Aalborg Universitet

Development of Pore Pressure and Material Damping during Cyclic Loading

Ibsen, Lars Bo

Published in:
Proceedings of "Dynamics of Structures"

Publication date:
1994

Document Version
Accepted author manuscript, peer reviewed version

Development of pore pressure and material damping during cyclic loading

L.B. Ibsen, Aalborg University, Denmark

ABSTRACT: The behaviour of sand during cyclic loading can be characterized as "stabilization", "instant stabilization", "pore pressure buildup" and "liquefaction". The terminologies can be defined exactly by a simple mathematical formulation based on the existence of a cyclic stable state. By introducing a mobilization index M it is possible to describe the strongly hysteretic behaviour during loading and unloading, even if the stress path is complicated.

INTRODUCTION

In the last thirty years a great number of test series with cyclic loading of sand have been performed, many phenomena have been described and important theories have been presented. Today the main problem is to gather all relevant information in a consistent mathematical formulation.

Laboratory testing gives a possibility to study soil behaviour in details. However, it is widely recognized that it is not possible to use laboratory test results for practical purposes without calibrating them against field tests and field observations.

For instance, the preparation of a sand specimen and the reconstruction of stress history and seismic history have a major effect on the cyclic behaviour. In most natural deposits, soil elements are subjected to shear stresses corresponding to the "earth pressure of rest" situation. In earth structures close to natural slopes or beneath foundations, soil elements are subjected to even larger shear stresses. The stress history is reconstructed by anisotropic consolidation before cyclic testing.

An old sand deposit in an earthquake region has been vibrated many times in its lifetime and the specimen should therefore be prepared by a vibration technique at a level corresponding to the seismic history.

The success of laboratory testing depends on the extent to which the in-situ characteristics are reestablished.

The purpose of this paper is therefore limited to describe in mathematical formulations the phenomena involved in soil response on cyclic loading, and to give a definition of the terminologies, which are already accepted, but not clearly defined. It combines the two different assumptions

i) Alternating loads build up pore pressure, and liquefaction will develop if the amplitude or the number of cycles are big enough (initiated by Seed and Lee in 1966).

ii) The initial effective stress state and the relative density of the soil play a definitive role for the behaviour of a soil. If the initial shear stress exceeds a certain value the pore pressure will be reduced by cyclic loading and the soil will stabilize (Casagrande 1976, Castro and Poulus 1977, Loung 1980).

The paper is based on triaxial tests on a uniform sand called Lund no 0. The mean diameter $d_{50} = 0.4\ mm$, the coefficient of uniformity $U = 1.7$, the initial void ratio 0.62 corresponding to a density index $I_D = 0.7$. The test specimens were prepared by a pluvial technique and carefully saturated in vacuum. A test consists of an anisotropic consolidation phase followed by cyclic loading at constant volume.

STATIC BEHAVIOUR OF DENSE SAND

The parameters, which describe the state of a soil under axisymmetrical stress conditions, are

$$\begin{aligned} &\text{the mean normal stress} & p' &= \frac{1}{3}(\sigma_1' + 2\sigma_3') \\ &\text{the deviator stress} & q' &= (\sigma_1' - \sigma_3') \\ &\text{the volumetric strain} & \varepsilon_v &= \varepsilon_1 + 2\varepsilon_3 \\ &\text{the distorsion} & \varepsilon_q &= \frac{2}{3}(\varepsilon_1 - \varepsilon_3) \end{aligned} \tag{1}$$

where σ_1 is the vertical and σ_3 the horizontal pressure.

The strength of a soil is normally described by the Mohr-Coulomb's failure criterion. "Failure" is defined as a state where q is maximum, and corresponds normally to a distorsion $\varepsilon_q = 5 - 10\%$. The strength parameters c' and φ' are assumed to depend on the void ratio only. In Figure 1 is shown the failure line corresponding to $e = 0.62$.

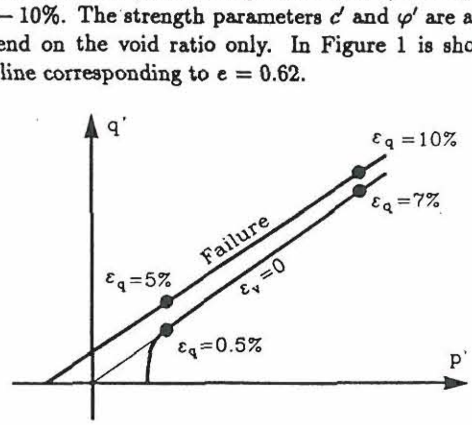

Figure 1. Stress path in an undrained test compared with the Mohr-Coulomb failure criterion.

