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Quasi-Isotropic Approximation of Geometrical Optics Method as Adequate Electrodynamical Basis for Tokamak Plasma Polarimetry

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Abstract

Basic principles and recent findings of quasi-isotropic approximation (QIA) of a geometrical optics method are presented in a compact manner. QIA was developed in 1969 to describe electromagnetic waves in weakly anisotropic media. QIA represents the wave field as a power series in two small parameters, one of which is a traditional geometrical optics parameter, equal to wavelength ratio to plasma characteristic scale, and the other one is the largest component of anisotropy tensor. As a result, ,, ,,QIA ideally suits to tokamak polarimetry/interferometry systems in submillimeter range, where plasma manifests properties of weakly anisotropic medium.

Keywords: Plasma; Polarimetry

1. Introduction

Quasi-isotropic approximation (QIA) of the geometrical optics method was developed in 1969 for the description of electromagnetic waves in weakly anisotropic media, and particularly in weakly anisotropic plasma [1]. The fact is that in submillimeter and IR bands of electromagnetic spectrum tokamak plasma manifest properties of weakly anisotropic medium in a wide range of magnetic fields and electron densities [2]. That is why QIA may serve as an ideal instrument of the wave theory, uniformly describing both polarimetric and interferometric components of tokamak diagnostics systems.

This paper presents in Sect. 2 the basic equations of QIA, related to amplitude and phase of an electromagnetic wave and to the ray configuration in plasma. Sect. 3 outlines QIA results related to refraction, absorption and diffraction of electromagnetic waves. Finally Sect. 3 summarizes the main features of QIA concept

related to tokamak plasma polarimetry.

2. Basic equations of QIA

2.1. Small parameters of QIA and power series for the wave field

QIA appeared as a modification of the geometrical optics method for weakly anisotropic media. Traditional geometric optics [3-6] represents the wave field as a power series in geometrical optics small parameter $\mu_{GO} = \lambda/L$, which is the ratio of a wavelength $\lambda = 2\pi/k$ (k is a wave number) to the characteristics scale of an inhomogeneous medium L. Besides μ_{GO}, QIA also uses the second small parameter $\mu_A = \max|\nu_{ik}|/\varepsilon_0$, where $\hat{\nu}$ is an anisotropic part of dielectric tensor:

$$\hat{\varepsilon} = \varepsilon_0 \hat{\delta} + \hat{\nu} \tag{2.1}$$

δ is a unit tensor and $\varepsilon_0 \hat{\delta}$ is a permittivity tensor of the isotropic background of weakly anisotropic medium. Thus, the anisotropic small parameter μ_A characterizes the degree of anisotropy of a studied medium. Within QIA the two small parameters μ_{GO} and μ_A are united into a joint small parameter

$$\mu = \max(\mu_{GO}, \mu_A) \tag{2.2}$$

The amplitude $\mathbf{E}$ of the wave field

$$\mathbf{\varepsilon} = \mathbf{E}\exp[ik\varphi] \tag{2.3}$$

is presented by a power series in parameter μ:

$$\mathbf{E} = \mathbf{E}_0 + \mu\mathbf{E}_1 + \mu^2\mathbf{E}_2 + ... \tag{2.4}$$

Substituting power expansion (2.4) into Maxwell equations and equalling the terms of the power μ^m in the left- and right-hand parts of Maxwell equations, we arrive at the equations of subsequent approximations for successive approximations $\mathbf{E}_0, \mathbf{E}_1, \mathbf{E}_2, ...$

2.2. Eikonal and ray equations, polarization independent phase and transverse nature of the wave field

A linear system of equations for components of the zeroth order field $\mathbf{E}_0$ will be consistent if the eikonal equation for isotropic plasma is satisfied:

$$(\nabla\varphi)^2 = \varepsilon_0 \tag{2.5}$$

It means that the phase $S = k\varphi$ of the zeroth order field $\mathbf{E}_0$ does not depend on the anisotropic part $\hat{\nu}$ of the permittivity tensor $\hat{\varepsilon}$, that is the zeroth order field $\mathbf{E}_0$ is identical to that in isotropic medium. As a result, phase S of the zeroth order field as well that of total field (2.3) happens to be polarization independent. This property seems to be quite important for plasma interferometry.

The ray pattern corresponding to the isotropic eikonal equation (2.5), also obeys isotropic ray equations [4-6]:

$$\frac{d^2\mathbf{r}}{d\tau^2} = \tfrac{1}{2}\nabla\varepsilon_0 \tag{2.6}$$

where the increment $d\tau$ of the parameter τ is connected with the elementary arc length $d\sigma$ by relation $d\tau = d\sigma/\sqrt{\varepsilon_0}$. Correspondingly, the eikonal φ is presented by the integral

$$\varphi = \int \varepsilon_0 d\tau = \int \sqrt{\varepsilon_0} \, d\sigma \tag{2.7}$$

along the ray.

