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Modeling a Superconducting Triplet Spin Valve with Several Layers of a Superconductor

R. R. Gaifullin[a], *, R. G. Deminov[a], V. N. Kushnir[b, c], M. Yu. Kupriyanov[d], A. A. Golubov[e, f], and L. R. Tagirov[a, g]

[a] *Institute of Physics, Kazan Federal University, Kazan, 420008 Russia*
[b] *Belarus State University of Informatics and Radioelectronics, Minsk, 220013 Belarus*
[c] *Belarusian State University, Minsk, 220030 Belarus*
[d] *Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, 119992 Russia*
[e] *University of Twente, Enschede 7500 AE, Netherlands*
[f] *HSE University, Moscow, 101000 Russia*
[g] *Zavoisky Physical-Technical Institute, Kazan Scientific Center, Russian Academy of Sciences, Kazan, 420029 Russia*
**e-mail: gaifullin.rashid@gmail.com*

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Abstract—A matrix solution to Usadel linearized equations is used to obtain the critical temperature and distribution of singlet pairing components of a superconductor/ferromagnetic/superconductor/ferromagnetic structure with nonideal boundaries. There is a transition from the π- to the 0-phase state between the superconductor layers upon varying the angle between the magnetizations of ferromagnetic layers in such a structure.

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INTRODUCTION

We studied the critical temperature of the transition to superconducting state T_{cr} and the distribution of singlet components of the superconducting pairing of symmetric Sc/Fm/Sc, asymmetric Sc1/Fm/Sc2, and Sc1/Fm1/Sc2/Fm2 heterostructures (where Sc is a singlet superconductor, and Fm is a ferromagnetic metal) with nonideal boundaries, in which the long-range triplet superconducting component is generated at noncolinear directions of the magnetizations of ferromagnetic layers [1]. It was shown in [2] that the critical temperature of a three-layer Sc/Fm1/Fm2 heterostructure can be a non-monotonous function of angle α between the magnetizations of ferromagnetic layers, in contrast to the monotonous behavior of $T_{cr}(\alpha)$ in a three-layer Fm1/Sc/Fm2 heterostructure [3]. The authors of [4] analyzed the effect an additional layer of a superconductor or a normal metal between ferromagnetic layers have on the direct and triplet regimes of a spin valve when these layers were considered in an approximation of ideal boundaries. The parameter of quantum mechanical transparency of boundary $\gamma_b = 0$, the constants of diffusion, and the specific resistances of all layers were taken the same. This work considers the effect of an additional layer of superconducting metal with parameters of the materials and boundaries from [5]. It is therefore relevant for both determining and modeling parameters of real structures, and for revealing features in their behavior.

MODEL AND NUMERICAL TECHNIQUE

The dependences of critical temperature T_{cr} of symmetrical Sc/Fm/Sc films [6] with axis x, perpendicular to a layer's surface and infinite in directions y and z, were initially modeled using the matrix approach [7–9] in the 0- and π-phase states to compare them to asymmetrical Sc1/Fm/Sc2 heterostructures. The critical temperature of Sc1/Fm1/Sc2/Fm2 structures was then modeled. The direction of the exchange field of the Fm1 layer lay in plane yz, and $\vec{h}(x) = E_{ex}\vec{m}(x) = (0, E_{ex}\sin\alpha, E_{ex}\cos\alpha)$. The exchange field of the Fm2 layer was directed along axis z, $\vec{h}(x) = (0, 0, E_{ex})$, $\vec{m}(x)$ is the unit vector codirected to the vector of magnetization, and E_{ex} is the energy of exchange interaction. Angle α varies from 0 (parallel orientation) to π (antiparallel orientation). The critical temperature of the spin valve is determined in the diffusion limit. The Usadel equations for the superconducting and ferromagnetic layer that can be used in this limit [7],

$$\begin{cases}\left(-\frac{D_{Sc}}{2}\partial_x^2+|\omega|\right)F_{0,\omega}(x)=\Delta(x)\\ \left(-\frac{D_{Sc}}{2}\partial_x^2+|\omega|\right)\vec{F}_{1,\omega}(x)=\vec{0},\end{cases}\quad(1)$$

$$\begin{cases}\left(-\frac{D_{Fm}}{2}\partial_x^2+|\omega|\right)F_{0,\omega}(x)+i\,\mathrm{sgn}(\omega)\left(\vec{h}(x),\vec{F}_{1,\omega}(x)\right)=0\\ \left(-\frac{D_{Fm}}{2}\partial_x^2+|\omega|\right)\vec{F}_{1,\omega}(x)+i\,\mathrm{sgn}(\omega)\vec{h}(x)F_{0,\omega}(x)=\vec{0},\end{cases}\quad(2)$$

