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B.1

Identification of Civil Engineering Structures using Vector ARMA Models

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SYNOPSIS: This paper describes the work which have been carried out in the project B.1: *Damage Detection in Structures under Random Loading*. The project is a part of the research programme *Dynamics of Structures* founded by the *Danish Technical Research Council*. The planned contents of and the requirements to the project prior to its start is described together with the results obtained during the project. The project was mainly carried out as a Ph.D. project by the first author from September 1993 to May 1997 under supervision of Professor Rune Brincker and Associate Professor Poul Henning Kirkegaard both from department of Building Technology and Structural Engineering, Aalborg University.

1. INTRODUCTION

In the mid eighties, researchers at the Department of Building Technology and Structural Engineering at Aalborg University, Denmark, started using time domain models for system identification of civil engineering structures. A common feature of the work has been the use of the so-called auto-regressive moving average (ARMA) models for time series modelling. The reason is the ability of these models to provide an accurate estimate of the modal parameters of a structural system on the basis of discretely sampled response. This ability makes them suitable as tool for vibration based inspection, i.e. damage detection. In the past decade the results of this work have been reported in several papers and in three Ph.D. theses. However, the link between the ARMA model and the mathematical description of civil engineering structures has not been addressed to the same extent as mathematical description of dynamic systems in fields such as electrical engineering and econometrics. Therefore, the model has been applied toas a greybox model in the above references. The relation between the auto-regressive part of the model and the modal parameters has been well understood, whereas the understanding of the moving average part has been limited.

In order to obtain a deeper understanding of the ARMA models, and how they are related to the modelling of civil engineering structures and damage detection, the project B.1: *Damage Detection in Structures under Random Loading*, with this as its primary objective, was granted as a part of the Danish Research Council frame programme "Dynamics of Structures". Another objective of the Ph.D. project was to implement the obtained knowledge as, especially designed, time domain software for system identification of civil engineering structures. The results have been reported in several papers and the Ph.D. thesis: *System Identification of Civil Engineering Structures using Vector ARMA Models*. In section 9, a list of references to papers and reports prepared during the project is presented.
This list together with the possibility to download some of these and the thesis can be seen at *http://www.civil.auc.dk/~i6pa*. A short description of the Ph.D. thesis can be seen at *http://www.civil.auc.dk/~i6pa/thesis.htm.*

2. PLAN FOR THE PROJECT

The following topics relating to system identification of ambient excited civil engineering structures were planned to be investigated.

1. Relation between system identification using ARMA models and vibration based inspection.
2. ARMA modelling of ambient excited civil engineering structures.
3. Estimation of ARMA models from measured system response.
4. Extraction of modal parameters and estimation of their uncertainties.
5. Software development for system identification using ARMA models.

In section 3, the relation between mathematical modelling and system identification is described. Also described are the relations between non-parametric and parametric system identification, and why it is desirable to apply the ARMA models in relation to vibration based inspection of ambient excited civil engineering structures. Finally, the scope of the work of the Ph.D. project is stated. Section 4 explains how the ambient excited civil engineering structure can be modelled in discrete-time by the ARMA model. In section 5, it is explained how the ARMA model can be estimated from measured system response. The extraction of the modal parameters and the estimation of their uncertainties are the topics of section 6. Section 7 briefly describes the software package developed during the project. Finally, in section 8 conclusions are made.

3. SYSTEM IDENTIFICATION OF CIVIL ENGINEERING STRUCTURES USING ARMAV MODELS

In this section the basic concepts of system identification are introduced. Also introduced, are the applications of system identification in civil engineering that are of interest to project B.1.

3.1 *System Identification*

A convenient way of describing a dynamic system is by use of mathematical models. These models can either be represented in continuous time as differential equation systems or in discrete-time as difference equation systems. There are in general two ways to construct mathematical models:

- ☞ *Physical modelling.*
- ☞ *System identification.*

In physical modelling the construction of a dynamic model is based on physical knowledge and fundamental laws, such as the Newton 2. law of motion. On the other hand, if the physical knowledge about a dynamic system is limited a model of the input / output behaviour of the system can be obtained through system identification based on calibration of a model using experimental data. If the structure of the calibrated model is chosen without regard to physical knowledge the calibrated model is called a black box model. If some parts of the model are based on physical knowledge the calibrated model is called a grey box model. On the other hand, if the calibrated model is based completely on the physical laws, i.e. if it originates from a physical modelling, then the calibrated model is called a white box model. Thus, system identification should not be thought of as a substitute of physical modelling, since identification can be based on model structures that have physical origin. Basically there are two categories of model structures :

- *Non-parametric model structures.*
- *Parametric model structures.*

In any case, physical modelling will always be linked with parametric model structures. Common to both categories of model structures is that they depend on the applied excitation, which may be one of the following

- *Instantaneous excitation.*
- *Periodic excitation.*
- *Pseudo-random periodic excitation.*
- *Stochastic excitation.*

In the case of instantaneous excitation, the system is either given an impulse or step excitation and the system is the left vibrating on its own. The excitation may or may not be measured. It is also possible to excite the system with a known periodic excitation, such as sinusoidal, or several periodic signals mixed to obtain a pseudo-random periodic excitation. Finally, as an alternative to the deterministic excitation, the excitation might also be a stationary stochastic process with either known or unknown statistical properties. In any case, the dynamic behaviour can be conceptually described as in figure 1.

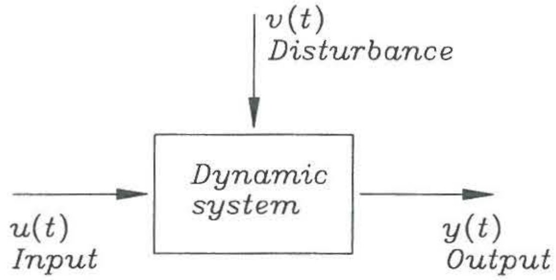

Figure 1: A dynamic system with input $u(t)$, output $y(t)$ and disturbance $v(t)$.

