Three Essays on the Labor Market Effects of Technological Change and Unemployment Benefits

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Contents

1 Introduction **1**

References . . . 10

2 Automation, Robots and Wage Inequality in Germany: A Decomposition Analysis **15**

2.1 Introduction . . . 16

2.2 Data . . . 20

2.3 Empirical Approach . . . 23

2.3.1 Method . . . 23

2.3.2 Model Specification . . . 27

2.4 Descriptive Evidence . . . 30

2.5 Decomposition Analysis and Discussion . . . 35

2.5.1 1996-2010 . . . 36

2.5.2 2012-2017 . . . 43

2.5.3 Robustness Check . . . 47

2.6 Conclusion . . . 51

Appendix 2.A . . . 54

2.A.1 Classification of Economic Activities . . . 54

2.A.2 Advantages of the Substitution Potential Provided by Dengler and Matthes (2015) . . . 54

2.A.3 Counterfactual Wage Distributions . . . 55

2.A.4 SOC 2010 - KldB 2010 Crosswalk . . . 56

Appendix 2.B . . . 57

Appendix 2.C 84
References 88

3 Technical Change, Task Allocation, and Labor Unions 93

3.1 Introduction 94
3.2 Related Literature 97
3.3 The Model 99
3.3.1 Firms 99
3.3.2 Labor Unions for Low-Skilled Workers 103
3.3.3 Task Reallocation and the Elasticity of Labor Demand 105
3.3.4 Solution of the Model in the Steady State 107
3.4 Comparative Statics: Technical Change 108
3.5 Calibration 116
3.5.1 Design 116
3.5.2 Calibrated parameters 118
3.5.3 Comparative Statics: Increase in A_H 123
3.6 Summary and Conclusions 125
Appendix 3.A 127
3.A.1 The Firm's Optimization Problem 127
3.A.2 Wage Setting of Labor Unions 129
3.A.3 Second-Order Condition for the Optimal Wage 131
3.A.4 Proof of Lemma 3.1 132
3.A.5 Special Cases of the Task Productivity Schedule $\bar{\alpha}(I)$ 133
3.A.6 Proof of Proposition 3.2 135
3.A.7 Stability Analysis 136
3.A.7.1 Model Dynamics and Saddle Path 136
3.A.7.2 Conditions for Saddle-Path Stability 140
3.A.8 Slopes and Curvatures of the JC, WC, and TAC 141
3.A.9 Comparative Statics: Effects on Other Variables 143
3.A.10 Data Description 144

References . . . 149

4 Unemployment Benefits in a Task-Based Matching Model: The Impact of Hartz IV Revisited 154

4.1 Introduction . . . 155

4.2 The Task-Based Matching Model . . . 160

4.2.1 Goods Market . . . 160

4.2.2 Firms . . . 161

4.2.3 Labor Unions and Wage Determination . . . 165

4.2.4 Low- and High-Skilled Labor Demand of the Firm . . . 166

4.2.5 Steady State Equilibrium . . . 169

4.3 The Effect of Unemployment Benefits on Labor Market Outcomes . . . 170

4.3.1 General Results in the Task-Based Matching Model . . . 170

4.3.2 The Task-Based Matching Model with Constant Low-Skilled Labor Demand Elasticity . . . 174

4.3.3 The Task-Based Matching Model with Exogenous and Constant Task Allocation . . . 177

4.4 Quantitative Analysis . . . 179

4.4.1 Parameter Selection . . . 179

4.4.2 The Long-Run Effects of the Hartz IV Reform . . . 183

4.4.3 Improvement in Matching Efficiency . . . 187

4.5 Conclusions . . . 188

Appendix 4.A . . . 190

4.A.1 The Household's Optimization Problem . . . 190

4.A.2 The Firm's Optimization Problem . . . 191

4.A.3 Elasticity of Substitution . . . 194

4.A.4 Derivation of Low- and High-Skilled Labor Demand . . . 195

4.A.5 Proof of Proposition 4.2 . . . 196

4.A.6 General Effects of Unemployment Benefits on Final Production and Wage Outcomes . . . 199

4.A.7 Proof of Lemma 4.1 201
4.A.8 Derivation of Slopes and Curvatures of JC and WC and the Shift in WC due to a Change in γ_L 202
4.A.9 Change in Matching Efficiency ζ 204
References 206

5 Conclusions **209**

List of Tables

2.1 Descriptive statistics . . . 33

2.B.1 Comparison between the estimated automation threat and the automation probability provided by Dengler and Matthes (2015), by sector . . 57

2.B.2 Descriptive statistics of the automation threat variable by groups . . . 58

2.B.3 Descriptive statistics of the alternative automation threat variable with automation probabilities provided by Frey and Osborne (2017) by groups 59

2.B.4 Decomposition of the 85-15 percentile wage gap and the Gini coefficient, 1996-2010 . . . 60

2.B.5 Decomposition of the 50-15 and the 85-50 percentile wage gap, 1996-2010 61

2.B.6 Decomposition of the 85-15 percentile wage gap and the Gini coefficient, 2012-2017 . . . 62

2.B.7 Decomposition of the 50-15 and the 85-50 percentile wage gap, 2012-2017 63

2.B.8 RIF-regressions 15th quantile, 1996 2010 2012 2017 . . . 64

2.B.9 RIF-regressions 50th quantile, 1996 2010 2012 2017 . . . 67

2.B.10 RIF-regressions 85th quantile, 1996 2010 2012 2017 . . . 70

2.B.11 RIF-regressions Gini coefficient, 1996 2010 2012 2017 . . . 73

2.B.12 RIF-regressions variance, 1996 2010 2012 2017 . . . 76

2.B.13 Decomposition of the variance, 1996-2010 and 2012-2017 . . . 79

2.B.14 Decomposition results using the probability of computerization by Frey and Osborne (2017), 1996-2010 . . . 80

2.B.15 Decomposition results using the probability of computerization by Frey and Osborne (2017), 2012-2017 . . . 81

2.B.16 Decomposition results without the automotive and other vehicles sector, 1996-2010 82

2.B.17 Decomposition results without the automotive and other vehicles sector, 2012-2017 83

3.1 Parameter values for Germany (DE) and France (FR) 117

3.2 Matched targets for Germany (DE) and France (FR) 118

3.3 Calibrated parameters 119

3.4 Labor market effects of an increase in high-skill productivity A_H by 10% (changes in variables expressed in percent). 124

4.1 Parameter values and targets 180

4.2 Calibrated parameters 181

4.3 Long-run effects of a decrease in the average low-skilled net replacement rate γ_L by 6.5% 184

List of Figures

2.1 The evolution of the 85-15, 50-15 and 85-50 percentile wage gap between 1996 and 2017 . . . 31

2.2 Gini coefficients in different automation threat groups, 1996-2017 . . . 35

2.3 Actual and counterfactual wage distributions, 1996-2010 . . . 37

2.4 Decomposition results of the composition and wage structure effect by percentile wage gaps, 1996-2010 . . . 39

2.C.1 Automation threat in Germany across sectors in the manufacturing industry from 1996 to 2017 . . . 84

2.C.2 Actual and counterfactual differences, 1996-2010 . . . 84

2.C.3 Actual and counterfactual 85-15 percentile wage gap and Gini coefficient from 1996 to 2010 . . . 85

2.C.4 Actual and counterfactual 50-15 and 85-50 percentile wage gap from 1996 to 2010 . . . 85

2.C.5 Actual and counterfactual wage distributions, 2012-2017 . . . 86

2.C.6 Actual and counterfactual differences, 2012-2017 . . . 86

2.C.7 Actual and counterfactual 85-15 percentile wage gap and Gini coefficient from 2012 to 2017 . . . 87

2.C.8 Actual and counterfactual 50-15 and 85-50 percentile wage gap from 2012 to 2017 . . . 87

3.1 Timeline with events during period t . . . 101

3.2 Effects of skill-biased technological progress (increase in the productivity of high-skilled workers A_H) on labor market outcomes and task allocation . . . 113
3.3 Calibrated productivity functions for Germany (DE) and France (FR) in periods 1995–2005 and 2010–2017. . . . 121
3.4 Calibrated productivity functions for Germany (DE) in two periods 1995–2005 and 2010–2017. . . . 123

4.1 Quarterly harmonized unemployment rate in Germany, France and Italy between 1991-2022 . . . 155
4.2 Graphical analysis of a reduction in the replacement rate for low-skilled workers . . . 176
4.3 Graphical illustration of the calibrated effects of a reduction in the replacement rate for low-skilled workers . . . 185

Chapter 1

Introduction

Since the industrial revolution, there have been concerns about the effects of technological change on the labor market. The rapid increase in new technologies since the 1980s, such as the rise of information and communication technologies or the recent advances in robotics and artificial intelligence, has given new impetus to the subject. The study by Frey and Osborne (2017) has contributed to the ongoing discussion, according to which 47% of all jobs in the U.S. could potentially be replaced by technology. This dark picture of the future of work is not shared by all economists. Changes in technology also complement labor, which raises productivity and increases output. Both lead to increased labor demand and, consequently, to higher labor compensation; see e.g., Autor (2015). Moreover, Arntz et al. (2016) argue that the majority of occupations encompass a significant proportion of tasks that remain challenging to automate. Nevertheless, technological change leads to structural transformations in the labor market, inevitably affecting the labor market outcomes of workers and entailing distributional issues. One possibility to address the challenges arising from technological change could involve labor market reforms, which might encompass a reduction in the welfare state or an improvement in matching efficiency. This thesis provides new insights into the impact of technological change on the labor market and revisits the effect of a reduction in unemployment benefits to stimulate employment.

A wide body of literature has identified technological change as a central cause for rising wage inequality in the last decades in industrialized countries, especially in the

U.S.; see, e.g., Acemoglu (1998), Autor et al. (1998) and Goldin and Katz (2008). The prevailing hypothesis to explain the rise in wage inequality is skill-biased technical change (SBTC), which states that changes in technology favour skilled over unskilled workers by increasing the relative productivity and consequently the relative demand and the relative wage of skilled workers; see, e.g., Acemoglu and Autor (2011), Biewen and Seckler (2019) and Katz and Autor (1999). Furthermore, SBTC stands as a potential factor for explaining employment differentials among different skill groups. For instance Stadler and Wapler (2004) demonstrate that SBTC reduces the unemployment level of high-skilled workers using a general equilibrium model with matching frictions.

Autor et al. (2003) and Acemoglu and Autor (2011) refine the canonical SBTC concept by introducing the task approach, which distinguishes between tasks and skills. A task refers to a distinct work activity, while a skill is the workers capability to execute several tasks. The production process is then interpreted as a set of tasks that are combined to produce output. The distinction between skills and tasks becomes crucial when workers with a given skill level can potentially perform multiple tasks and technology influences the allocation of skills to tasks. In the proposed task-based model, it becomes feasible for SBTC to reduce the wage level of specific worker groups, a scenario not captured in the canonical SBTC model.

The more recent rise in automation has shifted the academic consensus from labor-augmenting technological change, which includes SBTC, towards labor-replacing technological change. Empirical evidence suggest that routine tasks are more easy programmable using algorithms, implying that automation technologies primarily replace routine tasks while complementing non-routine tasks; see, e.g., Autor et al. (2003) and Goos et al. (2014). This hypothesis is summarized as routine-biased technical change (RBTC), or sometimes referred to routine-replacing technical change (RRTC). As routine tasks are often performed by middle-skilled workers, the rise in automation technologies has a polarizing effect on employment and wages, rather than an upgrading pattern. This recent phenomenon of job polarization is documented in a wide range of literature; see, e.g., Acemoglu and Autor (2011) for a cross-national evidence, Autor and Dorn (2013) for the U.S., Goos et al. (2014) for Europe, Dustmann et al. (2009) and Spitz-Oener (2006) for

Germany or Goos and Manning (2007) for the United Kingdom.

Acemoglu and Autor (2011) introduce middle-skilled task replacing technologies within the task-based model to provide a formal explanation for the phenomenon of employment and wage polarization. Building on this framework, Acemoglu and Restrepo (2018a, 2018b, 2022) and Hémous and Olsen (2022) develop task-based approaches to analyze the impact of automation, which involves the replacement of tasks previously performed by labor. Assuming perfect competition in the labor market, these studies focus on the effects of automation on wages and the labor share and provide evidence that automation increases wage inequality, reduces the labor share and potentially leads to wage declines.

Another branch of research analyzes the effect of automation using the Mortensen-Pissarides matching model. Jaimovich et al. (2021) consider a model with a frictional labor market for heterogeneous low-skilled workers and perfect competition for high-skilled workers, in which automation results in a considerable polarization of welfare. Cords and Prettner (2022) demonstrate that robot adoption leads to wage and employment losses for low-skilled workers, while high-skilled workers benefit from higher wages and employment. Leduc and Liu (2020) analyze the consequences of automation on labor market fluctuations and demonstrate that automation decreases the labor share, depresses wage growth and generate fluctuations in unemployment. Modifying a standard matching model with endogenous job destruction, Guimarães and Gil (2022) suggest that automation may moderately reduce long-run employment, but not to a significant extent due to the simultaneous increase in both job destruction and job creation.

There is a wide range of recent literature providing empirical evidence on the impact of industrial robots as a leading automation technology on wages and employment. Acemoglu and Restrepo (2020) find evidence that the rise in robot exposure reduces employment and wages in the U.S. from the 1990s to the mid-2000s. Similarly, Dauth et al. (2021) show that the rise in robot exposure in Germany decreases employment and increases wage inequality in the manufacturing industry between 1994 and 2014. Moreover, they provide evidence that the negative employment effect is offset by an increase in employment in the service sector, which is in line with the findings of Kariel (2021) for the UK. De Vries et al. (2020) find evidence that the increased use of industrial robots rise the employment share of especially non-routine analytic jobs, while it decreases the share of routine manual jobs. Similar to Dauth et al. (2021), they find no significant effect on aggregate employment, suggesting adjustment effects in the task content of occupations.

An important factor that has to be considered when analyzing the effect of technological change on the labor market is the influence of labor market institutions. It is often argued that the rise in wage inequality in the U.S. and the increase in unemployment in Europe are two sides of the same coin: the more flexible labor market in the U.S. forces workers to accept lower wages, while institutional rigidities in Europe, including employment protection, centralized wage bargaining, generous unemployment benefits and a stringent minimum wage regulation, result in higher unemployment rates; see, e.g., Hornstein et al. (2005), Katz and Autor (1999) and Krugman (1994). In a model of equilibrium unemployment, Mortensen and Pissarides (1999) demonstrate that SBTC can lead to different wage and employment responses when considering various systems of unemployment benefits and employment protection policies. By considering the institutional differences between the U.S. and Europe, the model suggests that SBTC leads to higher wage dispersion in the U.S. and higher unemployment in Europe. This view is supported by Davis (1998), who analyzes the effects of labor-saving technological change with rigid minimum wages in Europe, and by Weiss and Garloff (2011), who link the effect of SBTC on unemployment and wage inequality to the presence of wage-dependent unemployment benefits.

Acemoglu et al. (2001) and Açıkgöz and Kaymak (2014) argue that SBTC leads to a deunionization trend, which reinforces the direct effect of SBTC. The increase in the relative productivity of skilled workers leads to higher relative wages, thereby reducing the incentive for skilled workers to join unions. As labor unions typically compress the wage structure, the deunionization trend results in increasing wage inequality. More recently, Dauth et al. (2021) point to labor unions as an important intermediary in the effect of industrial robots on employment. Strong protection for incumbent workers shifts the job displacement caused by automation to labor market entrants and young workers. To retain workers at the same workplace, even though their tasks have been automated, they are reassigned to new occupations and tasks. This transition notably mitigates the displacement effects of automation.

So far, the discussion has covered various explanations regarding the impact of technological change on labor market outcomes of different skill groups and the task content of occupations. It has also emphasized the importance of labor market institutions in analyzing the effects of technological change. The subsequent discussion shifts attention to labor market reforms as potential solutions for addressing high unemployment, involving a reduction in the welfare state or an improvement in matching efficiency. In particular, the focus is on the impact of the most far-reaching labor market reforms in the history of the German welfare state, known as the Hartz reforms.

Germany, referred to as the "sick man of Europe" in the late 1990s into the early 2000s, struggled with rising unemployment rates, increasing wage inequality and low economic growth. To cope with the high unemployment level, the German government introduced the Hartz I-IV reforms between 2003 and 2005. Data provided by the Federal Employment Agency suggest that the German government has been quite successful in reducing the registered unemployment rate from around 11.7% in 2005 to 6.4% ten years later.

The core part of the reform package, Hartz IV, primarily involved a substantial cut in unemployment benefits. From a theoretical point of view, a reduction in unemployment benefits stimulates employment: in a standard search and matching model à la Mortensen and Pissarides (1994), a cut in unemployment benefits decreases the expected utility of an unemployed worker and, therefore, reduces wage pressure. This, in turn, stimulates firms' vacancy creation and increases employment; see, e.g., Pissarides (2000), Pries and Rogerson (2005) and Yashiv (2004). Including search and matching frictions in a dynamic stochastic general equilibrium (DSGE) model, Zanetti (2011) demonstrates the effects of unemployment benefits on aggregate fluctuations and finds that a reduction in unemployment benefits decreases the volatility of employment, output and job flows.

The Hartz IV reform was implemented in January 2005 and includes a cut in the level of long-term unemployment benefits and a reduction in the entitlement duration of short-term unemployment benefits. However, the extent of the reduction differed across household types. One of the key challenges in analyzing the effects of the Hartz IV reform is the magnitude to which Hartz IV reduced the average net replacement rate, which measures the generosity of unemployment benefits as the proportion of income obtained after a certain number of months of unemployment. The different estimation strategies explain, to some extent, the differences in the estimated reduction in the long-run unemployment rate resulting from Hartz IV, which ranges from less than 0.1 percentage points to 3.2 percentage points, see, Launov and Wälde (2013), Hartung et al. (2022), Hochmuth et al. (2021), Krause and Uhlig (2012) and Krebs and Scheffel (2013). In addition to the more or less pronounced decrease in the unemployment rate, Hartz IV contributes to declining wage levels and the continued increase in wage dispersion; see, e.g., Dustmann et al. (2014), Giannelli et al. (2016) and Krause and Uhlig (2012).

Moreover, the German government implemented two additional waves of the Hartz reforms in 2003 and 2004. The Hartz I-III reforms aimed to increase incentives to return to work by creating new job opportunities and improve the matching efficiency through the reconstruction of the Federal Employment Agency. Empirical evidence suggests that the first two waves of the Hartz reforms indeed substantially improved the matching efficiency, as shown by Fahr and Sunde (2009), Hertweck and Sigrist (2013) and Klinger and Rothe (2012). This led to a reduction of the unemployment rate, but in comparison to the effect of the Hartz IV reform, the decline was limited; see, e.g., Krause and Uhlig (2012) and Krebs and Scheffel (2013).

This thesis contributes to the understanding of *(i)* the impact of automation technologies on wage inequality, *(ii)* the labor market effects of SBTC and *(iii)* the consequences of a reduction in unemployment benefits on labor market outcomes. In particular, the thesis provides a new measure of automation threat to reveal the relative contribution of automation and robotization to wage inequality in Germany. Additionally, it revisits the effects of SBTC and the Hartz IV reform on labor market outcomes, using a novel general equilibrium model that integrates the task approach into a search and matching framework with wage setting by labor unions.

Chapter 2 (joint work with Ramona Schmid and already published in LABOUR, see Brall & Schmid, 2023), contributes to the existing literature by examining the relative importance of automation and robotization on wage inequality in the German manufacturing sector from 1996 to 2017. Our novel measure of automation threat combines occupation- and requirement-specific scores of automation risk with yearly sector-specific robot densities. This approach allows to encompass a broader dimension of automation and robotization than has been done so far. By employing the extended Oaxaca-Blinder decomposition method based on recentered influence function (RIF) regressions introduced by Firpo et al. (2018), we demonstrate that our measure of automation threat contributes significantly to wage inequality in the German manufacturing sector, besides the conventional demographic factors like age and education. Moreover, we present general findings on the development of wage inequality and the associated driving forces for the recent years until 2017, in which wage inequality stayed rather constant or even declined.

The empirical strategy allows distinguishing between composition and wage structure effects. We identify compositional effects due to automation threat as a non-negligible factor associated with changes in wage inequality in Germany. There is an observable trend towards occupations with medium automation threat, accompanied by a decreasing share of occupations with high and low automation threat. Due to the fact that within-group wage inequality is the lowest in occupations with the highest automation threat, those compositional changes are associated with an increase in overall wage inequality. In addition, we find evidence for growing wage dispersion between workers in occupations with low automation threat (containing especially non-routine tasks) and workers in occupations with high automation threat (containing especially routine tasks). This trend contributes to rising wage inequality as predicted by RBTC, where technology replaces routine tasks and complements non-routine tasks.

In Chapter 3 (joint work with Thomas Beißinger and Martyna Marczak), a new general equilibrium model for the analysis of SBTC is developed, combining the task approach, wage setting by labor unions, as well as search and matching frictions. So far, the focus has primarily been on perfect competition models that consider the impact of SBTC on the task allocation between low- and high-skilled workers; see e.g., Acemoglu and Autor (2011). Using the new model framework with labor market imperfections and collective bargaining, the analysis reveals that the wage elasticity of labor demand is influenced by the task threshold, which divides the range of tasks performed by low-and high-skilled workers, respectively. The effect of changes in the task threshold on the labor demand elasticity, and consequently on the labor unions' wage-setting, is in general ambiguous. This ambiguity arises from different shapes of the relative task productivity schedule, which reflects the substitutability of high- and low-skilled workers. This has consequences for the effects of SBTC. Unlike the conventional result that SBTC typically leads to a positive impact on employment and wages of low-skilled workers, the task-based matching model presents the possibility that low-skilled workers may instead either experience higher unemployment or lower real wages.

The model is calibrated using German and French data for the periods 1995-2005 and 2010-2017 to illustrate that the impact of SBTC may even change its sign over time. In the first period, SBTC leads to a strong decline in the labor demand elasticity, resulting in a significant rise in labor unions' wage markup. This contributes to an increase in low-skilled unemployment in both countries. In the second period, the effect of SBTC on the labor demand elasticity in France becomes weaker, implying only a moderate increase in wage pressure. Consequently, the unemployment rate for low-skilled workers in France decreases. For both countries and periods, real wages of high-skilled workers increase more than those for low-skilled workers, thus leading to an increase in the skill premium.

Chapter 4 sheds new light on the labor market effects of unemployment benefit reforms by demonstrating the importance of endogenous task allocation between low- and high-skilled labor in the evaluation of the Hartz IV reform. The analysis builds on a modified version of the task-based matching model developed in Chapter 3. I introduce monopolistic competition in the goods market and provide a specific function for the relative task productivity schedule, resulting in a constant labor demand elasticity of low-skilled labor. Within this extended model framework, I demonstrate that the reduction in low-skilled unemployment benefits triggers a reallocation of tasks towards low-skilled workers. As a consequence, this leads to additional effects on labor market outcomes that are disregarded in the prevailing literature, which assumes a conventional production function. To assess the importance of this task reallocation effect, the task-based matching model with exogenous and constant task allocation is considered for comparison.

Both model variants are calibrated to quantify the effects of Hartz IV and the importance of endogenous task allocation. In contrast to the existing literature, see, e.g., Hartung et al. (2022) and Krebs and Scheffel (2013), I calculate the average net replacement rate and its decline caused by Hartz IV in a more detailed way, using the OECD tax-benefit model (TaxBEN). The Hartz IV reform reduces the noncyclical low-skilled unemployment rate by 4 percentage points from its long-run value before the reform. Ignoring endogenous task allocation results in a lower decline of only 3.4 percentage points and the effect on low- and high-skilled wages is more pronounced, leading to a larger increase in the skill premium. Hence, disregarding the endogenous task allocation of firms within this calibration set-up would underestimate the effect of the Hartz IV reform on low-skilled unemployment and overestimate the effect on wages and the skill premium. Moreover, the calibration of the increase in matching efficiency resulting from Hartz I-III supports the findings in the literature that the first two waves of the reform package play a minor role in explaining the drop in the unemployment rate compared to Hartz IV.

References

Acemoglu, D. (1998). Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality. *The Quarterly Journal of Economics*, *113*(4), 1055-1089.

Acemoglu, D., Aghion, P., and Violante, G. L. (2001). Deunionization, Technical Change and Inequality. *Carnegie-Rochester Conference Series on Public Policy*, *55*, 229–264.

Acemoglu, D., and Autor, D. (2011). Skills, Tasks and Technologies: Implications for Employment and Earnings. In D. Card and O. Ashenfelter (Eds.), *Handbook of Labor Economics* (Vol. 4b, pp. 1043–1171). Elsevier.

Acemoglu, D., and Restrepo, P. (2018a). Modeling Automation. *AEA Papers and Proceedings*, *108*, 48–53.

Acemoglu, D., and Restrepo, P. (2018b). The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment. *American Economic Review*, *108*(6), 1488–1542.

Acemoglu, D., and Restrepo, P. (2020). Robots and Jobs: Evidence from US Labor Markets. *Journal of Political Economy*, *128*(6), 2188–2244.

Acemoglu, D., and Restrepo, P. (2022). Tasks, automation, and the rise in us wage inequality. *Econometrica*, *90*(5), 1973–2016.

Açıkgöz, Ö. T., and Kaymak, B. (2014). The Rising Skill Premium and Deunionization. *Journal of Monetary Economics*, *63*, 37–50.

Arntz, M., Gregory, T., and Zierahn, U. (2016). The Risk of Automation for Jobs in OECD Countries: A Comparative Analysis. *OECD Social, Employment and Migration Working Papers*, *189*.

Autor, D. H. (2015). Why are there still so many jobs? the history and future of workplace automation. *Journal of economic perspectives*, *29*(3), 3–30.

Autor, D. H., and Dorn, D. (2013). The growth of low-skill service jobs and the polarization of the us labor market. *American economic review*, *103*(5), 1553–1597.

Autor, D. H., Katz, L. F., and Krueger, A. B. (1998). Computing Inequality: Have

Computers Changed the Labor Market? *The Quarterly Journal of Economics*, *113*(4), 1169-1213.

Autor, D. H., Levy, F., and Murnane, R. J. (2003). The Skill Content of Recent Technological Change: An Empirical Exploration. *Quarterly Journal of Economics*, *118*(4), 1279–1333.

Biewen, M., and Seckler, M. (2019). Unions, Internationalization, Tasks, Firms, and Worker Characteristics: A Detailed Decomposition Analysis of Rising Wage Inequality in Germany. *The Journal of Economic Inequality*, *17*, 461-498.

Brall, F., and Schmid, R. (2023). Automation, robots and wage inequality in germany: A decomposition analysis. *LABOUR*, *37*(1), 33–95.

Cords, D., and Prettner, K. (2022). Technological Unemployment Revisited: Automation in a Search and Matching Framework. *Oxford Economic Papers*, *74*(1), 115–135.

Dauth, W., Findeisen, S., Südekum, J., and Wössner, N. (2021). The Adjustment of Labor Markets to Robots. *Journal of the European Economic Association*, *00*(0), 1-50.

Davis, D. R. (1998). Technology, unemployment, and relative wages in a global economy. *European Economic Review*, *42*(9), 1613–1633.

De Vries, G. J., Gentile, E., Miroudot, S., and Wacker, K. M. (2020). The rise of robots and the fall of routine jobs. *Labour Economics*, *66*, 101885.

Dustmann, C., Fitzenberger, B., Schönberg, U., and Spitz-Oener, A. (2014). From Sick Man of Europe to Economic Superstar: Germany's Resurgent Economy. *Journal of Economic Perspectives*, *28*(1), 167-188.

Dustmann, C., Ludsteck, J., and Schönberg, U. (2009, 05). Revisiting the German Wage Structure. *The Quarterly Journal of Economics*, *124*(2), 843-881.

Fahr, R., and Sunde, U. (2009). Did the Hartz reforms speed-up the matching process? A macro-evaluation using empirical matching functions. *German economic review*, *10*(3), 284–316.

Firpo, S., Fortin, N., and Lemieux, T. (2018). Decomposing Wage Distributions Using Recentered Influence Function Regressions. *Econometrics*, *6*(2), 1-40.

Frey, C. B., and Osborne, M. A. (2017). The Future of Employment: How Susceptible are

Jobs to Computerisation? *Technological Forecasting and Social Change*, *114*(C), 254–280.

Giannelli, G. C., Jaenichen, U., and Rothe, T. (2016). The evolution of job stability and wages after the implementation of the hartz reforms. *Journal for Labour Market Research*, *49*(3), 269–294.

Goldin, C., and Katz, L. F. (2008). *The Race between Education and Technology.* Cambridge, Massachusetts: Havard University Press.

Goos, M., and Manning, A. (2007). Lousy and lovely jobs: The rising polarization of work in britain. *The Review of Economics and Statistics*, *89*(1), 118-133.

Goos, M., Manning, A., and Salomons, A. (2014). Explaining job polarization: Routine-biased technological change and offshoring. *American economic review*, *104*(8), 2509–2526.

Guimarães, L., and Gil, P. M. (2022). Looking Ahead at the Effects of Automation in an Economy with Matching Frictions. *Journal of Economic Dynamics & Control*, *144*.

Hartung, B., Jung, P., and Kuhn, M. (2022). Unemployment insurance reforms and labor market dynamics. *mimeo*.

Hémous, D., and Olsen, M. (2022). The Rise of the Machines: Automation, Horizontal Innovation, and Income Inequality. *American Economic Journal: Macroeconomics*, *14*(1), 179–223.

Hertweck, M. S., and Sigrist, O. (2013). The aggregate effects of the hartz reforms in germany. *SOEPpapers on Multidisciplinary Panel Data Research 532*.

Hochmuth, B., Kohlbrecher, B., Merkl, C., and Gartner, H. (2021). Hartz IV and the Decline of German Unemployment: A Macroeconomic Evaluation. *Journal of Economic Dynamics and Control*, *127*, 104114.

Hornstein, A., Krusell, P., and Violante, G. L. (2005). Chapter 20 - The Effects of Technical Change on Labor Market Inequalities. In P. Aghion and S. N. Durlauf (Eds.), *Handbook of economic growth* (Vol. 1, p. 1275-1370). Elsevier.

Jaimovich, N., Saporta-Eksten, I., Siu, H., and Yedid-Levi, Y. (2021). The Macroeconomics of Automation: Data, Theory, and Policy Analysis. *Journal of Monetary*

Economics, *122*, 1–16.

Kariel, J. (2021). Job Creators or Job Killers? Heterogeneous Effects of Industrial Robots on UK Employment. *LABOUR*, *35*(1), 52-78.

Katz, L. F., and Autor, D. H. (1999). Chapter 26 - Changes in the Wage Structure and Earnings Inequality. In O. C. Ashenfelter and D. Card (Eds.), *Handbook of labor economics* (Vol. 3, p. 1463-1555). Elsevier.

Klinger, S., and Rothe, T. (2012). The impact of labour market reforms and economic performance on the matching of the short-term and the long-term unemployed. *Scottish journal of political economy*, *59*(1), 90–114.

Krause, M. U., and Uhlig, H. (2012). Transitions in the german labor market: Structure and crisis. *Journal of Monetary Economics*, *59*(1), 64-79.

Krebs, T., and Scheffel, M. (2013). Macroeconomic evaluation of labor market reform in germany. *IMF Economic Review*, *61*(4), 664–701.

Krugman, P. (1994). Past and prospective causes of high unemployment. *Reducing unemployment: Current issues and policy options*, 49–80.

Launov, A., and Wälde, K. (2013). Estimating incentive and welfare effects of nonstationary unemployment benefits. *International Economic Review*, *54*(4), 1159–1198.

Leduc, S., and Liu, Z. (2020). Robots or Workers? A Macro Analysis of Automation and Labor Markets. *Federal Reserve Bank of San Francisco Working Paper No. 2019-17*.

Mortensen, D. T., and Pissarides, C. A. (1994). Job Creation and Job Destruction in the Theory of Unemployment. *The Review of Economic Studies*, *61*(3), 397-415.

Mortensen, D. T., and Pissarides, C. A. (1999). Unemployment Responses to Skill-biased Technology Shocks: the Role of Labour Market Policy. *The Economic Journal*, *109*(455), 242-265.

Pissarides, C. A. (2000). *Equilibrium Unemployment Theory* (2nd ed.). Cambridge, Mass. and London: MIT Press.

Pries, M., and Rogerson, R. (2005). Hiring policies, labor market institutions, and labor market flows. *Journal of Political Economy*, *113*(4), 811–839.

Spitz-Oener, A. (2006). Technical change, job tasks, and rising educational demands:

Looking outside the wage structure. *Journal of labor economics*, *24*(2), 235–270.

Stadler, M., and Wapler, R. (2004). Endogenous skilled-biased technological change and matching unemployment. *Journal of Economics*, *81*, 1–24.

Weiss, M., and Garloff, A. (2011). Skill-biased technological change and endogenous benefits: the dynamics of unemployment and wage inequality. *Applied Economics*, *43*(7), 811–821.

Yashiv, E. (2004). Macroeconomic policy lessons of labor market frictions. *European Economic Review*, *48*(2), 259-284.

Zanetti, F. (2011). Labor market institutions and aggregate fluctuations in a search and matching model. *European Economic Review*, *55*(5), 644-658.

Chapter 2

Automation, Robots and Wage Inequality in Germany: A Decomposition Analysis

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Abstract. We conduct a decomposition analysis based on recentered influence function (RIF) regressions to disentangle the relative importance of automation and robotization for wage inequality in the manufacturing sector in Germany between 1996 and 2017. Our measure of automation threat combines occupation-specific scores of automation risk with sector-specific robot densities. We find that besides changes in the composition of individual characteristics, structural shifts among different automation threat groups are a non-negligible factor associated with wage inequality between 1996 and 2017. Moreover, the increase in wage dispersion among the different automation threat groups has contributed significantly to higher wage inequality in the 1990s and 2000s.

JEL classification: J31, C21, O33

Keywords: Automation, Decomposition Method, Linked Employer-Employee Data, RIF-regressions, Robots, Wage Inequality

2.1 Introduction

During the last decades, Germany experienced increasing wage inequality like many other industrialized countries all over the world. The considerable rise in German wage dispersion since the 1990s is well documented by a vast literature; see, e.g., Antonczyk et al. (2018), Card et al. (2013) and Dustmann et al. (2009). At the same time, automation technologies have entered virtually every area in the economy. The manufacturing sector uses widely automated processes that on the one hand increase the productivity of labor but on the other hand also enable the substitution of labor.

Although there is a lot of current research analysing the impact of automation on labor market outcomes, see, e.g., Acemoglu and Restrepo (2020), Dauth et al. (2021), De Vries et al. (2020) and Kariel (2021), we are one of the first who examine the relative contribution of automation and robotization on wage inequality using a decomposition analysis. In order to measure the contribution of automation and robotization, we implement a new measure of automation threat in which we combine the information about occupation- and requirement-specific automation probabilities with sector-specific robot densities. This allows us to take into account, that on the one hand working in a sector with lower robot density is associated with a lower automation threat than working in a sector with higher robot density, regardless of the occupation. On the other hand, working in the same sector but in different occupations or requirement levels naturally leads to a different threat of automation.

In addition, we enlarge the covered time period of the existing literature by considering wage inequality developments in the German manufacturing sector from 1996 to 2017. We find that the recent time period exhibits steady or even declining wage inequality developments. Nevertheless, even in this more recent period, we find evidence of an inequality-increasing contribution due to compositional changes in automation threat structures. Using the administrative linked employer-employee data provided by the German Institute for Employment Research (IAB), we are able to evaluate the importance of further individual-, firm- and industry-specific explanatory factors on German wage inequality. We apply the extended Oaxaca-Blinder decomposition method based on recentered influence function (RIF) regressions introduced by Firpo et al. (2018). Using this empirical estimation strategy, we are able to disentangle the relative contribution of several covariates on different inequality measures. Moreover, we are able to distinguish between composition and wage structure effects. It is important to note that the decomposition analysis enables us to identify sources that contribute to wage inequality, however, our results cannot be interpreted as causal effects.

We reveal that besides the commonly used demographic factors, our measure of automation threat contributes significantly to wage inequality in the German manufacturing sector. We identify compositional effects due to automation threat as a non-negligible factor associated with changes in wage inequality in Germany. There is an observable trend towards occupations with medium automation threat, accompanied by a decreasing share of occupations with high and low automation threat. Due to the fact that within-group wage inequality is the lowest in occupations with the highest automation threat, those compositional changes are associated with an increase in overall wage inequality.

Moreover, we find evidence that there is growing wage dispersion between workers in occupations with high and low automation threat that contributes to rising overall wage inequality between 1996 and 2010. This result is supported by the predictions of routine-biased technical change (RBTC), where technology is replacing labor in routine tasks and complements labor in non-routine tasks; see, e.g., Acemoglu and Autor (2011), Autor et al. (2003) and Goos and Manning (2007). An increase in technology would increase the relative demand for non-routine tasks compared to routine tasks, which leads to an increase in the relative wage returns of workers performing the former tasks. Our proposed automation threat variable captures different automation probabilities in occupations based on a task-based approach. Due to this, we can link the changes in relative wages between workers in occupations with high and low automation threat to RBTC, where the relative wage of non-routine tasks that are typically at low risk of automation is increasing compared to routine tasks that are usually faced with higher risk of automation, leading to a rise in wage dispersion between those two groups.

Regarding the general empirical approach and the applied data, this paper is related to Antonczyk et al. (2010), Biewen and Seckler (2019), Felbermayr et al. (2014) and

Baumgarten et al. (2020), who have implemented decomposition analyses of the wage distribution in Germany using linked employer-employee data. Antonczyk et al. (2010) and Biewen and Seckler (2019) analyze the increase in wage inequality in West Germany and show that firm effects, bargaining effects and personal characteristics mainly account for the rise in wage dispersion. Felbermayr et al. (2014) restrict the sample to the manufacturing sector and focus on the contribution of investment in new technologies and international trade to the increase in wage inequality from 1996 to 2010. Their results show that the change in the wage distribution can be explained to a large extent by composition effects, where the traditional factors such as age, education and collective bargaining agreements play the most important roles. Investment in new technologies as well as international trade had no significant influence on wage dispersion. More recently, Baumgarten et al. (2020) enlarge the covered time period up to 2014 and show that overall wage inequality in Germany has been rising up to 2010 before decreasing slightly thereafter.

There is a variety of theoretical and empirical literature that supports the implementation of automation threat as a factor of rising wage inequality. The endogenous growth models presented by Acemoglu and Restrepo (2018), Hémous and Olsen (2022) and Prettner and Strulik (2019) analyze labor-saving innovation and their impact on economic growth and inequality. While Acemoglu and Restrepo (2018) and Hémous and Olsen (2022) focus on the production sector in order to analyze under which conditions (low-skilled) workers could gain from automation, Prettner and Strulik (2019) endogenize education decisions of households in order to capture the race between education and technology. Beside conceptual differences, in all three endogenous growth models automation tend to increase wage inequality. Lankisch et al. (2019) present a variant of the Solow (1956) model with high-skill workers, low-skill workers and automation capital. In this simpler model, an increase in automation leads as well to a rise in the skill premium.

Turning to empirical literature, Autor et al. (2003) show that an increase in computerization goes along with a relative shift in labor demand towards college-educated workers. Furthermore, Acemoglu and Restrepo (2020) find evidence that a rise in robot exposure reduces employment and wages between 1990 and 2007 in the USA. In a similar way

Dauth et al. (2021) analyze the effect of robot exposure in Germany and show that a rise in robot exposure decreases employment of workers in the manufacturing industry. They provide evidence that the negative employment effect is offset by an increase in employment in the service sector. In addition, they show that robot exposure increases inequality within the manufacturing sector, because those who remain by their original employer experienced higher wages, while those who are forced to leave their original firm are faced with wage losses. Kariel (2021) introduces a new measure of automation that captures the regional exposure to automation innovation and finds evidence that automation has a negative impact on manufacturing employment in the UK, while it increases employment in other industries such as services. De Vries et al. (2020) analyze the impact of industrial robots on occupational shifts by task content and find evidence that the increased use of robots rise the employment share of especially non-routine analytic jobs, while it decreases the share of routine manual jobs. Aksoy et al. (2021) examine the impact of robots on the gender wage gap in European countries and find evidence that while both men and women receive an increase in earnings due to robotization, men at medium- and high-skill occupations benefit disproportionately. Kaltenberg and Foster-McGregor (2020) present related decomposition analyses on wage distributions in 10 European countries, where Germany is not included, and focus on the impact of automation risk of occupations between 2002 and 2014. They find evidence that the composition effect contributes to a large extent to automation related wage dispersion in all countries, while the wage effect explains automation related inequality in half of the countries.

The remainder of this paper proceeds as follows: Section 2.2 describes the different data sets used in our empirical analysis. In Section 2.3 we outline our empirical approach and define our variable quantifying automation threat. Descriptive evidence on the development of wage inequality and automation as well as descriptive statistics of our explanatory variables are revealed in Section 2.4. Finally, we present our empirical results in Section 2.5 before we conclude in Section 2.6.

2.2 Data

Labor Market Data. We use German linked employer-employee data (LIAB), provided by the Research Data Center of the Institute for Employment Research (IAB).[1] The data set combines information on the yearly representative employer survey (IAB Establishment Panel) with the corresponding establishment and individual data, drawn from labor administration and social security. The IAB Establishment Panel has been conducted since 1993 in West Germany as well as since 1996 in East Germany and contains establishments with at least one employee subject to social security. The sample size of the IAB Establishment panel increased from roughly 4,000 establishments in 1993 to more than 16,000 establishments in 2017. Due to the fact that larger establishments are overrepresented, the IAB provides appropriate weights to ensure a representative sample. This sample of establishments is matched with the social security data of workers who were employed in those establishments on June 30th of each year. Therefore, workers that do not contribute to social security are not included in the panel.

The main advantage of the LIAB data is the wide set of information of the workers characteristics and of the particular establishment in which they are employed. The data contains personal information of the workers such as gender, year of birth, nationality, vocational training, education and place of residence as well as information on their employment like daily wage, occupation, task level and number of days in employment. Moreover, the data set provides information on the establishments such as the classification of economic activities, total number of employees and region.

We restrict the data to male full-time workers in the manufacturing sector in their main employment between 18 and 65 years, who earned more than 10 Euros per day and consider the time period between 1996 and 2017.[2] Following the common literature on wage inequality in Germany, we restrict our analysis to West Germany, due to the

[1]In more detail, this study uses the LIAB cross-sectional model 2, version 1993-2017, of the Linked-Employer-Employee Data (LIAB) from the IAB. Data access was provided via on-site use at the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB) and subsequently remote data access. DOI: 10.5164/IAB.LIABQM29317.de.en.v1. For detailed data description see Schmidtlein et al. (2019).

[2]Due to the fact that the data do not contain any information on the number of working hours, we decide to consider only men working full-time. We are aware of this strong restriction, nevertheless it reduces noise and increases consistency in the analysis.

fact that East and West Germany are still faced with significantly different labor market and wage structures; see, e.g., Baumgarten et al. (2020), Biewen and Seckler (2019) and Dustmann et al. (2009). The wage earnings recorded by social security are right-censored at the contribution assessment ceiling of the social security system. To account for this problem, we use imputed wages following the approach by Gartner (2005).[3] Non-censored and imputed wages are converted into constant 2015 Euros with the Consumer Price Index provided by the German Federal Statistical Office.

Robot Data. The data on industrial robots is obtained from the International Federation of Robotics (IFR), which is commonly used in recent analysis; see, e.g., Acemoglu and Restrepo (2020), Anelli et al. (2021) and Dauth et al. (2021). The data contain the stock of industrial robots for 50 countries broken down at the industry level, where data availability differs across countries. German robot data is available from 1993 to 2017. An industrial robot is defined as "an automatically controlled, reprogrammable, multipurpose [machine]" International Federation of Robotics (2018).[4] The data rely on primary and secondary data sources. The primary source are yearly surveys of worldwide industrial robot suppliers that report their stock of industrial robots to the IFR. Additionally, the IFR uses secondary data collected by national robot associations to validate the survey data. Before 2004, the data on German industrial robots rely solely on collected data by national robot associations.

The industry classifications in the IFR data are very coarse and differ between the manufacturing and non-manufacturing sector, which is one of the main disadvantage of the data. Away from the manufacturing sector, industries are aggregated to very broad groups, while among the manufacturing sector the data are more disaggregated. Thus, our analysis focuses on the manufacturing sector in Germany due to better data availability and the predominant role of automation in this sector. Industrial robot data reported by the IFR is mainly based on the International Standard Industrial Classification of

[3] In order to circumvent estimations that are driven by the imputation procedure, the analysis provides results including only the uncensored part of the wage distribution represented by the inter-percentile ranges up to the 85th percentile.

[4] We consider only industrial robots in the analysis. Data on service robots is also available since 2002. However, the data is not available at the industry level during the considered time period.

All Economic Activities (ISIC) Rev. 4.[5] In total, we focus on 8 different manufacturing sectors: 10-12 food products, beverages and tobacco products, 13-15 textiles, wearing apparel, leather and related products, 16-18 wood (including furniture) and paper products, printing and reproduction of recorded media, 19-23 coke and refined petroleum products, chemical products, pharmaceutical products, rubber and plastics products, and other non-metallic mineral products, 24-25 basic metals and fabricated metal products, 26-27 computer, electronic and optical products, electrical equipment, 28 industrial machinery and equipment n.e.c., 29-30 automotive and other vehicles.[6] The IFR data can be matched with the LIAB data without any crosswalk, for further information see Appendix 2.A.1.

Automation Risk Data. We use an occupation- and requirement-specific score of automation risk. In contrast to the commonly used measure provided by Frey and Osborne (2017), we relate to specific estimations for occupations in Germany based on the task-based approach by Dengler and Matthes (2015). The resulting advantages are presented in the Appendix 2.A.2. Dengler et al. (2014) calculate the task composition for different occupations, based on BERUFENET Expert Database of the German Federal Employment Agency. The data set contains information of around 3,900 single occupations, such as the tasks to be performed in the respective occupation, the equipment or the working conditions. The so called requirement matrices classify 8,000 different requirements to each single occupation. Dengler et al. (2014) assign to each requirement one task type (analytical non-routine tasks, interactive non-routine tasks, cognitive routine tasks, manual routine tasks and manual non-routine tasks). The central criterion whether the task is routine or non-routine is the substitutability of computers or computer-controlled machines, based on the available technology in 2013.[7]

On the basis of this data, Dengler and Matthes (2015) estimate the share of routine

[5]Within the manufacturing sector there is one exception at the 2-digit level. The IFR classification uses the 2-digit code 16-Wood and furniture. This industry contains the ISIC Rev. 4 code 16 and 31.

[6]As Dauth et al. (2021) and Graetz and Michaels (2018), we exclude *All other manufacturing branches*, since it covers only 6.8% of the robot stock in the manufacturing sector in 1996 and the share declines to 1.7% in 2017.

[7]There are already updated versions of the automation probabilities based on the available technology in 2016, see Dengler and Matthes (2018), and 2019, see Dengler and Matthes (2021). Due to the fact that the considered time period in our analysis begins in 1996, we use the automation probabilities calculated on the basis of the available technology in 2013.

tasks to non-routine tasks for each single occupation, by dividing the core requirements, that are essential for the occupation, in each single occupation that have been assigned to a routine task by the total number of core requirements in the respective single occupation.[8] Next, they aggregate the shares of routine tasks for each single occupation into different occupation aggregates, using weights based on employment numbers from 2012. The weights ensure that single occupations with high employment are taken more into consideration, when determining the substitution potential at the aggregated occupational level. The share of routine activities is used to determine the substitution potential of the occupation.

The data is available in the 2-digit Classification of Occupations 2010 (Klassifizierung der Berufe 2010, KldB 2010). In addition, they distinguish for each 2-digit KldB 2010 code four different requirement levels.[9] In total, they estimate substitution potentials for 131 occupation-requirement level combinations. The LIAB data contains occupation codes and requirement levels in the KldB 2010 classification.[10] Therefore, merging both data sets is possible without any crosswalk.

2.3 Empirical Approach

2.3.1 Method

Oaxaca-Blinder Decomposition. The standard OB decomposition divides the overall mean wage gap, $\hat{\Delta}_O^\mu$, between two defined groups, in our case two points in time ($t = 0, 1$) (Firpo et al., 2018; Oaxaca, 1973). Assuming a linear wage equation, where w_t denotes the log wage and X is a vector of covariates, the following holds true:

$$\begin{aligned}\hat{\Delta}_O^\mu &= \bar{X}_1(\hat{\beta}_1 - \hat{\beta}_0) + (\bar{X}_1 - \bar{X}_0)\hat{\beta}_0 \\ &= \hat{\Delta}_S^\mu + \hat{\Delta}_X^\mu.\end{aligned} \tag{2.1}$$

[8] For example, if one single occupation contains three different core requirements, and one requirement is assigned to a routine task, then the share would be 1/3.

[9] The requirement levels correspond to the 5th digit KldB 2010 classification: 1-unskilled activities, 2-specialist activities, 3-complex activities, 4-highly complex activities.

[10] The occupational information before 2011 was reported using the occupation code KldB 1988. This older classification is less detailed than the occupation code KldB 2010, which leads to inaccuracies.

The first part of equation (2.1) denotes the wage structure effect, $\hat{\Delta}^{\mu}_{S}$, which is the result of holding the distribution of covariates constant and only modifying the conditional wage structure.[11] Thus, in other words this effect represents the differences in the estimated coefficients between the two groups and shows the way the specific characteristics are valued in the labor market. The second part is the composition effect, $\hat{\Delta}^{\mu}_{X}$, where the conditional wage structure is held constant and the distribution of covariates varies according to the observed changes between the two points in time (Fortin et al., 2011). In other words, this effect presents the differences in the distribution of the explanatory factors between the two points in time.

RIF-Regression Approach. The RIF-regression approach allows to quantify the impact of each covariate, conditional on all other factors, on the change in wage inequality measures, such as percentile wage gaps, the variance or the Gini coefficient (Firpo et al., 2018). Thus, the dependent variable, w, is replaced by the recentered influence function of the statistic of interest. The influence function, $IF(w; v)$, of an observed wage w for the distributional statistic $v(F_w)$, that is dependent on the wage distribution F_w, shows the influence of each observation on this distributional statistic. The conditional expectation of the $RIF(w; v)$ can be estimated using a linear function of the explanatory variables, i.e. $E[RIF(w; v)|X] = X\gamma$, where the parameters γ can be estimated by OLS (Fortin et al., 2011).

When it comes to quantiles, the estimated coefficients are interpreted as unconditional (quantile) partial effects (UQPE) of small location shifts in the covariates (Firpo et al., 2009). Using the RIF-regression approach it is possible to identify the effect of a changing explanatory variable on the τth quantile of the unconditional distribution of w. This procedure is different to the commonly used conditional quantile regressions.

With the estimated coefficients of the unconditional quantile regressions, $\hat{\gamma}_{t,\tau}$, for each

[11] Using categorical variables in a detailed decomposition, the estimated wage structure effect depends on the defined base group. Therefore, the effect of changes in the returns have to be interpreted based on this omitted group Fortin et al. (2011).

group of $t = 0, 1$ the OB decomposition can be written as:

$$\begin{aligned} \hat{\Delta}_O^\tau &= \bar{X}_1(\hat{\gamma}_{1,\tau} - \hat{\gamma}_{0,\tau}) + (\bar{X}_1 - \bar{X}_0)\hat{\gamma}_{0,\tau} \\ &= \hat{\Delta}_S^\tau + \hat{\Delta}_X^\tau, \end{aligned} \tag{2.2}$$

where $\hat{\Delta}_O^\tau$ defines the wage gap at the τth unconditional quantile. The first term of equation (2.2) corresponds to the wage structure effect that is obtained by holding the distribution of the covariates constant and only modifying the conditional wage structure represented by the RIF coefficients. The second term represents the composition effect, which is the result of holding the conditional wage structure constant and changing the distribution of the covariates according to the observed change between the points in time $t = 0$ and $t = 1$. The detailed decomposition can be computed similarly as in the case of the mean (Fortin et al., 2011).

However, as in the standard OB decomposition it could be the case that the linearity assumption does not hold.[12] Therefore, the two step procedure proposed by Firpo et al. (2018) is used in order to avoid this problem. In a first step, a counterfactual sample, which is defined by point in time $t = 01$, is estimated applying the reweighting function introduced by DiNardo et al. (1996). Using the reweighting function the hypothetical sample makes the characteristics of point in time $t = 0$ similar to those of point in time $t = 1$. In a second step, two OB decompositions are specified by using the three different samples.

The first OB decomposition uses the sample $t = 0$ and the counterfactual sample $t = 01$ to estimate the reweighted composition effect, $\hat{\Delta}_{X,R}^\tau$, as follows:

$$\begin{aligned} \hat{\Delta}_{X,R}^\tau &= (\bar{X}_{01} - \bar{X}_0)\hat{\gamma}_{0,\tau} + \bar{X}_{01}(\hat{\gamma}_{01,\tau} - \hat{\gamma}_{0,\tau}) \\ &= \hat{\Delta}_{X,p}^\tau + \hat{\Delta}_{X,e}^\tau, \end{aligned} \tag{2.3}$$

where the first part of the right-hand side of equation (2.3) corresponds to the pure composition effect, while the second part represents the specification error.

[12] As discussed by Barsky et al. (2002), if the linearity assumption does not hold, the estimated counterfactual mean wage would not be equal to $\bar{X}_1\hat{\beta}_0$ in the case of the standard OB decomposition.

The wage structure effect is estimated in a similar way using the sample $t = 1$ and the counterfactual sample $t = 01$:

$$\begin{aligned}\hat{\Delta}^{\tau}_{S,R} &= \bar{X}_1(\hat{\gamma}_{1,\tau} - \hat{\gamma}_{01,\tau}) + (\bar{X}_1 - \bar{X}_{01})\hat{\gamma}_{01,\tau} \\ &= \hat{\Delta}^{\tau}_{S,p} + \hat{\Delta}^{\tau}_{S,e},\end{aligned} \tag{2.4}$$

where the first term of the right-hand side of equation (2.4) defines the pure wage structure effect and the second part denotes the reweighting error. Since the counterfactual sample $t = 01$ is used to imitate the sample of point in time $t = 1$, in large samples it should be $plim(\bar{X}_{01}) = plim(\bar{X}_1)$.

The description of the RIF-regressions based OB decomposition is limited to specific percentiles of the wage distribution. In order to estimate effects on percentile wage gaps, the difference between the respective estimated coefficients of the corresponding percentiles has to be computed. Regarding other distributional statistics, like the variance or the Gini coefficient, the RIF-regressions have to be adjusted accordingly (see Firpo et al., 2018).

The following analysis is based on different inequality measures. Depending on which index is used, a specific part of the wage distribution is taken into focus. The commonly used Gini coefficient is one of the standard indices and measures inequality considering the whole wage distribution. However, it has to be taken into account that the Gini index is more sensitive to changes in the middle of wage distribution and less sensitive to changes at the top and the bottom of wage distribution (Atkinson, 1970). That is why we use in addition percentile wage gaps not only between the highest and the lowest wages (85th-15th wage gap) but also in relation to the median wages (50th-15th and 85th-50th wage gaps). Thus, it is also possible to observe changes separately for the lower and upper half of the wage distribution. Further results of the variance are presented in order to have comparative values for estimates of the whole distribution.

The fact that the method uses simple regressions that are easy to interpret provides a straightforward way of a detailed decomposition. Compared to the sequential decomposition introduced by DiNardo et al. (1996) (DFL-method), the RIF-regressions based detailed decomposition does not suffer from path dependence. However, the RIF-regression assumes the invariance of the conditional distribution and therefore does not take general equilibrium effects into account (Fortin et al., 2011). Moreover, this decomposition method ascribes the change in wage inequality completely to the considered covariates. Thus, the sum of all composition effects and wage structure effects defines the overall change in wage inequality over time.

2.3.2 Model Specification

The decomposition analyses consider a wide range of covariates that are determinants to changes in the wage distribution. Besides the commonly used personal and plant characteristics, we propose a measure of automation threat that is described in more detail below. The personal characteristics include the individual's age (five categories)[13]; education (three categories)[14]; tenure (five categories)[15]; and a dummy variable capturing German or foreign citizenship. Furthermore, we consider the following two plant characteristics: plant size (six categories)[16]; and the bargaining regime (three categories)[17]. In addition, we control for fixed effects of 8 different manufacturing sectors and include federal state dummies to capture regional shifts.[18]

The main factor of interest is our new introduced measure of automation threat, which captures two dimensions of automation. On the one hand, we take the various evolution of the sectoral robot density into account, which is often used as an approximation of automation exposure. On the other hand, we consider the different automation risk of workers due to the task content of their occupation. Therefore, we merge data on the substitution potential of an occupation provided by Dengler and Matthes (2015), which

[13](1) 18-25 years; (2) 26-35 years; (3) 36-45 years; (4) 46-55 years; (5) 56-65 years.

[14](1) Low: lower/middle secondary without vocational training; (2) Medium: lower/middle secondary with vocational training or upper secondary with or without vocational training; (3) High: university of applied sciences or traditional university.

[15](1) 0-2 years; (2) 2-4 years; (3) 4-8 years; (4) 8-16 years; (5) >16 years.

[16](1) 1-9 employees; (2) 10-49 employees; (3) 50-199 employees; (4) 200-999 employees; (5) 1000-4999 employees; (6) ≥5000 employees.

[17](1) Sector-level agreement; (2) Firm-level agreement; (3) No collective bargaining agreement.

[18]The base category is a medium-skilled worker between 26 and 35 years, with 0-2 years of tenure, with German citizenship and is exposed to low automation threat. Further, the worker is employed in an establishment with 200-999 employees, which has no collective bargaining agreement, belongs to the basic metals and fabricated metal products sector and is located in North Rhine-Westphalia.

we interpret as a proxy variable for the automation probability of an occupation, with the IFR robot data. This procedure combines the occupational information about the automation probability with the time varying sectoral information about the number of robots per 1,000 workers:[19]

$$automation\ threat_{j,s,t} = \theta_j * \frac{Robots_{s,t}}{emp_{s,1995}}. \tag{2.5}$$

where θ_j is the automation probability of occupation j, $Robots_{s,t}$ is the stock of operational robots in sector s in year t and $emp_{s,1995}$ is the number of employees in thousands in the corresponding sector s in the base year 1995.[20] Thus, each individual working in occupation j and sector s is confronted with the corresponding automation probability of its occupation and a specific sectoral robot density of a given year t.

For our decomposition analysis we have to define three groups of different automation threat in order to ensure the common support assumption.[21] In a first step we have a look at the total number of all combinations of the occupation specific automation probabilities with the sector specific robot densities in a specific year sorted by size. Then we define cut-off points in a way that the number of combinations in a specific year is divided into three groups.[22] As a consequence, we are able to assign every individual to either low, middle or high automation threat. This procedure is done separately for each year.

[19]In a familiar way, this approach is used in Anelli et al. (2019) to capture the individual exposure to automation. In a first step, a multinomial logit model is estimated using all available covariates to predict the probability of an individual being in a certain occupation. This probability is multiplied with the corresponding automation probability in that occupation to obtain an individual vulnerability to automation. In a last step, the individual vulnerability is multiplied with the national percentage change in total operational robots in a country. Due to the characteristics of our estimation strategy it is not possible to implement this kind of automation threat variable.

[20]The data on sectoral employment in 1995 is provided by EU KLEMS database, see Stehrer et al. (2019).