are solved using the matrix technique: $D_{Sc(Fm)}$ denotes the coefficients of diffusion of the superconducting and ferromagnetic materials; $\omega=\pi k_B T(2n+1)$ are the Matsubara frequencies; $n=0,1,\ldots,n_D$; n_D is the integer part of $\omega_D/2\pi k_B T-0.5$, where ω_D is the Debye frequency; $\Delta(x)=\pi k_B T\lambda\sum_\omega F_{0,\omega}(x)$ is the parameter of order in the superconductor; λ is the constant of effective electron–electron interaction; $\vec{\mathcal{F}}_\omega(\vec{r})=(F_{0,\omega}\sigma_0+\vec{F}_{1,\omega}\vec{\sigma})\sigma_3=(F_{0,\omega}\sigma_0+F_{12,\omega}\sigma_2+F_{13,\omega}\sigma_3)\sigma_3$ is the anomalous Green function; $F_{0,\omega}$, $F_{12,\omega}$, $F_{13,\omega}$ are the singlet and triplet components; and σ_i are the Pauli matrices; $\hbar=1$ (comment for the editor, symbol $\hbar$ is the reduced Planck constant). Equations (1), (2) are supplemented by conditions on the external boundaries,

$$\begin{aligned}\partial_x f_{0,n}(0)=\partial_x f_{0,n}(L)=0,\\ \partial_x\vec{f}_{1,n}(0)=\partial_x\vec{f}_{1,n}(L)=\vec{0},\end{aligned}\quad(3)$$

on contact boundaries

$$\begin{aligned}&\rho^{-1}(x_i+0)\partial_x f_{0,n}(x_i+0)=\rho^{-1}(x_i-0)\partial_x f_{0,n}(x_i-0),\\ &f_{0,n}(x_i+0)\\ &=f_{0,n}(x_i-0)+\gamma_{bFmSc}\xi_{Sc}\frac{\rho_{Fm}}{\rho(x_i-0)}\partial_x f_{0,n}(x_i-0),\end{aligned}\quad(4)$$

for the singlet component, and the same conditions for the triplet components; $\rho(x)=\rho_{Sc(Fm)}$ is the specific low temperature resistance of superconducting and ferromagnetic materials; $\xi_{Sc(Fm)}=\sqrt{D_{Sc(Fm)}/2\pi k_B T_{crSc}}$ are their coherence lengths, and T_{crSc} is the temperature of the transition to the superconducting state of massive superconductor. The characteristic equation is

$$\begin{aligned}&\psi\left(\frac{\omega_D}{2\pi k_B T}+1+\mu_{Sc}^{(k)}(T)\right)-\psi\left(\frac{1}{2}+\mu_{Sc}^{(k)}(T)\right)\\ &=\psi\left(\frac{\omega_D}{2\pi k_B T_{crSc}}+1\right)-\psi\left(\frac{1}{2}\right),\end{aligned}\quad(5)$$

where $\psi(x)$ is the digamma function. The values of the parameters in this work were $T_{crSc}=7$ K, $\rho_{Sc}=7.5\ \mu\Omega$ cm, $\xi_{Sc}=8.9$ nm, $E_{ex}=150$ K, $\rho_{Fm}=60\ \mu\Omega$ cm, and $\xi_{Fm}=7.6$ nm.

RESULTS AND DISCUSSION

The distribution of pair wave function characterizes the phase state and behavior of a critical temperature as a function of the thickness of the ferromagnetic layer of symmetric Sc/Fm/Sc, asymmetric Sc1/Fm/Sc2, and Sc1/Fm1/Sc2/Fm2 heterostructures (Figs. 1a–3a). The physically observed critical temperature is the higher of the two. The 0-phase state (where the phase difference between the superconducting pairing functions in superconducting layers is zero) and the π-phase state (with different signs of pairing functions in superconducting layers) are illustrated in Fig. 1b in the transverse cross section of the structure. The thickness of the Sc1 layer is constant for asymmetric Sc1/Fm/Sc2 structures, while the thickness of Sc2 layer is lower in the left side of Fig. 2a and higher in the right side. The distributions of the singlet component for each of the two critical temperatures are plotted for different thicknesses $d_{Fm}/\xi_{Fm}=0.75, 1.0$ of the ferromagnetic layer of asymmetric structure Sc1/Fm/Sc2. The solid line shows the distribution at higher T_{cr}, and the dotted line is for lower temperatures. Normalizing denominator $\sum f_0$ of the singlet components of the asymmetric structure Sc1/Fm/Sc2 at higher T_{cr} belongs to the thicker superconducting layer in Fig. 2b.