The system is driven by input $u(t)$ and affected by disturbance $v(t)$. In some cases the user can control the input $u(t)$ but not the disturbance $v(t)$. It might also be that the actual input is unknown and therefore uncontrollable in some applications. The output $y(t)$ describes how the system reacts or responds to the applied input and disturbance. Therefore, the output will be a mixture of dynamic response of the system and characteristics of the input and disturbance as well. In general, the input at previous time instances will also affect the current output. In other words, the dynamic system has memory.

3.1.1 *Non-Parametric Model Structures*

The non-parametric models are described by curves, functional relationships or tables. These analysis methods are :

- *Transient analysis.*
- *Frequency analysis.*
- *Correlation analysis.*
- *Spectral analysis.*

Transient analysis is applied when the system response is transient, i.e. generated on the basis of impulse or step excitation. The dynamic behaviour of the system is then identified on the basis of the impulse or step response. Frequency analysis is applied when the excitation is deterministic and either periodic, or pseudo-random and periodic. The measured excitation and the corresponding system response are transformed to frequency domain, and the frequency response function is obtained as the ratio of the transformed response and excitation. Correlation and spectral analysis are methods that are applied to a stationary stochastically excited system. In these cases, the excitation and the system response can be characterized either by the correlation functions in time domain or the spectral densities in frequency domain. Having estimated the correlation functions of the excitation and the response the impulse response function of the system can be obtained. On the other hand, if the spectral densities of the excitation and response are estimated instead, it is possible to obtain the frequency response function.

The traditional non-parametric system identification techniques are primarily based on the Fast Fourier Transform (FFT) techniques. However, the FFT has some limitations. The most obvious limitation is that the FFT assumes periodic data, which is certainly not the case for sampled response from stochastically excited structures. In principle, the Fourier transform assumes that the amount of data is infinite. In this case the frequency response functions or the spectral densities will have an infinite frequency resolution. However, in practice the available data records have a finite length, resulting in a finite frequency resolution. Since sampled stochastic signals in general exhibit non-periodicity, leakage errors will certainly be introduced. Further, in the case of closely spaced modes, it might be impossible to separate these if one of the resonance frequencies has a small amplitude compared to the other. In this case the resonance frequency with the smallest energy content may be masked completely by the resonance frequency with highest energy content. The leakage errors are compensated by windowing the data before the FFT is applied, to secure periodicity of the data by damping the discontinuities at the ends of the data record. The problem of windowing is, that it introduces an extra damping into the system, and thus creates its own leakage problem.

3.1.2 *Parametric Model Structures*

Parametric models are characterized by the assumption of a mathematical model constructed from a set of parameters. These parameters are then estimated during the system identification. The mathematical model of a linear and time-invariant continuous-time system is usually in the form of a differential equation system. The equivalent discrete-time parametric model is a difference equation system. In figure 1 an input / output system affected by noise was shown. The appearance of the discrete-time parametric model that describes such a system depends on whether the input is measured or not. If the input is measured, then the associated parametric model will have a deterministic term as well as a stochastic term that describes the unknown disturbance. If the actual input is unknown, it is treated stochastically. In this case the description of input and disturbance will be described by a single stochastic term.

Model Structures using Deterministic Input

The general input / output model structure used for modelling of linear and time-invariant dynamic systems excited by deterministic input, is Auto-Regressive Moving Average with eXternal input (ARMAX)

$$y(t) = \boldsymbol{G}(q)\boldsymbol{u}(t) + \boldsymbol{H}(q)\boldsymbol{e}(t) \tag{1}$$

where $\boldsymbol{G}(q)$ and $\boldsymbol{H}(q)$ are the transfer functions of the deterministic part and the stochastic part. The stochastic input $e(t)$ are the innovations, which are an equivalent process of the noise and prediction errors. If $H(q) = I$ (1) is called an output error (OE) model. In any case the dynamic properties of the system are modelled by $G(q)$. A parametric model structure is called multivariable when it includes several variables. If there are several outputs, it is characterized as a multivariate model structure. If it only has one output, it is termed a univariate model structure.

Model Structures using Stochastic Input

If the input is an unmeasurable stationary stochastic process, the ARMAX model is no longer the correct model structure to use. In this case an Auto-Regressive Moving Average (ARMA) model should be applied

$$y(t) = H(q)e(t) \tag{2}$$

The dynamical properties as well as the noise are now modelled by the same transfer function $H(q)$. In the multivariate case the model structure is called an Auto-Regressive Moving Average Vector (ARMAV) model. As observed the choice of model structure depends on whether the input is deterministic or stochastic, i.e. whether the excitation of the structural system is known and measured or unknown. It also depends on whether the system is stationarily excited, or excited by an impulse or step excitation.

3.2 *Applications of System Identification of Civil Engineering Structures*

In the field of civil engineering, system identification might be applied for several reasons. However, the following two areas have attracted much attention in the recent years

- ☞ *Modal analysis.*
- ☞ *Vibration based inspection.*

Modal analysis covers a variety of applications all based on the analysis of modal parameters. These parameters describe specific dynamic characteristics of the structure. One of the applications that uses the modal parameters as basis is vibration based inspection.

3.2.1 *Modal Analysis*

Modal analysis is based on the determination of modal parameters of a structural system. These parameters represent an optimal model, or basis, which can be used to describe the dynamics of a structural system. The modal parameters can be divided into the following four categories:

- ☞ *Modal frequencies.*
- ☞ *Modal damping.*
- ☞ *Modal vectors.*
- ☞ *Modal scaling.*

The modal frequencies are more explicitly eigenvalues, or angular or natural eigenfrequencies. Modal damping is characterized by the damping ratios, and modal vectors by the eigenvectors or mode shapes. Finally, modal masses and residues are typical parameters used to characterize modal scaling. Since the modal parameters are directly related to the impulse and frequency response functions, as well as the correlation functions and spectral densities, they can be extracted from the non-parametric system identification methods by applying different curve fitting procedures. In case of parametric system identification methods there are direct mathematical relationships between the modal parameters and the estimated model parameters.

