Directional Effects on Freak Wave Prediction

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1 Introduction

The last decade freak waves have become an important topic in engineering and science. The freak waves are sometimes featured by a single and steep crest causing severe damage to structures and ships. The freak wave studies started in the late 80's and the high-order nonlinear effects on the freak waves were discussed in the early 90's (*i.e.* Yasuda *et al.*, 1992). Owing to the many research efforts, the occurrence of freak waves, their mechanism and detailed dynamic properties are now becoming clear. Janssen (2003) showed that the deviations from the Gaussian pdf of the surface elevation are related to the presence of resonant and non-resonant four-wave interactions. The kurtosis, fourth-order moments of the surface elevation, is the measure for extreme events. Mori and Janssen (2006) investigated explicit formulation between the maximum wave height distribution and kurtosis change through the four-wave interactions. This give us the possibility of the freak wave prediction in the open ocean. However, there is no validation of the theory at present.

The purpose of this study is to verify the Mori and Janssen (2006) theory to compare with observed data and modification of the theory for making an operational prediction system of freak wave. First, the present theory is verified by the observed data. Second, the directional effects on freak wave occurrence is discussed by the numerical simulations. Finally, the modification of freak wave prediction theory including the directional effects is discussed.

2 Verification of Janssen's Theory

Freak waves most likely only occur for narrow band wave trains. This corresponds to situations where both the frequency and directional distribution of the wave is narrow. It is possible to simplify assuming the narrow-band approximation. Janssen (2003) proposed the kurtosis dependence on the square of the Benjamin-Feir Index (BFI) neglecting directional effects.

The dependence of kurtosis on BFI verified by the observed data. The wave data were measured by the Doppler-type Wave Directional Meter (DWDM) at the depth of 50 m. The DWDM is installed at 26 nation-wide stations at the end of the year 2003 in Japan and six of them in year of 2004 were picked up for the analysis in here. Fig.1 shows the relationship averaged $H_{max}/H_{1/3}$, number of waves N in the record and kurtosis μ_4. The observed data clearly shows the kurtosis dependence on $H_{max}/H_{1/3}$. The similar tendency was confirmed the other observed stations.

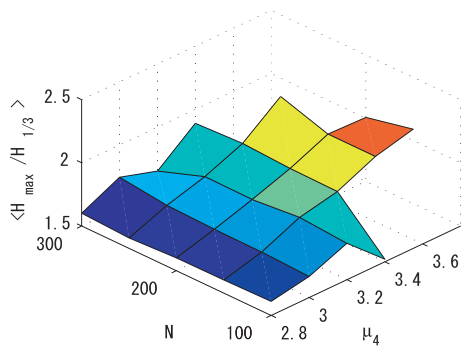

Figure 1: Dependence of kurtosis μ_4 and number of waves N on maximum wave height $H_{max}/H_{1/3}$

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Define the freak wave condition as $H_{max}/H_{1/3} \geq 2$, following Mori and Janssen (2006), we have the simple formula to predict the occurrence probability of a freak wave as function of number of waves N, and kurtosis μ_4 with unidirectional assumption.

$$P_{freak} = 1 - \exp\left[-\mathrm{e}^{-8} N(1 + 24\mu_4)\right] \qquad (1)$$

and $\mu_4 = \frac{\pi}{\sqrt{3}}\mathrm{BFI}^2 + 3$, where $\mathrm{BFI} = \varepsilon Q_p^2$ and $\varepsilon = k_0\sqrt{m_0}$. Using Eq.(1), the occurrence probability of freak wave can be estimated from the wave statistics.

The freak wave prediction theory was applied to operational wave forecasting system based on WAM at European Centre for Medium-Range Weather Forecasts (ECMWF). Fig.2 shows the comparison between predicted kurtosis and observed kurtosis. There is no correlation between the predicted and observed kurtosis ($R = 0$). Through the detail comparison between the predicted and observed directional spectra, it is found that the wave directionality effects significantly on kurtosis prediction. This is main reason why the predicted kurtosis so different from observed data.