In the Sc1/Fm/Sc2 structure, the π state (with the π-phase difference between the superconducting layers) can have higher critical temperatures than the 0 state (with zero phase difference) (Figs. 1 and 2). The Sc1/Fm1/Sc2/Fm2 structure probably also has spin valve regimes in the π state with higher critical temperatures than that in the Sc/Fm1/Fm2 structure. In the Sc1/Fm1/Sc2/Fm2 structure, the thickness of the Sc1 layer is the same as for the Sc1/Fm/Sc2 in Figs. 1 and 2, and the thicknesses of the Fm1 and Fm2 layers are the same (Fig. 3). The distribution of the singlet components is considered the main reason for the behavior of the critical temperature of the Sc1/Fm1/Sc2/Fm2 structure as a function of the angle α between the magnetic moments of the ferromagnetic layers. The thickness of the ferromagnetic layer between superconductors at which the phase state of the structure changes upon varying the angle between the magnetizations of the ferromagnetic layers is shifted toward higher values. There is a region of thickness $d_{Fm}/\xi_{Fm}=0.9-0.95$ in a ferromagnetic layer (Fig. 3a) at which the phase state changes from π to 0. Despite our expectations, the obtained regimes were characterized by lower critical temperatures than when the change of the phase state upon varying the angle between the magnetization of ferromagnetic layers

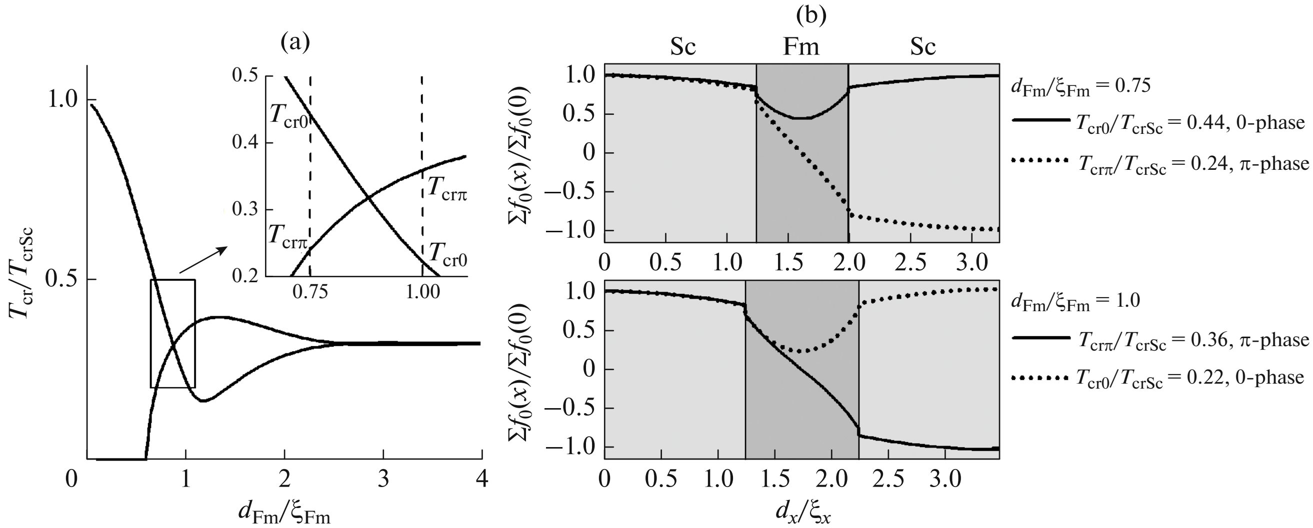

Fig. 1. (a) T_{cr} as a function of thickness d_{Fm} of the symmetric Sc/Fm/Sc structure, $d_{Sc}/\xi_{Sc} = 1.24$, $\gamma_{bFmSc} = 0.07$. (b) Distribution of spin singlet $\sum f_0(x)/\sum f_0(0)$ components of superconducting pairing in the transverse cross section of the structure in Fig. 1a.

