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ERRATUM

Erratum to: Methods of harmonic synthesis for global geopotential models and their first-, second- and third-order gradients

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The expression for the coefficient $A_m^{(rr)}$ given in the third row of the first column of Table 5 should be corrected to include the factor $\bar{C}_{nm}$ as follows:

$$A_m^{(rr)} = \sum_{n=m}^{N} \left(\frac{a}{r}\right)^n (n+1)(n+2)\bar{C}_{nm}\bar{P}_n^m.$$

Similarly, the expression for the coefficient $A_m^{(rrr)}$ given in the fourth row of the first column of Table 6 should be corrected to include the factor $\bar{C}_{nm}$ as follows:

$$A_m^{(rrr)} = \sum_{n=m}^{N} \left(\frac{a}{r}\right)^n (n+1)(n+2)(n+3)\bar{C}_{nm}\bar{P}_n^m.$$

The right-hand sides of the formulas appearing in Table 10 should be corrected to include the factor $(1/r)$ as follows:

$$a_1 = \frac{1}{r}\sum_{m=1}^{N} m\left(A_m^{(1)}c_{m-1} + B_m^{(1)}s_{m-1}\right)\cos^{m-1}\phi$$

$$a_2 = \frac{1}{r}\sum_{m=1}^{N} m\left(B_m^{(1)}c_{m-1} - A_m^{(1)}s_{m-1}\right)\cos^{m-1}\phi$$

$$a_3 = \frac{1}{r}\sum_{m=0}^{N} \left(A_m^{(2)}c_{m} + B_m^{(2)}s_{m}\right)\cos^{m}\phi$$

$$a_4 = -\frac{1}{r}\sum_{m=0}^{N} \left(A_m^{(3)}c_{m} + B_m^{(3)}s_{m}\right)\cos^{m}\phi.$$

The right-hand sides of the formulas appearing in Table 11 should be corrected to include the factor $(1/r)^2$. The expression of the coefficient a_{11} contains further misprints. Hence, the formulas of Table 11 should read:

$$a_{11} = \frac{1}{r^2}\sum_{m=2}^{N} m(m-1)\left(A_m^{(1)}c_{m-2} + B_m^{(1)}s_{m-2}\right)\cos^{m-2}\phi$$

$$a_{12} = \frac{1}{r^2}\sum_{m=2}^{N} m(m-1)\left(B_m^{(1)}c_{m-2} - A_m^{(1)}s_{m-2}\right)\cos^{m-2}\phi$$

$$a_{13} = \frac{1}{r^2}\sum_{m=1}^{N} m\left(A_m^{(2)}c_{m-1} + B_m^{(2)}s_{m-1}\right)\cos^{m-1}\phi$$

$$a_{14} = -\frac{1}{r^2}\sum_{m=1}^{N} m\left(A_m^{(3)}c_{m-1} + B_m^{(3)}s_{m-1}\right)\cos^{m-1}\phi$$

$$a_{22} = -a_{11}$$

$$a_{23} = \frac{1}{r^2}\sum_{m=1}^{N} m\left(B_m^{(2)}c_{m-1} - A_m^{(2)}s_{m-1}\right)\cos^{m-1}\phi$$

$$a_{24} = -\frac{1}{r^2}\sum_{m=1}^{N} m\left(B_m^{(3)}c_{m-1} - A_m^{(3)}s_{m-1}\right)\cos^{m-1}\phi$$

$$a_{33} = \frac{1}{r^2}\sum_{m=0}^{N} \left(A_m^{(4)}c_{m} + B_m^{(4)}s_{m}\right)\cos^{m}\phi$$

$$a_{34} = -\frac{1}{r^2}\sum_{m=0}^{N} \left(A_m^{(5)}c_{m} + B_m^{(5)}s_{m}\right)\cos^{m}\phi$$

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$$a_{44} = \frac{1}{r^2}\sum_{m=0}^{N}\left(A_m^{(6)}c_m + B_m^{(6)}s_m\right)\cos^m\phi.$$

The right-hand sides of the formulas appearing in Table 12 should be corrected by including the factor $(1/r)^3$. The expressions of the coefficients a_{111}, a_{143}, a_{222} and a_{444} contain further misprints. Hence, the formulas of Table 12 should read:

$$a_{111} = \frac{1}{r^3}\sum_{m=3}^{N} m(m-1)(m-2) \times \left(A_m^{(1)}c_{m-3} + B_m^{(1)}s_{m-3}\right)\cos^{m-3}\phi$$

$$a_{112} = -a_{222}$$

$$a_{113} = \frac{1}{r^3}\sum_{m=2}^{N} m(m-1)\left(A_m^{(2)}c_{m-2} + B_m^{(2)}s_{m-2}\right)\cos^{m-2}\phi$$

$$a_{114} = -\frac{1}{r^3}\sum_{m=2}^{N} m(m-1)\left(A_m^{(3)}c_{m-2} + B_m^{(3)}s_{m-2}\right)\cos^{m-2}\phi$$

$$a_{123} = \frac{1}{r^3}\sum_{m=2}^{N} m(m-1)\left(B_m^{(2)}c_{m-2} - A_m^{(2)}s_{m-2}\right)\cos^{m-2}\phi$$

$$a_{124} = -\frac{1}{r^3}\sum_{m=2}^{N} m(m-1)\left(B_m^{(3)}c_{m-2} - A_m^{(3)}s_{m-2}\right)\cos^{m-2}\phi$$

