Chapter 27
Comments

Abstract Conclusion of the examples of equations of motion investigated in Part II. The main points of the derivation of the equations of motion of a system are identified: the choice of the virtual velocities and the choice of the velocities which describe how the shape of the system changes. The possible choices are based on observation.

The derivation of the equations of motion with the principle of virtual work is productive. The key point is the choice of the power of the internal forces: a duality pairing between velocities of deformation and dual quantities, the internal forces which are abstract quantities. This choice is based on observation of the way the shape of the system changes. The examples show that this choice is not fixed once for all. Thus the dual quantities, the internal forces, are also not fixed once for all.

After the internal forces have been identified, it remains to get the constitutive laws which relate for each material the deformations quantities and the internal forces. In the example we have given in this part, we stress also that both theory and observation intervene. The theory relies on the Clausius Duhem inequality. The observation is a guide to choose the free energy and the pseudo-potential of dissipation.

Related both to theory and observation, let us mention *the internal constraints or the perfect constraints, les liaisons parfaites in French, i vincoli perfetti in Italian. They are not always easy to identify. A way to detect them is to check that the kinematic quantities which appear in the actual power or work of the internal forces span the whole linear spaces to which they belong.*

Reactions to the internal constraints intervene in the internal forces. They are not entirely defined by the constitutive laws. The part which is not given by the constitutive law, often the intensity of the reaction, is given by the equations of motion. This is the case of the Lagrangian Mechanics where the values of the perfect reactions are given by the equations of motion. One may say that *the direction of the reaction is given by the constitutive laws but its intensity is given by the equations of motion.* Words direction and intensity have to be defined precisely in each example.