[21]The common support assumption is one of the main conditions proposed by Fortin et al. (2011) that ensures a successful estimation of the decomposition. This assumption imposes the condition of common support on the covariates and makes sure that no observation can serve to identify the assignment into one specific group (Fortin et al., 2011). Due to this condition it is not possible to use a continuous variable measuring automation threat. The considerable increase over time would lead to exclusively present values in points in time $t = 0$ and $t = 1$, which contradicts this assumption.

[22]For example, if there are in total 300 possible occupation-sector combinations in one year, the first group includes the lowest 100 combinations, the second group the 100 combinations in the middle and the third group the 100 highest combinations. There are two cut-off points, namely the values of the 100th and the 200th combination. Of course, the values of these cut-off points increase over time as the values of the automation threat variable increases as well.

The estimation strategy of this variable is reasoned by the following considerations. First of all, since the automation probabilities are time constant, adding yearly information about the stock of robots in a given sector adds a time dimension to our proposed automation variable. Due to this, the significant increase in the use of robots is represented and considered in our subsequent analysis. Second, the sector specific robot densities influence the relative degree of automation threat, since there are substantial differences between economic sectors. In other words, the automation probability of an occupation exhibits a different importance depending on the specific sector.

The necessity of the combination between automation probabilities and sector specific robots densities is shown in Table 2.B.1 in Appendix 2.B. Here the distribution of the different economic sectors within the three groups of automation threat is compared to the shares of economic sectors within the groups based on the automation probabilities by Dengler and Matthes (2015).[23] The first thing that becomes apparent is the fact that in the medium and especially in the high automation threat group not all economic sectors are represented. Looking at the robot densities reveals that the missing sectors (textiles and wood, furniture and paper) indeed exhibit the lowest values. The low robot density weights the automation probability down, which leads to the result that no employee within this sector is faced with a high (or even medium) automation threat. Another striking feature is the relatively low share in the low and medium automation threat group within the automotive sector. This is due to the fact that the automotive sector is faced with a very high robot density which leads to an upweight of the automation probabilities. This takes into account that working in a sector with higher robot density is associated with a higher automation threat than working in a sector with lower robot density, regardless of the occupation. These findings validate the combination of automation probabilities of occupations and sector specific robot densities. Further descriptive information about our proposed variable is presented in the following.

[23]The group of low automation risk is given if a maximum of 30% of the occupation could be performed by computers. The medium automation risk captures those occupations, which are substitutable by automation between 30% and a maximum of 70% and high automation risk exists if more than 70% of the occupation could be performed by computers.

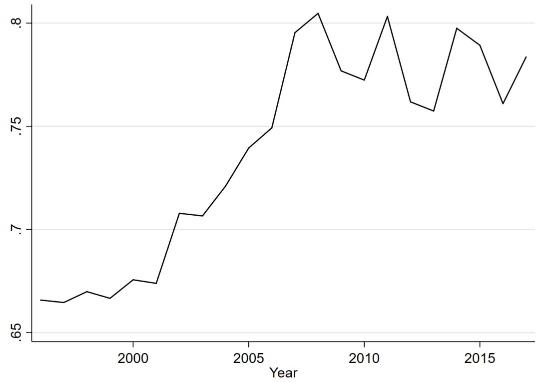
2.4 Descriptive Evidence

Developments in Wage Inequality. The development of wage inequality in the German manufacturing sector defined by the difference between the 85th and 15th percentiles of log real daily wages for men working full-time is displayed in Panel (a) of Figure 2.1. Starting with a short period of moderate increase in wage inequality, a significant rise in the wage gap is observable between 2001 and 2008. In the subsequent years, wage inequality shows an alternating behaviour but is not subjected to major increases as before. A similar pattern is observable by having a look at the development of the Gini coefficient, which measures the normalised average absolute difference between all wage pairs in the workforce. As a result of these observations, we divide our overall period of observation into two subperiods, 1996 to 2010 in which wage inequality is overall increasing and 2012 to 2017 in which wage inequality more or less stagnates.[24]

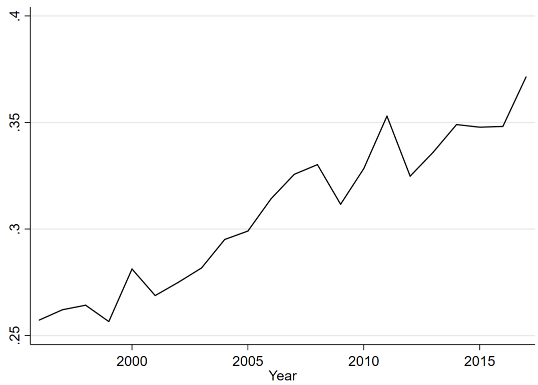
Since the 85-15 percentile wage gap only takes the top and bottom percentiles into account, developments in the middle of the distribution are omitted. Therefore, the wage gaps between the 50th and 15th percentiles as well as between the 85th and 50th percentiles are presented to account on the one hand for developments at the lower half and on the other hand for developments at the upper half of the wage distribution. The results presented in Panel (b) of Figure 2.1 suggest that in the manufacturing sector a significant increase in inequality at the lower part of the wage distribution is observable. This development is seen throughout the whole period of observation. Regarding the findings of the wage gap in the upper half of the distribution a different pattern is identified. Panel (c) of Figure 2.1 shows a noticeable increase between 2000 and 2008. However, in the following years inequality at the upper part of the wage distribution decreased significantly and ends up in 2017 almost at the same level as in 1996. These trends result in the consistent increase of the overall wage inequality until 2008. Thereafter, the observed developments in wage inequality at the lower and upper parts of the wage

[24] Baumgarten et al. (2020) consider similar time periods: 1996 to 2010 and 2010 to 2014. Due to a change in the reporting procedure of the social security agency, a considerable increase in the number of missing values occurs in the year 2011. In order to circumvent this possible source of misleading estimation results, we define 2012 as our starting point of the second period of observation. For more information see Schmidtlein et al. (2019).

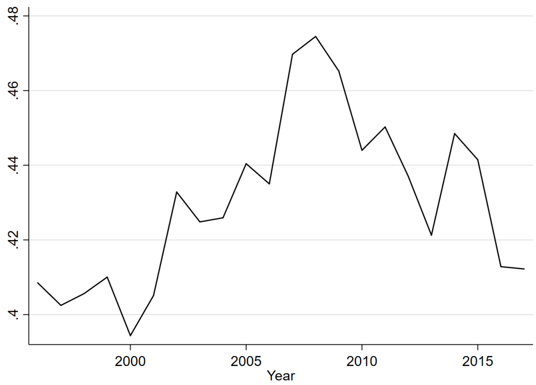
distribution balance each other out.



(a) 85-15 percentile wage gap



(b) 50-15 percentile wage gap



(c) 85-50 percentile wage gap

Figure 2.1: The evolution of the 85-15, 50-15 and 85-50 percentile wage gap between 1996 and 2017

Notes: The figure presents the evolution of the 85-15, 50-15 and 85-50 percentile wage gap between 1996 and 2017 for men working full-time in the manufacturing sector in Germany. The results are based on imputed real daily wages. Sampling weights are employed.
Source: LIAB QM2 9317, own calculations.

The Rise of Automation. At the same time, automation technology accelerated since the 1990s. This increase is also captured by our automation threat variable, despite sectoral differences. Figure 2.C.1 in Appendix 2.C illustrates the estimated automation threat variable in equation (2.5) summarized over all occupations in each manufacturing sector in Germany from 1996 to 2017. While most sectors experienced an increase in automation threat, the wood, furniture and paper sector and the textiles sector have seen a slight decrease in automation threat. It is striking that the automotive and other vehicles sector was faced with an extraordinarily increase compared to the other sectors.

Automation threat in the automotive and other vehicles sector was eight times higher in 1996 compared to the average of automation threat in the other manufacturing sectors. In 2017 automation threat was even almost twelve times higher than in the other sectors.

Descriptive Statistics of Explanatory Variables. Since one important part of the OB decomposition are changes in the composition of workers, we present in Table 2.1 the descriptive statistics of our considered explanatory variables for the years 1996, 2010, 2012 and 2017. The first column of each year gives the mean of the respective variable, whereas in the second column the corresponding standard deviation is listed. Looking at the first row, a clear trend towards higher real daily wages becomes apparent, where between 1996 and 2010 an increase by 9% and between 2012 and 2017 an increase by 7% is observed. The demographic factors regarding age and education reflect the often described trend in the literature towards an older and more educated workforce. The share of highly skilled workers increased in our sample from 9% in 1996 to more than 15% in 2017, whereas at the same time the low skilled group is halved, from 12% to 6%. In addition, workers tend to have a higher tenure. The group of workers with more than 16 years of employment increased by more than 16 percentage points over the whole period of observation, whereas all other groups decreased in size over time. Workers are denoted as foreigners or natives based on their nationality. During the observed time span the amount of workers with a foreign nationality decreased, which is presumably the result of a change in the German nationality law.

Regarding plant characteristics, one striking development is presented when it comes to collective bargaining coverage. Between 1996 and 2017 the group of workers that is not covered by any sort of collective bargaining agreement increased from 8% to 29%, whereas the group with sector level agreements decreased from 82% to 58%. The fraction of workers with firm level agreements slightly increased. Regarding the size of the plants, a tendency away from smaller firms with less than 200 employees becomes apparent. In total, the share of the group with more than 5,000 employees increased by 9 percentage points. Looking at compositional changes of the sectors and changes in employment shares of the different federal states no major differences over the years appear.

Table 2.1: Descriptive statistics

	1996		2010		2012		2017	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Real daily wage	126.42	(51.31)	137.52	(69.71)	137.19	(67.78)	147.33	(70.32)
Age: 18-25 years	7.39	(26.17)	5.73	(23.25)	6.65	(24.92)	5.84	(23.45)
Age: 26-35 years	32.19	(46.71)	18.04	(38.45)	18.77	(39.05)	20.17	(40.13)
Age: 36-45 years	28.62	(45.19)	30.87	(46.19)	26.58	(44.18)	22.49	(41.75)
Age: 46-55 years	22.29	(41.62)	33.88	(47.33)	34.04	(47.38)	33.68	(47.26)
Age: $\geq$ 56 years	9.51	(29.33)	11.48	(31.87)	13.96	(34.65)	17.81	(38.26)
Education: low	12.21	(32.73)	8.65	(28.10)	7.22	(25.89)	6.03	(23.80)
Education: middle	78.55	(41.04)	77.64	(41.66)	78.25	(41.25)	78.49	(41.09)
Education: high	9.23	(28.96)	13.71	(34.39)	14.53	(35.24)	15.48	(36.17)
Tenure: 0-2 years	5.11	(22.02)	2.45	(15.47)	3.24	(17.70)	2.61	(15.95)
Tenure: 2-4 years	5.33	(22.46)	3.38	(18.06)	3.78	(19.07)	3.95	(19.48)
Tenure: 4-8 years	16.94	(37.50)	9.03	(28.65)	9.48	(29.29)	9.35	(29.10)
Tenure: 8-16 years	25.32	(43.48)	22.15	(41.52)	21.18	(40.86)	20.10	(40.07)
Tenure: $\geq$ 16 years	47.30	(49.93)	62.99	(48.28)	62.32	(48.45)	63.99	(48.00)
Nationality	11.32	(31.69)	8.74	(27.91)	9.25	(28.97)	8.92	(28.50)
Automation threat: low	11.14	(31.46)	7.73	(26.70)	10.93	(31.21)	12.76	(33.36)
Automation threat: middle	17.26	(37.79)	25.45	(43.56)	23.41	(42.34)	25.12	(43.37)
Automation threat: high	71.60	(45.09)	66.82	(47.08)	65.66	(47.48)	62.12	(48.51)
No collective agreement	7.75	(26.73)	28.36	(45.07)	31.07	46.28	29.25	(45.49)
Firm level agreement	9.91	(29.88)	13.38	(34.04)	11.80	(32.26)	12.83	(33.43)
Sector level agreement	82.34	(38.13)	58.25	(49.31)	57.13	(49.49)	57.92	(49.36)
Plant size: 1-9 employees	5.30	(22.41)	3.08	(17.27)	3.09	(17.29)	2.19	(14.64)
Plant size: 10-49 employees	14.75	(35.46)	13.71	(34.39)	13.69	(34.37)	10.91	(31.17)
Plant size: 50-199 employees	21.86	(41.33)	23.56	(42.44)	23.02	(42.09)	19.05	(39.27)
Plant size: 200-999 employees	30.79	(46.16)	31.67	(46.52)	32.99	(47.01)	35.08	(47.72)
Plant size: 1000-4999 employees	17.14	(37.68)	18.48	(38.82)	16.68	(37.28)	13.59	(34.27)
Plant size: $\geq$ 5000 employees	10.16	(30.22)	9.50	(29.32)	10.53	(30.71)	19.17	(39.37)
Sector: Food and beverages	6.58	(24.79)	7.05	(25.59)	6.89	(25.33)	9.74	(29.64)
Sector: Textiles	2.93	(16.87)	1.33	(11.44)	1.30	(11.32)	0.76	(8.69)
Sector: Wood, furniture and paper	9.34	(2909)	8.38	(27.71)	7.36	(26.11)	7.01	(25.53)
Sector: Plastic and chemical products	14.20	(34.91)	14.24	(34.95)	13.93	(34.62)	10.46	(30.61)
Sector: Metal products	21.02	(40.75)	22.38	(41.68)	23.77	(42.56)	18.87	(39.13)
Sector: Electrical products	10.49	(30.64)	14.15	(34.86)	12.06	(32.57)	10.76	(30.98)
Sector: Industrial machinery	20.66	(40.48)	16.46	(37.08)	19.41	(39.55)	19.40	(39.54)
Sector: Automotive and other vehicles	14.77	(35.48)	16.01	(36.67)	15.28	(35.97)	23.00	(42.08)
Schleswig-Holstein	2.12	(14.39)	2.46	(15.48)	1.94	(13.78)	1.59	(12.51)
Hamburg	2.04	(14.18)	3.37	(18.04)	3.71	(18.90)	3.69	(18.85)
Lower Saxony	11.86	(32.33)	10.31	(30.40)	10.36	(30.47)	8.81	(28.34)
Bremen	1.18	(10.81)	0.52	(7.19)	1.01	(10.00)	0.74	(8.57)
North Rhine-Westphalia	30.29	(45.95)	27.83	(44.82)	27.93	(44.87)	22.87	(42.00)
Hesse	8.85	(28.39)	6.66	(24.93)	7.80	(26.81)	7.95	27.06
Rhineland-Palatinate	5.13	(22.05)	5.86	(23.49)	5.51	(22.81)	5.98	(23.71)
Baden-Wuerttemberg	18.69	(38.98)	20.88	(40.64)	19.52	(39.63)	17.46	(37.96)
Bavaria	18.04	(38.44)	20.38	(40.28)	21.25	(40.91)	30.07	(45.85)
Saarland	1.80	(13.28)	1.73	(13.05)	0.97	(9.82)	0.83	(9.09)
Observations	576,895		389,624		437,336		320,970	

Notes: The table presents the descriptive statistics for four time points, standards errors are given in parentheses. All variables, except the real wage, are reported in percent. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

When it comes to our proposed automation threat variable, there is an observable trend towards the medium group of automation between 1996 and 2010. At the same time, this observation is accompanied with a reduction by nearly 5 percentage points in the highest automation group and a decrease in the group with the lowest automation threat by more than 3 percentage points. From this one could conclude two movements. On the one hand, it seems that workers are displaced by automation in the groups of high automation threat. On the other hand, it becomes more and more impossible to resist automation in work life, which leads to a decrease in the share of the lowest automation threat group. In the second time period the share of workers which are faced with high automation threat decreased further, although at a smaller amount and the middle automation threat group is still increasing. In contrast to the first period, the share of workers in the lowest automation threat group slightly increased between 2012 and 2017.

To get a first impression about the relation between automation threat and changes in wage inequality, we provide descriptive evidence of differences in within-group wage inequality. In Figure 2.2 the estimated Gini coefficients for the respective groups of automation threat for the whole period of observation are illustrated. In all three groups the significant increase of wage inequality between 1996 and 2008 and the stagnation thereafter becomes apparent. However, there is a substantial difference in the level of wage inequality between the high automation threat group and the groups with middle and low automation threat. The lowest wage inequality is found in the highest group of automation threat. Table 2.B.2 in Appendix 2.B reveals that the average real daily wages of the high automation threat group are predominantly lower than those from the medium or lowest automation threat groups, however the distribution of wages within this group is the most equal.

In order to figure out the reasons behind these results, we have a closer look at the educational and occupational structures within these three groups. Table 2.B.2 in Appendix 2.B shows that the highest automation threat group exhibits a mainly similar level of education with more than 80% in the medium group throughout the entire period of observation. Thus, the two remaining educational groups play only a minor role in this case. A different picture emerges when it comes to the medium and lowest groups of au-

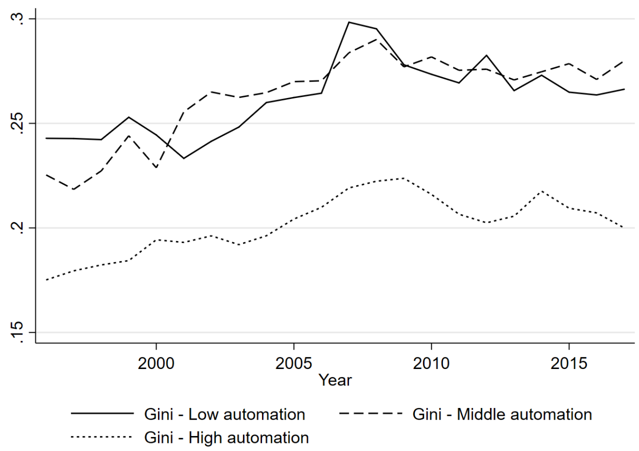


Figure 2.2: Gini coefficients in different automation threat groups, 1996-2017

Notes: The figure presents the evolution of the group-specific Gini coefficient estimations between 1996 and 2017. We distinguish between low, medium and high automation threat. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

tomation threat. Although the medium educational level still makes up the largest group in both cases, especially the highest educational level plays a more important role and therefore leads to a more diverse structure. When it comes to the requirement levels a similar picture emerges. A significant clustering of workers in the second requirement level of specialist activities in the highest group of automation threat is revealed. Other levels are much less present. Again the low and medium group of automation threat exhibit a more varied distribution of requirement levels and no extremely outstanding grouping as seen before occurs. As a result of these observations, we conclude that the more equal distribution of wages in the highest group of automation threat stems from the mainly identical levels of education and occupations with similar levels of requirements.

2.5 Decomposition Analysis and Discussion

The goal of this section is to identify the major factors associated with changes in wage inequality and their specific contribution in the two defined time periods (1996-2010 and 2012-2017). Our primary focus lies in quantifying the importance of automation and robotization on changes in the wage distribution using our measure of automation threat.

2.5.1 1996-2010

Counterfactual Analysis. Since we are mainly interested in the contribution of automation on changes in wage inequality, we first provide results of a ceteris paribus analysis. Multinomial logit estimations are used in order to derive counterfactual weights by which a counterfactual wage distribution is estimated. This distribution reflects the case where the distribution of all covariates is as in point in time 1 except for the distribution of the automation threat groups, which is shifted to that of point in time 0. This procedure is different to that proposed by DiNardo et al. (1996), where a counterfactual distribution is estimated shifting all available covariates. Thus, the conducted analysis makes it possible to show graphically the effect of a compositional change of one specific covariate. The multinomial logit model that estimates the possibility of belonging to one of the three possible types of automation threat is estimated accounting for all remaining covariates we used in the decomposition (for further information see Appendix 2.A.3).

Figure 2.3 illustrates the actual wage distributions of 1996 and 2010 using kernel density estimations of the log wage distributions of the respective years. In 2010 a lower peak and fatter tails compared to the one in 1996 are observed. Moreover, the widening of the wage distribution is not symmetric, since more mass is shifted to the upper half of the wage distribution. In addition, the counterfactual wage distribution of 2010 with the composition of the automation threat groups shifted back to 1996 is shown. We observe that the counterfactual distribution approaches the density in 1996. A higher peak and a narrower tail at the upper half of the distribution suggest an impact that contributes to a reduction in wage inequality if the composition of the automation threat groups would have been the same in 2010 as in 1996. The actual observed change in the wage distribution between 1996 and 2010 is compared to the difference between the counterfactual and the actual wage distribution in 2010 in Figure 2.C.2 in Appendix 2.C.

The analysis shows that the observed trend in automation threat contributes to the shift in the upper half of the wage distribution. However, since the counterfactual difference stays close to zero up to the middle of the distribution, a smaller contribution on

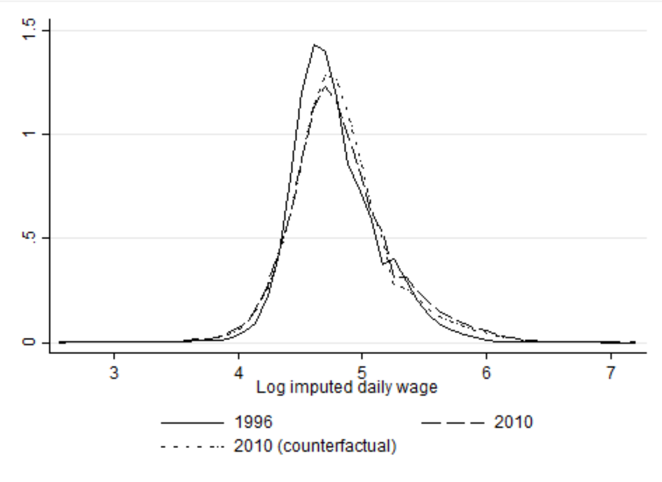


Figure 2.3: Actual and counterfactual wage distributions, 1996-2010

Notes: The figure presents the actual wage distributions in 1996 and 2010 as well as the counterfactual wage distribution that would have prevailed if automation and robotization had remained at the level of 1996. Counterfactual weights are estimated using multinomial logit estimations, see Appendix 2.A.3. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

lower wages is assumed. In Figure 2.C.3 we re-estimate the 85-15 percentile wage gap and the Gini coefficient using our counterfactual weights. Indeed, we are able to show that compositional changes in the automation threat groups have played an important role in the rise in wage inequality between 1996 and 2010 since the counterfactual estimates are at all time below the actual outcomes. Further, Figure 2.C.4 confirms the different impact along the wage distribution. Whereas the counterfactual line stays close to the actual line at the lower half of the distribution, a substantial gap between the two lines is shown for the upper half revealing a higher impact of automation to increasing wage inequality at this part of the wage distribution.

Decomposition Results. We now turn to the results of the RIF-regressions based OB decomposition for the period 1996 and 2010 for men working full-time in the manufacturing sector in West Germany. Figure 2.4 presents graphically the estimated results of different percentile wage gaps for the composition effect, see Panel (a), and the wage structure effect, see Panel (b).[25] First of all, we turn to the decomposition results of

[25] Comprehensive tables of the decomposition results, which also include specification and reweighting errors, can be found in Table 2.B.4 and Table 2.B.5 in Appendix 2.B.

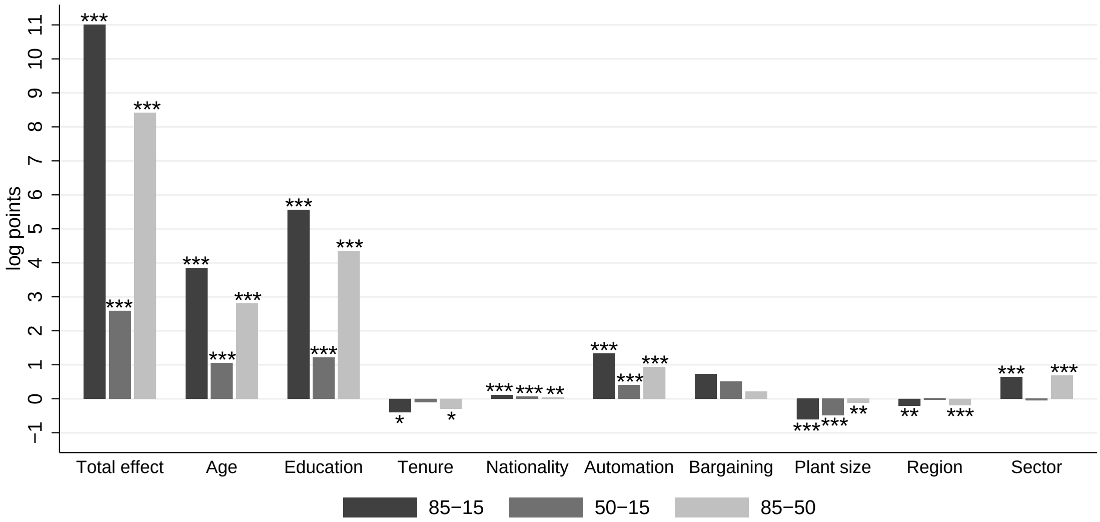
the wage gap between the 85th and the 15th percentile, which increased by 10.67 log points between 1996 and 2010. The aggregate composition effect mainly contributes to the increase in the wage gap, while the aggregate wage structure effect is not statistically different from zero. The estimated specification error is statistically insignificant and the reweighting error is sufficiently small.[26]

Among the composition effects, depicted in Panel (a) of Figure 2.4, the ones associated with educational levels (5.56 log points) and the age structure of workers (3.85 log points) have played the most important role, which correspond to a relative importance of 41%[27] and 29% of the composition effect, respectively. These findings are supported by the observed shift towards older and higher educated workers in the underlying data. The contribution of the automation-related composition effect has played a slightly smaller, but non-negligible role and amounts to 1.33 log points, which corresponds to a relative importance of roughly 10% of the composition effect. As shown in the descriptive analysis, there is an observable trend towards occupations with medium automation threat, accompanied by decreasing shares of occupations with high and low automation threat between 1996 and 2010. Due to the fact that within-group wage inequality is the lowest in the group with the highest automation threat, those compositional changes contribute to an increase in wage inequality. Less pronounced but still significant effects that contribute to wage dispersion are changes in the composition of the sector variable and the nationality variable. A factor that has played a small but highly significant role dampening the effect on wage inequality is provided by changes in the composition of the firm size.

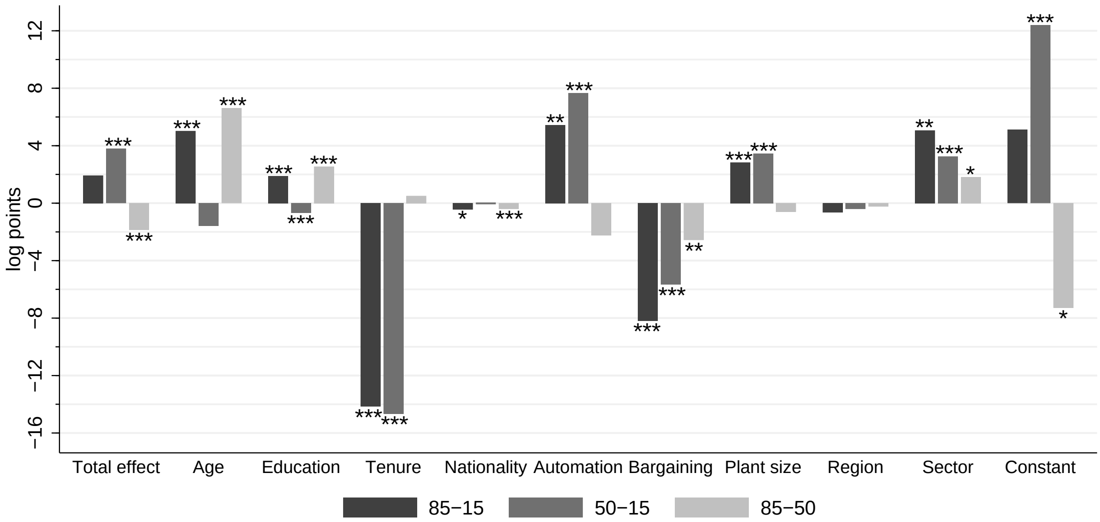
When we consider the detailed results of the wage structure effects, presented in Panel (b) of Figure 2.4, very different implications become evident. The interpretation of the wage structure effects of the respective factors depends on the choice of the base category. Due to this, the specific contribution of one covariate to a change in the wage structure

[26]In order to show that the main results are not affected by the definition of the used percentiles, the 90th-10th wage gap is estimated as a robustness check. The relative importance of the different explanatory variables in the detailed decomposition analysis does not shift as well as the signs and statistical significance.

[27]We interpret the specific estimated effect of a covariate as follows: in the observed case we have 5.56/13.42=0.41, where 13.42 is the sum of all detailed composition effects in absolute terms. Thus, we are able to provide percentages that show the respective relative importance in comparison to all other factors and which sum up to 100%.



(a) Composition effect



(b) Wage structure effect

Figure 2.4: Decomposition results of the composition and wage structure effect by percentile wage gaps, 1996-2010

Notes: The figure presents the results of the RIF-regressions based OB decomposition approach for the composition and wage structure effect based on log daily wages. The sample is restricted to male full-time workers in the manufacturing sector between 18 and 65 years, who earned more than 10 euros per day and work in West Germany. All coefficients above are multiplied by 100 for convenience. ***, **, and * indicate statistical significance at the 1, 5, and 10 percent level, respectively. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

has to be interpreted relative to its base category. Moreover, the wage structure effects capture both the between group and the within group inequality component. In other words, on the one hand direct changes in the return for individual factors are considered and on the other hand changes in the residual wage inequality within the observed group relative to the base group are observed. Thus, the constant of the wage structure effect can be interpreted as the change in residual wage inequality of the base category.

The most important factors that are associated with an increase in the 85-15 percentile wage gap are automation threat (5.43 log points), sector differences (5.07 log points) and the age structure (5.03 log points). The positive automation-related wage structure effect could be the result of changes in relative wage returns between workers in occupations with high and low automation threat, as predicted by RBTC. This would suggest an increase in the relative wage of non-routine tasks that are typically at low risk of automation compared to routine tasks that are usually faced with higher risk of automation. In this case, a change in between group wage inequality would be observed. Among the remaining wage structure effects, education profiles of workers and the firm size have played small but non-negligible roles. However, all effects that contribute to wage inequality are compensated by negative effects especially related to tenure and collective bargaining.

In order to show an appropriate comparison with the results of the 85-15 percentile wage gap, the decomposition results of the Gini coefficient are presented in the second column of Table 2.B.4 in Appendix 2.B. In contrast to the previous estimates, the total increase of the Gini coefficient can be divided in equal parts into the composition effect and the wage structure effect.

Among the composition effects, the same covariates like educational levels (1.64) and age (0.7) exhibit the largest statistically significant effects that are associated with an increase in wage inequality. A less pronounced but non-negligible role played collective bargaining (0.37) and automation threat (0.17), which contribute by around 11% and 5% to the composition effect, respectively. Using the Gini coefficient makes it easier to explain the movements behind the contribution of automation threat on wage inequality in the following. The automation-related positive composition effect stems from the observable trend towards occupations with medium automation threat, accompanied by decreasing shares of occupations with high and low automation threat. Due to the fact that within-group wage inequality is the highest in the lowest automation threat group, the estimated RIF coefficients on the middle and high automation threat groups are mainly negative, see Table 2.B.11.[28] Since the composition effect is defined as the change in the share of employment of the respective groups times the coefficient of the RIF-regression in 1996, it can be shown why compositional changes regarding the automation threat variable are associated with increasing wage inequality. In other words, in this case the composition effect consists of two negative components, which build together a positive effect that contributes to wage dispersion. As a result, we provide evidence that structural shifts in the workforce composition between occupations with different automation threat contributes to increasing wage inequality. Similar to the 85-15 percentile wage gap, changes in the composition of the firm size has played a small but non-negligible role to dampen wage inequality.

Looking at the wage structure effect, the same covariates like automation threat, age, education, sector and the firm size contribute the rise in wage inequality, where automation threat is the major factor, which amounts to 2.55. Again, a closer look at the results of the RIF-regressions explains this result, see Table 2.B.11. As already seen, in 1996 both coefficients of the middle and high automation threat group are negative. This suggests that an increase in the share of the highest automation threat group is associated with a decrease in the estimated Gini coefficient, since this group exhibits a lower within-group wage inequality than the base group of low automation risk. Moreover, regarding the wage structure effect it is important to observe how the coefficients change over time. We see that between 1996 and 2010, the RIF-regressions estimates for the medium and high automation risk group either decrease in absolute terms or even get positive. This means that in 1996 the contribution of the two groups on dampening wage inequality was more pronounced than in 2010, keeping everything else equal. Looking at the equation for the wage structure effect it can be seen that the change in the coefficients becomes positive and is multiplied by the positive employment share of 2010. As a result of this condition, a positive automation-related wage structure effect is estimated.

[28]All RIF-regressions estimation results of the applied inequality measures and percentiles are presented in Table 2.B.8 - 2.B.12 in Appendix 2.B.

Among the remaining wage structure effects, the same covariates such as tenure and collective bargaining are associated with a reduction in wage inequality. Other inequality decreasing wage structure effects are either weakly or not statistically significant. In summary, the main results of the two presented inequality measures concerning the whole wage distribution are comparable for most parts. Further, we provide decomposition results of the Variance in Table 2.B.13 in Appendix 2.B. Again, the automation-related composition effect has played a small but non-negligible role in rising wage inequality, which amounts to 0.17 log points and corresponds to a relative importance of 4% of the composition effect. The wage structure effect associated with automation threat is the major factor that contributes to wage inequality, amounting to 3.82 log points.

We now turn to the decomposition results of the two inequality measures considering either the lower part or the upper part of the wage distribution. The wage gap between the 50th and 15th percentile increased by 7.11 log points, whereas the 85-50 percentile wage gap increased only by 3.56 log points, see Table 2.B.5 in Appendix 2.B. The sum of both increases is again the whole increase of the 85-15 percentile wage gap. Looking at the aggregate composition and wage structure effects we observe different results. Whereas the 50-15 percentile wage gap can be divided roughly into equal positive parts, the 85-50 percentile wage gap exhibits a four times as big positive composition effect compared to the negative wage effect in absolute terms.

In general, the key results of the detailed composition effect are for both measures similar to the overall wage gap, see Panel (a) of Figure 2.4. Comparing the composition effects on the lower and upper half of the wage distribution, we find that the effects on the upper half of the wage distribution are more pronounced than on the lower part. This holds also true for the automation threat variable. Turning to the wage structure effects on the lower and upper half of the wage distribution different outcomes become apparent, see Panel (b) of Figure 2.4. Regarding our measure of automation and robotization, we can state that automation threat has a clear inequality-increasing and highly significant wage structure effect at the lower part of the wage distribution, while it has no statistically significant effect at the upper part of the wage distribution. Thus, the changes in relative wage returns between workers in occupations with high and low automation threat, as predicted by RBTC, are only observable at the lower part of the wage distribution.

2.5.2 2012-2017

Counterfactual Analysis. Figure 2.C.5 in Appendix 2.C shows the changes in the wage distribution and the corresponding difference between 2012 and 2017. The shift of the wage distribution to the right is more pronounced. Moreover, no major drop of the peak compared to the development between 1996 and 2010 is observed. In fact, a rather horizontal shift of the distribution where the peak is more located to the right becomes apparent. Furthermore, the counterfactual distribution in 2017, where the composition of the automation threat groups is shifted back to 2012, is illustrated. As seen before, the counterfactual density approaches to the actual distribution in 2012. However, it becomes evident that changes in the composition of automation threat are not responsible for the horizontal shift to the right.

The comparison of the counterfactual difference to the actual difference between 2012 and 2017 is illustrated in Figure 2.C.6 in Appendix 2.C. Again, changes in the lower part of the distribution are not affected by a large extent through compositional changes in the automation threat groups, which is represented by a counterfactual difference close to zero. In Figures 2.C.7 and 2.C.8 we re-estimate the standard inequality measures using counterfactual weights. In this case, we also find supporting results of the above described findings.

Decomposition Results. In the more recent time period, the rise in the wage gap between the 85th and the 15th percentile is less pronounced and increased by only 2.17 log points.[29] This is due to the fact that the positive aggregate composition effect is mitigated by the negative aggregate wage structure effect.

Among the composition effects for the 85-15 percentile wage gap, presented in Table 2.B.6 in Appendix 2.B, the one associated with the age structure of workers is no more statistically significant in comparison to the first time period. A still significant although

[29]Comprehensive tables of the decomposition results, which also include specification and reweighting errors, can be found in Table 2.B.6 and Table 2.B.7 in Appendix 2.B.

less pronounced composition effect comes from education (1.15 log points), which corresponds to a relative importance of 27% of the composition effect. The most important inequality-increasing composition effect is associated with automation threat (1.72 log points), which corresponds to a relative importance of 41%. Rather small but still significant effects that contribute to wage inequality are driven by changes in the composition of the firm size, sector and nationality variables. Composition effects that are associated with decreasing wage inequality are related to tenure and the bargaining regime, even if their contribution is relatively small.

When turning to the decomposition results of the wage structure effect for the 85-15 precentile wage gap, there are observable differences compared to the previous period. The wage structure effects related to collective bargaining (2.32 log points) and nationality (0.49 log points) contribute positively to rising wage dispersion in the more recent time period, while the ones associated with tenure, age, regional differences, education and the plant size dampen it. Again, the most important wage structure effect, which is associated with decreasing wage dispersion is related to tenure, which amounts to -9.63 log points. In comparison to the first time period, automation threat has no more a statistically significant wage structure effect. It seems that in the recent past the change in the composition of automation threat is the prominent channel through which automation contributes to rising wage dispersion.

The decomposition results for the Gini coefficient show a slightly decrease in the overall wage inequality by around 0.31 log points during the considered time period. The aggregate composition effect is positive, while the aggregate wage structure effect is negative, both are rather small (0.77 and 0.81, respectively). Among the composition effects, the ones associated with education (0.32), automation threat (0.23) and the plant size (0.22) contribute the most to the increase in wage inequality. The relative importance of automation threat belongs to 26% of the composition effect. The positive automation-related composition effect is supported by the observed shift from 2012 to 2017 towards occupations with low and middle automation threat, which are faced with significantly higher wage dispersion. The estimated RIF coefficients on the middle and high automation risk groups are again negative, see Table 2.B.11. Thus, we see the same dynamics behind the automation-related composition effect as in the first period. Sectoral differences have played a small but non-negligible role in rising wage dispersion with a relative importance of around 8% of the composition effect. Small but still significant effects that contribute to a decrease in wage inequality are associated with changes in the composition of tenure and the bargaining regime, which is in line with the decomposition results of the 85-15 percentile wage gap.

The results of the detailed wage structure effect are more or less equal to the results of the 85-15 percentile wage gap, although the automation-related wage structure effect is now the most important factor associated with decreasing wage dispersion, which amounts to -2.22. A closer look at the results of the RIF-regressions explains this result, see Table 2.B.11. In 2012 as well as in 2017 both coefficients of the middle and high automation threat group are negative. This suggests that an increase in the share of the middle and high automation threat group is associated with a decrease in the estimated Gini coefficient. Moreover, we see that between 2012 and 2017, the RIF-regressions estimates for the middle automation threat group increase in absolute terms, while the estimates for the high automation threat group decreases slightly in absolute terms. This means that the contribution of the middle automation threat group on dampening wage inequality was more pronounced in 2017 than in 2012, while the contribution of the high automation threat group on dampening wage inequality was more pronounced in 2012 than in 2017, keeping everything else equal. Due to the fact that the automation-related wage structure effect is negative, the contribution of the middle automation threat group overweigh the contribution of the high automation threat group, thus, the change in the coefficients becomes negative and is multiplied by the positive employment share of 2019 leading to a negative automation-related wage structure effect.

The decomposition results regarding the automation threat variable are comparable to the decomposition results for the Variance, see Table 2.B.13 in Appendix 2.B. The automation-related composition effect has played a major role in rising wage inequality, which amounts to 0.33 log points and corresponds to a relative importance of 24% of the composition effect. The wage structure effect associated with automation threat is an important factor, which contributes to a decrease in wage inequality, amounting to -1.96

log points.

We now turn to the decomposition results of the two inequality measures considering the lower and upper part of the wage distribution, see Table 2.B.7 in Appendix 2.B. It becomes obvious that the less pronounced total increase of the 85-15 percentile wage gap is due to the fact that the lower and upper part of the wage distribution are faced with different inequality trends during the last years. While the wage gap at the lower end of the wage distribution increased by 4.66 log points, the wage gap at the upper end of the wage distribution decreased by 2.48 log points.

The aggregate composition effect is for both inequality measures positive. At the lower part of the wage distribution the composition effects related to sectoral differences, automation threat and plant size contribute the most to rising wage inequality, while tenure, collective bargaining and regional fixed effects played a small but significant role to dampen wage dispersion. Those effects are more or less similar to the detailed composition effects at the upper part of the wage distribution. However, it is evident that the automation-related composition effect is more pronounced at the upper part of the wage distribution then at the lower part. This observed difference in the contribution of automation threat along the wage distribution confirms the results from the counterfactual analysis presented in Figure 2.C.5 in Appendix 2.C.

Among the wage structure effects, differences between the two inequality measures become apparent. The aggregate wage structure effect for the 50-15 percentile wage gap is positive, while it is negative for the 85-50 percentile wage gap. At the lower part of the wage distribution, collective bargaining and nationality are relatively small but highly significant factors associated with increasing wage dispersion, while those effects have played no significant role for the upper part of the wage distribution. Regional differences and the plant size contribute to a decrease in wage inequality at the lower part of the wage distribution. Turning to the upper part of the wage distribution, inequality-increasing wage structure effects are related only to the RIF constant. As explained earlier in this subsection, the constant of the wage structure effect can be interpreted as the change in residual wage inequality of the base category. However, this effect is fully compensated by inequality-decreasing factors associated with automation threat, tenure, age, education and regional fixed effects. Automation threat is the most important factor associated with a dampening effect on wage dispersion at the upper part of the wage distribution, which amounts to -5.98 log points, while it has no significant effect at the lower part of the wage distribution.

2.5.3 Robustness Check

The preceding decomposition analyses show a clear impact of automation threat on the development of wage inequality in the manufacturing sector in Germany. In order to validate our findings, we test the robustness using alternative model specifications. First, we replace the automation probabilities by Dengler and Matthes (2015) with the common used probabilities of computerization provided by Frey and Osborne (2017). In a second robustness check we test whether the automotive and other vehicles sector has a superior influence on the analysis and thus leads to biased estimates. Similar to this, we exclude the electronics sector and the plastic, chemicals and glass sector as further robustness checks.

Probability of Computerization by Frey and Osborne (2017). Frey and Osborne (2017) estimate the probability of computerization of different occupations in the US, which is a commonly used measure of automation risk. Using these estimated automation probabilities for German occupations creates several problems, which are described in Appendix 2.A.2. Those compatibility and conceptual problems have to be taken into account by interpreting the results.

Frey and Osborne (2017) provide three types of "engineering bottlenecks" to automation, which are (1) perception and manipulation, (2) creative intelligence and (3) social intelligence (Frey and Osborne, 2017, p. 264). The higher the relevance of these bottlenecks for a given occupation, the lower the probability for workers to be substituted by machines. In total, there are estimates for 702 occupations. The data are available at the 6-digit SOC 2010 classification, thus, we have to translate the data into the 3-digit German KldB 2010 classification, see Appendix 2.A.4. The alternative automation threat variable is estimated in a similar way as before, see equation (2.5), using the computerization probabilities provided by Frey and Osborne (2017) as θ_j.

The descriptive statistics of this alternative automation threat variable are presented in Table 2.B.3 in Appendix 2.B. Similar to the findings in section 2.4, the lowest within-group wage inequality is found in the group with the highest automation threat, because workers within the highest automation threat group tend to have similar education and requirement levels. However, the distinct differences in the level of within-group wage inequality are not that much pronounced as in our base variable, which is likely influenced by the different estimation strategies. In the case of Dengler and Matthes (2015), higher automation probabilities are associated with routine tasks, which are often conducted by workers with middle education and similar requirement levels, while lower automation probabilities are associated with non-routine tasks, which could be performed by low and high educated workers with a broader range of requirement levels. This would lead to lower within-group wage inequality in the high automation threat group and higher within-group wage inequality in the middle and low automation threat groups.

In contrast, Frey and Osborne (2017) define some bottlenecks to automation for given occupations. Those bottlenecks are more equally distributed over the whole range of workers. Thus, in all three automation threat groups, the distribution of education and requirement levels tend to be more equal, leading to smaller differences in within-group wage inequality between the automation threat groups. In addition, the employment share of the highest automation threat group decreases within the two time periods, while the employment shares in the low and middle automation threat groups stay rather constant or increase. Those compositional changes could contribute to an increase in wage inequality, although to a smaller amount as compared to our basic automation threat variable.

The decomposition results are presented in Table 2.B.14 and 2.B.15 in Appendix 2.B. Turning to the automation-related composition effect, smaller coefficients are now observable for almost all inequality measures during both periods. This underpins our results from the descriptive analysis. For the first time period, the automation-related wage structure effect at the 85-15 percentile wage gap and the Gini coefficient is positive, but no more significant. This is due to the fact that automation threat is now associated with a significant inequality-decreasing wage structure effect at the 85-50 percentile wage gap. This means that wages at the upper part of the wage distribution become more equally distributed between and within the three automation threat groups over the first time period.

In the second time period, the automation-related wage structure effect at the 85-15 percentile wage gap is now positive and significant. Thus, changes in wage dispersion between or within the automation threat groups lead to an increase in the 85-15 percentile wage gap. This is due to the fact that automation threat is now associated with a large positive and highly significant wage structure effect at the 50-15 percentile wage gap, while the automation-related wage structure effect contributes no more to a decline in the 85-50 percentile wage gap.

Due to the different estimation strategy of Frey and Osborne (2017), the contribution of the automation-related composition effect is smaller. In addition, changes in wage dispersion between or within the automation threat groups lead to an increase in wage inequality during the second time period. However, the compatibility and conceptual problems that occur by using the estimations of Frey and Osborne (2017) for German occupations lead to biased results, which we avoid by using the automation probabilities provided by Dengler and Matthes (2015).

Automotive and other Vehicles Sector. The automotive and other vehicles sector (in the following automotive sector) is by far the most affected sector by automation threat, as already seen in Figure 2.C.1. In order to check whether our results are mainly driven by the development in this sector, we exclude the automotive sector in Table 2.B.16 and Table 2.B.17 in Appendix 2.B.

For both periods, the automation-related composition effect at the 85-15 percentile wage gap, the Gini coefficient and the Variance is still positive and significant, but even larger than our basic decomposition results. This can be explained by the fact that most workers within the automotive sector belong to the high automation threat group, see Table 2.B.1. In addition, the employment share in the automotive sector increased over the whole period, see Table 2.1. Due to the fact that wage inequality is the lowest in the group with the highest automation threat, as it is depicted in Figure 2.2, those structural changes towards the automotive sector lead to a decrease in overall wage inequality. Therefore, this dampening effect on wage inequality is not existent if we exclude the automotive sector, leading to a higher automation-related composition effect. The same pattern becomes apparent if we have a look at the results of the lower and upper part of the wage distribution. As in the basic decomposition, the automation-related composition effect is more pronounced at the upper part of the wage distribution. But again, the contribution to the 50-15 and the 85-50 percentile wage gap is higher than in our basic decomposition analysis.

Turning to the wage structure effect in the first period, the positive contribution of automation to the 85-15 percentile wage gap is no more statistically significant. This is due to the fact that automation threat is now associated with a significant inequality-decreasing wage structure effect at the 85-50 percentile wage gap. This means that without the automotive sector wages at the upper part of the wage distribution become more equally distributed between and within the three automation threat groups over the first time period. This effect at the upper part of the wage distribution vanishes if we include the automotive sector in our basic decomposition analysis, leading to a positive and significant automation-related wage structure effect at the 85-15 percentile wage gap.

In the second time period, the automation-related wage structure effect at the 85-15 percentile wage gap is now positive and significant. Thus, changes in wage dispersion between or within the automation threat groups lead to an increase in the 85-15 percentile wage gap if the automotive sector is excluded. This is due to the fact that automation threat is now associated with a large positive and highly significant wage structure effect at the 50-15 percentile wage gap, while the automation-related wage structure effect contributes no more to a decline in the 85-50 percentile wage gap.

This robustness check shows, that the automotive sector plays an important role for the automation-related wage structure effect. It seems that the automotive sector exhibits a different evolution of the wage structure within and between the automation threat groups than other manufacturing sectors. However, the automation-related composition effect is still positive and significant and differs only in its magnitude.

Further Affected Sectors. As presented in Figure 2.C.1 in Appendix 2.C there are further sectors that are outstandingly affected by automation and robotization. Therefore, additional robustness checks are conducted in order to exclude possible misinterpretations. At first the observations of the electronics sector are dropped. The overall estimated results reveal slightly smaller sizes of changes in the used inequality measures. Thus, the effects of the automation threat variable are as well smaller in absolute terms. However, the relative size and statistical significance do not change. Further, since the plastic, chemicals and glass sector is also highly affected by our estimated automation threat we additionally conduct the robustness check excluding observations of this sector. The results reveal slightly higher sizes of changes in the used inequality measures, however as already seen before the relative effects and information regarding significance do not change. Concluding, it can be seen that the development of these two sectors do not bias the overall estimated results.

2.6 Conclusion

Germany is faced with one of the highest industrial robot density in the world. At the same time, wage inequality in Germany underwent substantial changes in the last 25 years. Thus, possible impacts of automation and robotization on wage inequality should be observable in Germany. We conduct a detailed decomposition analysis based on RIF regressions on several inequality indices considering automation threat. Using rich linked employer-employee data, we are able to account for further different individual-, firm- and industry-specific characteristics.

The analysis contributes to the existing literature in examining the relative importance of automation technologies on wage inequality in the German manufacturing sector. Our newly introduced measure of automation threat combines occupation- and requirement-specific scores of automation risk with yearly sector-specific robot densities to approximately cover the whole dimension of automation and robotization. We provide evidence that automation threat contributes significantly to rising wage inequality in the German manufacturing sector in the last two decades. Moreover, we present general findings on the development of wage inequality and the associated driving forces for the recent years until 2017, in which wage inequality stayed rather constant or even declined.

We distinguish between two channels through which automation threat contributes to rising wage inequality. First, there is an observable trend towards occupations with medium automation threat, accompanied by decreasing shares of occupations with high and low automation threat. Due to the fact that within-group wage inequality is the lowest in the group with the highest automation threat, those compositional changes contribute to an increase in wage inequality. This automation-related composition effect corresponds to a relative importance of roughly 10% of the overall composition effect between 1996 and 2010 and actually 41% in the time period until 2017. Second, we find evidence that there is a growing wage dispersion between occupations with low automation threat (containing especially non-routine tasks) and occupations with high automation threat (containing especially routine tasks). This trend contributes to rising wage inequality as predicted by RBTC, where technology increases the relative demand, and consequently the relative wages, for non-routine tasks compared to routine tasks. This automation-related wage structure effect is prevalent in the 1990s and 2000s, while there is no evidence that this effect has played a significant role in the more recent time period.

Dauth et al. (2021) confirm our findings that automation contributes to rising wage inequality within the manufacturing sector. They provide evidence that this increase stems from the fact that workers who remain by their employer experienced higher wages, whereas those who are forced to leave their original firm are faced with wage losses. Our findings according to the composition effect of automation threat are in line with the decomposition results of Kaltenberg and Foster-McGregor (2020). They find evidence that the composition effect of increasing automation contributes to a large extent to wage inequality across European countries, where the automation related impact occurs mainly at the upper part of the wage distribution.

The decomposition analysis enables us to identify automation threat as an important source that contributes to increasing wage inequality, however, our results cannot be interpreted as causal effects. An analysis of the sources of wage inequality, especially of automation and robotization, in a more causal sense is highly important for future research. Another interesting research area examines the effects of industrial robots by gender and on the gender wage gap. This could be a valuable extension of future research based on the approach presented in this analysis. Moreover, considering only wage inequality could underestimate the effect of automation and robotization on the earning capacity of the society. Due to our data structure we are not able to analyze if workers are forced into unemployment as a result of increasing automation in their occupational field. Future research could examine whether such displacement effects lead to even higher inequality.

Appendix 2.A

2.A.1 Classification of Economic Activities

The robot data can be matched with the LIAB data without using a crosswalk. The LIAB data are available in the Classification of Economic Activities for the Statistics of the Federal Employment Services, edition 2008 (Klassifikation der Wirtschaftszweige 2008, WZ 2008). WZ 2008 is equivalent to the Statistical Classification of Economic Activities in the European Community (NACE) Rev. 2 and this classification is equal to ISIC Rev. 4 at the 2-digit level. There is one drawback that has to be taken into account when using the industrial classification WZ 2008. The data provides original values between 2008 and 2017. However, before the classifications of the economic activity have been updated, the industry codes rely on prior editions. Thus, the IAB provides a variable for industry classification WZ 2008, where the industry codes have been extrapolated and imputed to obtain time-consistent information for the period prior 2008. The imputation procedure is described in Eberle et al. (2011).

2.A.2 Advantages of the Substitution Potential Provided by Dengler and Matthes (2015)

Frey and Osborne (2017) estimate the probability of computerization of different occupations in the US. Using these estimated automation probabilities for German occupations creates several problems. First, there are compatibility problems by mapping the occupation classification, used by Frey and Osborne (2017), into the German occupation classification, see Appendix 2.A.4. Second, it is not likely that occupations in the US have the same job profiles and thus the same automation probabilities than the corresponding occupations in Germany. Given the problems by establishing a similar concept for occupations practised in Europe, see Sloane (2008), it is unlikely that the job profiles in the US and Germany are so similar that a direct transformation of the US automation probabilities to Germany is appropriate. Third, Frey and Osborne (2017) estimate the automation probabilities using an occupation-based approach. This underlies the assumption that whole jobs are replaced by automation. As Arntz et al. (2016) argue, it is more realistic to assume that single job-tasks rather than whole occupations are substituted by automation, because high-risk occupations still contain some tasks that are difficult to automate. By applying the occupation-based approach, it is likely that they overestimate the probability of job automatibility; see, e.g., Arntz et al. (2016) and Bonin et al. (2015). In order to avoid those problems, it is necessary to investigate the probability of job automatibility directly for occupations in Germany, based on a task-based approach.

2.A.3 Counterfactual Wage Distributions

In total we consider three different groups of possible automation threat, $r = 1, 2, 3$. Following Hyslop and Maré (2005) and Biewen and Juhasz (2012), a multinomial logit model is estimated accounting for all remaining covariates of our main analysis in order to estimate counterfactual weights, ω_{0r}. With the resulting weights it is possible to establish a counterfactual distribution that accounts for changes in the composition of the automation groups. This counterfactual distribution illustrates the distribution, where the automation groups are shifted back to the level of point in time 0 and everything else is fixed at the level of point in time 1. As a result of this, we obtain counterfactual weights, which are multiplied with the initial sample weights provided by the LIAB data. For further details see DiNardo (2002). The counterfactual wage distribution is then estimated as follows:

$$f_1(w|t_r = 0) = \sum_{r=1}^{3} \omega_{0r} f_{1r}(w), \tag{2.A.1}$$

where $f_{1r}(w)$ is the initial wage distribution of point in time 1.

Using the weights ω_{0r}, it is also possible to estimate counterfactual values of our described inequality measures.

2.A.4 SOC 2010 - KldB 2010 Crosswalk

Mapping the occupations at the 6-digit SOC 2010 classification into the 3-digit KldB 2010 classification creates ambiguous cases, because one KldB 2010 occupation can be allocated to several SOC 2010 occupations. Brzeski and Burk (2015) and Bonin et al. (2015) (in a first step) transfer the occupations at the 6-digit SOC 2010 classification into the KldB 2010 classification by using the average of the automation probability, if the mapping is not unique. In order to improve the crosswalk we apply in those ambiguous cases a weighted average of the automation probability, using employment shares.

First, we use the crosswalk provided by the Bureau of Labor Statistics (BLS)[30] to map the data from the 6-digit SOC 2010 into the international 4-digit ISCO 2008 classification. We assign a weighted average of the job automation probability, using 2014 US employment weights provided by the BLS[31], in case that the mapping is not unique. Next, we map the international 4-digit ISCO 2008 classification into the German 5-digit KldB 2010 classification, where the crosswalk is provided by the German Federal Employment Agency[32], again applying 2014 US employment weights.[33] As a last step, we aggregate the 5-digit KldB 2010 classification into the 3-digit code, using 2014 German employment weights provided by the German Federal Employment Agency.[34]

[30] `https://www.bls.gov/soc/soccrosswalks.htm`

[31] `https://www.bls.gov/oes/tables.htm`

[32] `https://statistik.arbeitsagentur.de/Navigation/Statistik/Grundlagen/Klassifikationen/Klassifikation-der-Berufe/KldB2010/Arbeitshilfen/Umsteigeschluessel/Umsteigeschluessel-Nav.html`

[33] Due to the fact that US employment data are only available for SOC 2010 classification, we apply the crosswalk provided by the BLS to map the US employment data from the 6-digit SOC 2010 into the international 4-digit ISCO 2008 classification, using 2014 US employment weights.