In a stress state with very small deviatoric stresses the behaviour of a sand is contractive, but when q' increases a dense sand dilates. However, it is not possible to cut up the $q' - p'$ stress-space in a contractive and a dilative zone, because changes in ε_v depend on stress increments.

The stress path for an undrained test is shown in Figure 1. It is of particular interest because the first cycle on cyclic loading has to follow a similar stress path. It must be emphasized that for normal stress levels this stress path does not describe an undrained failure state, because the destorsion is too small, only $\varepsilon_q \approx 0.5 - 1.0\%$. At very high stress levels $\varepsilon_q = 5 - 10\%$ and failure can occur. We can conclude that the first cycle in an undrained cyclic test has limited strains.

MOBILIZATION INDEX M

The deviator stress q' can be normalized by introducing a mobilization index M

$$M = q' / \left|q'_f\right| \quad ; \quad -1 < M < 1 \tag{2}$$

where q'_f corresponds to the actual mean normal stress p'.

The advantage of using a mobilization index instead of the often used stress ratio $(\sigma'_1 - \sigma'_3)/\sigma'_3$ is obvious: It is possible to compare tests with different soil and densities, because q' is normalized with respect to the strength of the soil. M can be used with success even for curved failure envelopes and as mentioned later a mathematical description of hysteresis is possible even for large strains and complicated stress variations.

The mobilization index is introduced in Figure 2. The drained, anisotropic stress state (p'_o, q'_o) just before cyclic loading is then (p'_o, M_m°). During cyclic loading the amplitude A is constant and the maximum value of M at each cycle is:

$$M_{max} = \frac{q'_o + A}{A} M_m = k\, M_m \tag{3}$$

where k is the amplitude ratio and M_m is the mean value of M.

CYCLIC TRIAXIAL TESTS

In triaxial tests the loads, movements, volume, and pore pressure are measured on the outside of the test specimen, and it is essential to have homogeneous conditions inside the specimen in order to achieve correct values of stresses, strains, and void ratio. The height of the specimen is therefore equal to the diameter, and smooth pressure heads are used. But in extension it is impossible to avoid inhomogeneous strains at failure where "necking" occurs, preventing the strain and stresses from being calculated correctly.

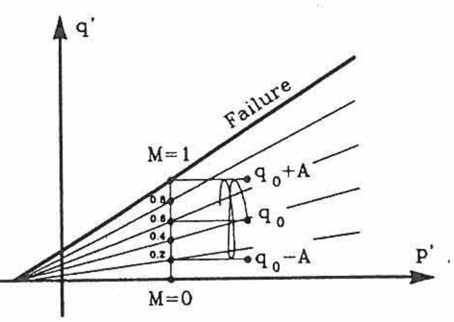

Figure 2. Normalization of the deviator stress q. Variation of M during cyclic loading.

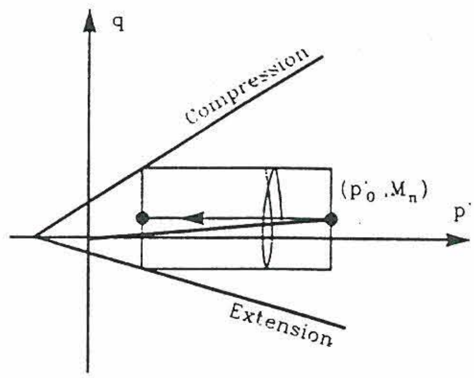

Figure 3. Simultaneous liquefaction in compression and extension. Risk for necking.

The Mohr-Coulomb failure criterion is unsymmetric in triaxial compression and extension, because the intermediate normal stress changes from σ'_{min} to σ'_{max}.

$$\begin{aligned} \text{In compression:} \quad q'_f &= \frac{6 sin\varphi'}{3 - sin\varphi'} (p' + c' cot\varphi') \\ \text{In extension:} \quad q'_f &= \frac{-6 sin\varphi'}{3 + sin\varphi'} (p' + c' cot\varphi') \end{aligned} \tag{4}$$

If q' varies symmetrically $M_m = sin\varphi'/(3 + sin\varphi')$ at failure. Failure then takes place simultaneously in compression and extension (Figure 3). If $M_m < sin\varphi'/(3 + sin\varphi')$ at failure necking takes place. The corresponding initial value of M_m is given by

$$M_m^\circ < M_n = \frac{1}{18}(3 - sin\varphi)\frac{A}{p'_o} \tag{5}$$

In tests with $M_m^\circ = M_n$ the strains in loading and reloading are almost identical, and the cyclic stress-strain curves are reversible.

