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A stable downward continuation by using the ISVD method*

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SUMMARY

Downward continuation of potential fields represents a very interesting way to enhance the information content of a gravity or magnetic map. In fact, apart from the increase of resolution, shared with many recent methods involving the use of directional derivatives, the downward continued data have the advantage of maintaining the physical dimensions of the original ones. This means that the interpretative tools that may be used are the same as for the untransformed data, but the obtainable models can fully exploit the benefits of the increased resolution of the field. However, because of its inherent instability, this method has progressively lost popularity. In this paper a stable downward continuation algorithm is presented. It is based on the computation of stable vertical derivatives obtained by the ISVD method and Taylor series expansion of the field. The algorithm uses both frequency and space domain transformations. Tests on synthetic examples proved its utility especially in cases where the signal is corrupted by noise, when the continuation level is greater than the data sampling step or when the needed continuation level is close to the source top level.

Its application to real gravity data of Southern Italy shows that a stable downward continuation of a potential field to a close-to-source level gives very valuable information. In this case, while our algorithm can give meaningful results even continuing the field to a level close to sources, the use of a standard method dramatically decreased the signal-to-noise ratio necessitating low-pass filtering of the transformed map.

Key words: data enhancement, downward continuation, potential fields.

INTRODUCTION

The shape of a potential field anomaly is intrinsically smoothed with respect to that of their sources. Geophysicists have therefore always attempted to enhance the potential fields measured on the ground surface or from airborne surveys. A number of different approaches have been used. In general, the computation of vertical or horizontal derivatives of potential fields is involved and, more recently, even combinations of the two. First and second vertical derivatives have been in common use since the early decades of the 20th century (i.e. Evjen 1936). Later methods focused mainly on the automatic definition of the horizontal boundaries of the anomaly sources and in some cases even of the depth to the density (or magnetization) contrast. Among them, we cite the study of the horizontal derivative (Cordell & Grauch 1985), the analytic signal (Nabighian 1984; Roest *et al.* 1992), the generalized analytic signal (Hsu *et al.* 1996, 1998), Euler deconvolution (Thompson 1982; Reid *et al.* 1990), the Source Parameter Imaging method (SPI™, Thurston & Smith 1997; Smith *et al.* 1998; Thurston *et al.* 1999) and the Enhanced Horizontal Derivative (Fedi & Florio 2001).

Another common approach to improve resolution of potential field maps is to compute the field to a level close to the sources by means of downward continuation. The main advantage of enhancing potential fields by downward continuation with respect to other derivative-based techniques is that the physical dimensions of the transformed field are the same as in the original data. So a number of interpretation tools can, in principle, still be applied to model the transformed field. The increased resolution may help to better define the shape of the source body and, in the magnetic case, can also provide very important information about the magnetization direction. In fact in the continued field the highs and lows of single anomalies are better separated from those of adjacent anomalies, and new patterns related to the magnetization direction can be revealed.

However, in practice these advantages can not be fully exploited because of the need to control and stabilize the amplitudes of high frequencies in the transformed fields. In fact, the main problem with the downward continuation, similar to that of all the methods involving the use of directional derivatives, is the well-known inherent instability of the transformation, defined by the downward continuation frequency response:

$$R_{\mathrm{down}}(u, v) = e^{+h\sqrt{u^2+v^2}}, \tag{1}$$

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where h is the difference in altitude between the original survey and the target level, u and v are spatial wavenumbers in the two orthogonal horizontal directions.

The theory of potential fields states that it is possible to perform downward continuations only in the harmonic region, that is in a source-free volume, where Laplace equation applies. So, a limiting case for downward continuation is represented by the level of the top of the source. It was noted (e.g. Roy 1967) that this level marks the limit of the convergence of the continuation process by using the FFT operator. Consistently, the recognition of an oscillatory character of the continued field was used to indicate the depth to the source. Ku *et al.* (1971) pointed out that, for discrete data, a prior filtering is performed by the choice of sampling step. This observation implies that the oscillatory character of a downward continued field is determined not only by the depth of the continuation level, but also by the sampling step, in the sense that the smaller the sampling step the nearer will be the continuation level at which the transformation becomes unstable.

Techniques to improve stability

The positive exponent in eq. (1) implies that the higher the frequency of a spectral component, the greater will be the corresponding amplification defined by the downward continuation filter. Moreover, the greater the continuation depth attempted, the stronger this amplification will be. In practice, the unavoidable presence of high frequency noise in real data may prevent the computation of meaningful downward continued fields, because it gets multiplied in the Fourier domain by a exponentially growing factor. So, different approaches have been devised to combat this divergent nature of the theoretical frequency response of downward continuation. The first of them is aimed at the definition of rules to define the maximum level of downward continuation attainable without an excessive amplification of noise, often expressed as a fraction of the inferred depth to sources (i.e. Henderson 1960). Other strategies are devoted to attenuate the highest frequencies of the potential field by means of some smoothing process (i.e. Dean 1958) that may be tuned in