[34] `https://statistik.arbeitsagentur.de/Statistikdaten/Detail/201406/iiia6/beschaeftigung-sozbe-bo-heft/bo-heft-d-0-201406-xlsx.xlsx?__blob=publicationFile&v=1`

Appendix 2.B

Table 2.B.1: Comparison between the estimated automation threat and the automation probability provided by Dengler and Matthes (2015), by sector

	Automation threat defined by equation (2.5)			Automation probability by Dengler and Matthes (2015)		
Economic Sector	Low automation	Medium automation	High automation	Low automation	Medium automation	High automation
1996						
Food and beverages	52.20	4.43	0	10.73	11.37	0.43
Textiles	5.08	13.73	0	3.40	2.32	3.50
Wood, furniture and paper	6.81	13.95	8.62	7.70	5.10	14.31
Plastic and chemical products	13.12	12.92	14.68	22.61	11.29	15.45
Metal products	5.37	6.85	26.88	18.10	14.08	29.23
Electrical products	5.18	8.44	11.81	10.15	12.94	7.91
Industrial machinery	11.37	38.91	17.71	13.38	25.68	16.87
Automotive and other vehicles	0.87	0.77	20.31	13.93	17.22	12.30
2010						
Food and beverages	14.68	20.30	1.11	12.60	11.37	0.45
Textiles	17.19	0	0	1.79	1.01	1.60
Wood, furniture and paper	22.99	25.94	0	9.28	4.26	13.11
Plastic and chemical products	6.85	5.75	18.33	19.69	9.86	18.17
Metal products	14.19	13.01	26.90	14.80	16.42	31.45
Electrical products	14.00	13.12	14.57	16.33	16.79	10.44
Industrial machinery	9.94	21.68	15.22	10.40	20.10	13.58
Automotive and other vehicles	0.16	0.19	23.87	15.11	20.20	11.19
2012						
Food and beverages	9.19	21.16	1.42	8.96	11.25	1.12
Textiles	11.86	0	0	1.44	1.05	1.54
Wood, furniture and paper	43.93	10.92	0	7.14	4.40	10.90
Plastic and chemical products	3.49	9.92	17.10	18.73	10.59	16.29
Metal products	7.53	10.84	31.09	14.40	17.47	34.18
Electrical products	11.29	17.16	10.37	17.35	14.00	8.07
Industrial machinery	12.36	29.78	16.89	18.26	22.15	16.58
Automotive and other vehicles	0.35	0.23	23.13	13.72	19.08	11.32
2017						
Food and beverages	13.49	30.40	0.61	13.03	14.47	1.12
Textiles	5.98	0	0	0.64	0.48	1.25
Wood, furniture and paper	54.94	0	0	7.45	3.95	11.50
Plastic and chemical products	2.49	7.33	13.37	13.50	7.69	13.41
Metal products	2.15	10.18	25.82	11.79	12.98	30.88
Electrical products	8.47	22.12	6.63	12.86	11.80	8.28
Industrial machinery	12.04	29.72	16.73	18.17	21.39	16.87
Automotive and other vehicles	0.46	0.24	36.83	22.57	27.25	16.69

Notes: The table presents the descriptive statistics for four time points separately for each automation threat group. It is a comparison of our proposed variable measuring automation threat and the automation probabilities provided by Dengler and Matthes (2015). All employment shares are reported in percent. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Table 2.B.2: Descriptive statistics of the automation threat variable by groups

	1996	Std. Dev.	2010	Std. Dev.	2012	Std. Dev.	2017	Std. Dev.
Automation Threat: low								
Real daily wage	127.62	(61.12)	141.74	(77.25)	143.28	(79.92)	140.54	(76.99)
Education: low	12.58	(33.16)	8.67	(28.14)	6.13	(23.99)	4.55	(20.84)
Education: middle	75.48	(43.01)	71.60	(45.09)	72.33	(44.74)	76.98	(42.09)
Education: high	11.94	(32.42)	19.73	(39.79)	21.55	(41.11)	18.47	(38.80)
Requirement level: unskilled activities	0.78	(8.79)	0.79	(8.85)	9.11	(28.77)	10.21	(30.28)
Requirement level: specialist activities	67.82	(46.72)	56.24	(49.61)	44.95	(49.74)	52.64	(49.93)
Requirement level: complex activities	17.25	(37.78)	15.78	(36.45)	25.12	(43.37)	20.51	(40.38)
Requirement level: highly complex activities	14.15	(34.85)	27.19	(44.49)	20.82	(40.60)	16.64	(37.25)
Automation Threat: middle								
Real daily wage	147.36	(65.55)	145.86	(82.24)	158.28	(84.46)	160.45	(85.43)
Education: low	9.60	(29.45)	7.56	(26.44)	4.86	(21.51)	4.88	(21.54)
Education: middle	68.56	(46.43)	70.59	(45.56)	65.93	(47.39)	66.37	(47.24)
Education: high	21.84	(41.31)	21.85	(41.32)	29.21	(45.47)	28.74	(45.25)
Requirement level: unskilled activities	1.07	(10.28)	2.77	(16.39)	10.92	(31.19)	12.42	(32.98)
Requirement level: specialist activities	55.65	(49.68)	59.56	(49.07)	36.35	(48.10)	36.24	(48.06)
Requirement level: complex activities	25.86	(43.78)	23.67	(42.50)	26.58	(44.17)	26.83	(44.31)
Requirement level: highly complex activities	17.43	(37.93)	14.01	(34.71)	26.15	(43.94)	24.52	(43.02)
Automation Threat: high								
Real daily wage	121.20	(43.90)	133.85	(62.96)	128.65	(56.16)	143.43	(60.81)
Education: low	12.78	(33.38)	9.06	(28.70)	8.25	(27.51)	6.80	(25.17)
Education: middle	81.44	(38.88)	81.03	(39.21)	83.63	(37.00)	83.69	(36.94)
Education: high	5.78	(23.34)	9.92	(29.88)	8.12	(27.32)	9.51	(29.33)
Requirement level: unskilled activities	3.10	(17.32)	1.16	(10.73)	16.34	(36.97)	12.61	(33.19)
Requirement level: specialist activities	83.53	(37.09)	84.88	(35.82)	68.39	(46.49)	68.86	(46.30)
Requirement level: complex activities	9.40	(29.18)	7.76	(26.75)	10.51	(30.67)	11.58	(32.00)
Requirement level: highly complex activities	3.98	(19.55)	6.19	(24.10)	4.75	(21.27)	6.94	(25.41)

Notes: The table presents the descriptive statistics for four time points separately for each automation threat group. All variables, except the wage are reported in percent. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Table 2.B.3: Descriptive statistics of the alternative automation threat variable with automation probabilities provided by Frey and Osborne (2017) by groups

	1996	2010	2012	2017
Automation Threat: low				
Employment share	18.29	21.38	16.90	16.53
Real daily wage	129.05	140.67	145.29	154.94
Gini coefficient	0.2142	0.2469	0.2644	0.2546
Education: low	9.63	5.31	5.13	3.61
Education: middle	80.13	77.55	71.04	72.04
Education: high	10.24	17.14	23.83	24.35
Requirement level: unskilled activities	0.08	2.67	10.66	7.38
Requirement level: specialist activities	59.77	59.36	45.59	46.68
Requirement level: complex activities	28.73	21.92	21.53	23.54
Requirement level: highly complex activities	11.42	16.04	22.22	22.40
Automation Threat: middle				
Employment share	37.77	37.73	35.45	36.06
Real daily wage	127.59	137.09	138.93	141.06
Gini coefficient	0.1950	0.2530	0.2370	0.2378
Education: low	10.01	8.28	5.24	5.34
Education: middle	79.15	77.21	79.04	79.87
Education: high	10.85	14.51	15.72	14.79
Requirement level: unskilled activities	5.36	1.95	10.09	12.05
Requirement level: specialist activities	74.65	75.12	60.27	60.32
Requirement level: complex activities	11.47	12.52	18.69	17.49
Requirement level: highly complex activities	8.52	10.41	10.96	10.15
Automation Threat: high				
Employment share	43.95	40.89	47.65	47.41
Real daily wage	124.34	136.28	133.01	149.46
Gini coefficient	0.1924	0.2241	0.2252	0.2218
Education: low	15.17	10.73	9.44	7.40
Education: middle	77.39	78.09	80.22	79.69
Education: high	7.44	11.18	10.34	12.91
Requirement level: unskilled activities	1.02	0.58	18.70	14.12
Requirement level: specialist activities	86.11	86.06	61.40	61.45
Requirement level: complex activities	8.03	7.38	11.76	13.40
Requirement level: highly complex activities	4.84	5.98	8.13	11.03

Notes: The table presents the descriptive statistics for four time points separately for each automation threat group. All variables, except the wage are reported in percent. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Frey and Osborne (2017), own calculations.

Table 2.B.4: Decomposition of the 85-15 percentile wage gap and the Gini coefficient, 1996-2010

Inequality measure	85-15		Gini coefficient	
	Coefficient	Standard Error	Coefficient	Standard Error
Total change	10.67***	(0.40)	4.24***	(0.10)
Pure composition effect				
Age	3.85***	(0.23)	0.70***	(0.05)
Education	5.56***	(0.31)	1.64***	(0.09)
Tenure	−0.39*	(0.22)	−0.04	(0.05)
Nationality	0.11***	(0.03)	0.01	(0.01)
Automation threat	1.33***	(0.16)	0.17***	(0.03)
Collective bargaining	0.73	(0.51)	0.37***	(0.11)
Plant size	−0.61***	(0.12)	−0.22***	(0.03)
Region	−0.20**	(0.08)	−0.03	(0.02)
Sector	0.64***	(0.09)	0.11***	(0.02)
Total	11.01***	(0.69)	2.71***	(0.15)
Specification error	−0.85	(0.62)	−0.57***	(0.10)
Pure wage structure effect				
Age	5.03***	(1.59)	1.57***	(0.44)
Education	1.88***	(0.58)	1.10***	(0.12)
Tenure	−14.16***	(5.08)	−2.56**	(1.17)
Nationality	−0.45**	(0.18)	−0.06	(0.04)
Automation threat	5.43**	(2.69)	2.55***	(0.80)
Collective bargaining	−8.18***	(1.25)	−1.46***	(0.26)
Plant size	2.84***	(0.68)	0.58***	(0.16)
Region	−0.65	(0.84)	−0.22	(0.21)
Sector	5.07***	(1.12)	0.70***	(0.26)
Constant	5.11	(6.02)	0.20	(1.47)
Total	1.92	(0.55)	2.38***	(0.15)
Reweighting error	−1.42***	(0.16)	−0.28***	(0.05)

Notes: The table presents the results of the RIF-regressions based OB decomposition approach based on log daily wages (85-15) and daily wages (Gini coefficient). The sample is restricted to male full-time workers in the manufacturing sector between 18 and 65 years, who earned more than 10 euros per day and work in West Germany. All coefficients above are multiplied by 100 for convenience. ***, **, and * indicate statistical significance at the 1, 5, and 10 percent level, respectively. Bootstrapped standard errors with 100 replications are presented in parentheses. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Table 2.B.5: Decomposition of the 50-15 and the 85-50 percentile wage gap, 1996-2010

Inequality measure	50-15		85-50	
	Coefficient	Standard Error	Coefficient	Standard Error
Total change	7.11***	(0.32)	3.56***	(0.27)
Pure composition effect				
Age	1.05***	(0.15)	2.80***	(0.17)
Education	1.21***	(0.07)	4.35***	(0.27)
Tenure	−0.10	(0.18)	−0.29*	(0.17)
Nationality	0.06***	(0.02)	0.04**	(0.02)
Automation threat	0.40***	(0.06)	0.93***	(0.13)
Collective bargaining	0.51	(0.35)	0.21	(0.37)
Plant size	−0.49***	(0.08)	−0.12**	(0.06)
Region	−0.01	(0.05)	−0.19***	(0.06)
Sector	−0.05	(0.06)	0.69***	(0.08)
Total	2.59***	(0.40)	8.42***	(0.57)
Specification error	1.17***	(0.36)	−2.02***	(0.57)
Pure wage structure effect				
Age	−1.57	(1.25)	6.61***	(1.31)
Education	−0.68***	(0.20)	2.55***	(0.56)
Tenure	−14.67***	(4.01)	0.51	(2.42)
Nationality	−0.03	(0.14)	−0.41***	(0.10)
Automation threat	7.67***	(1.65)	−2.24	(1.91)
Collective bargaining	−5.61***	(0.96)	−2.57**	(1.00)
Plant size	3.45***	(0.61)	−0.61	(0.64)
Region	−0.41	(0.72)	−0.24	(0.69)
Sector	3.26***	(0.99)	1.81*	(1.02)
Constant	12.39***	(4.80)	−7.28*	(3.85)
Total	3.79***	(0.46)	−1.87***	(0.49)
Reweighting error	−0.44***	(0.09)	−0.98***	(0.11)

Notes: The table presents the results of the RIF-regressions based OB decomposition approach based on log daily wages. The sample is restricted to male full-time workers in the manufacturing sector between 18 and 65 years, who earned more than 10 euros per day and work in West Germany. All coefficients above are multiplied by 100 for convenience. ***, **, and * indicate statistical significance at the 1, 5, and 10 percent level, respectively. Bootstrapped standard errors with 100 replications are presented in parentheses. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Table 2.B.6: Decomposition of the 85-15 percentile wage gap and the Gini coefficient, 2012-2017

Inequality measure	85-15		Gini coefficient	
	Coefficient	Standard Error	Coefficient	Standard Error
Total change	2.17***	(0.49)	−0.31***	(0.09)
Pure composition effect				
Age	−0.03	(0.10)	0.00	(0.02)
Education	1.15***	(0.17)	0.32***	(0.05)
Tenure	−0.19***	(0.04)	−0.04***	(0.01)
Nationality	0.02**	(0.01)	0.00	(0.00)
Automation threat	1.72***	(0.15)	0.23***	(0.02)
Collective bargaining	−0.11***	(0.04)	−0.02**	(0.01)
Plant size	0.68***	(0.14)	0.22***	(0.03)
Region	−0.08	(0.06)	0.00	(0.01)
Sector	0.24***	(0.09)	0.07***	(0.02)
Total	3.40***	(0.28)	0.77***	(0.06)
Specification error	1.24***	(0.13)	−0.16***	(0.01)
Pure wage structure effect				
Age	−6.12***	(1.74)	−1.39***	(0.31)
Education	−2.76***	(0.48)	−0.38***	(0.06)
Tenure	−9.63**	(4.10)	−1.99**	(0.82)
Nationality	0.49***	(0.18)	0.05*	(0.03)
Automation threat	−3.52	(2.78)	−2.22***	(0.45)
Collective bargaining	2.32**	(0.93)	0.39**	(0.18)
Plant size	−1.16*	(0.64)	−0.30**	(0.12)
Region	−4.23***	(0.81)	−0.86***	(0.20)
Sector	1.10	(0.99)	0.19	(0.20)
Constant	21.72***	(5.67)	5.70***	(1.02)
Total	−1.79***	(0.51)	−0.81***	(0.09)
Reweighting error	−0.69***	(0.06)	−0.11***	(0.02)

Notes: The table presents the results of the RIF-regressions based OB decomposition approach based on log daily wages (85-15) and daily wages (Gini coefficient). The sample is restricted to male full-time workers in the manufacturing sector between 18 and 65 years, who earned more than 10 euros per day and work in West Germany. All coefficients above are multiplied by 100 for convenience. ***, **, and * indicate statistical significance at the 1, 5, and 10 percent level, respectively. Bootstrapped standard errors with 100 replications are presented in parentheses. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Table 2.B.7: Decomposition of the 50-15 and the 85-50 percentile wage gap, 2012-2017

Inequality measure	50-15		85-50	
	Coefficient	Standard Error	Coefficient	Standard Error
Total change	4.66***	(0.39)	−2.48***	(0.24)
Pure composition effect				
Age	−0.03	(0.04)	0.00	(0.08)
Education	0.20***	(0.03)	0.95***	(0.14)
Tenure	−0.14***	(0.03)	−0.05**	(0.02)
Nationality	0.01	(0.01)	0.01***	(0.00)
Automation threat	0.58***	(0.06)	1.14***	(0.10)
Collective bargaining	−0.09***	(0.03)	−0.02**	(0.01)
Plant size	0.49***	(0.10)	0.19**	(0.08)
Region	−0.10**	(0.04)	0.02	(0.04)
Sector	0.61***	(0.05)	−0.37***	(0.08)
Total	1.54***	(0.15)	1.86***	(0.22)
Specification error	0.02	(0.09)	1.22***	(0.10)
Pure wage structure effect				
Age	−1.69	(1.31)	−4.42***	(1.07)
Education	0.13	(0.16)	−2.89***	(0.43)
Tenure	−4.05	(3.08)	−5.58**	(2.31)
Nationality	0.42***	(0.15)	0.07	(0.09)
Automation threat	2.47	(2.10)	−5.98***	(1.90)
Collective bargaining	1.79**	(0.78)	0.53	(0.58)
Plant size	−1.12**	(0.46)	−0.05	(0.42)
Region	−1.69***	(0.64)	−2.54***	(0.66)
Sector	0.07	(0.73)	1.03	(0.71)
Constant	6.86	(4.66)	14.86***	(3.29)
Total	3.19***	(0.39)	−4.98***	(0.33)
Reweighting error	−0.10***	(0.03)	−0.59***	(0.05)

Notes: The table presents the results of the RIF-regressions based OB decomposition approach based on log daily wages. The sample is restricted to male full-time workers in the manufacturing sector between 18 and 65 years, who earned more than 10 euros per day and work in West Germany. All coefficients above are multiplied by 100 for convenience. ***, **, and * indicate statistical significance at the 1, 5, and 10 percent level, respectively. Bootstrapped standard errors with 100 replications are presented in parentheses. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Table 2.B.8: RIF-regressions 15th quantile, 1996 2010 2012 2017

	1996	**2010**	**2012**	**2017**
Age: 18-25	-0.0406**	-0.0827***	-0.0618***	-0.0347*
	(0.0166)	(0.0171)	(0.0155)	(0.0208)
Age: 36-45	-0.0442***	-0.0442***	-0.0589***	-0.0582***
	(0.0086)	(0.0104)	(0.0104)	(0.0137)
Age: 46-55	-0.0442***	-0.0887***	-0.0962***	-.0692***
	(0.0104)	(0.0114)	(0.0117)	(.0164)
Age: $\geq$ 56	-0.0506***	-0.0946***	-0.0983***	-0.1165***
	(0.0115)	(0.0130)	(0.0126)	(0.0172)
Education: low	-0.1148***	-0.1534***	-0.1568***	-.1383***
	(0.0069)	(0.0084)	(0.0101)	(.0133)
Education: high	0.0779***	0.1227***	0.1197***	0.1277***
	(0.0060)	(0.0051)	(0.0051)	(0.0066)
Tenure: 2-4	0.0997***	0.0984***	0.1237***	0.0993***
years	(0.0219)	(0.0232)	(0.0179)	(0.0222)
Tenure: 4-8	0.2144***	0.1975***	0.1875***	0.2661***
years	(0.0189)	(0.0241)	(0.0177)	(0.0231)
Tenure: 8-16	0.2721***	0.3239***	0.2938***	0.3606***
years	(0.0189)	(0.0267)	(0.0187)	(0.0240)
Tenure: $\geq$ 16	0.3412***	0.4645***	0.4519***	0.4806***
years	(0.0198)	(0.0286)	(0.0207)	(0.0269)
Nationality	-0.0392***	-0.0311***	-0.0455***	-0.0881***
	(.0084)	(0.0084)	(0.0091)	(0.0127)
Automation	0.0021	-0.0563***	-0.0042	-0.0350
threat: middle	(0.0099)	(0.0117)	(0.0097)	(0.0244)
Automation	-0.0180*	-0.0673***	-0.0314***	-0.0191
threat: high	(0.0108)	(0.0109)	(0.0097)	(0.0221)

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Table 2.B.8 – *Continued from previous page*

	1996	**2010**	**2012**	**2017**
Firm level agreement	-0.0097	0.1666***	0.1377***	0.1274***
	(0.0198)	(0.0072)	(0.0069)	(0.0090)
Sector level agreement	0.0379**	0.1762***	0.1781***	0.1863***
	(0.0172)	(0.0071)	(0.0066)	(0.0074)
Plant size: 1-9 employees	-0.3037***	-0.6222***	-0.6038***	-0.5561***
	(0.0320)	(0.0358)	(0.0365)	(0.0481)
Plant size: 10-49 employees	-0.1651***	-0.2516***	-0.2717***	-0.2482***
	(0.0111)	(0.0111)	(0.0109)	(0.0152)
Plant size: 50-199 employees	-0.0472***	-0.1045***	-0.1064***	-0.1031***
	(0.0037)	(0.0048)	(0.0048)	(0.0072)
Plant size: 1000-4999 employees	0.0219***	0.0404***	0.0438***	0.0528***
	(0.0021)	(0.0025)	(0.0024)	(0.0045)
Plant size: $\geq$ 5000 employees	0.0313***	0.0386***	0.0326***	0.0796***
	(0.0029)	(0.0035)	(0.0026)	(0.0066)
Sector: Food and beverages	-0.1033***	-0.2407***	-0.2665***	-0.5079***
	(0.0179)	(0.0141)	(0.0135)	(0.0226)
Sector: Textiles	-0.1922***	-0.3958***	-0.3723***	-0.3915***
	(0.0181)	(0.0282)	(0.0330)	(0.0429)
Sector: Wood, furniture and paper	0.0229*	-0.1248***	-0.1117***	-0.1047***
	(0.0132)	(0.0129)	(0.0135)	(0.0270)
Sector: Plastic and chemical products	0.0418***	-0.0183**	0.0056	-0.0516***
	(0.0084)	(0.0076)	(0.0078)	(0.0099)
Sector: Electrical products	0.0307***	0.0217***	0.0198***	0.0188*
	(0.0093)	(0.0072)	(0.0077)	(0.0108)

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Table 2.B.8 – *Continued from previous page*

	1996	**2010**	**2012**	**2017**
Sector: Industrial machinery	0.0438***	0.0846***	0.0949***	0.0696***
	(0.0095)	(0.0065)	(0.0065)	(0.0077)
Sector: Automotive and other vehicles	0.0572***	0.0198***	0.0248***	-0.0373***
	(0.0066)	(0.0061)	(0.0057)	(0.0079)
Schleswig-Holstein	0.0035	-0.0655***	-0.0592***	-0.0093
	(0.0182)	(0.0149)	(0.0126)	(0.0161)
Hamburg	0.0527***	0.0145*	0.0601***	-0.0092
	(0.0117)	(0.0087)	(0.0094)	(0.0157)
Lower Saxony	-0.0676***	-0.0718***	-0.0359***	-0.0071
	(0.0100)	(0.0077)	(0.0076)	(.0091)
Bremen	-0.0119	-0.0056	0.0183**	-0.0805***
	(0.0231)	(0.0132)	(0.0079)	(0.0111)
Hesse	-0.0125	-0.0871***	-0.0636***	-0.0697***
	(0.0096)	(0.0092)	(0.0091)	(0.0096)
Rhineland-Palatinate	-0.0828***	-0.0487***	-0.0473***	0.0249
	(0.0147)	(0.0087)	(0.0095)	(0.0088)
Baden-Wuerttemberg	0.0019	0.0117*	0.0022	0.0264***
	(0.0074)	(0.0066)	(0.0069)	(0.0090)
Bavaria	-0.0565***	-0.0554***	-0.0396***	-0.0325***
	(0.0069)	(0.0074)	(0.0071)	(0.0092)
Saarland	0.0328**	-0.0854***	-0.1410***	-0.0695***
	(0.0132)	(0.0097)	(0.0128)	(0.0152)
Constant	4.2661***	4.1694***	4.1530***	4.1578***
	(0.0295)	(0.0275)	(0.0209)	(0.0334)
Observations	576,895	389,624	437,336	320,970

Notes: The table presents the RIF-regressions for the 15th quantile. The observed years are 1996, 2010, 2012 and 2017. Standard errors are given in parentheses. Sampling weights are employed.

Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Table 2.B.9: RIF-regressions 50th quantile, 1996 2010 2012 2017

	1996	**2010**	**2012**	**2017**
Age: 18-25	-0.0720***	-0.1081***	-0.0723***	-0.0496***
	(0.0071)	(0.0076)	(0.0070)	(0.0065)
Age: 36-45	0.0049	0.0033	-0.0019	-0.0043
	(0.0052)	(0.0051)	(0.0054)	(0.0053)
Age: 46-55	0.0198***	-0.0141**	-0.0251***	-0.0077
	(0.0064)	(0.0055)	(0.0061)	(0.0066)
Age: $\geq$ 56	0.0179**	-0.0187***	-0.0357***	-0.0348***
	(0.0082)	(0.0065)	(0.0068)	(0.0072)
Education: low	-0.1721***	-0.1666***	-0.1567***	-0.1059***
	(0.0038)	(0.0037)	(0.0042)	(0.0047)
Education: high	0.2435***	0.2769***	0.2668***	0.2792***
	(0.0050)	(0.0035)	(0.0038)	(0.0043)
Tenure: 2-4 years	0.0313***	0.0289***	0.0426***	0.0394***
	(0.0097)	(0.0105)	(0.0079)	(0.0080)
Tenure: 4-8 years	0.0608***	0.0529***	0.0763***	0.0870***
	(0.0092)	(0.0125)	(0.0077)	(0.0084)
Tenure: 8-16 years	0.1447***	0.1239***	0.1571***	0.1676***
	(0.0100)	(0.0144)	(0.0087)	(0.0089)
Tenure: $\geq$ 16 years	0.2259***	0.2172***	0.2607***	0.2489***
	(0.0107)	(0.0154)	(0.0100)	(0.0101)
Nationality	-0.0621***	-0.0660***	-0.0660***	-0.0602***
	(0.0046)	(0.0043)	(0.0045)	(0.0045)
Automation threat: middle	-0.0074	-0.0428***	-0.0613***	-0.0245***
	(0.0075)	(0.0070)	(0.0055)	(0.0090)
Automation threat: high	-0.1009***	-0.1390***	-0.2038***	-0.1649***
	(0.0074)	(0.0068)	(0.0058)	(0.0087)

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Table 2.B.9 – *Continued from previous page*

	1996	**2010**	**2012**	**2017**
Firm level agreement	0.0026	0.0477***	0.0397***	0.0475***
	(0.0119)	(0.0042)	(0.0049)	(0.0047)
Sector level agreement	0.0169*	0.0666***	0.0799***	0.1020***
	(0.0102)	(0.0037)	(0.0036)	(0.0038)
Plant size: 1-9 employees	-0.1746***	-0.2226***	-0.2331***	-0.2131***
	(0.0180)	(0.0154)	(0.0143)	(0.0196)
Plant size: 10-49 employees	-0.1350***	-0.1495***	-0.1387***	-0.1271***
	(0.0081)	(0.0058)	(0.0060)	(0.0074)
Plant size: 50-199 employees	-0.0319***	-0.0719***	-0.0801***	-0.0819***
	(0.0034)	(0.0032)	(0.0036)	(0.0041)
Plant size: 1000-4999 employees	0.0313***	0.1098***	0.1319***	0.1317***
	(0.0018)	(0.0021)	(0.0022)	(0.0028)
Plant size: $\geq$ 5000 employees	0.1607***	0.2317***	0.2211***	0.2282***
	(0.0024)	(0.0026)	(0.0024)	(0.0038)
Sector: Food and beverages	-0.0946***	-0.1487***	-0.1886***	-0.3234***
	(0.0106)	(0.0071)	(0.0074)	(0.0086)
Sector: Textiles	-0.1808***	-0.1958***	-0.2469***	-0.2124***
	(0.0113)	(0.0147)	(0.0161)	(0.0182)
Sector: Wood, furniture and paper	0.0129	-0.1461***	-0.2175***	-0.2177***
	(0.0087)	(0.0065)	(0.0075)	(0.0111)
Sector: Plastic and chemical products	0.0375***	0.0168***	0.0117**	-0.0274***
	(0.0059)	(0.0042)	(0.0048)	(0.0052)
Sector: Electrical products	0.0365***	0.0858***	0.0419***	0.0250***
	(0.0072)	(0.0043)	(0.0045)	(0.0065)

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Table 2.B.9 – *Continued from previous page*

	1996	**2010**	**2012**	**2017**
Sector: Industrial machinery	0.0557***	0.0929***	0.0876***	0.0811***
	(0.0058)	(0.0040)	(0.0041)	(0.0047)
Sector: Automotive and other vehicles	0.0955***	0.0841***	0.1013***	0.0733***
	(0.0046)	(0.0037)	(0.0038)	(0.0045)
Schleswig-Holstein	-0.0019***	-0.0350***	-0.0286***	-0.0302***
	(0.0139)	(0.0067)	(0.0067)	(0.0074)
Hamburg	0.0688***	0.0616***	0.0828***	0.0400***
	(0.0116)	(0.0052)	(0.0081)	(0.0078)
Lower Saxony	-0.0608***	-0.0271***	-0.0255***	0.0026
	(0.0057)	(0.0040)	(0.0046)	(0.0048)
Bremen	0.0375**	0.0749***	0.0577***	0.0575***
	(0.0189)	(0.0075)	(0.0049)	(0.0055)
Hesse	-0.0076	-0.0470***	-0.0477***	-0.0355***
	(0.0072)	(0.0045)	(0.0049)	(0.0049)
Rhineland-Palatinate	-0.0722***	-0.0548***	-0.0474***	0.0297***
	(0.0078)	(0.0043)	(0.0050)	(0.0052)
Baden-Wuerttemberg	0.0317***	0.0422***	0.0557***	0.0646***
	(0.0051)	(0.0037)	(0.0039)	(0.0049)
Bavaria	-0.0573***	-0.0538***	-0.0399***	-0.0240***
	(0.0045)	(0.0039)	(0.0039)	(0.0046)
Saarland	-0.0582***	-0.0721***	-0.0643***	-0.0019
	(0.0087)	(0.0061)	(0.0074)	(0.0090)
Constant	4.6348***	4.6585***	4.6683***	4.6855***
	(0.0150)	(0.0146)	(0.0102)	(0.0129)
Observations	576,895	389,624	437,336	320,970

Notes: The table presents the RIF-regressions for the 50th quantile. The observed years are 1996, 2010, 2012 and 2017. Standard errors are given in parentheses. Sampling weights are employed.

Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Table 2.B.10: RIF-regressions 85th quantile, 1996 2010 2012 2017

	1996	**2010**	**2012**	**2017**
Age: 18-25	0.1309	0.1533***	0.1564***	0.1391***
	(0.0106)	(0.0089)	(0.0107)	(0.0099)
Age: 36-45	0.1239	0.1355***	0.1406***	0.0914***
	(0.0085)	(0.0075)	(0.0083)	(0.0105)
Age: 46-55	0.2275	0.1785***	0.1940***	0.1518***
	(0.0101)	(0.0081)	(0.0095)	(0.0121)
Age: $\geq$ 56	0.2649	0.1902***	0.1939***	0.1332***
	(0.0127)	(0.0095)	(0.0107)	(0.0130)
Education: low	-0.1830***	-0.1191	-0.1187***	-0.0882***
	(0.0046)	(0.0038)	(0.0049)	(0.0055)
Education: high	0.9562***	1.0164	1.0184***	0.9216***
	(0.0146)	(0.0095)	(0.0109)	(0.0115)
Tenure: 3-4	0.0530***	-0.0044***	-0.0022	-0.0059
years	(0.0155)	(0.0106)	(0.0118)	(0.0138)
Tenure: 5-8	0.1376***	0.0658***	0.0755***	0.0684***
years	(0.0139)	(0.0111)	(0.0130)	(0.0133)
Tenure: 9-16	0.2469***	0.1902***	0.1943***	0.1734***
years	(0.0148)	(0.0126)	(0.0145)	(0.0150)
Tenure: $\geq$ 17	0.2671***	0.2219***	0.2505***	0.2529***
years	(0.0155)	(0.0139)	(0.0158)	(0.0174)
Nationality	-0.0759***	-0.0885***	-0.0863***	-0.0679***
	(0.0058)	(0.0049)	(0.0066)	(0.0065)
Automation	-0.0706***	-0.0209	-0.1377***	-0.1402***
threat: middle	(0.0166)	(0.0124)	(0.0112)	(0.0188)
Automation	-0.3492***	-0.2708***	-0.5368***	-0.4575***
threat: high	(0.0160)	(0.0126)	(0.0120)	(0.0179)

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Table 2.B.10 – *Continued from previous page*

	1996	2010	2012	2017
Firm level agreement	0.0060***	0.0045***	0.0194	0.0902***
	(0.0179)	(0.0066)	(0.0087)	(0.0077)
Sector level agreement	0.0079***	0.0454***	0.0673***	0.0912***
	(0.0161)	(0.0057)	(0.0061)	(0.0065)
Plant size: 1-9 employees	-0.1042***	-0.1108***	-0.1295***	-0.1155***
	(0.0264)	(0.0220)	(0.0210)	(0.0360)
Plant size: 10-49 employees	-0.1133***	-0.0923***	-0.1138***	-0.0643***
	(0.0130)	(0.0086)	(0.0097)	(0.0131)
Plant size: 50-199 employees	-0.0168***	-0.0526***	-0.0793***	-0.0725***
	(0.0062)	(0.0051)	(0.0067)	(0.0063)
Plant size: 1000-4999 employees	0.0261***	0.0771***	0.1119***	0.1179***
	(0.0032)	(0.0039)	(0.0046)	(0.0053)
Plant size: $\geq$ 5000 employees	0.1221***	0.2125***	0.2626***	0.1862***
	(0.0043)	(0.0047)	(0.0047)	(0.0064)
Sector: Food and beverages	-0.2853***	-0.1827***	-0.3242***	-0.3437***
	(0.0194)	(0.0108)	(0.0123)	(0.0138)
Sector: Textiles	-0.3253***	-0.2836***	-0.4845***	-0.3932***
	(0.0189)	(0.0237)	(0.0264)	(0.0307)
Sector: Wood, furniture and paper	-0.0649***	-0.2411***	-0.4705***	-0.4698***
	(0.0124)	(0.0106)	(0.0125)	(0.0204)
Sector: Plastic and chemical products	0.0176*	0.0308***	0.0500***	0.0056
	(0.0098)	(0.0064)	(0.0086)	(0.0081)
Sector: Electrical products	0.0355***	0.1485***	0.0398***	0.0017
	(0.0097)	(0.0075)	(0.0089)	(0.0129)

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Table 2.B.10 – *Continued from previous page*

	1996	**2010**	**2012**	**2017**
Sector: Industrial machinery	-0.0219**	0.0212***	-0.0178***	-0.0197***
	(0.0091)	(0.0062)	(0.0068)	(0.0072)
Sector: Automotive and other vehicles	0.0775***	0.0294***	0.0719***	0.0233***
	(0.0070)	(0.0053)	(0.0061)	(0.0075)
Schleswig-Holstein	-0.0851***	-0.0165	-0.0283**	-0.0234
	(0.0169)	(0.0113)	(0.0123)	(0.0145)
Hamburg	-0.0086	0.0186*	0.0525***	-0.0801***
	(0.0134)	(0.0098)	(0.0141)	(0.0176)
Lower Saxony	-0.0631***	-0.0124*	0.0008	-0.0334***
	(0.0089)	(0.0065)	(0.0081)	(0.0082)
Bremen	0.0214	0.0519***	-0.0440***	-0.0044
	(0.0180)	(0.0139)	(0.0097)	(0.0112)
Hesse	0.0045	-0.0313***	-0.0214**	0.0129
	(0.0137)	(0.0072)	(0.0085)	(0.0083)
Rhineland-Palatinate	-0.1184***	-0.0664***	-0.0317***	-0.0189**
	(0.0123)	(0.0066)	(0.0082)	(0.0090)
Baden-Wuerttemberg	-0.0088	0.0319***	0.0879***	0.0679***
	(0.0080)	(0.0063)	(0.0070)	(0.0093)
Bavaria	-0.0533***	-0.0383***	-0.0369***	-0.0815***
	(0.0064)	(0.0058)	(0.0065)	(0.0081)
Saarland	-0.0932***	-0.0862***	-0.0415***	0.0089
	(0.0136)	(0.0083)	(0.0122)	(0.0162)
Constant	5.0507***	4.9485***	5.1143***	5.1670***
	(0.0244)	(0.0177)	(0.0192)	(0.0235)
Observations	576,895	389,624	437,336	320,970

Notes: The table presents the RIF-regressions for the 85th quantile. The observed years are 1996, 2010, 2012 and 2017. Standard errors are given in parentheses. Sampling weights are employed.

Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Table 2.B.11: RIF-regressions Gini coefficient, 1996 2010 2012 2017

	1996	**2010**	**2012**	**2017**
Age: 18-25	0.0591***	0.0779***	0.0664***	0.0502***
	(0.0010)	(0.0021)	(0.0018)	(0.0020)
Age: 36-45	0.0306***	0.0368***	0.0371***	0.0227***
	(0.0007)	(0.0013)	(0.0012)	(0.0013)
Age: 46-55	0.0544***	0.0589***	0.0599***	0.0424***
	(0.0008)	(0.0015)	(0.0014)	(0.0015)
Age: $\geq$ 56	0.0635***	0.0628***	0.0613***	0.0489***
	(0.0010)	(0.0017)	(0.0015)	(0.0016)
Education: low	0.0059***	0.0248***	0.0278***	0.0231***
	(0.0007)	(0.0012)	(0.0012)	(0.0014)
Education: high	0.2834***	0.2996***	0.2582***	0.2298***
	(0.0008)	(0.0011)	(0.0009)	(0.0010)
Tenure: 3-4	-0.0139***	-0.0326***	-0.0332***	-0.0325***
years	(0.0014)	(0.0028)	(0.0023)	(0.0027)
Tenure: 5-8	-0.0134***	-0.0292***	-0.0331***	-0.0538***
years	(0.0012)	(0.0027)	(0.0022)	(0.0026)
Tenure: 9-16	0.0023	-0.0236***	-0.0251***	-0.0535***
years	(0.0012)	(0.0028)	(0.0023)	(0.0027)
Tenure: $\geq$ 17	-0.0099***	-0.0409***	-0.0442***	-0.0520***
years	(0.0013)	(0.0029)	(0.0025)	(0.0029)
Nationality	-0.0051***	-0.0095***	0.0004	0.0074***
	(0.0007)	(0.0012)	(0.0011)	(0.0012)
Automation	-0.0145***	0.0153***	-0.0158***	-0.0238***
threat: middle	(0.0010)	(0.0015)	(0.0012)	(0.0016)
Automation	-0.0509***	-0.0195***	-0.0648***	-0.0631***
threat: high	(0.0009)	(0.0015)	(0.0014)	(0.0017)

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Table 2.B.11 – *Continued from previous page*

	1996	**2010**	**2012**	**2017**
Firm level agreement	-0.0056***	-0.0322***	-0.0225***	-0.0092***
	(0.0011)	(0.0012)	(0.0011)	(0.0014)
Sector level agreement	-0.0162***	-0.0211***	-0.0228***	-0.0224***
	(0.0009)	(0.0009)	(0.0007)	(0.0009)
Plant size: 1-9 employees	0.0616***	0.1097***	0.1128***	0.0950***
	(0.0011)	(0.0020)	(0.0018)	(0.0024)
Plant size: 10-49 employees	0.0208***	0.0348***	0.0369***	0.0334***
	(0.0007)	(0.0011)	(0.0010)	(0.0012)
Plant size: 50-199 employees	0.0101***	0.0109***	0.0087***	0.0083***
	(0.0006)	(0.0009)	(0.0008)	(0.0010)
Plant size: 1000-4999 employees	0.0011	0.0163***	0.0205***	0.0187***
	(0.0007)	(0.0010)	(0.0009)	(0.0011)
Plant size: $\geq$ 5000 employees	0.0417***	0.0668***	.0639***	0.0306***
	(0.0009)	(0.0014)	(.0011)	(0.0014)
Sector: Food and beverages	-0.0066***	0.0269***	0.0125***	0.0622***
	(0.0013)	(0.0016)	(0.0014)	(0.0016)
Sector: Textiles	-0.0119***	0.0303***	-0.0023	0.0436***
	(0.0015)	(0.0032)	(0.0030)	(0.0043)
Sector: Wood, furniture and paper	-0.0153***	-0.0111***	-0.0395***	-0.0464***
	(0.0009)	(0.0015)	(0.0016)	(0.0022)
Sector: Plastic and chemical products	0.0007	0.0049***	0.0102***	0.0121***
	(0.0008)	(0.0011)	(0.0010)	(0.0013)
Sector: Electrical products	-0.0020**	0.0311***	-0.0021*	-0.0031**
	(0.0009)	(0.0011)	(0.0011)	(0.0014)

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Table 2.B.11 – *Continued from previous page*

	1996	**2010**	**2012**	**2017**
Sector: Industrial machinery	-0.0191***	-0.0228***	-0.0266***	-0.0289***
	(0.0007)	(0.0011)	(0.0009)	(0.0012)
Sector: Automotive and other vehicles	0.0039***	-0.0094***	0.0093***	0.0056***
	(0.0009)	(0.0012)	(0.0011)	(0.0014)
Schleswig-Holstein	0.0169***	0.0128***	0.0071***	-0.0062**
	(0.0016)	(0.0022)	(0.0022)	(0.0028)
Hamburg	-0.0026***	-0.0124***	-0.0002	-0.0394***
	(0.0016)	(0.0020)	(0.0017)	(0.0022)
Lower Saxony	0.0049***	0.0142***	0.0184***	-0.0051***
	(0.0008)	(0.0012)	(0.0011)	(0.0013)
Bremen	0.0081***	-0.0002	-0.0231***	-0.0029
	(0.0021)	(0.0046)	(0.0030)	(0.0040)
Hesse	0.0045***	0.0149***	0.0122***	0.0253***
	(0.0009)	(0.0014)	(0.0012)	(0.0014)
Rhineland-Palatinate	-0.0069***	-0.0057***	0.0052***	-0.0125***
	(0.0011)	(0.0015)	(0.0014)	(0.0015)
Baden-Wuerttemberg	0.0019***	0.0043***	0.0106***	0.0122***
	(0.0007)	(0.0009)	(0.0009)	(0.0011)
Bavaria	0.0017**	0.0094***	0.0032***	-0.0168***
	(0.0007)	(0.0009)	(0.0009)	(0.0010)
Saarland	-0.0241***	-0.0004	0.0215***	0.0187***
	(0.0017)	(0.0025)	(0.0031)	(0.0037)
Constant	0.1918***	0.1895***	0.2317***	0.2634
	(0.0018)	(0.0033)	(0.0028)	(0.0034)
Observations	576,895	389,624	437,336	320,970

Notes: The table presents the RIF-regressions for the Gini coefficients. The observed years are 1996, 2010, 2012 and 2017. Standard errors are given in parentheses. Sampling weights are employed.

Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Table 2.B.12: RIF-regressions variance, 1996 2010 2012 2017

	1996	**2010**	**2012**	**2017**
Age: 18-25	0.0567***	0.0818***	0.0709***	0.0488***
	(0.0013)	(0.0029)	(0.0025)	(0.0029)
Age: 36-45	0.0306***	0.0566***	0.0517***	0.0349***
	(0.0008)	(0.0018)	(0.0017)	(0.0019)
Age: 46-55	0.0573***	0.0854***	0.0829***	.0617***
	(0.0010)	(0.0020)	(0.0019)	(.0022)
Age: $\geq$ 56	0.0684***	0.0923***	0.0851***	0.0699***
	(0.0012)	(0.0023)	(0.0021)	(0.0024)
Education: low	0.0134***	0.0360***	0.0440***	0.0354***
	(0.0009)	(0.0017)	(0.0017)	(0.0020)
Education: high	0.2951***	0.3686***	0.3163***	0.2890***
	(0.0010)	(0.0014)	(0.0013)	(0.0014)
Tenure: 3-4	-0.0394***	-0.0883***	-0.0877***	-0.0861***
years	(0.0017)	(0.0039)	(0.0032)	(0.0038)
Tenure: 5-8	-0.0422***	0.0901***	-0.0976***	-0.1346***
years	(0.0015)	(0.0039)	(0.0031)	(0.0037)
Tenure: 9-16	-0.0222***	-0.1051***	-0.1015***	-0.1393***
years	(0.0015)	(0.0040)	(0.0033)	(0.0038)
Tenure: $\geq$ 17	-0.0326***	-0.1319***	-0.1295***	-0.1415***
years	(0.0016)	(0.0042)	(0.0035)	(0.0041)
Nationality	-0.0109***	-0.0159***	0.0001	0.0133***
	(0.0009)	(0.0017)	(0.0015)	(0.0017)
Automation	-0.0254***	0.0191***	-0.0201***	-0.0254***
threat: middle	(0.0012)	(0.0020)	(0.0018)	(0.0023)
Automation	-0.0654***	-0.0360***	-0.0929***	-0.0868***
threat: high	(0.0012)	(0.0021)	(0.0019)	(0.0025)

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Table 2.B.12 – *Continued from previous page*

	1996	**2010**	**2012**	**2017**
Firm level agreement	-0.0297***	-0.0432***	-0.0346***	-0.0144***
	(0.0013)	(0.0017)	(0.0016)	(0.0019)
Sector level agreement	-0.0428***	-0.0295***	-0.0347***	-0.0318***
	(0.0011)	(0.0012)	(0.0011)	(0.0013)
Plant size: 1-9 employees	0.0716***	0.2133***	0.1967***	0.1528***
	(0.0013)	(0.0028)	(0.0026)	(0.0034)
Plant size: 10-49 employees	0.0183***	0.0431***	0.0477***	0.0430***
	(0.0009)	(0.0016)	(0.0014)	(0.0017)
Plant size: 50-199 employees	0.0037***	0.0144***	0.0124***	0.0098***
	(0.0007)	(0.0013)	(0.0012)	(0.0014)
Plant size: 1000-4999 employees	-0.0008	0.0281***	0.0339***	0.0339***
	(0.0008)	(0.0014)	(0.0013)	(0.0016)
Plant size: $\geq$ 5000 employees	0.0469***	0.0936***	0.0977***	0.0617***
	(0.0011)	(0.0020)	(0.0017)	(0.0020)
Sector: Food and beverages	0.0090***	0.0504***	0.0292***	0.0797***
	(0.0016)	(0.0022)	(0.0020)	(0.0023)
Sector: Textiles	-0.0170***	0.0503***	0.0140***	0.0764***
	(0.0018)	(0.0046)	(0.0042)	(0.0061)
Sector: Wood, furniture and paper	-0.0181***	-0.0154***	-0.0573***	-0.0639***
	(0.0011)	(0.0022)	(0.0022)	(0.0032)
Sector: Plastic and chemical products	-0.0034***	0.0134***	0.0145***	0.0167***
	(0.0009)	(0.0016)	(0.0015)	(0.0019)
Sector: Electrical products	-0.0028***	0.0439***	0.0045***	-0.0045**
	(0.0010)	(0.0016)	(0.0016)	(0.0020)

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Table 2.B.12 – *Continued from previous page*

	1996	**2010**	**2012**	**2017**
Sector: Industrial machinery	-0.0146***	-0.0236***	-0.0300***	-0.0318***
	(0.0009)	(0.0015)	(0.0013)	(0.0016)
Sector: Automotive and other vehicles	0.0067***	0.0006	0.0198***	0.0141***
	(0.0010)	(0.0017)	(0.0016)	(0.0020)
Schleswig-Holstein	0.0118***	0.0262***	0.0203***	-0.0113***
	(0.0019)	(0.0031)	(0.0032)	(0.0039)
Hamburg	-0.0037*	-0.0044	0.0139***	-0.0328***
	(0.0019)	(0.0028)	(0.0024)	(0.0031)
Lower Saxony	0.0048***	0.0187***	0.0182***	-0.0112***
	(0.0009)	(0.0017)	(0.0016)	(0.0019)
Bremen	0.0138***	0.0102***	-0.0286***	0.0037
	(0.0026)	(0.0065)	(0.0043)	(0.0057)
Hesse	0.0228***	0.0171***	0.0119***	0.0265***
	(0.0010)	(0.0020)	(0.0017)	(0.0020)
Rhineland-Palatinate	-0.0121***	-0.0103***	0.0015	-0.0192***
	(0.0013)	(0.0021)	(0.0020)	(0.0022)
Baden-Wuerttemberg	0.0067***	0.0113***	0.0151***	0.0196***
	(0.0008)	(0.0013)	(0.0013)	(0.0015)
Bavaria	0.0005	0.0108***	0.0021	0.0256***
	(0.0008)	(0.0013)	(0.0012)	(0.0014)
Saarland	-0.0299***	-0.0059	0.0218***	0.0208***
	(0.0021)	(0.0036)	(0.0044)	(0.0053)
Constant	0.1681***	0.1757***	0.2318***	0.2696***
	(0.0022)	(0.0046)	(0.0039)	(0.0047)
Observations	576,895	389,624	437,336	320,970

Notes: The table presents the RIF-regressions for the log variance. The observed years are 1996, 2010, 2012 and 2017. Standard errors are given in parentheses. Sampling weights are employed.

Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Table 2.B.13: Decomposition of the variance, 1996-2010 and 2012-2017

	1996-2010		2012-2017	
	Coefficient	Standard Deviation	Coefficient	Standard Deviation
Total change	5.58***	(0.18)	−0.19	(0.17)
Pure composition effect				
Age	0.75***	(0.06)	−0.01	(0.03)
Education	1.67***	(0.09)	0.38***	(0.06)
Tenure	−0.04	(0.07)	−0.09***	(0.02)
Nationality	0.03	(0.02)	0.00	(0.00)
Automation threat	0.17***	(0.04)	0.33***	(0.03)
Collective bargaining	0.94***	(0.22)	−0.03***	(0.01)
Plant size	−0.24***	(0.05)	0.34***	(0.05)
Region	−0.07**	(0.03)	−0.03	(0.02)
Sector	0.08***	(0.03)	0.16***	(0.03)
Total	3.27***	(0.23)	1.06***	(0.08)
Specification error	−0.47***	(0.14)	−0.17***	(0.02)
Pure wage structure effect				
Age	2.55***	(0.70)	−2.12***	(0.52)
Education	1.52***	(0.18)	−0.55***	(0.11)
Tenure	−8.56***	(2.78)	−1.26	(1.59)
Nationality	−0.05	(0.06)	0.10*	(0.06)
Automation threat	3.82**	(1.81)	−1.96***	(0.68)
Collective bargaining	−0.39	(0.47)	0.97***	(0.33)
Plant size	1.99***	(0.24)	−0.48***	(0.18)
Region	−0.70*	(0.38)	−1.36***	(0.39)
Sector	1.22**	(0.59)	0.06	(0.39)
Constant	1.78	(3.55)	5.64***	(1.93)
Total	3.17***	(0.23)	−0.96***	(0.15)
Reweighting error	−0.39***	(0.06)	−0.12***	(0.02)

Notes: The table presents the results of the RIF-regressions based OB decomposition approach based on log daily wages. The sample is restricted to male full-time workers in the manufacturing sector between 18 and 65 years, who earned more than 10 euros per day and work in West Germany. All coefficients above are multiplied by 100 for convenience. ***, **, and * indicate statistical significance at the 1, 5, and 10 percent level, respectively. Bootstrapped standard errors with 100 replications are presented in parentheses. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Table 2.B.14: Decomposition results using the probability of computerization by Frey and Osborne (2017), 1996-2010

Inequality measure	85-15		Gini coefficient		50-15		85-50		Variance	
	Coefficient	Std Dev.	Coefficient	Std. Dev.	Coefficient	Std. Dev.	Coefficient	Std. Dev.	Coefficient	Std. Dev.
Total change	10.67***	(0.37)	4.24***	(0.10)	7.11***	(0.28)	3.56***	(0.26)	5.58***	(0.19)
Pure composition effect										
Age	4.03***	(0.23)	0.72***	(0.05)	1.10***	(0.15)	2.93***	(0.17)	0.78***	(0.07)
Education	5.59***	(0.35)	1.56***	(0.09)	1.23***	(0.07)	4.36***	(0.31)	1.60***	(0.10)
Tenure	−0.37*	(0.22)	−0.03	(0.05)	−0.11	(0.17)	−0.26**	(0.18)	−0.04	(0.07)
Nationality	0.13***	(0.03)	0.02	(0.00)	0.07***	(0.02)	0.07***	(0.02)	0.04***	(0.01)
Automation threat	0.13**	(0.06)	0.01	(0.01)	0.22***	(0.04)	−0.09*	(0.05)	0.03	(0.05)
Collective bargaining	1.28**	(0.54)	0.45***	(0.11)	0.68*	(0.37)	0.60	(0.38)	1.05***	(0.21)
Plant size	−0.57***	(0.09)	−0.21***	(0.02)	−0.49***	(0.07)	−0.08	(0.05)	−0.24***	(0.04)
Region	−0.19***	(0.07)	−0.02	(0.02)	0.00	(0.06)	−0.19***	(0.05)	−0.06**	(0.03)
Sector	0.05	(0.10)	0.08***	(0.02)	−0.11*	(0.06)	0.17**	(0.08)	0.09**	(0.04)
Total	10.09***	(0.80)	2.58***	(0.14)	2.59***	(0.40)	7.50***	(0.60)	3.23***	(0.29)
Specification error	−0.58	(0.60)	−0.50***	(0.08)	1.10***	(0.42)	−1.68***	(0.56)	−0.44***	(0.13)
Pure wage structure effect										
Age	4.66***	(1.69)	1.66***	(0.38)	−1.22	(1.34)	5.88***	(1.33)	2.75***	(0.66)
Education	1.87***	(0.59)	1.03***	(0.12)	−0.91***	(0.19)	2.79***	(0.59)	1.52***	(0.17)
Tenure	−13.83***	(5.01)	−2.67**	(1.10)	−13.31***	(4.33)	−0.52	(2.39)	−8.39***	(2.70)
Nationality	−0.43***	(0.15)	−0.05*	(0.03)	−0.05	(0.13)	−0.39***	(0.12)	−0.04	(0.06)
Automation threat	1.31	(1.63)	0.22	(0.29)	4.93***	(1.36)	−3.63**	(1.54)	0.54	(0.43)
Collective bargaining	−6.55***	(1.11)	−1.12***	(0.25)	−4.66***	(0.93)	−1.89*	(1.02)	0.06	(0.52)
Plant size	2.61***	(0.70)	0.40**	(0.18)	3.08***	(0.62)	−0.47	(0.64)	1.77***	(0.28)
Region	−1.01	(0.86)	−0.26	(0.22)	−0.90	(0.81)	-0.12	(0.59)	−0.77**	(0.37)
Sector	3.48***	(1.26)	0.17	(0.24)	3.58***	(1.06)	−0.10	(0.88)	0.71	(0.49)
Constant	10.09*	(5.92)	3.00**	(1.40)	13.11***	(4.60)	-3.03	(3.53)	4.93	(3.10)
Total	2.19***	(0.64)	2.37***	(0.18)	3.65***	(0.47)	−1.45***	(0.55)	3.09***	(0.29)
Reweighting error	−1.04***	(0.17)	−0.21***	(0.05)	−0.23**	(0.11)	−0.81***	(0.11)	−0.30***	(0.06)

Notes: The table presents the results of the RIF-regressions based OB decomposition approach based on log daily wages (85-15, 50-15, 85-50, Variance) and daily wages (Gini coefficient). The sample is restricted to male full-time workers in the manufacturing sector between 18 and 65 years, who earned more than 10 euros per day and work in West Germany. All coefficients above are multiplied by 100 for convenience. ***, **, and * indicate statistical significance at the 1, 5, and 10 percent level, respectively. Bootstrapped standard errors with 100 replications are presented in parentheses. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Frey and Osborne (2017), own calculations.

Table 2.B.15: Decomposition results using the probability of computerization by Frey and Osborne (2017), 2012-2017

Inequality measure	85-15		Gini coefficient		50-15		85-50		Variance	
	Coefficient	Std Dev.	Coefficient	Std. Dev.	Coefficient	Std. Dev.	Coefficient	Std. Dev.	Coefficient	Std. Dev.
Total change	2.17***	(0.57)	−0.31***	(0.10)	4.65***	(0.48)	−2.48***	(0.36)	−0.19***	(0.15)
Pure composition effect										
Age	−0.01	(0.09)	0.00	(0.02)	−0.02	(0.04)	0.01	(0.06)	0.02	(0.03)
Education	1.29***	(0.22)	0.33***	(0.06)	0.24***	(0.04)	1.05***	(0.18)	0.40***	(0.10)
Tenure	−0.18***	(0.04)	−0.04***	(0.01)	−0.13***	(0.03)	−0.05**	(0.02)	−0.09***	(0.02)
Nationality	0.02**	(0.01)	0.00	(0.00)	0.01**	(0.00)	0.01	(0.01)	0.00	(0.00)
Automation threat	0.06	(0.20)	0.00***	(0.00)	0.05	(0.03)	0.01	(0.01)	0.01	(0.01)
Collective bargaining	−0.11*	(0.04)	−0.02**	(0.01)	−0.09***	(0.03)	−0.02**	(0.01)	−0.03***	(0.01)
Plant size	0.71***	(0.13)	0.23***	(0.03)	0.48***	(0.09)	0.23**	(0.07)	0.34***	(0.05)
Region	−0.07	(0.06)	−0.00	(0.01)	−0.09**	(0.04)	0.02	(0.04)	−0.02***	(0.03)
Sector	0.05	(0.07)	0.04***	(0.02)	0.62	(0.06)	−0.57***	(0.05)	0.11***	(0.03)
Total	1.75***	(0.29)	0.54***	(0.07)	1.05***	(0.16)	0.70***	(0.20)	0.73***	(0.10)
Specification error	1.47	(0.16)	−0.07***	(0.01)	0.00	(0.10)	1.47***	(0.13)	−0.08***	(0.02)
Pure wage structure effect										
Age	−6.88***	(1.65)	−1.40***	(0.32)	−1.08	(1.37)	−5.80***	(1.04)	−2.06***	(0.62)
Education	−3.90***	(0.59)	−0.42***	(0.06)	0.33*	(0.18)	−4.24***	(0.57)	−0.56***	(0.19)
Tenure	−11.53***	(4.24)	−2.13*	(0.90)	−4.72	(3.44)	−6.82***	(2.14)	−1.62	(1.97)
Nationality	0.54***	(0.16)	0.06*	(0.03)	0.38	(0.14)	0.16***	(0.10)	0.11	(0.16)
Automation threat	3.87	(2.09)	−0.13	(0.38)	4.33***	(1.13)	−0.46	(1.64)	−0.22	(0.57)
Collective bargaining	1.83*	(0.94)	0.42**	(0.18)	1.88***	(0.77)	−0.05	(0.66)	0.96	(0.29)
Plant size	−1.58***	(0.57)	−0.35***	(0.12)	−1.22***	(0.48)	−0.36	(0.36)	−0.52***	(0.20)
Region	−3.31	(0.92)	−0.78	(0.19)	−1.16	(0.57)	−2.16***	(0.72)	−1.23***	(0.36)
Sector	4.33***	(1.08)	0.84***	(0.22)	2.15***	(0.77)	2.18***	(0.79)	0.82**	(0.40)
Constant	16.08***	(4.97)	3.21***	(1.04)	2.75	(3.58)	13.33	(3.04)	3.58*	(2.12)
Total	−0.57	(0.51)	−0.69	(0.09)	3.64***	(0.40)	−4.21*	(0.28)	−0.76***	(0.14)
Reweighting error	−0.48***	(0.06)	−0.09***	(0.01)	−0.04**	(0.02)	−0.44***	(0.05)	−0.09***	(0.02)

Notes: The table presents the results of the RIF-regressions based OB decomposition approach based on log daily wages (85-15, 50-15, 85-50, Variance) and daily wages (Gini coefficient). The sample is restricted to male full-time workers in the manufacturing sector between 18 and 65 years, who earned more than 10 euros per day and work in West Germany. All coefficients above are multiplied by 100 for convenience. ***, **, and * indicate statistical significance at the 1, 5, and 10 percent level, respectively. Bootstrapped standard errors with 100 replications are presented in parentheses. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Frey and Osborne (2017), own calculations.