CYCLIC PHENOMENA

The stable state M_s

It is now postulated that a stable state exists at a certain mobilization index M_s, where the positive and negative pore pressure generated during a loading cycle neutralize each other, provided that $|M_{max}| < 1$. In the stable state the stress variation during a loading cycle does not change anymore. The stable state M_s has been verified in an extensive test series, shown in Figure 4. The initial value of M_m, the confining pressure, the amplitude and the number of cycles vary from test to test, but the number of cycles, N, is large enough to ensure that the last hundreds of cycles takes place in the stable state, where the stress paths do not change.

Stabilization

It is seen that when $M_m^\circ > M_s$ then negative pore pressure will develop and the effective stress level will increase until the stable state is reached. This phenomenon is called "stabilization". If $M_m^\circ >> M_s$ then "instant stabilization" takes place.

Pore presssure buildup and liquefaction

If $M_m < M_s$ and $|M_{max}| < 1$ a positive pore pressure will be generated and the effective stress level will decrease until the stable state is reached. This phenomenon is called "pore pressure buildup".

If M_{max} equals 1 during pore pressure buildup, the hysteretic strains get very large ($\varepsilon_q = \pm 10\%$), the testing equipment looses all control, and the peak pore pressure in each cycle rises to the confining pressure. This ultimate state is called "liquefaction". It is well documented in many test series.

Mathematical formulation

A simple description of this phenomenon is given by:

$$\begin{aligned} M_m &= M_m^\circ + (M_s - M_m^\circ)\, f(N) \\ M_{max} &= k \cdot M_m \leq 1 \end{aligned} \tag{6}$$

where $f(N)$ is a function of the number of cycles. $f(N) = 0$ for $N = 0$ and $f(N) \rightarrow 1$ for $N \rightarrow \infty$. Thus

$$f(N) = \left(\frac{N}{N + N_o}\right)^{\ell} \tag{7}$$

ℓ is rather close to 1. In analysis of liquefaction risks during earthquakes an advantageous value of ℓ is 1.25.

N_o depends on M_m° as indicated in Figure 5, which is based on results from a larger number of tests than shown in Figure 4. It is seen that

$$N_o = 4 \cdot \left(\frac{1 - M_m^\circ}{M_m^\circ}\right) \tag{8}$$

M_m° is at the actual stress level limited upwards: $M_m^\circ < 0.8$.

Figure 6 shows the three possible developments of M_m and M_{max} during cyclic loading as given by formulas (6), (7), (8).

STRESS VARIATION DURING CYCLIC LOADING

Figure 7 shows some examples of stress variations during cyclic loading, which corresponds to the phenomena defined earlier.

The first loading ($N = 1$) always follows the stress path in a static undrained test. and the corresponding value of M is always lesser than one. This causes in Figure 7 b) a negative pore pressure big enough to stabilize the sand almost immediately.

As the cyclic loading goes on the distorsion grows bigger and bigger and when $\varepsilon_q \approx 5 - 10\%$ the maximum value of M is able to reach 1. In Figure 7 d) liquefaction can occur.

In tests where $M_m^\circ < M_n$, necking can give big distorsions and liquefaction after a few cycles.

HYSTERETIC BEHAVIOUR OF SAND

The behaviour of sand during cyclic loading is strongly hysteretic and irreversibility occurs and causes permanent deformations.

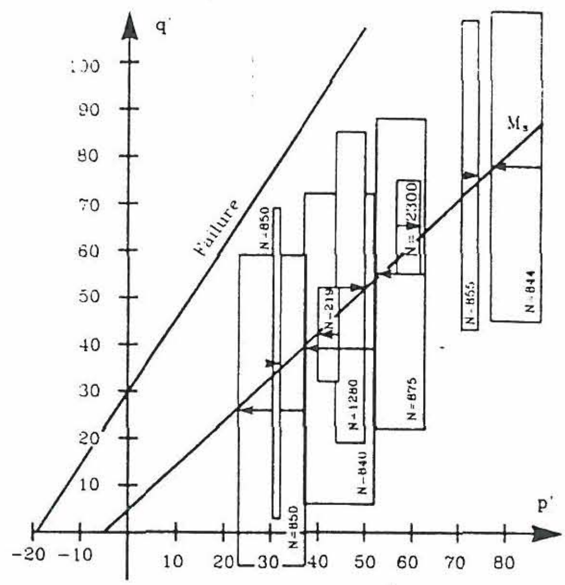

Figure 4. Verification of the stable static M_s.