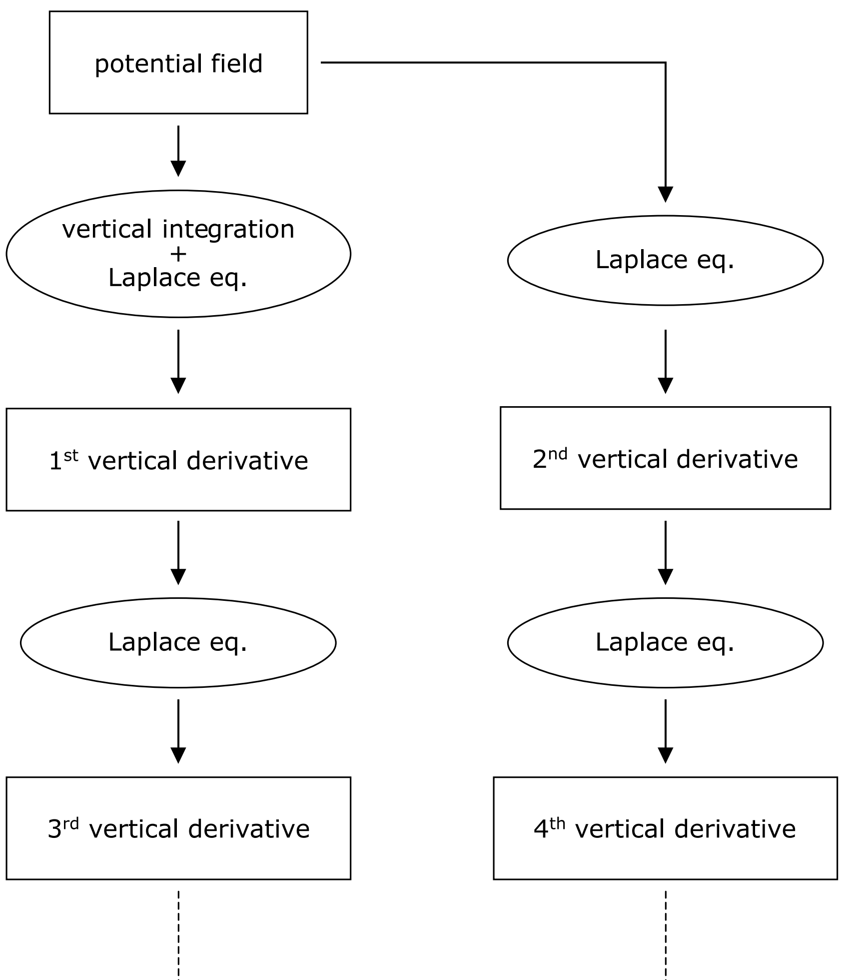

Figure 1. Flow diagram showing the ISVD scheme for the computation of stable vertical derivatives. Second vertical derivative is obtained through the computation of second horizontal derivatives of the potential field following Laplace equation. Higher order even vertical derivatives are computed in the same way starting from the second vertical derivative. First vertical derivative is obtained through the computation of second horizontal derivatives of the vertical integrated potential field and through application of Laplace equation. Third order vertical derivative is computed through the computation of second horizontal derivatives of the first vertical derivative following Laplace equation. Successive odd vertical derivatives are computed in the same way starting from the third vertical derivative.

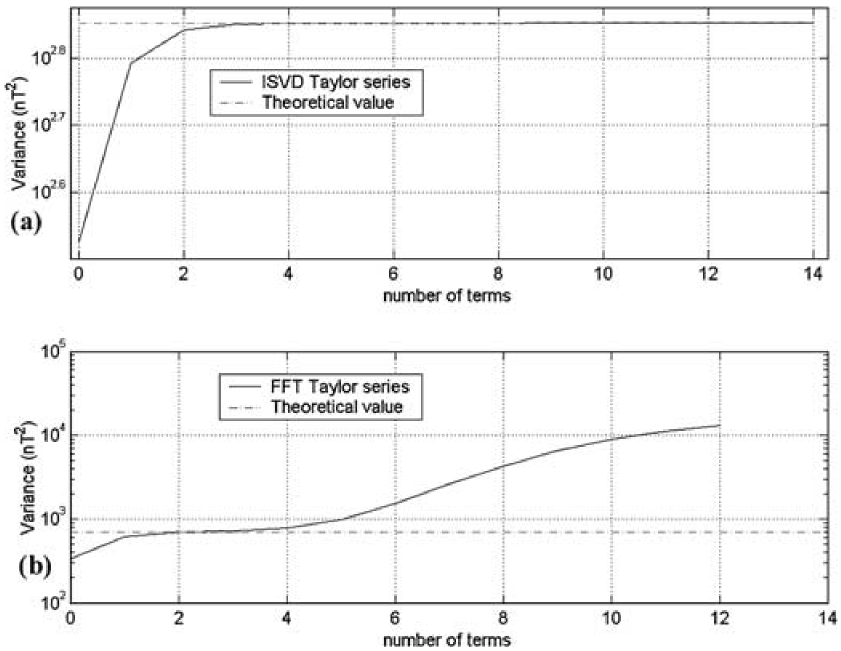

Figure 2. Analysis of variance for the 3 m Taylor series downward continuations of the total field magnetic anomaly due to a prismatic body (20 × 20 × 10 m) at 8 m from its top. The variance of the downward continued field is plotted against the number of derivative terms considered. (a) ISVD vertical derivatives; (b) FFT vertical derivatives. Dashed line represents the theoretical value of variance of the field at the same level.

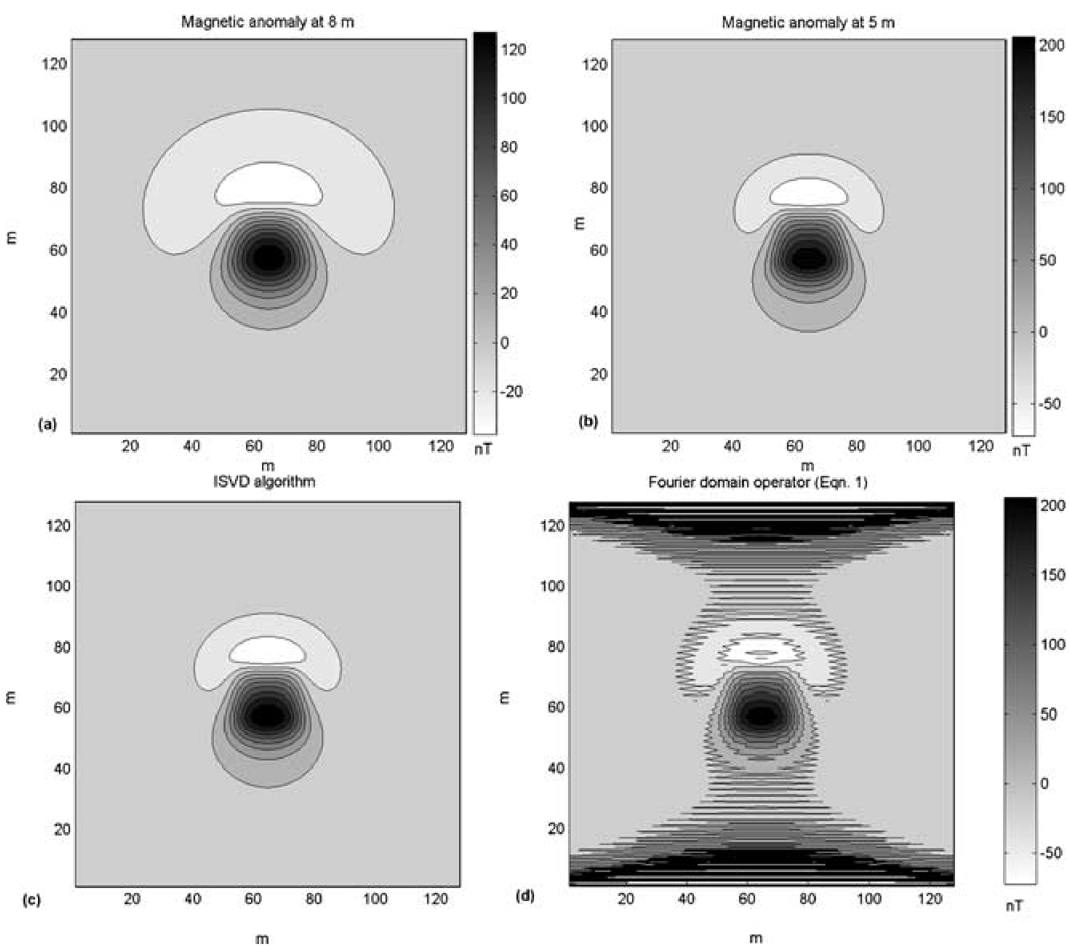

Figure 3. (a) Total field magnetic anomaly due to a prismatic body (20 × 20 × 10 m) at 8 m from its top. The grid has 128 × 128 nodes, with spacing of 1 m. (b) Magnetic anomaly due to the same source computed at 5 m from its top. (c, d) Downward continuations of field in (a) of 3 m performed using respectively the ISVD algorithm (5 terms in the summation) and FFT linear filter.

