

On the moments of Cox rate-and-state models

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ABSTRACT: Rate-and-state models are widely used physical models for the relation between changes in pore pressure due to fluid injection or gas extraction and the induced seismic hazard in a field. We consider the modification where the pore pressure measurements are affected by noise and provide explicit expressions for the first and second moments of the state variable. We show that when the pressure increases, there is positive correlation. In the case of decreasing pressure, both positive and negative correlation is possible. Using the delta method, approximate first and second moments of the rate variable are derived and compared to empirical moments.

Keywords: Cox process, induced seismicity, pore pressure, rate-and-state model.

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1 Introduction

The study of induced earthquakes caused by extraction or injection of fluids or gases is an important research topic. In the Netherlands, the Groningen gas field discovered in the late 1950s was viewed in a mostly positive light for decades until the occurrence of earthquakes from the 1990s onward caused a shift in public opinion that eventually led to the closing of the field. Thus, for planning purposes, it is important to be able to predict seismic hazard

based on field measurements, for instance of pore pressure or, equivalently, Coulomb stress. One of the most widely used methodologies to do so is the rate-and-state model [1, 4, 7] which is now considered to be the state-of-the-art technique [5].

In the rate-and-state model, the earthquake intensity $\lambda(t)$, the rate at time t , is assumed to be inversely proportional to a state variable $\Gamma(t)$, that is,

$$\lambda(t) = \frac{1}{\Gamma(t)}, \quad t \geq 0.$$

The state variable $\Gamma(t)$ is defined by the ordinary differential equation

$$d\Gamma(t) = \alpha [dt + \Gamma(t)dX(t)],$$

where $X(t) \geq 0$ is the pore pressure at time t and $\alpha > 0$ is a parameter. Multiplying both sides by $\exp(-\alpha X(t))$ and discretising in time steps of length $\Delta > 0$, we obtain the corresponding Euler difference equation

$$\Gamma((k+1)\Delta) = (\Gamma(k\Delta) + \Delta\alpha) \exp[\alpha \{X((k+1)\Delta) - X(k\Delta)\}]. \quad (1)$$

Additionally, a spatial location may be added to the model. For a fuller discussion, we refer to [5].

In this paper, we consider stochastic rate-and-state models in which noise is added to the pore pressure X . After the formal definition of this model in Section 2, we give explicit expressions of the first and second moments of the random state variable. We then apply the delta method to approximate the first and second moments of the random rate variable. The paper finishes with an illustration of the accuracy of the approximations for various pore pressure scenarios.

2 Cox rate-and-state models

Let Ψ be a Cox process (cf. [3, Section 5.2]) on $k\Delta$, $k = 0, 1, 2, \dots$, with driving random measure

$$\Lambda(k\Delta) = \frac{1}{\Gamma(k\Delta)}, \quad k \in \mathbb{N}_0. \quad (2)$$

In contrast to the classic deterministic case discussed in Section 1, we assume that the state Γ is a stochastic process defined by equation (1) for a deterministic starting state $\gamma_0 > 0$ and random pore pressure X of the form $X(k\Delta) = m(k\Delta) + E(k\Delta)$. Here, $m \geq 0$ is a deterministic mean and E is Gaussian white noise. In other words, each $E(k\Delta)$ is normally distributed with mean zero and variance σ^2 , and for $k \neq k'$, $E(k\Delta)$ and $E(k'\Delta)$ are mutually independent.

The Euler difference equation (1) can be solved explicitly. Indeed, for each $k \in \mathbb{N}$,

$$\Gamma(k\Delta) = \exp[\alpha X(k\Delta)] \left\{ \alpha\Delta \sum_{i=0}^{k-1} \exp[-\alpha X(i\Delta)] + \gamma_0 \exp[-\alpha X(0)] \right\} \quad (3)$$

under the convention that $\Gamma(0) = \gamma_0$. The Cox rate-and-state model description is complete upon requiring that $N(k\Delta)$, the number of induced earthquakes during the time period $[k\Delta, (k+1)\Delta)$, is Poisson distributed with rate parameter $\Lambda(k\Delta)\Delta$, $k = 0, 1, \dots$

3 Moments of the state variable

Since the randomness in the driving random measure (2) of our Cox process is induced by the state process Γ , we investigate its first and second-moment properties first.

