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Optical soliton solutions of the generalized non-autonomous nonlinear Schrödinger equations by the new Kudryashov's method

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ABSTRACT

In this work, we study the optical soliton solutions of the generalized non-autonomous nonlinear Schrödinger equation (NLSE) by means of the new Kudryashov's method (NKM). The aforesaid model is examined with time-dependent coefficients. We considered three interesting non-Kerr laws which are respectively the quadratic-cubic law, anti-cubic law, andtriple power law. The proposed method, as a newly developed mathematical tool, is efficient, reliable, and a simple approach for computing new solutions to various kinds of nonlinear partial differential equations (NLPDEs) in applied sciences and engineering.

Introduction

The Optical fibers are commonly used incommunication, broadcasting, medical field, industries, lighting and decorations, mechanical inspections, defense, and have many commercial and scientific applications. Telecommunication companies totally work on optical fibers such as in the transmission of telephone signals, internet communication and cable television signals. It is very thin, lighter, and having highly flexible nature. That is why it carries more data compared to copper wires with high-speed having more accuracy. In medical industries, it gives us information and images from the internal parts of the human bodies by entering the hollow parts of the body. Surgeries, endoscopy, microscopy, lasers, and biomedical research are working on optical fibers. It is also used as sensors to pressure and temperature measurement. Economically, it is used for safety purposes, lighting internally and externally in automobiles, for decorations and research purposes in testing. Engineers used the optical fibers to detect damages and faults in pipes and for inspection in hard regions. These are used in aircraft wirings and for the transmission of high-level confidential data.

Additionally, the field of optical soliton solutions of some nonlinear equations are very interesting and has an important role in mathematics and scientific applications. That is why different techniques shall be used for the solutions of such problems [1–27]. Bright optical solitons have been studied earlier for various non-Kerr law nonlinearities by several scientists [28–32]. In this paper, ringing solitons are studied along with nonlinear optics and nonlinear non-Kerr law. The four kinds of bizarre non-Kerr laws deliberated in our work are the quadratic-cubic law [33–36], an anti-cubic law [37–42] and a triple power law [43,44]. For the better understanding of optical solitons solutions, different techniques were used by Arnous et al. [45,46]. Many other mathematicians briefly discussed the different cases of optical solitons solutions along with different conditions for finding the exact solutions such as Biswas discussed for resonant optical solitons with nonlinear cubic law and cubic-quintic-septic law nonlinearities [47], Bulut and Pandir studied the nonlinear fractional Sharma-Tasso-Olver equation [48], Demiray and Bulut worked on generalized Gardner equation [49], while Fazli and Adibi used NLSE [50].

The following equation is NLSE having time-dependent coefficients

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[14,44].

$$iq_t+\alpha(t)q_{xx}+\beta(t)F\left(|q|^2\right)q+\gamma(t)\left(\frac{|q|_{xx}}{|q|}\right)q$$
$$=i\left(\delta(t)q_x+\lambda(t)\left(|q|^{2n}q\right)_x+\mu(t)\left(|q|^{2n}\right)_x q\right), \tag{1}$$

where $\delta(t)$, $\lambda(t)$, $\mu(t)$ and n represent the inter-modal dispersion, coefficient of self-steepening, coefficient of higher-order dispersion and the nonlinearity, respectively. The spatial variable x and temporal variable$t$ are the independent variables. The soliton profile is related to a complex valued function $q(x,t)$. The following transformation helps us in the solution of Eq. (1)

$$q(x,t)=g(z)e^{(i\phi(x,t))},\qquad z=x+2\kappa\int_0^t\alpha(t)dt,\qquad \phi=-\kappa x+\omega(t)t, \tag{2}$$

where κ is the soliton frequency. Putting Eq. (2) in(1) and then separating the real and imaginary parts as

$$\delta(t)+((2n+1)\lambda(t)+2n\mu(t))g^{2n}=0, \tag{3}$$

$$(\alpha(t)+\gamma(t))g''-\left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t)\right)g-\kappa\lambda(t)g^{2n+1}+\beta(t)F\left(g^2\right)g=0. \tag{4}$$

In this work, we will study the optical soliton solutions of the generalized non-autonomous NLSE in optical fibers with respect to the three types of Ffunction that are given by

$$\beta(t)F\left(|q|^2\right)=\begin{cases}\beta_1(t)\sqrt{q}+\beta_2(t)q, & \text{Quadratic - cubic law},\\ \beta_1(t)q^{-2}+\beta_2(t)q+\beta_3(t)q^2, & \text{Anti - cubic law},\\ \beta_1(t)q^a+\beta_2(t)q^{2a}+\beta_3(t)q^{3a}, & \text{Triple power law},\end{cases}$$

where $\beta_1(t)$, $\beta_2(t)$ and $\beta_3(t)$ are the coefficients depend upon time. The primary objective of this study is to discuss certain exact optical soliton structures to the NLSEs having full nonlinearity using the NKM with the help of the Maple. The optical solutions will be analyzed through a few supportive illustrations. This paper is partitioned as follows: Sec. 1 present the introduction, Sec. 2 is devoted to the description of NKM. Sec. 3 is described the application of NKM on generalized non-autonomous NLSE in optical fibers, while the paper is concluded in Sec. 4.

