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Fatigue Reliability Analysis of a Mono-Tower Platform

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Abstract - In this paper, a fatigue reliability analysis of a Mono-tower platform is presented. The failure mode, fatigue failure in the butt welds, is investigated with two different models. The one with the fatigue strength expressed through SN relations, the other with the fatigue strength expressed through linear-elastic fracture mechanics (LEFM). In determining the cumulative fatigue damage, Palmgren-Miner's rule is applied. Element reliability as well as systems reliability is estimated using first-order reliability methods (FORM). The sensitivity of the systems reliability to various parameters is investigated. The systems reliability index, estimated by using the fatigue elements with the fatigue strength expressed through SN relations, is found to be smaller than the systems reliability index estimated by using LEFM. It is shown that the systems reliability index is very sensitive to variations of the natural period, damping ratio, current, stress spectrum and parameters describing the fatigue strength. Further, soil damping is shown to be significant for the Mono-tower.

Key words: Reliability analysis, first-order reliability methods (FORM), sensitivity analysis, fatigue failure, random loads, offshore, Mono-tower platform.

1 INTRODUCTION

For a Mono-tower platform and other flexible and dynamically sensitive offshore structures, fatigue failure is often found to govern the overall configuration of the structures. However, calculation of fatigue life is subjected to large uncertainty due to uncertainties in the computation of loads, the dynamic response, fatigue strength and damage accumulation. In order to analyse these uncertainties a reliability analysis, which provide the tools for efficient uncertainty analysis, can be used. Reliability methods have been extensively applied in the last decade, where considerable progress has been made in the area of structural reliability theory. Especially, the development of the so- called first-order reliability methods (FORM) and the second- order reliability methods (SORM) have been very important, see e.g. Madsen, Krenk & Lind [1] and Thoft-Christensen & Murotsu [2]. These methods are especially developed to estimate the reliability of structural elements and systems. The reliability methods are also an excellent tool to determine important sources of uncertainty.

In this paper, a fatigue reliability analysis of a Mono-tower platform is performed. The Mono-tower structure, considered, has been described in Petersen, Lyngberg, Eskesen & Larsen [3], where data for the environmental conditions also have been stated.

Originally, the structure had been designed as an attractive solution for a marginal oil and gas field (Rolf field) in the Danish Sector of the North Sea, but the plans for this field were changed to a traditional 4 - legs jacket structure. A short description of the Mono-tower platform is given in section 2. Then, in section 3, first-order reliability methods (FORM) are briefly summarized. Next, in section 4, modelling of two different fatigue failure elements is performed. The one with the fatigue strength expressed through SN relations, the other with the fatigue strength expressed through linear-elastic fracture mechanics (LEFM). In determining the fatigue damage, Palmgren-Miner's rule is applied. In the fatigue models, the structural response is calculated on the basis of a modal spectral analysis, where the structure is modelled as a one-dimensional, lightly damped, linear, continuous single degree of freedom system. Finally, in section 5, results of the reliability analysis are presented. Reliability calculations are performed by first-order reliability methods. The sensitivity of the reliability to various parameters is estimated and important sources of uncertainties are determined. The influence on the fatigue life by neglecting the current and assuming a Rayleigh distribution of the stress amplitudes is also investigated.

The reliability calculations in this paper are performed with the computer program PRADSS (Program for Reliability Analysis and Design of Structural Systems), see Sørensen [4].

2 DESCRIPTION OF MONO-TOWER PLATFORM

The single pile platform, Mono-tower, investigated, throughout this paper, is a remotely operated platform, with provision for four wells, designed for 33.7 m. of water in the Danish part of the North sea. The platform is a single steel cylinder driven into the seabed, supporting a topside facility deck.

The structure consists of three different sections

- A cylindrical section driven 28 m. into the seabed and ranging up to 7 m above mudline. This section has an external diameter of 4.5 m.
- A tapered section from 7 m above mudline to 3 m above still water level (SWL), elevation (el.) 0.
- A cylindrical section from 3 m above SWL up to main deck located at el. +19 m. This section has an external diameter of 2 m.

The wall thickness of the Mono-tower platform is 80 mm; except for a 7 m long, 100 mm thick, section from el. -4 to el.+3.

The topside structure consists of an emergency deck at el. +15.6, a main deck at el. +19.0, a mezzanine deck at el. +21.7 and a helideck at el. +26.0. The total weight of the topside is 200 tonnes including the deck structure and all the equipment necessary for four wells. The total tower is weighting approximately 700 tonnes. The well conductors have been placed inside the pile, while an oil export riser, a ladder, a boat-landing and anodes have been placed outside the pile.

3 RELIABILITY ANALYSIS METHODS.

A reliability analysis is based on a reliability model of the structural system. The elements in the reliability model are failure elements, modelling potential failure modes of the structural system, e.g. fatigue failure of a weld. Each failure element is described by a failure function $g(\overline{x},\overline{p}) = 0$ in terms of a realization $\overline{x}$ of a random vector $\overline{X} = (X_1, X_2, .., X_n)$, and deterministic parameters $\overline{p}$, i.e. deterministic design parameters and parameters describing the stochastic variables, (expected value and standard deviation). $\overline{X}$ is assumed to contain n stochastic variables, e.g. variables describing the loads, strength, geometry, model uncertainty etc. Realizations $\overline{x}$ of $\overline{X}$ where $g(\overline{x},\overline{p}) \leq 0$ correspond to failure states in the n-dimensional basic variable space, while $g(\overline{x},\overline{p}) > 0$ correspond to safe states.