Theorem 1. *Let Γ be defined by (3). Set $c = e^{\alpha^2\sigma^2}$ and define, for $i, j \in \mathbb{N}_0$,*

$$f_{ij} = \exp[\alpha \{m(i\Delta) - m(j\Delta)\}].$$

Then, for $k \in \mathbb{N}$,

$$\begin{aligned}\mathbb{E}\Gamma(k\Delta) &= c \left(\alpha\Delta \sum_{i=0}^{k-1} f_{ki} + \gamma_0 f_{k0} \right), \\ \text{Var}\Gamma(k\Delta) &= \alpha^2 \Delta^2 c^2 (c^2 - 1) \sum_{i=0}^{k-1} f_{ki}^2 + \alpha^2 \Delta^2 c^2 (c - 1) \sum_{i=0}^{k-1} \sum_{i \neq j=0}^{k-1} f_{ki} f_{kj} \\ &\quad + 2\alpha\Delta\gamma_0 c^2 f_{k0}^2 \left(c^2 - 1 + (c - 1) \sum_{i=1}^{k-1} f_{0i} \right) + \gamma_0^2 f_{k0}^2 c^2 (c^2 - 1)\end{aligned}$$

and, for $0 < k < l$,

$$\begin{aligned}\text{Cov}(\Gamma(k\Delta), \Gamma(l\Delta)) &= \alpha^2 \Delta^2 c^2 \sum_{i=0}^{k-1} \left[f_{ki} f_{li} (c - 1) - f_{li} \left(1 - \frac{1}{c} \right) \right] \\ &\quad + (2\alpha\Delta\gamma_0 + \gamma_0^2) c^2 f_{k0} f_{l0} (c - 1) - \alpha\Delta\gamma_0 c^2 f_{l0} \left(1 - \frac{1}{c} \right).\end{aligned}\quad (4)$$

Proof. Recall the moment-generating function of a normal distribution to conclude that

$$\mathbb{E}e^{tZ} = \exp(\mu t + \sigma^2 t^2 / 2)$$

when Z is normally distributed with mean μ and variance σ^2 . The formula for the mean follows immediately. Moreover, for $0 < k \leq l$,

$$\begin{aligned}\text{Cov}(\Gamma(k\Delta), \Gamma(l\Delta)) &= \alpha^2 \Delta^2 \sum_{i=0}^{k-1} \sum_{j=0}^{l-1} \text{Cov} \left(e^{\alpha[X(k\Delta) - X(i\Delta)]}, e^{\alpha[X(l\Delta) - X(j\Delta)]} \right) \\ &\quad + \alpha\Delta\gamma_0 \sum_{i=0}^{k-1} \text{Cov} \left(e^{\alpha[X(k\Delta) - X(i\Delta)]}, e^{\alpha[X(l\Delta) - X(0)]} \right) \\ &\quad + \alpha\Delta\gamma_0 \sum_{j=0}^{l-1} \text{Cov} \left(e^{\alpha[X(k\Delta) - X(0)]}, e^{\alpha[X(l\Delta) - X(j\Delta)]} \right) \\ &\quad + \gamma_0^2 \text{Cov} \left(e^{\alpha[X(k\Delta) - X(0)]}, e^{\alpha[X(l\Delta) - X(0)]} \right).\end{aligned}\quad (5)$$

Let us work out the terms in the expression on the right-hand side of equation (5) one by one.

First, consider the double sum over i and j . Because of the independence of the components of the random vector E , summands for which k, l, i and j are all different do not contribute. Therefore, for $0 < k = l$, the total contribution of the double sum is

$$\alpha^2 \Delta^2 c^2 \sum_{i=0}^{k-1} f_{ki}^2 (c^2 - 1) + \alpha^2 \Delta^2 c^2 \sum_{i=0}^{k-1} f_{ki} \sum_{j \neq i; j=0}^{k-1} f_{kj} (c - 1).$$

For $0 < k < l$, the double sum contributes non-zero entries for $i = j$ (which cannot be equal to k or l) and for $i \neq j = k$. Their total contribution is

$$\alpha^2 \Delta^2 c^2 \sum_{i=0}^{k-1} f_{ki} f_{li} (c - 1) - \alpha^2 \Delta^2 c^2 \sum_{i=0}^{k-1} f_{li} \left(1 - \frac{1}{c}\right).$$

Next consider the two single sums on the right-hand side of (5). If $0 < k = l$, they are identical and each one is equal to

$$\alpha \Delta \gamma_0 c^2 \left[f_{k0}^2 (c^2 - 1) + (c - 1) \sum_{i=1}^{k-1} f_{ki} f_{k0} \right].$$