Table 2.B.16: Decomposition results without the automotive and other vehicles sector, 1996-2010

Inequality measure	85-15		Gini coefficient		50-15		85-50		Variance	
	Coefficient	Std Dev.	Coefficient	Std. Dev.	Coefficient	Std. Dev.	Coefficient	Std. Dev.	Coefficient	Std. Dev.
Total change	12.04***	(0.50)	4.56***	(0.13)	7.35***	(0.33)	4.69***	(0.36)	5.83***	(0.24)
Pure composition effect										
Age	3.56***	(0.25)	0.69***	(0.06)	0.91***	(0.18)	2.64***	(0.17)	0.72***	(0.07)
Education	4.96***	(0.32)	1.55***	(0.10)	1.15***	(0.09)	3.81***	(0.27)	1.57***	(0.10)
Tenure	−0.41	(0.26)	−0.02	(0.06)	−0.05	(0.21)	−0.37**	(0.17)	−0.02	(0.08)
Nationality	0.04	(0.03)	0.00	(0.01)	0.05**	(0.02)	-0.01	(0.02)	0.02	(0.02)
Automation threat	1.74***	(0.20)	0.33***	(0.04)	0.39***	(0.06)	1.34***	(0.16)	0.32***	(0.05)
Collective bargaining	0.96*	(0.55)	0.34***	(0.13)	0.69*	(0.40)	0.27	(0.38)	0.94***	(0.21)
Plant size	−0.42***	(0.11)	−0.15***	(0.04)	−0.24***	(0.08)	−0.18**	(0.08)	−0.16***	(0.05)
Region	−0.10	(0.08)	−0.02	(0.02)	0.00	(0.06)	−0.10	(0.07)	−0.08***	(0.03)
Sector	0.80***	(0.11)	0.19***	(0.02)	−0.09	(0.07)	0.89***	(0.10)	0.16***	(0.03)
Total	11.12***	(0.83)	2.90***	(0.18)	2.81***	(0.54)	8.30***	(0.63)	3.48***	(0.24)
Specification error	−0.77	(0.66)	−0.57***	(0.11)	0.91**	(0.46)	−1.68***	(0.59)	−0.44***	(0.14)
Pure wage structure effect										
Age	5.07**	(2.45)	1.42***	(0.54)	0.03	(1.75)	5.04***	(1.64)	2.36***	(0.83)
Education	1.93***	(0.59)	1.16***	(0.13)	−0.62***	(0.20)	2.55***	(0.57)	1.59***	(0.19)
Tenure	−16.37***	(5.67)	−2.46*	(1.28)	−17.57***	(4.72)	1.20	(2.49)	−7.75**	(3.13)
Nationality	−0.67***	(0.20)	−0.11***	(0.04)	−0.04	(0.16)	−0.63***	(0.13)	−0.10	(0.08)
Automation threat	3.68	(2.28)	1.84**	(0.81)	7.46***	(1.68)	−3.78**	(1.82)	3.00	(1.85)
Collective bargaining	−6.44***	(1.11)	−1.00***	(0.26)	−3.84***	(0.80)	−2.61**	(1.04)	0.25	(0.44)
Plant size	2.70***	(0.84)	0.58***	(0.21)	3.31***	(0.62)	−0.61	(0.70)	1.93***	(0.33)
Region	−0.65	(1.05)	0.05	(0.23)	−1.11	(0.88)	0.46	(0.72)	−0.45	(0.38)
Sector	4.90***	(1.28)	0.91***	(0.34)	2.63***	(1.00)	2.27**	(1.05)	1.34*	(0.70)
Constant	9.14	(6.19)	0.18	(1.51)	14.10***	(4.78)	-4.96	(3.44)	1.11	(3.60)
Total	3.27***	(0.70)	2.57***	(0.19)	4.34***	(0.48)	−1.07*	(0.56)	3.28***	(0.29)
Reweighting error	−1.57***	(0.20)	−0.35***	(0.05)	−0.71***	(0.13)	−0.86***	(0.13)	−0.49***	(0.06)

Notes: The table presents the results of the RIF-regressions based OB decomposition approach based on log daily wages (85-15, 50-15, 85-50, Variance) and daily wages (Gini coefficient). The sample is restricted to male full-time workers in the manufacturing sector between 18 and 65 years, who earned more than 10 euros per day and work in West Germany. All coefficients above are multiplied by 100 for convenience. ***, **, and * indicate statistical significance at the 1, 5, and 10 percent level, respectively. Bootstrapped standard errors with 100 replications are presented in parentheses. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Table 2.B.17: Decomposition results without the automotive and other vehicles sector, 2012-2017

Inequality measure	85-15		Gini coefficient		50-15		85-50		Variance	
	Coefficient	Std Dev.	Coefficient	Std. Dev.	Coefficient	Std. Dev.	Coefficient	Std. Dev.	Coefficient	Std. Dev.
Total change	3.41***	(0.60)	0.03	(0.12)	2.96***	(0.50)	0.44	(0.40)	−0.05	(0.20)
Pure composition effect										
Age	−0.07	(0.09)	0.00	(0.02)	−0.04	(0.04)	−0.03	(0.07)	0.01	(0.03)
Education	0.67***	(0.18)	0.20***	(0.06)	0.09***	(0.03)	0.59***	(0.15)	0.23***	(0.07)
Tenure	−0.12***	(0.04)	−0.03***	(0.01)	−0.08**	(0.04)	−0.04**	(0.02)	−0.06***	(0.02)
Nationality	0.00	(0.01)	0.00	(0.00)	0.00	(0.01)	0.00	(0.00)	0.00	(0.00)
Automation threat	3.98***	(0.22)	0.70***	(0.03)	1.07***	(0.08)	2.91***	(0.17)	0.93***	(0.05)
Collective bargaining	0.24***	(0.04)	0.05***	(0.01)	0.22***	(0.04)	0.02	(0.02)	0.07***	(0.01)
Plant size	−0.97***	(0.12)	−0.26***	(0.03)	−0.75***	(0.09)	−0.22***	(0.04)	−0.37***	(0.05)
Region	−0.05	(0.08)	−0.03	(0.02)	-0.05	(0.06)	0.00	(0.06)	−0.05*	(0.03)
Sector	−0.64***	(0.11)	−0.12***	(0.02)	0.19***	(0.07)	−0.84***	(0.10)	−0.11***	(0.03)
Total	3.04***	(0.32)	0.50***	(0.07)	0.65***	(0.17)	2.40***	(0.23)	0.64***	(0.10)
Specification error	1.16**	(0.50)	−0.04***	(0.01)	0.11	(0.07)	1.05**	(0.49)	−0.03	(0.02)
Pure wage structure effect										
Age	−6.36***	(2.20)	−1.06***	(0.34)	−0.57	(1.65)	−5.79***	(1.34)	−1.38**	(0.65)
Tenure	−8.16*	(4.97)	−1.23	(1.02)	0.45	(3.67)	−8.61***	(2.69)	0.55	(2.17)
Nationality	0.37	(0.23)	0.00	(0.04)	0.43**	(0.18)	−0.06	(0.12)	0.02	(0.07)
Education	−3.79***	(0.56)	−0.37***	(0.07)	0.29	(0.20)	−4.08***	(0.53)	−0.41***	(0.13)
Collective bargaining	1.53*	(0.90)	0.42***	(0.14)	2.45***	(0.74)	−0.91	(0.57)	0.85***	(0.27)
Automation threat	5.48**	(2.67)	−1.99***	(0.50)	4.92***	(1.89)	0.56	(2.15)	−1.78***	(0.68)
Plant size	1.13*	(0.68)	−0.05	(0.13)	−1.11**	(0.50)	2.24***	(0.48)	−0.16	(0.21)
Region	−2.92***	(1.01)	−0.45**	(0.19)	−0.42	(0.69)	−2.50***	(0.84)	−0.75**	(0.36)
Sector	2.39**	(1.05)	−0.35*	(0.19)	0.58	(0.79)	1.81**	(0.91)	−0.48	(0.32)
Constant	10.19*	(6.14)	4.76***	(1.24)	−4.63	(4.41)	14.82***	(3.76)	3.01	(2.44)
Total	−0.14	(0.82)	−0.33***	(0.11)	2.38***	(0.46)	−2.52***	(0.70)	−0.53***	(0.17)
Reweighting error	−0.65***	(0.08)	−0.10***	(0.02)	−0.17***	(0.03)	−0.48***	(0.06)	−0.13***	(0.02)

Notes: The table presents the results of the RIF-regressions based OB decomposition approach based on log daily wages (85-15, 50-15, 85-50, Variance) and daily wages (Gini coefficient). The sample is restricted to male full-time workers in the manufacturing sector between 18 and 65 years, who earned more than 10 euros per day and work in West Germany. All coefficients above are multiplied by 100 for convenience. ***, **, and * indicate statistical significance at the 1, 5, and 10 percent level, respectively. Bootstrapped standard errors with 100 replications are presented in parentheses. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

Appendix 2.C

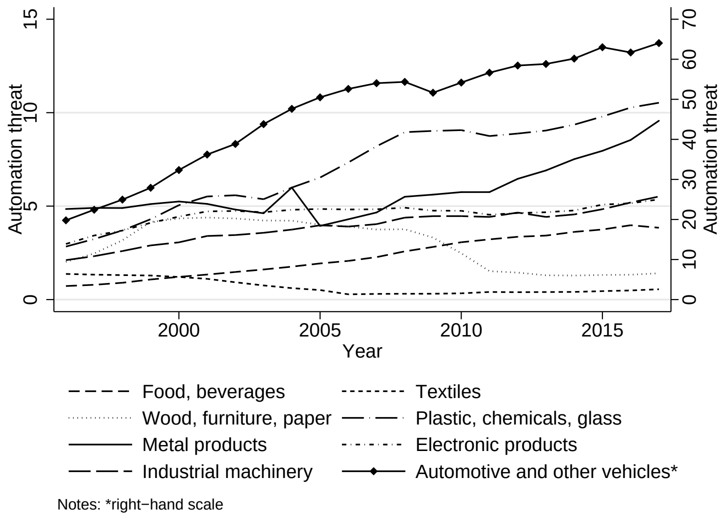


Figure 2.C.1: Automation threat in Germany across sectors in the manufacturing industry from 1996 to 2017

Notes: The figure presents the evolution of the automation threat variable across sectors in the German manufacturing industry. In the case of the automotive and other vehicles sector the development is right-hand scaled. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

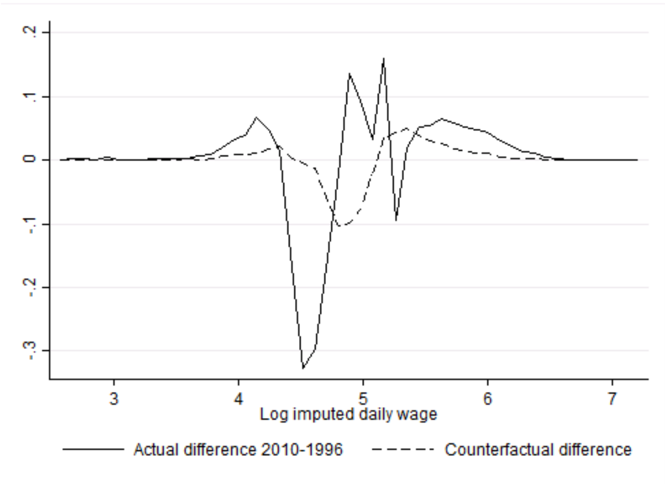


Figure 2.C.2: Actual and counterfactual differences, 1996-2010

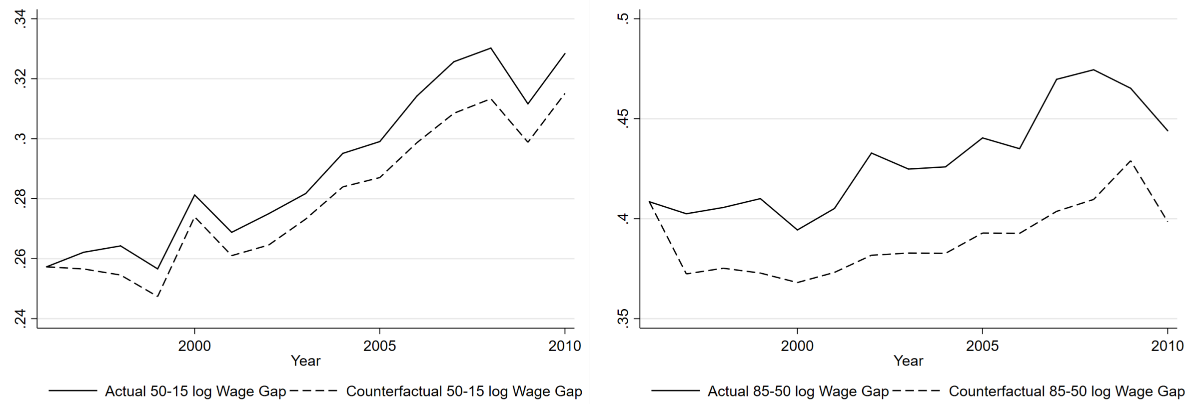
Notes: The figure presents the comparison between the actual and counterfactual differences between 1996 and 2010. Counterfactual weights are estimated using multinomial logit estimations, see Appendix 2.A.3. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

(a) 85-15 percentile wage gap

(b) Gini coefficient

Figure 2.C.3: Actual and counterfactual 85-15 percentile wage gap and Gini coefficient from 1996 to 2010

Notes: Panel (a) (Panel (b)) of the figure presents the evolution of the actual 85-15 percentile wage gap (Gini coefficient estimations) as well as the counterfactual 85-15 percentile wage gap (Gini coefficient estimations) that would have prevailed if automation and robotization had remained at the level of 1996. Counterfactual weights are estimated using multinomial logit estimations, see Appendix 2.A.3. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.



(a) 50-15 percentile wage gap

(b) 85-50 percentile wage gap

Figure 2.C.4: Actual and counterfactual 50-15 and 85-50 percentile wage gap from 1996 to 2010

Notes: Panel (a) (Panel (b)) of the figure presents the evolution of the actual 50-15 (85-50) percentile wage gap as well as the counterfactual 50-15 (85-50) percentile wage gap that would have prevailed if automation and robotization had remained at the level of 1996. Counterfactual weights are estimated using multinomial logit estimations, see Appendix 2.A.3. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

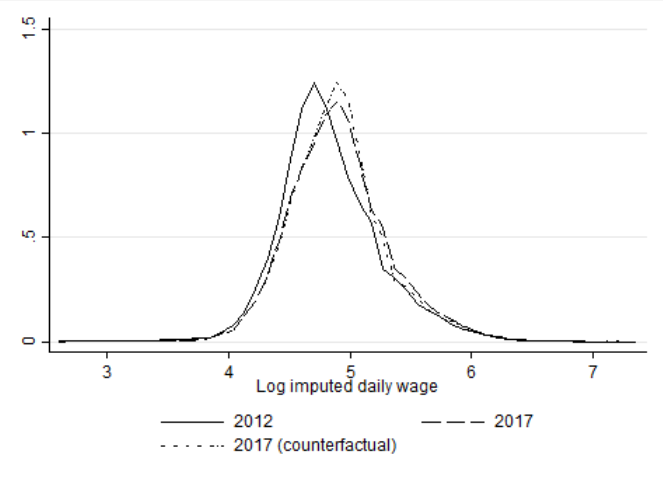


Figure 2.C.5: Actual and counterfactual wage distributions, 2012-2017

Notes: The figure presents the actual wage distributions in 2012 and 2017 as well as the counterfactual wage distribution that would have prevailed if automation and robotization had remained at the level of 2012. Counterfactual weights are estimated using multinomial logit estimations, see Appendix 2.A.3. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

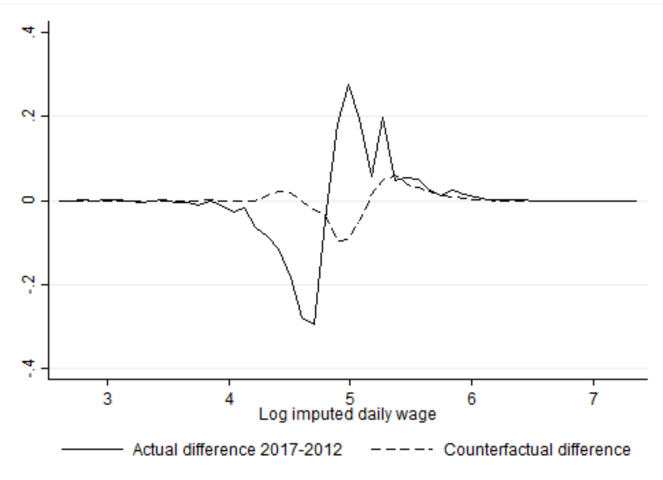


Figure 2.C.6: Actual and counterfactual differences, 2012-2017

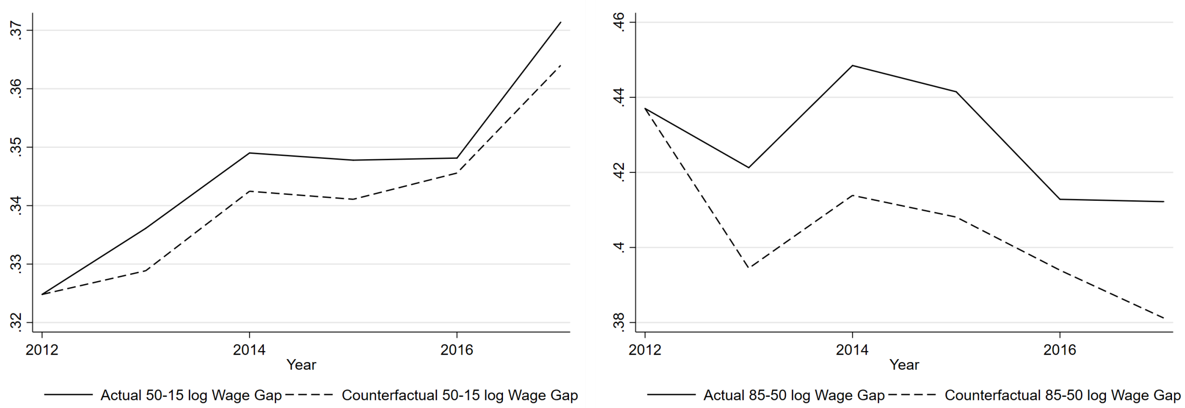
Notes: The figure presents the comparison between the actual and counterfactual differences between 2012 and 2017. Counterfactual weights are estimated using multinomial logit estimations, see Appendix 2.A.3. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

(a) 85-15 percentile wage gap

(b) Gini coefficient

Figure 2.C.7: Actual and counterfactual 85-15 percentile wage gap and Gini coefficient from 2012 to 2017

Notes: Panel (a) (Panel (b)) of the figure presents the evolution of the actual 85-15 percentile wage gap (Gini coefficient estimations) as well as the counterfactual 85-15 percentile wage gap (Gini coefficient estimations) that would have prevailed if automation and robotization had remained at the level of 2012. Counterfactual weights are estimated using multinomial logit estimations, see Appendix 2.A.3. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.



(a) 50-15 percentile wage gap

(b) 85-50 percentile wage gap

Figure 2.C.8: Actual and counterfactual 50-15 and 85-50 percentile wage gap from 2012 to 2017

Notes: Panel (a) (Panel (b)) of the figure presents the evolution of the actual 50-15 (85-50) percentile wage gap as well as the counterfactual 50-15 (85-50) percentile wage gap that would have prevailed if automation and robotization had remained at the level of 2012. Counterfactual weights are estimated using multinomial logit estimations, see Appendix 2.A.3. Sampling weights are employed.
Source: LIAB QM2 9317, International Federation of Robotics (2018) and Dengler and Matthes (2015), own calculations.

References

Acemoglu, D., and Autor, D. (2011). Skills, Tasks and Technologies: Implications for Employment and Earnings. In D. Card and O. Ashenfelter (Eds.), *Handbook of labor economics* (Vol. 4b, pp. 1043–1171). Elsevier.

Acemoglu, D., and Restrepo, P. (2018). The Race Between Man and Machine: Implications of Technology for Growth, Factor Shares and Employment. *American Economic Review*, *108*(6), 1488–1542.

Acemoglu, D., and Restrepo, P. (2020). Robots and Jobs: Evidence from US Labor Markets. *Journal of Political Economy*, *128*(6), 2188–2244.

Aksoy, C. G., Özcan, B., and Philipp, J. (2021). Robots and the gender pay gap in europe. *European Economic Review*, *134*, 103693.

Anelli, M., Colantone, I., and Stanig, P. (2019). We Were the Robots: Automation and Voting Behavior in Western Europe. *BAFFI CAREFIN Centre Research Paper*, *2019-115*.

Anelli, M., Giuntella, O., and Stella, L. (2021). Robots, Marriageable Men, Family, and Fertility. *The Journal of Human Resources*, *published ahead of print November 15, 2021*.

Antonczyk, D., DeLeire, T., and Fitzenberger, B. (2018). Polarization and Rising Wage Inequality: Comparing the U.S. and Germany. *Econometrics*, *6*(2), 1-33.

Antonczyk, D., Fitzenberger, B., and Sommerfeld, K. (2010). Rising Wage Inequality, the Decline of Collective Bargaining, and the Gender Wage Gap. *Labour Economics*, *17*(5), 835 - 847.

Arntz, M., Gregory, T., and Zierahn, U. (2016). The Risk of Automation for Jobs in OECD Countries: A Comparative Analysis. *OECD Social, Employment and Migration Working Papers*, *189*.

Atkinson, A. B. (1970). On the measurement of inequality. *Journal of economic theory*, *2*(3), 244–263.

Autor, D. H., Levy, F., and Murnane, R. J. (2003). The Skill Content of Recent Technological Change: An Empirical Exploration. *Quarterly Journal of Economics*, *118*(4), 1279–1333.

Barsky, R., Bound, J., Charles, K. K., and Lupton, J. P. (2002). Accounting for the Black-White Wealth Gap. *Journal of the American Statistical Association*, *97*(459), 663-673.

Baumgarten, D., Felbermayr, G., and Lehwald, S. (2020). Dissecting Between-Plant and Within-Plant Wage Dispersion: Evidence from Germany. *Industrial Relations: A Journal of Economy and Society*, *59*(1), 85-122.

Biewen, M., and Juhasz, A. (2012). Understanding Rising Income Inequality in Germany, 1999/2000-2005/2006. *Review of Income and Wealth*, *58*(4), 622-647.

Biewen, M., and Seckler, M. (2019). Unions, Internationalization, Tasks, Firms, and Worker Characteristics: A Detailed Decomposition Analysis of Rising Wage Inequality in Germany. *The Journal of Economic Inequality*, *17*, 461-498.

Bonin, H., Gregory, T., and Zierahn, U. (2015). *Übertragung der Studie von Frey/Osborne (2013) auf Deutschland* (ZEW Kurzexpertise No. 57). Mannheim.

Brall, F., and Schmid, R. (2023). Automation, robots and wage inequality in germany: A decomposition analysis. *LABOUR*, *37*(1), 33–95.

Brzeski, C., and Burk, I. (2015). Die Roboter kommen: Folgen der Automatisierung für den deutschen Arbeitsmarkt. *ING DiBa Economic Research*.

Card, D., Heining, J., and Kline, P. (2013). Workplace Heterogeneity and the Rise of West German Wage Inequality. *The Quarterly Journal of Economics*, *128*(3), 967-1015.

Dauth, W., Findeisen, S., Südekum, J., and Wössner, N. (2021). The Adjustment of Labor Markets to Robots. *Journal of the European Economic Association*, *00*(0), 1-50.

De Vries, G. J., Gentile, E., Miroudot, S., and Wacker, K. M. (2020). The rise of robots and the fall of routine jobs. *Labour Economics*, *66*, 101885.

Dengler, K., and Matthes, B. (2015). *Folgen der Digitalisierung für die Arbeitswelt: Substituierbarkeitspotenziale von Berufen in Deutschland* (IAB-Forschungsbericht No. 11/2015). Nürnberg.

Dengler, K., and Matthes, B. (2018). *Substituierbarkeitspotenziale von Berufen: Wenige Berufsbilder halten mit der Digitalisierung Schritt* (IAB-Kurzbericht No. 4/2018).

Nürnberg.

Dengler, K., and Matthes, B. (2021). *Auch komplexere Tätigkeiten könnten zunehmend automatisiert werden* (IAB-Kurzbericht No. 13/2021). Nürnberg.

Dengler, K., Matthes, B., and Paulus, W. (2014). *Occupational Tasks in the German Labour Market* (IAB-Forschungsbericht No. 12/2014). Nürnberg.

DiNardo, J. (2002). Propensity Score Reweighting and Changes in Wage Distributions. *University of Michigan, mimeograph*.

DiNardo, J., Fortin, N. M., and Lemieux, T. (1996). Labor Market Institutions and the Distribution of Wages, 1973-1992: A Semiparametric Approach. *Econometrica*, *64*(5), 1001–1044.

Dustmann, C., Ludsteck, J., and Schönberg, U. (2009, 05). Revisiting the German Wage Structure. *The Quarterly Journal of Economics*, *124*(2), 843-881.

Eberle, J., Jacobebbinghaus, P., Ludsteck, J., and Witter, J. (2011). *Generation of Time-Consistent Industry Codes in the Face of Classification Changes: Simple heuristic based on the Establishment History Panel (BHP)* (FDZ Methodenreport No. 201105_en). Institut für Arbeitsmarkt- und Berufsforschung (IAB), Nürnberg.

Felbermayr, G., Baumgarten, D., and Lehwald, S. (2014). Increasing Wage Inequality in Germany: What Role Does Global Trade Play? *Docbase - Publication Database of the CESifo Group*.

Firpo, S., Fortin, N., and Lemieux, T. (2018). Decomposing Wage Distributions Using Recentered Influence Function Regressions. *Econometrics*, *6*(2), 1-40.

Firpo, S., Fortin, N. M., and Lemieux, T. (2009). Unconditional Quantile Regressions. *Econometrica*, *77*(3), 953-973.

Fortin, N., Lemieux, T., and Firpo, S. (2011). Decomposition Methods in Economics. In O. Ashenfelter and D. Card (Eds.), *Handbook of Labor Economics* (Vol. 4, p. 1 -102). Elsevier.

Frey, C. B., and Osborne, M. A. (2017). The Future of Employment: How Susceptible are Jobs to Computerisation? *Technological Forecasting and Social Change*, *114*(C), 254–280.

Gartner, H. (2005). The Imputation of Wages Above the Contribution Limit with the German IAB Employment Sample. *FDZ Methodenreport 02/2005, Institut für Arbeitsmarkt- und Berufsforschung (IAB), Nürnberg*.

Goos, M., and Manning, A. (2007). Lousy and lovely jobs: The rising polarization of work in britain. *The Review of Economics and Statistics*, *89*(1), 118-133.

Graetz, G., and Michaels, G. (2018). Robots at Work. *The Review of Economics and Statistics*, *100*(5), 753-768.

Hémous, D., and Olsen, M. (2022). The rise of the machines: Automation, horizontal innovation, and income inequality. *American Economic Journal: Macroeconomics*, *14*(1), 179–223.

Hyslop, D. R., and Maré, D. C. (2005). Understanding New Zealand's Changing Income Distribution, 1983-1998: A Semi-parametric Analysis. *Economica*, *72*(287), 469-495.

International Federation of Robotics. (2018). World robotics industrial robots 2018. *Frankfurt am Main: IFR Statistical Department*.

Kaltenberg, M., and Foster-McGregor, N. (2020). The Impact of Automation on Inequality across Europe. *UNU-MERIT Working Paper*.

Kariel, J. (2021). Job Creators or Job Killers? Heterogeneous Effects of Industrial Robots on UK Employment. *LABOUR*, *35*(1), 52-78.

Lankisch, C., Prettner, K., and Prskawetz, A. (2019). How can robots affect wage inequality? *Economic Modelling*, *81*, 161-169.

Oaxaca, R. (1973). Male-Female Wage Differentials in Urban Labor Markets. *International Economic Review*, *14*(3), 693–709.

Prettner, K., and Strulik, H. (2019). Innovation, Automation, and Inequality: Policy Challenges in the Race Against the Machine. *Journal of Monetary Economics*.

Schmidtlein, L., Seth, S., and Umkehrer, M. (2019, July). *Linked EmployerEmployee Data from the IAB: LIAB Cross-Sectional Model 2 (LIAB QM2) 1993-2017* (FDZ-Datenreport, 06/2019 (en)). Nürnberg.

Sloane, P. F. (2008). Zu den Grundlagen eines Deutschen Qualifikationsrahmens (DQR): Konzeptionen, Kategorien, Konstruktionsprinzipien. *W. Bertelsmann Verlag*.

Solow, R. M. (1956, 02). A Contribution to the Theory of Economic Growth. *The*

Quarterly Journal of Economics, *70*(1), 65-94.

Stehrer, R., Bykova, A., Jäger, K., Reiter, O., and Schwarzhappel, M. (2019). Industry Level Growth and Poductivity Data with Special Focus on Intangible Assets. *wiiw Statistical Report No. 8*.

Chapter 3

Technical Change, Task Allocation, and Labor Unions

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Abstract. We develop a general equilibrium model combining the task approach, wage setting by labor unions, as well as search and matching frictions to show that skill-biased technical change (SBTC) affects labor unions' wage setting through changes in the firms' assignment of tasks to high- and low-skilled workers. This effect of task reallocation depends on the shape of the relative task productivity schedule. In case of a downward-sloping wage curve, SBTC harms low-skilled workers either by a reduction of real wages or an increase in unemployment. To complement the theoretical analysis we calibrate the model to German and French data.

JEL classification: J64, J51, E23, E24, O33

Keywords: Task Approach, Labor Unions, Wage Setting, Skill-Biased Technical Change, Search and Matching, Labor Demand

3.1 Introduction

In conventional production theory the production process is usually considered to be a black box that is left largely unspecified – except for some assumptions regarding marginal products of production factors, returns to scale, and the elasticity of substitution. For many economic applications such a coarse modeling of production has proven to be sufficient. However, when it comes to a discussion of skill-biased technical change (SBTC), the conventional approach may make us blind to some negative implications of technological progress on labor market outcomes.

To get a fuller picture of the consequences of SBTC, it has been suggested to interpret the production process as a set of tasks that are combined to produce output. The tasks are then allocated to production factors based on the principles of comparative advantage and the theory of optimum assignment. The distinction between skills and tasks becomes of central importance when workers with a given skill level can potentially perform multiple tasks and technological progress influences the equilibrium mapping between production tasks and workers of different skill groups, see Autor (2013) for an overview.

So far it is mostly perfect competition models considering the impact of SBTC on the task allocation between low- and high-skilled workers that in turn influences the wage levels of both skill groups in general equilibrium, see for example the seminal work of Acemoglu and Autor (2011). However, labor markets are subject to numerous imperfections, such as search and matching frictions. Moreover, especially in European labor markets, collective bargaining still plays a significant role for wage determination. Therefore, it appears sensible to take account of these aspects when analyzing the effects of SBTC within the framework of the task approach.

This paper tries to fill this gap by proposing a new modeling framework for the analysis of SBTC that combines the task approach, wage setting by labor unions, as well as search and matching frictions. The central focus of our paper is the research question of how labor unions respond to the firms' incentives to modify the task allocation because of SBTC. Our model involves two important channels through which SBTC and the resulting task reallocation affects the behavior of labor unions and hence wage determination which are absent in task-based perfect competition models. First, in a world with matching frictions, firms intending to change their task composition will quite likely have to adjust their posted vacancies, thereby affecting aggregate labor market tightness. This in turn affects the outside option of labor unions and the extent of wage pressure in the economy as well as labor market flows. Second, changes in task assignments trigger changes in the wage-setting behavior of labor unions through the effect on the wage elasticity of labor demand thereby affecting unions' wage markups. This channel has not yet been explored in the literature and deserves more attention since it decides to a large extent on whether low-skilled workers are ultimately harmed by SBTC.

Whereas it is well known that wage claims of labor unions negatively depend on the labor demand elasticity, the relationship between labor demand elasticity and task allocation is less obvious. In general equilibrium models with union wage setting the analysis is often simplified by assuming a Cobb-Douglas production function because it leads to a constant labor demand elasticity and a constant union wage markup; see e.g., Layard et al. (1991). However, we show that even with a simple Cobb-Douglas structure at the task level, the labor demand elasticity is in general not constant but depends on the task threshold that divides the range of tasks assigned to low-skilled and high-skilled workers. The way how changes in the task threshold affect the wage elasticity of labor demand is determined by the shape of the relative task productivity schedule that describes the comparative advantage of the two skill groups in performing the various tasks. Since we allow for both, concave and convex shapes of the relative task productivity schedule, the impact of changes in the task threshold on the labor demand elasticity and union wage setting is in general ambiguous and can be explained by different degrees of substitutability of high- and low-skilled workers. Taking these mechanisms into account, SBTC may harm low-skilled workers either in terms of higher unemployment or lower real wages – in contrast to the predictions of "canonical models", i.e models using the conventional production function approach.

To establish the general equilibrium effects of SBTC, in analogy to standard search-and-matching models, we derive from the model a two-equations system describing the job creation of firms and the wage-setting behavior of labor unions. Whereas the job creation curve has the usual negative slope, the wage curve can be either upward- or downward-sloping. In the case of a positively sloped wage curve low-skilled workers benefit from an increase in the productivity of high-skilled workers because of higher employment as well as higher real wages. However, with a downward-sloping wage curve low-skilled workers may be harmed either by an increase in unemployment or a reduction of real wages. The key factor governing the slope of the wage curve is the response of the labor demand elasticity to changes in the task threshold. The slope gets negative if the labor demand elasticity increases with increasing task threshold and this reaction is strong thus implying strong upward wage pressure if the task threshold decreases. Our results make it clear that the SBTC channel working through task allocation to wage setting of labor unions is relevant *(i)* per se as it could provide a technological explanation for different degrees of wage rigidity across sectors and countries as reflected in different slopes of the wage curve, as well as *(ii)* because of the consequences of SBTC for the labor market outcomes for low-skilled workers.

To demonstrate the applicability of our framework, we calibrate the model to the data of Germany and France for two time periods: 1995 to 2005 and 2010 to 2017. The reason for selecting Germany and France is that both countries are comparably large economies with (still) high union coverage rates. We interpret the mean values of selected labor market variables over each period and country as steady-state values and examine the consequences of a ten percent increase in the skill bias of production technologies on the respective steady state. We consider two different periods to show that SBTC may have opposing impact on, for example, labor market tightness in the same country depending on the steady state. According to the results, for the first period SBTC leads to a strong decline in the labor demand elasticity that strongly raises labor unions' wage markup. This contributes to a decline in labor market tightness and an increase in unemployment for low-skilled workers in both countries. For Germany, a similar result is obtained also for the second period, whereas in France the impact of SBTC on the labor demand elasticity becomes weaker implying only a moderate increase in wage pressure. As a consequence, low-skilled labor market tightness increases and the unemployment rate decreases. For both countries and periods, low- and high-skilled workers benefit from real wage increases but the increase is much stronger for the high-skilled, implying a pronounced increase in the skill premium. We can conclude that the strength of the decline in the labor demand elasticity decides on the labor market effects of SBTC in our simulation exercise. It itself depends on the examined initial steady state, in particular the task productivity schedule and the task allocation in this steady state.

The rest of the paper is organized as follows: Section 3.2 presents the literature that is related to our paper. Section 3.3 introduces the model and discusses the implications of changes in the task threshold on the wage elasticity of labor demand for low-skilled workers and on labor unions' wage-setting. Section 3.4 performs a comparative-static analysis of the labor market consequences of SBTC. Section 3.5 calibrates the model to data of Germany and France. Section 3.6 contains a summary of the results and some conclusions.

3.2 Related Literature

Our paper is related to the literature modeling the production process with the task approach. In an early contribution, Rosen (1978) explains the structure of work activities within firms by developing a theory of optimum assignment of workers to tasks based on the principle of comparative advantage. Our paper is influenced by the seminal contribution of Acemoglu and Autor (2011) who demonstrate how the firms' optimum assignment decisions can be integrated into a general equilibrium model of SBTC and compare the consequences of SBTC in the canonical and task-based model. Follow-up papers, such as Acemoglu and Restrepo (2018a, 2018b, 2022), and Hémous and Olsen (2022) develop task-based approaches for the analysis of the labor-market consequences of automation. Assuming perfect competition in the labor market, these papers focus on the impact of SBTC or automation on wages and the labor share but do not consider the effects on collective bargaining, unemployment, or labor market tightness.[1]

As outlined in the introduction, our paper focuses on the impact of SBTC on labor

[1] The task-based approach has also been used in the literature on the labor-market effects of offshoring; see, e.g., Grossman and Rossi-Hansberg (2008) and Costinot and Vogel (2010).

unions' wage-setting brought about by changes in the firms' task allocation. A few papers also look at the impact of SBTC on labor unions but focus on the question of whether SBTC may be responsible for the deunionization observed in the US and other countries. In their theoretical analysis, Acemoglu et al. (2001) start with the well-known fact that unions compress the wage structure. If SBTC is limited, skilled workers accept to work at unionized firms because of the benefits provided by unions. However, with strong SBTC, the outside option, i.e. the competitive market return, for skilled workers increases. This weakens their incentive to join the unionized sector. In that sense, SBTC leads to deunionization which amplifies the original effect of SBTC on inequality. The hypothesis of Acemoglu et al. (2001) is supported by the model and calibration of Açıkgöz and Kaymak (2014) who use similar arguments to explain the deunionization in the US. Dinlersoz and Greenwood (2016) go one step further and argue that SBTC is not only responsible for the deunionization observed in more recent decades, but also for the increase in union density in the first half of the 20th century. Neto et al. (2019) consider the "reverse" question of whether labor unions influence the direction of technical change. They build an endogenous growth model of directed technical change in which only low-skilled workers are organized in labor unions. The unions' impact on SBTC depends on whether the unions are more wage-oriented or more employment-oriented. It turns out that in the first case firms have a higher incentive to invest in high-skilled technologies than in the second one.

Another related strand of literature integrates, similarly to our paper, union wage determination in the Mortensen-Pissarides matching model. In a matching model with monopoly unions, Pissarides (1986) shows that an efficient level of unemployment is obtained if unions only care about unemployed workers. The efficiency effects of firm-level collective bargaining in a search economy with concave production are analyzed in Bauer and Lingens (2014). Boeri and Burda (2009) show that preferences for rigid wages and collective bargaining may endogenously emerge if there are distortions of the separation decision in the form of a firing tax. Ebell and Haefke (2006) analyze how the bargaining regime affects the impact of product market competition on unemployment. Delacroix (2006) considers a segmented labor market with union and non-union sectors and analyzes the interaction of unemployment benefits and union wage setting. Krusell and Rudanko (2016) discuss the holdup problem and show that without unions' credible commitment to future wages the firms' hiring is too low. Morin (2017) analyzes how unions affect the volatility of wages over the business cycle. As is evident from this overview, these papers are interested in research questions unrelated to SBTC and do consider the impact of changes in the firms' task assignment on labor unions' wage claims.

Finally, some papers include automation in the Mortensen-Pissarides matching model. Similar to our model framework, Jaimovich et al. (2021) consider a model with a frictional labor market for low-skilled workers and perfect competition for high-skilled workers. In contrast to our paper, they assume that low-skilled workers are heterogeneous and labor markets are fully segmented by ability and by type of produced good. Segmented labor markets are also assumed in Cords and Prettner (2022) who show that robot adoption leads to falling wages and rising unemployment of low-skilled workers. In Leduc and Liu (2020) firms draw an i.i.d. cost of automation and then decide whether to automate vacancies. Guimarães and Gil (2022) modify a standard matching model with endogenous job destruction and distinguish between firms using only workers and those using only machines. None of these papers, however, addresses the problem of assigning tasks to workers or the impact of task assignment on labor union wage setting.

3.3 The Model

3.3.1 Firms

There is a mass one of identical firms in the economy. Timing is discrete and will be explained in more detail below. At the end of period t the representative firm produces the final good Y_t by using the services of a continuum of tasks $y_t(i)$, measured on the unit interval, according to the Cobb-Douglas-function

$$Y_t = \exp\left[\int_0^1 \ln y_t(i)\mathrm{d}i\right]. \tag{3.1}$$

Index $0 < i < 1$ refers to the complexity of a particular task. The firm assigns L_t low-skilled workers and H_t high-skilled workers to the different tasks according to the task-specific production function

$$y_t(i) = A_{Lt}\, \alpha_L(i)\, l_t(i) + A_{Ht}\, \alpha_H(i)\, h_t(i), \tag{3.2}$$

where $l_t(i)$ and $h_t(i)$ denote the low-skilled and high-skilled labor input assigned to task with index i in period t, respectively, and

$$L_t = \int_0^1 l_t(i)\mathrm{d}i \qquad \text{and} \qquad H_t = \int_0^1 h_t(i)\mathrm{d}i. \tag{3.3}$$

A_{Lt} and A_{Ht} denote factor-augmenting technology, whereas the functions $\alpha_H(i)$ and $\alpha_L(i)$ describe the productivity of high- and low-skilled workers in task with index i, respectively. We make an important assumption regarding the task-related productivities. We define $\bar{\alpha}(i) \equiv \alpha_H(i)/\alpha_L(i)$, hereafter also referred to as the relative task productivity schedule, and assume that $\bar{\alpha}'(i) > 0$. This implies that the comparative advantage of high-skilled (low-skilled) workers in performing the different tasks is increasing (decreasing) in the task index i.

The goods market and the labor market for high-skilled workers are competitive, hence high-skilled workers are always fully employed. In contrast, the low-skilled labor market is characterized by matching frictions and monopoly unions at the firm level. The matching frictions are described by the linear homogeneous matching function $M_{Lt} = M(V_{Lt}, U_{Lt})$, where V_{Lt} denotes vacant jobs for low-skilled workers and U_{Lt} the low-skilled unemployed persons.

The timing is as follows. At the beginning of period t there are L_{t-1} employed low-skilled workers and U_{Lt} unemployed workers. The total labor force is normalized to one, hence $U_{Lt} = 1 - L_{t-1} - H_{t-1}$. The representative labor union chooses a wage w_{Lt} anticipating that the respective firm may adjust the employment level by posting vacancies accordingly. This timing contrasts with that used in standard search and matching models but is in line with studies incorporating trade unions into the search and matching framework, such as Delacroix (2006) and Morin (2017). For simplicity, the inflow of unemployed workers into jobs and exogenous job separations happen simultaneously in such a way that a further change of a worker's employment/unemployment status within the same period is not possible. At the end of the period, production takes place as outlined above.

If the representative firm wants to increase the number of low-skilled workers it has to post vacant jobs first and bear the (constant) search costs s_L for each vacant job. With rate $M_{Lt}/V_{Lt} \equiv m(\theta_{Lt})$ job vacancies are filled, where $\theta_{Lt} \equiv V_{Lt}/U_{Lt}$ describes labor market tightness in the low-skilled labor market in period t. The single firm considers labor market tightness and thus the job filling rate as given. With the exogenous rate q_L low skilled jobs are destroyed. The dynamics for low-skilled employment is therefore described by

$$L_t = (1 - q_L)L_{t-1} + m(\theta_{Lt})V_{Lt}. \tag{3.4}$$

Figure 3.1 illustrates the timing of the events during period t.

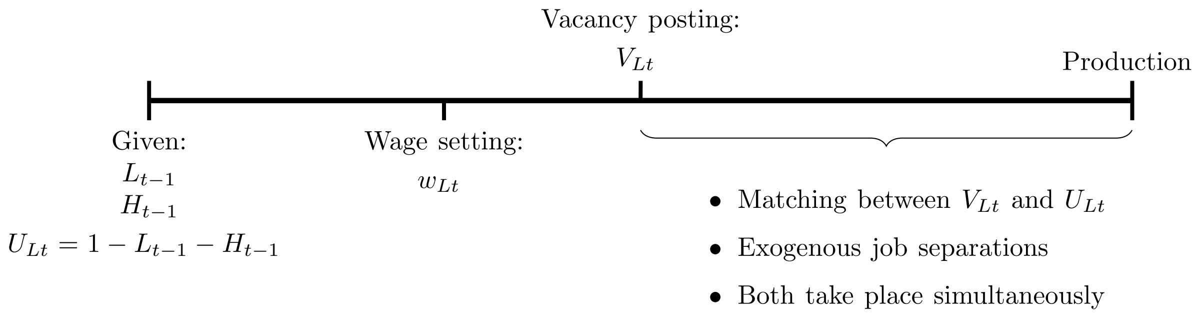


Figure 3.1: Timeline with events during period t

To simplify the analysis we follow Pissarides (2000, p. 68) in assuming that each firm is large enough to eliminate all uncertainty about the flow of labor. Moreover, the final good is chosen as the numeraire. The representative firm maximizes profits

$$\sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} [Y_t - w_{Lt}L_t - w_{Ht}H_t - s_L V_{Lt}] \tag{3.5}$$

s.t. eqs. (3.1) – (3.4) and the conditions

$$l_t(i) \geq 0 \qquad \text{and} \qquad h_t(i) \geq 0, \tag{3.6}$$

where r is the constant real interest rate, and $l_0(i)$ and $h_0(i)$ are given. There are no productivity differences within the group of high- or low-skilled workers. Due to perfect competition in the high-skilled labor market and the fact that no high-skilled worker would supply labor to tasks paying lower wages, all high-skilled workers obtain the same real wage w_{Ht}. Similarly, the representative labor union sets a uniform real wage w_{Lt} for low-skilled workers.

The first-order conditions of this optimization problem are derived in Appendix 3.A.1. The analysis focuses on the steady state in which $A_{Lt} = A_L$, $A_{Ht} = A_H$, $L_{t-1} = L_t = L$ and the time index on all variables can be omitted. Similar to the perfect competition model of Acemoglu and Autor (2011), there exists a task threshold $0 < I < 1$ such that unit labor costs of low-skilled workers are equal to those of high-skilled workers at I:

$$\frac{\widetilde{w}_L}{A_L \alpha_L(I)} = \frac{w_H}{A_H \alpha_H(I)}, \qquad \text{with} \quad \widetilde{w}_L \equiv w_L + \frac{(q_L + r)}{(1+r)} \frac{s_L}{m(\theta_L)}. \tag{3.7}$$

The modified wage $\widetilde{w}_L$ represents the low-skilled labor costs relevant to the representative firm, i.e. the low-skilled real wage w_L plus the labor adjustment costs. In the tasks $i < I$ only low-skilled workers are employed, i.e. $h(i) = 0$, whereas in tasks $i > I$ only high-skilled workers are employed, i.e. $l(i) = 0$. Eq. (3.7) can be written as:

$$\bar{\alpha}(I) = \frac{\widetilde{\omega}}{\bar{A}}, \quad \text{with} \quad \bar{\alpha}(I) = \frac{\alpha_H(I)}{\alpha_L(I)}, \quad \widetilde{\omega} \equiv \frac{w_H}{\widetilde{w}_L}, \quad \text{and} \quad \bar{A} \equiv \frac{A_H}{A_L}. \tag{3.8}$$

This leads to:

$$I = I\left(\widetilde{\omega}, \bar{A}\right), \qquad \text{with} \qquad \frac{\partial I}{\partial \widetilde{\omega}} > 0 \quad \text{and} \quad \frac{\partial I}{\partial \bar{A}} < 0. \tag{3.9}$$

As shown in Appendix 3.A.1, from the first-order conditions it follows that $\widetilde{w}_L l(i) = Y = w_H h(i)$. This has two important implications. First, the same labor input is used in all low-skilled and high-skilled tasks, respectively, i.e. $l(i) = l = L/I$ for $i < I$ and $h(i) = h = H/(1-I)$ for $i > I$. Second, it holds that $I = \widetilde{w}_L L/Y$ and $1 - I = w_H H/Y$ so that I represents the modified labor share of the low-skilled workers (as it refers to labor costs $\widetilde{w}_L$ and not w_L), and $1 - I$ corresponds to the high-skilled labor share. It follows that:

$$L = \frac{I}{1-I}\,\widetilde{\omega}\,H. \tag{3.10}$$

Wage changes have two effects on L: a direct effect at a given threshold I and an indirect effect due to a change of this threshold. Eq. (3.10) in combination with eq. (3.9) can be interpreted as the labor demand function for low-skilled workers for given H:

$$L = L\left(\widetilde{\omega}, I\left(\widetilde{\omega}, \bar{A}\right), H\right) \equiv L^d\left(\widetilde{\omega}, \bar{A}, H\right). \tag{3.11}$$

Moreover, taking into account the optimality conditions, the production function for the final good takes the following Cobb-Douglas form:

$$\begin{aligned} Y &= B\left(A_L \frac{L}{I}\right)^I \left(A_H \frac{H}{1-I}\right)^{1-I}, \\ B &\equiv e^{\xi(I)}, \quad \xi(I) \equiv \int_0^I \ln \alpha_L(i)\mathrm{d}i + \int_I^1 \ln \alpha_H(i)\mathrm{d}i. \end{aligned} \tag{3.12}$$

3.3.2 Labor Unions for Low-Skilled Workers

With the timing assumption outlined in the last section, the present discounted utility of a low-skilled worker being employed at the end of period t is

$$\Psi_{EL,t} = w_{Lt} + \frac{1}{1+r}\left[q_L\,\Psi_{UL,t+1} + (1-q_L)\,\Psi_{EL,t+1}\right], \tag{3.13}$$

where $\Psi_{UL,t+1}$ denotes the present discounted utility of a low-skilled worker being unemployed at the end of period $t+1$. The job separation rate q_L refers to period $t+1$ but since it is assumed to be constant the time index is omitted. A low-skilled worker being unemployed at the end of period t has the present discounted utility

$$\Psi_{UL,t} = z_{Lt} + \frac{1}{1+r}\left[p_{L,t+1}\,\Psi_{EL,t+1} + (1-p_{L,t+1})\,\Psi_{UL,t+1}\right], \tag{3.14}$$

where z_L denotes net unemployment benefits and p_L is the exit rate out of unemployment which positively depends on labor market tightness, i.e. $p_L \equiv M_L/U_L = \theta_L m(\theta_L)$. As we are not interested in the implications of different tax systems on the wage-setting process we assume for simplicity that unemployment benefits are financed by lump-sum taxes.

The low-skilled wage w_{Lt} is determined by firm-level monopoly unions. Assuming Nash bargaining between firms and unions would not change the qualitative results. We choose the present setup as it is our intention to explain the mechanisms of the model in the simplest possible way.[2] Similar to Manning (1991) we assume as a starting point that the single union sets the wage for n periods. The union thereby considers the aggregate labor market tightness to be given and constant which is consistent with an analysis of steady-state wage pressure. If the wage w_{Lt} is set in period t for n periods, it will affect the utility difference $\Psi_{EL} - \Psi_{UL}$ from period t until period $t+n-1$, but not from period $t+n$ onwards. Running forward $\Psi_{EL,t} - \Psi_{UL,t}$ for n periods leads to

$$\Psi_{EL,t} - \Psi_{UL,t} = \left(\frac{1-\delta^n}{1-\delta}\right)(w_{Lt} - z_{Lt}) + \delta^n(\Psi_{EL,t+n} - \Psi_{UL,t+n}), \tag{3.15}$$

where $\delta \equiv (1 - q_L - p_L)/(1 + r) < 1$. The representative labor union at the firm level maximizes the rent of the employed low-skilled workers:

$$\max_{w_{Lt}} \left(\Psi_{EL,t} - \Psi_{UL,t}\right) L_t \tag{3.16}$$

subject to the labor demand equation. In line with, among others, Pissarides (1985), Layard and Nickell (1990), and Beissinger and Egger (2004), we restrict our steady state analysis to the case $n \to \infty$. Omitting time indices for steady-state values, the rent-maximizing wage w_L implies the following wage costs $\widetilde{w}_L$ relevant to the firm (see Appendix 3.A.2):

$$\widetilde{w}_L = \kappa_L \, \widetilde{z}_L, \quad \text{with} \quad \kappa_L \equiv \frac{\varepsilon_{L,\widetilde{w}_L}}{\varepsilon_{L,\widetilde{w}_L} - 1} \quad \text{and } \widetilde{z}_L \equiv z_L + \frac{(q_L + r)}{(1+r)} \frac{s_L}{m(\theta_L)}, \tag{3.17}$$

where κ_L denotes the wage markup on $\widetilde{z}_L$ consisting of unemployment benefits and labor adjustment costs. The wage markup is negatively related to the wage elasticity of the demand for low-skilled labor (in absolute values), $\varepsilon_{L,\widetilde{w}_L}$. The next subsection takes a closer look at this elasticity and shows that $\varepsilon_{L,\widetilde{w}_L} > 1$, implying $\kappa_L > 1$.

[2] The simplifying assumption of monopoly unions is also made in other studies, such as Pissarides (1986), Delacroix (2006), or Dinlersoz and Greenwood (2016).

3.3.3 Task Reallocation and the Elasticity of Labor Demand

The wage elasticity of the demand for low-skilled labor (in absolute values) can be written as

$$\varepsilon_{L,\widetilde{w}_L} \equiv \left| \frac{\partial \ln L^d(\cdot)}{\partial \ln \widetilde{w}_L} \right| = 1 + \frac{1}{1-I} \frac{\partial \ln I}{\partial \ln \widetilde{\omega}} = 1 + \frac{1}{(1-I) \cdot \varepsilon_{\bar{\alpha},I}(I)} > 1, \tag{3.18}$$

where

$$\varepsilon_{\bar{\alpha},I}(I) \equiv \frac{\mathrm{d} \ln \bar{\alpha}(I)}{\mathrm{d} \ln I} > 0$$

denotes the elasticity of the relative task productivity schedule at the task threshold with respect to a one-percent change in I.

The wage elasticity of the demand for low-skilled labor is the sum of a direct wage effect (equal to one) for a given task allocation, and a task reallocation effect caused by the change in the task threshold I due to a change in relative labor costs $\widetilde{\omega}$. The task reallocation effect implies that with an increase in $\widetilde{w}_L$ fewer tasks are allocated to low-skilled labor. The *strength* of this effect depends on the task threshold I in two ways. The more tasks are allocated to low-skilled labor the larger is $1/(1-I)$ which *cet. par.* increases the task reallocation effect and thereby $\varepsilon_{L,\widetilde{w}_L}$. However, the size of the task reallocation effect also negatively depends on $\varepsilon_{\bar{\alpha},I}$. In general, $\varepsilon_{\bar{\alpha},I}$ is a function of I with the sign of $\mathrm{d} \ln \varepsilon_{\bar{\alpha},I} / \mathrm{d} \ln I$ depending on the functional form of $\bar{\alpha}(I)$, i.e. $\mathrm{d} \ln \varepsilon_{\bar{\alpha},I} / \mathrm{d} \ln I \lesseqgtr 0$ is possible.[3] This leads to

Proposition 3.1. *An increase in the task threshold I leads to the following change in the wage elasticity of the demand for low-skilled labor:*

$$\frac{\mathrm{d} \ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d} \ln I} \begin{cases} > 0, & \textit{if} \quad \mathrm{d} \ln \varepsilon_{\bar{\alpha},I} / \mathrm{d} \ln I < I/(1-I) \\ = 0, & \textit{if} \quad \mathrm{d} \ln \varepsilon_{\bar{\alpha},I} / \mathrm{d} \ln I = I/(1-I) \\ < 0, & \textit{if} \quad \mathrm{d} \ln \varepsilon_{\bar{\alpha},I} / \mathrm{d} \ln I > I/(1-I). \end{cases}$$

[3] As shown in Appendix 3.A.3, the second-order condition for the optimization problem of the representative labor union puts a restriction on $\mathrm{d} \ln \bar{\alpha} / \mathrm{d} \ln I$ equivalent to $\bar{\alpha}''(I)\, I / \bar{\alpha}'(I) > -2$ so that $\bar{\alpha}(I)$ must not be "too concave".

Proof. Taking into account eq. (3.18) and the definition of κ_L in eq. (3.17), $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I$ can be written as:

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = \frac{1}{\kappa_L}\left(\frac{I}{1-I} - \frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I}\right). \tag{3.19}$$

Since $\kappa_L > 0$, Proposition 3.1 immediately follows from eq. (3.19). $\square$

Since the wage markup κ_L is negatively related to $\varepsilon_{L,\widetilde{w}_L}$, Proposition 3.1 can be directly applied to establish the effect of the threshold I on κ_L:

$$\varepsilon_{\kappa_L,I} \equiv \frac{\mathrm{d}\ln\kappa_L}{\mathrm{d}\ln I} = -(\kappa_L - 1)\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}, \quad \text{with} \quad \mathrm{sgn}(\varepsilon_{\kappa_L,I}) = -\mathrm{sgn}\left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\right). \tag{3.20}$$

The important insight from this analysis is that changes in the task allocation have an impact on the effective wage-setting power of labor unions. The way in which a change in the task allocation affects the labor demand elasticity and thus the labor unions' wage markup crucially depends on the shape of the relative task productivity schedule.

To provide some intuition, consider, for example, a negative $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I$. This implies a concave shape of $\bar{\alpha}(I)$ suggesting that at lower thresholds I the low- and high-skilled workers are more apart in term of their productivities than at higher I. At a low I, a reduction of the task threshold leads to a pronounced drop in the relative high-skilled productivity $\bar{\alpha}(I)$, thereby making the substitution of low-skilled by those with higher skills harder. Therefore, an increase in $\widetilde{w}_L$ requires only a small decline in I to induce such an increase in the relative productivity of low-skilled workers $1/\bar{\alpha}(I)$ that the unit labor costs of both skill groups are equal again in optimum. As a result, labor demand decreases only slightly with increasing $\widetilde{w}_L$ at a low I. In contrast, with low-skilled workers being more substitutable at a high I, the response of labor demand to increasing $\widetilde{w}_L$ is more pronounced. This explains why a concave $\bar{\alpha}(I)$ function leads to a rise in the labor demand elasticity with an increase in the task threshold I. The opposite applies if $\bar{\alpha}(I)$ is convex and $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I$ is sufficiently large. These results can be relevant when comparing the outcomes of the wage-setting process in different sectors of the economy. Some sectors may encompass a range of tasks which rapidly increase in their complexity so that the relative productivity of high-skilled workers increases in a more exponential manner. Other sectors may display a task complexity profile that is prone to stronger substitution of different skills.

One special case of Proposition 3.1 is especially worth mentioning – the case of a constant wage elasticity of labor demand. The corresponding functional form of $\bar{\alpha}(I)$ is specified in

Lemma 3.1. *The wage elasticity of labor demand for unskilled workers $\varepsilon_{L,\tilde{w}_L}$ is constant if and only if the relative task productivity schedule of high- and low-skilled workers is given by*

$$\bar{\alpha}(I) = b\left(\frac{I}{1-I}\right)^{\eta}, \quad \textit{with} \quad \eta > 0 \quad \textit{and} \quad b > 0, \tag{3.21}$$

leading to $\varepsilon_{L,\tilde{w}_L} = 1 + 1/\eta$. For the special case $\eta = 1$, we have $\varepsilon_{L,\tilde{w}_L} = 2$.

Proof. See Appendix 3.A.4. □

A more generic function $\bar{\alpha}(I)$ that nests function (3.21) as a special case and also allows for different responses of the labor demand elasticity summarized in Proposition 3.1 is:

$$\bar{\alpha}(I) = b\,\frac{I^{\eta_H}}{(1-I)^{\eta_L}}, \quad \text{with} \quad \eta_H \geq 0, \eta_L \geq 0, \eta_H + \eta_L > 0. \tag{3.22}$$

In Appendix 3.A.5 we outline the properties of the above function as well as its three special cases.

3.3.4 Solution of the Model in the Steady State

In the steady state the inflows into low-skilled employment are equal to the outflows from low-skilled employment. From eq. (3.4) follows $m(\theta_L)V_L = q_L L$. Equivalently, the inflows into low-skilled jobs are equal to the flows out of unemployment, i.e. $m(\theta_L)V_L = p_L(\theta_L)U_L$, where again $p_L(\theta_L) \equiv m(\theta_L)\theta_L$. With a mass one of individuals, $L + U_L + H = 1$. Hence,

$$q_L L = p_L(\theta_L)\,(1 - H - L). \tag{3.23}$$

Since the final good is chosen as numeraire, its price equals one. This implies

$$\int_0^I \ln\left(\frac{\widetilde{w}_L}{A_L \alpha_L(i)}\right) \mathrm{d}i + \int_I^1 \ln\left(\frac{w_H}{A_H \alpha_H(i)}\right) \mathrm{d}i = 0, \tag{3.24}$$

which closes the model. The solution of the model is described in

Definition 3.1. *The general equilibrium values of the task threshold I, low-skilled employment L, labor market tightness θ_L, and the firm's wage costs $\widetilde{w}_L$ and w_H are determined by eqs. (3.7), (3.10), (3.17), (3.23) and (3.24). From the definition of $\widetilde{w}_L$ in eq. (3.7) the low-skilled wage w_L is obtained. The solution for output follows from eq. (3.12).*

3.4 Comparative Statics: Technical Change

In the following, we will analyze the implications of skill-biased technical change (SBTC) in the task-based matching model. To keep things as simple as possible, we consider a one-time increase in A_H while A_L remains constant ($\mathrm{d}\ln \bar{A} = \mathrm{d}\ln A_H$). The high-skilled and low-skilled labor force is assumed to be given. With perfect competition in the high-skilled labor market this assumption implies that high-skilled employment remains constant, i.e. $\mathrm{d}\ln H = 0$.