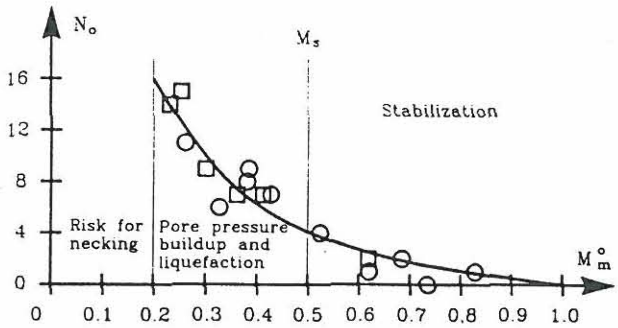

Figure 5. Estimation of N_o as a function of M_m° formula 8.

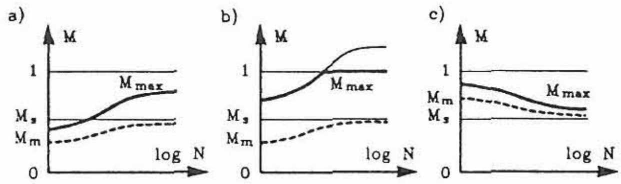

Figure 6. Development of the degree of mobilization under cyclic loading.

a) Increasing M_{max} resulting in pore pressure build-up.

b) Increasing M_{max} resulting in liquefaction.

c) Increasing M_{max} resulting in stabilization.

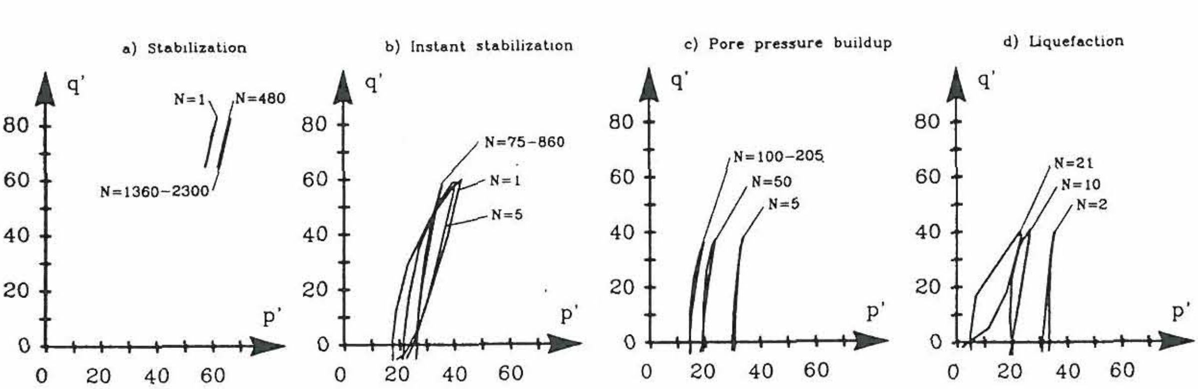

Figure 7. Stress path in cyclic loading.

The irreversibility depends on M_m° and does not occur for $M_m^\circ = M_n$. It depends on $(M_m^\circ - M_n), (M_s - M_m^\circ)$ and the number of cycles. In Figure 7 a) the irreversibility dominates the hysteresis, in Figure 7 d) only small irreversibility occurs.

Two stress cycles from the sequence in Figure 7 d) are shown in Figure 10. It shows that the behaviour of sand in this case is strongly hysteretic. The shape of the two curves are considerably different, corresponding to the different variations in p' and q', and it seems very complicated for a mathematical description.

However, by introducing the mobilization index M the curves become very regular and the mathematical formulation rather easy.

A performance curve for a first loading is shown in Figure 8 with $M_m^\circ \approx 0$. It can be described by

$$\frac{\partial M}{\partial \varepsilon_q} = G_M\left(1 - M^n\right)$$

where G_M is a normalized shear modulus, and n is a parameter which describes the curvature. In unloading the formula is modified to

$$\frac{\partial M}{\partial \varepsilon} = G_M\left(1 - |M|^n\right)$$

and a hysteric cyclic curve can then be described by:

$$\frac{\partial M}{\partial \varepsilon} = G_M\left(1 - sign\left(\frac{M}{d\varepsilon}\right)|M^n|\right) \qquad (9)$$

This shows continuity and differentiability for $M = 0$, (Figure 9). The formula is a simplified Bouc-Wen formula.