In first-order reliability methods (FORM) a transformation $\overline{T}$ of the generally correlated and non-normally distributed variables $\overline{X}$ into standardized, normally distributed variables $\overline{U} = (U_1, U_2, .., U_n)$ is defined. Let $\overline{U} = \overline{T}^{-1}(\overline{X},\overline{p})$. In the $\overline{u}$-space the reliability index β_i is defined as

$$\beta_i = \min_{g(\overline{T}(\overline{u}),\overline{p})=0} (\overline{u}^T\overline{u})^{\frac{1}{2}} \tag{1}$$

If the whole structural system is modelled, as a series system, by m failure elements, and failure of the system is defined as failure of one failure element, then a generalized systems reliability index β^s of this series system can be estimated from, see e.g. Thoft-Christensen & Murotsu [2]

$$\beta^s = -\Phi^{-1}(1 - \Phi_m(\overline{\beta};\overline{\overline{\rho}})) \tag{2}$$

where $\Phi_m(\cdot)$ is the m-dimensional normal distribution function. $\overline{\beta} = (\beta_1,\beta_2,..,\beta_m)$ is the reliability indices of the m most significant failure elements determined by the FORM analysis. The elements in the correlation coefficient matrix $\overline{\overline{\rho}}$ are determined by the FORM analysis.

Besides the absolute values of the element reliability indices β_i and the systems reliability index β^s , it is often of interest to know the sensitivity of the element reliability indices and the systems index to variations of parameters $\overline{p}$.

The derivatives of β_i and β^s become, see Sørensen [4]

$$\frac{\partial \beta_i}{\partial p_j} = \frac{1}{\beta_i} \sum_{l=1}^{n} u_{il}^{\star} \frac{\partial\{T_l^{-1}(\overline{x}_i^{\star},\overline{p})\}}{\partial p_j} \tag{3}$$

$$\frac{\partial \beta^s}{\partial p_j} \approx \frac{1}{\varphi(\beta^s)} \sum_{i=1}^{s} \Phi_{s-1}(\overline{\beta}_i^a;\overline{\overline{\rho}}_i^a)\varphi(\beta_i)\frac{\partial \beta_i}{\partial p_j} \tag{4}$$

where it is assumed that the s significant failure modes are numbered $1, 2, ..., s$. $\overline{\beta}_i^a$ and $\overline{\overline{\rho}}_i^a$ are the conditional reliability indices and correlation coefficients, respectively, see Sørensen [5]. $\varphi(\cdot)$ is the normal density function.

4. RELIABILITY MODELLING OF MONO-TOWER

In this paper, two different fatigue failure elements are used. The first, fatigue element no.1, with the fatigue strength expressed through SN relations, the second, fatigue element no.2, with the fatigue strength expressed through linear-elastic fracture mechanics (LEFM).

Of the dynamic loads, which produce stress fluctuations and with that fatigue damages in the Mono-tower, only the load due to wave action is taken into account. Contribution of large long period storm waves to fatigue is excluded.

Wind loads are ignored, because for fixed offshore structures, these represent only a contribution of about 5 % to the total environmental loading, Watt [6].

Current loads are also ignored, because the frequencies of current loads are not sufficient to excite the structure. Normally, this assumption is used in a dynamic analysis of an offshore structure. In this paper, the assumption is investigated, as the effect of the current on the fatigue lifetime is approximated in a simple way, see section 5.

Because the Mono-tower platform is flexible, vortex shedding can contribute to the fatigue damage. However, according to Jacobsen, Hansen & Petersen [7] this contribution is not taken into account.

In Petersen, Lyngberg, Eksesen & Larsen [3] it is stated that the Mono-tower will only sustain minor damage in a collision with a supply boat, wherefore this contribution to the damage is neglected.

4.1 Fatigue failure element no. 1

In Kirkegaard, Enevoldsen, Sørensen & Brincker [8], the fatigue element no. 1 has been formulated. In this section, the formulation will only be briefly described.

Failure function

The failure function for the fatigue element no.1 is written

$$g(\overline{x}, \overline{p}) = D_{Fail} - (D_{Driving} + D_{wave}) \tag{5}$$

where D_{Fail} is the value of Palmgren-Miner's sum at failure. $D_{Driving}$ is the damage from the driving of the Mono-tower into the seabed and D_{Wave} is the damage from wave action.

Calculation of damage

The cumulative fatigue damage D_{wave} due to wave action is assumed to be given by the Palmgren-Miner's rule, where experimentally determined SN-curves are used to calculate the fatigue strength. It is assumed that the stress range at a time is double of the stress amplitude. Further, it is assumed that stress variation is a zero-mean narrow-band Gaussian process.

Under these assumptions, the total wave induced fatigue damage D_{wave} is calculated by summing up the mean fatigue damage per stress cycle within one sea state $\overline{D}_i$ over the service lifetime of the structure T_L, which is assumed to be 25 years, and weighting the mean fatigue damage for each sea state according to the long- term sea state probability density function for the significant wave height $f_{H_s}(h_s)$, which is assumed to be well represented by a Weibull density function. The coefficients in the Weibull distribution are estimated from a Wave-scatter diagram for the Danish part of the North sea. This leads to that the failure function (5) can be written

$$g(\overline{x},\overline{p}) = \ln(D_{Fail} - D_{Driving}) + \ln(K) - \ln(T_L) - k\ln(SCF2\sqrt{2}) - \ln(\Gamma(1+\frac{k}{2}))$$

$$-\ln(\int_0^\infty \int_{-\pi}^{\pi} \frac{(\sigma_s(h_s))^k}{T_0(h_s)} f_{H_s}(h_s) f_{\Phi_s}(\varphi_s) dh_s d\varphi_s) - \frac{k}{4}\ln(\frac{t}{22}) \qquad (6)$$

where $\Gamma(\cdot)$ is the gamma function. $\sigma_s(h_s)$ is the standard deviation of the stress response and $T_0(h_s)$ is the zero-upcrossing period of the stress cycles. $f_{\Phi_s}(\varphi_s)$ is the probability density function for the predominant wave direction. k and K are the parameters in the SN-curves to be determined from experimental data. Here, two different SN-curves are chosen by using criteria stated in Lotsberg & Andersson [9]. A so-called C-curve is used in the cone/cylinder transitions and below level -25.7. Otherwise, there is used a F2-curve. The SN-curves, used, have been intended for joints exposed to sea water and cathodic protected. The stress concentration factor SCF is assumed to be 1; expect at the cone/cylinder transitions, where SCF is calculated by a formula stated in API RP 2A [10]. Since the fatigue strength of welded joints decreases with increasing plate thickness t, see Berge [11], equation (6) has been corrected (the last term in (6)) for thicknesses greater than 22 mm, which the basic SN-curves have been related to.