2.3. Coupled QIA equations for the wave amplitude and energy conservation law

QIA equations for energy flow and polarization state evolution stem from consistence conditions for the first order field equations. It is convenient to present zero order amplitude in the form:

$$\mathbf{E}_0 = A\mathbf{\Gamma} = A(\mathbf{\Gamma}_n n + \mathbf{\Gamma}_b b) \tag{2.8}$$

where $\mathbf{\Gamma}$ is a unit vector: $|\mathbf{\Gamma}| = 1$ and A is the amplitude of the electric field. Differential equations of a complex vector $\mathbf{\Gamma}$ evolution in weakly anisotropic media were written down first in a pioneer paper [1] and then reproduced in a review paper [7], in the book [8], on electrodynamics of waves in weakly anisotropic media, as well as in short sections on QIA approach in the books [4-6] and in successive publications [9-11]. Here we make use of consistency conditions, presented in the form [7-10]

$$\begin{cases} \dfrac{d\mathbf{\Gamma}_n}{d\sigma} = \tfrac{1}{2} ik\varepsilon_o^{-\frac{1}{2}}(\nu_{nn}\mathbf{\Gamma}_n + \nu_{nb}\mathbf{\Gamma}_b) - \kappa\mathbf{\Gamma}_b \\ \dfrac{d\mathbf{\Gamma}_b}{d\sigma} = \tfrac{1}{2} ik\varepsilon_o^{-\frac{1}{2}}(\nu_{bn}\mathbf{\Gamma}_n + \nu_{bb}\mathbf{\Gamma}_b) + \kappa\mathbf{\Gamma}_n \end{cases} \tag{2.9}$$

where κ is a ray torsion. In a cold plasma without dissipation, equations (2.9) could be written in the form

$$\begin{cases} \dfrac{d\mathbf{\Gamma}_n}{d\sigma} = -\tfrac{1}{2} i(2\Omega_0 - \Omega_\perp - \Omega_1)\mathbf{\Gamma}_n + \tfrac{1}{2} i(\Omega_2 + i\Omega_3)\mathbf{\Gamma}_b - \kappa\mathbf{\Gamma}_b \\ \dfrac{d\mathbf{\Gamma}_b}{d\sigma} = \tfrac{1}{2} i(\Omega_2 - i\Omega_3)\mathbf{\Gamma}_n - \tfrac{1}{2} i(2\Omega_0 - \Omega_\perp + \Omega_1)\mathbf{\Gamma}_b + \kappa\mathbf{\Gamma}_n \end{cases} \tag{2.10}$$

Here $\Omega_{1,2,3}$ are the components of the vector $\mathbf{\Omega}$ introduced in [12] and widely used in plasma polarimetry [13]. $\Omega_\perp$ with Ω_0 are auxiliary parameters: $\Omega_\perp = \sqrt{\Omega_1^2 + \Omega_2^2}_{\|} = \Omega_0 \sin^2 \alpha_\|$, where $\alpha_\|$ is the angle between the ray propagation direction and the magnetic field vector. The parameters Ω_1 and Ω_2, quadratically related to the magnetic field B_0, characterize the Cotton-Mouton effect, and Ω_3, linearly related to the magnetic field B_0, corresponds to the Faraday phenomenon. Starting from Eqs. (2.10) evolution equations for other variables representing polarization state of e-m beam were obtained in the following publications: Stokes vector [14], angular variables sets (ψ, χ) [15] or (α, δ) [16], complex polarization angle (CPA) [15] or complex amplitude ratio (CAR) [16]. Using the proper form of a permittivity tensor ε it is also possible to rewrite Eqs. (2.9) for any weakly anisotropic and weakly inhomogeneous medium (e.g. medium where dissipation [14] or relativistic effects play an important role [17]).

It follows from (2.8) and (2.9) that a squared modulus of amplitude (2.8) satisfies energy flux conservation law in the from

$$\nabla \cdot (|\mathbf{A}_0|^2 \mathbf{p}) = 0 \tag{2.11}$$

where $\mathbf{p}$ is the ray "momentum". Equation (2.11) can be presented also as energy conservation in the narrow ray tube of cross section da [4-6]:

$$A^2 da = 0 \tag{2.12}$$

3. Other aspects of QIA

3.1. Account for diffraction

The ability of the geometrical optics method to describe diffraction phenomena for the Gaussian beam was revealed in the paper [18] on complex geometrical optics (CGO). A modern form of CGO for Gaussian beams, developed in [19,20], deals with a complex eikonal along the central ray of the Gaussian beam

$$\varphi_c = \sigma + \tfrac{1}{2} B\rho^2 \tag{3.1}$$

where ρ is a distance between the observational point and the central ray, and B is a complex curvature of the Gaussian beam: the real part of B characterizes the curvature of the phase front, whereas the imaginary part describes the beam width. The complex parameter obeys a nonlinear equation of Riccati type:

$$\frac{\mathrm{d}B}{\mathrm{d}\sigma} + B^2 = \alpha \tag{3.2}$$

where the parameter α for an axially symmetric medium is given by

$$\alpha = \tfrac{1}{2}\frac{\mathrm{d}^2\varepsilon}{\mathrm{d}\rho^2}\bigg|_{\rho=0} \tag{3.3}$$

A solution of Eq. (3.2) determines the evolution of the beam width and phase front curvature along the ray.