3.2.2 *Vibration Based Inspection*

The accumulation of damages in a civil engineering structure will cause a change in the dynamic characteristics of the structure. The basic idea in Vibration Based Inspection (VBI) is to measure these dynamic characteristics during the lifetime of the structure and use them as a basis for identification of structural damages. Typically, a VBI programme uses the modal parameters to describe the dynamic characteristics of a structure. A synonym for the dynamic characteristics used as basis for the VBI programme is damage indicators. In other words:

☞ *A damage indicator is a dynamic quantity, which can be used to identify the existence of damage in a structure.*

Often VBI has at random been referred to as damage detection. VBI can be divided into the following four levels:

☞ *Level 1 - Detection.*
☞ *Level 2 - Localization.*
☞ *Level 3 - Assessment.*
☞ *Level 4 - Consequence.*

Methods of the first level give a qualitative indication that a structure might be damaged. Level two methods give information about the probable location of the damage as well. Methods of the third level provide information about the size of the damage, and finally the level four methods give information about the actual safety of the structure given a certain damage state. The use of a damage indicator primarily gives a qualitative indication of the existence of damage, and should therefore be characterized as a level 1 method. However, some of the damage indicators will in some cases give rough estimates for the locations of damage, which is equivalent to a primitive level 2 method. Changes in natural eigenfrequencies are no doubt the most used damage indicators. One of the reasons for this is that the natural eigenfrequencies are rather easy to determine with a high level of accuracy. Another reason is that they are sensitive to both global and local damages. So comparison of estimates of natural eigenfrequencies is usually an effective level 1 method. A local damage will cause changes in the derivatives of the mode shapes at the position of the damage. This means that a mode shape having many coordinates or measurement points will be a fast way to locate the approximate position of a damage. They can therefore be characterized as a simple level 2 method. The introduction of damage in a structure will usually cause changes in the damping capacity of the structure. It has been shown that the damping ratios are extremely sensitive to the introduction of even small cracks in a cantilever beam. However, dealing with real structures, the estimation of the damping ratios of the individual modes is highly sensitive to time-varying and non-physical sources. Thus, a satisfactory accuracy of the estimates of the damping ratios will in general be impossible to obtain. Therefore, the damping is applicable as a damage indicator, but it cannot and should not be used as the only damage indicator.

As explained, all modal parameters are in principle applicable as damage indicators. This means that they can be used at least for detection of damage, and as such be characterized as level 1 methods. However, the key to a successful VBI is the use of unbiased and low-variance modal parameter estimates as damage indicators. If the estimates are biased they might cause a false alarm, i.e. indicate a damage that does not exist. If the estimation inaccuracies are too dominant, it might be impossible to detect any significant changes. Thus, the existence of a damage might be hidden.

So in conclusion :

☞ *Successful VBI based on modal parameters requires accurate and unbiased modal parameter estimates.*

In this context the computational effort spent in obtaining reliable estimates is not so important. Further, the limitations and systematic errors of the traditional FFT-based non-parametric system identification techniques motivates the use of other techniques.

This motivation can be stated as:

☞ *The need for a more accurate estimation of the modal parameters from sampled data, compared to what traditional FFT-based non-parametric techniques can provide.*

This need is basically the reason for using the parametric models in the system identification, since the physical knowledge about a dynamic system in this way is incorporated into the system identification process.

3.2.3 *Excitation of Civil Engineering Structures*

In the case of civil engineering structures there will most likely be a natural excitation of the structure such as wind or waves. These natural forms of excitation are commonly called ambient excitation and the vibrations of the structure caused by them are called ambient vibrations. System identification of structural dynamics on the basis of ambient excitation is also referred to as ambient testing. From an experimental point of view, the simplest approach to measure the dynamic parameters of a structure is to detect the response due to ambient excitation. In the case of very large structures this approach is the only practical way of performing dynamic tests, it is simply impossible to excite such structures artificially.

The ambient excitation is stochastic in nature. Therefore, it cannot be described by an explicit time-dependent function, but must be characterized by certain statistical parameters, such as its mean and covariance function. Since the structural system can be seen as a linear transformation of the applied input, this means that the response will also be stochastic, and may as such also be represented by its statistical characteristics. Several researches have shown that ambient excitation provides a quick, inexpensive and reliable way for testing of large civil engineering structures, such as buildings and offshore structures. It has been concluded that parameter estimates obtained by ambient excitation are as good as parameter estimates obtained by external excitation. This conclusion is based on a study of several published results of ambient versus forced vibration tests of high-rise structures in the USA. Because of the nature of dynamic testing under ambient excitation conditions this method has advantages over others, such as the impulse and periodic excitation. Ambient excitation has a broad frequency range, and thus theoretically excites all relevant modes of a structure. Also, the use of ambient excitation in dynamic testing does not disturb the normal functioning of a structure and no excitation equipment is required for ambient testing. However, the disadvantage of ambient excitation is that its characteristics cannot be controlled and measured directly.

Since ambient excitation cannot be measured directly, it can be constructed from other measurements such as the surface elevation if system identification of an offshore structure is considered. From these measurements, sea state characteristics such as significant wave height and average zero-upcrossing period, can be estimated. These characteristics can then be used as input to models, which have been developed to describe the waves either as time series or spectral densities. The connection between the theoretical description of the waves and the forces on the structure is established using a load model, which could e.g. be the Morrison equation. In the case where the ambient excitation is generated by fluctuating wind pressure forces, numerous measurement projects have shown that the fluctuations may be described by a stationary ergodic Gaussian stochastic process with regard to short-term conditions.