Description of new Kudryashov's method

The key steps of the NKM for the solutions of Eq. (1) are:

Step 1. We consider the nonlinear PDE for $q(x,t)$ of the form

$$F(q,\ q_t,\ q_x,\ q_{xx},\ \ldots)=0, \tag{5}$$

where $q=q(x,t)$ is an unknown function.

Step 2. We introduce the wave transformation as

$$q(x,\ t)=g(z)e^{i\phi(x,t)},\qquad z=x-bt, \tag{6}$$

where b is an unknown constant. Eq. (5) can be converted to the following nonlinear ODE with the help of Eq. (6) as

$$H(g,\ g',\ g'',\ \ldots)=0, \tag{7}$$

where the prime denotes the derivative concerning z.

Step 3. Along with NKM, solutions of (7) in terms of $Q(z)$ can be written as the following finite series

$$g(z)=\sum_{j=0}^{N}c_j(Q(z))^j,\qquad c_N\neq 0, \tag{8}$$

where $N$is a positive integer, and $Q(z)$ is defined as

$$Q(z)=\frac{P\varphi(z)}{P+R(\varphi(z)-1)},\qquad R\neq 0, \tag{9}$$

where $c_0,c_1,c_2,c_3,\ldots,c_N,P$ and R are the arbitrary constants to be determined. The function $\varphi(z)$ satisfying the following nonlinear ODE

$$\frac{d\varphi}{dz}=\varphi(z)(\varphi(z)-1). \tag{10}$$

The general solution of Eq. (10) is

$$\varphi(z)=\frac{1}{1+de^z}, \tag{11}$$

where d is an arbitrary parameter.

Step 4. The N can be finding from Eq. (7). For this homogeneous balance principle helps in equating the nonlinear terms in Eq. (7) with higher derivatives. For $N=1$, Eq. (8) can be written as

$$g(z)=c_0+c_1Q(z). \tag{12}$$

Step 5. Putting Eq. (8) in (7) and then equating the coefficients of $Q(z)^j$ to zero, we get a nonlinear algebraic system containing the constants$c_0,c_1,c_2,c_3,\ldots,c_N$ and b.

Step 6. Putting Eq. (11) in Eq. (9) and then putting Eq. (9) along with constants $b,c_0,c_1,c_2,c_3,\ldots,c_N$ in Eq. (8), we get optical soliton solutions of Eq. (7) with the help of Maple.

Optical soliton solutions

Here NKM is used to find the new optical soliton solutions for the generalized nonautonomous NLSE in optical fibers. The following three cases are solved with the help of Maple.

Quadratic-cubic law

By substituting $\beta(t)F\left(|q|^2\right)=\beta_1(t)\sqrt{g}+\beta_2(t)g$ in (4), for $n=1$, we get

$$(\alpha(t)+\gamma(t))g''-\left(t\omega's(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t)\right)g-\kappa\lambda(t)g^3+\beta_1(t)g^2+\beta_2(t)g^3=0. \tag{13}$$

By balancing principle, we obtain $N=1$. Therefore, the solution of Eq. (13) can be considered as Eq. (12). Substituting Eq. (12) into Eq. (13) and then equating the coefficients of $Q^j(z)$ to zero, the following nonlinear algebraic system is obtained as follows:

$$-t\omega'(t)c_0+\beta_2(t)c_0^3-\kappa^2\alpha(t)c_0-\kappa\delta(t)c_0-\kappa\lambda(t)c_0^3-\omega(t)c_0+\beta_1(t)c_0^2=0, \tag{14}$$

$$-\kappa^2\alpha(t)c_1-3\kappa\lambda(t)c_0^2c_1+3\beta_2(t)c_0^2c_1-\omega(t)c_1-\kappa\delta(t)c_1+c_1\gamma(t)-t\omega'(t)c_1+c_1\alpha(t)+2\beta_1(t)c_0c_1=0, \tag{15}$$

$$-3c_1\gamma(t)-3\kappa\lambda(t)c_0c_1^2+3\beta_2(t)c_0c_1^2-3c_1\alpha(t)+\beta_1(t)c_1^2=0, \tag{16}$$

$$\beta_2(t)c_1^3-\kappa\lambda(t)c_1^3+2c_1\omega+2c_1\alpha(t)=0. \tag{17}$$

This system is solved with the help of Maple to get following two sets: Set 1.