function of the value of h (i.e. Grant & West 1965). Baranov (1975) suggests that a downward continuation to depths greater than the horizontal sampling interval should be avoided; otherwise, it may be necessary to apply a low-pass filter to the data to eliminate the high frequency effects arising from shallow sources. More recently, Pawlowski (1995) proposed a downward continuation operator that behaves like the theoretical filter only at low wavenumbers, while at higher wavenumbers the amplitude spectrum is attenuated according to a function determined by Wiener filtering theory. The result is that the field due to deep sources is actually downward continued, while the contribution of shallower sources is left unchanged.

Computational approaches

The downward continuation may be performed by using a number of different approaches. The most common method used is perhaps the linear transformation obtained by multiplying the frequency response in eq. (1) by the Fourier transform of the potential field at the measurement level and inverse-transforming the result. This approach has its main limitations in the errors generated in the high-wavenumber components of the data spectrum by the aliasing and Gibbs' phenomenon. Moreover, the highest wavenumbers are also influenced by random and/or geological noise. So, the use of the downward continuation Fourier domain operator (eq. 1) can lead to an unstable process and to meaningless transformed maps, in which the highest wavenumber components become excessively amplified.

Some authors have instead proposed a formulation of analytical continuation as an inverse problem, by computing an equivalent layer and by multiplying the obtained density/magnetization distribution with the appropriate Green's function (e.g. Leão & Silva 1989). The inversion is generally unstable in presence of noise and the use of a damping factor is needed; this damping factor has the same role as low-pass wavenumber filtering (Leão & Silva 1989). Other inverse approaches to the analytical continuation of potential fields were proposed by (Courtillot *et al.* 1978; Achache *et al.* 1987; Ivan 1994). Fedi *et al.* (1999) formulated an inverse method which allows continuation of data not arranged on a regular grid.

A different approach, first proposed by Evjen (1936), consists in approximating the downward continued field by a Taylor series expansion:

$$f(x,y,z) = f(x,y,z_0) + \left[\frac{\partial f}{\partial z}\right]_{z_0}(z-z_0) + \frac{1}{2!}\left[\frac{\partial^2 f}{\partial z^2}\right]_{z_0}(z-z_0)^2 + \cdots + \frac{1}{m!}\left[\frac{\partial^m f}{\partial z^m}\right]_{z_0}(z-z_0)^m \qquad (2)$$

where $f(x, y)$ is the potential field, z_0 is the measurement level and z the continuation level.

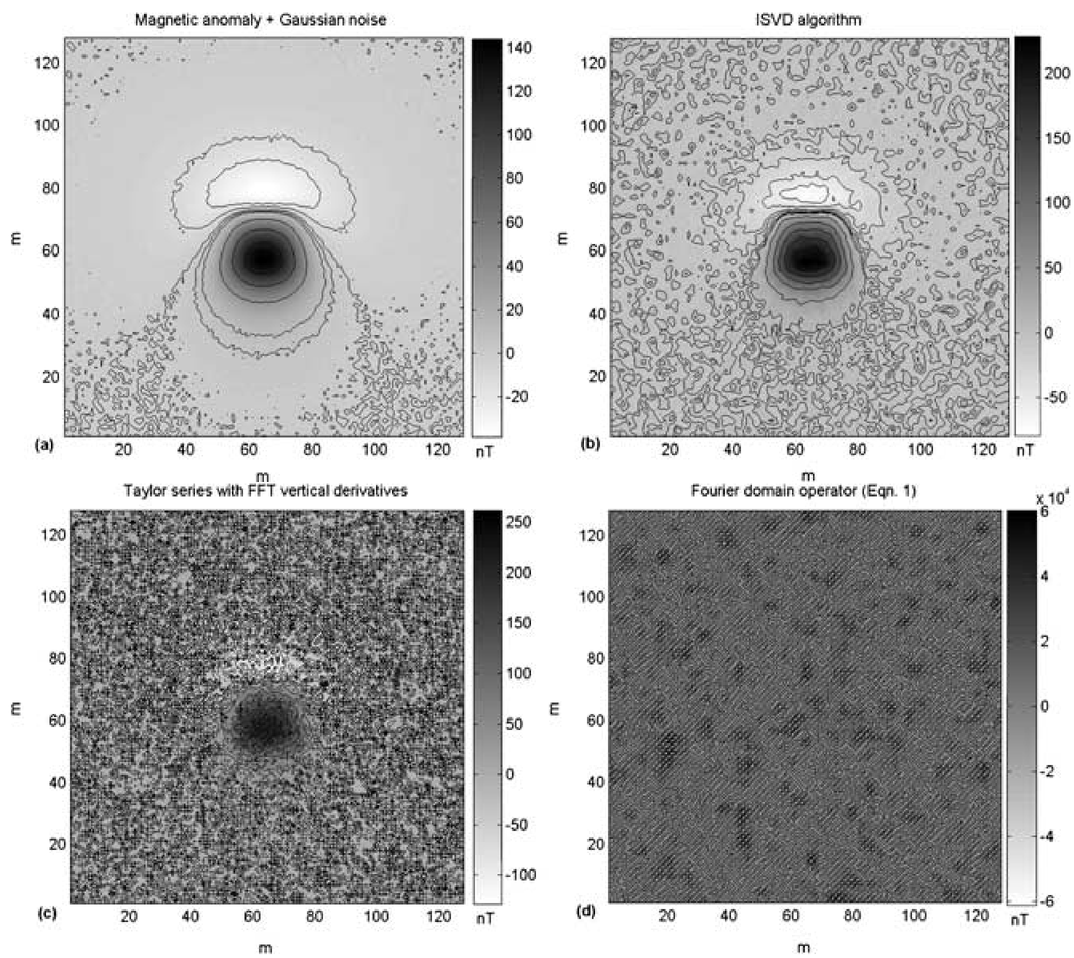

Figure 4. (a) Total field magnetic anomaly as in Fig. 2(a) corrupted with Gaussian noise (2.5 per cent of the anomaly amplitude). (b, c, d) Downward continuation to 5 m (3 grid units from original level) performed using respectively ISVD algorithm (5 terms in the summation), Taylor series with FFT vertical derivatives (2 terms in the summation) and FFT linear filter.