3.3 *Scope of the Ph.D. Thesis*

From the above the following can be stated concerning system identification of civil engineering structures :

- ☞ *The dynamic behaviour of a civil engineering structure is usually modelled by a linear and time-invariant model.*
- ☞ *The excitation of civil engineering structures is typically unknown ambient excitation.*
- ☞ *If this unknown ambient excitation is e.g. the wind, it is often modelled as a stationary Gaussian stochastic process.*
- ☞ *For applications such as VBI a high degree of estimation accuracy of the modal parameters is required.*
- ☞ *Adequate parametric model structures are not limited by frequency resolution, and can as such be more accurate than FFT-based non-parametric model structures.*

These statements imply that the parametric models are applicable to system identification of civil engineering structures when a high degree of accuracy is needed. Therefore, the main aim of this thesis has been to investigate how to represent ambient excited civil engineering structural systems by stochastic time-domain models, and how to estimate these on the basis of sampled structural response data. Since measurements of the true ambient excitation are not available, system identification using standard multivariate input / output ARMAX model cannot be applied. Therefore, the focus has been put on the use of stochastic models of the Auto-Regressive Moving Average Vector (ARMAV) type. Particular emphasis has been be put on computationally accurate system identification methods, since the intention is to provide a more accurate alternative to the traditional non-parametric system identification methods typically applied in the field of civil engineering. A secondary purpose of the thesis has been to make the theory of system identification using stochastic time domain models more accessible to civil engineers. Since system identification is a relatively new field for civil engineers, it has been natural to search for applicable theory and methods within disciplines that are at the research front, such as automatic control engineering, mathematical system theory, econometrics, and aerospace engineering. It is shown how an ARMAV model equivalent to the continuous-time mathematical model of a stochastically excited structural system arises. In this context, an explanation of the purpose of the moving average is emphasized. It has also been shown how to account for the presence of disturbance, and how ARMAV models are directly related to the so-called stochastic state space systems. Since modal analysis is one of the main reasons for system identification of civil engineering structures, a thorough treatment of the modal decomposition and extraction of modal parameters has been given together with guidelines for estimation of the uncertainties of the estimated modal parameters will be given. During the Ph.D. project emphasis has been put on the practical implementation of user-friendly system identification software.

4. CONTINUOUS-TIME STRUCTURAL SYSTEMS

Structures can be regarded as distributed parameter systems characterized by the distribution of the mass, damping and stiffness properties. However, parameter identification of such systems is in general not easy. Thus, with a few exceptions, in most of the literature on testing of structures, the data are analysed based on the assumption that the system is described by one or more linear ordinary differential equations. Because of their simplicity the linear time-invariant lumped parameter models are the most widely used models in structural identification. More complex models such as the linear continuous parameter models and non-linear models are used only when the lumped parameter model cannot be used to provide an adequate representation of the structural behaviour. In general, system identification concerns the determination of modal parameters. If the excitation of a structure e.g. is the wind, then the only available information about the dynamic behaviour of a structure is the measured vibrations of it. As will be shown later, system identification using ARMAV models is in these situations capable of providing good estimates of the modal parameters of the structure. However, it is necessary to obtain some kind of understanding of how the discrete-time ARMAV model relates to the modal parameters, i.e. how it relates to the continuous-time lumped parameter model. It is the purpose of this section to provide this understanding. This chapter is restricted to the continuous-time modelling of civil engineering structures.

4.1 *ARMAV Modelling of Ambient Excited Structural Systems*

Experience has led to the following mathematical mass-spring-dashpot lumped parameter model for a structure subjected to external loading

$$\boldsymbol{M}\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}\dot{\boldsymbol{y}}(t) + \boldsymbol{K}\boldsymbol{y}(t) = \boldsymbol{f}(t) \tag{3}$$

$\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices all of dimensions $p \times p$. $\boldsymbol{y}(t)$ and $\boldsymbol{f}(t)$ are the $p \times 1$ displacement and $p \times 1$ force vectors at the mass points, respectively. From a system identification point of view a generalization of the mathematical model is necessary, since the number of measurement channels is usually less than the number of identified modes. A generalized multivariate model can be formulated as, Andersen [1]

$$D^s\boldsymbol{y}(t) + \boldsymbol{A}_{y,s-1}D^{s-1}\boldsymbol{y}(t) + \dots + \boldsymbol{A}_{y,0}\boldsymbol{y}(t) = \boldsymbol{B}_{f,s-2}D^{s-2}\boldsymbol{f}(t) + \dots + \boldsymbol{B}_{f,0}\boldsymbol{f}(t) \tag{4}$$

where D is a differential operator. The matrices $\boldsymbol{A}_{y,i}$ and $\boldsymbol{B}_{f,i}$ are all of the dimension $p \times p$. The displacement vector $\boldsymbol{y}(t)$ and its derivatives are all of the dimension $p \times 1$. The $p \times 1$ vector $\boldsymbol{f}(t)$ describes the forces applied to the system. The modes of a structural system will typically be underdamped, which implies that each mode is described by a pair of complex conjugated eigenvalues. In this situation the order s will be defined as $s = \frac{2N}{p}$ with N being the number of underdamped modes. It is often assumed that the ambient excitation $\boldsymbol{f}(t)$ is given as the output of a linear time-invariant shaping filter subjected to Gaussian white noise. Due to the Gaussian assumption, it is implicitly assumed that the true ambient excitation is at least weakly stationary. If the ambient excitation can be described by filtered white noise, it is possible to derive a model for it. Assume that the excitation $\boldsymbol{f}(t)$ of the structural system is obtained as the output of an mth-order p-variate linear time-invariant continuous-time shaping filter

$$D^m \boldsymbol{f}(t) + \boldsymbol{A}_{f,m-1} D^{m-1} \boldsymbol{f}(t) + \ \dots \ + \boldsymbol{A}_{f,0} \boldsymbol{f}(t) = \boldsymbol{w}(t) \tag{5}$$