$$c_0=0,\qquad c_1=c_1,\qquad \omega(t)=\frac{1}{t}\int_0^t\left(-\kappa^2\alpha(t)-\kappa\delta(t)+\gamma(t)+\alpha(t)\right)dt,$$
$$\beta_1(t)=\frac{3(\alpha(t)+\gamma(t))}{c_1},\qquad \beta_2(t)=-\frac{-\kappa\lambda(t)c_1^2+2\alpha(t)+2\gamma(t)}{c_1^2}. \tag{18}$$

Substituting Eq. (11) in (9) and then putting Eqs. (9), (18) into (12), we have

$$q_1(x,t)=\frac{c_1 P e^{(i\phi(x,t))}}{(1+de^{z})\left(P+R\left(\frac{1}{1+de^{z}}-1\right)\right)},\tag{19}$$

where

$$z=x+2\kappa\int_0^t \alpha(t)dt,\qquad \phi=-\kappa x+\omega(t)t.$$

Fig. 1 illustrates the dark soliton solution $|q_1|$ for $\kappa=1.5, c_1=1, d=2.5, P=1, R=2, \gamma(t)=2t, \delta(t)=t+e^t$ and $\alpha(t)=tanh(3t)$.

Set 2.

$$c_0=-c_1,\quad c_1=c_1,\quad \omega(t)=\frac{1}{t}\int_0^t(-\kappa(\kappa\alpha(t)-\delta(t))+\gamma(t)+\alpha(t))dt,$$
$$\beta_1(t)=-\frac{3(\alpha(t)+\gamma(t))}{c_1},\qquad \beta_2(t)=-\frac{-c_1^2\kappa\lambda(t)+2(\alpha(t)+\gamma(t))}{c_1^2}.\tag{20}$$

Substituting Eq. (11) in (9) and then putting Eqs. (9), (20) into (12), we have

$$q_2(x,t)=-c_1\left[1-\frac{P}{(1+de^{z})\left(P+R\left(\frac{1}{1+de^{z}}-1\right)\right)}\right]e^{(i\phi(x,t))},\tag{21}$$

where

$$z=x+2\kappa\int_0^t \alpha(t)dt,\qquad \phi=-\kappa x+\omega(t)t.$$

Anti-cubic law

By substituting $\beta(t)F\left(|q|^2\right)=\beta_1(t)g^{-2}+\beta_2(t)g+\beta_3(t)g^2$ in (4), we get the below equation

$$\begin{aligned}&(\alpha(t)+\gamma(t))g''-\left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)\right.\\&\left.+\kappa\delta(t)\right)g-\kappa\lambda(t)g^{2n+1}+\beta_1(t)g^{-3}+\beta_2(t)g^3+\beta_3(t)g^5\\&=0.\end{aligned}\tag{22}$$

Balancing $g''$and g^5 in Eq. (22) leads to $N=\frac{1}{2}$. Then by virtue of the transformation