This method involves the computation of m-order vertical derivatives of the potential field $f(x, y)$. Evjen (1936) suggested to use the Laplace equation to compute the derivatives from degree two on, while the computation of the first vertical derivative was implemented as a graphical method. Peters (1949) found a very clever method to eliminate the odd derivative terms from the Taylor's summation by combining upward and downward continuation expansion terms. However, since the definition of the FFT algorithm (Cooley & Tukey 1965) most implementations of the Taylor series approach to downward continuation make use of the Fourier domain approach to perform the vertical derivatives (e.g. Gunn 1975). Cordell & Grauch (1985) used a Taylor series expansion to downward continue a magnetic field from a single level to a non planar surface and vice versa. They calculated the vertical derivatives using the discrete Fourier transform, but to get a stable continued field they had to truncate the series to just two or three terms (first and second vertical derivative respectively).

Fedi & Florio (2001) proposed the Integrated Second Vertical Derivative (ISVD) method to compute any order vertical derivative. This scheme involves both wavenumber and space domain transformations and gives more stable results than the usual Fourier method (Fig. 1). The first vertical derivative of the gravity or magnetic field is computed in two steps: (a) vertically integrating the field by using a frequency domain operator; (b) computing its second vertical derivative by means of the sum of its second horizontal derivatives, according to the Laplace equation. All the other vertical derivatives are again computed by the Laplace equation starting from the field (even derivatives) and its first vertical derivative (odd derivatives). Such a computation involves: an intrinsically stable transformation in the Fourier domain (a) and the use of methods like finite

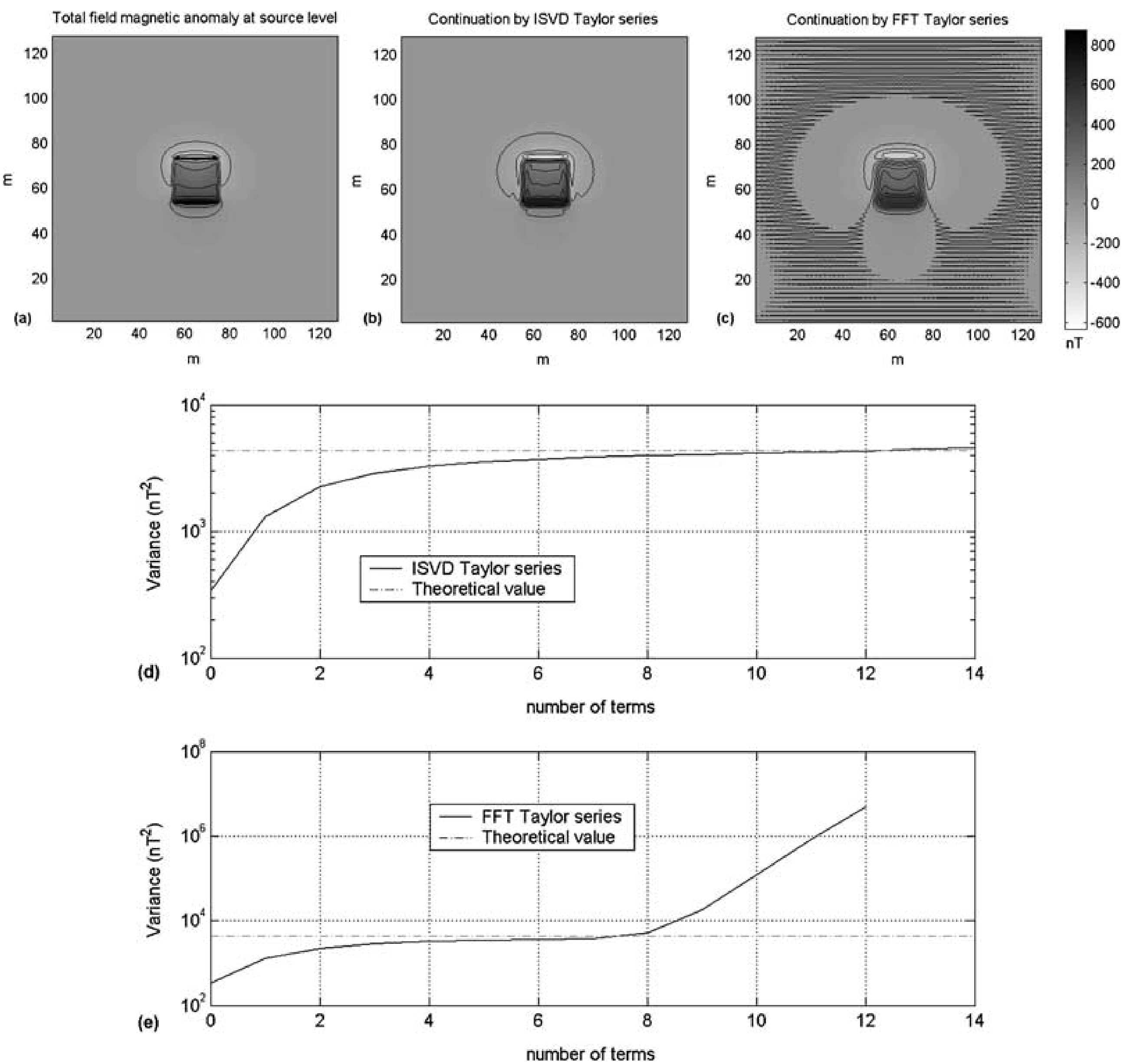

Figure 5. (a) Total field magnetic anomaly due to the same prismatic body as in Fig. 2, at the source level. (b) ISVD downward continuation of 8 grid units to the level of source (12 terms in the summation). (c) Same continuation by using Taylor series with FFT vertical derivatives (6 terms in the summation). (d, e) Variance plots for the Taylor series with ISVD and FFT vertical derivatives. In this case a slow convergence toward the theoretical values occur.

differences to the computation of second horizontal derivatives (b). This is quite different from previous approaches using Taylor series (Evjen 1936; Cordell & Grauch 1985), since the computation of the first vertical derivative, and consequentially of all the higher order odd derivatives, is stable and accurate. The sum of stable vertical derivatives ensures the control of high wavenumber amplification in the downward continued field.

SYNTHETIC EXAMPLES

The ISVD method requires the computation of second horizontal derivatives. These derivatives may be easily computed by three-points finite-differences relations that are comparatively free from the inconveniences that affect the computation in the Fourier domain. The computation of stable horizontal derivatives may also be performed by means of spline interpolating functions.

The first example illustrates the downward continuation of a noise-free total field magnetic anomaly (sampling step of 1 m) generated by a prismatic source (20 × 20 × 10 m) from 8 grid units above its top to 5 grid units. The computation was performed on a window of 128 × 128 grid nodes.

To choose the maximum number of terms to include in the summation, the inspection of the plot of the L_2 norm or of the variance of the continued field (by the Taylor series) vs. the number of derivative terms used, may be useful. However, we performed several tests that indicated that to study the behaviour of the series the most useful parameter is the variance. From this plot (Figs 2a and b) the convergent or divergent character of the series appears quite evident, and it is easy to determine the derivative order beyond which there is no real gain in approximating the function.