For simplicity, it is assumed that $\boldsymbol{f}(t)$ and $\boldsymbol{w}(t)$ have the same dimensions as $\boldsymbol{z}(t)$. This implies that the matrices $\boldsymbol{A}_{f,i}$ all have the dimension $p \times p$. The stochastic process $\boldsymbol{w}(t)$ is a zero-mean Gaussian white noise, fully described by its covariance function. This covariance function is defined in terms of the $p \times p$ intensity matrix $\boldsymbol{W}$ as

$$E[\boldsymbol{w}(t)] = \boldsymbol{0}, \quad E[\boldsymbol{w}(t)\boldsymbol{w}^T(t-\tau)] = \delta(\tau)\boldsymbol{W} \tag{6}$$

where $\delta(\tau)$ is the Dirac delta function. These statistical properties are abbreviated $NID(\boldsymbol{0},\boldsymbol{W})$. It is obvious that the response $\boldsymbol{y}(t)$ of the system will contain a mixture of the dynamic behaviour of the structural system and of the excitation. It is also intuitively clear that during a system identification the dynamic modes of the shaping filter will also be estimated. These modes are, together with any noise modes, called non-physical modes. In this way they can be distinguished from the physical modes of the structural system. The structural system can then be combined with the shaping filter of the excitation by means of convolution into a resulting linear system subjected to a Gaussian white noise. The resulting differential equation system will be of the order $n = s+m$. Such a differential equation system can be represented by the following state space system, Andersen [1]

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{F}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{w}(t) , \quad \boldsymbol{w}(t) \in NID(\boldsymbol{0},\boldsymbol{W}) \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) \end{aligned} \tag{7}$$

where $\boldsymbol{F}$ is the $np \times np$ state matrix, $\boldsymbol{B}$ is the $np \times p$ input matrix, and $\boldsymbol{C}$ the $p \times np$ observation matrix. Define a discrete time instance as $t_k = kT$, where k is an integer and T is the sampling interval. A sampling of the solution of (6) then leads to

$$\begin{aligned} \boldsymbol{x}(t_{k+1}) &= \boldsymbol{A}\boldsymbol{x}(t_k) + \tilde{\boldsymbol{w}}(t_k), \quad \tilde{\boldsymbol{w}}(t_k) \in NID(\boldsymbol{0},\boldsymbol{\Omega}) \\ \boldsymbol{y}(t_k) &= \boldsymbol{C}\boldsymbol{x}(t_k) \end{aligned} \tag{8}$$

where the process $\tilde{\boldsymbol{w}}(t_k)$ is a discrete-time Gaussian white noise that is completely described by the covariance matrix $\boldsymbol{\Omega}$, given by

$$\boldsymbol{\Omega} = \int_0^T e^{\boldsymbol{F}t} \boldsymbol{B} \boldsymbol{W} \boldsymbol{B}^T e^{\boldsymbol{F}^T t} dt \tag{9}$$

The transition matrix $\boldsymbol{A}$ is defined as

$$\boldsymbol{A} = e^{\boldsymbol{F}T} \tag{10}$$

whereas C is unaffected by the sampling. In Andersen [1] and Andersen et al. [7], it is shown how the state space system (6) can be represented in discrete time by a covariance equivalent p-variate ARMAV(n,n-1) model, i.e. an ARMAV model with an nth order autoregressive part and a moving average part of order n-1. However, (7) does not account for the presence of noise which will most certainly always be present. A way to incorporate a noise description into the ARMAV model is to add process and measurement noise to the sampled state space system, Andersen [1] and Andersen et al. [12]

$$\begin{aligned} x(t_{k+1}) &= Ax(t_k) + \tilde{w}(t_k) + w(t_k) \\ y(t_k) &= Cx(t_k) + v(t_k) \end{aligned} \tag{11}$$

These noise terms are all assumed zero-mean with a joint second-order moment given by

$$E\left[\begin{bmatrix} \tilde{w}(t) \\ w(t) \\ v(t) \end{bmatrix} \begin{bmatrix} \tilde{w}^T(t) & w^T(t) & v^T(t) \end{bmatrix}\right] = \begin{bmatrix} \Omega & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & Q & S^T \\ \mathbf{0} & S & R \end{bmatrix} \tag{12}$$

Since external noise is now present in the system the system response at a given time step cannot be calculated explicitly, but only predicted. This prediction is performed by the means of a Kalman filter. From this filter a p-variate ARMAV(n,n) model that describes the system dynamics as well as the noise, can be derived, Andersen [1].

$$\begin{aligned} & y(t_k) + A_1 y(t_{k-1}) + \ldots + A_n y(t_{k-n}) = \\ & \quad e(t_k) + C_1 e(t_{k-1}) + \ldots + C_n e(t_{k-n}), \quad e(t_k) \in NID(\mathbf{0}, \Lambda) \end{aligned} \tag{13}$$

The left-hand side of this difference equation system is the auto-regressive part that describes the system dynamics. The left-hand side is the moving average part that describes the external noise as well as the white noise excitation, and secures stationarity of the system response. $e(t_k)$ is a stationary zero-mean Gaussian white noise innovation process, described by the covariance matrix Λ. The matrices A_i and C_i are the auto-regressive and the moving average coefficient matrices, respectively. The auto-regressive coefficient matrices are obtained as

$$\begin{bmatrix} A_n & A_{n-1} & . & . & A_1 \end{bmatrix} = -CA^n \begin{bmatrix} C \\ CA \\ . \\ CA^{n-1} \end{bmatrix}^{-1} \tag{14}$$

This follows directly from the following relation that links the auto-regressive coefficient matrices to the state space matrices, Andersen [1]

$$CA^n + A_1 CA^{n-1} + ... + A_{n-1} CA + A_n C = 0 \tag{15}$$

The conversion from the ARMAV model back to state space is not unique. Several ways to realize the model in state space exist. These realizations can e.g. be balanced or canonical forms, Andersen [1].