$$g=\sqrt{\gamma},\tag{23}$$

and$n=1$, Eq. (22) modifies to

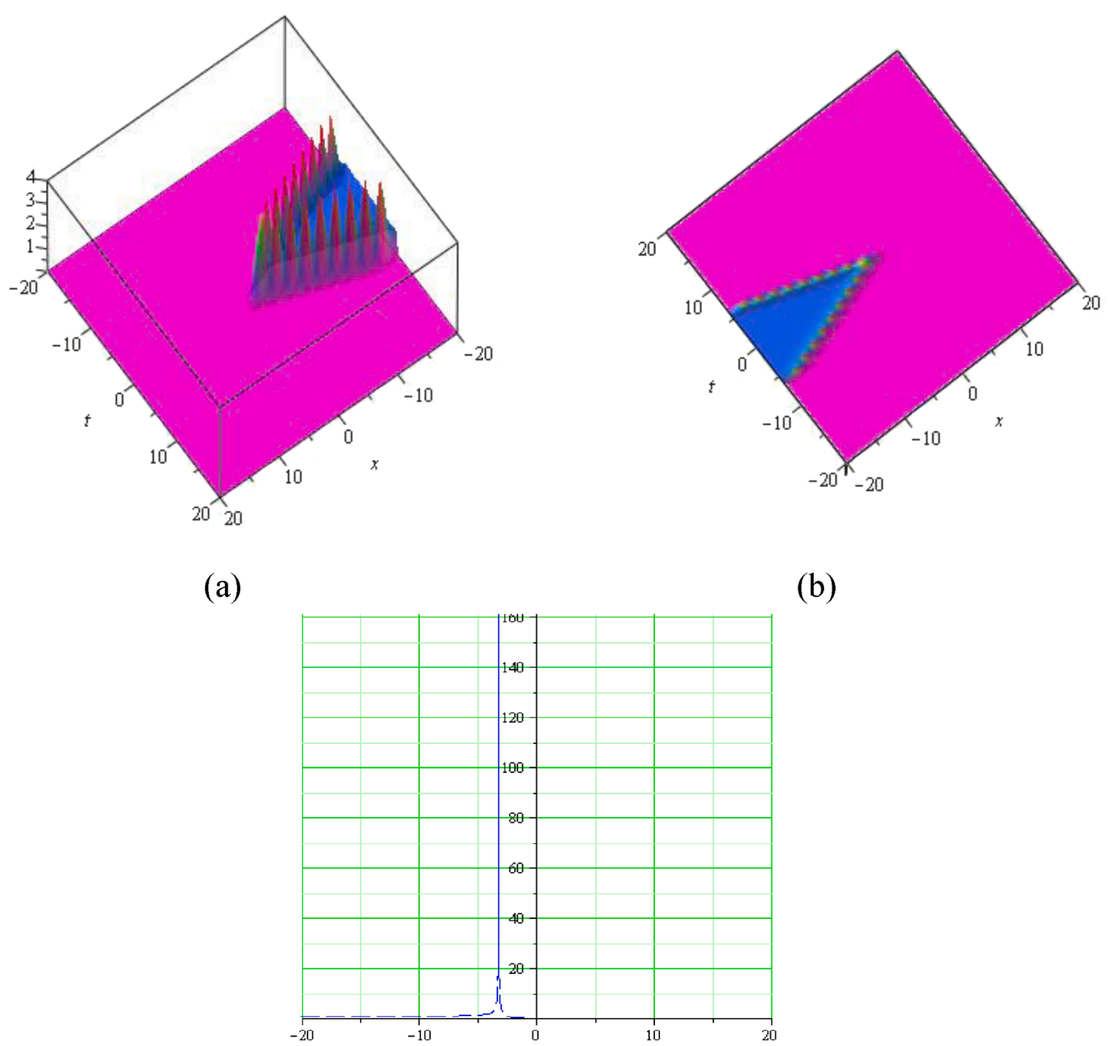

Fig. 1. The 3D (a), contour (b) and 2D (c) surfaces of the soliton solution of Eq. (19) given by $|q_1|$ for $\kappa=1.5, c_1=1, d=2.5, P=1, R=2, \alpha(t)=tanh(3t), \gamma(t)=2t, \delta(t)=t+e^t$, and $t=1$ for the 2D graphics.

$$\begin{aligned}&(\alpha(t)+\gamma(t))\left(-(\gamma)^2+2\gamma\gamma''\right)-4\left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)\right.\\&\left.+\kappa\delta(t)\right)\gamma^2-4\kappa\lambda(t)\gamma^3+4\left(\beta_1(t)+\beta_2(t)\gamma^3+\beta_3(t)\gamma^4\right)\\&=0.\end{aligned}\tag{24}$$

By balancing principle, we obtain $N=1$. Therefore, the solution of Eq. (24) can be considered as (12). Substituting Eq. (12) into Eq. (24) and then equating the coefficients of $Q^j(z)$ to zero, the following nonlinear algebraic system is obtained

$$\begin{aligned}&-4\kappa\delta(t)c_0^2-4t\omega'(t)c_0^2+4\beta_3(t)c_0^4-4\kappa^2\alpha(t)c_0^2-4\omega(t)c_0^2-4\kappa\lambda(t)c_0^3+4\beta_1(t)\\&+4\beta_2(t)c_0^3=0,\end{aligned}\tag{25}$$

$$\begin{aligned}&-8\kappa\delta(t)c_1c_0-8t\omega'(t)c_1c_0-8\kappa^2\alpha(t)c_1c_0+2\alpha(t)c_1c_0-8\omega(t)c_1c_0\\&+12\beta_2(t)c_1c_0^2+16\beta_3(t)c_1c_0^3+2\gamma(t)c_1c_0-12\kappa\lambda(t)c_1c_0^2=0,\end{aligned}\tag{26}$$