Calculation of structural response

In order to estimate the statistical measures of stress variations, $\sigma_s^2(h_s)$, $T_0(h_s)$, the modal spectral analysis method is applied. It is assumed that the long-term sea state can be accurately modelled as a piecewise zero-mean stationary Gaussian process.

Here, Pierson-Moskowitz sea spectrum is used. The transfer function from water elevation to wave forces on the Mono- tower is calculated by using linear Airy wave theory and Morison's equation, where the non-linear drag term is linearized by the "minimun square error method". Hydrodynamic coefficients for the combined tube and riser have been estimated in Jacobsen,Hansen & Petersen [7]. In order to take diffraction into account, the basic value for the inertia coefficient C_M is changed as function of the wave length.

Because the modal spectral analysis is a very time-consuming process, the total computing time required for the reliability analysis tends to be long. To reduce the computing time only first mode is taken into account. This is assumed to be reasonable, as the structure is assumed lightly damped and the second lowest natural frequency is not coinciding with the peak of the sea-state spectrum. Further, the two lowest natural frequencies, $f_1 = 0.49$ Hz., $f_2 = 2.19$ Hz, have been well-separated. The natural frequencies have been estimated by modelling the structure, including the soil, in a finite element program. The eigen-value analysis, which shall in principle have to be performed for each calculation of the failure function $g(\overline{x}, \overline{p})$ is not included in the reliability analysis. Here, the estimated eigen-mode shape vector is used for all the calculations of the failure function, i.e. the variation in the eigen mode shape to variation in the safety problems parameters is disregarded. On the other hand, the variation in the natural frequency to variation in the mass of the structure is taken into account by using Rayleigh's quotient to calculate an equivalent stiffness $Equi$ for the Mono-tower.

Stochastic variables

In table 1, the statistical characteristics of the basic variables are fully enumerated. Further, there is shown the deterministic design parameters, which are investigated in a sensitivity analysis. In this paper, statistical characteristics of the basic variables for both the fatigue failure elements are mainly from published information . In Enevoldsen & Kirkegaard [12], the stipulation of the statistical characteristics for fatigue element no. 1 has been discussed in details. The SI units system is used.

Variable	Designation	Distribution	Expected value	Coeff. of var.
C_D	Drag coefficient	N	1.0*	0.2
C_M	Inertia coefficient	N	1.0*	0.2
TM	Mass of topside	N	200000	0.1
t	Wall thickness	N	1.0*	0.05
SCF	Stress concent. factor	N	1.0*	0.1
B	Parameters in long-term	N	2.35	0.1
C	distribution of H_s	N	1.89	0.1
$Equi$	Equivalent stiffness	N	1.0*	0.1
m_1	Thickness correction	LN	1.0*	0.1
λ	Coeff. for added mass	N	0.9	0.1
$D_{Driving}$	Damage from "driving"	LN	1.0*	0.15
ζ	Damping ratio	LN	0.015	0.5
K	Constant in SN-curve	LN	1.0*	0.65
D_{fail}	Damage at failure	LN	1.0	0.3
Z_1	Model uncertainty	N	1.0	0.2
d	Tube diameter	D	1.0*	
d_1	Marine growth	D	1.0*	
G	Acceleration of gravity	D	9.82	
ρ_w	Density of sea water	D	1025	
h	Water depth	D	33.7	

Table 1: Statistical characteristics (EX1 : Extreme type 1, N : Normal, LN : Lognormal, D : Deterministic.)

Expected values represented by 1.0* indicate that the expected value varies along the structure. In the reliability calculations, the expected value 1.0* is multiplied with the real expected value of the stochastic variable at the given level.

The expected value of TM includes permanent loads and not live loads. $m_1 = \frac{k}{4}$ model the uncertainty with the plate thickness reduction factor. In order to take into account the uncertainty of the stiffness of the soil and structure, respectively, the equivalent stiffness $Equi$ is modelled stochastic. A direct stochastic modelling of the stiffnesses is not possible, as the eigenvalue analysis has been excluded from the reliability calculations. Uncertainties in the calculation of added mass, due to surrounding water, is modelled by λ. Uncertainties of the different contributions to the damping of the structure are taken into account by modelling the modal damping ratio as a stochastic variable. It is assumed that the damping of a Mono-tower consists of structural damping, viscous hydrodynamic damping, radiation damping and soil damping. Further, it is seen in table 1 that only K in the SN relation is modelled as a stochastic variable. It is proposed by Wirsching [13], where statistical characteristics of K are stated, too. D_{fail} is a model uncertain variable, which models the uncertainty connected by Palmgren- Miner's rule. The other model uncertainty variable Z_1 models the uncertainties connected by the models, which are used to calculate the variance and the zero-upcrossing period of the stress process. The statistical characteristics of this stochastic variable have been chosen according to Wirsching [13]. C_D and C_M are assumed to be mutually correlated with the correlation coefficient ρ = -0.9. All the other stochastic variables are assumed to be independent.

4.2 Fatigue failure element no.2

In this section, a failure function based on fracture mechanics and Palmgren-Miner's rule is established for the butt welds.

Failure function

The failure function for fatigue element no. 2 is written as (5). The difference between the failure function for fatigue element no. 1 and no. 2 is due to the calculation of the damage from wave action D_{Wave}.

Calculation of Damage

The cumulative damage due to wave action D_{wave} is calculated, using Palmgren-Miner's rule. The formulation of the fatigue strength is based on a linear-elastic fracture mechanics (LEFM) approach. LEFM is applicable for brittle materials, when the plastic zone of the crack tip is small compared to the crack size and other dimensions of the component. In general, this is the case, if the fracture occurs at stresses, which are considerably lower than the yield stress and at plane strain conditions. Here, this is supposed to be satisfied.