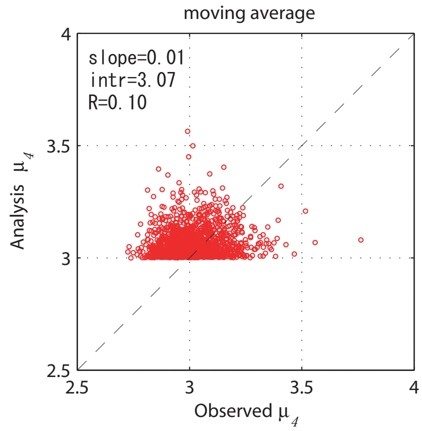

Figure 2: Comparison of the predicted and observed kurtosis

3 Directional Effects on Freak Wave Occurrence

The cubic Nonlinear Schrödinger equation (CNLS) was used to verify the directional effects on kurtosis. The Monte Carlo simulations were conducted to avoid statistical sensitivity changing directional wave spectra and initial phases (500 set of phases each wave condition). Fig.3(a) shows the numerical results of Mote Carlo CNLS. The vertical axis indicates the fourth cumulant of the surface elevations κ_4 which is equivalent to $\kappa_4 = \mu_4 - 3$. For a narrow-band case ($\sigma_y \to 0$), the value of kurtosis is increased as BFI increased monotonically. Remarkably, the kurtosis is decreased as directional spread become larger ($\sigma_y \to 1$) and becomes the Gaussian state ($\mu_4 = 3$). Therefore, the occurrence of freak wave is only increased the wave which has a narrow directionality and high nonlinearity. The modification of theory of freak wave prediction was investigated including directional effects assuming only principal direction of the spectral components contributes to the kurtosis change. Fig.3(b) shows the theoretical relationship between kurtosis, BFI and directional spread σ_y. The modified theory shows nice agreement with the numerical results. This results is promising to establish the freak wave prediction.

The details of the theory, numerical results and comparison with the data will be presented at the conference.

References

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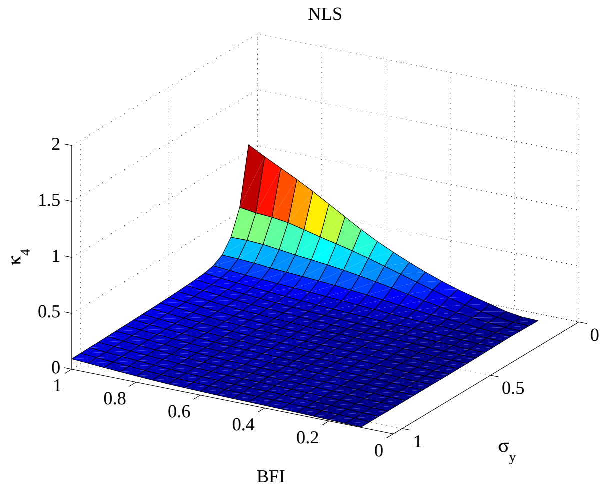
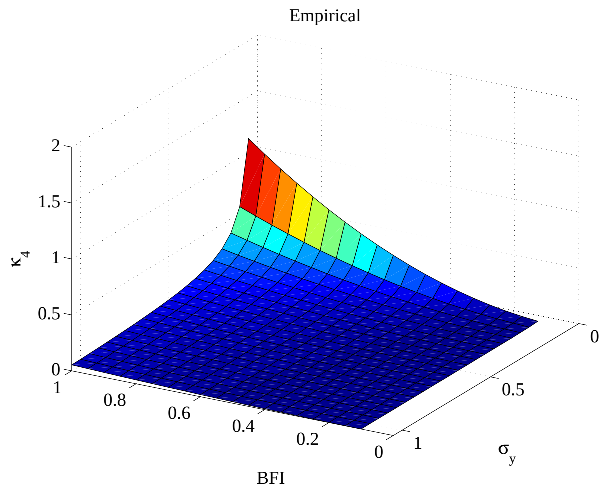


(a) Numerical results by MC-QNLS



(b) Modified theory

Figure 3: Dependence of BFI and directional spread σ_y on kurtosis ($\kappa_4 = \mu_4 - 3$, $\varepsilon = 0.1$)