$$\begin{aligned}&-4\kappa^2\alpha(t)c_1^2-6\alpha(t)c_1c_0-4\omega(t)c_1^2-4t\omega'(t)c_1^2-12\kappa\lambda(t)c_0c_1^2-6\gamma c_1c_0\\&+24\beta_3(t)c_0^2c_1^2+\alpha(t)c_1^2+\gamma(t)c_1^2-4\kappa\delta(t)c_1^2+12\beta_2(t)c_0c_1^2=0,\end{aligned}\tag{27}$$

$$4\gamma c_1c_0+16\beta_3(t)c_0c_1^3-4\gamma c_1^2+4\alpha(t)c_1c_0-4\kappa\lambda(t)c_1^3-4\alpha(t)c_1^2+4\beta_2(t)c_1^3=0,\tag{28}$$

$$4\beta_3(t)c_1^4+3\alpha(t)c_1^2+3\gamma(t)c_1^2=0.\tag{29}$$

This system is solved by using Maple to get following:

$$\begin{aligned}&c_0=0,\quad c_1=c_1,\quad \beta_1(t)=\left(\frac{c_0(c_1+c_0)}{2c_1}\right)^2((\alpha(t)+\gamma(t))),\\&\beta_2(t)=\frac{(c_1+2c_0)(\gamma(t)+\alpha(t))+\kappa\lambda(t)c_1^2}{c_1^2},\quad \beta_3(t)=-\frac{3}{4}\frac{\alpha(t)+\gamma(t)}{c_1^2},\\&\omega(t)=\frac{1}{t}\int_0^t\left(\frac{1}{4c_1^2}\left(-4c_1^2\kappa(\kappa\alpha(t)+\delta(t))+\left(c_1^2+6c_1c_0+6c_0^2\right)(\alpha(t)+\gamma(t))\right)\right)dt.\end{aligned}\tag{30}$$

Substituting Eq. (11) into (9) and then putting Eqs. (9), (30) into (12) along with (23), we have

$$q_3(x,t)=\left(\sqrt{c_0+\frac{c_1P}{(1+de^z)\left(P+R\left(\frac{1}{1+de^z}-1\right)\right)}}\right)e^{(i\phi(x,t))},\tag{31}$$

where

$$z=x+2\kappa\int_0^t\alpha(t)dt,\qquad \phi=-\kappa x+\omega(t)t.$$

Fig. 2 demonstrates the bright soliton solution $|q_3|$for $\kappa=1.5, c_1=2$, $c_0=1, d=2.5, P=1.5, R=0.75, \gamma(t)=e^{2t}, \delta(t)=tanh(t)$ and $\alpha(t)=$ sech $(-t)$.