5. ESTIMATION OF ARMAV MODELS USING THE PREDICTION ERROR METHOD

The parameter estimates, based on N samples, and returned in $\hat{\theta}_N$ can be obtained as the global minimum point of the criterion function

$$V_N(\theta) = det\left(\frac{1}{N}\sum_{k=1}^{N} \varepsilon(t_k,\theta)\varepsilon^T(t_k,\theta)\right) \tag{16}$$

In other words, as $\hat{\theta}_N = arg \min_{\theta} V_N(\theta)$. The model parameter vector θ is determined so that the prediction error, defined as

$$\varepsilon(t_k,\theta) = y(t_k) - \hat{y}(t_k|t_{k-1};\theta) \tag{17}$$

is as small as possible. $\hat{y}(t_k|t_{k-1};\theta)$ is the one-step ahead predicted system response. The $m \times 1$ parameter vector θ is organized in the following way

$$\theta = col\left(\begin{bmatrix} A_1 & . & . & A_n & , & C_1 & . & . & C_n \end{bmatrix}\right) \tag{18}$$

where *col* means stacking of all columns of the argument matrix. The total number of adjustable parameters in θ is as such $m = 2np^2$. The predictor of the ARMAV(n,n) model is defined as

$$\begin{aligned} \hat{y}(t_k|t_{k-1};\theta) = \\ -A_1(\theta)y(t_{k-1}) - ... - A_n(\theta)y(t_{k-n}) + \\ C_1(\theta)\varepsilon(t_{k-1},\theta) + ... + C_n(\theta)\varepsilon(t_{k-nc},\theta) \end{aligned} \tag{19}$$

This relation reveals that the predictor of the ARMAV model is non-linear, since the prediction errors themselves depend on the parameter vector θ. This implies that an iterative minimization procedure such as the following Gauss-Newton search scheme has to be applied.

$$
\begin{aligned}
\hat{\theta}_N^{i+1} &= \hat{\theta}_N^i + \mu_i \boldsymbol{R}_N^{-1}(\hat{\theta}_N^i)\boldsymbol{F}_N(\hat{\theta}_N^i) \\
\boldsymbol{R}_N(\theta) &= \sum_{k=1}^{N} \psi(t_k,\theta)\boldsymbol{Q}_N^{-1}(\theta)\psi^T(t_k,\theta) \\
\boldsymbol{F}_N(\theta) &= \sum_{k=1}^{N} \psi(t_k,\theta)\boldsymbol{Q}_N^{-1}(\theta)\varepsilon(t_k,\theta) \\
\boldsymbol{Q}_N(\theta) &= \frac{1}{N}\sum_{k=1}^{N} \varepsilon(t_k,\theta)\varepsilon^T(t_k,\theta)
\end{aligned} \tag{20}
$$

The dimensions of $\boldsymbol{R}_N(\theta)$ and $\boldsymbol{F}_N(\theta)$ are $m \times m$ and $m \times 1$, respectively. μ_i is a bisection constant that adjusts the step size. $\psi(t_k,\theta)$ is the gradient of the predictor (19), i.e. the derivative of (19) with respect to each of the adjustable parameters of the ARMAV model. At each time step this gradient forms an $m \times p$ dimensional matrix. The estimate of the parameters of the ARMAV model can as such be calculated by supplying an initial parameter estimate. On the basis of this the prediction errors can be calculated, the matrix $\boldsymbol{R}_N(\theta)$ and the vector $\boldsymbol{F}_N(\theta)$ a can be calculated. An updated estimate can then be calculated using (19). This method is called the prediction error method (PEM) since it is the prediction errors that are minimized.

For Gaussian distributed prediction errors this method is asymptotically efficient. In this case an estimate of the uncertainties of the estimate is provided by the covariance matrix, Andersen [3]

$$
\hat{P}(\hat{\theta}_N) = \boldsymbol{R}_N^{-1}(\hat{\theta}_N) \tag{21}
$$

When the amount of data is limited the estimator will not be efficient. However, the performance can in this situation be improved by a backward forecasting approach, Andersen et al. [4].

6. EXTRACTING MODAL PARAMETERS AND ESTIMATION OF THEIR UNCERTAINTIES

The free vibrations of an ARMAV model realized in state space are described by the deterministic part of (10) as

$$
\begin{aligned}
\boldsymbol{x}(t_{k+1}) &= \boldsymbol{A}\boldsymbol{x}(t_k) \\
\boldsymbol{y}(t_k) &= \boldsymbol{C}\boldsymbol{x}(t_k)
\end{aligned} \tag{22}
$$

The solution of this system is assumed to be of the form $\boldsymbol{x}(t_k) = \psi\mu^k$, where ψ is an $np \times 1$ complex vector and μ is a complex constant. Insertion into (21) yields

$$
\begin{aligned}
\psi\mu^{k+1} &= \boldsymbol{A}\psi\mu^k \\
\boldsymbol{y}(t_k) &= \boldsymbol{C}\psi\mu^k
\end{aligned} \tag{23}
$$

showing that $x(t_k) = \psi\mu^k$ only is a solution if and only if ψ is a solution to the first-order eigenvalue problem

$$(I\mu - A)\psi = 0 \tag{24}$$

This eigenvalue problem only has non-trivial solutions if its characteristic polynomial is satisfied. The order of this real-valued polynomial is np. Thus, there will be np roots μ_j that are the eigenvalues of A. For each of these eigenvalues there is a non-trivial solution vector ψ_j which is the corresponding eigenvector. The mode shape Φ_j is then obtained from (22) as

$$\Phi_j = C\psi_j \, , \quad j = 1, 2, \ldots , np \tag{25}$$

The continuous-time eigenvalues λ_i, the natural eigen-frequencies f_i, and damping ratios ζ_i can be extracted from the discrete-time eigenvalues as

$$\left.\begin{aligned} \lambda_j &= \frac{\log(\mu_j)}{T} \\ f_j &= \frac{|\lambda_j|}{2\pi} \\ \zeta_j &= -\frac{Re(\lambda_j)}{|\lambda_j|} \end{aligned}\right\} \quad j = 1, 2, \ldots, np \tag{26}$$