A further study shows that when $M_m \neq 0$ unrealistic irreversibilities occur except for small stress amplitudes. In order to separate the hysteretic behaviour from irreversibility, the formula is modified

$$\frac{\partial (M - M_m)}{\partial \varepsilon} = G_M\left(1 - sign\left(\frac{M - M_m}{d\varepsilon}\right)\left|\frac{M - M_m}{1 - M_m}\right|^n\right) \qquad (10)$$

In Figure 10 formula (10) is fitted to test results with small values of M_m by the method of least squares. Characteristic values of G_n and n are $G_M = 900$ and $n = 0.5$.

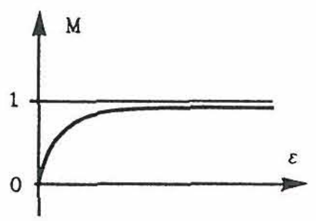

Figure 8. Normalized performance curve.

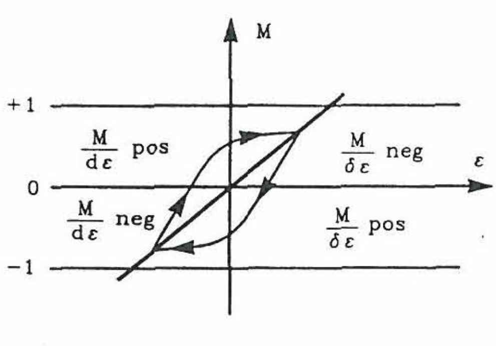

Figure 9. Hysteretic curve (eq 9).

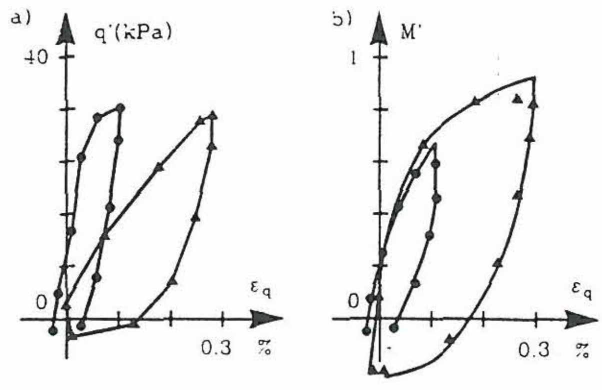

Figure 10. Hysteretic curves estimated from eq (10) and measured in triaxial tests.

Using formula (6), (7), (8) and (10) the development of hysteresis during cyclic loading can be followed and the damping ratio D calculated. For small values of M_m the damping ratio depends on M_{max} only:

$$D \approx 0.5\ M \qquad (11)$$

Hardin and Drnevich propose for a clean dense sand $D = 0.28 - 0.015\ log(N)$, which is seen to correspond to a natural state with stabilisation.

CONCLUSION

The behaviour of sand subjected to cyclic loading is described in simple mathematical formulations by introducing a normalized deviator stress, called the mobilization index. This paper shows how stabilization, instant stabilization, pore pressure buildup, and liquefaction develop and how hysteretic curves and damping ratios can be calculated. The damping ratio agrees well with expected values for a saturated sand.

REFERENCES

Casagrande, A. : *Liquefaction and cyclic deformations of sand. A critical review.* Harvard Soil Mechanics series 88. Harvard University, Cambridge. Mass. 1976.

Castro, G. : *Liquefaction and cyclic mobility of saturated sand.* J. Geo. Eng. Div., ASCE, Vol. 101, pp 551-569, 1975.

Castro, G. and Poulos, S.J. : *Factors affecting liquefaction and cyclic mobility.* Proc. ASCE, Vol. 103, GT6, 1977.

Hardin, B.O. and Drnevich, V.P. (1972) : *Shear modolus and damping in soils: Design equations and curves.* J. of SMFD, ASCE 98 (SM7) pp 667-692, 1972.

Jacobsen, H.M. and Ibsen, L.B. : *Development of pore pressure in cohesionless soil with initial shear stresses during cyclic loading.* YGEC III, Minsk, 1989.

Loung, M. : *Stress-strain aspects of cohesionless soil under cyclic and transient loading.* Int. Symp. on Soils under Cyclic and Transient Loading. Swansea, Jan. 1980.

Nielsen, S.R.K., Thoft-Christensen, P., Jacobsen, H.M. : *Reliability of soil sublayers under earthquake excitation: Markov Approach.* IV Int. Conf. on Soil Dynamics and Earthquake Engineering. Mexico, 1989.

Seed, H.B., Lee, K.L. : *Liquefaction of saturated sands during cyclic loading.* J. SMFD, ASCE, Vol. 92, No. SM6, Nov. 1966, pp 105-134.

Seed, H.B. : *Soil liquefaction and cyclic mobility evaluation for level ground during earthquake.* Proc. ASCE, No. GT2, Feb. 1979, pp 200-255.