To ease the exposition, the matching function is assumed to be of the Cobb-Douglas type:

$$M_L = M(V_L, U_L) = V_L^{1-\beta_L} U_L^{\beta_L}, \qquad \text{with} \quad 0 < \beta_L < 1, \tag{3.25}$$

which implies $m(\theta_L) = \theta_L^{-\beta_L}$. Therefore, β_L is the (constant) elasticity of the job filling rate $m(\theta_L)$ with respect to labor market tightness θ_L (in absolute values). The elasticity of the job finding rate p_L with respect to θ_L is $(1-\beta_L)$.

It is useful to write the model equations in log differences. They can be condensed into a three-equations system for θ_L, $\widetilde{w}_L$ and I summarized in

Proposition 3.2 (Comparative Statics)**.** *Let u_L denote the low-skilled unemployment rate, and let $\varepsilon_{\widetilde{z}_L,\theta_L}$ be the elasticity of $\widetilde{z}_L$ with respect to θ_L, where $\widetilde{z}_L$ is defined in eq. (3.17) and $0 < \varepsilon_{\widetilde{z}_L,\theta_L} < \beta_L$. Moreover, $\varepsilon_{\kappa_L,I}$ denotes the elasticity of the wage markup κ_L with* *respect to the task threshold I, as defined in eq. (3.20). Then*

$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)\,u_L}\left[\mathrm{d}\ln A_H - \frac{1}{1-I}\mathrm{d}\ln\widetilde{w}_L + \frac{1}{1-I}\mathrm{d}\ln I\right], \tag{3.26}$$

$$\mathrm{d}\ln\widetilde{w}_L = \varepsilon_{\widetilde{z}_L,\theta_L}\mathrm{d}\ln\theta_L + \varepsilon_{\kappa_L,I}\,\mathrm{d}\ln I, \tag{3.27}$$

$$\mathrm{d}\ln I = -(\varepsilon_{L,\widetilde{w}_L} - 1)\,\mathrm{d}\ln\widetilde{w}_L. \tag{3.28}$$

Proof. See Appendix 3.A.6.

Eq. (3.26) represents the job creation condition, eq. (3.27) is the wage equation for low-skilled workers, and eq. (3.28) can be interpreted as the "task allocation" equation (respectively in log differences). It is evident that both, job creation and wage setting, are influenced by changes in the task threshold I. The equations in Proposition 3.2 represent general equilibrium relationships in which the adjustment of high-skilled wages necessary for full-employment of high-skilled workers has already been taken into account.[4]

According to the job creation equation, an increase in A_H *cet. par.* leads to higher labor market tightness. As can be seen from the wage equation, an increase in labor market tightness leads to higher wage pressure and *cet. par.* increases $\widetilde{w}_L$. However, this increase in labor costs induces firms to reduce the range of tasks allocated to low-skilled labor, which reduces labor market tightness and has ambiguous effects on wage setting as explained above. Inserting the task allocation equation in the other two equations leads to

$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)\,u_L}\left[\mathrm{d}\ln A_H - \frac{\varepsilon_{L,\widetilde{w}_L}}{1-I}\,\mathrm{d}\ln\widetilde{w}_L\right], \tag{3.29}$$

$$\mathrm{d}\ln\widetilde{w}_L = \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}\,\mathrm{d}\ln\theta_L, \tag{3.30}$$

where for the latter expression eq. (3.20) and the definition of κ_L in eq. (3.17) have been used, and $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I \neq 1$ must hold. In this version of the job creation and wage-setting equation the adjustment of the task threshold I due to a change in firm's low-skilled labor costs is already taken into account. This two-equations system can be

[4]This is, for instance, the reason why changes in A_H have been "netted out" of the task allocation equation.

graphically represented by a job creation curve (JC) and wage curve (WC) in $\theta_L - \widetilde{w}_L$ space. As can be seen from eq. (3.29), increases in A_H lead to rightward shift of the JC. Moreover, the JC is downward-sloping, i.e.

$$\Phi \equiv \left.\frac{\mathrm{d}\ln \widetilde{w}_L}{\mathrm{d}\ln \theta_L}\right|_{\mathrm{JC}} = -\frac{(1-I)(1-\beta_L)u_L}{\varepsilon_{L,\widetilde{w}_L}} < 0. \tag{3.31}$$

As regards the WC described by eq. (3.30), the relationship between θ_L and $\widetilde{w}_L$ is not unambiguous. The slope of the WC is

$$\Gamma \equiv \left.\frac{\mathrm{d}\ln \widetilde{w}_L}{\mathrm{d}\ln \theta_L}\right|_{\mathrm{WC}} = \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1-\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}} \gtreqless 0. \tag{3.32}$$

Quite similar to standard matching models, the slope of the WC positively depends on $\varepsilon_{\widetilde{z}_L,\theta_L}$ which is a function of r, q_L, β_L, and s_L, as shown in eq. (3.A.19) in Appendix 3.A.6. In addition to these parameters, the slope of the WC in the task-based model also depends on $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I$, i.e. on how changes in the task allocation affect the wage elasticity of labor demand.

In a conventional matching model an increase in labor market tightness leads to higher wage claims of workers, implying an upward–sloping WC in θ_L–$\widetilde{w}_L$ space. In eq. (3.30) this situation arises if $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$. However, as is evident from Proposition 3.1, the case $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$ is also possible, implying that the corresponding WC would be downward-sloping. In that case, two situations can be distinguished depending on whether the JC is steeper or flatter than the WC, i.e. depending on whether $|\Phi| \gtreqless |\Gamma|$. In Appendix 3.A.7 we first demonstrate that, irrespectively of the slope of the WC, a steady state equilibrium exists in all situations. Moreover, we show that all steady state equilibria can be in principle (saddle-path) stable so that we cannot rule out the possibility of a downward-sloping WC in a general comparative-static analysis. The results of this analysis are summarized in

Proposition 3.3 (Comparative-Static Results)**.** *High-skilled labor-augmenting technical change has the following effects on the labor market equilibrium:*

(i) Low-skilled labor market tightness:

$$\frac{\mathrm{d}\ln\theta_L}{\mathrm{d}\ln A_H}\begin{cases} >0, & \textit{if} \quad \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}<1 \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}>1 \;\wedge\; |\Phi|>|\Gamma|\right) \\ <0, & \textit{otherwise.} \end{cases}$$

(ii) Low-skilled labor costs:

$$\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln A_H}\begin{cases} >0, & \textit{if} \quad \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}<1 \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}>1 \;\wedge\; |\Phi|<|\Gamma|\right) \\ <0, & \textit{otherwise.} \end{cases}$$

(iii) Task threshold:

$$\frac{\mathrm{d}\ln I}{\mathrm{d}\ln A_H}\begin{cases} <0, & \textit{if} \quad \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}<1 \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}>1 \;\wedge\; |\Phi|<|\Gamma|\right) \\ >0, & \textit{otherwise.} \end{cases}$$

Proof. Solving eqs. (3.29) and (3.30) leads to:

$$\begin{aligned} \frac{\mathrm{d}\ln\theta_L}{\mathrm{d}\ln A_H} &= \frac{1-I}{\varepsilon_{L,\widetilde{w}_L}}\,\frac{1}{|\Phi|+\Gamma}, \\ \frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln A_H} &= \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}\,\frac{\mathrm{d}\ln\theta_L}{\mathrm{d}\ln A_H} = \frac{(1-I)\,\Gamma}{\varepsilon_{L,\widetilde{w}_L}}\,\frac{1}{|\Phi|+\Gamma}. \end{aligned}$$

Because of eq. (3.28) it holds:

$$\frac{\mathrm{d}\ln I}{\mathrm{d}\ln A_H} = -(\varepsilon_{L,\widetilde{w}_L}-1)\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln A_H} = -\frac{(1-I)\,\Gamma}{\kappa_L}\,\frac{1}{|\Phi|+\Gamma}.$$

□

Figure 3.2 illustrates the comparative-static results. If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I<1$, the WC is upward-sloping (see Figure 3.2a). In this case an increase in A_H leads to an increase in labor market tightness θ_L and in firms' labor costs $\widetilde{w}_L$. Figure 3.2b depicts the situation where both curves are downward-sloping and the WC is steeper than the JC. In that case an increase in A_H still leads to an increase in $\widetilde{w}_L$, but θ_L is declining. If the JC is steeper than the WC, as depicted in Figure 3.2c, the opposite results are obtained, i.e. θ_L increases whereas $\widetilde{w}_L$ decreases.

To give some intuition to the comparative-static results, it is useful to distinguish between the decisions at the firm level and the general equilibrium results. At the firm level, the high-skilled wage and labor market tightness are considered as given. An increase in A_H lowers the unit labor costs for high-skilled workers relative to low-skilled workers at the old task threshold I. Hence, the firm has an incentive to reduce the range of tasks performed by low-skilled workers. The firm's labor union chooses a wage w_L that implies $\widetilde{w}_L = \kappa_L \widetilde{z}_L$, where $\widetilde{z}_L$ is taken as given. Depending on the curvature of the task productivity schedule $\bar{\alpha}$, the decline in I affects the labor demand elasticity as outlined in Proposition 3.1. In response to that, the labor unions' wage claims may rise, remain unchanged or fall.

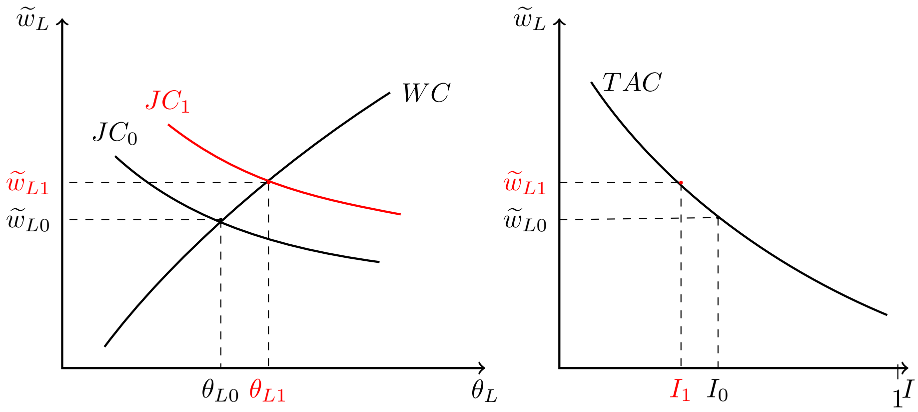
In the general equilibrium, the increase in the firms' demand for high-skilled workers *cet. par.* leads to a rise in high-skilled wages and in the firms' relative wage costs $\widetilde{\omega} = w_H/\widetilde{w}_L$.[5] The increase in $\widetilde{\omega}$ *cet. par.* increases labor demand for low-skilled workers. As shown in Appendix 3.A.6,

$$\mathrm{d}\ln L = \frac{1}{1-I}\,\mathrm{d}\ln I + \mathrm{d}\ln\widetilde{\omega}, \tag{3.33}$$

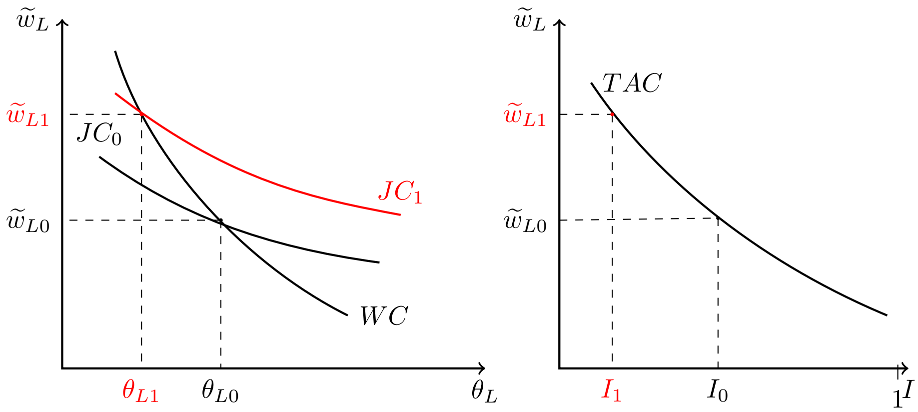
$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)u_L}\mathrm{d}\ln L. \tag{3.34}$$

Hence, whether L and therefore θ_L increase relative to the initial equilibrium depends on whether the positive effect on labor demand caused by the increase in $\widetilde{\omega}$ is larger than the negative effect caused by the decline in the task threshold I. Of course, changes in θ_L lead to changes in $\widetilde{z}_L$ which lead to further adjustments in labor unions' wage claims for low-skilled workers.

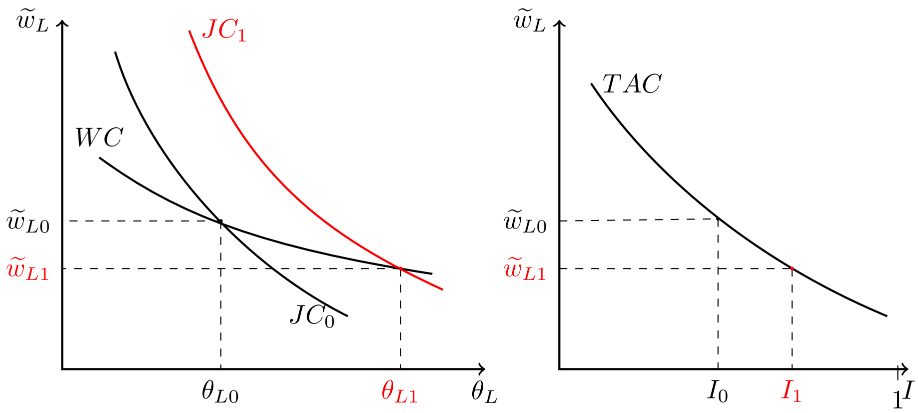
[5] By how much w_H and $\widetilde{\omega}$ rise also depends on the change in low-skilled wage costs $\widetilde{w}_L$.



(a) Case 1: $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$



(b) Case 2: $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$; WC steeper than JC



(c) Case 3: $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$; JC steeper than WC

Figure 3.2: Effects of skill-biased technological progress (increase in the productivity of high-skilled workers A_H) on labor market outcomes and task allocation

Notes: The effects depend on the size of $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I$ and the relative slopes of the job creation curve (JC) and the wage curve (WC). Graphical illustration of the JC, WC and TAC (task allocation curve) follows from the formal analysis of the slopes and curvatures of these curves based on the general function $\bar{\alpha}(I) = bI^{\eta_H}/(1-I)^{\eta_L}$; see Appendix 3.A.8. The diagrams should nevertheless be interpreted as a sketch. The axes scale is allowed to differ across cases and may encompass different ranges of values for $\widetilde{w}_L$, θ_L, and I.

In Figure 3.2a the increase in $\mathrm{d}\ln\widetilde{\omega}$ dominates, and θ_L and $\widetilde{w}_L$ increase. Despite the decline in the task threshold I, labor demand for low-skilled workers is higher in the new equilibrium because more workers are employed in each of the remaining low-skilled tasks. The WC is relatively steep for $0 < \mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$ because the decline in I then leads to a lower labor demand elasticity and hence higher wage pressure. Vice versa, for $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 0$ the WC is relatively flat because the increase in labor unions' wage claims (due to higher θ_L) is dampened by the increase in the labor demand elasticity. For $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I = 0$ the slope of the resulting WC lies in between the other two cases. Since the slope of the WC is related to the concept of real wage rigidity, this analysis offers additional explanations for different degrees of real wage rigidity between countries or industries. In the literature, the degree of real wage rigidity is often explained by institutional factors such as the unemployment compensation system or the degree of centralization of wage bargaining; see e.g., Layard et al. (1991), Chapter 9. According to our analysis, changes in the task composition also affect the real wage response to changes in labor market tightness depending on the curvature of the task productivity schedule. In that sense, the production technology may also influence the extent of real wage rigidity in an industry or country.

In Figure 3.2b, $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$ and the slope of the JC is smaller than the slope of WC (in absolute values). Because of eq. (3.31), a relatively small $|\Phi|$ arises if the low-skilled unemployment rate u_L is relatively low and I is relatively high, i.e. many tasks are allocated to low-skilled workers, implying that the labor share of low-skilled workers is relatively high. The firm's reduction in I leads to a strong decline in the labor demand elasticity and thus to a strong increase in wage pressure. Since the labor share of low-skilled workers is high, the increase in low-skilled wages raises each firm's labor costs significantly, implying a relatively small increase in output, the labor demand for high-skilled workers, the high-skilled wage and $\widetilde{\omega}$. As a consequence, in eq. (3.33) the effect on L due to a decline in I dominates. The resulting decline in θ_L would *cet. par.* lead to lower wage pressure. However, this effect is overcompensated by the decline in the labor demand elasticity and thus the rising wage markup.

In Figure 3.2c, $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$ and the slope of the JC is larger than the slope of the WC (in absolute values). In comparison to the previous figure, the situation is now reversed. The slope $|\Phi|$ is the larger, the lower the task threshold I, i.e. the lower the low-skilled labor share in the initial equilibrium. Despite the initial increase in $\widetilde{w}_L$ caused by the decline in I, the rise of the firm's labor cost is this time comparably small, leading to a relatively strong increase in production, the demand for high-skilled workers, the high-skilled wage and $\widetilde{\omega}$. These general equilibrium effects lead to a strong increase in L and θ_L and even to an increase in the task threshold I that reduces wage pressure. The rising wage pressure due to higher θ_L is overcompensated by the strongly declining wage pressure as the response of the labor demand elasticity and thus the markup is very strong. As a consequence, $\widetilde{w}_L$ even falls in comparison to the initial equilibrium.

Appendix 3.A.9 summarizes how an increase in A_H affects other variables of the model. If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$, SBTC will unambiguously increase $\widetilde{\omega}$, w_H, L, and Y, and decrease u_L. In most of the cases, for example always when $1 > \mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 0$, low-skilled workers will benefit from an increase in A_H in terms of their wages w_L as well. However, for negative responses of the labor demand elasticity it is even possible that w_L will decline implying a pronounced increase in wage inequality. If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$ the relative sizes of the slopes of the JC and WC again play a key role. In majority of the cases, the effects for w_L match qualitatively those for $\widetilde{w}_L$ discussed above. Therefore, if the low-skilled workers experience higher unemployment rates due to SBTC the rise in inequality is less pronounced than in a situation of declining unemployment rates.

The analysis of this section shows that, other than in a canonical model with a conventional modeling of a production process, SBTC can in our model harm low-skilled workers. This is always the case if the labor demand elasticity increases with higher I and this response is strong. Whether low-skilled workers lose in such a case in terms of their wages or their labor market tightness is to a large extent governed by the initial task composition. This insight could be relevant in sectoral context when different sectors may face similarly strong responses to increasing task thresholds but different division of tasks between low- and high-skilled workers.

3.5 Calibration

3.5.1 Design

To quantify the effects of SBTC in our framework, we calibrate the model to German and French data for two time periods: 1995 to 2005 and 2010 to 2017. We choose Germany and France for our quantitative analysis since both countries are comparably large economies with high union coverage rates. They amount to 70.8% and 57.5% in Germany in the first and second period, respectively, and the corresponding numbers for France are 95.5% and 98% (data from the OECD/AIAS ICTWSS database). We exclude the period 2006–2009 that covers the Great Recession, and treat the two considered periods as two separate steady states. More specifically, we interpret the mean values of the labor market variables used as targets in the calibration as steady-state values for the respective period.[6] We consider two different steady states to demonstrate that even for the same country SBTC can have opposing effects on some labor market variables depending on the initial steady state.

Since our model distinguishes between only two skills, in the calibration the group of low-skilled comprises not only low-skilled but also medium-skilled workers. To better fit the model to the data, we introduce a scale parameter ζ in the Cobb-Douglas matching function, $M(V_L, U_L) = \zeta\, V_L^{1-\beta_L}\, U_L^{\beta_L}$, which indexes the efficiency of the matching process. Moreover, we choose the concave relative task productivity schedule $\bar{\alpha}(i) = bi^{\eta_H}$ for $0 < i < 1$, with $\eta_H < 1$ and $b = 1$. The advantage of the concave functional form for $\bar{\alpha}(i)$ is that it allows to generate any of the cases described by Figures 3.2a–3.2c. The parameterization of the model then decides which case is obtained. Note that throughout this section the task productivity schedule is formulated as a function of task index i describing task complexity. In the previous sections, we put the focus on $\bar{\alpha}(I)$, i.e. $\bar{\alpha}(i)$ evaluated at a specific task threshold I. For the properties of the function $\bar{\alpha}(I) = bI^{\eta_H}$ see Appendix 3.A.5.

The model is characterized by 18 exogenous parameters: $\{\beta_L, q_L, \zeta_c, r_c, z_{L,c}, s_{L,c}, H_c,$

[6] Even though the period 1995–2005 covers the 2001/2002 recession in Germany and France, this recession was less severe than the Great Recession. Including both years in the first period allows us to consider a longer time span as a steady-state period.

$A_{H,c}, A_{L,c}, \eta_{H,c}\}$, with $c \in \{DE, FR\}$. We take nine parameters from the data or the literature, see Table 3.1, and calculate nine parameters to match German and French data during the periods 1995–2005 and 2010–2017, see Tables 3.2 and 3.3. One period in the model corresponds to one quarter, so all parameters are interpreted quarterly.

Table 3.1: Parameter values for Germany (DE) and France (FR)

Parameter	**Country**	**Value**		**Source**
		95–05	10–17	
Parameters without country variation				
Matching elasticity: β_L	–	0.5	0.5	Petrongolo and Pissarides (2001)
Quarterly separation rate: q_L	–	0.0873	0.0873	Battisti et al. (2018)
Parameters with country variation				
Real interest rate: r	DE	0.0117	0.0016	Deutsche Bundesbank, FRED
	FR	0.0104	0.0047	Banque de France, FRED
Unemployment benefits: z_L	DE	0.42	0.35	Values are set to target net replacement rates from the OECD
	FR	0.52	0.49	
Share of high-skilled: H	DE	0.24	0.285	EU-LFS
	FR	0.263	0.359	
Skill bias: $\frac{A_H}{A_L}$	DE	1.121	1.344	Own calculations based on data from EU-LFS, EU-SILC, Destatis

Notes: For detailed data description see Appendix 3.A.10.

The first two parameters are without country and time variation. We set the matching elasticity β_L to 0.5, which is within the range of estimates reported in Petrongolo and Pissarides (2001). The quarterly separation rate q_L is assumed to be 0.0873 based on the monthly separation rate of 0.03 calibrated by Battisti et al. (2018). The real interest rate r and the share of high-skilled workers H are calculated from the data and vary over time and country. Unemployment benefits are chosen in such a way that they imply average net replacement rates calculated with OECD data (OECD Benefits, Taxes and Wages Dataset) for each country and time period.[7] Lastly, we calculate the time-dependent skill bias A_H/A_L for Germany. For detailed description of the data and the corresponding

[7]The calibrated low-skilled wage in Germany and France in 1995–2005 (2010–2017) is 0.685 (0.696) and 0.832 (0.851), respectively. To obtain net average replacement rates from OECD data, we first take the average over replacement rates for two previous in-work earnings, two family situations, including social assistance benefits. Then, we compute the weighted average of the net replacement rates for short-term (less than one year) and long-term unemployed (more than one year). The final replacement rates for Germany and France in 2001–2005 (2010–2017) are 0.61 (0.50) and 0.62 (0.58), respectively.

calculations see Appendix 3.A.10.

Table 3.2: Matched targets for Germany (DE) and France (FR)

Target	**Country**	**Value**		**Source**
		95–05	10–17	
Low-skilled unemployment rate: u_L	DE	0.106	0.062	EU-LFS
	FR	0.112	0.113	
Low-skilled labor market tightness: θ_L	DE	0.204	0.471	IAB-JVS, EU-LFS, BA
	FR	0.279	0.301	DARES, Pôle emploi, EU-LFS, IAB-JVS
Skill premium: $\frac{w_H}{w_L}$	DE	1.32	1.49	EU-SILC, Destatis
	FR	1.43	1.41	EU-SILC
Task threshold: I	DE	0.6499	0.6229	WIOD SEA Release 2013, EU Klems Release 2017
	FR	0.6333	0.5436	
Relative real total factor productivity (RTPF): $\frac{RTFP_{DE}}{RTFP_{FR}}$	–	0.9494	1.0023	Penn World Tables 10.0 by Feenstra et al. (2015)

Notes: For detailed data description see Appendix 3.A.10.

We jointly calibrate the remaining nine parameters by matching nine targets obtained from German and French data over the two periods. The targets are summarized in Table 3.2 and the parameters that are obtained by matching these targets are shown in Table 3.3. The most important target is the task threshold I which is calculated as the relative share of low-skilled and middle-skilled labor compensation in total labor compensation. The comparative-static results depend on whether $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I$ is below or above one, i.e. whether the task threshold I is smaller or larger than the specific boundary value $I_b = 1-(\sqrt{1+\eta_H}-1)/\eta_H$. The value for I_b for each country and time period is estimated from the calibrated η_H. To allow for model calibration for both countries jointly, we exploit the information on the real total factor productivity (RTFP) in Germany relative to France taking into account that RTFP in our model is given by $A_L^I A_H^{1-I} B$, with B defined in eq. (3.12).

3.5.2 Calibrated parameters

To be confident that our model can produce meaningful predictions of labor market effects of SBTC, we first want to make sure that it is well calibrated. In the following we discuss our calibration results.

Table 3.3: Calibrated parameters

Parameter	**Germany**		**France**	
	95–05	10–17	95–05	10–17
Matching efficiency parameter: ζ	0.452	0.686	0.528	0.549
Search cost: s_L	3.291	4.850	2.223	2.979
High-skill biased technology: A_H	–	–	1.741	1.874
Low-skill biased technology: A_L	1.030	1.097	1.000	1.000
Parameter of function $\bar{\alpha}(I)$: η_H	0.684	0.924	0.990	0.963

Notes: Calibration has been done by matching the targets in Table 3.2. The model parameters satisfy the stability conditions described in Appendix 3.A.7.

Matching efficiency

According to our calibration, the matching efficiency in Germany is higher in the period 2010–2017 than in 1995–2005 which is in line with evidence showing that the Hartz reforms introduced between 2003 and 2005 have considerably improved the matching process in the German labor market; see, e.g., Fahr and Sunde (2009) and Klinger and Rothe (2012). In contrast, our calibration points to a nearly constant matching efficiency in France. This is consistent with the finding in Arpaia et al. (2014) that, differently than in Germany, the matching efficiency in France does not display any trend. Moreover, the calibration in Bentolila et al. (2012) infers that, in contrast to Spain, the 2008/2009 recession did not impair the matching efficiency in France.

Search costs

Our calibration implies an increase in firms' search costs in Germany. We treat this parameter as all-encompassing costs for the firms associated with hiring. Lochner et al. (2021) document a steady increase in recruiting intensity in Germany between 1992 and 2017 as measured by the number of search channels used by establishments for their most recent hire. Assuming that a higher recruiting intensity increases hiring costs, this finding corroborates our calibration results. At a given labor market tightness, the recruiting intensity per vacancy raises the job filling rate (Davis et al., 2012). Using German data, Carrillo-Tudela et al. (2020) show that the recruiting intensity has a positive impact on the matching efficiency.[8] A positive relationship between these two measures is also present

[8]Such a relationship is also found for the US, see Davis et al. (2013) and Gavazza et al. (2018) who

in our calibration results for Germany. To the best of our knowledge, there is no empirical evidence on the recruiting intensity and its impact on the matching efficiency for France. Our calibration results for France can nevertheless be interpreted as a confirmation of the empirical evidence from other countries. Matching efficiency being nearly constant across both periods in France is consistent with calibrated search costs that increased considerably less than in Germany.

Skill bias in France

Skill bias $\bar{A} = A_H/A_L$ is higher in France than in Germany which can be primarily explained by the composition of skill groups in both countries. The share of medium-skilled in the total group of low-skilled is higher in Germany and amounts to about 60% in both periods compared to about 45% in France. Along with the fact that medium-skilled workers obtain more education and can be considered as more productive than less educated workers, this implies a higher A_L in Germany than in France. Moreover, both countries also differ in the way how the educational system is linked to the work organization. Traditionally, France is considered as "organizational" space and Germany as "qualificational" space—this classification has been introduced in Maurice et al. (1986). Whereas in France hiring criteria are based on workers' general education, in Germany specific jobs require qualifications tailored to those jobs. Such qualifications are obtained primarily by medium-skilled workers through vocational education which is more prevalent in Germany than in France. This type of education creates stronger school-to-work linkages which, in turn, lead to higher productivity; see, e.g., Elbers et al. (2021) and DiPrete et al. (2017). This factor additionally contributes to a higher A_L and ultimately lower skill bias in Germany than in France. As regards the comparison across both periods, calibrated skill bias in France is higher in the more recent period which can be explained with a strong increase in high-skilled labor. This positive effect on skill bias clearly dominates the negative effect of a slightly lower skill premium in the second period.

provide evidence on a declining recruiting intensity in the US during the Great Recession and its negative impact on the matching efficiency, thus contributing to an outward shift of the post-recession Beveridge curve for the US.

Productivity function

By setting $\bar{\alpha}(i) = bi^{\eta_H}$, with $b = 1$, total relative productivity in our quantitative analysis can be described by the function $\bar{A}\bar{\alpha}(i) = \bar{A}i^{\eta_H}$. Figure 3.3 depicts the calibrated productivity functions for both periods and countries. For Germany, the calibrated total relative productivity depends on the skill bias calculated from the data and on the calibrated η_H value. For France, both these parameters are obtained in the calibration. Differences in the positions and/or shapes of the total relative productivity curves in each country do not represent a simulated increase in $\bar{A}$ but represent differences in technology parameters in two different initial steady states. Simulated effects of an increase in $\bar{A}$ on labor market variables in each of both steady states will be covered in the next subsection.

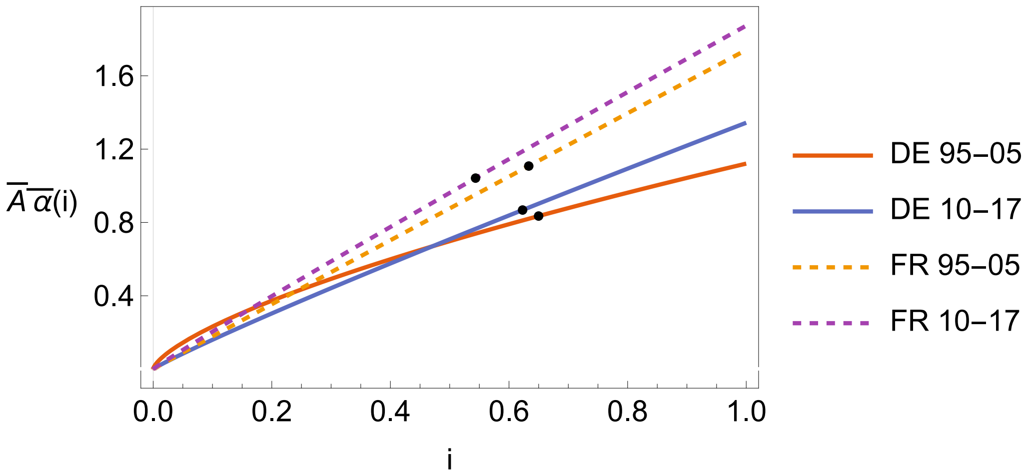


Figure 3.3: Calibrated productivity functions for Germany (DE) and France (FR) in periods 1995–2005 and 2010–2017.

Notes: The relative productivity function is described by $\bar{A}\bar{\alpha}(i) = \bar{A}i^{\eta_H}$. The circle marker on each of the plotted curves indicates the position of the corresponding task threshold $i = I$ set as a target in the calibration; see Table 3.2.

The initial productivity curve for France is virtually a straight line whereas that for Germany displays a higher degree of concavity due to a lower calibrated η_H. The initial curve for France is also steeper which reflects higher skill bias in France as discussed above. Its slope increases in the more recent period because of increasing skill bias in France but there is practically no change in the concavity of the displayed function. Our calibration suggests that the gap in the relative productivity of high-skilled workers between the two periods increases with increasing task complexity. For Germany, the pattern is different—the relative productivity of high-skilled workers in period 2010–2017 is lower than in period 1995–2005 for lower indexed tasks with the gap decreasing up to a value of i of around 0.45. Starting from this value, $\bar{A}\bar{\alpha}(i)$ is higher in the more recent period and the gap widens with increasing i.

Figure 3.4 provides a more detailed picture on the differences in the productivity profile across the periods for Germany. With η_H held constant, the properties of the $\bar{\alpha}$-function remain unchanged and, similarly as for France, a higher $\bar{A}$ in the period 2010–2017 shifts the entire relative productivity profile upwards, leading to a growing discrepancy between the old and new productivity function. Keeping $\bar{A}$ at the level of the period 1995–2005, a higher η_H at the level of 2010–2017 induces an increase in the relative productivity of low-skilled that could be alternatively interpreted as a shift in task complexity i at the given relative productivity $\bar{\alpha}$. This shift is consistent with the empirically documented increase in occupation complexity in Germany, especially for rather less educated workers; see, e.g., Pikos and Thomsen (2016) regarding the increase in categories of performed tasks, and Spitz-Oener (2006) regarding the change of task composition of occupations towards more non-routine tasks. Bachmann et al. (2022) demonstrate that jobs being previously more intense in routine tasks but becoming more intense in cognitive tasks are associated with more training which in turn increases productivity of workers employed in these jobs. For France, in contrast, Bittarello et al. (2018) show that low- plus middle-skilled workers experienced a similar increase in routine and social tasks as high-skilled workers from 1991 and 2013 but either no change (low- plus medium-skilled) or decrease (high-skilled) in cognitive tasks.[9] This may justify why the calibrated French relative productivity function does not exhibit a similar shift as the German one. However, an accurate interpretation of our result would require a rigorous empirical investigation tailored to our model and is left for future research.

[9] With expanding high education the skill premium has decreased and highly educated workers have been assigned more routine tasks, as Bittarello et al. (2018) illustrate using a comparison of task categories of a bank clerk in 1991 and 2013 as an example.

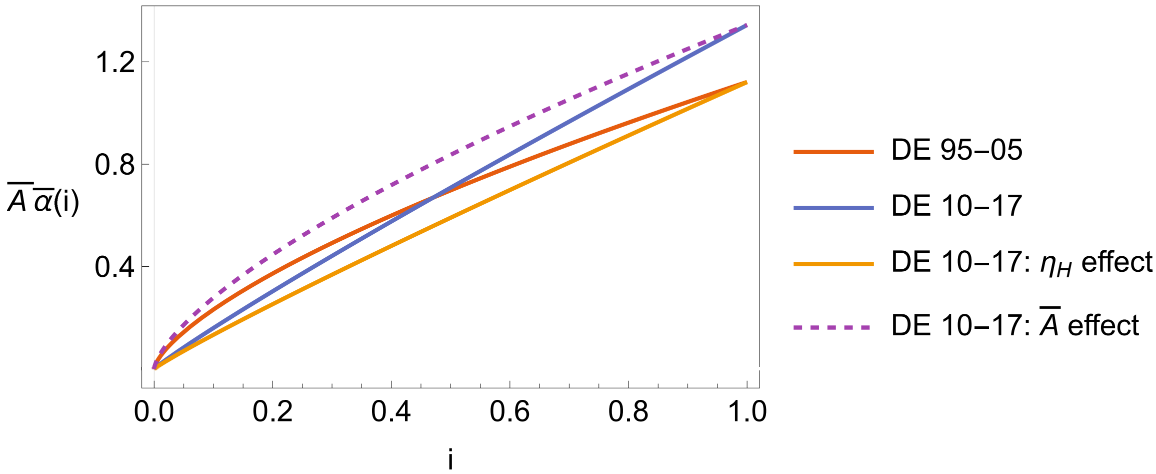


Figure 3.4: Calibrated productivity functions for Germany (DE) in two periods 1995–2005 and 2010–2017.

Notes: The relative productivity function is described by $\bar{A}\bar{\alpha}(i) = \bar{A}i^{\eta_H}$. The solid yellow line (dashed violet line) corresponds to the productivity function for the period 2010–2017 when keeping $\bar{A}$ (η_H) at the level of the period 1995–2005.

3.5.3 Comparative Statics: Increase in A_H

The simulated results of an increase in A_H by 10% on different labor market variables are summarized in Table 3.4. In the period 1995–2005, the effect on θ_L is negative in both countries, with the decrease in Germany being weaker than in France. The negative effect on θ_L corresponds to an increase in the unemployment rate of low-skilled workers in both countries and occurs due to two factors. First, it holds for both countries that with the calibrated values of the task productivity schedule parameter η_H the target values of I lie above the boundary value I_b in this period. This implies $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$, meaning that when the firms reallocate tasks towards more high-skilled workers due to higher A_H the labor demand elasticity decreases strongly, thereby leading to a strong improvement in the wage-setting power of labor unions. In this case the WC is downward-sloping as has been explained in Section 3.4. Second, with values for the model parameters and targets it also holds for both countries in this period that $|\Phi| < |\Gamma|$ so that the WC steeper than the JC; see also Figure 3.2b.

In the period 2010–2017, the effect on θ_L remains negative for Germany but it is weaker than in the previous period. Correspondingly, also in this period the unemployment rate increases due to SBTC in Germany, albeit to a lesser extent (0.02 percentage points

Table 3.4: Labor market effects of an increase in high-skill productivity A_H by 10% (changes in variables expressed in percent).

Variable	**Germany**		**France**	
	95-05	10-17	95-05	10-17
Low-skilled labor market tightness: θ_L	–1.56	–0.80	–1.80	1.33
Low-skilled unemployment rate: u_L	0.70	0.37	0.80	–0.59
Task threshold: I	–2.85	–2.80	–2.72	–3.15
Low-skilled wage: w_L	1.37	1.83	1.47	1.61
High-skilled wage: w_H	8.73	8.39	8.30	8.35
Skill premium: $\frac{w_H}{w_L}$	7.36	6.56	6.82	6.74

Notes: Changes in u_L in percentage points are for Germany 0.07 (period 1995–2005) and 0.02 (2010–2017); the corresponding numbers for France are: 0.09 and –0.07.

compared to 0.07 in the previous period). In contrast, in France the effect on θ_L (u_L) becomes positive (negative). The boundary value I_b stays virtually constant in France across both periods due to almost identical calibrated η_H. However, a strong reduction in I reflecting educational expansion in France and thus an increasing share of high-skilled in total income results in this case in $I < I_b$, for which we have $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$. Even though the wage-setting power of labor unions increases in this case as well with a SBTC-induced task reallocation the markup increase is more moderate this time. The WC is upward-sloping and the SBTC effect on θ_L correspond to the scenario in Figure 3.2a.

As regards the effect of an increase in A_H on the task threshold, there is a stronger reallocation of tasks towards high-skilled workers in Germany than in France in the first period, while in the second period this effect is stronger in France. Low-skilled wages rise due to SBTC in both countries in both periods which is due to a lower labor demand elasticity and thus stronger wage-setting power of labor unions induced by the decline in I. The increase in w_L is, however, moderate in both countries and periods compared to the significant increase in wages of high-skilled workers which translates into a pronounced increase in the skill premium in both countries and periods.

This quantitative exercise illustrates the most important mechanisms of our model—*(i)* how task reallocation affects the labor demand elasticity and thus the wage pressure in the economy, and *(ii)* how these resulting changes in the labor demand elasticity impact labor market outcomes. Our simulation shows that two factors decide on the direction and the strength of the labor demand elasticity to changes in I: the relative task productivity schedule and the task threshold in the economy determined by the share of high-skilled workers in total income. Since these both factors can differ across different steady states, the effects of SBTC on labor market variables can be completely different as well. Opposed signs of SBTC effects on labor market tightness in France in the 1995–2005 and 2010–2017 steady states perfectly exemplify this insight.

3.6 Summary and Conclusions

This paper combines the task approach, labor union wage setting and the matching framework to analyze the impact of skill-biased technical change (SBTC) on labor union wage setting that is brought about by changes in the firms' assignment of tasks to low- and high-skilled workers. In labor union models the wage elasticity of labor demand plays a crucial role for the extent of wage pressure in the economy. We show that this elasticity is influenced by the task threshold that divides the range of tasks performed by low- and high-skilled workers, respectively. More specifically, we demonstrate that the labor demand elasticity for low-skilled workers consists of a direct wage effect and a task reallocation effect. The latter effect implies that with an increase in low-skilled labor costs fewer tasks are allocated to low-skilled labor. The strength of the task reallocation effect depends on the intensity with which low-skilled workers are used in the production process and on the shape of the relative task productivity schedule that reflects the substitutability of high- and low-skilled workers. Since both convex and concave shapes of the relative task productivity schedule are theoretically possible, the effect of a change in the task allocation on the labor demand elasticity remains ambiguous.

This ambiguity carries over to the general equilibrium that is condensed into a two equation system reflecting job creation by firms and wage claims of labor unions. Whereas in standard matching models an increase in labor market tightness leads to higher wage pressure along a positively sloped wage curve, in our model the wage curve can also be downward-sloping. This has consequences for the effects of SBTC. In contrast to the standard result that an increase in the factor productivity of high-skilled workers also has a positive impact on employment and wages of low-skilled workers, in our model it is possible that low-skilled workers may instead either experience higher unemployment or lower real wages.

In the calibration of the model to German and French data for the two time periods 1995–2005 and 2010–2017, we find that the impact of SBTC may even change its sign over time. With the parameterization for the first period, SBTC increases low-skilled unemployment in both countries. With the parameterization for the second period, SBTC still increases low-skilled unemployment in Germany, but reduces it in France. For both countries and periods real wages of high-skilled workers increase more than those for low-skilled workers, hence the skill premium increases in both countries. The driving force behind these simulation outcomes is the decline in the labor demand elasticity due to SBTC, in particular the strength of this decline. It depends on the shape of the task productivity schedule and the task allocation of low- and high-skilled workers, and both these factors differ across countries and also across different periods.

These results point to interesting extensions of our model that could be considered in future research. For example, our modeling framework could also be relevant when comparing the outcomes of the wage-setting process in different sectors of the economy. Some sectors may encompass a range of tasks which rapidly increase in their complexity so that the relative productivity of high-skilled workers increases in a more exponential manner. Other sectors may display a task complexity profile that is prone to stronger substitution of different skills. Moreover, different sectors can be also characterized by different task allocation of different skill groups. Applying the insights of our model could in this case explain differing sectoral real wage developments for workers exhibiting the same skill level and facing the same extent of SBTC. Moreover, one could include capital, especially automation capital, into the model and in this way analyze the impact of automation on the bargaining power of labor unions and unemployment.

Appendix 3.A

3.A.1 The Firm's Optimization Problem

Combining eqs. (3.1)–(3.5), and considering the restrictions (3.6) for $l_t(i)$ and $h_t(i)$, the firm's optimization problem can be written as

$$\begin{aligned}\max_{\{l_t(i),h_t(i),V_{Lt},\mu_{Lt}\}} \mathcal{L} = \sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1}&\left\{\exp\left[\int_0^1 \ln\left(A_{Lt}\alpha_L(i)l_t(i) + A_{Ht}\alpha_H(i)h_t(i)\right)\mathrm{d}i\right]\right.\\ &\left.-w_{Lt}\int_0^1 l_t(i)\mathrm{d}i - w_{Ht}\int_0^1 h_t(i)\mathrm{d}i - s_L V_{Lt}\right\}\\ +\sum_{t=0}^{\infty}\mu_{Lt}\left(\frac{1}{1+r}\right)^{t-1}&\left[m(\theta_{Lt})V_{Lt} + (1-q_L)\int_0^1 l_{t-1}(i)\mathrm{d}i - \int_0^1 l_t(i)\mathrm{d}i\right]\\ \text{s.t.}\quad l_t(i)\geq 0,\qquad h_t(i)\geq 0,&\qquad \text{and}\qquad l_0(i), h_0(i)\ \text{given},\end{aligned}$$

where μ_{Lt} denotes the shadow price of L_t in period t. The single firm takes aggregate labor market tightness θ_{Lt} as given. The first-order conditions are $\partial\mathcal{L}/\partial\mu_{Lt} = 0$, $\partial\mathcal{L}/\partial V_{Lt} = 0$ (which gives $\mu_{Lt} = s_L/m(\theta_{Lt})$), and the complementary slackness conditions

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial h_t(i)}\leq 0,\qquad h_t(i)\geq 0,\qquad \frac{\partial\mathcal{L}}{\partial h_t(i)}h_t(i) = 0,\\ \frac{\partial\mathcal{L}}{\partial l_t(i)}\leq 0,\qquad l_t(i)\geq 0,\qquad \frac{\partial\mathcal{L}}{\partial l_t(i)}l_t(i) = 0.\end{aligned}$$

This leads to

$$\frac{Y_t}{y_t(i)}A_{Ht}\alpha_H(i)\leq w_{Ht},\qquad h_t(i)\geq 0, \tag{3.A.1}$$

$$\frac{Y_t}{y_t(i)}A_{Lt}\alpha_L(i)\leq \widetilde{w}_{Lt}\equiv w_{Lt} + \frac{s_L}{m(\theta_{Lt})} - \frac{s_L}{m(\theta_{L,t+1})}\frac{(1-q_L)}{(1+r)},\qquad l_t(i)\geq 0. \tag{3.A.2}$$

Due to complementary slackness in each equation only one inequality can hold at the same time. As can be seen from eq. (3.A.2), the low-skilled labor costs relevant to the firm, $\widetilde{w}_{Lt}$, consist of the wage w_{Lt} plus the search costs incurred in period t, which are reduced by the vacancy posting costs that are saved in period $t+1$ if the employment relationship continues. For the discussion of the different cases we focus on the steady state in which $\theta_{L,t+1} = \theta_{Lt} = \theta_L$. In that case, $\widetilde{w}_{Lt} = \widetilde{w}_L$, where

$$\widetilde{w}_L \equiv w_L + \frac{(q_L + r)}{(1 + r)} \frac{s_L}{m(\theta_L)}. \tag{3.A.3}$$

Case 1: $l(i) > 0$ and $h(i) = 0$. Due to eq. (3.2) in the main text $y(i) = A_L \alpha_L(i) l(i)$, implying $Y/l(i) = \widetilde{w}_L$ in eq. (3.A.2). The marginal product of unskilled labor in task i with respect to output Y equals the low-skilled labor costs relevant to the firm. It follows that $l(i) = l$, i.e. the same labor input l is chosen in all low-skilled tasks. From eq. (3.A.1) follows

$$\frac{\widetilde{w}_L}{A_L \alpha_L(i)} < \frac{w_H}{A_H \alpha_H(i)},$$

if the constraint on $h(i)$ is binding. Hence, low-skilled workers are employed in those tasks in which their unit labor costs are lower than those of high-skilled workers. At the margin where $\partial \mathcal{L}/\partial h(i) = 0$, there is a specific task $i = I$ for which $\widetilde{w}_L/(A_L \alpha_L(I)) = w_H/(A_H \alpha_H(I))$.

Case 2: $h(i) > 0$ and $l(i) = 0$. From eq. (3.2) follows $y(i) = A_H \alpha_H(i) h(i)$, implying $Y/h(i) = w_H$ in eq. (3.A.1) which is interpreted analogously. It follows that $h(i) = h$, i.e. the same labor input h is chosen in all high-skilled tasks. From eq. (3.A.2) follows

$$\frac{\widetilde{w}_L}{A_L \alpha_L(i)} > \frac{w_H}{A_H \alpha_H(i)},$$

if the constraint on $l(i)$ is binding. Hence, high-skilled workers are employed in those tasks in which their unit labor costs are lower than those of low-skilled workers. At the margin where $\partial \mathcal{L}/\partial l(i) = 0$, there is a specific task $i = I$ for which $\widetilde{w}_L/(A_L \alpha_L(I)) = w_H/(A_H \alpha_H(I))$.

Case 3: $h(i) > 0$ and $l(i) > 0$. Because of eq. (3.2) $y(i) = A_L \alpha_L(i) l(i) + A_H \alpha_H(i) h(i)$. In eq. (3.A.2) it holds that $(Y/y(i)) A_L \alpha_L(i) = \widetilde{w}_L$. In eq. (3.A.1) it holds that $(Y/y(i)) A_H \alpha_H(i) = w_H$. Hence,

$$\frac{\widetilde{w}_L}{A_L \alpha_L(I)} = \frac{w_H}{A_H \alpha_H(I)}.$$

Case 4: $h(i) = 0$ and $l(i) = 0$. In that case $y(i) = 0$ which due to the production function in eq. (3.1) is not possible.

In cases 1-3 the task threshold I is defined as the task where unit labor costs for high- and low-skilled workers are equal. This condition can be written as

$$\bar{\alpha}(I) \equiv \frac{\alpha_H(I)}{\alpha_L(I)} = \frac{A_L\, w_H}{A_H \widetilde{w}_L}. \tag{3.A.4}$$

Since $\bar{\alpha}'(i) > 0$, there is only one task $i = I$ where unit labor costs of both worker types are equal. It must hold that $I < 1$, because values $I \geq 1$ would imply that no high-skilled workers are used in the production process, in contradiction to our assumption that high-skilled workers are fully employed. Moreover, if unemployment benefits are not too high, it is never optimal for labor unions to demand such high wages that no unskilled workers are employed. In that case it must also hold that $I > 0$. As a consequence, $0 < I < 1$.

3.A.2 Wage Setting of Labor Unions

To simplify the notation, we define $R_{Lt} \equiv \Psi_{EL,t} - \Psi_{UL,t}$. With the wage w_{Lt} being set for n periods and unemployment benefits being equal to z_{Lt} in all periods,

$$R_{Lt} = \left(\frac{1-\delta^n}{1-\delta}\right)(w_{Lt} - z_{Lt}) + \delta^n R_{L,t+n},$$

where $\delta \equiv (1 - q_L - p_L)/(1 + r) < 1$. The representative labor union maximizes

$$\max_{w_{Lt}} V_{Lt} = R_{Lt}\, L_t \tag{3.A.5}$$

s.t. to the labor demand equation (3.11)

$$L_t = L^d\left(\widetilde{\omega}_t, \cdot\right), \qquad \text{with} \qquad \widetilde{\omega}_t \equiv \frac{w_{Ht}}{\widetilde{w}_{Lt}}.$$

The labor union considers aggregate labor market tightness to be given and constant, in line with steady-state considerations. Therefore, $\widetilde{w}_{Lt}$ corresponds to the expression in eq. (3.A.3). The first-order condition $\mathrm{d}V_{Lt}/\mathrm{d}w_{Lt} = 0$ gives

$$\frac{\partial R_{Lt}}{\partial w_{Lt}} L_t + R_{Lt} \frac{\partial L^d}{\partial \widetilde{\omega}_t} \frac{\partial \widetilde{\omega}_t}{\partial w_{Lt}} = 0. \tag{3.A.6}$$

Multiplying by $\widetilde{w}_{Lt}/L_t$ and defining

$$\varepsilon_{L\widetilde{w}_L,t} \equiv \left| \frac{\partial \ln L^d(\cdot)}{\partial \ln \widetilde{w}_{Lt}} \right| = \frac{\partial \ln L^d(\cdot)}{\partial \ln \widetilde{\omega}_t} \tag{3.A.7}$$

leads to

$$\left(\frac{1-\delta^n}{1-\delta} \right) \widetilde{w}_{Lt} - R_{Lt}\, \varepsilon_{L\widetilde{w}_L,t} = 0.$$

Defining

$$\widetilde{z}_{Lt} \equiv z_{Lt} + \frac{(q_L + r)}{(1+r)} \frac{s_L}{m(\theta_L)}$$

and noting that $w_{Lt} - z_{Lt} = \widetilde{w}_{Lt} - \widetilde{z}_{Lt}$ gives

$$\frac{(1-\delta^n)}{1-\delta} \widetilde{w}_{Lt} - \left[\frac{(1-\delta^n)}{1-\delta} (\widetilde{w}_{Lt} - \widetilde{z}_{Lt}) + \delta^n R_{L,t+n} \right] \varepsilon_{L\widetilde{w}_L,t} = 0.$$

Therefore, the wage w_{Lt} set in period t for n periods implies the following wage costs $\widetilde{w}_{Lt}$ in period t:

$$\widetilde{w}_{Lt} = \frac{\varepsilon_{L\widetilde{w}_L,t}}{\varepsilon_{L\widetilde{w}_L,t} - 1} \left(\widetilde{z}_{Lt} - \frac{(1-\delta)\delta^n}{1-\delta^n} R_{L,t+n} \right). \tag{3.A.8}$$

In the steady state $R_{L,t+n} = (\widetilde{w}_{Lt} - \widetilde{z}_{Lt})/(1-\delta)$. Hence,

$$\widetilde{w}_{Lt} = \frac{\varepsilon_{L\widetilde{w}_L,t}}{\varepsilon_{L\widetilde{w}_L,t} + \delta^n - 1} \widetilde{z}_{Lt}. \tag{3.A.9}$$

This result is in line with the result in Manning (1991) that wage pressure is the higher the longer the duration of the wage contract. We focus on the situation in which $n \to \infty$ and therefore $\delta^n \to 0$. Omitting the time index, this leads to

$$\widetilde{w}_L = \frac{\varepsilon_{L,\widetilde{w}_L}}{\varepsilon_{L,\widetilde{w}_L} - 1} \widetilde{z}_L. \tag{3.A.10}$$

3.A.3 Second-Order Condition for the Optimal Wage

The following exposition builds on Appendix 3.A.2. With $n \to \infty$ and the definition of δ, $\mathrm{d}V_L/\mathrm{d}w_L$ can be written as

$$\frac{\mathrm{d}V_L}{\mathrm{d}w_L} = \frac{(1+r)L^d(\cdot)}{r+q_L+p_L}\left[1 - \frac{(\widetilde{w}_L - \widetilde{z}_L)}{\widetilde{w}_L}\varepsilon_{L,\widetilde{w}_L}\right],$$

where $\varepsilon_{L,\widetilde{w}_L}$ is defined in eq. (3.A.7), and the time index has been omitted. Therefore,

$$\begin{aligned}\frac{\mathrm{d}^2V_L}{\mathrm{d}w_L^2} = &-\frac{(1+r)\varepsilon_{L,\widetilde{w}_L}L^d(\cdot)}{(r+q_L+p_L)\widetilde{w}_L}\left[1 - \frac{(\widetilde{w}_L - \widetilde{z}_L)\varepsilon_{L,\widetilde{w}_L}}{\widetilde{w}_L} + \frac{\widetilde{z}_L}{\widetilde{w}_L}\right.\\ &\left.+(\widetilde{w}_L - \widetilde{z}_L)\frac{\mathrm{d}\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}I}\frac{\partial I}{\partial \widetilde{w}_L}\frac{1}{\varepsilon_{L,\widetilde{w}_L}}\right].\end{aligned}$$

The last expression takes into account that $\varepsilon_{L,\widetilde{w}_L}$ is a function of the task threshold I which in turn depends on $\widetilde{w}_L$, as is evident from eq. (3.18). For a maximum, next to $\mathrm{d}V_L/\mathrm{d}w_L = 0$, the condition $\mathrm{d}^2V_L/\mathrm{d}w_L^2 < 0$ has to be satisfied. This implies that the term in brackets must be positive. Hence,

$$1 - \frac{(\widetilde{w}_L - \widetilde{z}_L)\varepsilon_{L,\widetilde{w}_L}}{\widetilde{w}_L} + 1 - \frac{\widetilde{w}_L - \widetilde{z}_L}{\widetilde{w}_L} + \frac{(\widetilde{w}_L - \widetilde{z}_L)}{\widetilde{w}_L}\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\frac{\partial \ln I}{\partial \ln \widetilde{w}_L} > 0$$

must hold. Since

$$\frac{\partial \ln I}{\partial \ln \widetilde{w}_L} = -\frac{\partial \ln I}{\partial \ln \widetilde{\omega}} = -\frac{1}{\varepsilon_{\bar{\alpha},I}},$$

this condition is equivalent to

$$1 + \varepsilon_{L,\widetilde{w}_L} + \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\frac{1}{\varepsilon_{\bar{\alpha},I}} < 2\frac{\widetilde{w}_L}{\widetilde{w}_L - \widetilde{z}_L}.$$

From $\mathrm{d}V_L/\mathrm{d}w_L = 0$ follows $\varepsilon_{L,\widetilde{w}_L} = \widetilde{w}_L/(\widetilde{w}_L - \widetilde{z}_L) > 1$. Therefore,

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < (\varepsilon_{L,\widetilde{w}_L} - 1)\varepsilon_{\bar{\alpha},I}\ .$$

Taking account of eq. (3.18), the second-order condition for an optimum therefore requires

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < \frac{1}{1-I} \quad . \tag{3.A.11}$$

Because of eq. (3.18) and the definition of κ_L in eq. (3.17)

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = \frac{1}{\kappa_L}\left(\frac{I}{1-I} - \frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I}\right),$$

which will play an important role in Proposition 3.1. Moreover, it holds

$$\kappa_L \equiv \frac{\varepsilon_{L,\widetilde{w}_L}}{\varepsilon_{L,\widetilde{w}_L}-1} = 1 + (1-I)\varepsilon_{\bar{\alpha},I} \quad .$$

Therefore, the condition (3.A.11) can be alternatively written as

$$\frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I} > -(1+\varepsilon_{\bar{\alpha},I}). \tag{3.A.12}$$

Since $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I = 1 - \varepsilon_{\bar{\alpha},I} + I\bar{\alpha}''/\bar{\alpha}'$, (3.A.12) implies

$$\bar{\alpha}''\frac{I}{\bar{\alpha}'(I)} > -2. \tag{3.A.13}$$

3.A.4 Proof of Lemma 3.1

Since $\varepsilon_{\bar{\alpha},I} \equiv \bar{\alpha}'(I)\, I/\bar{\alpha}(I)$, eq. (3.18) for the wage elasticity of labor demand for low-skilled workers can be written as

$$(\varepsilon_{L,\widetilde{w}_L} - 1)\,\frac{\bar{\alpha}'(I)}{\bar{\alpha}(I)} = \frac{1}{I\,(1-I)},$$

where $\varepsilon_{L,\widetilde{w}_L} > 1$ is (in this case) constant by assumption. Therefore,

$$(\varepsilon_{L,\widetilde{w}_L} - 1)\int\frac{\bar{\alpha}'(I)}{\bar{\alpha}(I)}\,\mathrm{d}I = \int\frac{1}{1-I}\,\mathrm{d}\ln I.$$

Hence,

$$(\varepsilon_{L,\widetilde{w}_L} - 1)\ln\bar{\alpha}(I) + c_1 = \ln\frac{I}{1-I} + c_2,$$

where c_1 and c_2 are integration constants. Applying the exponential function, one arrives at

$$\bar{\alpha}(I) = b\left(\frac{I}{1-I}\right)^{\eta}, \tag{3.A.14}$$

where $\eta \equiv 1/(\varepsilon_{L,\widetilde{w}_L} - 1) > 0$ and $b \equiv e^{\eta\,(c_2-c_1)}$ which must be positive for $\bar{\alpha}(I) > 0$. With this specific functional form for $\bar{\alpha}(I)$, it holds that

$$\varepsilon_{\bar{\alpha},I} = \eta\,\frac{1}{1-I} \quad \text{and} \quad \frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I} = \frac{I}{1-I},$$

where the second equation is in line with Proposition 3.1 for the case of a constant $\varepsilon_{L,\widetilde{w}_L}$. Inserting the expression for $\varepsilon_{\bar{\alpha},I}$ in eq. (3.18) leads to $\varepsilon_{L,\widetilde{w}_L} = 1 + 1/\eta$ which concludes the proof of Lemma 3.1. □

3.A.5 Special Cases of the Task Productivity Schedule $\bar{\alpha}(I)$

Starting from Lemma 3.1, a more general function for $\bar{\alpha}(I)$ that allows to consider all three cases of Proposition 3.1 is given by eq. (3.22) that is repeated here:

$$\bar{\alpha}(I) = b\,\frac{I^{\eta_H}}{(1-I)^{\eta_L}},$$

where $\bar{\alpha}'(I) > 0$ requires $\eta_H \geq 0$, $\eta_L \geq 0$, and $\eta_H + \eta_L > 0$.
Then $\varepsilon_{L,\widetilde{w}_L} = 1 + 1/[I\,\eta_L + (1-I)\,\eta_H]$, and

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} \begin{cases} > 0, & \text{if} \quad \eta_H > \eta_L \\ = 0, & \text{if} \quad \eta_H = \eta_L \\ < 0, & \text{if} \quad \eta_H < \eta_L. \end{cases}$$

Proof. From eq. (3.22) follows

$$\frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I} = \frac{I}{1-I} - \frac{(\eta_H - \eta_L)I}{I\eta_L + (1-I)\eta_H}.$$

Inserting this expression into eq. (3.19) leads to:

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = \frac{1}{\kappa_L}\frac{(\eta_H-\eta_L)I}{I\eta_L+(1-I)\eta_H}.$$

With κ_L and I being positive, and with the restrictions for η_L and η_H, the above result for $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I$ holds, which completes the proof. □

Moreover, if $\eta_L > \eta_H$, it holds that $|(\eta_H-\eta_L)I| < I\eta_L+(1-I)\eta_H$, and since $1/\kappa_L < 1$, this implies $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > -1$ in the case of the above general task productivity schedule $\bar{\alpha}(I)$.