The stable character of the ISVD downward continuation is quite evident, while the FFT vertical derivatives determine a divergent series from 4th term on. These plots indicate that taking more than 4 or 5 ISVD derivative terms will not help in get a better result, but do not affect its quality. On the contrary, taking more than 2 or 3 FFT derivative terms will result in a continued field dominated by excessively amplified high wavenumbers.

In Figs 3(a) and (b) the theoretical total field anomaly at 8 and 5 grid units are respectively shown. In Figs 3(c) and (d) the magnetic anomaly of Fig. 3(a) downward continued to 5 grid units by the Taylor series using ISVD vertical derivatives and direct application of Fourier downward continuation filter are respectively shown. Strong disturbances in the direct application of Fourier downward continuation filter appear. This noise-free example demonstrates that the method using unstable transformation in the Fourier domain is more affected by the window size than the ISVD method. ISVD downward continuation involves only an inherently stable vertical integration in the Fourier domain.

To simulate a more realistic case, the total magnetic field in Fig. 3(a) was corrupted with Gaussian noise, its amplitude being the 2.5 per cent of the maximum amplitude of the starting anomaly (Fig. 4a). We will attempt now to perform the same downward continuation (from 8 to 5 grid units above the top of source) by using Taylor series with ISVD or FFT vertical derivatives, and by using the Fourier domain transformation with the continuation operator (eq. 1).

Following the indications given by the variance plot (not shown here), the Taylor series were truncated to 5 and 2 terms when using ISVD (Fig. 4b) and FFT (Fig. 4c) vertical derivatives respectively. Results show that both the Fourier domain transformation with the

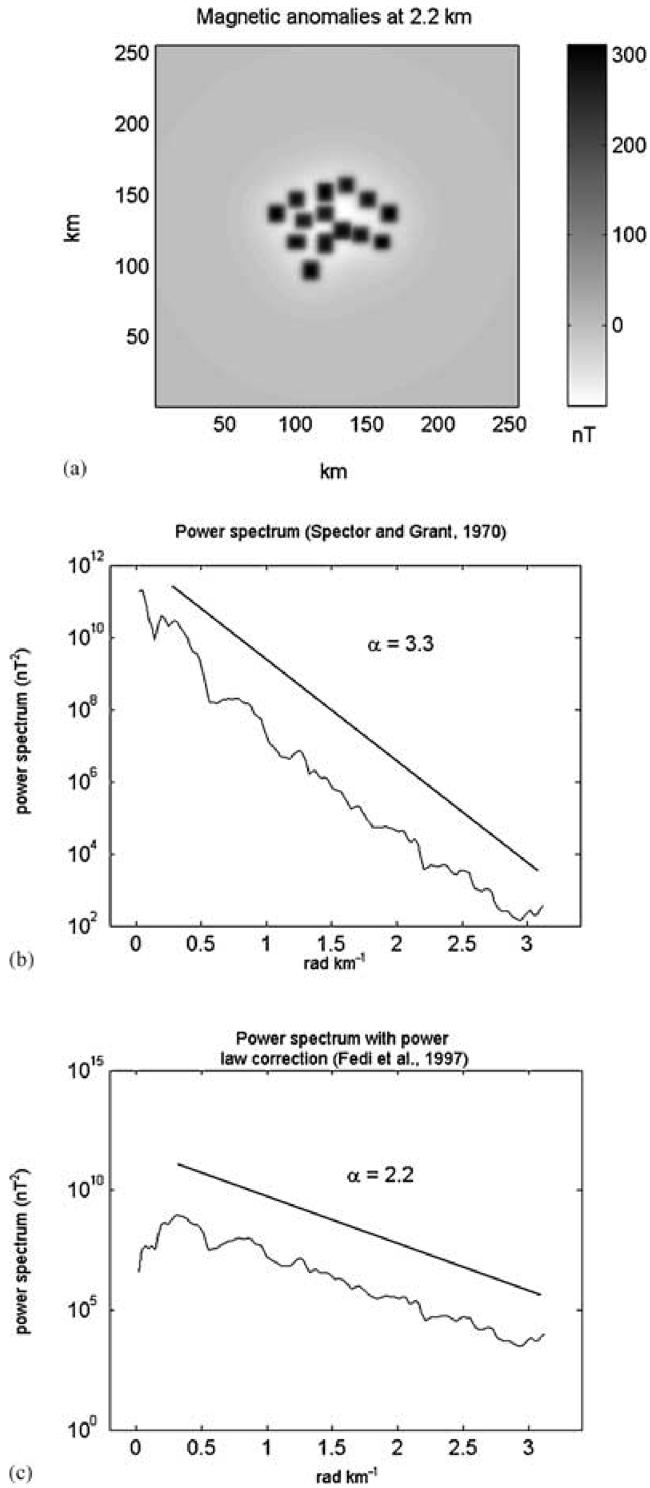

Figure 6. (a) Total field magnetic field produced by an ensemble of 14 prismatic sources. Average prism dimensions: $x = 11.5$ km, $y = 11.3$ km, $z = 10.1$ km; average depth to top $d = 2.2$ km; average magnetization: $M = 1$ A m^{-1}. Magnetization and ambient field are vertical. Grid interval is 1 km. (b) Logarithm of the radial power spectrum of the magnetic field in (a). The half slope (α) of a linear regression approximating the curve gives an estimation of depth to top of magnetic sources (Spector & Grant 1970). This value is in general overestimated. (c) Following the suggestion of Fedi *et al.* (1997), the application of a correcting factor to the power spectrum substantially improves the depth estimation.

continuation operator of eq. (1) (Fig. 4d) and the Taylor series based on FFT vertical derivatives are dominated by an excessive amplification of the high frequency noise. These cases point out the need for low-pass filtering of the data before or after the continuation using FFT unstable operators. On the contrary, the ISVD continuation responded in a very stable way (Fig. 4b) and the resulting downward continued anomaly has an amplitude in a range similar to that of the theoretical undisturbed anomaly at the target elevation (Fig. 4a).

Fig. 5 illustrates the case of continuation down to the top of the source, by considering as a starting anomaly the same noise-free anomaly of the first example, that is a total field magnetic anomaly at 8 grid units from the top of source. Note that now the continuation step is very large with respect to the grid spacing (8 grid units); moreover we want to continue the field at the measurement level down to the maximum depth theoretically possible. To minimize the edge effects the computation is made in a window of 256 × 256 grid nodes. The variance plots (Figs 5d and e) exhibit a slower convergence toward the theoretical value. If ISVD vertical derivatives are used, the series has a convergent character and the maximum useful derivative order is about 12. In the FFT vertical derivative case, the series diverges from the addition of the 8th derivative term on, so the maximum derivative order that it is possible to use in the Taylor series is about 6.