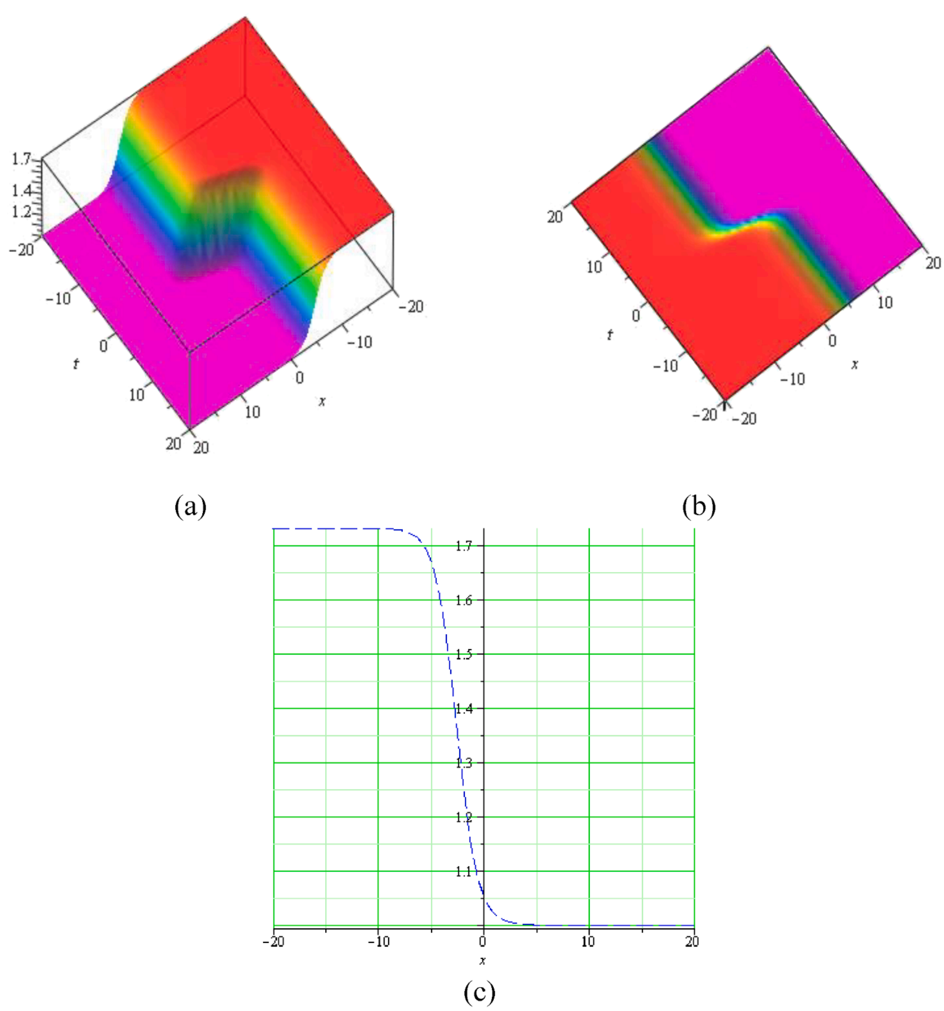

Fig. 2. The 3D (a), contour (b) and 2D (c) surfaces of the soliton solution of Eq. (31) given by $|q_3|$ for $\kappa=1.5, c_1=1$, $c_0=1, d=2.5, P=1.5, R=0.75, \alpha(t)=$ sech$(-t), \gamma(t)=e^{2t}, \delta(t)=tanh(t)$, and $t=1$ for the 2D graphics.

Triple-power law

By substituting $\beta(t)F\left(|q|^2\right) = \beta_1(t)g^a + \beta_2(t)g^{2a} + \beta_3(t)g^{3a}$ in (4), we get the following equation

$$(\alpha(t)+\gamma(t))g'' - \left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t) + \kappa\delta(t)\right)g - \kappa\lambda(t)g^{2n+1} + \beta_1(t)g^{2a+1} + \beta_2(t)g^{4a+1} + \beta_3(t)g^{6a+1} = 0. \quad (32)$$

By virtue of the transformation

$$g = \sqrt[2a]{\gamma}, \quad (33)$$

and $n = 3a$, Eq. (32) modifies to

$$(\alpha(t)+\gamma(t))\left((1-2a)(\gamma')^2 + 2a\gamma''\right) - 4a^2\left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t) + \kappa\delta(t)\right)\gamma^2 - 4a^2\kappa\lambda(t)\gamma^5 + 4a^2\left(\beta_1(t)\gamma^3+\beta_2(t)\gamma^4+\beta_3(t)\gamma^5\right) = 0. \quad (34)$$

By balancing principle, we obtain $N = 1$. Therefore, the solution of Eq. (34) can be considered as (12). Substituting Eq. (12) into Eq. (34) and then equating the coefficients of $Q^j(z)$ to zero, the following nonlinear algebraic equations are obtained

$$-4a^2\kappa^2\alpha(t)c_0^2 - 4a^2\kappa\delta(t)c_0^2 - 4a^2\kappa\lambda(t)c_0^5 + 4a^2\beta_2(t)c_0^4 + 4a^2\beta_3(t)c_0^5 - 4a^2\omega(t)c_0^2 + 4a^2\beta_1(t)c_0^3 - 4a^2t\omega'(t)c_0^2 = 0, \quad (35)$$

$$12a^2\beta_1(t)c_0^2c_1 + 20a^2\beta_3(t)c_0^4c_1 - 8a^2\kappa^2\alpha(t)c_0c_1 - 20a^2\kappa\lambda(t)c_0^4c_1 + 2\gamma(t)ac_1c_0 - 8a^2t\omega'(t)c_0c_1 - 8a^2\omega(t)c_0c_1 - 8a^2\kappa\delta(t)c_0c_1 + 2\alpha(t)ac_1c_0 + 16a^2\beta_2(t)c_0^3c_1 = 0, \quad (36)$$

$$-4a^2\omega(t)c_1^2 - 6\gamma(t)ac_1c_0 - 6\alpha(t)ac_1c_0 - 40a^2\kappa\lambda(t)c_0^3c_1^2 + 12a^2\beta_1(t)c_0c_1^2 + 40a^2\beta_3(t)c_0^3c_1^2 - 4a^2\kappa\delta(t)c_1^2 - 4a^2t\omega'(t)c_1^2 - 4a^2\kappa^2\alpha(t)c_1^2 + 24a^2\beta_2(t)c_0^2c_1^2 + \gamma(t)c_1^2 + \alpha(t)c_1^2 = 0, \quad (37)$$

$$4\gamma(t)ac_1c_0 + 4a^2\beta_1(t)c_1^3 - 2\gamma(t)c_1^2 + 4\alpha(t)ac_1c_0 + 16a^2\beta_2(t)c_0c_1^3 - 2\alpha(t)c_1^2a - 2\gamma(t)c_1^2a + 40a^2\beta_3(t)c_0^2c_1^3 - 2\alpha(t)c_1^2 - 40a^2\kappa\lambda(t)c_0^2c_1^3 = 0, \quad (38)$$