We can consider three special cases of the above general function $\bar{\alpha}(I)$:

Case 1: $\eta_H = \eta_L = \eta$. This leads to the $\bar{\alpha}$ function in eq. (3.21) implying a constant labor demand elasticity.

Case 2: $\bar{\alpha}(I) = b\,I^{\eta_H}, \quad \eta_H < 1$

In this case $\bar{\alpha}(I)$ is isoelastic and concave. We have:

$$\varepsilon_{L,\widetilde{w}_L} = 1+\frac{1}{\eta_H(1-I)}, \quad \kappa_L = 1+\eta_H(1-I), \quad \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = \frac{1}{1+\eta_H(1-I)}\frac{I}{1-I} > 0.$$

Moreover, there exists a value $I_b = 1-(\sqrt{1+\eta_H}-1)/\eta_H$ such that:

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} \lesseqgtr 1, \quad \text{if} \quad I \lesseqgtr I_b.$$

Case 3: $\bar{\alpha}(I) = b\,(1-I)^{-\eta_L}$

In this case $\bar{\alpha}(I)$ is convex. We have:

$$\varepsilon_{L,\widetilde{w}_L} = 1+\frac{1}{\eta_L I}, \quad \kappa_L = 1+\eta_L I, \quad \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = -\frac{1}{1+\eta_L I}.$$

3.A.6 Proof of Proposition 3.2

From eqs. (3.7) and (3.10) follows

$$\mathrm{d}\ln I = \frac{1}{\varepsilon_{\bar{\alpha},I}}\left(\mathrm{d}\ln\widetilde{\omega} - \mathrm{d}\ln\bar{A}\right) \tag{3.A.15}$$

and

$$\mathrm{d}\ln L = \frac{1}{1-I}\,\mathrm{d}\ln I + \mathrm{d}\ln\widetilde{\omega}, \tag{3.A.16}$$

where it has been taken into account that $\mathrm{d}\ln H = 0$, and

$$\mathrm{d}\ln\widetilde{\omega} = \mathrm{d}\ln w_H - \mathrm{d}\ln\widetilde{w}_L. \tag{3.A.17}$$

From eq. (3.17) follows

$$\mathrm{d}\ln\widetilde{w}_L = \varepsilon_{\widetilde{z}_L,\theta_L}\mathrm{d}\ln\theta_L + \varepsilon_{\kappa_L,I}\,\mathrm{d}\ln I, \tag{3.A.18}$$

where

$$\varepsilon_{\widetilde{z}_L,\theta_L} \equiv \frac{\mathrm{d}\ln\widetilde{z}_L}{\mathrm{d}\ln\theta_L} = \beta_L\frac{\widetilde{z}_L - z_L}{\widetilde{z}_L} = \beta_L\kappa_L\frac{\widetilde{w}_L - w_L}{\widetilde{w}_L} = \beta_L\frac{\frac{(q_L+r)}{1+r}s_L\theta_L^{\beta_L}}{z_L + \frac{(q_L+r)}{1+r}s_L\theta_L^{\beta_L}} < \beta_L \tag{3.A.19}$$

and

$$\varepsilon_{\kappa_L,I} \equiv \frac{\mathrm{d}\ln\kappa_L}{\mathrm{d}\ln I} = -(\kappa_L - 1)\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = -\frac{\kappa_L - 1}{\kappa_L}\left(\frac{I}{1-I} - \frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I}\right). \tag{3.A.20}$$

Because of eq. (3.23)

$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)u_L}\mathrm{d}\ln L, \tag{3.A.21}$$

where $u_L \equiv (1-H-L)/(1-H)$ denotes the low-skilled unemployment rate.

The price index equation (3.24) can be written as

$$I(\ln\widetilde{w}_L - \ln A_L) + (1-I)(\ln w_H - \ln A_H) - \xi(I) = 0,$$

where

$$\xi(I) \equiv \int_0^I \ln \alpha_L(i)\mathrm{d}i + \int_I^1 \ln \alpha_H(i)\mathrm{d}i.$$

The total differential of this equation is

$$\begin{aligned} & I(\mathrm{d}\ln \widetilde{w}_L - \mathrm{d}\ln A_L) + (1-I)(\mathrm{d}\ln w_H - \mathrm{d}\ln A_H) \\ & - [(\ln w_H - \ln \widetilde{w}_L) - (\ln A_H - \ln A_L) - \ln \bar{\alpha}(I)]\,\mathrm{d}I = 0, \end{aligned} \tag{3.A.22}$$

where it has been taken into account that $\xi'(I) = -\ln \bar{\alpha}(I)$. Since the task threshold is endogenously determined from profit maximization, eq. (3.7) must hold, implying $\ln \bar{\alpha}(I) = (\ln w_H - \ln \widetilde{w}_L) - (\ln A_H - \ln A_L)$. Hence, the term in brackets in the second line is zero, leading to

$$\mathrm{d}\ln w_H = \mathrm{d}\ln A_H + \frac{I}{1-I}\mathrm{d}\ln A_L - \frac{I}{1-I}\mathrm{d}\ln \widetilde{w}_L. \tag{3.A.23}$$

The job creation equation (3.26) in Proposition 3.2 is obtained by combining eqs. (3.A.23), (3.A.17), (3.A.16), and (3.A.21), and by assuming $\mathrm{d}\ln A_L = 0$. The wage-setting equation corresponds to eq. (3.A.18) and the task allocation equation follows from eqs. (3.A.15), (3.A.17) and (3.A.23), where $\mathrm{d}\ln \bar{A} = \mathrm{d}\ln A_H$ if $\mathrm{d}\ln A_L = 0$. This concludes the proof of Proposition 3.2. □

3.A.7 Stability Analysis

3.A.7.1 Model Dynamics and Saddle Path

Starting point for the stability analysis is the set of dynamic model equations:

$$\begin{aligned} & \theta_{Lt} = \frac{V_{Lt}}{U_{Lt}}, \quad U_{Lt} = 1 - L_{t-1} - H, \quad M_{Lt} = V_{Lt}^{1-\beta_L} U_{Lt}^{\beta_L}, \quad L_t = (1-q_L)L_{t-1} + M_t, \\ & \frac{\widetilde{w}_{Lt}}{A_L \alpha_L(I_t)} = \frac{w_{Ht}}{A_H \alpha_H(I_t)}, \quad L_t = \frac{w_{Ht}}{\widetilde{w}_{Lt}} \frac{I_t}{1-I_t} H, \\ & \widetilde{w}_{Lt} = \frac{\varepsilon_{L\widetilde{w}_L,t}}{\varepsilon_{L\widetilde{w}_L,t} - 1}, \quad \widetilde{z}_{Lt} \equiv z_{Lt} + \frac{s_L}{m(\theta_{Lt})} - \frac{1-q_L}{1+r} \frac{s_L}{m(\theta_{L,t+1})}, \\ & \int_0^{I_t} \ln\left(\frac{\widetilde{w}_{Lt}}{A_L \alpha_L(i)}\right) \mathrm{d}i + \int_{I_t}^1 \ln\left(\frac{w_{Ht}}{A_H \alpha_H(i)}\right) \mathrm{d}i = 0. \end{aligned}$$

Letting $\widehat{x}_t$ denote the percentage deviation of x_t from its steady-state level ($\widehat{x}_t \equiv \ln x_t - \ln x$), the linearized equation system is:

$$\widehat{\theta}_{Lt} = \widehat{V}_{Lt} - \widehat{U}_{Lt},\ \widehat{U}_{Lt} = \frac{u_L - 1}{u_L}\widehat{L}_{t-1},\ \widehat{M}_{Lt} = (1-\beta_L)\widehat{V}_{Lt} + \beta_L\widehat{U}_{Lt},\ \widehat{L}_t = (1-q_L)\widehat{L}_{t-1} + q_L\widehat{M}_t,$$
$$\widehat{I}_t = \frac{1}{\varepsilon_{\bar{\alpha},I}}\ (\widehat{w}_{Ht} - \widehat{\widetilde{w}}_{Lt}), \quad \widehat{L}_t = \widehat{w}_{Ht} - \widehat{\widetilde{w}}_{Lt} + \frac{1}{1-I}\ \widehat{I}_t,$$
$$\widehat{\widetilde{w}}_{Lt} = \varepsilon_{\kappa_L,I}\ \widehat{I}_t + \frac{s_L\beta_L\theta_L^{\beta_L}}{\widetilde{z}_L}\ \left(\widehat{\theta}_{Lt} - \frac{1-q_L}{1+r}\ \widehat{\theta}_{L,t+1}\right),$$
$$\widehat{w}_{Ht} = -\frac{I}{1-I}\ \widehat{\widetilde{w}}_{Lt}.$$

After appropriate substitutions, we can eliminate $\widehat{V}_{Lt}$, $\widehat{U}_{Lt}$, $\widehat{L}_t$, $\widehat{M}_t$, and $\widehat{w}_{Ht}$, and we obtain a dynamic version of the job creation (3.26) and wage curve (3.27):

$$\widehat{\theta}_{Lt} = -\frac{\varepsilon_{L,\widetilde{w}_L}}{(1-\beta_L)(1-I)q_L}\ \left(\widehat{\widetilde{w}}_{Lt} - \frac{u_L - q_L}{u_L}\widehat{\widetilde{w}}_{L,t-1}\right), \tag{3.A.24}$$
$$\widehat{\widetilde{w}}_{Lt} = \frac{1+r}{r+q_L}\ \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}\left(\widehat{\theta}_{Lt} - \frac{1-q_L}{1+r}\ \widehat{\theta}_{L,t+1}\right). \tag{3.A.25}$$

In this version, A_H is assumed to be constant as our focus is to examine stability of the model and not the effects of A_H. Finally, the above equation system in two endogenous variables $\widehat{\theta}_{Lt}$ and $\widehat{\widetilde{w}}_{Lt}$ can be reduced to a second-oder difference equation in $\widehat{\theta}_{Lt}$:

$$\widehat{\theta}_{Lt} = \phi_1\widehat{\theta}_{L,t-1} + \phi_2\widehat{\theta}_{L,t-2}, \quad \text{where}$$
$$\phi_1 \equiv \frac{q_L(r+q_L)}{(1-q_L)u_L}\ \frac{(1-\beta_L)(1-I)u_L}{\varepsilon_{L,\widetilde{w}_L}}\ \frac{1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}{\varepsilon_{\widetilde{z}_L,\theta_L}} + \frac{1+r}{1-q_L} + u_L - q_Lu_L, \tag{3.A.26}$$
$$\phi_2 \equiv -\frac{1+r}{1-q_L}\ \frac{u_L - q_L}{u_L}.$$

The existence of the steady state and the dynamic properties of the model can be then studied based on the coefficients ϕ_1 and ϕ_2. In particular, for the steady state to exist it must hold $1-\phi_1-\phi_2 \neq 0$. To ensure that the model solution (path for θ_{Lt}) is unique, the model has to be saddle-path stable. See Krause and Lubik (2010) for a detailed discussion of stability and determinacy aspects in matching models. We could have also derived a first-order equation system for $\widehat{\theta}_{Lt}$ (jump variable) and $\widehat{L}_t$ (state variable), instead of a second-order difference equation for $\widehat{\theta}_{Lt}$. However, we are not interested in the solution for, e.g., θ_{Lt} or L_t, but rather in stability properties.

Saddle-path stability is fulfilled if for the eigenvalues of the system (3.A.26), λ_1 and λ_2, with $\lambda_{1,2} = (\phi_1 \pm \sqrt{\phi_1^2 + 4\phi 2})/2$, holds:

$$(|\lambda_1| < 1 \wedge |\lambda_2| > 1) \quad \vee \quad (|\lambda_1| > 1 \wedge |\lambda_2| < 1). \tag{3.A.27}$$

In the case of a second-order difference equation, saddle-path stability requires real eigenvalues only, i.e. $\Delta = \phi_1^2 + 4\phi_2 \geq 0$, as complex eigenvalues are complex conjugates with identical modulus. The following theorem summarizes conditions equivalent to (3.A.27) which do not demand explicit computation of eigenvalues.

Theorem 3.A.1. *Difference equation $\widehat{\theta}_{Lt} = \phi_1\widehat{\theta}_{L,t-1} + \phi_2\widehat{\theta}_{L,t-2}$ has real solutions and is saddle-path stable, if $\Delta \geq 0$ and one of the following two sets of conditions is fulfilled:*

(i) $1 - \phi_1 - \phi_2 > 0 \quad \wedge \quad 1 + \phi_1 - \phi_2 < 0,$

(ii) $1 - \phi_1 - \phi_2 < 0 \quad \wedge \quad 1 + \phi_1 - \phi_2 > 0.$

An interested reader can find the proof of these conditions in the next subsection.

In the next step, we apply previously discussed conditions in our model. We begin by examining the existence of the steady state. Condition $1 - \phi_1 - \phi_2 \neq 0$ can be represented in terms of slopes of the job creation and the wage curve as follows:

$$\begin{aligned} -\frac{(1-\beta_L)(1-I)u_L}{\varepsilon_{L,\widetilde{w}_L}} &\neq \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1 - \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}, \ \text{ if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1, \\ \frac{(1-\beta_L)(1-I)u_L}{\varepsilon_{L,\widetilde{w}_L}} &\neq \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{|1 - \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}|}, \ \text{ if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1. \end{aligned}$$

In presence of an upward-sloping wage curve, i.e., if $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$, steady state always exists, whereas for a downward-sloping wage curve, steady state exists if slopes of these both curves differ. As regards saddle-path stability, we can distinguish between two cases. In the first case, the wage curve is upward-sloping or it is downward-sloping and steeper than the job creation curve:

$$
\begin{aligned}
-\frac{(1-\beta_L)(1-I)u_L}{\varepsilon_{L,\widetilde{w}_L}} &< \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}, \text{ if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1,\\
\frac{(1-\beta_L)(1-I)u_L}{\varepsilon_{L,\widetilde{w}_L}} &< \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{|1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}|}, \text{ if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1.
\end{aligned}
$$

These inequalities are equivalent to $1-\phi_1-\phi_2<0$, which is the first part of condition (ii) of Theorem 3.A.1. Both parts of condition (ii) imply a restriction for u_L:

$$
\begin{aligned}
&u_L > \left[\frac{2}{q_L}+\frac{r+q_L}{r+q_L+2(1-q_L)\ \Upsilon}\right]^{-1}, \text{ if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1,\\
&\frac{1}{\Upsilon} > u_L > \left[\frac{2}{q_L}+\frac{r+q_L}{r+q_L+2(1-q_L)\ \Upsilon}\right]^{-1}, \text{ if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1, \quad \text{with}\\
&\Upsilon \equiv \frac{(1-\beta_L)(1-I)}{\varepsilon_{L,\widetilde{w}_L}}\ \frac{|1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}|}{\varepsilon_{\widetilde{z}_L,\theta_L}}.
\end{aligned}
$$

The second case corresponds to a downward-sloping wage curve being flatter than the job creation curve and represents the first inequality in condition (i) of Theorem 3.A.1:

$$
\frac{(1-\beta_L)(1-I)u_L}{\varepsilon_{L,\widetilde{w}_L}} > \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{|1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}|}, \text{ with } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1.
$$

According to both conditions (i), the resulting restriction for u_L is:

$$
\frac{1}{\Upsilon} < u_L < \left[\frac{2}{q_L}+\frac{r+q_L}{r+q_L+2(1-q_L)\ \Upsilon}\right]^{-1} < \frac{q_L}{2}.
$$

All these results can be summarized as follows. The model is saddle-path stable if:

(i) for an upward-sloping wage curve: $u_L > q_L/2$ (sufficient condition),

(ii) for a downward-sloping wage curve: either $1/\Upsilon > u_L > q_L/2$ (sufficient condition) or $1/\Upsilon < u_L < q_L/2$ (necessary condition).

3.A.7.2 Conditions for Saddle-Path Stability

Proof of Theorem 3.A.1. For $\Delta \geq 0$ to be satisfied, ϕ_2 must be either nonnegative or, if $\phi_2 < 0$, $|\phi_2| < \phi_1^2/4$. In the following, these two cases will be distinguished.

Case 1: $\phi_2 \geq 0$

We have $\lambda_1 \geq 0$ and $\lambda_2 \leq 0$, with $\lambda_1 + \lambda_2 \neq 0$. If condition (i) is satisfied, then $\phi_1 < 1 - \phi_2 \leq 1$.

$$\lambda_1 < \frac{1}{2}\left(\phi_1^2 + \sqrt{\phi_1^2 + 4(1-\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 + \sqrt{(\phi_1 - 2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 - 2|) = 1$$

$$\begin{aligned}\lambda_2 &< \frac{1}{2}\left(\phi_1^2 - \sqrt{\phi_1^2 + 4(1+\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 - \sqrt{(\phi_1 + 2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 + 2|) \\ &= \begin{cases} -1, & \text{if} \quad \phi_1 > -2 \\ \phi_1 + 1 < -1, & \text{if} \quad \phi_1 < -2 \end{cases}\end{aligned}$$

If condition (ii) is satisfied, then $\phi_1 > \phi_2 - 1 \geq -1$.

$$\begin{aligned}\lambda_1 &> \frac{1}{2}\left(\phi_1^2 + \sqrt{\phi_1^2 + 4(1-\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 + \sqrt{(\phi_1 - 2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 - 2|) \\ &= \begin{cases} \phi_1 - 1 > 1, & \text{if} \quad \phi_1 > 2 \\ 1, & \text{if} \quad \phi_1 < 2 \end{cases}\end{aligned}$$

$$\lambda_2 > \frac{1}{2}\left(\phi_1^2 - \sqrt{\phi_1^2 + 4(1+\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 - \sqrt{(\phi_1 + 2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 + 2|) = -1$$

Case 2: $\phi_2 < 0$

We have $\lambda_1 \lambda_2 > 0$. If condition (i) is satisfied, then $\phi_1 < \phi_2 - 1 < -1$.

$$\begin{aligned}\lambda_1 &> \frac{1}{2}\left(\phi_1^2 + \sqrt{\phi_1^2 + 4(1+\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 + \sqrt{(\phi_1 + 2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 + 2|) \\ &= \begin{cases} \phi_1 + 1 > -1, & \text{if} \quad \phi_1 > -2 \\ -1, & \text{if} \quad \phi_1 < -2 \end{cases}\end{aligned}$$

$$\lambda_2 < \frac{1}{2}\left(\phi_1^2 - \sqrt{\phi_1^2 + 4(1+\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 - \sqrt{(\phi_1+2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 + 2|)$$
$$= \begin{cases} -1, & \text{if} \quad \phi_1 > -2 \\ \phi_1 + 1 < -1, & \text{if} \quad \phi_1 < -2 \end{cases}$$

If condition (ii) is satisfied, then $\phi_1 > 1 - \phi_2 > 1$.

$$\lambda_1 > \frac{1}{2}\left(\phi_1^2 + \sqrt{\phi_1^2 + 4(1-\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 + \sqrt{(\phi_1-2)^2}\right) = \frac{1}{2}(\phi_1^2 + |\phi_1 - 2|)$$
$$= \begin{cases} \phi_1 - 1 > 1, & \text{if} \quad \phi_1 > 2 \\ 1, & \text{if} \quad \phi_1 < 2 \end{cases}$$

$$\lambda_2 < \frac{1}{2}\left(\phi_1^2 - \sqrt{\phi_1^2 + 4(1-\phi_1)}\right) = \frac{1}{2}\left(\phi_1^2 - \sqrt{(\phi_1-2)^2}\right) = \frac{1}{2}(\phi_1^2 - |\phi_1 - 2|)$$
$$= \begin{cases} 1, & \text{if} \quad \phi_1 > 2 \\ \phi_1 - 1 < 1, & \text{if} \quad \phi_1 < 2 \end{cases}$$

□

3.A.8 Slopes and Curvatures of the JC, WC, and TAC

The following analysis is based on the general function $\bar{\alpha}(I) = bI^{\eta_H}/(1-I)^{\eta_L}$ introduced in Section 3.3.3 and discussed in more detail in Appendix 3.A.5.

The slope of the JC in the $\theta_L - \widetilde{w}_L$ space is given by:

$$\left.\frac{\mathrm{d}\widetilde{w}_L}{\mathrm{d}\theta_L}\right|_{\mathrm{JC}} = \Phi\,\frac{\widetilde{w}_L}{\theta_L} < 0.$$

It can be shown that the JC is convex in the $\theta_L - \widetilde{w}_L$ space:

$$\begin{aligned}\left.\frac{\mathrm{d}^2\widetilde{w}_L}{\mathrm{d}\theta_L^2}\right|_{\mathrm{JC}} &= \Phi\frac{\widetilde{w}_L}{\theta_L^2}\left(1 + \Phi - \frac{\mathrm{d}\ln\Phi}{\mathrm{d}\ \ln\theta_L}\right) \\ &= \Phi\frac{\widetilde{w}_L}{\theta_L^2}\left[1 + (1-\beta_L)\left(u_L(1-I)\left(\frac{1}{\kappa_L}\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} + \frac{1}{\varepsilon_{L,\widetilde{w}_L}}\right) + (1-u_L)\left(1 + \frac{I}{\kappa_L}\right)\right)\right] > 0.\end{aligned}$$

The positive sign is due to the fact that $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > -1$.

The slope of the WC in the $\theta_L - \widetilde{w}_L$ space is given by:

$$\left.\frac{\mathrm{d}\widetilde{w}_L}{\mathrm{d}\theta_L}\right|_{\mathrm{WC}} = \Gamma\,\frac{\widetilde{w}_L}{\theta_L}\begin{cases} > 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \\ < 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1. \end{cases}$$

The curvature of the WC is:

$$\begin{aligned}\left.\frac{\mathrm{d}^2\widetilde{w}_L}{\mathrm{d}\theta_L^2}\right|_{\mathrm{WC}} &= -\Gamma\frac{\widetilde{w}_L}{\theta_L^2}\left(1-\Gamma-\frac{\mathrm{d}\ln\Gamma}{\mathrm{d}\ \ln\theta_L}\right) \\ &= -\Gamma\frac{\widetilde{w}_L}{\theta_L^2}\left[1-\beta_L+\Gamma\,\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\left(\frac{1}{1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}\frac{\mathrm{d}\ln\left|\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\right|}{\mathrm{d}\ln I}(\varepsilon_{L,\widetilde{w}_L}-1)-1\right)\right].\end{aligned}$$

If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$ (upward-sloping WC), for the WC to be concave the expression in square brackets has to be positive.

(i) If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I = 0$, this condition is satisfied.

(ii) If $0 < \mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$, it is sufficient if the expression in round brackets is positive, which is satisfied in the special case of a concave isoelastic task productivity schedule $\bar{\alpha}(I) = bI^{\eta_H}$ introduced in Appendix 3.A.5.

(iii) If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 0$, it is sufficient if the expression in round brackets is negative, which is satisfied in the special case of a convex task productivity schedule $\bar{\alpha}(I) = b(1-I)^{-\eta_L}$; see Appendix 3.A.5.

If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$, on the other hand, the WC is downward-sloping and convex.

The TAC is downward-sloping and convex in the $I - \widetilde{w}_L$ space:

$$\left.\frac{\mathrm{d}\widetilde{w}_L}{\mathrm{d}I}\right|_{\mathrm{TAC}} = -\frac{1}{\varepsilon_{L,\widetilde{w}_L}-1}\frac{\widetilde{w}_L}{I} < 0,$$

$$\begin{aligned}\left.\frac{\mathrm{d}^2\widetilde{w}_L}{\mathrm{d}I^2}\right|_{\mathrm{TAC}} &= \frac{1}{\varepsilon_{L,\widetilde{w}_L}-1}\frac{\widetilde{w}_L}{I^2}\left(1+\frac{1}{\varepsilon_{L,\widetilde{w}_L}-1}+\frac{\mathrm{d}\ln(\varepsilon_{L,\widetilde{w}_L}-1)}{\mathrm{d}\ln I}\right) \\ &= \frac{1}{\varepsilon_{L,\widetilde{w}_L}-1}\frac{\widetilde{w}_L}{I^2}\kappa_L\left(1+\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\right) > 0.\end{aligned}$$

3.A.9 Comparative Statics: Effects on Other Variables

$$\frac{\mathrm{d}\ln\widetilde{\omega}}{\mathrm{d}\ln A_H} = \left(|\Phi| + \frac{1}{\kappa_L}\Gamma\right)\frac{1}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \vee \\ & \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| < \frac{1}{\kappa_L}|\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln w_H}{\mathrm{d}\ln A_H} = \left(|\Phi| + \frac{\varepsilon_{L,\widetilde{w}_L} - I}{\varepsilon_{L,\widetilde{w}_L}}\Gamma\right)\frac{1}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \vee \\ & \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| < \frac{\varepsilon_{L,\widetilde{w}_L}-I}{\varepsilon_{L,\widetilde{w}_L}}|\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln w_L}{\mathrm{d}\ln A_H} = \left(\frac{\kappa_L}{1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}} - 1\right)\frac{\varepsilon_{\widetilde{z}_L,\theta_L}\,\widetilde{z}_L}{\varepsilon_{L,\widetilde{w}_L}\,w_L}\,(1-I)\,\frac{1}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } 1 > \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1-\kappa_L \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| < |\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln L}{\mathrm{d}\ln A_H} = \frac{|\Phi|}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln u_L}{\mathrm{d}\ln A_H} = -\frac{1-u_L}{u_L}\,\frac{|\Phi|}{|\Phi|+\Gamma}$$

$$\begin{cases} < 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \\ > 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}\ln A_H} = \Big(|\Phi| + (1-I)\Gamma\Big)\frac{1}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \vee \\ & \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| < (1-I)\,|\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

3.A.10 Data Description

In our quantitative analysis we use different data sets to calculate some of the parameters of the model and the targets for the calibration of the remaining parameters. In the case of any skill-related quantity, we follow Battisti et al. (2018), Chassamboulli and Palivos (2014), and Krusell et al. (2000), and define high-skilled workers as workers with at least a Bachelor degree. This corresponds to levels 5–8 of the International Standard Classification of Education (ISCED 2011). Workers with less than a Bachelor degree (levels 0–4 of ISCED 2011) belong, in fact, to the group of low- plus medium-skilled workers but they are classified as low-skilled workers for the purpose of calibration of our model that distinguishes between only two skills.

Real interest rate r. We follow Chassamboulli and Palivos (2014) to obtain real interest rates for both countries. First, we average over the government bond rates at 30-year constant maturity in the periods 1995–2005 and 2010–2017. Then, we translate the resulting annualized average bond rates into quarterly interest rates. In the next step, we calculate the quarterly GDP deflator as a ratio of nominal and real GDP for the periods 1994-2005 and 2009-2017. Based on the GDP deflator series, we generate quarterly inflation rates. Finally, for both periods we subtract the average quarterly inflation rate from the quarterly interest rate to obtain quarterly real interest rates. Data on the quarterly GDP series for both countries have been retrieved from the Federal Reserve Economic Data (FRED) of the Federal Reserve Bank of St. Louis. The data source for the German government bond rates at 30-year constant maturity is Deutsche Bundesbank (series BBK01.WT3030). The French government bond rates at 30-year constant maturity are provided by Banque de France (series FM.D.FR.EUR.FR2.BB.FR30YT_RR.YLD).

Share of high-skilled workers H. We restrict our sample to individuals between 20 and 64 years. For both countries, we calculate the share of high-skilled workers in periods 1995–2005 and 2010–2017 as the average ratio of high-skilled workers (levels 5–8 of ISCED 2011) to the total labor force in the respective periods using data from the EU Labor Force Statistic (EU-LFS).

Skill bias A_H/A_L. We follow Katz and Murphy (1992) to calculate the implied skill-biased technical change in Germany as a function of relative wages and relative skill supply for a given elasticity of substitution between high- and low-skilled workers, $\sigma > 0$:

$$\frac{A_{Ht}}{A_{Lt}} = \exp\left[\frac{1}{\sigma - 1}\ln\left(\frac{H_t}{L_t}\right) + \frac{\sigma}{\sigma - 1}\ln\left(\frac{w_{Ht}}{w_{Lt}}\right)\right].$$

Relative skill supply in Germany is calculated using data from EU-LFS as the ratio of high-skilled workers to low-skilled workers in each year. For the calculation of the skill premium, see paragraph *Skill premium w_H/w_L* below. We assume an elasticity of substitution between high- and low-skilled workers of 6, which is within the range between 4.9 and 6.9 estimated by Fitzenberger and Kohn (2006) for Germany. This value for σ is notably higher than the estimates of Katz and Murphy (1992) for the US for the period 1963–1987, ranging from 1.4 to 1.8. However, more recent literature uncovered several issues when applying the Katz and Murphy (1992) procedure to later time periods. Bowlus et al. (2021) argue that, among others, neglecting permanent effects of recessions on SBTC (large changes in skill demand due to the implementation of new technologies during recessions) leads to underestimation of σ. Using more elaborate measures for relative skill prices and supplies and taking into account recession effects, they find σ-estimates for the US between 4.0 and 5.2. Due to data restrictions in the time series for the skill premium in Germany (not available before 2001), we calculate the average skill bias for the periods 2001–2005 and 2010–2017.

Low-skilled unemployment rate u_L. As for the high-skilled share, we use data from EU-LFS. For Germany, data on unemployment levels by education are available only since 2005. Before 2005, only unemployment rates by education are available. Since we need the unemployment rate jointly for education levels 0–4, we weigh the unemployment rates for levels 0–2 and 3–4 by the share of persons in the corresponding education level in 2005. Next, we compute the averages of the low-skilled unemployment rates in periods 1995–2005 and 2010–2017. For France, data on active population and employment by education are available since 1995. We calculate the number of unemployed persons as the difference between active population and employed persons and sum over education levels 0–2 and 3–4 to obtain the number of unemployed low-skilled persons. Finally, we compute the average ratio of unemployed low-skilled persons to the low-skilled active population in the periods 1995–2005 and 2010–2017.

Low-skilled labor market tightness θ_L. For Germany, we use quarterly data on job vacancies by education level from the IAB-JVS, which start only in 2011, and quarterly data on unemployment levels by education from the EU-LFS. We calculate the quarterly low-skilled labor market tightness as the ratio of low-skilled vacancies (0–4 ISCED levels) to unemployed low-skilled persons, and then average over the values from 2011 to 2017. As regards the earlier time period, we use monthly data on registered vacancies from the Federal Employment Agency (BA). Due to the break in the classification in 2000 (before 2000: "gemeldete Stellen"; after 2000: "gemeldete Arbeitsstellen"), we restrict the sample to 2000–2005. Since the numbers for registered vacancies provided by the BA are much lower than the number of vacancies according to the IAB-JVS, the registered vacancies in the period 2000-2005 are adjusted using the average ratio of the IAB-JVS vacancies to the registered vacancies in the period 2011–2017. Moreover, since the registered vacancies are not disaggregated by education levels, the adjusted vacancies from the previous step are weighted with the average share of low-skilled vacancies (0–4 ISCED levels) in total vacancies in Germany between 2011 and 2017 from the IAB-JVS dataset. Finally, based on the obtained data we construct the average low-skilled labor market tightness between 2000 and 2005.

To estimate the low-skilled labor market tightness in France in the earlier time period, we use data provided by DARES (Direction de l'animation, de la recherche, des études et des statistiques) of the Ministry of Labour and Social Affairs using information from the national employment office (Pôle emploi), which are available since 1996. The data are given on a monthly basis, hence, we transform them into quarterly data. For the more recent time period, we use quarterly data on job offers from Pôle emploi, which are available since 2010. Due to the fact that the job offers in both data sets are not disaggregated by education, we multiply aggregate vacancies in France with the share of low-skilled vacancies (0–4 ISCED levels) in total vacancies in Germany between 2011 and 2017 from the IAB-JVS. Quarterly data on unemployment levels by education are obtained from the EU-LFS, see paragraph *Low-skilled unemployment rate* u_L for details. Next, we calculate the average low-skilled labor market tightness as the ratio of the estimated low-skilled vacancies to unemployed low-skilled persons. As a last step, we calculate the average low-skilled labor market tightness in France in the periods 1996–2005 and 2010–2017.

Skill premium w_H/w_L. For both countries, we employ data from the European Union Statistics on Income and Living Conditions (EU-SILC). The data relevant in this context is the mean equivalised net income of individuals between 18 and 64 years by educational level. We calculate the skill premium for each year as the mean income of high-skilled (5-8 ISCED levels) relative to the mean income of low-skilled (0-4 ISCED levels). For Germany, the underlying data start only in 2005, so we use in addition data from the Federal Statistical Office of Germany (Destatis) on gross monthly earnings by education and type of workers in former federal territory and "Neue Länder", which are available since 2001. First, we calculate the weighted average of monthly earnings over white and blue collar workers. Next, we compute the weighted average over former federal territory and "Neue Länder" to obtain average monthly earnings in total Germany by education. We calculate then the yearly skill premium as the weighted average monthly earnings of high-skilled workers (5-8 ISCED levels) relative to weighted average monthly earnings of low-skilled workers (0-4 ISCED levels). In order to obtain a consistent time series of the German skill premium from 2001 onwards, we calculate the skill premium in 2005 obtained from EU-SILC relative to the skill premium in 2005 calculated from Destatis and multiply the data prior 2005 with this ratio. As a last step, we average over the values of skill premia in periods 2001-2005 and 2010-2017. The EU-SILC data for France start in 2004, so we calculate the average skill premia in periods 2004-2005 and 2010-2017.

Task threshold I. According to our model, the task threshold can be obtained as the ratio of labor costs of low-skilled workers to aggregate output, with aggregate output being equal to total labor costs. By assuming that labor costs are equal to labor compensation, the task threshold is calculated from the data as the average ratio of low-skilled and middle-skilled labor compensation to total labor compensation over the corresponding periods. For the period 1995-2005 the data for both countries are obtained from the Socio-Economic Accounts (SEA) which is a part of the World Input-Output Database (WIOD) Release 2013; see Timmer et al. (2015). Since the SEA data are available until 2009, for the second period we use data on labor compensation for both countries from EU Klems Release 2017 (available until 2015) to calculate the average task threshold from 2010–2015.

Relative real total factor productivity (RTFP). We calculate the time series of RTFP of Germany relative to France using Penn World Tables 10.0 provided by Feenstra et al. (2015) as follows:

$$\frac{RTFP_{t,DE}}{RTFP_{t,FR}} = relTFP_{2017} * \frac{RTFPInd_{t,DE}}{RTFPInd_{t,FR}},$$

where $relTFP_{2017}$ denotes the value of the TFP in current prices in Germany relative to France in the base year 2017, and $RTFPInd_{t,c}$ is the RTFP index series for country $c \in \{DE, FR\}$. As the final step, we calculate the average RTFP of Germany relative to France over the years 1995 to 2005 and 2010 to 2017, respectively.

References

Acemoglu, D., Aghion, P., and Violante, G. L. (2001). Deunionization, Technical Change and Inequality. *Carnegie-Rochester Conference Series on Public Policy*, *55*, 229–264.

Acemoglu, D., and Autor, D. (2011). Skills, Tasks and Technologies: Implications for Employment and Earnings. In D. Card and O. Ashenfelter (Eds.), *Handbook of Labor Economics* (Vol. 4b, pp. 1043–1171). Elsevier.

Acemoglu, D., and Restrepo, P. (2018a). Modeling Automation. *AEA Papers and Proceedings*, *108*, 48–53.

Acemoglu, D., and Restrepo, P. (2018b). The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment. *American Economic Review*, *108*(6), 1488–1542.

Acemoglu, D., and Restrepo, P. (2022). Tasks, automation, and the rise in us wage inequality. *Econometrica*, *90*(5), 1973–2016.

Açıkgöz, Ö. T., and Kaymak, B. (2014). The Rising Skill Premium and Deunionization. *Journal of Monetary Economics*, *63*, 37–50.

Arpaia, A., Kiss, A., and Turini, A. (2014). Is Unemployment Structural or Cyclical? Main Features of Job Matching in the EU after the Crisis. *European Economy – Economic Papers 527, European Commission*.

Autor, D. H. (2013). The "Task Approach" to Labor Markets: An Overview. *Journal of Labor Market Research*, *46*, 185–199.

Bachmann, R., Demir, G., Green, C., and Uhlendorff, A. (2022). The Role of Within-Occupation Task Changes in Wage Development. *IZA Discussion Paper No. 15647*.

Battisti, M., Felbermayr, G., Peri, G., and Poutvaara, P. (2018). Immigration, Search and Redistribution: A Quantitative Assessment of Native Welfare. *Journal of the European Economic Association*, *16*(4), 1137–1188.

Bauer, C., and Lingens, J. (2014). Does Collective Wage Bargaining Restore Efficiency in a Search Model with Large Firms? *The Economic Journal*, *124*, 1066–1085.

Beissinger, T., and Egger, H. (2004). Dynamic Wage Bargaining if Benefits are Tied to

Individual Wages. *Oxford Economic Papers*, *56*(3), 437-460.

Bentolila, S., Cahuc, P., Dolado, J. J., and Le Barbanchon, T. (2012). Two-Tier Labour Markets in the Great Recession: France versus Spain. *The Economic Journal*, *122*(562), 155–187.

Bittarello, L., Kramarz, F., and Maitre, A. (2018). The Task Content of Occupations. *IZA Discussion Paper No. 12006*.

Boeri, T., and Burda, M. C. (2009). Preferences for Collective versus Individualised Wage Setting. *The Economic Journal*, *119*, 1440–1463.

Bowlus, A., Lochner, L., Robinson, C., and Suleymanoglu, E. (2021). Wages, Skills, and Skill-Biased Technical Change: The Canonical Model Revisited. *Journal of Human Resources*, 0617–8889R1.

Carrillo-Tudela, C., Gartner, H., and Kaas, L. (2020). Recruitment Policies, Job-Filling Rates and Matching Efficiency. *IZA Discussion Paper No. 13240*.

Chassamboulli, A., and Palivos, T. (2014). A Search-Equilibrium Approach to the Effects of Immigration on Labor Market Outcomes. *International Economic Review*, *55*(1), 111–129.

Cords, D., and Prettner, K. (2022). Technological Unemployment Revisited: Automation in a Search and Matching Framework. *Oxford Economic Papers*, *74*(1), 115–135.

Costinot, A., and Vogel, J. (2010). Matching and Inequality in the World Economy. *Journal of Political Economy*, *118*(4), 747–786.

Davis, S. J., Faberman, R. J., and Haltiwanger, J. C. (2012). Recruiting Intensity during and after the Great Recession: National and Industry Evidence. *American Economic Review: Papers and Proceedings*, *103*(2), 584–588.

Davis, S. J., Faberman, R. J., and Haltiwanger, J. C. (2013). The Establishment-Level Behavior of Vacancies and Hiring. *The Quarterly Journal of Economics*, *128*(2), 581–622.

Delacroix, A. (2006). A Multisectorial Matching Model of Unions. *Journal of Monetary Economics*, *53*, 573–596.

Dinlersoz, E., and Greenwood, J. (2016). The Rise and Fall of Unions in the United States. *Journal of Monetary Economics*, *83*, 129–146.

DiPrete, T. A., Eller, C. C., Bol, T., and van de Werfhorst, H. G. (2017). School-to-Work Linkages in the United States, Germany, and France. *American Journal of Sociology*, *122*(6), 1869–1938.

Ebell, M., and Haefke, C. (2006). Product Market Regulation and Endogenous Union Formation. *IZA Discussion Paper No. 2222*.

Elbers, B., Bol, T., and DiPrete, T. A. (2021). Training Regimes and Skill Formation in France and Germany: An Analysis of Change Between 1970 and 2010. *Social Forces*, *99*(3), 1113–1145.

Fahr, R., and Sunde, U. (2009). Did the Hartz Reforms Speed-Up the Matching Process? A Macro-Evaluation Using Empirical Matching Functions. *German Economic Review*, *10*(3), 284–316.

Feenstra, R. C., Inklaar, R., and Timmer, M. P. (2015). The Next Generation of the Penn World Table. *American Economic Review*, *105*(10), 3150–82.

Fitzenberger, B., and Kohn, K. (2006). Skill Wage Premia, Employment, and Cohort Effects: Are Workers in Germany all of the Same Type? *IZA Discussion Paper No. 2185*.

Gavazza, A., Mongey, S., and Violante, G. L. (2018). Aggregate Recruiting Intensity. *American Economic Review*, *108*(8), 2088–2127.

Grossman, G. M., and Rossi-Hansberg, E. (2008). Trading Tasks: A Simple Theory of Offshoring. *American Economic Review*, *98*(5), 1978–1997.

Guimarães, L., and Gil, P. M. (2022). Looking Ahead at the Effects of Automation in an Economy with Matching Frictions. *Journal of Economic Dynamics & Control*, *144*.

Hémous, D., and Olsen, M. (2022). The Rise of the Machines: Automation, Horizontal Innovation, and Income Inequality. *American Economic Journal: Macroeconomics*, *14*(1), 179–223.

Jaimovich, N., Saporta-Eksten, I., Siu, H., and Yedid-Levi, Y. (2021). The Macroeconomics of Automation: Data, Theory, and Policy Analysis. *Journal of Monetary Economics*, *122*, 1–16.

Katz, L. F., and Murphy, K. M. (1992). Changes in Relative Wages, 1963–1987: Supply and Demand Factors. *The Quarterly Journal of Economics*, *107*(1), 35–78.

Klinger, S., and Rothe, T. (2012). The Impact of Labour Market Reforms and Economic Performance on the Matching of the Short-Term and the Long-Term Unemployed. *Scottish Journal of Political Economy*, *59*(1), 90–114.

Krause, M. U., and Lubik, T. A. (2010). Instability and Indeterminacy in a Simple Search and Matching Model. *Federal Reserve Bank of Richmond Economic Quarterly*, *96*(3), 259–272.

Krusell, P., Ohanian, L. E., Ríos-Rull, J.-V., and Violante, G. L. (2000). Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis. *Econometrica*, *68*(5), 1029–1053.

Krusell, P., and Rudanko, L. (2016). Unions in a Frictional Labor Market. *Journal of Monetary Economics*, *80*, 35–50.

Layard, R., and Nickell, S. (1990). Is Unemployment Lower If Unions Bargain Over Employment? *The Quarterly Journal of Economics*, *105*, 773–787.

Layard, R., Nickell, S., and Jackman, R. (1991). *Unemployment.* Oxford: Oxford University Press.

Leduc, S., and Liu, Z. (2020). Robots or Workers? A Macro Analysis of Automation and Labor Markets. *Federal Reserve Bank of San Francisco Working Paper No. 2019-17*.

Lochner, B., Merkl, C., Stüber, H., and Gürtzgen, N. (2021). Recruiting Intensity and Hiring Practices: Cross-Sectional and Time-Series Evidence. *Labour Economics*, *68*, 101939.

Manning, A. (1991). The Determinants of Wage Pressure: Some Implications of a Dynamic Model. *Economica*, *58*(231), 325–339.

Marczak, M., Beissinger, T., and Brall, F. (2022a). Technical change, task allocation, and labor unions. *GLO Discussion Paper No. 1177*.

Marczak, M., Beissinger, T., and Brall, F. (2022b). Technical change, task allocation, and labor unions. *Hohenheim Discussion Papers in Business, Economics and Social Sciences No. 05-2022*.

Marczak, M., Beissinger, T., and Brall, F. (2022c). Technical change, task allocation, and labor unions. *IZA Discussion Paper No. 15632*.

Maurice, M., Sellier, F., and Silvestre, J.-J. (1986). *The Social Foundations of Industrial Power: A Comparison of France and Germany.* Cambridge, MA: MIT Press.

Morin, A. (2017). Cyclicality of Wages and Union Power. *Labour Economics*, *48*, 1–22.

Neto, A., Afonso, O., and Silva, S. T. (2019). How Powerful Are Trade Unions? A Skill-Biased Technological Change Approach. *Macroeconomic Dynamics*, *23*, 730–774.

Petrongolo, B., and Pissarides, C. A. (2001). Looking into the Black Box: A Survey of the Matching Function. *Journal of Economic Literature*, *39*(2), 390–431.

Pikos, A., and Thomsen, S. L. (2016). Rising Work Complexity but Decreasing Returns. *IZA Discussion Paper No. 9878*.

Pissarides, C. A. (1985). Dynamics of Unemployment, Vacancies, and Real Wages with Trade Unions. *The Scandinavian Journal of Economics*, *87*(2), 386–403.

Pissarides, C. A. (1986). Trade Unions and the Efficiency of the Natural Rate of Unemployment. *Journal of Labor Economics*, *4*(4), 582–595.

Pissarides, C. A. (2000). *Equilibrium Unemployment Theory* (2nd ed.). Cambridge, Mass. and London: MIT Press.

Rosen, S. (1978). Substitution and Division of Labor. *Economica*, *45*(179), 235–250.

Spitz-Oener, A. (2006). Technical Change, Job Tasks, and Rising Educational Demands: Looking outside the Wage Structure. *Journal of Labor Economics*, *24*(2), 235–270.

Timmer, M. P., Dietzenbacher, E., Los, B., Stehrer, R., and de Vries, G. J. (2015). An Illustrated User Guide to the World Input-Output Database: the Case of Global Automotive Production. *Review of International Economics*, *23*(3), 575-605.

Chapter 4

Unemployment Benefits in a Task-Based Matching Model: The Impact of Hartz IV Revisited

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The paper has not yet been published.

Abstract. This paper revisits the labor market effects of a cut in unemployment benefits by investigating the effects of the Hartz IV reform using a task-based matching model. This model combines task assignment with search and matching frictions in the labor market, creating incentives for firms to reallocate tasks between high- and low-skilled labor based on differences in unit labor costs. To assess the importance of endogenous task allocation, the task-based matching model is compared to the model with exogenous and constant task allocation. The calibration suggests that Hartz IV reduced the noncyclical low-skilled unemployment rate by 4 percentage points. Ignoring task reallocation in the assessment of Hartz IV results in a lower decline of only 3.4 percentage points, highlighting the importance of endogenous task allocation in the evaluation of labor market reforms.

JEL classification: E24, J64, E60, E23

Keywords: Unemployment Benefit Reforms, Task Approach, Search and Matching, Labor Unions, Hartz Reforms

4.1 Introduction

In the 1990s and early 2000s, core European countries such as France, Italy and Germany experienced persistent and rising unemployment rates, see Figure 4.1. The origins of these high unemployment rates lie in a combination of the institutional framework and shocks; see, e.g., Blanchard and Wolfers (2000), Ljungquist and Sargent (1998) and Mortensen and Pissarides (1999). Possible solutions to address high unemployment include labor market reforms to be targeted on reducing the welfare state or improving the matching efficiency. With the same goal in mind, the German government introduced far-reaching labor market reforms between 2003 and 2005, known as the Hartz reforms. The core part of the reform package, Hartz IV, was implemented in January 2005 and primarily involved a substantial reduction in unemployment benefits. The data suggest that the German government has been quite successful in reducing the unemployment rate from around 11% in 2005 to almost 3% in 2022, despite sharp increases during the financial crisis in 2009 and the Corona crisis in 2020 and 2021.[1]

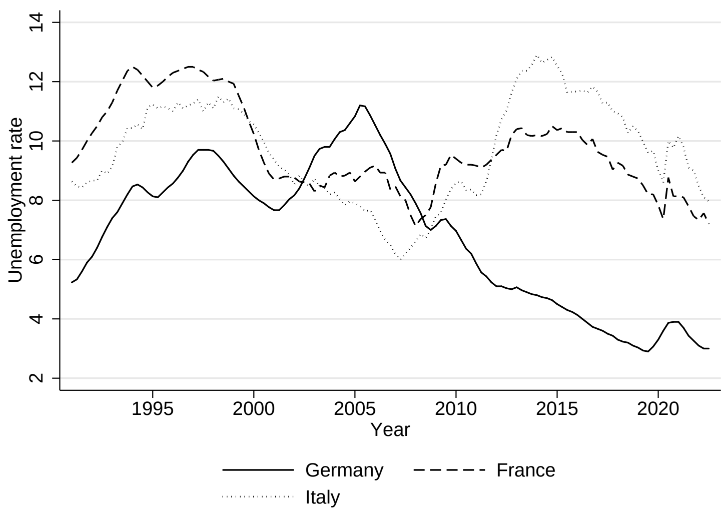

Figure 4.1: Quarterly harmonized unemployment rate in Germany, France and Italy between 1991-2022
Source: OECD.

This paper sheds new light on an old question about the impact of unemployment benefit reforms on labor market outcomes by investigating the effects of the Hartz IV re-

[1]The registered unemployment rate in Germany provided by the Federal Employment Agency is somewhat higher and amounts to 11.7% in 2005 and 5.1% at the beginning of 2022.

form within the framework of a task-based matching model based on Marczak, Beissinger, and Brall (2022). The model combines the task approach, see e.g., the pioneering work of Acemoglu and Autor (2011), with a search and matching model à la Mortensen and Pissarides (1994). The task approach implies that output is produced using a set of tasks. The tasks are assigned to production factors based on their comparative advantage in performing those tasks. In contrast to the prevailing theoretical literature on the consequences of the Hartz IV reform, which assumes a conventional production function, the more detailed modeling of the production process in this analysis reveals that the Hartz IV reform affects the equilibrium allocation between production tasks and workers with different skill levels, leading to additional effects on labor market outcomes.

In the model framework, the labor market for high-skilled workers is competitive, while the labor market for low-skilled workers is characterized by search frictions and monopoly unions. In contrast to Marczak et al. (2022), the goods market exhibits monopolistic competition, which is an important extension given the growing market power of firms in recent decades.[2] Moreover, monopolistic competition in the goods market allows to derive explicit labor demand functions for both worker groups from the profit maximization of firms and the monopolistic power positively impacts the wage markup of labor unions. The comparative advantage of both skill groups in performing specific tasks is defined by the relative task productivity schedule. Without specifying the relative task productivity schedule in more detail, the slope of the wage curve is ambiguous as it depends on the reaction of the labor demand elasticity to changes in the task threshold, which divides the range of tasks performed by low-skilled and high-skilled workers. This results in ambiguous effects of a cut in unemployment benefits on labor market outcomes. However, a negative relation between unemployment benefits and unemployment is implausible and requires unrealistic parameter values within the model framework. Moreover, the overwhelming majority of the literature identifies a positive relation between unemployment benefits and unemployment.[3] Thus, the analysis focuses on the relevant case where a cut in unemployment benefits results in a decreasing unemployment rate. For simplification,

[2]The global (German) markup has increased from 1.17 (1.07) in 1980 to 1.6 (1.35) in 2016, see De Loecker and Eeckhout (2018).

[3]See, e.g., Pries and Rogerson (2005), Yashiv (2004) and Zanetti (2011).

a specific relative task productivity schedule is derived, resulting in a constant labor demand elasticity of low-skilled labor. This leads to a typical upward-sloping wage curve. Similar to a standard search and matching model, declining low-skilled unemployment benefits reduce low-skilled wages, increase low-skilled labor market tightness and lead to a reduction in the unemployment rate of low-skilled workers. However, due to the specific modeling of the production process, the reduction in low-skilled labor costs induces a reallocation of tasks towards low-skilled workers, which, in turn, creates further effects on labor market outcomes that are disregarded in the existing literature.

To assess the importance of the task reallocation effect, the task-based matching model with exogenous and constant task allocation is considered for comparison, reflecting the standard literature with a conventional production function. This implies that task allocation is no longer endogenously determined from profit maximization and there is no reallocation of tasks due to changes in unit labor costs. The wage elasticity of the demand for low-skilled labor is larger than one but lower than in the case of endogenous task allocation. This follows from the fact that with endogenous task allocation, a change in low-skilled labor costs induces a reallocation of tasks between low-and high-skilled labor, which leads to an additional effect on low-skilled labor demand. In terms of the sign, the comparative static results in the task-based matching model with exogenous and constant task allocation are the same as those of the task-based matching model with constant labor demand elasticity.

To quantify the labor market effects of the Hartz IV reform and to illustrate the importance of task reallocation, both model variants are calibrated to German data before 2005. The Hartz IV reform reduced the level of long-term unemployment benefits and the entitlement duration of short-term unemployment benefits, but the extent of the reduction differed across household types.[4] To capture this heterogeneity, the average net replacement rate is used, which measures the generosity of unemployment benefits as the proportion of income obtained after a certain number of months of unemployment. In

[4]Prior to Hartz IV, the unemployment system comprised three stages: unemployment benefit ("Arbeitslosengeld"), unemployment assistance ("Arbeitslosenhilfe") and social assistance ("Sozialhilfe"). In the course of Hartz IV, unemployment assistance and social assistance were merged to a wage-independent unemployment benefit known as "Arbeitslosengeld II", while short-term unemployment benefits ("Arbeitslosengeld I") remained largely unchanged.

contrast to the existing literature, see, e.g., Battisti et al. (2018), Hartung et al. (2022) or Krebs and Scheffel (2013), the average net replacement rate is calculated in a more detailed way. This is possible by using the output from the OECD tax-benefit model (TaxBEN), which offers a wider range of settings in contrast to the frequently used OECD Benefits, Taxes and Wages Dataset. The average net replacement rate for low-skilled workers is calculated for different family situations and unemployment durations, taking into account the average earnings level of low-skilled workers. The decline in the average low-skilled net replacement rate in 2005 amounts to 6.5%, resulting in a noncyclical low-skilled unemployment rate after the Hartz IV reform of 5.8%, which is a reduction by 4 percentage points from its long-run value before the reform. In the case of the task-based matching model with exogenous and constant task allocation, the impact of the Hartz IV reform on the low-skilled unemployment rate is lower with a decrease of only 3.4 percentage points. Moreover, the effect of the cut in the net replacement rate on low- and high skilled wages is more pronounced, resulting in a stronger increase in the skill premium by 14%, compared to 11.6% in case of endogenous task allocation. Hence, ignoring the endogenous task allocation of firms due to changes in unit labor costs within this calibration set-up would underestimate the effect of the Hartz IV reform on low-skilled unemployment and overestimate the effect on wages and the skill premium.

In addition to Hartz IV, the German government implemented two additional waves of the Hartz reforms in 2003 and 2004, which substantially improved the matching efficiency. The calibration results show that the increase in the matching efficiency due to the Hartz I-III reforms leads to an additional reduction in the noncyclical low-skilled unemployment rate by 0.9 percentage points. However, in comparison to the effect of the Hartz IV reform, the first two waves play a minor role in explaining the drop in the unemployment rate.

The paper is related to two distinct strands of the literature. First, the analysis belongs to the literature on the task approach, in which tasks are assigned to production factors based on their comparative advantage in performing those tasks. Rosen (1978) formulates a theory concerning the optimal allocation of workers to tasks by using the concept of comparative advantage. Acemoglu and Autor (2011) integrates the task approach in a general equilibrium model to analyze the effect of skill-biased technical change. The task approach is used in various research domains, such as investigating the impact of automation on outcomes in the labor market, see, e.g., Acemoglu and Restrepo (2018a, 2018b, 2021) and Hémous and Olsen (2022), or analyzing the labor market effects of offshoring, see, e.g., Costinot and Vogel (2010) and Grossman and Rossi-Hansberg (2008). The model used in this analysis is primarily based on Marczak et al. (2022), who develop the task-based matching model to examine how skill-biased technical change affects labor unions' wage-setting through changes in the firms' assignment of tasks to high- and low-skilled workers.

The second strand of the literature is related to the theoretical analysis of the Hartz IV reforms. Most of macroeconomic papers use a search and matching model based on Mortensen and Pissarides (1994): Hartung et al. (2022), Krause and Uhlig (2012) and Launov and Wälde (2013) implement heterogenous agents that differ in their skill endowment and assume single-worker firms. While in Hartung et al. (2022) and Krause and Uhlig (2012) workers bargain individually, in Launov and Wälde (2013) the wage for each skill group is determined through collective bargaining. Hochmuth et al. (2021) introduce unemployed workers with different unemployment durations, contact efficiencies and fixed hiring costs. As in the task-based matching model used in this analysis, Hochmuth et al. (2021) assume multi-worker firms. Moreover, the study presents results of both – when wages are bargained individually and collectively. The quantitative effects of the Hartz IV reform on the decline in the aggregate noncyclical unemployment rate range from less than 0.1 percentage points in Launov and Wälde (2013), 2.1 percentage points in Hochmuth et al. (2021) and 2.8 percentage points in Krause and Uhlig (2012) to 3.2 percentage points in Hartung et al. (2022). Compared to the findings in the literature, the calibrated decline within the framework of the task-based matching model is somewhat more pronounced, because the effects correspond to the reduction in the low-skilled unemployment rate.

Krebs and Scheffel (2013) combine a search model based on Ljungquist and Sargent (1998) with incomplete markets and human capital, distinguishing between short-term and long-term unemployed. The calibration demonstrates that Hartz IV reduced the aggregate long-run unemployment rate by 1.4 percentage points, which is in the range of estimates reported before. Bradley and Kügler (2019) analyze the Hartz reforms using a sequential auction model based on Postel-Vinay and Robin (2002), which includes search frictions, forward-looking agents and workers with different education levels. They find no effect of the Hartz reforms on employment but observed decreasing wages, particularly driven by the cut in unemployment benefits due to Hartz IV.

The paper is structured as follows: Section 2 introduces the model framework. In Section 3, the comparative static results of a change in unemployment benefits are presented and the two specific model variants are introduced. In Section 4, both model variants are calibrated to German data and the long-run effects of Hartz IV on labor market outcomes are analyzed. Section 5 contains concluding remarks.