It can be noted that the ISVD continuation (Fig. 5b) gives still fairly good results: the amplitude range is only a little bit narrower than the theoretical case while the overall shape is very similar. On the contrary, the FFT Taylor series continued field (Fig. 5c) is worse than the ISVD continuation both with respect to the amplitude range and the signal-to-noise ratio. The same continuation performed by using the FFT linear filter (eq. 1) results in a field totally dominated by high-frequency noise, similarly to Fig. 4(d), and it is not shown here.

This last test demonstrates that the ISVD continuation is stable even at the source level. Actually, as a paradox, it will be shown that it is possible to get stable downward continued fields even into the region of sources. Such fields are obviously meaningless and in this case the stability of the continuation may constitute an unexpected obstacle to the interpretation. To help understand whether ISVD downward continuation is still in the harmonic region or not, we propose to use the Spector & Grant (1970) method revised according to Fedi *et al.* (1997). As well-known, this method yields the average depth to the top of magnetic (and gravity) sources from

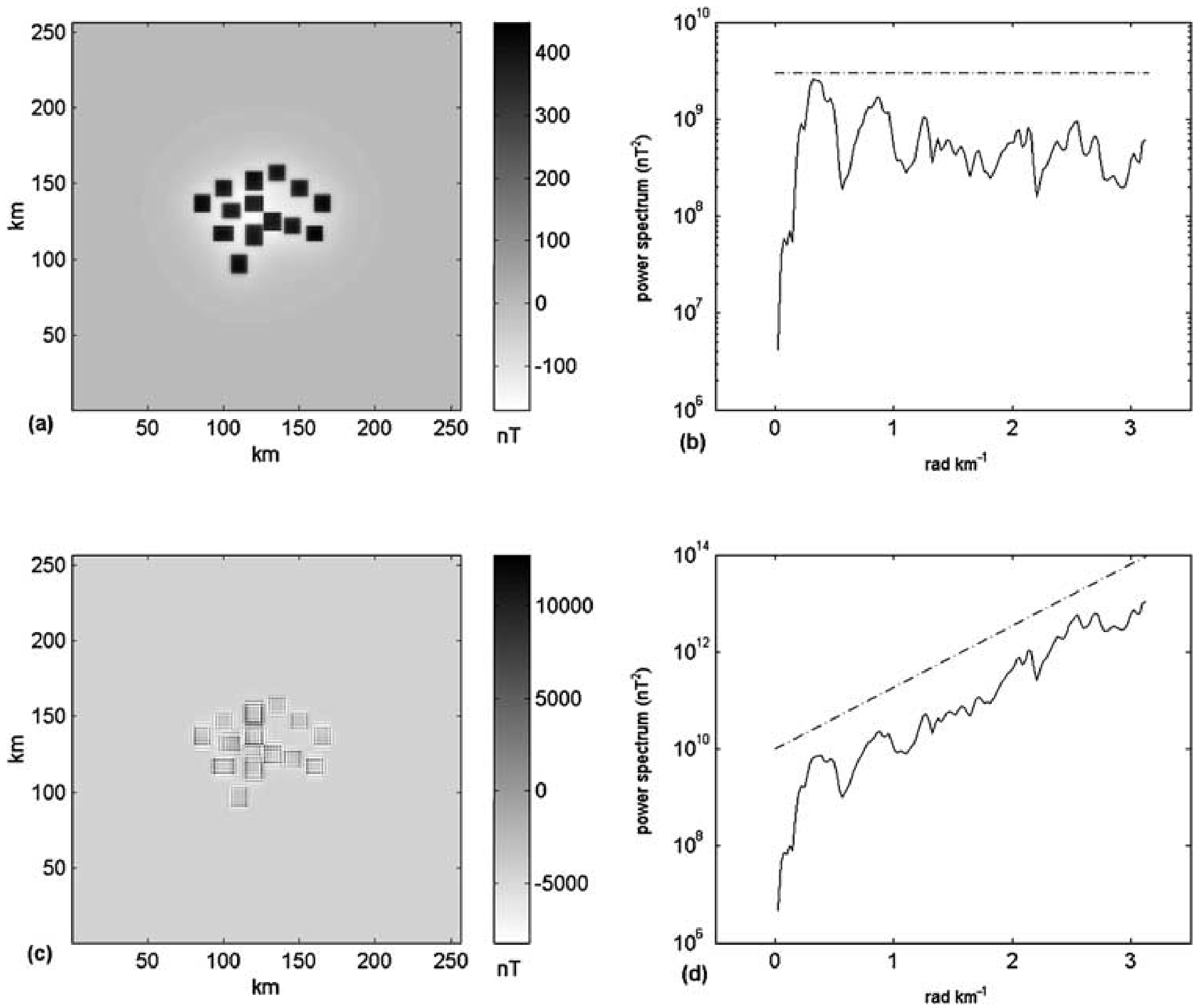

Figure 7. (a) ISVD downward continuation of field in Fig. 6(a) to the average level of top of sources (2.2 km; 14 terms in the summation). (b) despite the stable continued magnetic field, its power spectrum is white, indicating that it is relative to the source level. (c) ISVD downward continuation below the top of sources (14 terms in the summation). The power spectrum (d) reveals a blue character and its computation may help to understand that the field in (c) is meaningless.

the decay rate of the power spectrum of the field. Fedi *et al.* (1997) improved this determination with respect to the classical Spector & Grant (1970) method, by noting that even at the depth of the top of sources the spectrum has a distinct red character, following a power-law behaviour. It represents the cause of the known overestimation of depth to sources by the Spector and Grant's method. Fedi *et al.* (1997) showed that removing this power-law behaviour, by division of the power spectrum with a correcting factor ($\rho^{-2.9}$, where ρ