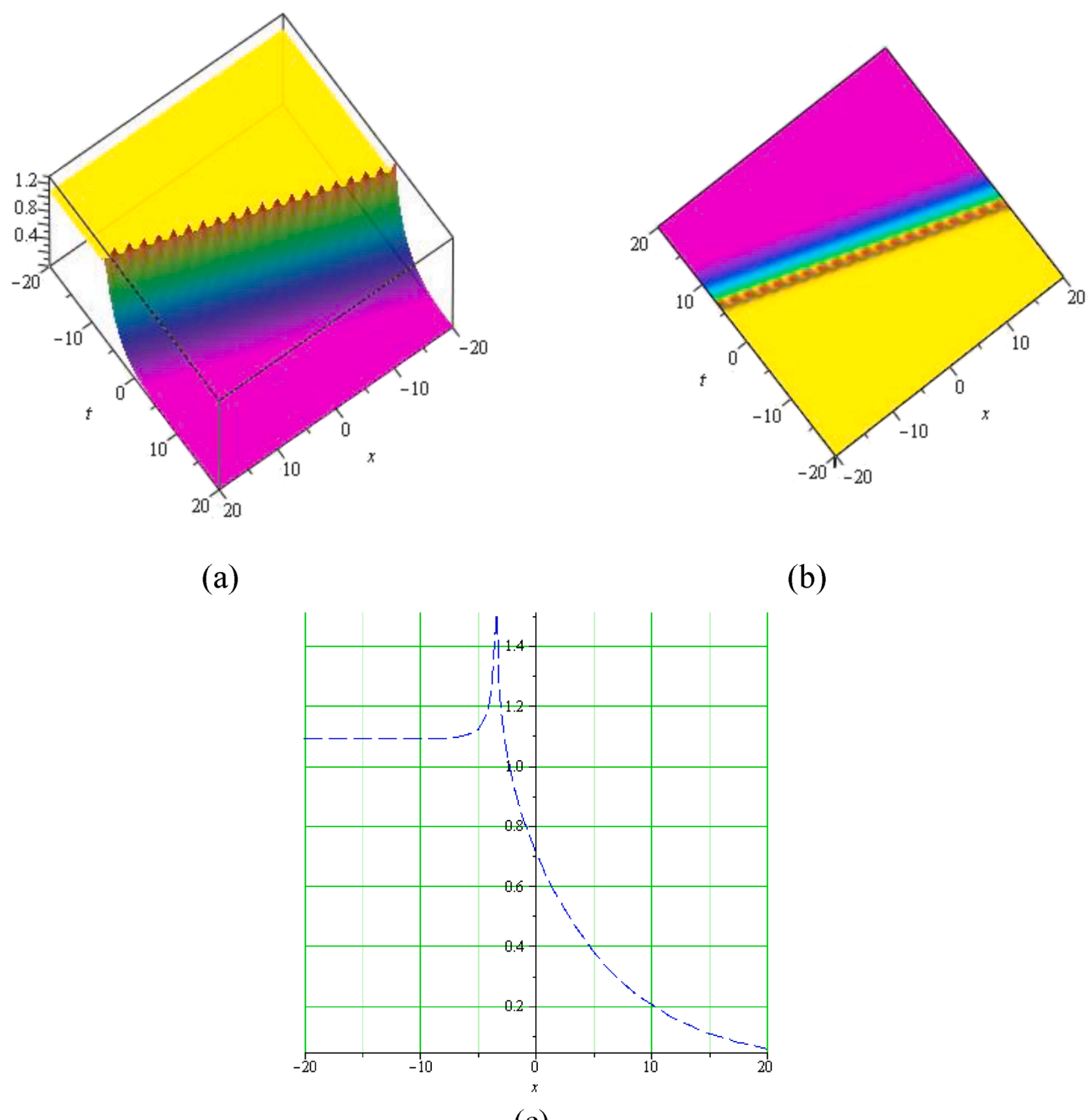

Fig. 3. The 3D (a), contour (b) and 2D (c) surfaces of the soliton solution of Eq. (42) given by $|q_4|$ for $\kappa = 0.75, c_1 = 2, d = 1.5, P = 0.5, R = 1, a = 4, \alpha(t) = 2, \gamma(t) = 2sin(t), \delta(t) = sinh(t)$, and $t = 1$ for the 2D graphics.