4.2 The Task-Based Matching Model

4.2.1 Goods Market

The household's optimization problem is static and the model abstracts from savings decisions as in the basic model in Pissarides (2000). Therefore, the time index in the goods market is neglected for now. The economy consists of a continuum of households indexed by $k \in [0, m]$ and a mass one of identical firms indexed by $j \in [0, 1]$. Each firm produces a specific variety, which implies that the number of firms is equal to the number of varieties. Households inelastically supply labor in the labor market and simultaneously consume goods in the goods market. They have Dixit-Stiglitz preferences over a continuum of differentiable goods and are faced with the following optimization problem:

$$\max_{c_{jk}} \left(\int_0^1 c_{jk}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \mathrm{d}j \qquad \text{s.t.} \quad X_k = \int_0^1 c_{jk} \frac{P_j}{P} \, \mathrm{d}j, \tag{4.1}$$

where c_{jk} gives the household k's consumption of the differentiated good j and X_k gives the real income of household k. The firm's price relative to the aggregate price level is captured by $p_j = P_j/P$. Solving the optimization problem, see Appendix 4.A.1, leads to the following Blanchard-Kiyotaki goods demand:

$$Y_j^d \equiv \int_0^m c_{jk} \, \mathrm{d}k = \left(\frac{P_j}{P} \right)^{-\eta} X, \tag{4.2}$$

where $X \equiv \int_0^m X_k \, \mathrm{d}k$ denotes aggregate real income, $P \equiv (\int_0^1 P_j^{1-\eta} \, \mathrm{d}j)^{\frac{1}{1-\eta}}$ is the price index and $\eta > 1$ is the goods demand elasticity. Taking into account that $Y_j^d = Y_j$, $X = Y$ and introducing the discrete time index t, the firm's inverse goods demand is derived from eq. (4.2) and given by

$$p_{jt}(Y_{jt}) = \left(\frac{Y_{jt}}{Y_t}\right)^{-\frac{1}{\eta}}. \tag{4.3}$$

4.2.2 Firms

Similar to Marczak et al. (2022), the representative firm j produces the final good Y_{jt} using a continuum of tasks $y_{jt}(i)$ indexed by $i \in [0, 1]$ given the following Cobb-Douglas production function

$$Y_{jt} = \exp\left[\int_0^1 \ln y_{jt}(i) \, \mathrm{d}i\right], \tag{4.4}$$

with the following production function for each task

$$y_{jt}(i) = A_{Lt} \, \alpha_L(i) \, l_{jt}(i) + A_{Ht} \, \alpha_H(i) \, h_{jt}(i). \tag{4.5}$$

The firm assigns $l_{jt}(i)$ low-skilled labor and $h_{jt}(i)$ high-skilled labor to task i in period t. In total the firm hires L_{jt} low-skilled workers and H_{jt} high-skilled workers:

$$L_{jt} = \int_0^1 l_{jt}(i) \, \mathrm{d}i \qquad \text{and} \qquad H_{jt} = \int_0^1 h_{jt}(i) \, \mathrm{d}i. \tag{4.6}$$

The task-related productivity of low-skilled and high-skilled labor is described by $\alpha_L(i)$ and $\alpha_H(i)$, respectively. Assuming that the relative task productivity schedule $\bar{\alpha}(i) \equiv \alpha_H(i)/\alpha_L(i)$ is continuously differentiable and strictly increasing specifies the structure of the comparative advantage of both skill groups across tasks. The assumption can be interpreted to mean that a higher task index i is associated with a higher task complexity in which high-skilled labor has a comparative advantage over low-skilled labor. The high-skilled and low-skilled augmenting technologies are denoted by A_{Ht} and A_{Lt}, respectively.

The labor market for low-skilled workers is characterized by search frictions and monopoly unions at the firm level, while the labor market for high-skilled workers is competitive.[5] The matching of workers and firms is described by the general matching function $M_{Lt} = M(V_{Lt}, U_{Lt})$, where V_{Lt} denotes the total number of vacant jobs for low-skilled workers in period t and U_{Lt} represents the total number of low-skilled unemployed persons in period t. The low-skilled labor market tightness in period t is denoted by $\theta_{Lt} \equiv V_{Lt}/U_{Lt}$ and is taken as given by the individual firm. Job vacancies are filled with rate $M_{Lt}/V_{Lt} \equiv m(\theta_{Lt})$ and $M_{Lt}/U_{Lt} \equiv \theta_{Lt} m(\theta_{Lt})$ denotes the job finding rate. The low-skilled employment flow in firm j is given by

$$L_{jt} = (1 - q_L)L_{j,t-1} + m(\theta_{Lt})V_{jLt}, \tag{4.7}$$

where L_{jt} denotes the number of workers employed in firm j in period t, V_{jLt} represents the vacant job postings of firm j in period t and low-skilled jobs are destroyed by a constant exogenous rate q_L. At the beginning of period t there are $L_{j,t-1}$ low-skilled workers employed in firm j. Thus, the first term on the right-hand side gives the number of workers who are still employed at the end of time period t and the second term gives the number of unemployed persons finding a job in firm j in period t. The inflow of unemployed persons into jobs and job destructions happen simultaneously. This implies that a worker who becomes unemployed cannot find a job again in the same period.

The representative firm j maximizes profits

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} [p_{jt}(Y_{jt})Y_{jt} - w_{jLt}L_{jt} - w_{jHt}H_{jt} - s_L V_{jLt}] \tag{4.8}$$

subject to eqs. (4.3) - (4.7) and the conditions

$$l_{jt}(i) \geq 0 \qquad \text{and} \qquad h_{jt}(i) \geq 0, \tag{4.9}$$

where the real interest rate is constant and denoted by r, the cost of posting a vacancy is s_L and $l_0(i)$ and $h_0(i)$ are given. The firm-specific labor union sets a uniform real wage w_{jLt} for all low-skilled workers in the firm. Due to perfect competition in the labor

[5] For tractability, Jaimovich et al. (2021) also assume search frictions in the low-skilled labor market and perfect competition in the high-skilled labor market.

market, all high-skilled workers in firm j obtain the same real wage w_{jHt}.

Since there is a mass one of identical firms in the economy, the firm index j is omitted in the following to simplify notation. The focus of the analysis is on the steady state for which it holds that $L_{t-1} = L_t = L$ and the time index is neglected. From the first-order conditions of the firm's maximization problem, see Appendix 4.A.2, it is possible to derive the condition for the task threshold $0 < I < 1$ at which the unit labor costs of low-skilled labor equal those of high-skilled labor:

$$\frac{\widetilde{w}_L}{A_L \alpha_L(I)} = \frac{w_H}{A_H \alpha_H(I)} \qquad \text{with} \quad \widetilde{w}_L \equiv w_L + \frac{q_L + r}{1+r} \frac{s_L}{m(\theta_L)}. \tag{4.10}$$

The labor costs for low-skilled workers are given by $\widetilde{w}_L$, which is the low-skilled real wage w_L plus the adjustment costs for labor. The firm employs only low-skilled labor for tasks $i < I$ and only high-skilled labor for tasks $i > I$. Eq. (4.10) implies

$$I = I\left(\widetilde{\omega}, \bar{A}\right) \qquad \text{with} \quad \frac{\partial I}{\partial \widetilde{\omega}} > 0 \quad \text{and} \quad \frac{\partial I}{\partial \bar{A}} < 0, \tag{4.11}$$

where

$$\widetilde{\omega} \equiv \frac{w_H}{\widetilde{w}_L} \qquad \text{and} \qquad \bar{A} \equiv \frac{A_H}{A_L}.$$

Eq. (4.11) states that with increasing $\widetilde{\omega}$, i.e. a decrease in $\widetilde{w}_L$ or an increase in w_H, the tasks are reallocated towards low-skilled labor, which implies an increase in the task threshold I. An increase in $\bar{A}$, i.e. an increase in high-skilled productivity A_H or a decrease in low-skilled productivity A_L, leads to a reallocation of tasks towards high-skilled labor, which implies a decrease in the task threshold I.

From the first-order conditions in Appendix 4.A.2 follows that the same labor input is used in all low-skilled and high-skilled tasks, respectively, i.e. $l(i) = l = L/I$ for $i < I$ and $h(i) = h = H/(1-I)$ for $i > I$. In contrast to perfect competition in the goods market, the modified low-skilled labor share is then given by $\frac{\widetilde{w}_L L}{Y} = \frac{\eta-1}{\eta} I$ and the high-skilled labor share is $\frac{w_H H}{Y} = \frac{\eta-1}{\eta}(1-I)$. Similarly to perfect competition, the relative factor input is $\frac{L}{H} = \widetilde{\omega}\frac{I}{1-I}$. Using the optimality conditions derived from the firm's maximization problem, the production function for the final good is given by the following Cobb-Douglas form:

$$
\begin{aligned}
Y &= B\left(A_L\frac{L}{I}\right)^I\left(A_H\frac{H}{1-I}\right)^{1-I}, \\
B &\equiv e^{\xi(I)}, \quad \xi(I)\equiv\int_0^I \ln\alpha_L(i)\mathrm{d}i+\int_I^1 \ln\alpha_H(i)\mathrm{d}i.
\end{aligned}
\tag{4.12}
$$

Moerover, it is possible to derive the elasticity of substitution between L and H, see Appendix 4.A.3, as given by

$$
\sigma\equiv\frac{\mathrm{d}(L/H)}{\mathrm{d}\widetilde{\omega}}\cdot\frac{\widetilde{\omega}}{L/H}=1+\frac{1}{(1-I)\varepsilon_{\bar{\alpha},I}(I)}>1, \tag{4.13}
$$

with

$$
\bar{\alpha}(I)\equiv\frac{\alpha_H(I)}{\alpha_L(I)} \quad \text{and} \quad \varepsilon_{\bar{\alpha},I}(I)\equiv\frac{\mathrm{d}\ln\bar{\alpha}(I)}{\mathrm{d}\ln I}>0. \tag{4.14}
$$

The elasticity of substitution is larger than one and depends negatively on the elasticity of the relative task productivity schedule $\varepsilon_{\bar{\alpha},I}$. But the effect of I on $\varepsilon_{\bar{\alpha},I}$ is ambiguous, $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I \lesseqgtr 0$, and depends on the shape of $\bar{\alpha}(I)$. In the case of $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I<0$, the relative task productivity schedule $\bar{\alpha}(I)$ is concave. This means that at lower thresholds of I, the disparity in productivity between low-skilled and high-skilled labor is greater compared to higher thresholds of I. At lower I, a change in the task threshold entails a pronounced change in the relative task productivity schedule $\bar{\alpha}(I)$, which makes it relatively hard to substitute between low-and high-skilled labor. If the task threshold is high, a change in this threshold leads to a comparatively small change in $\bar{\alpha}(I)$, making it relatively easy to substitute between the two types of labor. This results in an increasing degree of substitutability between low- and high-skilled labor with increasing I. In contrast, with a sufficiently large $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I$, the relative task productivity schedule $\bar{\alpha}(I)$ is convex, and the opposite result is obtained. That is, at lower I, it is easy to substitute between low-and high-skilled labor in comparison to higher I, which results in a decreasing degree of substitutability between low- and high-skilled labor with increasing I. The strength of the effect through $\varepsilon_{\bar{\alpha},I}$ depends on the task threshold I. If more tasks are allocated to low-skilled labor, i.e. I is high, the effect through $\varepsilon_{\bar{\alpha},I}$ is more pronounced.

4.2.3 Labor Unions and Wage Determination

All low-skilled workers are members of a firm-specific monopoly union. The labor union determines the low-skilled wage w_{Lt} in period t for n periods, considering the aggregate labor market tightness to be given and constant. The utility of a union member is given by the difference between the expected utility of being employed and the expected utility of being unemployed. The firm-specific monopoly union maximizes its utility

$$\max_{w_{Lt}} \left(\Psi_{EL,t} - \Psi_{UL,t}\right) L_t, \tag{4.15}$$

subject to the firm's low-skilled labor demand given in eq. (4.20). The expected value of a low-skilled worker being employed at the end of period t is given by

$$\Psi_{EL,t} = w_{Lt} + \frac{1}{1+r}\left[q_L\, \Psi_{UL,t+1} + (1-q_L)\, \Psi_{EL,t+1}\right]. \tag{4.16}$$

The wage rate w_{Lt} represents the instantaneous utility of being employed. $\Psi_{EL,t+1}$ and $\Psi_{UL,t+1}$ denotes the present discounted utility of an employed and an unemployed low-skilled worker at the end of period $t+1$, respectively. The second term reflects the discounted utility of either becoming unemployed or remaining employed in the next period, weighted by the corresponding probability.

The expected value of being unemployed at the end of period t is given by

$$\Psi_{UL,t} = z_{Lt} + \frac{1}{1+r}\left[p_{L,t+1}\, \Psi_{EL,t+1} + (1-p_{L,t+1})\, \Psi_{UL,t+1}\right], \tag{4.17}$$

where z_{Lt} denotes earnings-related unemployment benefits in period t which are financed by lump-sum taxes. For tractability, unemployment benefits are modeled as a fraction of the average wage level in the economy which is a common assumption in the literature; see, e.g., Beissinger and Büsse (2001), Pissarides (2000) and Yashiv (2004).[6] Using the average wage level implies that unemployment benefits are exogenous for the firm-specific labor union. The job finding rate in the next period $t+1$ is denoted by $p_{L,t+1} \equiv \theta_{L,t+1} m(\theta_{L,t+1})$.

[6] Beissinger and Egger (2004) show the consequences of relaxing this assumption and linking unemployment benefits to individuals' previous income.

Thus, the second term reflects the discounted utility of either becoming employed or remaining unemployed in the next period, weighted by the corresponding probability.

Running forward $\Psi_{EL,t} - \Psi_{UL,t}$ for n periods leads to

$$\Psi_{EL,t} - \Psi_{UL,t} = \left(\frac{1-\delta^n}{1-\delta}\right)(w_{Lt} - z_{Lt}) + \delta^n(\Psi_{EL,t+n} - \Psi_{UL,t+n}), \tag{4.18}$$

where $\delta \equiv (1 - q_L - p_L)/(1 + r) < 1$.

Restricting the steady state analysis to the case $n \to \infty$ and neglecting the time index for steady-state values, the wage w_L that maximizes the rent leads to the wage $\widetilde{w}_L$ that is relevant to the firm:

$$\widetilde{w}_L = \kappa_L \, \widetilde{z}_L, \qquad \text{with } \kappa_L \equiv \frac{\varepsilon_{L,\widetilde{w}_L}}{\varepsilon_{L,\widetilde{w}_L} - 1} \quad \text{and } \widetilde{z}_L \equiv z_L + \frac{q_L + r}{1+r}\,\frac{s_L}{m(\theta_L)}. \tag{4.19}$$

The wage markup is denoted by κ_L and the unemployment benefits plus labor adjustment costs are captured by $\widetilde{z}_L$. The wage markup increases with a decreasing wage elasticity of low-skilled labor demand, $\varepsilon_{L,\widetilde{w}_L}$. The following section discusses this labor demand elasticity in more detail and demonstrates that $\varepsilon_{L,\widetilde{w}_L} > 1$, implying that $\kappa_L > 1$.

4.2.4 Low- and High-Skilled Labor Demand of the Firm

With monopolistic competition in the goods market, it is possible to derive explicitly the firm's labor demand for low-skilled and high-skilled labor from the first order conditions of the firm's optimization problem, see Appendix 4.A.4:

$$L \equiv L^d = \frac{A_L^{(\eta-1)I} A_H^{(\eta-1)(1-I)} I \exp\left[(\eta-1)\xi(I)\right](\frac{\eta-1}{\eta})^\eta Y}{\widetilde{w}_L^{1+(\eta-1)I} w_H^{(\eta-1)(1-I)}}, \tag{4.20}$$

$$H \equiv H^d = \frac{A_L^{(\eta-1)I} A_H^{(\eta-1)(1-I)} (1-I) \exp\left[(\eta-1)\xi(I)\right](\frac{\eta-1}{\eta})^\eta Y}{\widetilde{w}_L^{(\eta-1)I} w_H^{\eta(1-I)+I}}, \tag{4.21}$$

where $\xi(I) \equiv \int_0^I \ln \alpha_L(i)\mathrm{d}i + \int_I^1 \ln \alpha_H(i)\mathrm{d}i$. The wage elasticity of the low-skilled labor demand in absolute values is

$$\begin{aligned}\varepsilon_{L,\widetilde{w}_L} \equiv \left|\frac{\partial \ln L^d(\cdot)}{\partial \ln \widetilde{w}_L}\right| = \left|1 + (\eta-1)I + \frac{\partial \ln I}{\partial \ln \widetilde{w}_L}\right| &= \underbrace{1 + (\eta-1)I}_{\substack{\text{direct}\\ \text{wage effect}}} + \underbrace{\frac{1}{\varepsilon_{\bar{\alpha},I}(I)}}_{\substack{\text{indirect}\\ \text{wage effect}}} > 1 \\ &= (1-I)\sigma + I\eta. \end{aligned} \tag{4.22}$$

The labor demand elasticity consists of a direct wage effect, which is greater than one, and an indirect wage effect through task reallocation, see the first line of eq. (4.22). Since the goods demand elasticity η is negatively related to the monopoly power of firms in the goods market, the second term of the direct wage effect captures the impact of the firm's monopoly power on the labor demand elasticity. An increase in η, implying a reduction in the firm's monopoly power, results in a higher direct wage effect and a higher $\varepsilon_{L,\widetilde{w}_L}$. The strength of the effect depends on the task threshold $I(\widetilde{\omega}, \bar{A})$. The more tasks are allocated towards low-skilled workers, the larger I, the greater is the impact of the firm's monopoly power and the larger $\varepsilon_{L,\widetilde{w}_L}$. The indirect effect through task reallocation indicates that with higher low-skilled labor costs $\widetilde{w}_L$, more tasks are reallocated towards high-skilled labor, resulting in a decrease in I. The size of this effect negatively depends on the elasticity of the relative task productivity schedule $\varepsilon_{\bar{\alpha},I}$. The higher $\varepsilon_{\bar{\alpha},I}$, meaning a stronger reaction of the relative task productivity schedule $\bar{\alpha}(I)$ due to a change in the task threshold I, the lower $\varepsilon_{L,\widetilde{w}_L}$.

Another interpretation of the labor demand elasticity becomes apparent if the notation for the elasticity of substitution, given in eq. (4.13), is considered. The second line of eq. (4.22) indicates that the labor demand elasticity is a weighted mean of the elasticity of substitution σ and the goods demand elasticity η. A higher degree of substitutability between low- and high-skilled labor, i.e. σ is large, or lower monopoly power of firms, i.e. η is large, implies a higher labor demand elasticity. The strength of both effects depends on the task threshold I. The larger I, the stronger (weaker) the effect of η (σ) on $\varepsilon_{L,\widetilde{w}_L}$.

The change in the labor demand elasticity of low-skilled labor due to a change in the task threshold I is ambiguous, because $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I \lesseqgtr 0$, and given by

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = \frac{1}{\varepsilon_{L,\widetilde{w}_L}}\frac{1}{\varepsilon_{\bar{\alpha},I}}\left((\eta-1)I\,\varepsilon_{\bar{\alpha},I} - \frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I}\right). \tag{4.23}$$

This leads to

Proposition 4.1. *An increase in the task threshold I leads to the following change in the labor demand elasticity of low-skilled labor:*

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\begin{cases} >0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I} < (\eta-1)I\,\varepsilon_{\bar{\alpha},I} \\ =0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I} = (\eta-1)I\,\varepsilon_{\bar{\alpha},I} \\ <0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I} > (\eta-1)I\,\varepsilon_{\bar{\alpha},I} \end{cases}$$

Proof. This follows immediately from eq. (4.23). □

From the definition of κ_L in eq. (4.19) follows that κ_L is negatively related to $\varepsilon_{L,\widetilde{w}_L}$. Thus, the lower the monopoly power of firms, i.e. the higher η, and the higher the degree of substitutability between L and H, i.e. the higher σ, the lower the wage-setting power of labor unions. The effect of the task threshold on the wage markup follows directly from Proposition 4.1 and is given by

$$\varepsilon_{\kappa_L,I} \equiv \frac{\mathrm{d}\ln\kappa_L}{\mathrm{d}\ln I} = -(\kappa_L-1)\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}, \qquad \operatorname{sgn}(\varepsilon_{\kappa_L,I}) = -\operatorname{sgn}\left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\right), \tag{4.24}$$

which indicates that changes in the task threshold impact the wage markup of labor unions through changes in the labor demand elasticity. In which way the task threshold affects the labor demand elasticity is essentially related to the shape of the relative task productivity schedule $\bar{\alpha}(I)$. As explained above, the effect of I on $\varepsilon_{\bar{\alpha},I}$ is ambiguous. In the case of $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I < 0$, resulting in a concave $\bar{\alpha}(I)$-function, the elasticity of substitution increases with increasing I. An increase in $\widetilde{w}_L$ leads to a reallocation of tasks towards high-skilled labor, i.e. a decrease in I. At lower I, this decrease in I leads to a comparably strong decrease in the relative task productivity schedule $\bar{\alpha}(I)$. Thus, only a small increase in $\widetilde{w}_L$ is necessary in order that the unit labor costs of both worker groups are equal again, resulting in a small decrease in low-skilled labor demand. At higher I, the decrease in I leads to a less pronounced decrease in the relative task productivity schedule. Thus, a strong increase in $\widetilde{w}_L$ is required to satisfy the unit labor cost condition, resulting in a more pronounced decrease in low-skilled labor demand. This is why an increase in the task threshold results in an increase in the elasticity of labor demand, if $\bar{\alpha}(I)$ is concave. In contrast, with a sufficiently large $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I$, the relative task productivity schedule $\bar{\alpha}(I)$ is convex, and the opposite result is obtained.

4.2.5 Steady State Equilibrium

Since there is a mass one of identical firms in the economy, the firm's price is equal to the aggregate price level and eq. (4.2) becomes $Y_j^d = Y^d = X$. In equilibrium aggregate real income X is equal to aggregate production Y, implying $Y^d = Y$. Additionally, the flows into and out of low-skilled employment in the steady state are equal, i.e. $L_{t-1} = L_t = L$. From eq. (4.7) follows $m(\theta_L)V_L = q_L L$. Considering a mass one of individuals $L + U_L + H = 1$ and $V_L = \theta_L \cdot U_L$ leads to:

$$q_L L = p_L(\theta_L)\,(1 - H - L), \tag{4.25}$$

where the left-hand side gives the inflow into unemployment and the right-hand side represents the outflow of unemployment. The steady state equilibrium is specified by

Definition 4.1. *The key endogenous variables $\{I, L, w_H, \widetilde{w}_L, \theta_L, Y\}$ in the steady state equilibrium of the task-based matching model are determined by the following equations:*

(i) the unit labor cost equation (4.10),

(ii) the labor demand equations (4.20) and (4.21),

(iii) the equation for low-skilled workers wage rate (4.19),

(iv) the flow equation given in (4.25),

(v) the production function given in (4.4).

4.3 The Effect of Unemployment Benefits on Labor Market Outcomes

4.3.1 General Results in the Task-Based Matching Model

The aim of this analysis is to examine the effects of a change in low-skilled unemployment benefits on labor market outcomes. As mentioned above, low-skilled unemployment benefits are modeled as a fraction of the average wage level. In more detail,

$$z_L = \gamma_L w_L, \qquad \text{with } 0 < \gamma_L < 1, \tag{4.26}$$

where γ_L represents the replacement rate for low-skilled workers which is the ratio of unemployment benefits to low-skilled wages. In the following analysis, a change in the low-skilled replacement rate γ_L is analyzed.

For simplification, the matching frictions are described by a Cobb-Douglas matching function:

$$M_L = M(V_L, U_L) = \zeta\, V_L^{(1-\beta_L)}\, U_L^{\beta_L}, \tag{4.27}$$

where ζ describes an efficiency parameter of the matching process. This specific matching function implies that the job filling rate becomes $m(\theta_L) = \zeta\theta_L^{-\beta_L}$, where $0 < \beta_L < 1$ represents the constant elasticity of the job filling rate with respect to θ_L in absolute values. The job finding rate becomes $p_L = \zeta\theta_L^{1-\beta_L}$, where $1 - \beta_L$ is the constant elasticity of the job finding rate with respect to θ_L.

In a first step, the model equations are written in log differences in order to condense the model into a three-equations system for θ_L, $\widetilde{w}_L$ and I, keeping r, q_L, s_L, ζ, A_H and A_L constant. Due to perfect competition in the labor market for high-skilled workers, high-skilled employment also remains constant. The model is summarized in

Proposition 4.2. *Denote the low-skilled unemployment rate by u_L , the elasticity of $\widetilde{z}_L$ with respect to $\widetilde{w}_L$ by $\varepsilon_{\widetilde{z}_L,\widetilde{w}_L}$, where $\widetilde{z}_L$ is defined in eq. (4.19) and $0 < \varepsilon_{\widetilde{z}_L,\widetilde{w}_L} < 1$. $\varepsilon_{\widetilde{z}_L,\gamma_L}$ is the elasticity of $\widetilde{z}_L$ with respect to γ_L, where $0 < \varepsilon_{\widetilde{z}_L,\gamma_L} < 1$, and $\varepsilon_{\widetilde{z}_L,\theta_L}$ represents the elasticity of $\widetilde{z}_L$ with respect to θ_L, where $0 < \varepsilon_{\widetilde{z}_L,\theta_L} < \beta_L$. The elasticity of the wage* *markup κ_L with respect to the task threshold I, as defined in eq. (4.24), is given by $\varepsilon_{\kappa_L,I}$. Then*

$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)\,u_L}\left[-\frac{1}{1-I}\mathrm{d}\ln\widetilde{w}_L + \frac{1}{1-I}\mathrm{d}\ln I\right] \tag{4.28}$$

$$\mathrm{d}\ln\widetilde{w}_L = \frac{1}{1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L}}\left[\varepsilon_{\widetilde{z}_L,\gamma_L}\,\mathrm{d}\ln\gamma_L + \varepsilon_{\widetilde{z}_L,\theta_L}\mathrm{d}\ln\theta_L + \varepsilon_{\kappa_L,I}\,\mathrm{d}\ln I\right] \tag{4.29}$$

$$\mathrm{d}\ln I = -(\sigma-1)\,\mathrm{d}\ln\widetilde{w}_L. \tag{4.30}$$

Proof. See Appendix 4.A.5. □

The job creation condition in log differences is represented in eq. (4.28) and shows the inverse relationship between $\widetilde{w}_L$ and θ_L as in the standard matching model. An increase in low-skilled labor costs reduces the profit prospects of a filled low-skilled vacancy. Therefore, firms post fewer vacancies, leading to decreasing labor market tightness for low-skilled workers. Additionally, θ_L now also depends positively on the task threshold I. A reallocation of tasks towards low-skilled workers, i.e. an increase in I, leads *cet. par.* to more vacant jobs for low-skilled workers, thus increasing low-skilled labor market tightness. Eq. (4.29) is the wage equation for low-skilled workers in log differences. Similar to the standard matching model, the wage equation represents a positive relationship between θ_L and $\widetilde{w}_L$ as well as a positive relationship between γ_L and $\widetilde{w}_L$. With increasing θ_L, it becomes easier for low-skilled workers to find a job, and as a consequence, the threat of unemployment decreases and labor unions claim higher wages. Similarly, more generous unemployment benefits increase the outside option of labor unions and raise their wage claims. Unlike the standard matching model, $\widetilde{w}_L$ now depends on the change in the task threshold I. However, the sign of $\varepsilon_{\kappa_L,I}$ is ambiguous, i.e. $\varepsilon_{\kappa_L,I} \lesseqgtr 0$. Eq. (4.30) can be interpreted as the "task allocation" equation in log differences. As the cost of low-skilled labor $\widetilde{w}_L$ increases, tasks are reallocated from low-skilled to high-skilled labor, implying a negative relationship between $\widetilde{w}_L$ and I.

Inserting the task allocation equation in the job creation condition and wage equation and taking the definition of $\varepsilon_{\kappa_L,I}$ given in eq. (4.24) into account leads to:

$$\mathrm{d}\ln\theta_L = -\frac{\sigma}{(1-I)(1-\beta_L)u_L}\mathrm{d}\ln\widetilde{w}_L, \tag{4.31}$$

$$\mathrm{d}\ln\widetilde{w}_L = \frac{1}{1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L}-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\frac{\sigma-1}{\varepsilon_{L,\widetilde{w}_L}-1}}\Big[\varepsilon_{\widetilde{z}_L,\gamma_L}\,\mathrm{d}\ln\gamma_L+\varepsilon_{\widetilde{z}_L,\theta_L}\mathrm{d}\ln\theta_L\Big]. \tag{4.32}$$

The first equation corresponds to the job creation curve (JC) and the second equation represents the wage curve (WC) in a $\widetilde{w}_L-\theta_L$ space with the following slopes:

$$\Phi \equiv \left.\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\theta_L}\right|_{\mathrm{JC}} = -\frac{(1-I)(1-\beta_L)u_L}{\sigma} < 0, \tag{4.33}$$

$$\Gamma \equiv \left.\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\theta_L}\right|_{\mathrm{WC}} = \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L}-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\frac{\sigma-1}{\varepsilon_{L,\widetilde{w}_L}-1}} \gtrless 0. \tag{4.34}$$

The JC is downward-sloping, while the slope of the WC is ambiguous because it depends on whether $\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} \gtrless (1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)/(\sigma-1)$. The comparative static results are ambiguous and depend on the slopes of the JC and WC, see

Corollary 4.2.1. *Changes in the replacement rate for low-skilled workers have the following effects on the labor market equilibrium:*

(i) Low-skilled labor costs:

$$\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\gamma_L} = \begin{cases} >0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \\ & \vee \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \;\wedge\; |\Phi|<|\Gamma|\right) \\ <0, & \text{otherwise.} \end{cases}$$

(ii) Low-skilled labor market tightness:

$$\frac{\mathrm{d}\ln\theta_L}{\mathrm{d}\ln\gamma_L} = \begin{cases} <0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \\ & \vee \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \;\wedge\; |\Phi|<|\Gamma|\right) \\ >0, & \text{otherwise.} \end{cases}$$

(iii) Task threshold:

$$\frac{\mathrm{d}\ln I}{\mathrm{d}\ln\gamma_L} = \begin{cases} < 0, & \textit{if} \;\; \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < \frac{(1-\varepsilon_{\tilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \\ & \vee \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > \frac{(1-\varepsilon_{\tilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \;\; \wedge \;\; |\Phi| < |\Gamma| \right) \\ > 0, & \textit{otherwise.} \end{cases}$$

(iv) Low-skilled unemployment rate:

$$\frac{\mathrm{d}\ln u_L}{\mathrm{d}\ln\gamma_L} = \begin{cases} > 0, & \textit{if} \;\; \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < \frac{(1-\varepsilon_{\tilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \\ & \vee \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > \frac{(1-\varepsilon_{\tilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \;\; \wedge \;\; |\Phi| < |\Gamma| \right) \\ < 0, & \textit{otherwise.} \end{cases}$$

Proof. Solving eqs. (4.31) and (4.32) leads to:

$$\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\gamma_L} = \frac{\varepsilon_{\tilde{z}_L,\gamma_L}}{\varepsilon_{\tilde{z}_L,\theta_L}} \, |\Phi| \, \frac{\Gamma}{|\Phi| + \Gamma},$$
$$\frac{\mathrm{d}\ln\theta_L}{\mathrm{d}\ln\gamma_L} = -\frac{1}{|\Phi|} \, \frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\gamma_L} = -\frac{\varepsilon_{\tilde{z}_L,\gamma_L}}{\varepsilon_{\tilde{z}_L,\theta_L}} \, \frac{\Gamma}{|\Phi| + \Gamma}.$$

Considering eq. (4.30) leads to:

$$\frac{\mathrm{d}\ln I}{\mathrm{d}\ln\gamma_L} = -(\sigma-1) \, \frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\gamma_L} = -(\sigma-1) \, \frac{\varepsilon_{\tilde{z}_L,\gamma_L}}{\varepsilon_{\tilde{z}_L,\theta_L}} \, |\Phi| \, \frac{\Gamma}{|\Phi| + \Gamma}.$$

Rearrange eq. (4.25) and write the equation in log differences, keeping q_L and H constant, gives

$$\frac{\mathrm{d}\ln u_L}{\mathrm{d}\ln\gamma_L} = -(1-\beta_L)(1-u_L)\frac{\mathrm{d}\ln\theta_L}{\mathrm{d}\ln\gamma_L} = (1-\beta_L)(1-u_L)\frac{\varepsilon_{\tilde{z}_L,\gamma_L}}{\varepsilon_{\tilde{z}_L,\theta_L}} \, \frac{\Gamma}{|\Phi| + \Gamma}.$$

□

Using the findings in Corollary 4.2.1, it is possible to derive the effects of a cut in low-skilled unemployment benefits on final output, low- and high-skilled wages and the skill premium, see Appendix 4.A.6.

Without specifying $\bar{\alpha}(I)$ in more detail, it would be possible that a cut in the low-skilled replacement rate leads to an increasing unemployment rate for low-skilled workers. However, to obtain this result requires unrealistic parameter values within the model framework. Moreover, the majority of the existing literature identifies a positive relation between unemployment benefits and unemployment; see, e.g., Pries and Rogerson (2005), Yashiv (2004) and Zanetti (2011) or, in case of specific analysis of the Hartz reforms, Hochmuth et al. (2021) and Krause and Uhlig (2012). Thus, the following analysis focuses on the relevant case for which a cut in the low-skilled replacement rate results in a decreasing unemployment rate for low-skilled workers, i.e. $\frac{\mathrm{d}\ln\varepsilon_{L,\tilde{w}_L}}{\mathrm{d}\ln I} < \frac{(1-\varepsilon_{\tilde{z}_L,\tilde{w}_L})(\varepsilon_{L,\tilde{w}_L}-1)}{\sigma-1}$ or $\frac{\mathrm{d}\ln\varepsilon_{L,\tilde{w}_L}}{\mathrm{d}\ln I} > \frac{(1-\varepsilon_{\tilde{z}_L,\tilde{w}_L})(\varepsilon_{L,\tilde{w}_L}-1)}{\sigma-1} \;\wedge\; |\Phi| < |\Gamma|$. To keep things as simple as possible, in the following, a specific function of $\bar{\alpha}(I)$ is derived, resulting in a constant labor demand elasticity of low-skilled labor, e.g. $\frac{\mathrm{d}\ln\varepsilon_{L,\tilde{w}_L}}{\mathrm{d}\ln I} = 0$.

4.3.2 The Task-Based Matching Model with Constant Low-Skilled Labor Demand Elasticity

From Proposition 4.1 follows, that the labor demand elasticity of low-skilled labor is only constant if $\frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I} = (\eta-1)I\,\varepsilon_{\bar{\alpha},I}$. The functional form of $\bar{\alpha}(I)$ for this result is given in

Lemma 4.1. *The labor demand elasticity of low-skilled labor is only constant if $\bar{\alpha}(I)$ has the following functional form:*

$$\bar{\alpha}(I) = b\left(\frac{I}{\frac{1}{\rho}-(\eta-1)I}\right)^{\rho} \quad \textit{with} \quad b>0 \quad \textit{and} \quad 0<\rho<\frac{1}{(\eta-1)I}, \tag{4.35}$$

leading to $\varepsilon_{\bar{\alpha},I} = \left[\frac{1}{\rho}-(\eta-1)I\right]^{-1} > 0$ *and* $\varepsilon_{L,\tilde{w}_L} = 1+\frac{1}{\rho} > 1$.

Proof. See Appendix 4.A.7. □

This functional form for $\bar{\alpha}(I)$, i.e. $\mathrm{d}\ln\varepsilon_{L,\tilde{w}_L}/\mathrm{d}\ln I = 0$ and $\varepsilon_{\kappa_L,I} = 0$, implies that the wage markup no more depends on the task threshold and thus changes in I do not directly influence labor union wage setting. In this case, a reduction in low-skilled unemployment benefits leads to the following labor market outcomes summarized by

Corollary 4.2.2. *In the task-based matching model with constant low-skilled labor demand elasticity, changes in the replacement rate for low-skilled workers have the following effects on the labor market equilibrium:*

(i) Low-skilled labor costs, low-skilled labor market tightness, task threshold and low-skilled unemployment rate:

$$\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\gamma_L} > 0, \quad \frac{\mathrm{d}\ln\theta_L}{\mathrm{d}\ln\gamma_L} < 0, \quad \frac{\mathrm{d}\ln I}{\mathrm{d}\ln\gamma_L} < 0, \quad \frac{\mathrm{d}\ln u_L}{\mathrm{d}\ln\gamma_L} > 0,$$

(ii) Final output and wage outcomes:

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}\ln\gamma_L} < 0, \quad \frac{\mathrm{d}\ln w_L}{\mathrm{d}\ln\gamma_L} > 0, \quad \frac{\mathrm{d}\ln w_H}{\mathrm{d}\ln\gamma_L} < 0, \quad \frac{\mathrm{d}\ln\omega}{\mathrm{d}\ln\gamma_L} < 0.$$

Proof. Using the findings of Lemma 4.1 in Corollary 4.2.1 and in the general results derived in Appendix 4.A.6 leads to the findings in Corollary 4.2.2. □

Figure 4.2 illustrates the effect of a decline in the replacement rate for low-skilled workers on $\widetilde{w}_L$ and θ_L within the framework of the task-based matching model with constant labor demand elasticity. The JC and WC are the graphical representation of the relations given in eq. (4.31) and eq. (4.32), and thus take already into account the endogenous task allocation. For a formal analysis of the slopes and curvatures of the JC and WC, see Appendix 4.A.8. In the case of a constant labor demand elasticity, i.e. $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I = 0$, the WC is upward-sloping, see eq. (4.34). Due to a cut in low-skilled unemployment benefits, the outside option of labor unions decreases, which leads to a reduction in their wage claims and a downward shift of the WC. The decrease in low-skilled labor costs induces an increase in the expected profits of the firms. This leads to higher labor demand for low-skilled workers, coming along with a rise in low-skilled labor market tightness and a reduction in low-skilled unemployment rate. Compared to the standard matching model, the reduction in low-skilled labor costs additionally increases the incentives of firms to reallocate tasks towards low-skilled workers, resulting in an increase in the task threshold that leads to a further creation of vacancies and an increase in low-skilled labor market tightness. This increase in the task threshold is already taken into account in the JC and thus the change in the task threshold does not lead to a change in the position of the JC.

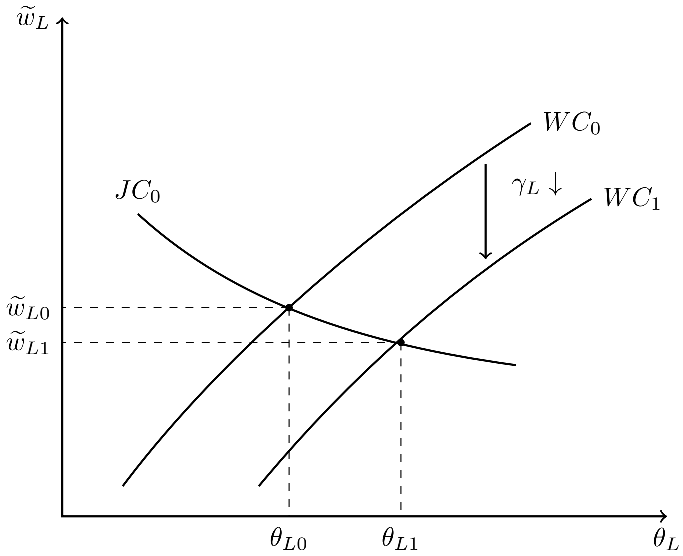

Figure 4.2: Graphical analysis of a reduction in the replacement rate for low-skilled workers

Notes: The figure illustrates the effect of a reduction in γ_L on $\widetilde{w}$ and θ_L in the task-based matching model with constant labor demand elasticity. For the formal analysis of the slopes and curvatures of the JC and WC, see Appendix 4.A.8.

Due to perfect competition in the high-skilled labor market, there is no reduction in total employment of high-skilled workers caused by the reallocation of tasks towards low-skilled workers, since H is assumed to be constant. Therefore, the rise in the task threshold increases labor input in each high-skilled task $h(i) = h = H/(1 - I)$ for $i > I$. The increase in low-skilled employment with a simultaneous constant employment level of high-skilled workers (and no skill-biased technical change) leads to higher output, as shown in eq. (4.A.29). At first glance, it may seem counterintuitive that high-skilled wages increase despite the reallocation of tasks towards low-skilled labor, which implies lower high-skilled labor demand and lower wages for high-skilled workers. However, this reallocation effect is dominated by the scale effect. Due to an increase in output, demand for high-skilled workers increases resulting in higher wages for high-skilled workers. In summary, a reduction in the replacement rate for low-skilled workers leads to a decrease in low-skilled wages and an increase in high-skilled wages, resulting in a higher skill premium.

4.3.3 The Task-Based Matching Model with Exogenous and Constant Task Allocation

In order to assess the importance of the task reallocation effect, the task-based matching model with exogenous and constant task allocation is considered, which reflects the literature with a conventional production function. This implies that the task threshold I is no longer endogenously determined from profit maximization, meaning that eq. (4.10) no longer has to be taken into account, and there is no reallocation of tasks, i.e. $\mathrm{d}\ln I = 0$. In this case, the low- and high-skilled labor shares, $\frac{\widetilde{w}_L L}{Y} = \frac{\eta-1}{\eta} I$ and $\frac{w_H H}{Y} = \frac{\eta-1}{\eta}(1-I)$, consist, besides η, of the exogenous and constant task threshold I. From eq. (4.12), it becomes clear that in the case of an exogenous I, the exponents of the production function are exogenous and constant and the elasticity of substitution is equal to one, see eq. (4.13), as in a conventional Cobb-Douglas production function.

In the task-based matching model with exogenous and constant task allocation, the labor demand elasticity of low-skilled labor (in absolute values) is given by:

$$\varepsilon_{L,\widetilde{w}_L} \equiv \left| \frac{\partial \ln L^d(\cdot)}{\partial \ln \widetilde{w}_L} \right| = 1 + (\eta - 1)I \quad > 1. \tag{4.36}$$

In this case, the labor demand elasticity only depends on the direct wage effect. Therefore, it is lower than in case of endogenous task allocation, see eq. (4.22). As in the case of endogenous task allocation, low-skilled labor demand elasticity is high if η is high, implying low monopoly power of firms in the goods market, and if the range of tasks assigned to low-skilled labor is high, i.e. $I(\widetilde{\omega}, \bar{A})$ is high.

The comparative-static results of a change in the replacement rate for low-skilled workers in the task-based matching model with exogenous and constant task allocation are derived in a similar way as in the general model, but eq. (4.10) no longer has to be taken into account and $\mathrm{d}\ln I = 0$. Then the results are summarized by

Corollary 4.2.3. *In the task-based matching model with exogenous and constant task allocation, changes in the replacement rate for low-skilled workers have the following effects on the labor market equilibrium:*

(i) Low-skilled labor costs, low-skilled labor market tightness and low-skilled unemployment rate:

$$\frac{\mathrm{d}\ln \widetilde{w}_L}{\mathrm{d}\ln \gamma_L} > 0, \quad \frac{\mathrm{d}\ln \theta_L}{\mathrm{d}\ln \gamma_L} < 0, \quad \frac{\mathrm{d}\ln u_L}{\mathrm{d}\ln \gamma_L} > 0,$$

(ii) Final output and wage outcomes:

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}\ln \gamma_L} < 0, \quad \frac{\mathrm{d}\ln w_L}{\mathrm{d}\ln \gamma_L} > 0, \quad \frac{\mathrm{d}\ln w_H}{\mathrm{d}\ln \gamma_L} < 0, \quad \frac{\mathrm{d}\ln \omega}{\mathrm{d}\ln \gamma_L} < 0.$$

Proof. See Appendix 4.A.5 and consider that eq. (4.10) no longer holds and $\mathrm{d}\ln I = 0$. This leads to the following job creation condition and wage equation:

$$\begin{aligned}
\mathrm{d}\ln \theta_L &= -\frac{1}{(1-\beta_L)\, u_L\, (1-I)}\, \mathrm{d}\ln \widetilde{w}_L, \\
\mathrm{d}\ln \widetilde{w}_L &= \frac{1}{1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L}} \Big[\varepsilon_{\widetilde{z}_L,\gamma_L}\, \mathrm{d}\ln \gamma_L + \varepsilon_{\widetilde{z}_L,\theta_L} \mathrm{d}\ln \theta_L\Big].
\end{aligned}$$

Combining those two equations gives

$$\begin{aligned}
\frac{\mathrm{d}\ln \widetilde{w}_L}{\mathrm{d}\ln \gamma_L} &= \frac{\varepsilon_{\widetilde{z}_L,\gamma_L}}{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L}) + \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{(1-I)(1-\beta_L)u_L}}, \\
\frac{\mathrm{d}\ln \theta_L}{\mathrm{d}\ln \gamma_L} &= -\frac{\varepsilon_{\widetilde{z}_L,\gamma_L}}{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(1-I)(1-\beta_L)u_L + \varepsilon_{\widetilde{z}_L,\theta_L}}.
\end{aligned}$$

Insert the results for $\frac{\mathrm{d}\ln \widetilde{w}_L}{\mathrm{d}\ln \gamma_L}$ and $\frac{\mathrm{d}\ln \theta_L}{\mathrm{d}\ln \gamma_L}$ in eqs. (4.A.32)–(4.A.35) in Appendix 4.A.6 leads to the findings in Corollary 4.2.3. □

Regarding the sign of the effects, the comparative static results in the task-based matching model with exogenous and constant task allocation are the same as those in the task-based matching model with constant labor demand elasticity. However, the model with exogenous and constant task allocation does not allow to capture reallocation effects arising from changes in unit labor costs. The quantitative comparison of the comparative static results in both model variants is discussed in the next section, where the models are calibrated to the data.

4.4 Quantitative Analysis

4.4.1 Parameter Selection

In order to quantify the labor market effects of the Hartz IV reform and to illustrate the importance of task reallocation, both model variants are calibrated to German data before 2005. This requires finding long-run values of different macroeconomic variables. To balance out business cycle fluctuations, the long-run values are calculated as the average over 1995 to 2002.[7] The functional form of the relative task productivity schedule in both model variants is given by $\bar{\alpha}(I) = bI^{\rho}\left(\frac{1}{\rho} - (\eta - 1)I\right)^{-\rho}$, with $0 < \rho < \frac{1}{(\eta-1)I}$ and $b = 1$, see Lemma 4.1. The model variants are characterized by 11 exogenous parameters: $\{\beta_L, q_L, \zeta, s_L, \gamma_L, \eta, r, H, A_H, A_L, \rho\}$. Eight parameters are taken from the data or the literature and three parameters are calibrated to match the long-run values in Germany before the reform, see Tables 4.1 and 4.2. The time period refers to one quarter and thus the parameters are interpreted quarterly. Following Battisti et al. (2018), Chassamboulli and Palivos (2014) and Krusell et al. (2000), high-skilled workers are defined as workers with at least a Bachelor degree. This corresponds to levels 5-8 of the International Standard Classification of Education (ISCED 2011). The model distinguishes between only two skill groups. Therefore, low-skilled workers correspond to low- plus medium-skilled workers (levels 0-4 of ISCED 2011) in the following analysis.[8]

Low-skilled labor market tightness θ_L is set as a target, which guarantees the same initial equilibrium value in both model variants, but low-skilled labor costs $\widetilde{w}_L$ are implicitly determined by the calibrated values. To ensure the same initial values in both model variants, the task-based matching model with constant labor demand elasticity (Model I)

[7]The years 2003 and 2004 are not included because the Hartz I-III reforms were already implemented during those years.

[8]This implies that the impact of Hartz IV is analyzed on low- plus medium-skilled workers. This restriction does not considerably influence the results, because Hartz IV mostly affect long-term unemployed and the share of high-skilled long-term unemployed in the total number of long-term unemployed amounts to less than 5% in 2017, see data provided by the German Federal Employment Agency.

Table 4.1: Parameter values and targets

Parameter	**Value**	**Source**
Matching elasticity: β_L	0.5	Petrongolo and Pissarides (2001)
Matching efficiency parameter: ζ	0.426	Pins down the quarterly job finding rate to 19.5% as estimated by Hobijn and Şahin (2009)
Separation rate: q_L	0.0873	Battisti et al. (2018)
Goods demand elasticity: η	5.4	Own calculations based on data from De Loecker and Eeckhout (2018)
Interest rate: r	0.013	Deutsche Bundesbank, FRED
Share of high-skilled: H	0.236	EU-LFS
Skill bias: $\frac{A_H}{A_L}$	1.104	Own calculations based on data from EU-LFS, EU-SILC, Federal Statistical Office
Replacement rate: γ_L	0.61	Own calculations based on output from the OECD tax-benefit model (Model version 2.5.0)
Target		
Low-skilled unemployment rate: u_L	0.098	EU-LFS
Low-skilled labor market tightness: θ_L	0.21	IAB-JVS, EU-LFS, Federal Employment Agency
Skill premium: $\frac{w_H}{w_L}$	1.31	Federal Statistical Office, EU-SILC
Task threshold: I	0.663	WIOD SEA Release 2013

Notes: All parameters and targets are interpreted quarterly.

is calibrated first to set the benchmark value of $\widetilde{w}_L$. Next, the value of $\widetilde{w}_L$ is used in the calibration of the task-based matching model with exogenous and constant task allocation (Model II). This procedure guarantees the same initial equilibrium values of $\widetilde{w}_L$ and θ_L.

The matching elasticity β_L is set to 0.5, which is in line with the estimates reported in Petrongolo and Pissarides (2001). The matching efficiency parameter ζ is set to 0.426, which pins down the quarterly job finding rate to 19.5% as estimated by Hobijn and Şahin (2009). The monthly separation rate q_L of 0.03 is taken from Battisti et al. (2018) and corresponds to a quarterly separation rate of 0.0873. The goods demand elasticity is calculated from data on price markups in Germany estimated by De Loecker and Eeckhout (2018). The average price markup in Germany between 1995 and 2002 is 1.23, which means that prices are on average 23% higher than marginal costs. This relates to a goods demand elasticity η of 5.4. This is in line with Delacroix (2006), who set the goods demand elasticity for the European market to 5, implying a price markup of 25%. The remaining parameter values and targets, except for the replacement rate, are calculated from the data in a similar way to Marczak et al. (2022), but the averages of the variables are calculated from 1995 to 2002.

Table 4.2: Calibrated parameters

Parameter	**Values**	
	Model I	Model II
Search cost: s_L	1.53	1.72
Low-skill biased technology: A_L	1.0	1.07
Parameter of function $\bar{\alpha}(I)$: ρ	0.28	0.33

Notes: Own calibration to match the targets in Table 4.1. Model I corresponds to the task-based matching model with constant labor demand elasticity and endogenous task allocation and Model II represents the task-based matching model with exogenous and constant task allocation. The value of $\widetilde{w}_L$ in Model I is used within the calibration of Model II to ensure the same initial equilibrium value.

One of the key parameter values is the net replacement rate, which measures the generosity of unemployment benefits as the proportion of income that is obtained after a certain number of months of unemployment. Although the measure is often used in the literature, the net replacement rate is calculated very differently; see, e.g., Battisti et al. (2018), Hartung et al. (2022) and Krebs and Scheffel (2013). Battisti et al. (2018) estimate the average net replacement rate for two earnings levels and three household types averaged over 60 months of unemployment. Hartung et al. (2022) use the net replacement rate of single households without children with previous in-work earnings equivalent to the average wage. They calculate the replacement rate during the first year of unemployment and the average replacement rate for the following four years of unemployment. Krebs and Scheffel (2013) take into account single households without children and with two children, weighted by the corresponding population share, and assume median earnings before job loss.

Without doubt, the Hartz IV reform had substantially different effects on the net replacement rate for different groups of unemployed persons: households without children or long-term unemployed are more affected by the reform than households with children or short-term unemployed; see, e.g., Hartung et al. (2022) and Krebs and Scheffel (2013). Thus, it is crucial to consider different household situations and unemployment durations when calculating the effect of the Hartz IV reform on the average net replacement rate.

In contrast to the literature, the average net replacement rate is calculated in a more detailed way. This is possible by using the output from the OECD tax-benefit model (TaxBEN), which offers a wider range of settings in contrast to the frequently used OECD Benefits, Taxes and Wages Dataset. For example, in the OECD Benefits, Taxes and Wages Dataset it is possible only to distinguish between households without children and with two children. But the net replacement rate of households with one child is more affected by the reform than the net replacement rate of households with two children. Thus, taking only households with two children into account would understate the effect of Hartz IV on the net replacement rate, see Krebs and Scheffel (2013). Using TaxBen offers the opportunity to obtain net replacement rates of households with only one child or more than two children. Moreover, specific previous in-work earnings as a percentage of the average wage can be chosen instead of three pre-defined values as in OECD Benefits, Taxes and Wages Dataset.

To capture the heterogenous effects of the Hartz IV reform on the net replacement rate, a range of different family situations and unemployment durations are considered to compute the net replacement rates in the years 2004 and 2005. In this model set-up, the net replacement rate for low-skilled persons is required. Therefore, using data from the Federal Statistical Office in Germany, the earnings level of low-skilled workers in 2004 and 2005 is estimated to be 74% of the average wage. As a first step, the net replacement rates for a 40-year-old person with previous earnings equivalent to 74% of the average wage are calculated for unemployment durations ranging from 1 to 60 months, taking into account six family situations.[9] Next, the net replacement rates are averaged over the unemploy-

[9] In detail, the following family situations are included: single person without children, with one child and with two children, couple without children, with one child and with two children where the partner is out of work. Social assistance benefits are included because in the course of Hartz IV, unemployment assistance and social assistance were merged. Housing benefits are excluded, which is in line with Battisti et al. (2018). It has to be noted that housing and heating allowances ("Leistungen für Unterkunft und Heizung"), which are a supplement to unemployment assistance, are included in housing benefits and are thus not taken into account.

ment durations by taking the share of low-skilled short- and long-term unemployed in 2004 into account.[10] To obtain the weighted average net replacement rates for singles (couples) with children, the net replacement rates are multiplied by the share of single households (couples) with one and with two children in 2020, respectively. Finally, the net replacement rates of singles with and without children and couples with and without children are weighted by the corresponding share of the household type in 2004.[11] This leads to a weighted average low-skilled net replacement rate γ_L of 0.61 in the year 2004.

4.4.2 The Long-Run Effects of the Hartz IV Reform

To assess the effect of the implementation of Hartz IV in January 2005, the change in the average low-skilled net replacement rate between 2004 and 2005 is calculated to be -6.5%. The effect of this decrease in the net replacement rate on labor market outcomes is presented in Table 4.3. The calibrated changes in the variables can be interpreted as long-run effects of the reform because the comparative static results give the difference in the steady-state variables before and after the reform.

In the benchmark model with endogenous task allocation (Model I), the 6.5% drop in the net replacement rate leads to a modest decrease in low-skilled labor costs of 0.5%, which induces that slightly more tasks are reallocated from high-skilled to low-skilled labor, resulting in an increase in the task threshold by 1%. The substantial increase in low-skilled labor market tightness by over 90% results in a remarkable reduction in the low-skilled unemployment rate by around 41%. This decrease leads to a noncyclical low-skilled unemployment rate of 5.8% after the Hartz IV reform, which is a reduction by 4 percentage points from its pre-reform value. The decline is somewhat more pronounced compared to the findings in the existing literature, where the quantitative effects of the Hartz IV reform on the decline in the noncyclical unemployment rate range from less than

[10] Long-term unemployed persons have been unemployed for more than 12 months. The data is obtained from the Federal Employment Agency. Due to the fact that data on long-term unemployed by education is only available since 2017, the total number of (long-term) unemployed in 2004 is multiplied by the share of low-skilled (long-term) unemployed to total (long-term) unemployed in 2017.

[11] The share of children for singles and couples as well as the share of household types with and without children are provided by the Federal Statistical Office of Germany.

Table 4.3: Long-run effects of a decrease in the average low-skilled net replacement rate γ_L by 6.5%

Variable	**Pre-reform**		**Post-reform**		**Difference in %**	
	Model I	Model II	Model I	Model II	Model I	Model II
$\widetilde{w}_L$	0.904		0.900	0.893	-0.5%	-1.3%
I	0.663		0.670	0.663	1.0%	0.0%
θ_L	0.21		0.402	0.372	91.6%	77.3%
u_L	9.8%		5.8%	6.4%	-41.3%	-34.8%
p_L	19.5%		28.5%	27.1%	45.8%	38.6%
w_H/w_L	1.31		1.46	1.49	11.6%	14.0%
w_L	0.741	0.721	0.662	0.638	-10.7%	-11.4%
w_H	0.971	0.944	0.981	0.968	1.0%	2.5%
Y	1.046	1.036	1.078	1.062	3.0%	2.5%

Notes: The decrease in γ_L corresponds to the change between the average low-skilled net replacement rate due to the Hartz IV reform in 2005. Model I corresponds to the task-based matching model with constant labor demand elasticity and endogenous task allocation and Model II represents the task-based matching model with exogenous and constant task allocation.

0.1 percentage points to 3.2 percentage points, see, Hartung et al. (2022), Hochmuth et al. (2021), Krause and Uhlig (2012), Krebs and Scheffel (2013) and Launov and Wälde (2013). However, it should be noted that those effects correspond to the reduction in the aggregate unemployment rate. Looking at the data, the low-skilled unemployment rate drops from 13.1% in 2005 to 8.9% in 2008, which is a reduction of 4.2 percentage points, while the aggregate unemployment rate is only reduced by around 3.6 percentage points in the same period, see Figure 4.1.

Another reason for the more moderate effects of Hartz IV in the literature could be the fact that none of the models include reallocation effects between production factors due to differences in unit labor costs. If task allocation is exogenous and constant (Model II), the reduction in low-skilled labor costs is more pronounced, while the increase in low-skilled labor market tightness and, thus, the reduction in the low-skilled unemployment rate is smaller. This leads to a reduction in the long-run value of the low-skilled unemployment rate by only 3.4 percentage points. Hence, ignoring the incentives of firms to reallocate tasks towards low-skilled labor would underestimate the effect of the Hartz IV reform on low-skilled unemployment and overestimate the effect on low-skilled labor costs.

Figure 4.3 depicts the difference between the two model variants and provides explanations for the difference in magnitude. The calibration set-up ensures that in both model variants the same initial values of $\widetilde{w}_L$ and θ_L exist. While the initial WC is the same in both model variants, the JC is steeper in case of exogenous and constant task allocation (dotted curve), see Appendix 4.A.8 and consider the parameter values in Table 4.1. This can be explained by the fact that the labor demand elasticity is lower when task allocation is exogenous and constant. This indicates that the reaction of firm's labor demand to changes in low-skilled labor costs, and therefore the adjustment of vacancies, is lower. As a result, the effect of $\widetilde{w}_L$ on θ_L is smaller.

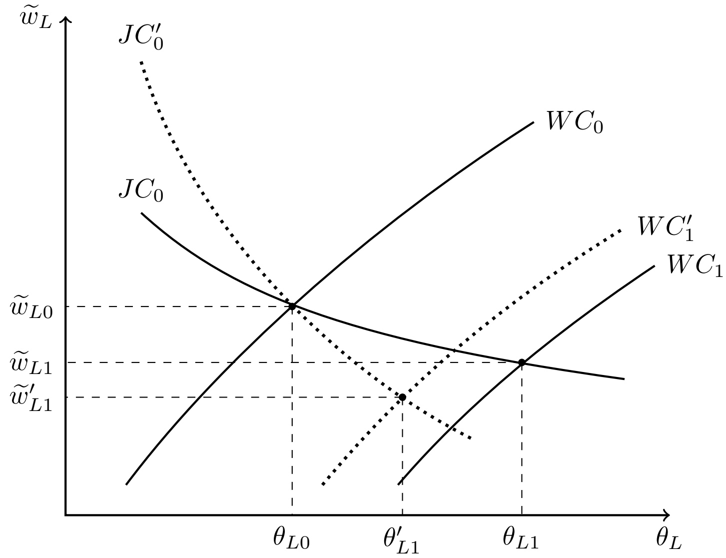

Figure 4.3: Graphical illustration of the calibrated effects of a reduction in the replacement rate for low-skilled workers

Notes: The figure illustrates the effects of a reduction in γ_L in the task-based matching model with constant labor demand elasticity (solid curves) and in the task-based matching model with exogenous and constant task allocation (dotted curves). The illustration is based on the calibration framework, which ensures the same initial equilibrium values of $\widetilde{w}_L$ and θ_L in both model variants. Considering the parameters given in Table 4.1 in the formal analysis of the slopes and curvatures in Appendix 4.A.8 leads to different slopes of the JC, while the slope of the WC is the same in both model variants. In the case of the task-based matching model with constant labor demand elasticity, the shift in the WC due to a reduction in γ_L is more pronounced, see Appendix 4.A.8 and consider the parameter values in Table 4.1 and Table 4.2.