Due to the relation (15) these modal parameters are also the modal parameters of the ARMAV model. Above, it was established that the PEM estimator for Gaussian distributed prediction errors would be statistically efficient. A standard for the estimation errors of a statistically efficient estimator is provided by the Cramer-Rao lower bound. This standard was utilized by the model parameter covariance matrix $P_\theta(\hat\theta_N) = E[(\theta_0-\hat\theta_N)(\theta_0-\hat\theta_N)^T]$ of the difference between the true parameters θ_0 and estimated parameters $\hat\theta_N$ as N tends to infinity. In general, the change of para-meterization from a set of model parameters, given in an $m \times 1$ dimensional vector θ, to another set of physical parameters, given in an $r \times 1$ dimensional vector κ, can be performed by a known r-dimensional functional relation

$$\kappa = f(\theta) \tag{27}$$

Since the number of physical parameters is less than the number of model parameters, obviously the accuracy and thus the sensitivity of κ is more significant than that of θ. In addition, the functional relation (27) will in general be non-linear as in the case of the modal decomposition. Thus, to obtain a practically applicable approach, (27) is usually linearized using a first-order generalized Taylor expansion at the operating point $(\hat\kappa_N, \hat\theta_N)$, Andersen [1]

$$\kappa = \hat\kappa_N + \left(\frac{\partial f(\theta)}{\partial\theta}\right)_{|\theta=\hat\theta_N}\left(\theta - \hat\theta_N\right) = \hat\kappa_N + J(\hat\theta_N)\left(\theta - \hat\theta_N\right) \tag{28}$$

where $J(\hat{\theta}_N)$ is a Jacobian matrix of partial derivatives which should be evaluated at the operating point $\hat{\theta}_N$. The covariance matrix $P_\kappa(\hat{\kappa}_N)$ of the deviation of $\hat{\kappa}_N$ from the true parameters can be estimated by

$$\hat{P}_\kappa(\hat{\kappa}_N) = E\left[(\kappa_0 - \hat{\kappa}_N)(\kappa_0 - \hat{\kappa}_N)^T\right] = J(\hat{\theta}_N)P_\theta(\hat{\theta}_N)J^T(\hat{\theta}_N) \tag{29}$$

The estimated covariance matrix $\hat{P}_\theta(\hat{\theta}_N)$ obtained from (20) can then be inserted instead of $P_\theta(\hat{\theta}_N)$. This expression will only be accurate if $\hat{P}_\theta(\hat{\theta}_N)$ is a good estimate of $P_\theta(\hat{\theta}_N)$ and if the error due to the linear approximation is small, Andersen [1]. What remains is to calculate the Jacobian matrix $J(\hat{\theta}_N)$. This is in general impossible to do analytically even for small model structures, when the physical parameters of interest are the modal parameters. It is therefore necessary to rely on numerical differentiation. As an example, the elements of $\hat{\kappa}_N$ can be defined as the estimated natural eigenfrequencies and associated damping ratios of the model.

$$\hat{\kappa}_N = \left[f_1 \;\; \zeta_1 \;\; f_2 \;\; \zeta_2 \;\; . \; . \; . \;\; f_s \;\; \zeta_s\right]^T \tag{30}$$

The functional relationship between these parameters and the model parameters is given by the eigenvalue problem followed by the calculation of the modal parameters given in (26). This means that the resulting functional relation between the model and modal parameters is highly non-linear, and numerical differentiation must be applied. For further information on the practical considerations and the estimation of the uncertainties of the mode shapes, see Andersen [1].

7. THE STRUCTURAL TIME DOMAIN IDENTIFICATION TOOLBOX FOR USE WITH MATLAB

As a part of the Ph.D. project, a MATLAB based toolbox for identification of especially ambient excited civil engineering structures using multivariate stochastic time domain models has been developed. The toolbox is called the *Structural Time Domain Identification (STDI) toolbox*. This toolbox has been developed in accordance with the theory and notation of the Ph.D. thesis. It has been the intention to make it completely independent of other official as well as unofficial MATLAB toolboxes. In this section the different routines in the toolbox will briefly be presented. The purpose of the routines can be divided into the following categories that cover all part of a system identification session:

- ☞ *Information importing.*
- ☞ *Data preprocessing.*
- ☞ *Parameter estimation.*
- ☞ *Model validation.*
- ☞ *Structural mode selection.*
- ☞ *Assessment of uncertainties.*
- ☞ *Information exporting.*

The presentation of the different routines will include a short description of its purpose. The first step in a system identification session is to set up the bookkeeping involved in the identification process and to acquire data. Table 7.1 shows that the STDI toolbox offers functions for the project management and preprocessing of the geometry of the structure, which is necessary for e.g. animation of mode shapes.

Information Importing

Project bookkeeping (Create, load, save....).
Structural geometry preprocessing.
Data acquisition (Interface to Data Translation plug-in AD-boards, load/save data files acquired in other data acquistion environments).

Table 7.1: Information importing.

For preprocessing of the data the STDI toolbox contains a whole range of functions, see table 7.2. The goal of such a preprocessing is to make the measured signals suitable for system identification. Further, the preprocessing functions can be used to show and to estimate statistical characteristics of the measured data.

Preprocessing

Scaling and trend removal.
Show measured data.
Show FFT-based spectral densities of the measured data.
Low, high or bandpass filtering .
Decimation and resampling.
Split the data into identification and validation data sets.

Table 7.2: Preprocessing of the measured data.

For the actual identification the toolbox implements algorithms for identifications of multivariate ARMAV models as well as multivariate stochastic state space systems. The essential algorithms are based on the PEM approach described in section 5. However, several other identification algorithms exist. These can either be used as stand-alone routines or to provide resonable initial estimates for the PEM routines. In the thesis it is shown how to convert an ARMAV model to a stochastic state space realization, and how to convert a stochastic state space realization to an ARMAV model. These conversion schemes make it possible to initialize the ARMAV PEM routine with the parameters of a stochastic state space realization, or to initialize the PEM routine of a stochastic state space realization with initial parameters of an ARMAV model. The different ARMAV estimation routines are listed in table 7.3a, whereas the different stochastic state space realization estimators are listed in table 7.3b.