In the case that $0 < k < l$, the sum over i has a non-zero contribution only for $i = 0$, and the sum over j has non-vanishing contributions for $j = 0$ and $j = k$. Adding these up, we obtain

$$\alpha \Delta \gamma_0 c^2 f_{l0} \left[2f_{k0} (c - 1) - \left(1 - \frac{1}{c}\right) \right].$$

Finally, the last term in the expression on the right-hand side of equation (5) reads $\gamma_0^2 f_{k0}^2 c^2 (c^2 - 1)$ for $0 < k = l$ and $\gamma_0^2 f_{k0} f_{l0} c^2 (c - 1)$ when $0 < k < l$. Expression (4) now follows from tallying up the various contributions. \square

When fluid is injected into a field, the pore pressure typically increases. In this case, the state variables are positively correlated.

Corollary 1. *Let Γ be as in Theorem 1. Then, if the function $m(k\Delta)$ is increasing in $k \in \mathbb{N}_0$, $\text{Cov}(\Gamma(k\Delta), \Gamma(l\Delta)) \geq 0$ for all $k, l \in \mathbb{N}_0$.*

Proof. Obviously, the claim holds for $k = l$. For $0 \leq i < k < l$, $\alpha^2 \Delta^2 f_{li} \{ f_{ki} c^2 (c - 1) - c(c - 1) \} \geq 0$ if $\sigma^2 = 0$ or $c f_{ki} \geq 1$. The latter condition is equivalent to

$$\alpha^2 \sigma^2 + \alpha \{ m(k\Delta) - m(i\Delta) \} \geq 0$$

and is implied by the assumption that m is increasing. By the same argument, for increasing pore pressure, $\alpha \Delta \gamma_0 f_{l0} \{ f_{k0} c^2 (c - 1) - c(c - 1) \} \geq 0$ and an appeal to (4) completes the proof. \square

When the pore pressure decreases, for example due to gas extraction, the picture is more varied.

Corollary 2. *Let Γ be as in Theorem 1 and assume that the function $m(k\Delta)$ is decreasing in $k \in \mathbb{N}_0$.*

(i) *If $\alpha \sigma^2 > m(0)$, then $\text{Cov}(\Gamma(k\Delta), \Gamma(l\Delta)) \geq 0$ for all $k, l \in \mathbb{N}_0$.*

(ii) *If*

$$\alpha \sigma^2 < \min_{i \in \mathbb{N}_0} \{ m(i\Delta) - m((i + 1)\Delta) \},$$

the minimal drop in pressure in between observation epochs,

$$\text{Cov}(\Gamma(k\Delta), \Gamma(l\Delta)) - (\alpha \Delta \gamma_0 + \gamma_0^2) c^2 (c - 1) f_{k0} f_{l0} \leq 0$$

for all $0 \leq k < l$.

Proof. For $0 \leq i < k < l$, consider $\alpha^2 \Delta^2 f_{li} c (c - 1) \{ c f_{ki} - 1 \}$. The term in between curly brackets is negative if and only if

$$\alpha \{ m(k\Delta) - m(i\Delta) \} + \alpha^2 \sigma^2 < 0.$$

Years							
1995–2001	179.81	177.39	174.86	172.20	169.42	166.50	163.48
2002–2008	160.32	157.05	153.65	150.13	146.49	142.72	138.82
2009–2015	134.81	130.68	126.43	122.04	117.53	112.91	108.16
2016–2021	103.28	98.29	93.17	87.94	82.56	77.08	

Table 1: Estimated pore pressure in bara on January 1st in the years 1995–2021 near the town of Slochteren, The Netherlands.

Since m is decreasing, under assumption (ii),

$$\alpha^2 \sigma^2 < m(i\Delta) - m((i+1)\Delta) \leq m(i\Delta) - m(k\Delta).$$

We conclude that $\alpha^2 \Delta^2 f_{li} c(c-1) \{cf_{ki} - 1\} \leq 0$. Since the same argument can be used to show that $\alpha \Delta \gamma_0 f_{l0} c(c-1) \{cf_{k0} - 1\} \leq 0$, (ii) is seen to hold. Under assumption (i),

$$\alpha^2 \sigma^2 > m(0) \geq m(i\Delta) \geq m(i\Delta) - m(k\Delta)$$

and therefore $\alpha^2 \Delta^2 f_{li} c(c-1) \{cf_{ki} - 1\} \geq 0$. Similarly, $\alpha \Delta \gamma_0 f_{l0} c(c-1) \{cf_{k0} - 1\} \geq 0$. The observation that $\Gamma(0) = \gamma_0$ is deterministic completes the proof of (i). \square

Example 1. Table 1 lists estimated pore pressure values near the town of Slochteren in the Groningen gas field in The Netherlands for January 1st, 1995–2021 [6]. The estimated standard deviation is $\sigma = 7.17$.