The reduction in γ_L leads to a downward shift of the WC in both model variants, but the shift is more pronounced in the case of the task-based matching model with constant labor demand elasticity, see Appendix 4.A.8 and consider the parameter values in Table 4.1 and Table 4.2. This indicates a stronger direct effect of γ_L on $\widetilde{w}_L$ (keeping θ_L unchanged), which together with a higher labor demand elasticity and task reallocation, leads to a more pronounced increase in θ_L. The larger increase in θ_L induces a stronger increase in $\widetilde{w}_L$, which dampens its first decrease. With the calibrated parameters, this dampening effect is larger, which leads to a smaller reduction in $\widetilde{w}_L$ compared to the task-based matching model with exogenous and constant task allocation.

The considerable increase in low-skilled labor market tightness within the benchmark model translates into a pronounced increase in the low-skilled job finding rate p_L by 9 percentage points, reaching a post-reform steady-state value of 28.5%. Without endogenous task allocation, the more moderate increase in low-skilled labor market tightness is related to a somewhat weaker increase in the job finding rate by 7.6 percentage points. By comparison, Krause and Uhlig (2012) estimate that Hartz IV leads to an increase of 6 percentage points in the aggregate job finding rate, resulting in a post-reform job finding rate of 32%. In contrast, Krebs and Scheffel (2013) provides a more conservative estimate, stating that Hartz IV induces an increase of 3.7 percentage points in the job finding rate for short-term unemployed, resulting in a new steady-state job finding rate of 27.7%.

Turning to the other labor market outcomes, in the case of the benchmark model with constant labor demand elasticity, the 6.5% drop in the net replacement rate results in a 10.7% reduction in low-skilled wages and a 1% increase in high-skilled wages. In the task-based matching model with exogenous and constant task allocation, the effect of the cut in the net replacement rate on low- and high skilled wages is more pronounced, amounting to -11.4% and 2.5%, respectively. The larger increase in high-skilled wages in Model II can be explained by the missing reallocation effect, which would dampen the increase in high-skilled wages resulting from higher production (scale effect). Therefore, the percentage increase in high-skilled wages in Model II corresponds to the percentage increase in output. In the benchmark model with constant labor demand elasticity the output increases somewhat stronger by 3%. The increase in the skill premium is smaller in the case of endogenous task allocation and amounts to 11.6%, in contrast to a 14% increase in the case of exogenous and constant task allocation. The calibrated effects of the Hartz IV reform on low- and high-skilled wages are consistent with the findings in Krause and Uhlig (2012), who estimate almost no effect on high-skilled wages, while low-skilled workers experience a wage decrease of 12.5%, resulting in an increase in the skill premium by 14.3%.

The calibration highlights the importance of taking task reallocation into account in the quantitative evaluation of labor market reforms. Ignoring task reallocation in the assessment of the Hartz IV reform results in a 0.6 percentage point higher low-skilled unemployment rate and a 2.1% higher skill premium in the post-reform steady-state, compared to the post-reform values in case of endogenous task allocation. Thus, abstracting from endogenous task allocation underestimates the effect of the Hartz IV reform on unemployment and overestimates the effect on wages and the skill premium.

4.4.3 Improvement in Matching Efficiency

So far, the focus of the calibration has been on the reduction in the low-skilled net replacement rate due to the Hartz IV reform in January 2005. In addition to Hartz IV, the German government implemented two additional waves of the Hartz reforms in 2003 and 2004. The Hartz I-III reforms had the goal to increase the incentives to return to work by creating new job opportunities and improve the matching efficiency by reconstructing the Federal Employment Agency. Empirical evidence shows that the first two waves of the Hartz reforms substantially improved the matching efficiency. The estimated effects range from 5-10% in Fahr and Sunde (2009) and Klinger and Rothe (2012) to more than 20% in Hertweck and Sigrist (2013).

To account for the first two waves of the Hartz reforms in the model calibration, an increase in the matching efficiency parameter ζ of 10% is used, which represents the middle of the range of values estimated from the literature. For the analytical results of a change in ζ, see Appendix 4.A.9. In the benchmark framework with constant labor demand elasticity, an increase in the matching efficiency by 10% reduces the noncyclical low-skilled unemployment rate by 0.9 percentage points.[12] Overall, the reduction in the

[12] Using the lowest and highest values from the literature, an increase of either 5% or 20% leads to a decrease in the low-skilled unemployment rate of 0.4 percentage points or 1.7 percentage points, respectively.

noncyclical low-skilled unemployment rate due to the Hartz I-IV reforms amounts to 4.9 percentage points. In comparison, Krause and Uhlig (2012) calibrate that a 10% increase in the matching efficiency reduces the long-run unemployment rate by a slightly smaller amount of 0.7 percentage points, resulting in an overall effect of the Hartz reforms on the long-run unemployment rate of around 3.5 percentage points.

The quantitative results show that the increase in the matching efficiency due to the Hartz I-III reforms leads to an additional reduction in the unemployment rate. However, in comparison to the effect of the Hartz IV reform, the first two waves play a minor role in explaining the drop in the unemployment rate.

4.5 Conclusions

This paper sheds new light on an old question about the impact of unemployment benefit reforms on labor market outcomes, particularly unemployment. The existing literature is silent on adjustment effects in the task allocation within the production process of firms resulting from a cut in unemployment benefits. This paper contributes to the literature by investigating the impact of the Hartz IV reform on the basis of the task-based matching model introduced in Marczak et al. (2022), in which tasks are assigned to low- and high-skilled labor based on their comparative advantage in performing those tasks. The model is extended by monopolistic competition in the goods market and earnings-related unemployment benefits. In addition, the paper introduces a specific relative task productivity schedule, resulting in a constant labor demand elasticity of low-skilled labor. To highlight the importance of task allocation in the evaluation of the Hartz IV reform, the task-based matching model with exogenous and constant task allocation is considered for comparison, reflecting the standard literature with a conventional production function. This implies that task allocation is no more endogenously determined from profit maximization and there is no reallocation of tasks due to changes in unit labor costs.

Both model variants are calibrated to evaluate the long-run effects of the Hartz IV reform. In the task-based matching model with constant labor demand elasticity, Hartz IV leads to a remarkable decrease in the long-run unemployment rate for low-skilled workers by 4 percentage points. In the case of exogenous and constant task allocation, the low-skilled unemployment rate decreases by only 3.4 percentage points. Moreover, the effect of the cut in the net replacement rate on low- and high skilled wages is more pronounced with exogenous and constant task allocation, resulting in a stronger increase in the skill premium. Hence, ignoring the adjustment of firms due to changes in unit labor costs within this calibration set-up would underestimate the effect of the Hartz IV reform on low-skilled unemployment and overestimate the effect on wages and the skill premium. Therefore, the calibration results emphasize the importance of considering endogenous task allocation in the evaluation of labor market reforms.

The main focus of this analysis is on the impact of endogenous task allocation and its implications for the evaluation of labor market reforms. Therefore, the model framework is kept as simple as possible and abstracts from specific modeling of different unemployment duration groups or a more complex structure of the unemployment benefit system. Future research on labor market reforms in the task-based matching model should take these aspects into account.

Appendix 4.A

4.A.1 The Household's Optimization Problem

The household $k's$ optimization problem, captured by eq. (4.1), can be written as

$$\max_{\{c_{jk},\phi\}} \mathcal{L} = \left(\int_0^1 c_{jk}^{\frac{\eta-1}{\eta}}\,\mathrm{d}j\right)^{\frac{\eta}{\eta-1}} + \phi\left(X_k - \int_0^1 c_{jk}\frac{P_j}{P}\,\mathrm{d}j\right),$$

where ϕ gives the Lagrange multiplier. The first-order conditions are

$$\frac{\partial\mathcal{L}}{\partial c_{jk}} = 0 \quad \Leftrightarrow \quad P_j = \frac{1}{\phi}\, c_k^{\frac{1}{\eta}}\, c_{jk}^{-\frac{1}{\eta}}, \tag{4.A.1}$$

$$\frac{\partial\mathcal{L}}{\partial\phi} = 0 \quad \Leftrightarrow \quad X_k = \int_0^1 c_{jk}\,\frac{P_j}{P}\,\mathrm{d}j, \tag{4.A.2}$$

where $c_k = \left(\int_0^1 c_{jk}^{\frac{\eta-1}{\eta}}\,\mathrm{d}j\right)^{\frac{\eta}{\eta-1}}$. Take eq. (4.A.1) to the power of $1-\eta$, integrate over all j and take the result to the power of $\frac{1}{1-\eta}$ leads to the inverse shadow price of wealth, which can be interpreted as the price index:

$$\left(\int_0^1 P_j^{1-\eta}\,\mathrm{d}j\right)^{\frac{1}{1-\eta}} = \frac{1}{\phi} \equiv P. \tag{4.A.3}$$

Insert eq. (4.A.3) in eq. (4.A.1) leads to

$$c_{jk} = \left(\frac{P_j}{P}\right)^{-\eta} c_k. \tag{4.A.4}$$

Insert eq. (4.A.4) in eq. (4.A.2) and consider the price index gives $X_k = c_k$. Taking this into account, eq. (4.A.4) becomes

$$c_{jk} = \left(\frac{P_j}{P}\right)^{-\eta} X_k. \tag{4.A.5}$$

Aggregate demand is given by $Y_j^d \equiv \int_0^m c_{jk}\,\mathrm{d}k$. Insert eq. (4.A.5) and consider $X \equiv \int_0^m X_k\,\mathrm{d}k$, leads to the Blanchard-Kiyotaki goods demand given in eq. (4.2).

4.A.2 The Firm's Optimization Problem

Combining eqs. (4.3) – (4.7) and taking the restriction in (4.9) into account, the firm j's optimization problem can be written as

$$\begin{aligned}
\max_{\{l_{jt}(i),h_{jt}(i),V_{jLt},\lambda_{jLt}\}} \mathcal{L} = & \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^{t-1} \\
& \times\left\{\left(\frac{Y_{jt}}{Y_t}\right)^{-\frac{1}{\eta}} \exp\left[\int_0^1 \ln\left(A_{Lt}\alpha_L(i)l_{jt}(i)+A_{Ht}\alpha_H(i)h_{jt}(i)\right)\mathrm{d}i\right]\right. \\
& \left. -w_{jLt}\int_0^1 l_{jt}(i)\mathrm{d}i - w_{jHt}\int_0^1 h_{jt}(i)\mathrm{d}i - s_L V_{jLt}\right\} \\
& +\sum_{t=0}^{\infty}\mu_{jLt}\left(\frac{1}{1+r}\right)^{t-1}\left[m(\theta_{Lt})\,V_{jLt}+(1-q_L)\int_0^1 l_{j,t-1}(i)\mathrm{d}i-\int_0^1 l_{jt}(i)\mathrm{d}i\right] \\
& \text{s.t.}\quad l_{jt}(i)\geq 0,\quad h_{jt}(i)\geq 0 \qquad \text{and} \qquad l_{j0}(i),\quad h_{j,0}(i)\ \text{given,}
\end{aligned}$$

where μ_{jLt} denotes the shadow price of L_{jt} and θ_{Lt} is given for each firm j. The first-order conditions $\partial\mathcal{L}/\partial V_{jLt}=0$ and $\partial\mathcal{L}/\partial\mu_{jLt}=0$ lead to

$$\mu_{jLt}=\frac{s_L}{m(\theta_{Lt})} \tag{4.A.6}$$

$$m(\theta_{Lt})\,V_{jLt}+L_{j,t-1}-L_{jt}=q_L L_{j,t-1}, \tag{4.A.7}$$

and the complementary slackness conditions are

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial l_{jt}(i)}\leq 0, \qquad & l_{jt}(i)\geq 0, \qquad & \frac{\partial\mathcal{L}}{\partial l_{jt}(i)}l_{jt}(i)=0 \\
\frac{\partial\mathcal{L}}{\partial h_{jt}(i)}\leq 0, \qquad & h_{jt}(i)\geq 0, \qquad & \frac{\partial\mathcal{L}}{\partial h_{jt}(i)}h_{jt}(i)=0.
\end{aligned}$$

This gives

$$\frac{Y_{jt}}{y_t(i)} A_{Lt}\alpha_L(i) \frac{\eta-1}{\eta}\left(\frac{Y_{jt}}{Y_t}\right)^{-\frac{1}{\eta}} \leq \widetilde{w}_{jLt} \equiv w_{jL} + \frac{s_L}{m(\theta_{Lt})} - \frac{s_L}{m(\theta_{L,t+1})}\frac{1-q_L}{1+r}, \qquad l_j(i) \geq 0, \tag{4.A.8}$$

$$\frac{Y_{jt}}{y_{jt}(i)} A_{Ht}\alpha_H(i) \frac{\eta-1}{\eta}\left(\frac{Y_{jt}}{Y_t}\right)^{-\frac{1}{\eta}} \leq w_{jHt}, \qquad h_{jt}(i) \geq 0. \tag{4.A.9}$$

The relevant low-skilled labor costs for the firm, $\widetilde{w}_{jLt}$, are given by the wage w_{jLt} plus the search costs in period t minus the costs of posting a vacancy that are saved in period $t+1$ in case the employment position is still occupied, see eq. (4.A.8). The different cases due to complementary slackness are discussed in the following. In the steady state it holds that $\theta_{L,t+1} = \theta_{Lt} = \theta_L$ and $\widetilde{w}_{jLt} = \widetilde{w}_{jL}$, where

$$\widetilde{w}_{jL} \equiv w_{jL} + \frac{(q_L+r)}{(1+r)}\frac{s_L}{m(\theta_L)}. \tag{4.A.10}$$

Case I: $l_j(i) > 0$ and $h_j(i) = 0$. From eq. (4.5) follows $y_j(i) = A_L\alpha_L(i)l_j(i)$, implying that eq. (4.A.8) becomes:

$$\underbrace{\frac{Y_j}{l_j(i)}}_{\text{marginal product of low-skilled}} \times \underbrace{\frac{\eta-1}{\eta}\left(\frac{Y_j}{Y}\right)^{-\frac{1}{\eta}}}_{\text{marginal revenues}} = \widetilde{w}_{jL}. \tag{4.A.11}$$

With monopolistic competition in the goods market, the marginal revenue product of unskilled labor in task i with respect to output Y_j equals the unskilled labor costs relevant to the firm, $\widetilde{w}_{jL}$. Nevertheless, it has the same implications than under perfect competition: It follows that $l_j(i) = l_j$, i.e. the same labor input l_j is chosen in all low-skilled tasks of firm j and from eq. (4.A.9) follows

$$\frac{\widetilde{w}_{jL}}{A_L\alpha_L(i)} < \frac{w_{jH}}{A_H\alpha_H(i)},$$

if the constraint on $h_j(i)$ is binding. At the point where $\partial\mathcal{L}/\partial h_j(i) = 0$, there is a specific task $i = I$ for which $\widetilde{w}_{jL}/(A_L\alpha_L(I)) = w_{jH}/(A_H\alpha_H(I))$.

Case II: $h_j(i) > 0$ and $l_j(i) = 0$. From eq. (4.5) follows $y_j(i) = A_H\alpha_H(i)h_j(i)$, implying that eq. (4.A.9) becomes:

$$\frac{Y_j}{h_j(i)}\frac{\eta-1}{\eta}\left(\frac{Y_j}{Y}\right)^{-\frac{1}{\eta}} = w_{jH}, \tag{4.A.12}$$

which is interpreted analogously. It follows that $h_j(i) = h_j$, i.e. the same labor input h_j is chosen in all high-skilled tasks. From eq. (4.A.8) follows

$$\frac{\widetilde{w}_{jL}}{A_L\alpha_L(i)} > \frac{w_{jH}}{A_H\alpha_H(i)},$$

if the constraint on $l_j(i)$ is binding. At the point where $\partial\mathcal{L}/\partial l_j(i) = 0$, there is a specific task $i = I$ for which $\widetilde{w}_{jL}/(A_L\alpha_L(I)) = w_{jH}/(A_H\alpha_H(I))$.

Case III: $h_j(i) > 0$ and $l_j(i) > 0$. Because of eq. (4.5): $y_j(i) = A_L\alpha_L(i)l_j(i) + A_H\alpha_H(i)h_j(i)$. In eq. (4.A.8) it holds that

$$\frac{Y_j}{y_j(i)}A_L\alpha_L(i)\frac{\eta-1}{\eta}\left(\frac{Y_j}{Y}\right)^{-\frac{1}{\eta}} = \widetilde{w}_{jL}.$$

In eq. (4.A.9) it holds that

$$\frac{Y_j}{y_j(i)}A_H\alpha_H(i)\frac{\eta-1}{\eta}\left(\frac{Y_j}{Y}\right)^{-\frac{1}{\eta}} = w_{jH}.$$

Hence,

$$\frac{\widetilde{w}_{jL}}{A_L\alpha_L(I)} = \frac{w_{jH}}{A_H\alpha_H(I)}.$$

Case IV: $h_j(i) = 0$ and $l_j(i) = 0$. This leads to $y_j(i) = 0$ which is not possible because of the production function in eq. (4.4).

Similar to the task-based matching model under perfect competition in the goods market, in cases I to III the task threshold I is defined by

$$\bar{\alpha}(I) \equiv \frac{\alpha_H(I)}{\alpha_L(I)} = \frac{A_L\, w_{jH}}{A_H \widetilde{w}_{jL}}, \tag{4.A.13}$$

with $0 < I < 1$.

It follows from eqs. (4.A.12) and (4.A.11) that $\widetilde{w}_{jL} l_j(i) = Y_j \frac{\eta-1}{\eta} \left(\frac{Y_j}{Y}\right)^{-\frac{1}{\eta}} = w_{jH} h_j(i)$. Omitting the firm index j due to the fact that there is a mass one of identical firms in the economy, implies that the same labor input is used in all low-skilled and high-skilled tasks, respectively, i.e. $l(i) = l = L/I$ for $i < I$ and $h(i) = h = H/(1-I)$ for $i > I$. Taking this into account and dividing eq. (4.A.12) by eq. (4.A.11) leads to the relative factor input given by

$$\frac{L}{H} = \frac{I(\widetilde{\omega}, \bar{A})}{1 - I(\widetilde{\omega}, \bar{A})} \widetilde{\omega}. \tag{4.A.14}$$

4.A.3 Elasticity of Substitution

In general, the elasticity of substitution between the production factors is given by

$$\sigma = \frac{\mathrm{d}(L/H)}{\mathrm{d}\widetilde{\omega}} \cdot \frac{\widetilde{\omega}}{L/H}. \tag{4.A.15}$$

Considering $\widetilde{\omega} = \frac{1-I}{I}\frac{L}{H} = g(\frac{L}{H}, I(\frac{L}{H}))$ leads to

$$\mathrm{d}\widetilde{\omega} = \Big(\frac{\partial g(\cdot)}{\partial(L/H)} + \frac{\partial g(\cdot)}{\partial I}\frac{\partial I}{\partial(L/H)}\Big) \cdot \mathrm{d}(L/H),$$

with $\frac{\partial g(\cdot)}{\partial(L/H)} = \frac{1-I}{I}$ and $\frac{\partial g(\cdot)}{\partial I} = -\frac{1}{I^2}\frac{L}{H}$. Considering $\widetilde{\omega} = \bar{\alpha}(I)\bar{A}$ gives $\frac{L}{H} = \bar{\alpha}(I)\frac{I}{1-I}$, where $\bar{A}$ is normalized to unity. Then,

$$\frac{\partial I}{\partial(L/H)} = \frac{1}{\frac{\partial(L/H)}{\partial I}} = \frac{\bar{\alpha}(I) + (1-I)I\bar{\alpha}'(I)}{(1-I)^2}.$$

This leads to

$$\mathrm{d}\widetilde{\omega} = \frac{(1-I)^2 \varepsilon_{\bar{\alpha},I}}{I(1 + (1-I)\varepsilon_{\bar{\alpha},I})}\mathrm{d}(L/H).$$

Insert this expression into eq. (4.A.15) and consider $\widetilde{\omega} = \frac{1-I}{I}\frac{L}{H}$ leads to the elasticity of substitution given in eq. (4.13).

4.A.4 Derivation of Low- and High-Skilled Labor Demand

Insert eq. (4.4) in eq. (4.A.11), take the logarithm, define $\xi(I) \equiv \int_0^I \ln \alpha_L(i)\mathrm{d}i + \int_I^1 \ln \alpha_H(i)\mathrm{d}i$ and solve for $\ln L^d$ leads to:

$$\begin{aligned}\ln L^d =& -\frac{\eta}{\eta-(\eta-1)I}\ln \widetilde{w}_L + \frac{\eta-1}{\eta-(\eta-1)I}(1-I)\ln H^d + \frac{\eta-1}{\eta-(\eta-1)I}I\ln A_L \\ &+ \frac{\eta-1}{\eta-(\eta-1)I}(1-I)\ln A_H + \ln I - \frac{\eta-1}{\eta-(\eta-1)I}(1-I)\ln(1-I) \\ &+ \frac{\eta-1}{\eta-(\eta-1)I}\xi(I) + \frac{\eta}{\eta-(\eta-1)I}\ln\rho + \frac{1}{\eta-(\eta-1)I}\ln Y.\end{aligned} \tag{4.A.16}$$

Insert eq. (4.4) in eq. (4.A.12), take the logarithm, consider the definition for $\xi(I)$ and solve for $\ln H^d$ leads to:

$$\begin{aligned}\ln H^d =& -\frac{\eta}{1+(\eta-1)I}\ln w_H + \frac{\eta-1}{1+(\eta-1)I}I\ln L^d + \frac{\eta-1}{1+(\eta-1)I}(1-I)\ln A_H \\ &+ \frac{\eta-1}{1+(\eta-1)I}I\ln A_L + \ln(1-I) - \frac{\eta-1}{1+(\eta-1)I}I\ln I \\ &+ \frac{\eta-1}{1+(\eta-1)I}\xi(I) + \frac{\eta}{1+(\eta-1)I}\ln\rho + \frac{1}{1+(\eta-1)I}\ln Y.\end{aligned} \tag{4.A.17}$$

Insert eq. (4.A.17) in eq. (4.A.16) and solve for $\ln L^d$ gives the labor demand for low-skilled workers in logarithms:

$$\begin{aligned}\ln L \equiv \ln L^d =& -(1+(\eta-1)I)\ln\widetilde{w}_L - (\eta-1)(1-I)\ln w_H + (\eta-1)I\ln A_L \\ &+ (\eta-1)(1-I)\ln A_H + \ln I + (\eta-1)\xi(I) + \eta\ln\Big(\frac{\eta-1}{\eta}\Big) + \ln Y,\end{aligned} \tag{4.A.18}$$

or analogously in levels:

$$L \equiv L^d = \frac{A_L^{(\eta-1)I} A_H^{(\eta-1)(1-I)} I \exp\left[(\eta-1)\xi(I)\right](\frac{\eta-1}{\eta})^{\eta} Y}{\widetilde{w}_L^{1+(\eta-1)I} w_H^{(\eta-1)(1-I)}}.$$

Insert eq. (4.A.16) in eq. (4.A.17) and solve for $\ln H^d$ gives the labor demand for high-skilled workers in logarithms:

$$\begin{aligned}\ln H \equiv \ln H^d = &-\left(\eta(1-I)+I\right)\ln w_H - (\eta-1)I\ln\widetilde{w}_L + (\eta-1)I\ln A_L \\ &+ (\eta-1)(1-I)\ln A_H + \ln(1-I) + (\eta-1)\xi(I) + \eta\ln\Big(\frac{\eta-1}{\eta}\Big) \\ &+ \ln Y,\end{aligned} \tag{4.A.19}$$

or analogously in levels:

$$H \equiv H^d = \frac{A_L^{(\eta-1)I} A_H^{(\eta-1)(1-I)} (1-I) \exp\left[(\eta-1)\xi(I)\right](\frac{\eta-1}{\eta})^{\eta} Y}{\widetilde{w}_L^{(\eta-1)I} w_H^{\eta(1-I)+I}}.$$

To derive the wage elasticity of the low-skilled labor demand in absolute values given in eq. (4.22), consider eq. (4.A.18) and take into account that $\xi'(I) = -\ln\bar{\alpha}(I)$. Moreover, the task threshold is endogenously determined from profit maximization and eq. (4.10) must hold, thus implying $\ln\bar{\alpha}(I) = (\ln w_H - \ln\widetilde{w}_L) - (\ln A_H - \ln A_L)$.

4.A.5 Proof of Proposition 4.2

From eq. (4.10) follows

$$\mathrm{d}\ln I = \frac{1}{\varepsilon_{\bar{\alpha},I}}\left(\mathrm{d}\ln w_H - \mathrm{d}\ln\widetilde{w}_L - \mathrm{d}\ln A_H + \mathrm{d}\ln A_L\right). \tag{4.A.20}$$

From eq. (4.A.18) follows

$$\begin{aligned}\mathrm{d}\ln L = &-(1+(\eta-1)I)\mathrm{d}\ln\widetilde{w}_L-(\eta-1)(1-I)\mathrm{d}\ln w_H+(\eta-1)I\mathrm{d}\ln A_L\\&+(\eta-1)(1-I)\mathrm{d}\ln A_H+\mathrm{d}\ln I+\mathrm{d}\ln Y\\&+(\eta-1)I\mathrm{d}\ln I[(\ln w_H-\ln\widetilde{w}_L)-(\ln A_H-\ln A_L)-\ln\bar{\alpha}(I)],\end{aligned}$$

where it has been taken into account that in equilibrium $\mathrm{d}\ln L^d=\mathrm{d}\ln L$ and $\xi'(I)=-\ln\bar{\alpha}(I)$. The task threshold is endogenously determined from profit maximization and thus eq. (4.10) must hold, implying $\ln\bar{\alpha}(I)=(\ln w_H-\ln\widetilde{w}_L)-(\ln A_H-\ln A_L)$. Hence, the term in brackets in the third line is zero. This leads to

$$\begin{aligned}\mathrm{d}\ln L = &-(1+(\eta-1)I)\mathrm{d}\ln\widetilde{w}_L-(\eta-1)(1-I)\mathrm{d}\ln w_H+(\eta-1)I\mathrm{d}\ln A_L\\&+(\eta-1)(1-I)\mathrm{d}\ln A_H+\mathrm{d}\ln I+\mathrm{d}\ln Y.\end{aligned} \tag{4.A.21}$$

From eq. (4.A.19) follows

$$\begin{aligned}0 = &-(\eta-(\eta-1)I)\mathrm{d}\ln w_H-(\eta-1)I\mathrm{d}\ln\widetilde{w}_L+(\eta-1)(1-I)\mathrm{d}\ln A_H+(\eta-1)I\mathrm{d}\ln A_L\\&-\frac{I}{1-I}\mathrm{d}\ln I+\mathrm{d}\ln Y+(\eta-1)I\mathrm{d}\ln I[(\ln w_H-\ln\widetilde{w}_L)-(\ln A_H-\ln A_L)-\ln\bar{\alpha}(I)],\end{aligned}$$

where it has been taken into account that in equilibrium $\mathrm{d}\ln H^d=\mathrm{d}\ln H=0$ and $\xi'(I)=-\ln\bar{\alpha}(I)$ and the second term in the second line becomes zero. This leads to

$$\begin{aligned}0 = &-(\eta-(\eta-1)I)\mathrm{d}\ln w_H-(\eta-1)I\mathrm{d}\ln\widetilde{w}_L+(\eta-1)(1-I)\mathrm{d}\ln A_H\\&+(\eta-1)I\mathrm{d}\ln A_L-\frac{I}{1-I}\mathrm{d}\ln I+\mathrm{d}\ln Y.\end{aligned} \tag{4.A.22}$$

Keeping r, q_L, s_L and ζ constant, it follows from eq. (4.19) and eq. (4.26)

$$\mathrm{d}\ln\widetilde{w}_L=\frac{1}{1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L}}\Big[\varepsilon_{\widetilde{z}_L,\gamma_L}\,\mathrm{d}\ln\gamma_L+\varepsilon_{\widetilde{z}_L,\theta_L}\mathrm{d}\ln\theta_L+\varepsilon_{\kappa_L,I}\,\mathrm{d}\ln I\Big], \tag{4.A.23}$$

where

$$\varepsilon_{\widetilde{z}_L,\widetilde{w}_L} \equiv \frac{\mathrm{d}\ln\widetilde{z}_L}{\mathrm{d}\ln\widetilde{w}_L} = \gamma_L\frac{\widetilde{w}_L}{\widetilde{z}_L} = \frac{w_L + \frac{q_L+r}{1+r}s_L\frac{1}{\zeta}\theta_L^{\beta_L}}{w_L + \frac{1}{\gamma_L}\frac{q_L+r}{1+r}s_L\frac{1}{\zeta}\theta_L^{\beta_L}} < 1, \tag{4.A.24}$$

$$\varepsilon_{\widetilde{z}_L,\gamma_L} \equiv \frac{\mathrm{d}\ln\widetilde{z}_L}{\mathrm{d}\ln\gamma_L} = \gamma_L\frac{w_L}{\widetilde{z}_L} = \frac{\gamma_L w_L}{\gamma_L w_L + \frac{q_L+r}{1+r}s_L\frac{1}{\zeta}\theta_L^{\beta_L}} < 1, \tag{4.A.25}$$

$$\varepsilon_{\widetilde{z}_L,\theta_L} \equiv \frac{\mathrm{d}\ln\widetilde{z}_L}{\mathrm{d}\ln\theta_L} = (1-\gamma_L)\beta_L\frac{\widetilde{z}_L - z_L}{\widetilde{z}_L} = (1-\gamma_L)\beta_L\frac{\frac{q_L+r}{1+r}s_L\frac{1}{\zeta}\theta_L^{\beta_L}}{z_L + \frac{q_L+r}{1+r}s_L\frac{1}{\zeta}\theta_L^{\beta_L}} < \beta_L, \tag{4.A.26}$$

$$\varepsilon_{\kappa_L,I} \equiv \frac{\mathrm{d}\ln\kappa_L}{\mathrm{d}\ln I} = -(\kappa_L - 1)\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}. \tag{4.A.27}$$

Because of eq. (4.25)

$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)u_L}\mathrm{d}\ln L, \tag{4.A.28}$$

where $u_L \equiv (1-H-L)/(1-H)$ denotes the low-skilled unemployment rate.

Using the optimality conditions derived from the firm's maximization problem, the production function for the final good is given by the following Cobb-Douglas form:

$$\ln Y = \left(A_L\frac{L}{I}\right)^I + \left(A_H\frac{H}{1-I}\right)^{1-I} + \xi(I),$$

or analogously in levels

$$Y = e^{\xi(I)}\left(A_L\frac{L}{I}\right)^I\left(A_H\frac{H}{1-I}\right)^{1-I},$$

where it has been taken into account that $l_j(i) = l = L/I$, $h_j(i) = h = H/(1-I)$ and $\xi(I) \equiv \int_0^I \ln\alpha_L(i)\mathrm{d}i + \int_I^1 \ln\alpha_H(i)\mathrm{d}i$. This leads to

$$\begin{aligned}\mathrm{d}\ln Y =& I\mathrm{d}\ln L + I\mathrm{d}\ln A_L + (1-I)\mathrm{d}\ln A_H \\ &+ I\mathrm{d}\ln I[(\ln w_H - \ln\widetilde{w}_L) - (\ln A_H - \ln A_L) - \ln\bar{\alpha}(I)],\end{aligned}$$

with $\mathrm{d}\ln H = 0$, $\xi'(I) = -\ln\bar{\alpha}(I)$ and $\ln(L/H) + \ln((1-I)/I) = \ln(w_H/\widetilde{w}_L)$, see eq. (4.A.14). Taking into account eq. (4.A.13) results in

$$\mathrm{d}\ln Y = I\mathrm{d}\ln L + I\mathrm{d}\ln A_L + (1-I)\mathrm{d}\ln A_H. \tag{4.A.29}$$

Insert eq. (4.A.29) in eq. (4.A.21) and consider $\mathrm{d}\ln A_L = \mathrm{d}\ln A_H = 0$ leads to

$$\mathrm{d}\ln L = -\frac{1}{1-I}(1+(\eta-1)I)\mathrm{d}\ln \widetilde{w}_L - (\eta-1)\mathrm{d}\ln w_H + \frac{1}{1-I}\mathrm{d}\ln I. \tag{4.A.30}$$

Insert eq. (4.A.29) in eq. (4.A.22) and consider $\mathrm{d}\ln A_L = \mathrm{d}\ln A_H = 0$ leads to

$$\begin{aligned}\mathrm{d}\ln w_H = &-\frac{\eta-1}{\eta-(\eta-1)I}I\mathrm{d}\ln \widetilde{w}_L + \frac{1}{\eta-(\eta-1)I}I\mathrm{d}\ln L\\ &-\frac{1}{\eta-(\eta-1)I}\frac{I}{1-I}\mathrm{d}\ln I.\end{aligned} \tag{4.A.31}$$

Insert eq. (4.A.31) in eq. (4.A.30) and insert the result for $\mathrm{d}\ln L$ in eq. (4.A.28) leads to the job creation equation (4.28) in Proposition 4.2. The wage-setting equation (4.29) corresponds to eq. (4.A.23) and the task allocation equation (4.30) is derived by inserting eq. (4.A.30) in eq. (4.A.31), solve for $\mathrm{d}\ln w_H$, insert the result in eqs. (4.A.20) and consider $\mathrm{d}\ln A_L = \mathrm{d}\ln A_H = 0$.

4.A.6 General Effects of Unemployment Benefits on Final Production and Wage Outcomes

Insert eq. (4.A.28) in eq. (4.A.29) and take into account that $\mathrm{d}\ln A_H = \mathrm{d}\ln A_L = 0$ leads to

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}\ln \gamma_L} = (1-\beta_L)\,u_L\,I\,\frac{\mathrm{d}\ln \theta_L}{\mathrm{d}\ln \gamma_L}. \tag{4.A.32}$$

Consider the result for $\frac{\mathrm{d}\ln\theta_L}{\mathrm{d}\ln\gamma_L}$ in Corollary 4.2.1 gives

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}\ln\gamma_L} = -\,(1-\beta_L)\,u_L\,I\,\frac{\varepsilon_{\widetilde{z}_L,\gamma_L}}{\varepsilon_{\widetilde{z}_L,\theta_L}}\,\frac{\Gamma}{|\Phi|+\Gamma}$$
$$\begin{cases} <0, & \text{if} \quad \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \\ & \quad\vee \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \quad \wedge \quad |\Phi|<|\Gamma|\right) \\ >0, & \text{otherwise.} \end{cases}$$

Write eq. (4.10) in log differences and keep r, q_L and s_L constant leads to

$$\frac{\mathrm{d}\ln w_L}{\mathrm{d}\ln\gamma_L} = \frac{\widetilde{w}_L}{w_L}\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\gamma_L} - \beta_L\frac{\widetilde{w}_L - w_L}{w_L}\frac{\mathrm{d}\ln\theta_L}{\mathrm{d}\ln\gamma_L}. \tag{4.A.33}$$

Consider the results in Corollary 4.2.1 gives

$$\frac{\mathrm{d}\ln w_L}{\mathrm{d}\ln\gamma_L} = \Big[\widetilde{w}_L\,|\Phi| + \beta_L(\widetilde{w}_L - w_L)\Big]\,\frac{1}{w_L}\,\frac{\varepsilon_{\widetilde{z}_L,\gamma_L}}{\varepsilon_{\widetilde{z}_L,\theta_L}}\,\frac{\Gamma}{|\Phi|+\Gamma}$$
$$\begin{cases} >0, & \text{if} \quad \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \\ & \quad\vee \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \quad \wedge \quad |\Phi|<|\Gamma|\right) \\ <0, & \text{otherwise.} \end{cases}$$

Insert eq. (4.A.30) in eq. (4.A.31) leads to

$$\frac{\mathrm{d}\ln w_H}{\mathrm{d}\ln\gamma_L} = -\,\frac{I}{1-I}\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\gamma_L}. \tag{4.A.34}$$

Consider the result for $\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\gamma_L}$ in Corollary 4.2.1 gives

$$\frac{\mathrm{d}\ln w_H}{\mathrm{d}\ln\gamma_L} = -\,\frac{I}{1-I}\,\frac{\varepsilon_{\widetilde{z}_L,\gamma_L}}{\varepsilon_{\widetilde{z}_L,\theta_L}}\,|\Phi|\,\frac{\Gamma}{|\Phi|+\Gamma}$$
$$\begin{cases} < 0, & \text{if} \quad \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \\ & \quad \vee \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \quad \wedge \quad |\Phi| < |\Gamma|\right) \\ > 0, & \text{otherwise.} \end{cases}$$

Consider the skill premium $\omega \equiv w_H/w_L$ in log differences

$$\frac{\mathrm{d}\ln\omega}{\mathrm{d}\ln\gamma_L} = \frac{\mathrm{d}\ln w_H}{\mathrm{d}\ln\gamma_L} - \frac{\mathrm{d}\ln w_L}{\mathrm{d}\ln\gamma_L}. \tag{4.A.35}$$

Inserting the results for $\frac{\mathrm{d}\ln w_H}{\mathrm{d}\ln\gamma_L}$ and $\frac{\mathrm{d}\ln w_L}{\mathrm{d}\ln\gamma_L}$ gives

$$\frac{\mathrm{d}\ln\omega}{\mathrm{d}\ln\gamma_L} = -\left[\frac{I}{1-I}\,|\Phi| + \big(\widetilde{w}_L\,|\Phi| + \beta_L(\widetilde{w}_L - w_L)\big)\frac{1}{w_L}\right]\frac{\varepsilon_{\widetilde{z}_L,\gamma_L}}{\varepsilon_{\widetilde{z}_L,\theta_L}}\,\frac{\Gamma}{|\Phi|+\Gamma}$$
$$\begin{cases} < 0, & \text{if} \quad \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \\ & \quad \vee \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \quad \wedge \quad |\Phi| < |\Gamma|\right) \\ > 0, & \text{otherwise.} \end{cases}$$

4.A.7 Proof of Lemma 4.1

Taking into account that $\varepsilon_{\bar{\alpha},I} \equiv \bar{\alpha}'(I)\,I/\bar{\alpha}(I)$, from eq. (4.22) follows

$$\frac{\bar{\alpha}'(I)}{\bar{\alpha}(I)} = \frac{1}{\frac{1}{\rho} - (\eta-1)I}\,\frac{1}{I},$$

with $\rho \equiv 1/(\varepsilon_{L,\widetilde{w}_L} - 1) > 0$.

Remember that $\varepsilon_{L,\widetilde{w}_L}$ is constant by assumption and integrate over I leads to

$$\int \frac{\bar{\alpha}'(I)}{\bar{\alpha}(I)}\,\mathrm{d}I = \int \frac{1}{\frac{1}{\rho}-(\eta-1)I}\,\frac{1}{I}\,\mathrm{d}I$$
$$\ln\bar{\alpha}(I)+c_1 = \rho\ln\frac{I}{\frac{1}{\rho}-(\eta-1)I} + c_2,$$

where c_1 and c_2 are integration constants. Applying the exponential function gives

$$\bar{\alpha}(I) = b\left(\frac{I}{\frac{1}{\rho}-(\eta-1)I}\right)^{\rho}, \quad \textit{with} \quad b \equiv \exp\left(c_2-c_1\right).$$

The restriction for $\bar{\alpha}(I)>0$ is $b>0$ and $\frac{1}{\rho}>(\eta-1)I$. Then it holds for the specific function for $\bar{\alpha}(I)$ that

$$\varepsilon_{\bar{\alpha},I} = \frac{1}{\frac{1}{\rho}-(\eta-1)I} > 0 \quad \text{and} \quad \frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I} = (\eta-1)I\frac{1}{\frac{1}{\rho}-(\eta-1)I} > 0,$$

which is in line with Proposition 4.1 for the case of a constant $\varepsilon_{L,\widetilde{w}_L}$. Inserting the expression for $\varepsilon_{\bar{\alpha},I}$ in eq. (4.22) leads to $\varepsilon_{L,\widetilde{w}_L} = 1+1/\rho$.

4.A.8 Derivation of Slopes and Curvatures of JC and WC and the Shift in WC due to a Change in γ_L

The task-based matching model with constant labor demand elasticity

Using the findings of Lemma 4.1 in eq. (4.33), the JC has the following slope in the $\theta_L-\widetilde{w}_L$ space:

$$\left.\frac{\mathrm{d}\widetilde{w}_L}{\mathrm{d}\theta_L}\right|_{\mathrm{JC}} = -\frac{(1-I)(1-\beta_L)u_L}{1+\frac{1}{1-I}\left(\frac{1}{\rho}-(\eta-1)I\right)}\,\frac{\widetilde{w}_L}{\theta_L} < 0.$$

The JC is convex in the $\theta_L - \widetilde{w}_L$ space, because

$$\left.\frac{\mathrm{d}^2\widetilde{w}_L}{\mathrm{d}\theta_L^2}\right|_{\mathrm{JC}} = -\frac{(1-I)(1-\beta_L)u_L}{1+\frac{1}{1-I}\left(\frac{1}{\rho}-(\eta-1)I\right)}\frac{\widetilde{w}_L}{\theta_L^2}$$
$$\cdot\left(\frac{\mathrm{d}\ln u_L}{\mathrm{d}\ln\theta_L} - \frac{\widetilde{w}_L}{\theta_L}\left(\frac{(1-I)(1-\beta_L)u_L}{1+\frac{1}{1-I}\left(\frac{1}{\rho}-(\eta-1)I\right)}\frac{\widetilde{w}_L}{\theta_L}+1\right)\right) > 0,$$

with $\frac{\mathrm{d}\ln u_L}{\mathrm{d}\ln\theta_L} = -(1-\beta_L)(1-u_L) < 0$. Using the findings of Lemma 4.1 in eq. (4.34), the WC has the following slope in the $\theta_L - \widetilde{w}_L$ space:

$$\left.\frac{\mathrm{d}\widetilde{w}_L}{\mathrm{d}\theta_L}\right|_{\mathrm{WC}} = \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L}}\frac{\widetilde{w}_L}{\theta_L} = \beta_L\frac{\widetilde{w}_L}{\theta_L} > 0.$$

The WC is concave in the $\theta_L - \widetilde{w}_L$ space, because

$$\left.\frac{\mathrm{d}^2\widetilde{w}_L}{\mathrm{d}\theta_L^2}\right|_{\mathrm{WC}} = \beta_L\frac{\widetilde{w}_L}{\theta_L^2}(\beta_L-1) < 0.$$

The shift of the WC due to a change in γ_L is given by

$$\left.\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\gamma_L}\right|_{\mathrm{d}\ln\theta_L=0} = \frac{\varepsilon_{\widetilde{z}_L,\gamma_L}}{1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L}} = \frac{\gamma_L}{1-\gamma_L}\frac{w_L}{\widetilde{w}_L-w_L} > 0.$$

<u>The task-based matching model with exogenous and constant task allocation</u>

Using the findings in Corollary 4.2.3, the JC has the following slope in the $\theta_L - \widetilde{w}_L$ space:

$$\left.\frac{\mathrm{d}\widetilde{w}_L}{\mathrm{d}\theta_L}\right|_{\mathrm{JC}} = -(1-I)(1-\beta_L)u_L\frac{\widetilde{w}_L}{\theta_L} < 0.$$

The JC is convex in the $\theta_L - \widetilde{w}_L$ space, because

$$\left.\frac{\mathrm{d}^2\widetilde{w}_L}{\mathrm{d}\theta_L^2}\right|_{\mathrm{JC}} = -(1-I)(1-\beta_L)u_L\frac{\widetilde{w}_L^2}{\theta_L^3}\left(\frac{\mathrm{d}\ln u_L}{\mathrm{d}\ln\theta_L} - (1-I)(1-\beta_L)u_L\frac{\widetilde{w}_L}{\theta_L} - 1\right) > 0,$$

with $\frac{\mathrm{d}\ln u_L}{\mathrm{d}\ln\theta_L} = -(1-\beta_L)(1-u_L) < 0$. Using the findings in Corollary 4.2.3, it becomes clear that the expression of the WC is similar to the WC in case of the task-based matching model with constant labor demand elasticity. Thus, the expressions for slope, curvature and the shift in the WC due to a change in γ_L are also equal.

4.A.9 Change in Matching Efficiency ζ

Keeping r, q_L, s_L and γ constant, the modified wage equation in log differences follows from eq. (4.19) and eq. (4.26) as

$$\mathrm{d}\ln\widetilde{w}_L = \frac{1}{1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L}}\Big[\varepsilon_{\widetilde{z}_L,\theta_L}\mathrm{d}\ln\theta_L - \frac{1}{\beta_L}\varepsilon_{\widetilde{z}_L,\theta_L}\mathrm{d}\ln\zeta + \varepsilon_{\kappa_L,I}\,\mathrm{d}\ln I\Big], \tag{4.A.36}$$

where $\varepsilon_{\widetilde{z}_L,\widetilde{w}_L}$, $\varepsilon_{\widetilde{z}_L,\theta_L}$ and $\varepsilon_{\kappa_L,I}$ are defined in eqs. (4.A.24) - (4.A.27). Keeping in mind that $p_L = \zeta\theta_L^{1-\beta_L}$, from eq. (4.25) follows

$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)u_L}\mathrm{d}\ln L - \frac{1}{1-\beta_L}\mathrm{d}\ln\zeta. \tag{4.A.37}$$

Insert eq. (4.A.31) in eq. (4.A.30) and insert the result for $\mathrm{d}\ln L$ in eq. (4.A.37) leads to the following modified job creation equation:

$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)\,u_L}\left[-\frac{1}{1-I}\mathrm{d}\ln\widetilde{w}_L + \frac{1}{1-I}\mathrm{d}\ln I\right] - \frac{1}{\beta_L}\mathrm{d}\ln\zeta. \tag{4.A.38}$$

Combining eqs.(4.A.36), (4.A.38) and (4.30) leads to the following comparative static results:

$$\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\zeta} = -\frac{1}{(1-\beta_L)\beta_L}\,|\Phi|\frac{\Gamma}{|\Phi|+\Gamma}$$
$$\begin{cases} <0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \\ & \vee \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \quad\wedge\quad |\Phi|<|\Gamma|\right) \\ >0, & \text{otherwise.} \end{cases}$$

$$\frac{\mathrm{d}\ln\theta_L}{\mathrm{d}\ln\zeta} = \frac{1}{1-\beta_L}\left[\frac{1}{\beta_L}\frac{\Gamma}{|\Phi|+\Gamma} - 1\right]$$
$$\begin{cases} > 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \quad \wedge \quad \beta_L|\Phi| < (1-\beta_L)\Gamma \\ < 0, & \text{otherwise.} \end{cases}$$

$$\frac{\mathrm{d}\ln I}{\mathrm{d}\ln\zeta} = -\frac{1}{1-I}\frac{1}{\varepsilon_{\bar{\alpha},I}}\frac{\mathrm{d}\ln\widetilde{w}_L}{\mathrm{d}\ln\zeta} = \frac{1}{1-I}\frac{1}{\varepsilon_{\bar{\alpha},I}}\frac{1}{(1-\beta_L)\beta_L}|\Phi|\frac{\Gamma}{|\Phi|+\Gamma}$$
$$\begin{cases} > 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \\ & \vee \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > \frac{(1-\varepsilon_{\widetilde{z}_L,\widetilde{w}_L})(\varepsilon_{L,\widetilde{w}_L}-1)}{\sigma-1} \quad \wedge \quad |\Phi| < |\Gamma|\right) \\ < 0, & \text{otherwise.} \end{cases}$$

References

Acemoglu, D., and Autor, D. (2011). Skills, Tasks and Technologies: Implications for Employment and Earnings. In D. Card and O. Ashenfelter (Eds.), *Handbook of Labor Economics* (Vol. 4b, pp. 1043–1171). Elsevier.

Acemoglu, D., and Restrepo, P. (2018a). Modeling Automation. *AEA Papers and Proceedings*, *108*, 48–53.

Acemoglu, D., and Restrepo, P. (2018b). The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment. *American Economic Review*, *108*(6), 1488–1542.

Acemoglu, D., and Restrepo, P. (2021). Task, Automation and the Rise in US Wage Inequality. *NBER Working Papers*(28920).

Battisti, M., Felbermayr, G., Peri, G., and Poutvaara, P. (2018). Immigration, Search and Redistribution: A Quantitative Assessment of Native Welfare. *Journal of the European Economic Association*, *16*(4), 1137–1188.

Beissinger, T., and Büsse, O. (2001). Bismarck Versus Beveridge: Which Unemployment Compensation System Is More Prone to Labor Market Shocks? *FinanzArchiv*, *58*, 78–102.

Beissinger, T., and Egger, H. (2004). Dynamic Wage Bargaining if Benefits are Tied to Individual Wages. *Oxford Economic Papers*, *56*(3), 437-460.

Blanchard, O. J., and Wolfers, J. (2000). The Role of Shocks and Institutions in the Rise of European Unemployment: The Aggregate Evidence. *Economic Journal*, *110*, 1–33.

Bradley, J., and Kügler, A. (2019). Labor market reforms: An evaluation of the hartz policies in germany. *European Economic Review*, *113*, 108–135.

Chassamboulli, A., and Palivos, T. (2014). A Search-Equilibrium Approach to the Effects of Immigration on Labor Market Outcomes. *International Economic Review*, *55*(1), 111–129.

Costinot, A., and Vogel, J. (2010). Matching and Inequality in the World Economy. *Journal of Political Economy*, *118*(4), 747–786.

Delacroix, A. (2006). A Multisectorial Matching Model of Unions. *Journal of Monetary Economics*, *53*, 573–596.

De Loecker, J., and Eeckhout, J. (2018). Global market power. *NBER Working Paper Series*(24768).

Fahr, R., and Sunde, U. (2009). Did the Hartz reforms speed-up the matching process? A macro-evaluation using empirical matching functions. *German economic review*, *10*(3), 284–316.

Grossman, G. M., and Rossi-Hansberg, E. (2008). Trading Tasks: A Simple Theory of Offshoring. *American Economic Review*, *98*(5), 1978–1997.

Hartung, B., Jung, P., and Kuhn, M. (2022). Unemployment insurance reforms and labor market dynamics. *mimeo*.

Hémous, D., and Olsen, M. (2022). The Rise of the Machines: Automation, Horizontal Innovation, and Income Inequality. *American Economic Journal: Macroeconomics*, *14*(1), 179–223.

Hertweck, M. S., and Sigrist, O. (2013). The aggregate effects of the hartz reforms in germany. *SOEPpapers on Multidisciplinary Panel Data Research 532*.

Hobijn, B., and Şahin, A. (2009). Job-finding and separation rates in the OECD. *Economics Letters*, *104*(3), 107–111.

Hochmuth, B., Kohlbrecher, B., Merkl, C., and Gartner, H. (2021). Hartz IV and the Decline of German Unemployment: A Macroeconomic Evaluation. *Journal of Economic Dynamics and Control*, *127*, 104114.

Jaimovich, N., Saporta-Eksten, I., Siu, H., and Yedid-Levi, Y. (2021). The Macroeconomics of Automation: Data, Theory, and Policy Analysis. *Journal of Monetary Economics*, *122*, 1–16.

Klinger, S., and Rothe, T. (2012). The impact of labour market reforms and economic performance on the matching of the short-term and the long-term unemployed. *Scottish journal of political economy*, *59*(1), 90–114.

Krause, M. U., and Uhlig, H. (2012). Transitions in the german labor market: Structure and crisis. *Journal of Monetary Economics*, *59*(1), 64-79.

Krebs, T., and Scheffel, M. (2013). Macroeconomic evaluation of labor market reform in germany. *IMF Economic Review*, *61*(4), 664–701.

Krusell, P., Ohanian, L. E., Ríos-Rull, J.-V., and Violante, G. L. (2000). Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis. *Econometrica*, *68*(5), 1029–1053.

Launov, A., and Wälde, K. (2013). Estimating incentive and welfare effects of nonstationary unemployment benefits. *International Economic Review*, *54*(4), 1159–1198.

Ljungquist, L., and Sargent, T. J. (1998). The European Unemployment Dilemma. *Journal of Political Economy*, *106*, 514–550.

Marczak, M., Beissinger, T., and Brall, F. (2022). Technical change, task allocation, and labor unions. *IZA Discussion Paper No. 15632*.

Mortensen, D. T., and Pissarides, C. A. (1994). Job Creation and Job Destruction in the Theory of Unemployment. *The Review of Economic Studies*, *61*(3), 397-415.

Mortensen, D. T., and Pissarides, C. A. (1999). New developments in models of search in the labor market. *Handbook of labor economics*, *3*, 2567–2627.

Petrongolo, B., and Pissarides, C. A. (2001). Looking into the Black Box: A Survey of the Matching Function. *Journal of Economic Literature*, *39*(2), 390–431.

Pissarides, C. A. (2000). *Equilibrium Unemployment Theory* (2nd ed.). Cambridge, Mass. and London: MIT Press.

Postel-Vinay, F., and Robin, J.-M. (2002). Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica*, *70*(6), 2295–2350.

Pries, M., and Rogerson, R. (2005). Hiring policies, labor market institutions, and labor market flows. *Journal of Political Economy*, *113*(4), 811–839.

Rosen, S. (1978). Substitution and Division of Labor. *Economica*, *45*(179), 235–250.

Yashiv, E. (2004). Macroeconomic policy lessons of labor market frictions. *European Economic Review*, *48*(2), 259-284.

Zanetti, F. (2011). Labor market institutions and aggregate fluctuations in a search and matching model. *European Economic Review*, *55*(5), 644-658.

Chapter 5

Conclusions

The recent developments in robotics and artificial intelligence have once again raised the question about the labor market effects of technological change. The discussion ranges from a rather pessimistic perspective, envisioning a future where human labor becomes entirely obsolete, to a relatively optimistic view in which technological advancements create new tasks in which human labor retains a comparative advantage and adjusts to the new requirements. Nevertheless, technological change, whether augmenting or replacing labor, creates winners and losers, at least in relative terms. From a political perspective, it is crucial to address the challenges arising from technological change like rising wage inequality or unemployment. This thesis provides new insights into the impact of technological change and a cut in unemployment benefits on labor market outcomes. It contributes to the empirical literature on sources of wage inequality in Germany by uncovering the relative importance of automation and robotization on wage dispersion. Moreover, the focus in the literature has primarily been on perfect competition models that consider the impact of SBTC on the allocation of task among different worker groups. Assuming search and matching frictions and collective bargaining, the thesis demonstrates that SBTC influences the wage-setting power of labor unions, which substantially affects the labor market outcomes of SBTC. Furthermore, the thesis contributes to the literature on the impact of unemployment benefit reforms by reexamining the labor market effects of the Hartz IV reform under consideration of endogenous task allocation.

The second chapter of the thesis reveals the contribution of automation technologies on wage inequality within the German manufacturing sector by using a detailed decomposition analysis based on RIF regressions. The analysis provides descriptive evidence on rising wage inequality during the mid-1990s and 2000s and rather constant or even declining wage dispersion for the recent years until 2017. The proposed measure of automation threat combines occupation- and requirement-specific scores of automation risk with sector-specific robot densities to comprehensively address the dimensions of automation and robotization. Using this novel measure of automation threat, the analysis reveals substantial differences in the level of wage inequality between occupations with high automation threat and occupations with middle and low automation threat. The wage distribution in the group with the highest automation threat is the most equal and the share of this group declines over time, indicating that this trend contributes to overall wage inequality. The detailed decomposition analysis confirms these findings and identifies two channels through which automation threat affects wage inequality. First, the observed compositional changes among different automation threat groups lead to increasing wage inequality. This automation-related composition effect is apparent between 1996 and 2010 and becomes even more important in the time period until 2017. Second, an increasing wage dispersion between workers in occupations with low automation threat (containing especially non-routine tasks) and workers in occupations with high automation threat (containing especially routine tasks) contributes to rising wage inequality. This is in line with the predictions of RBTC, where technology substitutes routine tasks and complements non-routine tasks. This automation-related wage structure effect is prevalent in the 1990s and 2000s, while there is no evidence that this effect has played a significant role in the more recent time period.

In Chapter 3, a novel modeling framework for the analysis of SBTC is developed, combining the task approach, wage setting by labor unions, as well as search and matching frictions. The important insight from this analysis is that changes in the task allocation between low- and high-skilled labor have an impact on the effective wage-setting power of labor unions. The effect of such a change in task allocation on the labor demand elasticity, and consequently on the labor unions' wage markup, is ambiguous and crucially depends on the shape of the relative task productivity schedule. This ambiguity carries over to the general equilibrium that is condensed into a two equation system reflecting job creation by firms and wage claims of labor unions. While standard matching models predict that an increase in labor market tightness results in higher wage pressure along a positively sloped wage curve, the task-based matching model also allows for a downward-sloping wage curve. This implies that SBTC may either harm low-skilled workers in terms of unemployment or lower real wages, unlike the results in the conventional SBTC model. Calibrating the model to German and French data for the two time periods 1995-2005 and 2010-2017, suggests that the impact of SBTC may even change its sign over time. In the first period, SBTC increases low-skilled unemployment in both countries. In the second period, SBTC still increases low-skilled unemployment in Germany, but reduces it in France. For both countries and periods, the skill premium increases. The driving force behind these simulation outcomes is the decline in the labor demand elasticity caused by SBTC, particularly the strength of this decline. It depends on the shape of the task productivity schedule and the task allocation of low- and high-skilled workers. These factors differ across both countries and time periods.

The fourth chapter revisits the labor market impacts of the Hartz IV reform by using a modified version of the task-based matching model developed in Chapter 3. The model is extended by monopolistic competition in the goods market and earnings-related unemployment benefits. Additionally, the analysis introduces a specific relative task productivity schedule, resulting in a constant labor demand elasticity. In contrast to the prevailing theoretical literature on unemployment benefit reforms that assumes conventional production functions, the more detailed modeling of the production process in this analysis uncovers task reallocation effects resulting from a reduction in unemployment benefits. To highlight the importance of endogenous task allocation, the task-based matching model with exogenous and constant task allocation is considered for comparison. In this case, task allocation is no longer determined endogenously from profit maximization and there is no task reallocation between the two types of labor. Both model variants are calibrated to quantify the long-run effects of the Hartz IV reform and to emphasize the role of endogenous task allocation. The quantitative analysis reveals a remarkable decrease in the low-skilled unemployment rate by 4 percentage points resulting from Hartz IV. In the case of exogenous and constant task allocation, the decrease is limited to 3.4 percentage points and the effect on low- and high skilled wages is more pronounced, resulting in a larger increase in the skill premium. Consequently, disregarding endogenous task allocation within this calibration framework would underestimate the effect of the Hartz IV reform on low-skilled unemployment and overestimate the effect on wages and the skill premium. These calibration results emphasize the importance of considering endogenous task allocation in the evaluation of labor market reforms.