It is adopted that the surface crack grows in a semi-elliptical shape. It is not practical to model, in an analysis, the arbitrary crack front shape as it is. An idealization of an arbitrary surface crack front by an elliptical curve is recommended , see e.g. Engesvik [14].

Using the linear-elastic fracture mechanics, the stress cycles with constant stress range required for propagation of a crack from the initial size to the final size can be calculated using a relation proposed by Paris & Erdogan [15]. It is assumed that the stress intensity factor at the surface K_c and the deepest point K_a will determine the shape of the growing semi-elliptical crack front following the Paris-Erdogan crack growth law in direction of the two semi-axes

$$\frac{da}{dN} = C_a(\Delta K_a)^m \quad , \quad \Delta K_a = S_i F(a,c)\sqrt{\pi a} > 0 \tag{7}$$

$$\frac{dc}{dN} = C_c(\Delta K_c)^m \quad , \quad \Delta K_c = S_i F(a,c)\sqrt{\pi c} > 0 \tag{8}$$

where a and $2c$ are the crack depth and length at the surface of the semi-elliptical crack, respectively. N is the number of stress cycles. C_a, C_c, and m are material parameters. ΔK_a and ΔK_c are the ranges of the stress intensity factors, where S_i is the far-field stress range and $F(a,c)$ is the geometry function. The geometry function depends on the over-all geometry including the geometry of the crack and the geometry of a possible weld. The stress intensity factors are computed by linear-elastic fracture mechanics theory, see e.g. Engesvik [14]. Here, the crack growth equations (7,8) are used without a non-zero lower threshold on ΔK_a and ΔK_c below which no crack growth occurs. The exponent m is assumed to be the same for the crack growth in the depth and surface direction. Based on fatigue tests Newman & Raju [16] have suggested $C_c = (0.9)^m C_a$. The relation indicates that the crack growth at the surface tends to be slower, which is probably caused by the larger size of the plastic zone at the free surface.

In order to calculate the crack extension using equations (7,8), the stress intensity factor ranges for both directions are estimated with expressions proposed by Newman & Raju [16]. Their solution has been critically evaluated in Hosseini & Mahmoud [17], where it is concluded that the solution appears to be in good agreement with experimental data and to be useful in predicting fatigue life of surface cracks under cyclic tension loading.

The surface crack is supposed to be loaded in axial tension perpendicular to the crack plane and constant over the thickness of the tube wall. Further, it is assumed that compressive stresses do not contribute to crack propagation. However, the welded joints along the Mono-tower structure are assumed to contain residual stresses, and the whole stress range is therefore applied. In general, local residual stresses will be relaxed by cyclic loading, if the total stress, (applied plus residual), exceeds the yield stress. This is leaving the stress range philosophy over-conservative.

It is assumed that the fatigue life consists of stable crack propagation under the cyclic loading. In general, one might subdivide the fatigue life in a crack initiation period and a crack growth period, ending with fatigue failure. However, the crack initiation period in welded joints, which are not stress relieved, usually occupies a small part of the total fatigue life, and therefore is neglected.

Based on the various aspects discussed above, the stress cycles required for propagation of a surface crack from initial size to final size can be calculated. If the fatigue failure is assumed, when the crack depth a becomes greater than a critical crack depth a_c, the total cumulative fatigue damage due to wave action is estimated by

$$D_{wave} = \sum_{i=1}^{q} \frac{n(S_i)}{N(S_i)} = \frac{\sum_{i=1}^{q} S_i^m n(S_i)}{\int_{a_o}^{a_c} \frac{dx}{C_a(F(a,c)\sqrt{\pi x})^m}} \tag{9}$$

where $n(S_i)$ is the number of stress cycles of stress range S_i in the stress history and $N(S_i)$ is the number of stress cycles of stress range S_i necessary to cause failure. The summation is over all stress ranges q. The denominator in (9) is calculated by numerical integration of the coupled differential equations (7,8). Non-stress interaction model is adopted, which means that crack retardation and acceleration effects are completely ignored. However, for structures under environmental conditions the sequence effect of the load cycles does not cause acceleration or retardation of crack growth in general, see Yao, Kozin, Wen, Yang, Schuëller & Ditlevsen [18].

The critical crack depth a_c is expressed according to a CTOD-design curve, as a_c is calculated from c_{max} by assuming that cracks of equal severity have the same stress intensity factors for uniform tensile loading, see Slatcher & Lereim [19].

$$c_{max} = \frac{\delta_{crit}E}{2\pi(\frac{\sigma}{\sigma_{y,w}})^2\sigma_{y,w}} \qquad for \qquad \frac{\sigma}{\sigma_{y,w}} \leq 0.5 \tag{10}$$

$$c_{max} = \frac{\delta_{crit}E}{2\pi(\sigma - 0.25\sigma_{y,w})} \qquad for \qquad \frac{\sigma}{\sigma_{y,w}} \geq 0.5 \tag{11}$$

c_{max} is the half-length of a through-thickness crack, where E is modulus of elasticity, δ_{crit} is the critical CTOD value at failure and $\sigma_{y,w}$ is the yield stress, (in weld material). The local stress σ is calculated by

$$\sigma = SCF\sigma_n + \sigma_{res} \tag{12}$$

where σ_n and σ_{res} are the maximal nominal stress and the residual stress, respectively.

It is taken into account that the surface crack can grow through the tube wall thickness and thereafter propagate as a through thickness crack, before it becomes critical.

Calculation of the stress range distribution

In order to calculate the total cumulative fatigue damage due to wave action, the long-term distribution of the stress ranges is established. It is done by using the modal spectral analysis method, see section 4.1, to calculate the variance of the stress response. The stress range S_i corresponding to $n_i(S_i)$ stress cycles is then calculated for all sea-states by using a Rayleigh distribution of the stress amplitudes and assuming that the stress range at a time is double of the stress amplitude.