Note that the pore pressure values are decreasing due to gas extraction. Since the intensity of induced earthquakes was very low in 1995, γ_0 can be considered infinite. Equation (4) then implies that the covariance matrix of the random vector $\Gamma(k\Delta)_k$ has positive entries only.

Example 2. Next, let us suppose that – in contrast to the previous example – the initial seismicity is very high, i.e. $\gamma_0 = 0$. Assume a linearly decreasing sequence of pore pressures $m(0) = 3$, $m(1) = 2$ and $m(2) = 1$ and set $\alpha = 1$. Then the covariance matrix of the random vector $(\Gamma(1), \Gamma(2))$ is readily calculated. Indeed, $\text{Var}\Gamma(1) = e^{-2}(e^{4\sigma^2} - e^{2\sigma^2})$ and

$\text{Var}\Gamma(2) = (e^{-2} + e^{-4})(e^{4\sigma^2} - e^{2\sigma^2}) + 2e^{-3}(e^{3\sigma^2} - e^{2\sigma^2})$. As for the off-diagonal entry,

$$\begin{aligned} \text{Cov}(\Gamma(1), \Gamma(2)) &= \text{Cov}\left(e^{-1}e^{E(1)-E(0)}, e^{-2}e^{E(2)-E(0)} + e^{-1}e^{E(2)-E(1)}\right) \\ &= e^{2\sigma^2} \left\{ e^{-3}(e^{\sigma^2} - 1) - e^{-2}(1 - e^{-\sigma^2}) \right\} \end{aligned}$$

is negative for $\sigma^2 < 1$ and positive for $\sigma^2 > 1$.

4 Approximate moments of the rate variable

Recall that in the rate-and-state model defined in Section 2, the rate of induced earthquakes is inversely proportional to the state. However, due to its form (3), the moments of $1/\Gamma(k\Delta)$ are intractable. Thus, in this section, we use the delta method to approximate them in terms of the tractable moments of $\Gamma(k\Delta)$ derived in the previous section.

Theorem 2. *Let Γ be defined by (3). Then, for $k \in \mathbb{N}_0$,*

$$\mathbb{E} \left[\frac{1}{\Gamma(k\Delta)} \right] \approx \frac{1}{\mathbb{E}\Gamma(k\Delta)} + \frac{\text{Var}\Gamma(k\Delta)}{(\mathbb{E}\Gamma(k\Delta))^3}.$$

Note that the approximation of the expectation of $\Gamma(k\Delta)^{-1}$ is at least as large as its ‘plug-in estimator’ $1/\mathbb{E}\Gamma(k\Delta)$.

Proof. Apply the delta method based on the Taylor expansion

$$\frac{1}{x_0 + h} \approx \frac{1}{x_0} - \frac{h}{x_0^2} + \frac{1}{2!} \frac{2h^2}{x_0^3}$$

around x_0 equal to the expectation of $\Gamma(k\Delta)$. Upon taking the expectation, one obtains that

$$\mathbb{E} \left[\frac{1}{\Gamma(k\Delta)} \right] \approx \frac{1}{\mathbb{E}\Gamma(k\Delta)} - \mathbb{E} \left[\frac{\Gamma(k\Delta) - \mathbb{E}\Gamma(k\Delta)}{(\mathbb{E}\Gamma(k\Delta))^2} \right] + \mathbb{E} \left[\frac{(\Gamma(k\Delta) - \mathbb{E}\Gamma(k\Delta))^2}{(\mathbb{E}\Gamma(k\Delta))^3} \right].$$

The middle term on the right-hand side is zero, and the claim follows. \square

Theorem 3. Let Γ be defined by (3). Then, for $k, l \in \mathbb{N}_0$,

$$\begin{aligned} \mathbb{E} \left[\frac{1}{\Gamma(k\Delta)\Gamma(l\Delta)} \right] &\approx \frac{1}{\mathbb{E}[\Gamma(k\Delta)] \mathbb{E}[\Gamma(l\Delta)]} + \frac{\text{Cov}(\Gamma(k\Delta), \Gamma(l\Delta))}{(\mathbb{E}\Gamma(k\Delta))^2(\mathbb{E}\Gamma(l\Delta))^2} \\ &+ \frac{\text{Var}\Gamma(k\Delta)}{\mathbb{E}\Gamma(l\Delta)(\mathbb{E}\Gamma(k\Delta))^3} + \frac{\text{Var}\Gamma(l\Delta)}{\mathbb{E}\Gamma(k\Delta)(\mathbb{E}\Gamma(l\Delta))^3}. \end{aligned}$$