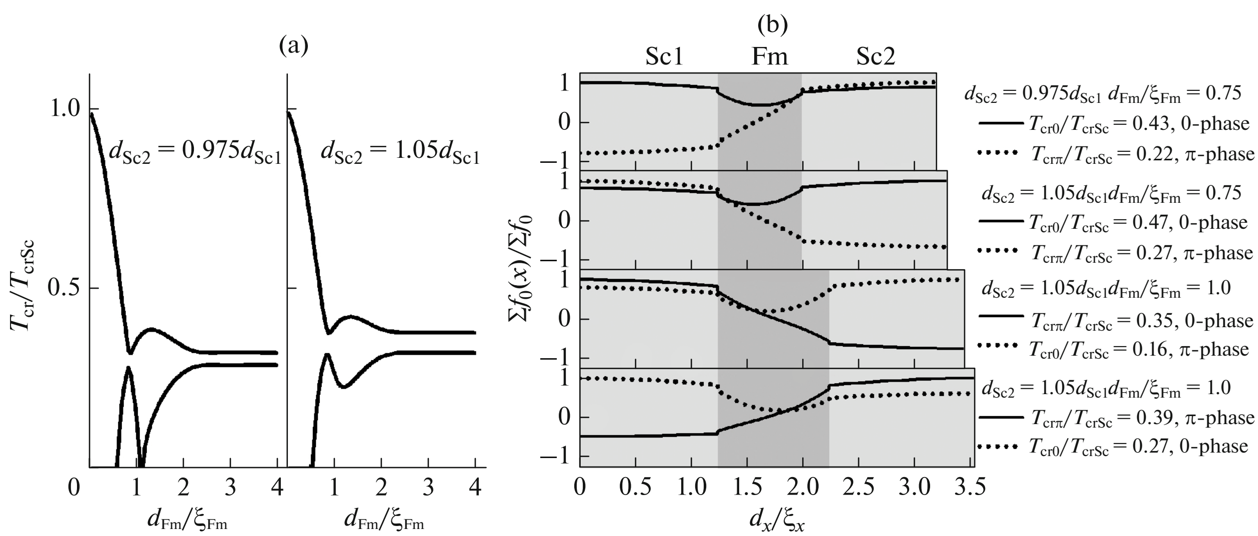

Fig. 2. (a) T_{cr} as a function of thickness d_{Fm} of the asymmetric Sc/Fm/Sc structure, $d_{Sc1}/\xi_{Sc1} = 1.24$, $\gamma_{bFmSc} = 0.07$. (b) Distribution of spin singlet $\sum f_0(x)/\sum f_0(0)$ components of superconducting pairing in the transverse cross section of the structure in Fig. 2a.

would shift toward smaller thickness, and the phase state would change from 0 to π. Adding the Sc2 layer to the Sc/Fm1/Fm2 structure varied the direct, triplet, and inverse regimes to a direct regime with higher critical temperature than in the Sc/Fm1/Fm2 structure (insets in Fig. 3).

The triplet regime is obtained by transition from the π- to the 0-phase state (Fig. 4a). Distributions of the singlet component in the transverse cross section for the higher of the two critical temperatures are plotted for the parallel and antiparallel orientations of the magnetizations of the ferromagnetic layers of the Sc1/Fm1/Sc2/Fm2 structure (Fig. 4b).

CONCLUSIONS

We determined the critical temperature of a nonsymmetrical Sc1/Fm1/Sc2 three-layer heterostructure as a function of layer thickness and the distribution of the spin-singlet component of the condensate

