village."*	*"Playing a Part in a Grand Story."*
Top Mot.	Competition + Challenge	Destruction + Fantasy	Strategy + Completion	Design + Community + Fantasy
Pop Games	Street Fighter, StarCraft, LoL	Mass Effect, Far Cry, Saints Row	Europa Universalis, Civ VI, Banished	The Secret World, FFXIV, LoTRO

Survey Results

- Quantic Foundry Player Motivation Profile – Segments Summary
  - Discovery Type – Acrobat, Gladiator, Bounty Hunter, Architect, and Bard
  - Player Type affects mental mapping ability



6. Conclusions

Conclusion

- **Majority** of players **did not feel lost** in this complicated, non-linear level
    - The methodology cannot **entirely prevent** loss of direction, but it can ensure the **fun of looking for paths** by providing exploration choices
- The ability to mentally map a level's structure is affected by **player types**
    - The methodology **ensures fun** for the player who **can subconsciously memorize** the level spaces **(Discovery Type)**
- **Preliminarily**, the methodology can help a non-linear level to maintain players' engagement **even if they sometimes feel lost**

Lessons Learned

- The height and positioning of landmarks affect player's ability to memorize them
- Ideally, need **control groups** to prove the methodology's effectiveness further
- Include more participants for **each Quantic Foundry Player Motivation Style**
  - Sufficient Samples
- For **future study:**
  - The height and positioning of landmarks
  - Distinctiveness
  - Content fitting different types of players


Q&A


Thanks for listening.

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