Stochastic variables

Stochastic variables for failure element no.2 are shown in table 2. Besides that, all the stochastic variables and deterministic design parameters for failure element no. 1 are also taken into account; except the stochastic variables K and m_1.

Variable	Designation	Distribution	Expected value	Coeff. of var.
δ_{crit}	CTOD value at failure	LN	0.19	0.32
Ψ	Residual stress factor	N	0.9	0.1
σ_y	Tube yield stress	LN	316000000	0.1
$\sigma_{y,w}$	Weld yield stress	LN	347000000	0.1
F	Geometry function	N	1.0*	0.1
a_o	Initial crack size	EXP	0.0005	
c_o	Initial crack size	EXP	0.004	
$Ln\, C_1$	Crack growth parameter	N	-31.0	0.016
m	Crack growth parameter	N	3.5	0.09
U	Random variable	N	0.	
H	Wave height	EX1	17.1	0.08
Z_1	Model uncertainty	N	1.0	0.2
Z_2	Model uncertainty	N	2.0	0.2
Z_3	Model uncertainty	N	1.0	0.2
Z_4	Model uncertainty	N	1.0	0.2

Table 2: Statistical characteristics (EX1 : Extreme type 1, N : Normal, LN : Lognormal, D : Deterministic, EXP : Exponential.)

In table 2, $ln\, C_1$ presupposes the length calculated in mm. The SI units system is used for all the others variables. Ψ models the residual stress, due to the welding procedure, as a part of base material yield stress. For welds that have not been stress relieved, it is common practice to take $\sigma_{res} = \Psi \sigma_y$ as 80-100 % of the yield stress of the base material, see e.g. Slatcher & Lereim [19]. The yield stress of the weld material $\sigma_{y,w}$ is assumed to be 10 % larger than the yield stress of the base material. The initial size of the surface crack is modelled as a stochastic variable with an initial depth at 0.5 mm, and a length 5-10 times the depth, which is normally assumed, as no other information is available. The parameters in (9) C_a, m are modelled stochastic according to Tanaka, Ichikawa & Akita [20], where it is shown that ln C_a and m are jointly normally distributed random variables, with a strong negative correlation. According to Madsen,[21] and Ortiz & Kiremidjian [22], (9) can be written, when C_a is randomized

$$D_{wave} = \frac{\sum_{i=1}^{q} S_i^m n(S_i) C_1}{\int_{a_o}^{a_c} \frac{C_2(a)dx}{(F(a,c)\sqrt{\pi x})^m}} \qquad C_a(a) = \frac{C_1}{C_2(a)} \tag{13}$$

where C_1 is a random variable describing the variation in the parameter C_a from specimen to specimen. Conferring Ortiz & Kiremidjian [22], $C_2(a)$ is modelled as a stationary random log-normally process, which is describing the variations from the mean value along the crack path within each specimen.

The mean value of $C_2(a)$ is taken as one and the process is assumed homogeneous. According to Madsen [21], the denominator in (13), damage function, is a conditioned random variable $\chi(a)|(a_o, c_o, F, m)$, which is a sum of many independent random variables of approximately the same variance. Therefore, the distribution is well approximated by a normal distribution and $\chi(a)|(a_o, c_o, F, m)$ is expressed as

$$\chi(a)|(a_o, c_o, F, m) = \int_{a_o}^{a_c} \frac{dx}{(F(a,c)\sqrt{\pi x})^m} + U\sqrt{r_{C_2}\sigma_{C_2}^2 \int_{a_o}^{a_c} \frac{dx}{F(a,c)^{2m}(\pi x)^m}} \quad (14)$$

where U is a standard normally distributed stochastic variable. r_{C_2}, $\sigma_{C_2(a)}^2$ are the correlation radius and the variance, respectively, for the stochastic process $C_2(a)$. These statistical characteristics have been estimated in Ortiz & Kiremidjian [22] for the data obtained of Virkler, Hillberry & Goel [23]. Here, these estimated statistical characteristics are used, in spite of the fact that the values are for crack growth in base material in an aluminium alloy.

H is modelled as a stochastic variable, as it is used to calculate the nominal stress σ_n in (12). It is assumed that the nominal stress is due to the extreme load, described in Kirkegaard, Enevoldsen, Sørensen & Brincker [8]. The model uncertainty variable Z_1 takes into account the uncertainty in the determination of the statistical characteristics of the initial crack size. Z_2 models the uncertainty, by the determination of δ_{crit}, due to the difference between measured fracture strength and the real fracture strength, respectively. Z_3 and Z_4 model the uncertainties by the calculation of the nominal stress σ_n and the stress range distribution, respectively. C_D, C_M and $\ln C_1$, m are calculated mutually correlated with the correlation coefficient ρ = -0.9, respectively. All the others stochastic variables are assumed to be independent.

5. RESULTS

For both of the fatigue failure elements, formulated in section 4, element as well as the systems reliabilities of the Mono-tower platform is estimated.

5.1 Reliability calculated by using the fatigue failure element no. 1

The Mono-tower platform is modelled as a series system with eighteen fatigue failure elements no.1, between level -33.7 and +15, see figure 1. Each element is assumed to model the damage at that point in the butt weld, where the greatest fatigue damage will occur. Between the failure elements, the stochastic variable K is assumed to be correlated with the correlation coefficient $\rho = 0.5$. The same assumption is also made for D_{fail}. All the others stochastic variables are separately assumed fully correlated between the failure elements. The variation of the element reliability index β_i along the structure is shown in figure 1.