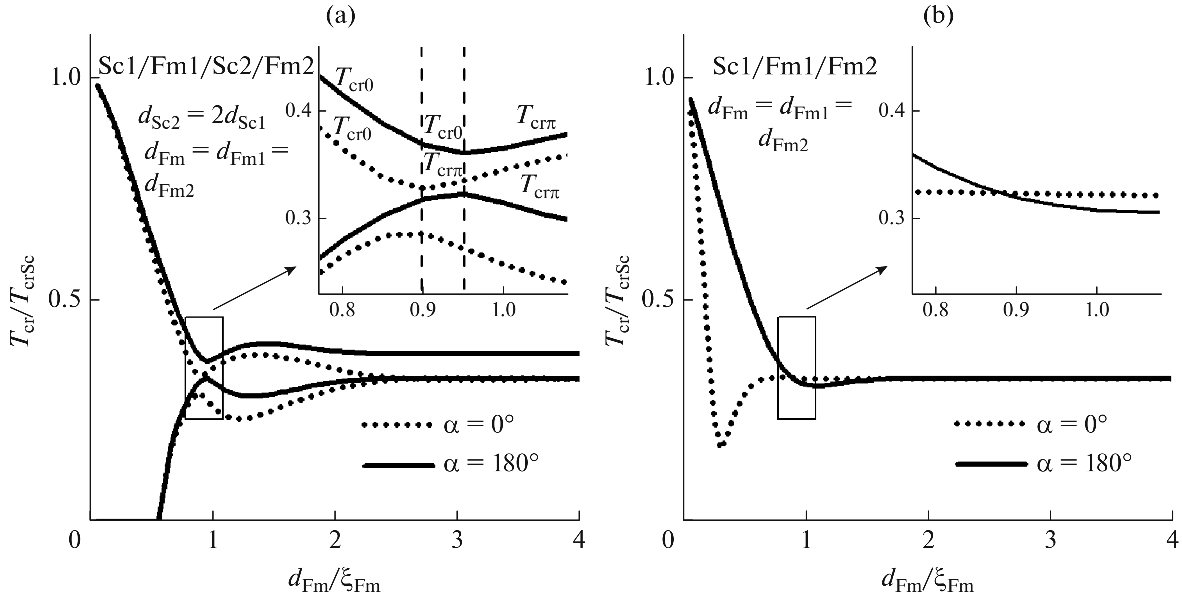

Fig. 3. T_{cr} as a function of thickness d_{Fm} for parallel and antiparallel configuration of magnetic moments of ferromagnetic layers for the (a) Sc1/Fm1/Sc2/Fm2 and (b) Sc/Fm1/Fm2 structures. The other parameters are $d_{Sc1}/\xi_{Sc1} = 1.24$, $\gamma_{bFmSc} = 0.07$.

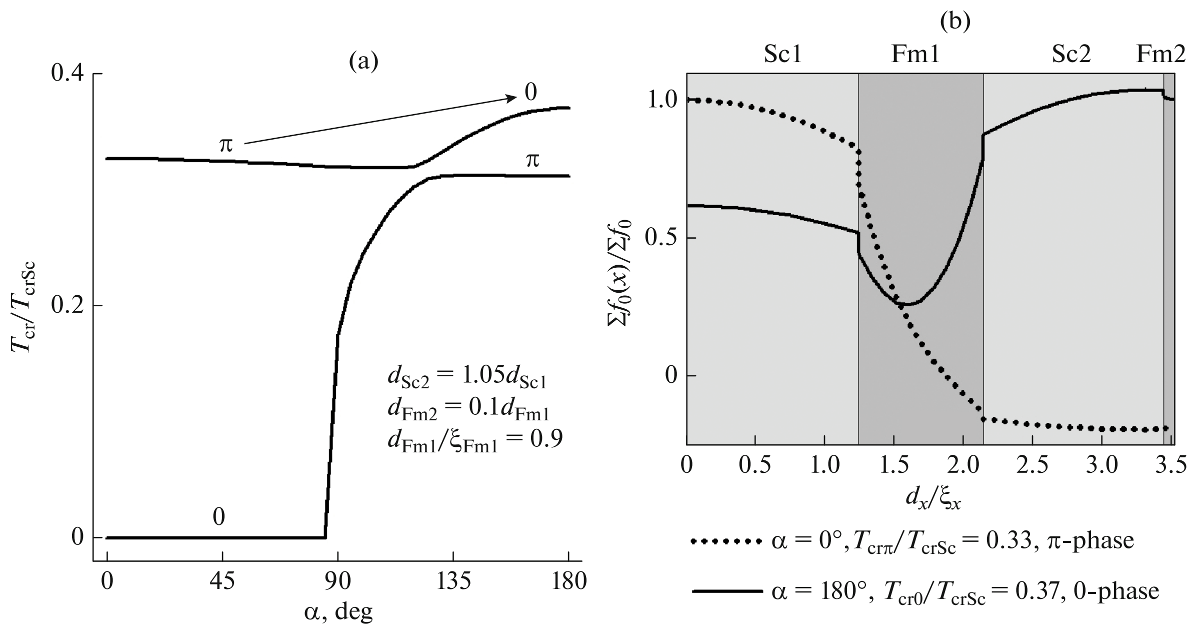

Fig. 4. (a) Critical temperature T_{cr} of the Sc1/Fm1/Sc2/Fm2 structure as a function of angle α, $d_{Sc1}/\xi_{Sc1} = 1.24$, $\gamma_{bFmSc} = 0.07$. (b) Distribution of spin singlet components $\sum f_0(x)/\sum f_0$ of superconducting pairing in the transverse cross section of the Sc1/Fm1/Sc2/Fm2 structure in Fig. 4a.

function across the layers of the structure. We also obtained the transition from the π- to the 0-phase state between the superconductor layers upon varying angle α between the magnetizations of the ferromagnetic layers in the Sc1/Fm1/Sc2/Fm2 heterostructure.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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