$$a_{143} = -\frac{1}{r^3}\sum_{m=1}^{N} m\left(A_m^{(5)}c_{m-1} + B_m^{(5)}s_{m-1}\right)\cos^{m-1}\phi$$

$$a_{221} = -a_{111}$$

$$a_{222} = -\frac{1}{r^3}\sum_{m=3}^{N} m(m-1)(m-2) \times \left(B_m^{(1)}c_{m-3} - A_m^{(1)}s_{m-3}\right)\cos^{m-3}\phi$$

$$a_{223} = -a_{113}$$

$$a_{224} = -a_{114}$$

$$a_{243} = -\frac{1}{r^3}\sum_{m=1}^{N} m\left(B_m^{(5)}c_{m-1} - A_m^{(5)}s_{m-1}\right)\cos^{m-1}\phi$$

$$a_{331} = \frac{1}{r^3}\sum_{m=1}^{N} m\left(A_m^{(4)}c_{m-1} + B_m^{(4)}s_{m-1}\right)\cos^{m-1}\phi$$

$$a_{332} = \frac{1}{r^3}\sum_{m=1}^{N} m\left(B_m^{(4)}c_{m-1} - A_m^{(4)}s_{m-1}\right)\cos^{m-1}\phi$$

$$a_{333} = \frac{1}{r^3}\sum_{m=0}^{N}\left(A_m^{(7)}c_m + B_m^{(7)}s_m\right)\cos^m\phi$$

$$a_{334} = -\frac{1}{r^3}\sum_{m=0}^{N}\left(A_m^{(8)}c_m + B_m^{(8)}s_m\right)\cos^m\phi$$

$$a_{441} = \frac{1}{r^3}\sum_{m=1}^{N} m\left(A_m^{(6)}c_{m-1} + B_m^{(6)}s_{m-1}\right)\cos^{m-1}\phi$$

$$a_{442} = \frac{1}{r^3}\sum_{m=1}^{N} m\left(B_m^{(6)}c_{m-1} - A_m^{(6)}s_{m-1}\right)\cos^{m-1}\phi$$

$$a_{443} = \frac{1}{r^3}\sum_{m=0}^{N}\left(A_m^{(9)}c_m + B_m^{(9)}s_m\right)\cos^m\phi$$

$$a_{444} = -\frac{1}{r^3}\sum_{m=0}^{N}\left(A_m^{(10)}c_m + B_m^{(10)}s_m\right)\cos^m\phi.$$

The expression for the coefficient $A_m^{(2)}$ in Table 13 should read:

$$A_m^{(2)} = \sum_{n=m}^{N}\rho_n\bar{C}_{nm}\mathrm{d}\bar{H}_n^m/\mathrm{d}u.$$

There are missing terms in the expression for the function $\bar{W}_n^m$ in Table 14. The formula should read:

$$\bar{W}_n^m = (n+m+1)(n+m+2)(n+m+3)\bar{H}_n^m + 3u(n+m+2)(n+m+3)\mathrm{d}\bar{H}_n^m/\mathrm{d}u + 3u^2\mathrm{d}^2\bar{H}_n^m/\mathrm{d}u^2 + u^3\mathrm{d}^3\bar{H}_n^m/\mathrm{d}u^3.$$

The letter l in Eqs. (76) and (77) should be replaced by the letter r, so that the two equations read:

$$V_{nm} = \frac{(-1)^n}{(n-m)!}\left(\frac{\partial}{\partial x} + \mathrm{i}\frac{\partial}{\partial y}\right)^m\left(\frac{\partial}{\partial z}\right)^{(n-m)}\left(\frac{1}{r}\right),$$

$$V_{nn} = (-1)^n\left(\frac{\partial}{\partial x} + \mathrm{i}\frac{\partial}{\partial y}\right)^n\left(\frac{1}{r}\right),$$

respectively.

A $+$ sign should be replaced by a $-$ sign in Eq. (94), in the following way:

$$\bar{S}_{nm}^{(\alpha\beta\gamma)} = \frac{(-1)^{\beta/2}}{(-2)^{\alpha+\beta}}\sum_{p=0}^{\alpha}\sum_{q=0}^{\beta}\left[\epsilon^+A^+\bar{S}_{n-\eta,m+\sigma} - (1-\delta_{0m})\epsilon^-A^-\bar{S}_{n-\eta,-m+\sigma}\right].$$

Finally, the term 3η appearing in the upper limits of the summations in Eqs. (101), (102) and (103) and related text should be replaced by η:

$$\frac{\partial^\eta V}{\partial x^\alpha\partial y^\beta\partial z^\gamma} = \frac{GM}{a^{1+\eta}}\sum_{m=0}^{N+\eta}\left(A_m^{(\alpha\beta\gamma)}\cos m\lambda + B_m^{(\alpha\beta\gamma)}\sin m\lambda\right),$$

$$A_m^{(\alpha\beta\gamma)} = \sum_{n=m}^{N+\eta} \bar{C}_{nm}^{(\alpha\beta\gamma)} \bar{E}_{nm},$$

$$B_m^{(\alpha\beta\gamma)} = \sum_{n=m}^{N+\eta} \bar{S}_{nm}^{(\alpha\beta\gamma)} \bar{E}_{nm}.$$

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