Advantages of CGO approach can be united with AVT equations for angular parameters evolution. The idea of such a "marriage" was suggested and put forward in the paper [21]. Thus, a combination of QIA and CGO enables the description of both polarization and "diffraction" states along the ray.

It is worth noticing that a diffraction of the Gaussian beam does not affect the polarization state, because all the components of the electrical vector change proportionally to the common amplitude, with amplitude ratios remaining unchanged.

4. Conclusions

The paper deals with the basic principles of the quasi-isotropic approximation of geometrical optics, which describes the propagation of electromagnetic waves in weakly anisotropic plasma. In the submillimeter range of an electromagnetic spectrum, where tokamak plasma manifests properties of weakly anisotropic medium, QIA provides all essential information on the ray structure, amplitude and phase behaviour as well as on polarization evolution in polarimetry/interferometry systems of modern tokamaks. Using a proper form of the permittivity tensor, the presented method can be easily applied to any other weakly anisotropic medium, e.g. dissipative, relativistic plasma. Finally, a simple combination of QIA and complex geometrical optics is suggested which enables to account of diffraction effects for electromagnetic beams of Gaussian profile.

Thus QIA serves as an adequate electrodynamical basis for tokamak polarimetry/interferometry systems, which play an important role in plasma diagnostics.

Acknowledgements

References

1. Yu.A.Kravtsov, SovietPhysics-Doklady 13 (1969) 1125.
2. F.DeMarco, S.E.Segre, PlasmaPhysics 14 (1972) 245.
3. M.Born, E.Wolf, Principles of Optics, seventh ed., Cambridge University Press, Cambridge, Oxford, 1999.
4. Yu.A.Kravtsov, I.Orlov, Geometrical Optics of Inhomogeneous Media, Springer, Berlin, 1990.
5. Yu.A.Kravtsov, Geometrical Optics in Engineering Physics, Alpha Science International, Harrow, 2005.
6. Yu.A.Kravtsov, N.Y.Zhu, Theory of Diffraction. Heuristic Approaches. Alpha Science Internet, Oxford, UK, 2010.
7. Yu.A.Kravtsov, O.N.Naida, A.A.Fuki, Physics:Uspekhi 39 (1996) 129.
8. A.A.Fuki, Yu.A.Kravtsov, O.N.Naida, Geometrical Optics of Weakly Anisotropic Media, Gordon&Breach, London, NewYork, 1997.
9. Yu.A.Kravtsov, B.Bieg, K.Yu.Bliokh, Journal of the Optical Society of America 24 (2007) 3388.
10. Yu.A.Kravtsov, B.Bieg, K.Yu.Bliokh, M.Hirsch, AIP Conference Proceedings 993 (2008) 143.
11. Z.H.Czyz, B.Bieg, Yu.A.Kravtsov, Physics Letters A 368 (2007) 101.
12. S.E.Segre, V.Zanza, Physics of Plasmas 18 (2011) 092107.

13. R.Imazawa, Y.Kawano, Y.Kusama, Plasma Phys. Control. Fusion 54 (2012) 055005.
14. Yu.A.Kravtsov, B.Bieg, J.Chrzanowski, Fusion Engineering and Design 88 (2013) 1449.
15. Yu.A.Kravtsov, J.Chrzanowski, The European Physical Journal D - Atomic, Molecular, Optical and Plasma Physics 63 (2011) 129.
16. K.Guenther, et al., Plasma Phys. Control. Fusion 46 (2004) 1423.
17. S.E.Segre, V.Zanza, Plasma Phys. Control. Fusion 48 (2006) 339.
18. J.D. Jackson, Classical Electrodynamics, Wiley, NY, 1998.
19. S.E.Segre, Plasma Physics and Controlled Fusion 41 (1999) R57.
20. Yu.A. Kravtsov, Radiophys. Quantum Electron. 10 (1967) 719.
21. P.Berczynski, K.Yu.Bliokh, Yu.A.Kravtsov, A.Stateczny, J. Opt. Soc. Am. A 23 (2006) 1442.
22. P.Berczynski, Yu.A.Kravtsov, G.Zeglinski, Cent. Eur. J. Phys. 6 (2008) 603.
23. Yu.A.Kravtsov, P.Berczynski, B.Bieg , Phys. Let. A 373 (2009) 2979.
24. S.E.Segre, J. Phys. D: Appl. Phys. 36 (2003) 2806.