$$20a^2\beta_3(t)c_0c_1^4+\alpha(t)c_1^2-20a^2\kappa\lambda(t)c_0c_1^4+4a^2\beta_2(t)c_1^4+2\gamma(t)c_1^2a+\gamma(t)c_1^2+2\alpha(t)c_1^2a=0, \tag{39}$$

$$-4a^2\kappa\lambda(t)c_1^5+4a^2\beta_3(t)c_1^5=0. \tag{40}$$

The above equations are solved by using Maple to get following two sets:

Set 1.

$$\begin{aligned}
&c_0=0,\quad c_1=c_1,\quad \beta_3(t)=\kappa\,\lambda(t),\\
&\beta_1(t)=\frac{1}{2a^2c_1}(\gamma(t)+\alpha(t)+\gamma(t)a+\alpha(t)a),\\
&\beta_2(t)=-\frac{1}{4a^2c_1^2}(2\alpha(t)a+\gamma(t)+2\gamma(t)a+\alpha(t)),\\
&\omega(t)=\frac{1}{t}\int_0^t\left(-\frac{1}{4a^2}\left(4a^2\kappa^2\alpha(t)-\alpha(t)+4a^2\kappa\delta(t)-\gamma(t)\right)\right)dt.
\end{aligned} \tag{41}$$

Substituting Eq. (11) into (9) and then putting Eqs. (9), (41) into (12) along with (33), we have

$$q_4(x,t)=\left(\sqrt[2a]{\frac{c_1P}{(1+de^z)\left(P+R\left(\frac{1}{1+de^z}-1\right)\right)}}\right)e^{(i\phi(x,t))}, \tag{42}$$

where

$$z=x+2\kappa\int_0^t\alpha(t)dt,\qquad \phi=-\kappa x+\omega(t)t.$$

Fig. 3 illustrates the bright soliton solution $|q_4|$ for $\kappa=0.75, c_1=2, d=1.5, P=0.5, R=1, a=4, \gamma(t)=2sin(t), \delta(t)=sinh(t)$ and $\alpha(t)=2$.

Set 2.

$$\begin{aligned}
&c_0=-c_1,\ \ c_1=c_1,\ \ \beta_1(t)=-\frac{(a+1)}{2a^2c_1}(\alpha(t)+\gamma(t)),\\
&\beta_2(t)=-\frac{(2a+1)}{4a^2c_1^2}(\gamma(t)+\alpha(t)),\quad \beta_3(t)=\kappa\,\lambda(t),\\
&\omega(t)=\frac{1}{t}\int_0^t\left(-\frac{1}{4a^2}\left(4a^2\kappa(\delta(t)+\kappa\alpha(t))-\gamma(t)-\alpha(t)\right)\right)dt.
\end{aligned} \tag{43}$$

Substituting Eq. (11) in (9) and then putting Eqs. (9), (43) into (12) along with (33), we have

$$q_5(x,t)=\left(\sqrt[2a]{-c_1\left[1-\frac{P}{(1+de^z)\left(P+R\left(\frac{1}{1+de^z}-1\right)\right)}\right]}\right)e^{(i\phi(x,t))}, \tag{44}$$

where

$$z=x+2\kappa\int_0^t\alpha(t)dt,\qquad \phi=-\kappa x+\omega(t)t.$$

Conclusion

In this work, we investigate the optical soliton solutions of the generalized non-autonomous NLSE with time-dependent coefficients through the use of the new Kudryashov's method. The approach is a very powerful scheme that first transforms the generalized NLSE to an ODE through a complex wave transformation and then the coefficients of equal powers are compared in the obtained ODEs, to get some nonlinear algebraic equations which are later solved by Maple. We considered three interesting non-Kerr laws which are categorized as the quadratic-cubic law, anti-cubic law, and triple power law. From our results, we suggest that the method is very effective, powerfully, and a well-defined algorithm.

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CRediT authorship contribution statement

Hadi Rezazadeh: Data curation, Formal analysis. **Najib Ullah:** Conceptualization. **Lanre Akinyemi:** Formal analysis. **Abdullah Shah:** Writing - review & editing. **Seyed Mehdi Mirhosseini-Alizamin:** Validation. **Yu-Ming Chu:** Funding acquisition. **Hijaz Ahmad:** Writing - original draft, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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