Proof. The Taylor expansion of the function $(x, y) \mapsto 1/(xy)$ around the point $(\mathbb{E}\Gamma(k\Delta), \mathbb{E}\Gamma(l\Delta))$ yields the approximation

$$\begin{aligned} \frac{1}{\Gamma(k\Delta)} \frac{1}{\Gamma(l\Delta)} &\approx \frac{1}{\mathbb{E}\Gamma(k\Delta)} \frac{1}{\mathbb{E}\Gamma(l\Delta)} + \frac{(\Gamma(k\Delta) - \mathbb{E}\Gamma(k\Delta)) (\Gamma(l\Delta) - \mathbb{E}\Gamma(l\Delta))}{(\mathbb{E}\Gamma(k\Delta))^2(\mathbb{E}\Gamma(l\Delta))^2} \\ &- \frac{\Gamma(k\Delta) - \mathbb{E}\Gamma(k\Delta)}{(\mathbb{E}\Gamma(k\Delta))^2\mathbb{E}\Gamma(l\Delta)} - \frac{\Gamma(l\Delta) - \mathbb{E}\Gamma(l\Delta)}{(\mathbb{E}\Gamma(l\Delta))^2\mathbb{E}\Gamma(k\Delta)} \\ &+ \frac{(\Gamma(k\Delta) - \mathbb{E}\Gamma(k\Delta))^2}{(\mathbb{E}\Gamma(k\Delta))^3\mathbb{E}\Gamma(l\Delta)} + \frac{(\Gamma(l\Delta) - \mathbb{E}\Gamma(l\Delta))^2}{\mathbb{E}\Gamma(k\Delta)(\mathbb{E}\Gamma(l\Delta))^3} \end{aligned}$$

up to second-order moments. Taking expectations proves the claim. \square

Combining Theorems 2 and 3, we obtain the following corollary.

Corollary 3. Let Γ be defined by (3). Then, for $k, l \in \mathbb{N}_0$,

$$\text{Cov} \left(\frac{1}{\Gamma(k\Delta)}, \frac{1}{\Gamma(l\Delta)} \right) \approx \frac{\text{Cov}(\Gamma(k\Delta), \Gamma(l\Delta))}{(\mathbb{E}\Gamma(k\Delta))^2(\mathbb{E}\Gamma(l\Delta))^2}.$$

5 Simulation examples

To investigate the accuracy of the approximations in Section 4, we compare the formulae in Theorem 2 and Corollary 3 with population estimates. Recall that for an i.i.d. sample X_1, \dots, X_n of some random variable X_1 with mean μ and variance σ^2 , approximate confidence intervals for μ and σ^2 take the form

$$\left(\bar{X}_n - \frac{S_n}{\sqrt{n}}\xi_{1-\alpha/2}, \quad \bar{X}_n + \frac{S_n}{\sqrt{n}}\xi_{1-\alpha/2} \right)$$

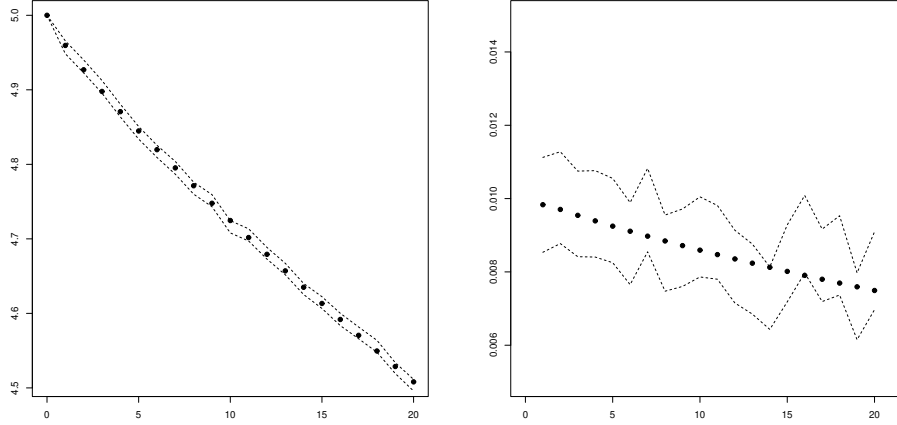


Figure 1: 95% pointwise confidence intervals for mean (left) and variance (right) of $\Gamma(k\Delta)^{-1}$ as a function of k when $m(\Delta k) = 6 - 1/(0.5k + 1)$, $\alpha = 0.01$, $\Delta = 0.1$, $\gamma_0 = 0.2$ and $\sigma^2 = 2.0$. The dots correspond to the approximation values of Theorem 2 and Corollary 3.