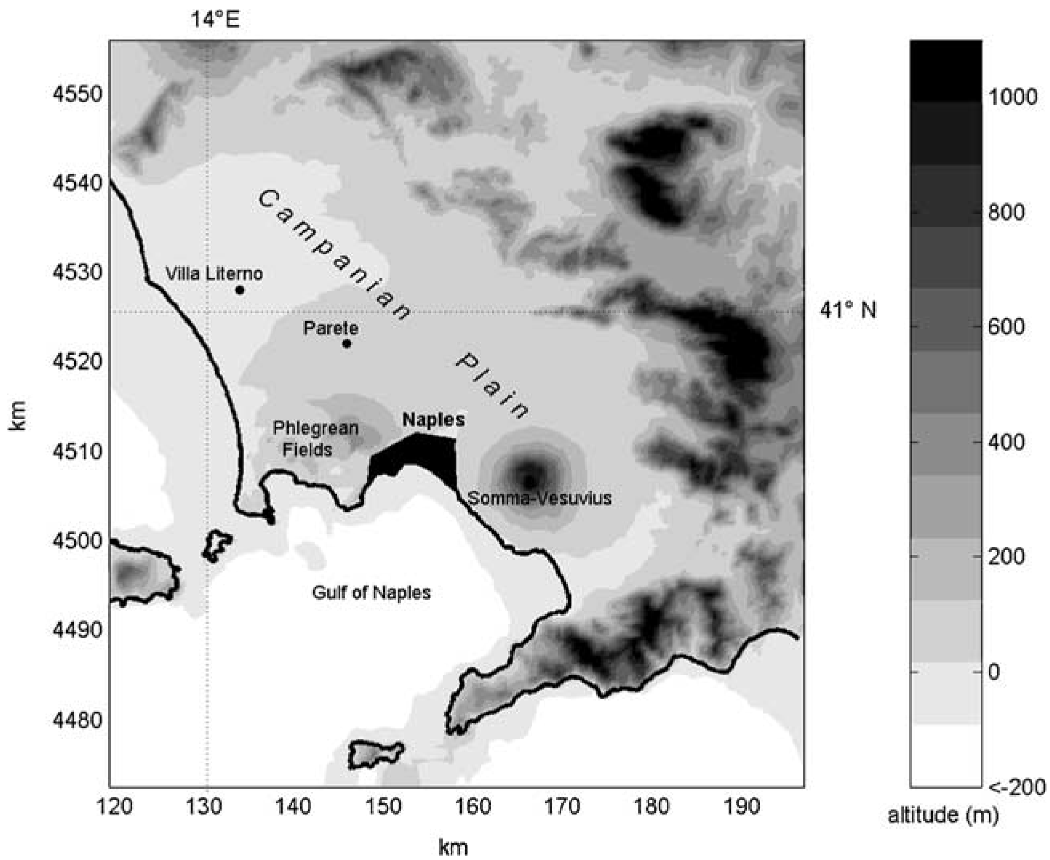

Figure 8. Topographic map of the Campanian plain (Southern Italy).

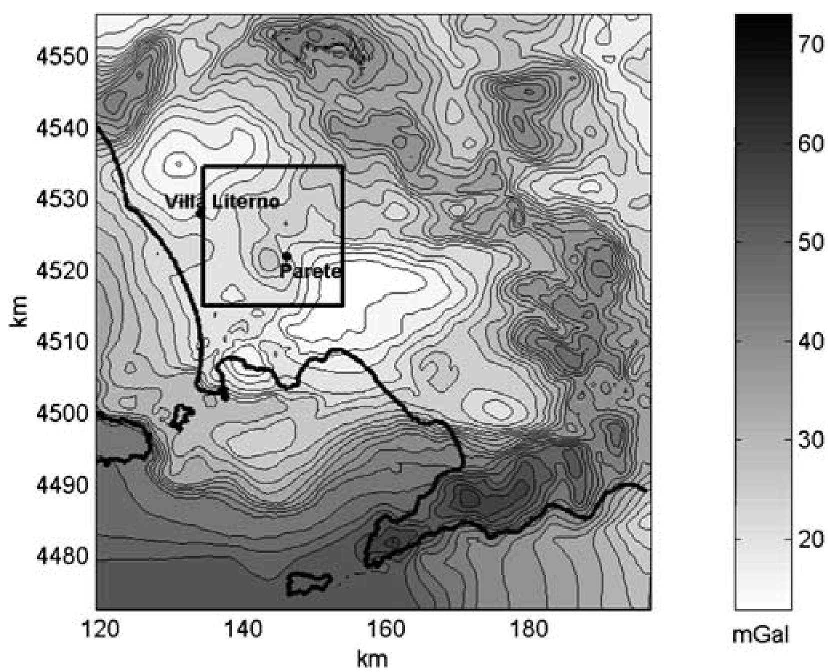

Figure 9. Bouguer gravity map of the Campanian plain. Reduction density 2.2 g cm^{-3}, grid interval 0.5 km. In the box is evidenced the studied area, mapped in Fig. 10(a).

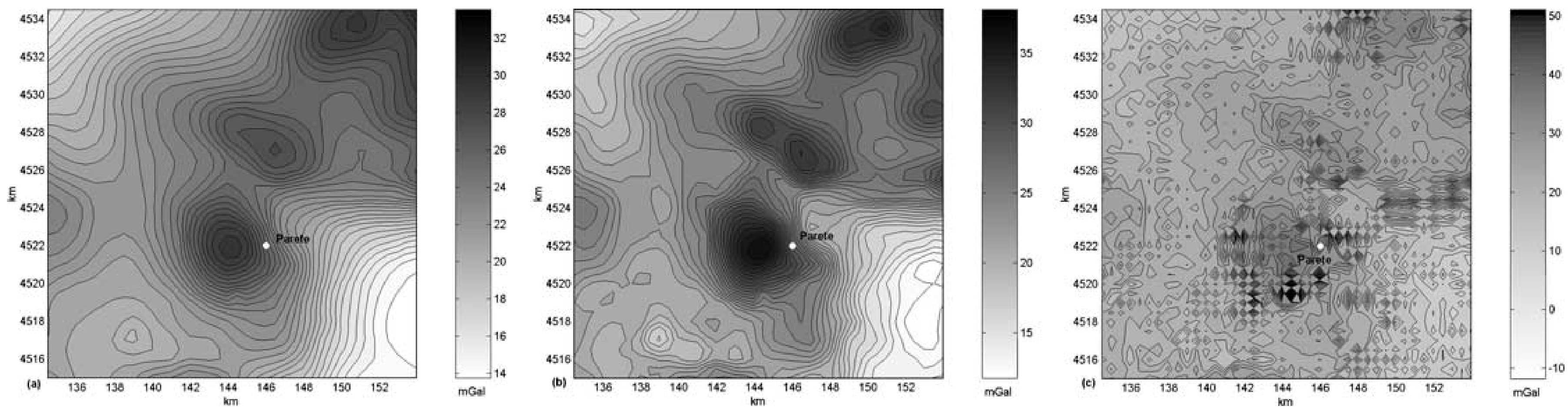

Figure 10. (a) Gravity anomaly in the Parete area and its downward continuation to 0.7 km by ISVD method (b, 5 terms in the summation) and using the frequency response in eq. (1) (c). ISVD method was successful in avoiding the excessive magnification of high frequencies visible in (c).

indicates radial frequency), substantially improves depth estimations with respect to the Spector and Grant method.