Parameter Estimation - ARMAV Models

Multi-Stage Least-Square estimation of an ARMAV model.
Non-linear Least-Square (PEM) estimation of an ARMAV model.
Least-Square (PEM) estimation of an ARV model.
Estimation of an ARMAV(n,n) model using different stochastic subspace identification algorithms.

Table 7.3a: Estimation of the parameters of ARMAV related models.

Parameter Estimation - Stochastic State Space Realizations
Non-linear Least-Square (PEM) estimation of a stochastic state space realization. Estimation of a stochastic state space realization using different stochastic subspace identification algorithms.

Table 7.3b: Estimation of the parameters of stochastic state space realizations.

To validate or assess the quality of a set of identified models the toolbox offers several functions, see table 7.4. It is possible to obtain the Akaike Information theoretic Criterion,AIC and the Akaike Final Prediction Error criterion, FPE of the models, plot the spectral densities and correlation functions of the prediction errors, and compare the predicted and measured response. Further, it is also possible to compare the FFT-based spectral densities of the measured response with the spectral densities obtained from the model.

Model Validation
FPE / AIC - Akaike's Final Prediction Error / Information theoretic Criterion. Plot spectral densities and correlation functions of prediction errors. Plot measured and predicted response. Plot poles and zeroes and their uncertainty elipsoides. Compare spectral densities of measured response with spectral densities of one or more identified models.

Table 7.4: Model validation functions.

To select the structural or fundamental modes of an optimal model the toolbox offers different mode selection functions, see table 7.5. One of the simplest ways to identify the structural modes is to compute the modal parameters and compare these with the spectral densities of the model and e.g. the MAC (Modal Assurance Criterion) values of the mode shapes. Another efficient way is to plot stability diagrams with different plotting criteria.

Structural Mode Selection
Compute the modal parameters. Compute the Modal Assurance Criterion (MAC). Plot the spectral densities of the model. Plot various stability diagrams.

Table 7.5: Mode selection functions.

When the optimal model has been selected it is important to be able to quantify the uncertainties of the estimated model and modal parameters, see table 7.8. The estimation of the uncertainties of the modal parameters is performed according to the principles explained in section 6.

Assessment of Uncertainties
Return estimated covariance matrix of estimated model parameters. Return estimated standard deviations of estimated modal parameters.

Table 7.6: Assessment of uncertainties.

Having finished an identification session, the results of it have to be presented and exported. The toolbox offers various functions especially made for these purposes. These functions make tables with all estimated modal parameters and their uncertainties, animate mode shapes, and generate a final report of the identification session, see table 7.7.

Information Exporting
Save tables with estimated modal parameters and their uncertainties to a file. Animate the estimated mode shapes Generate a final documentation report of the identification session.

Table 7.7: Information exporting.

Besides implementing routines that solve the primary tasks presented in the tables the toolbox implements a variety of necessary auxiliary functions. These routines can be divided into the categories shown in table 7.8.

Auxiliary Functions
Manipulation of matrix polynomials (Multiplication, modal decomposition, filtering). Manipulation of state space system (Modal decomposition, filtering, balancing , solving Algebraic Riccati and Lyapunov equations etc.). Relating continuous-time to equivalent discrete-time multivariate systems (Zero-order hold and covariance equivalence techniques).

Table 7.8: Auxiliary functions of the toolbox.

A more comprehensive description of the performance of the routines and how they are used can be found in Andersen [1], Kirkegaard et al. [15], and in the *Structural Time Domain Identification Users Guide*, Andersen et al. [17].

8. CONCLUSION

In this Ph.D. project, system identification of civil engineering structures using parametric stochastic models has been considered. It has been shown that if the structural system can be assumed to be a linear and time-invariant lumped parameter system, and if the excitation can be assumed generated by a linear and time-invariant shaping filter subjected to Gaussian white noise, then the ARMAV model will be an adequate model.

The relation between the combined continuous-time system and the ARMAV model has been derived by assuming a covariance equivalence of the response of the continuous-time system and the ARMAV model for all discrete time steps. If the measured response of a linear and time-invariant structural system is Gaussian distributed and if the excitation is unknown, then the covariance equivalence technique results in a discrete-time model correctly describing the dynamic properties of the structural system as well as the statistical properties of the response.

For an nth order multivariate continuous-time system subjected to Gaussian white noise and affected by external noise the covariance equivalent discrete-time model is an ARMAV(n,n) model. In other words, a model that is constructed from an nth order auto-regressive matrix polynomial and an nth order moving average matrix polynomial. It is shown that an ARMAV(n,n) model reduces to an ARMAV(n,n-1) model when no noise is present. In this project the accuracy of an estimated ARMAV model has been emphasized. For this reason the applied estimation technique is the Prediction Error Method (PEM). The advantage of this estimation technique is that it is asymptotically unbiased and efficient if the prediction errors are Gaussian distributed and if the true system is contained in the estimated model. The use of the PEM estimator has made it possible to estimate the standard deviations associated with the modal parameter estimates.

So to recapitulate the results of the project. The ARMAV model estimated using the PEM should be applied in modal analysis if :

- ☞ *The structural system is linear and time-invariant.*
- ☞ *The excitation is unknown.*
- ☞ *The measured response is stationary and can be assumed Gaussian distributed.*
- ☞ *Uncertainty estimates of the modal parameters are needed.*

For a large variety of practical system identification problems in civil engineering, the assumptions concerning linearity, Gaussianity and stationarity are fulfilled. In these cases, system identification using ARMAV models can serve as a reliable and valuable alternative to the traditional non-parametric system identification techniques.

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