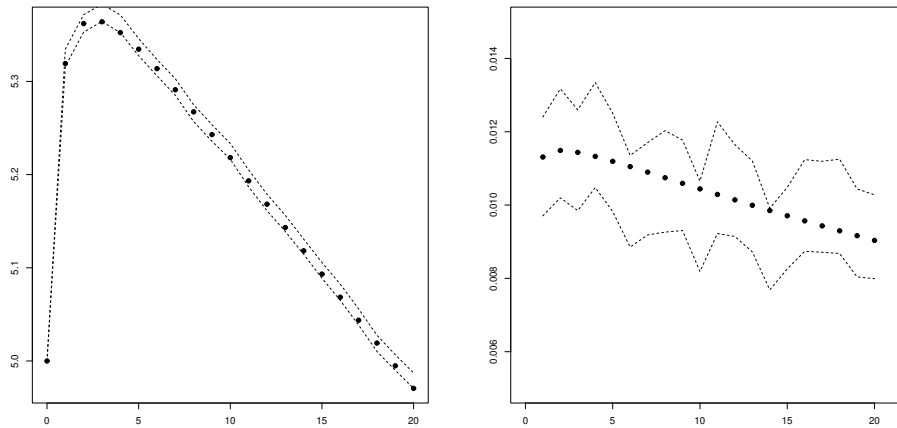


Figure 2: 95% pointwise confidence intervals for the mean (left) and variance (right) of $\Gamma(k\Delta)^{-1}$ as a function of k when $m(k\Delta) = 5 + 10/(2k + 1)$, $\alpha = 0.01$, $\Delta = 0.1$, $\gamma_0 = 0.2$ and $\sigma^2 = 2.0$. The dots correspond to the approximation values of Theorem 2 and Corollary 3.

for μ , and

$$\left(\frac{S_n^2}{1 + \xi_{1-\alpha/2} \sqrt{\frac{2\zeta}{n-1}}}, \frac{S_n^2}{1 - \xi_{1-\alpha/2} \sqrt{\frac{2\zeta}{n-1}}} \right)$$

for σ^2 . Here $\xi_{1-\alpha/2}$ is the $(1-\alpha/2)$ -quantile of the standard normal distribution. Furthermore, $\zeta = \frac{1}{2}(\gamma_4 - 1)$ for γ_4 the ratio of the fourth central moment and the squared variance. Since γ_4 is unknown, we estimate it by $\hat{\gamma}_4 = n \sum (X_i - m)^4 / ((n-1)^2 S_n^4)$ where m is the sample median [2]. In our case, $X_i = 1/\Gamma_i(k\Delta)$. A similar approach may be taken for the covariance, although we do not pursue it here.

We consider two cases: increasing and decreasing pore pressure. For Figure 1, define the pore pressure by the increasing function

$$m(\Delta k) = 6 - \frac{1}{1 + k/2}.$$

With parameter values $\alpha = 0.01$, $\Delta = 0.1$, $\gamma_0 = 0.2$ and $\sigma^2 = 2.0$, the 95% pointwise confidence intervals for the mean and variance are given in the left- and right-most panels. In both cases, the sample size was $n = 500$. It can be seen that the approximations are quite adequate.

For Figure 2, take the decreasing function

$$m(\Delta k) = 5 + \frac{10}{2k + 1}.$$

With the same parameter values and sample size, the 95% pointwise confidence intervals for the mean and variance are given in the left- and right-most panels. Again the approximations are satisfactory.

6 Conclusion

In this paper, we modified the rate-and-state model proposed in the earth sciences to describe the relation between changes in pore pressure and induced seismic hazard [1, 4, 7] in such a

way that measurement error in the pore pressure can be taken into account. For the time series of state variables Γ in the resulting Cox process [3], closed-form expressions for the first and second moments were derived. Additionally, we provided sufficient conditions for positive and negative correlation. On top of that, we gave approximate expressions for the first two moments of the rate variable and verified in simulated examples that the approximations are satisfactory.

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