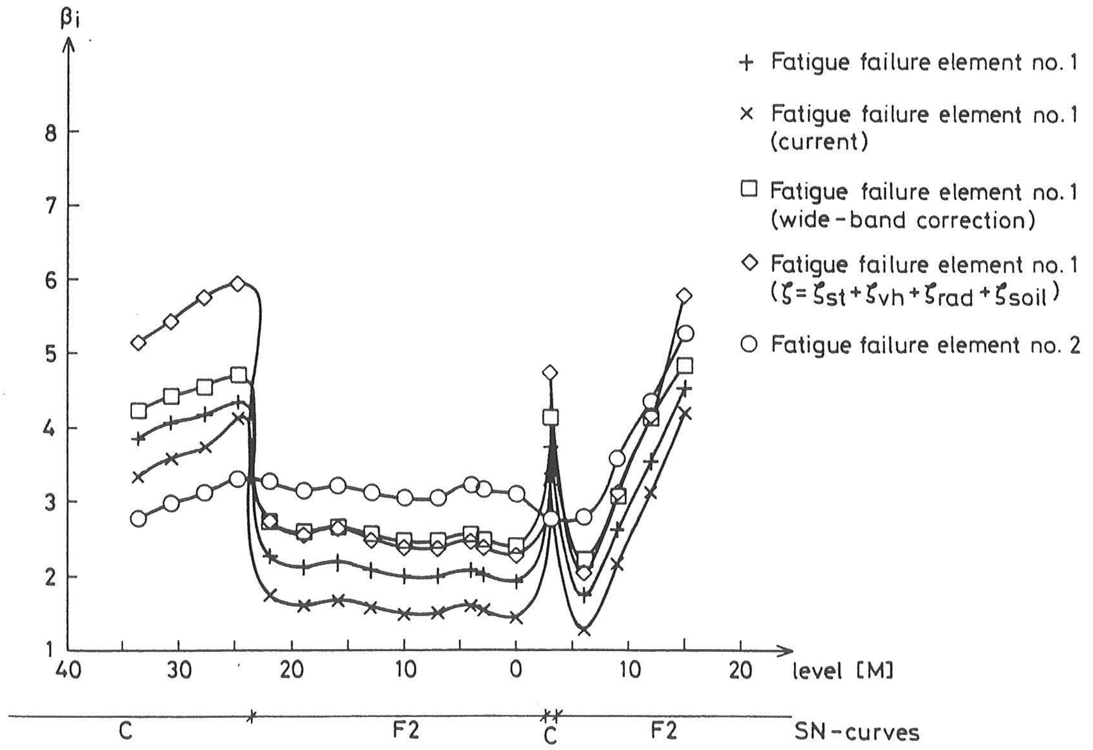

Figure 1: The variation of the element reliability index β_i along the Mono-tower platform.(Notice, the influence of the stipulation of SN-curves on β_i.)

To estimate the series systems reliability in (2) a number of methods can be used, see e.g. Thoft-Christensen & Murotsu [2].

Using the Hohenbichler approximation, the systems reliability index becomes $\beta^s = 1.432$

It is seen from figure 1 that the element reliability index, calculated by using fatigue element no.1, is very sensitive to different SN-curves, as the reliability index is significantly changed, when the SN-curve is changed. This is also seen from the results of the sensitivity analysis, see figure 2.

Figure 2 shows the sensitivity of the systems reliability index β^s to variations of the expected values of the stochastic variables $\frac{\partial \beta^s}{\partial \mu_j}$ and standard deviations $\frac{\partial \beta^s}{\partial \sigma_j}$. The sensitivities can be relatively compared, as each derivative is multiplied with a hundredth parameter.The sensitivity of the systems reliability to variations in deterministic parameter is estimated by modelling the deterministic parameters as fixed stochastic variables. In table 1, the designation of the stochastic variables has been stated.

Figure 2 shows that many of the stochastic variables contribute to the overall uncertainty. Especially, K, C_M, SCF, ζ, D_{fail} and Z_1 contribute to the uncertainty. The systems reliability is also seen to be very sensitive to variations of the deterministic design parameters, except the marine growth. As $Equi$ and TM do not turn out to be very important, it is seen in this example that the exclusion of the eigen-value analysis form the reliability calculations do not influence the results significantly.

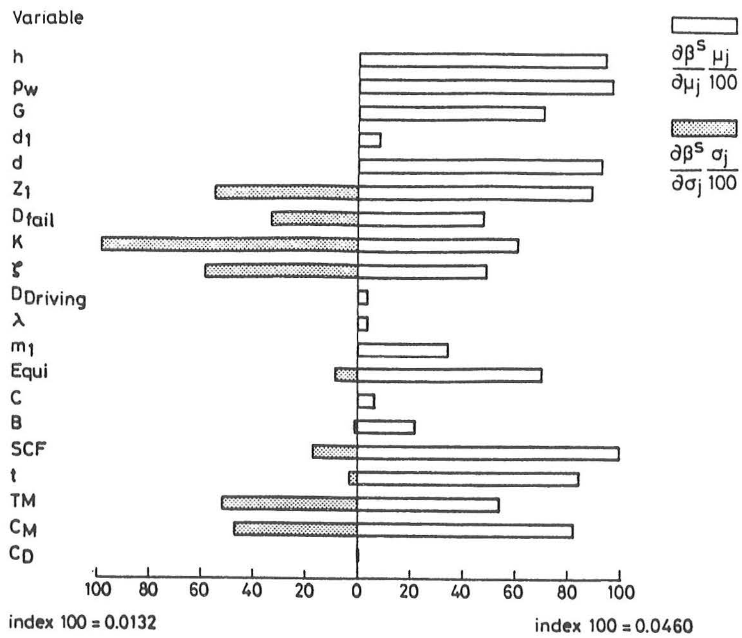

Figure 2: Sensitivity of the systems reliability to variations of the parameters of the stochastic variables, fatigue element no. 1

The sensitivity of the systems reliability to variations of the modal damping ratio and natural period is shown in figure 3.