An application of this method of depth determination to the field of a group of 14 magnetized sources is shown in Fig. 6(a). The average depth to top of sources is 2.2 ± 0.11 km, varying between 2.06 and 2.36 km. The improvement in the depth determination using the method of Fedi *et al.* (1997) is evident (Figs 6b and c). In Figs 7(a) and (c) the ISVD downward continuations at the source level and below the top of the source are shown together with their respective power spectra (Figs 7b and d). In both cases 14 vertical derivative terms were used in the Taylor summation. At the source level the spectrum looses the red character (negative slope) and becomes white (flat). Below this level the power spectrum slope increases in such a way to assume a blue character (positive slope). Such a slope cannot correspond to any physical source in the harmonic region. The spectral analysis can therefore help to correctly understand that the downward continued field of Fig. 6(c) is continued outside the harmonic region, and so to discard it. Note that the only visual analysis of the field may not have allowed us to distinguish it from shallower, meaningful continuations.

In conclusion, the ISVD downward continuation algorithm was tested on synthetic magnetic fields and it was shown to be very stable, even in extreme cases, either in the presence of high frequency noise or when the continuation is attempted at the source level. We expect that this algorithm should be especially useful in the cases (a) where the signal is corrupted by noise, (b) when the continuation level is 2 or more times the data sampling step or (c) when the needed continuation level is close to the source top level.

APPLICATION TO A REAL CASE

In previous sections we illustrated the advantages of ISVD downward continuation with test examples. Now we want to illustrate an application of the algorithm to the real case of a gravity data set from the Campanian Plain, Southern Italy.

The Campanian Plain is a Plio–Pleistocene tectonic depression limited at NW, NE and SE by Mesozoic carbonate platforms and at SW by the Tyrrhenian coast (Fig. 8). The maximum sinking of the carbonate platforms is about 5 km (Ippolito *et al.* 1973). The plain is filled with sediments of alluvial and volcanic origin. Scandone (1979) related the genesis of the Campanian Plain to the tectonic events accompanying the opening of the Tyrrhenian Sea and the anticlockwise rotation of the Italian peninsula. These events produced a regional rise of the mantle and caused an intense phase of potassic volcanism along the Italian peri-Tyrrhenian border, specifically in some subsided areas. In the Campanian Plain the main Quaternary volcanic areas are the Somma–Vesuvius and the Phlegrean volcanic district, including the island of Ischia. Older evidence of volcanism was found in geothermal boreholes near Parete, where calc-alkaline andesitic and basaltic lavas were drilled for more than 1.5 km (Ortolani & Aprile 1978), and northward, in the Villa Literno area.

A gravity data set is available for the whole Campanian Plain, consisting of the Bouguer anomaly map by Cassano & La Torre (1987). The density used for the reductions is 2.2 g cm^{-3}. The anomalies were digitized with a sampling step of 0.5 km (Fig. 9). The Plain is characterized by a wide gravity low bounded by intense highs which correspond to the carbonatic reliefs. This gravity low is interpreted as evidence of tectonic foundering occurring in this area with respect to its borders. The low is split into three parts by the relative highs of Parete and Vesuvius.

The gravity high near Parete was interpreted as due to a buried volcanic structure (Baldi *et al.* 1976; Aprile & Ortolani 1979) or as due to a buried carbonate horst with magmatic intrusions along its bordering faults, this being demonstrated by a local magnetic signature (Carrara *et al.* 1973). It would be helpful to have a more detailed image of the gravity field in the area of the Parete high to help reconstruct the buried structure generating the gravity high.Following the insights obtained in the previous section, it is, however, advisable to have an idea of the depth to the top of the gravity sources to prevent us making any downward continuation into the source region.

We considered a 20×20 km area centred on the Parete anomaly and computed its vertical derivative in order to apply the depth determination technique of Fedi *et al.* (1997). To reduce any edge effect in the computation of the radial power spectrum we applied a Hanning tapering window to the vertical derivative data. An average depth to sources of about 0.7 km was estimated in the selected area. So we can assume that the maximum allowed level of downward continuation is about 0.7 km. We have to observe that the underground gravity sources are at varying depths. So, although a stable algorithm of continuation is used, the gravity field continued to a single level may be correct in some areas while elsewhere it may be not meaningful.

The ISVD downward continuation to 0.7 km is shown in Fig. 10(b). Analysis of variance plot show that the ISVD Taylor series, that approximates the downward continued field, converges and reaches saturation when the 5th term is added on. Thus we chose to use a maximum number of terms in the Taylor series equal to 5. By comparing the results with the original gravity field (Fig. 10a) the increase in resolution is quite evident. In particular, the two highs in the central part of the map are now well separated. The north-eastern one assumes a clear elongated shape in the NW–SE direction, while the south-western one tends to lose the rounded shape of the original field, showing an elongation towards E and SE. The gradients between the highs and the SE low have increased considerably and their orientation is now clearly recognizable as W–E in the southern part and N–S to the north. All these features are better displayed in the downward continued map and may greatly improve the resolution of results of later interpretative steps. The downward continuation to the same level (0.7 km, that is about one and a half times the data sampling step) obtained through use of the frequency response in eq. (1) is shown for comparison (Fig. 10c). The large enhancement of noise in the data is the prominent feature of this continued field. It means that low-pass filtering must be used before any interpretation of this map may be attempted.

CONCLUSIONS

The enhancement of gravity and magnetic maps may be performed by means of many techniques, generally based on combinations of spatial derivatives. An interesting approach is however represented by the downward continuation. This method has the great advantage that the resultant map still represents a gravity or magnetic field and so it is interpretable with the variety of methods nowadays available. The more detailed field obtained by downward continuation will surely improve the quality of any other subsequent enhancement (e.g. horizontal and vertical derivatives) and the resolution of models obtained through field data inversion.

Unfortunately the downward continuation process is inherently unstable and any high-frequency noise present in the data gets strongly magnified in the transformed map in such a way to mask any useful signal. In this paper we present an algorithm to perform downward continuation, based on a Taylor's series approximation and on the use of stable vertical derivatives. When tested on synthetic examples, this algorithm was shown to provide better results than other techniques based on the Fourier domain transformation or on a Taylor's series and Fourier domain vertical derivatives. ISVD downward continuation is particularly useful in cases where gravity or magnetic fields are corrupted by Gaussian noise, when the continuation level is at an altitude difference of several times the sampling step, or when the continuation level is very close to the top of sources.

Actually, using this algorithm it is possible to get stable downward continued fields even into the region of sources. Such fields are obviously meaningless and in this case the stability of the continuation may constitute an unexpected obstacle to interpretation. The deepest level of continuation may be estimated by the use of spectral techniques such as the Spector & Grant (1970) method.

Another advantage of the ISVD downward continuation algorithm is given by the Taylor series approach that allows one to easily implement a scheme of continuation from a single level to a complex surface and vice versa, similar to that proposed by Cordell & Grauch (1985). This application may be particularly relevant for the analysis of fields measured on a single level, as in the case of gravity or magnetic data from aerogeophysical or ship-borne surveys.

The application to the real situation of the gravity field of the Campanian Plain allowed us to compute a map of the field very close to the source level estimated by the spectral method of Fedi *et al.* (1997), and provided a better definition of source geometry due to the increased resolution.

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