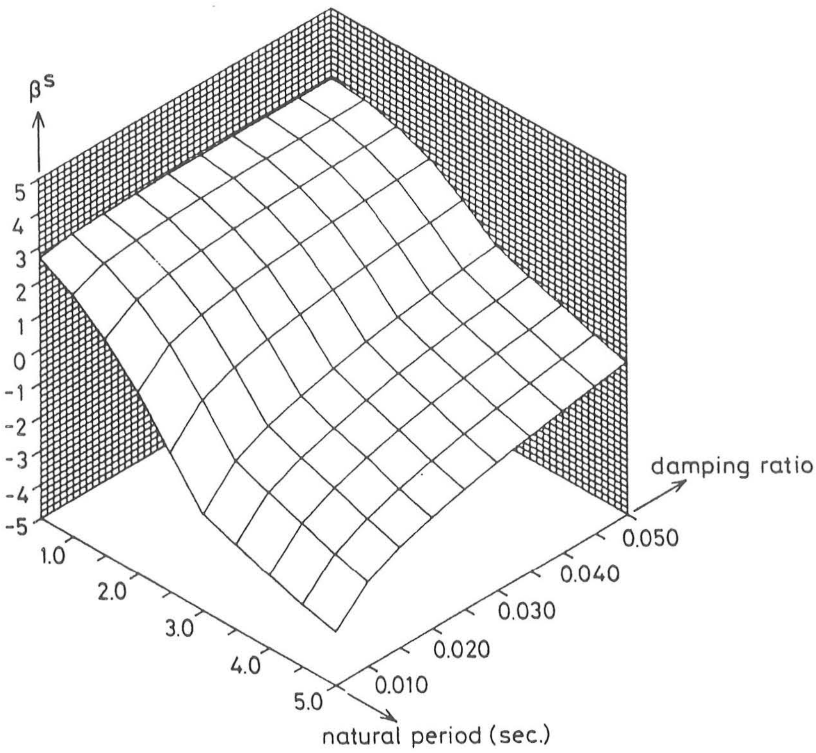

Figure 3: Sensitivity of the systems reliability index to variations of natural period and modal damping ratio

Figure 3 proclaims that the systems reliability for a Mono-tower platform with natural period greater than 1.5 sec. is very sensitive to variations of the damping ratio. Especially for damping ratios less than 0.03-0.04.

In order to investigate the modal damping ratio of the 1st mode, closer, it is assumed that ζ can be written as a sum of structural damping ζ_{st}, soil damping ζ_{soil}, radiation damping ζ_{rad} and viscous hydrodynamic damping ζ_{vh}. These variables are modelled stochastic with the statistical characteristics shown in table 3.

Variable	Designation	Distribution	Expected value	Coeff. of var.
ζ_{st}	Structural damping	LN	0.0024	0.1
ζ_{soil}	Soil damping	LN	0.006	0.4
ζ_{rad}	Radiation damping	LN	1.0*	0.3
ζ_{vh}	Viscous damping	LN	1.0*	0.2

Table 3: Statistical characteristics (LN : Lognormal)

Expected values represented by 1.0* indicate that the expected value varies along the structure. In the reliability calculations, the expected value 1.0* is multiplied with the real expected value of the stochastic variable at the given level.

The expected values of ζ_{rad} and ζ_{vh} are estimated by analytic expressions, stated in Cook [24]. The expected values for ζ_{st} and ζ_{soil} have been chosen according to Cook [24]. The coefficients of variation of the four damping variables have been adopted on an intuitive base. Besides these variables, only stochastic variables, which turned out to be important in the sensitivity analysis above, are taken into account, i.e K, SCF, TM, $Equi$, Z_1, t and D_{fail}. Using these stochastic variables, the reliability index variation along the structure becomes, see figure 1.

Using the Hohenbichler approximation the systems reliability index becomes $\beta^s = 1.741$.

In figure 4, the sensitivity of the systems reliability index to variations of parameters of the four damping variables is shown.

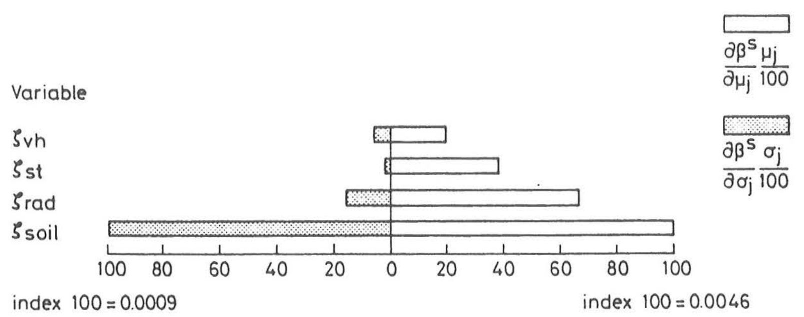

Figure 4: Sensitivity of the systems reliability index to variations of parameters of the four damping variables.
(Notice that it is only the four damping variables which are relatively compared)

Using this modelling of the damping, it is seen that the largest contribution to the overall uncertainty is from ζ_{soil}, but the expected values of ζ_{rad} and ζ_{st} turn also out to be important.

The influence of current, which has been neglected in the formulation of fatigue failure element no. 1, is investigated by taking the current into account as proposed in Peters & Boonstra [25], for a Mono-tower, Europlatform, placed in the south of the North sea. It is assumed that the current velocity is 0.8 m/s, and about 6 per cent of the fatigue lifetime, waves and current are acting in same direction. In the remaining 94 per cent of the lifetime, it is assumed that the waves have been unaffected of the current. When the current and waves are acting in same direction, the frequency of encounter of the waves is changed (decrease) due to the current. Calculating the reliability by fatigue element no.1 by taking into account the current and modelling the current velocity stochastic gives the element reliability index variation shown in figure 1.

Using the Hohenbichler approximation, the systems reliability index becomes $\beta^s =$ 0.926.

It is thus seen that the reliability is increased considerably, if the current is neglected. This is also concluded in Peters & Boonstra [25].

The fatigue element no.1 has been formulated by assuming a narrow-band spectrum of the stress process. However, the spectrum is not narrow-band. According to Wirsching [13], the damage from the wide-band stress process can be estimated by $D = \lambda(\epsilon, k)D_{nb}$, where λ is a correction factor for D_{nb}, which is the damage computed assuming an "equivalent narrow-band stress process". ϵ is the spectral width. Using this correction in fatigue element no.1, the element reliability index variation becomes, see figure 1.

Using the Hohenbichler approximation, the systems reliability index becomes $\beta^s =$ 1.943.

This systems reliability index is seen to be larger than the systems reliability index calculated by assuming a narrow-banded stress spectrum, i.e it is conservative to calculate the reliability by assuming a narrow-banded stress spectrum.

5.2 Reliability calculated by using fatigue element no. 2

The Mono-tower platform is modelled as a series system with eighteen fatigue failure elements no.2, between level -33.7 and +15, see figure 1. Each element is assumed to model the crack propagation at that point in a butt weld, where the greatest fatigue damage will occur. Between the failure elements, the stochastic variable D_{fail} is assumed to be correlated with the correlation coefficient ρ= 0.5. Z_1 are assumed to be independent between the failure elements, which is also assumed for Z_2, a_0 and c_o. All the others stochastic variables are separately assumed fully correlated between the failure elements. ln C_1 and m, respectively, are calculated fully correlated between the failure elements. In principle, this is not correct; partial correlation should have to be adopted instead. However, such an approach require an increase in the number of stochastic variables and thus add considerably to the computing time involved.

The variation of the element reliability index along the structure is shown in figure 1.

Using the Hohenbichler approximation, the systems reliability index becomes $\beta^s = 2.388$.

Figure 5 shows the sensitivity of the systems reliability index β^s to variations of the expected values of the stochastic variables. In table 2, the designation of the stochastic variables has been stated.

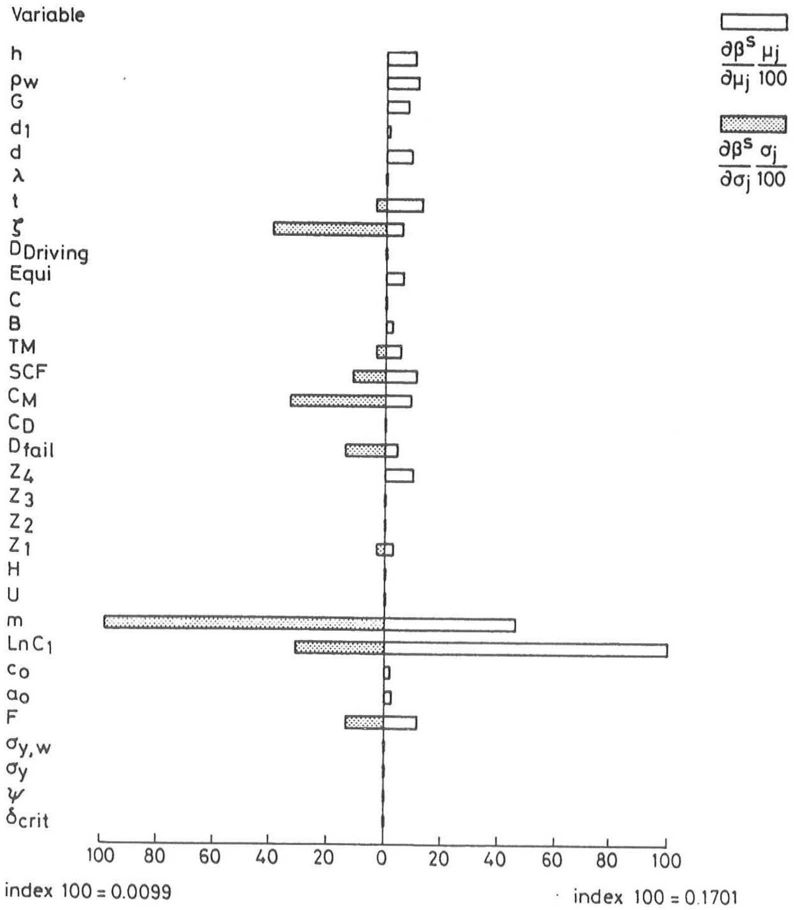

Figure 5: Sensitivity of the systems reliability to variations of the parameters of the stochastic variables, fatigue element no. 2

It is seen that it is only few variables $ln\, C_1$, m, ζ, C_M and SCF, which turn out to be important. ζ, SCF, C_M and the parameter/parameters describing the fatigue strength were also found important by the sensitivity analysis using fatigue element no.1. The sensitivity analysis shows that the deterministic design parameters and the model uncertainties do not turn out to be very important.

6 CONCLUSIONS

Based on the reliability analysis of the Mono-tower platform the following conclusions can be stated

1) It has been shown, how reliability methods can be used in an uncertainty analysis to calculate a nominal element reliability level as well as a systems reliability level. It has also been shown, how the reliability methods can be used to estimate the sensitivity of the reliability in order to identify the most important uncertainties, thereby pointing at problems for closer investigations.

2) The systems reliability index has been estimated to $\beta^s = 1.432$ by fatigue elements with the fatigue strength formulated by SN-curve, and $\beta^s = 2.388$ by the fatigue elements with the fatigue strength formulated by LEFM.

3) A sensitivity analysis with respect to the systems reliability index, calculated by using the two formulated fatigue elements, showed that the largest contributions to the overall uncertainty are due to the damping ratio, the inertia coefficient, the stress concentration factor and parameters describing the fatigue strength.

4) For a Mono-tower platform, the systems reliability index has been shown to be very sensitive to variations of the natural period and the damping ratio.

5) In order to investigate closer the modal damping ratio ζ of the 1st mode, ζ was modelled as a sum of structural damping ζ_{st}, soil damping ζ_{soil}, radiation damping ζ_{rad} and viscous hydrodynamic damping ζ_{vh}. The given modelling of the damping showed that the largest contribution to the overall uncertainty was from ζ_{soil}, but the expected values of ζ_{rad} and ζ_{st} also turned out to be important.

6) It has been shown that the systems reliability is increased considerably, if the current is neglected by a fatigue analysis of a Mono-tower platform. Further, the systems reliability index, calculated by assuming a narrow-banded stress spectrum, has been shown to be conservative.

ACKNOWLEDGEMENTS

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