A numerical model based on closed form solution for elastic stability of thin plates

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A numerical model based on closed form solution for elastic stability of thin plates.

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Abstract. An analytical approach for studying the elastic stability of thin rectangular plates under arbitrary boundary conditions is presented. Because the solution is given in closed-form, the approach can be regarded as "exact" under the Kirchhoff-Love assumption. The proposed procedure allows us to obtain the buckling load and modal displacements that do not depend on the number of elements adopted in the numerical discretization using, say, the finite element method.

1. Introduction

The buckling behavior of plates subjected to in-plane forces has been an important area of investigation for numerous researchers, due to their wide use in engineering applications. Aimed at the elastic buckling analysis, numerical methods have been employed over the past years, most of which dealt with rectangular plates. Many of the useful results have been summarized in texts and handbooks [1-3].

The finite element method (FEM) has appeared as a powerful tool, able to solve the buckling load, buckling mode and postbuckling behaviour of structural members, under different loading and boundary conditions[3]. Consequently, a large number of FEM software packages is now available. However, to provide accurate prediction of the buckling response, a large number of elements is required, which adds to the effort of computation (besides numerical instabilities), inefficiency in data preparation, and numerical errors arising especially because of the presence of close buckling modes. For structures with regular geometries (i.e. open ruled surface), more efficient techniques can be adopted successfully. The finite strip method (FSM), based on the discretization of the structure along the transverse direction only, has been systematically employed in buckling analysis.

It was found to be a more effective method for determining the critical loads of thin plates because of the reduction of both computation times and numerical instabilities. The FSM for analyzing the large deflection response of simply supported rectangular functionally graded plates under normal pressure loading was conducted by Ovesy and Ghannadpour[4], where the material properties of the functionally graded plates are assumed to vary continuously through the thickness of the plate, according to the simple power law and exponential law distribution. The cost of reduced computational effort is that the FSM-based procedure can be applied only to structures with simple geometry and boundary conditions, namely, it works only on prismatic structures, with simply supported edges.

On the other hand, semi-analytical, or exact, approach can be considered as a variation of the FSM. In fact, with the usual FSM, the lengthwise variation of displacements is represented by harmonic functions, and a polynomial shape function is retained to model the transverse variation of displacements. Semi-analytical approach makes use of the harmonic functions to describe the longitudinal displacement as well. By means of that, it is possible to reduce the partial differential equilibrium equations (PDEE), into a set of one-dimensional ordinary differential equilibrium equations (ODEE), suitable for analytical solution. The closed form solutions so obtained can be used finally as the shape function for the transverse displacement in the numerical approach to calculate the exact buckling load and modal displacements using very coarse meshes. The semi-analytical method for buckling analysis was extensively studied by Williams et al., [5]. They proposed a FEM-like procedure for analyzing the critical and post-critical behavior of isotropic and homogeneous rigidly connected plates assembly, and solved the equilibrium equations obtained via the perturbation technique [6]. In the work by Shukla et al. [7], an analytical postbuckling approach was adopted to solve the response of functionally graded rectangular plates subjected to thermomechanical loading, based on the hypotheses of Reissner-Mindlin shear deformable plates and von-Karman kinematics. In the present paper, the longitudinal variation of displacements is not restricted to be trigomometric, but a more complete function is used to circumvent the limitation mentioned above. The result has been obtained by coupling two 1D models to get a 2D model capable of enforcing a complete set of boundary conditions. The quality of the proposed method is demonstrated by means of comparison with results obtained using the commercial FEM package ANSYS and other analytical solutions available in literature.

2. Theory

Consider an isotropic thin rectangular plate with length *a*, width *b* and thickness *h*, subject to mono-axial in-plane compressive applied load N_L in the *x*-direction (Fig. 1).

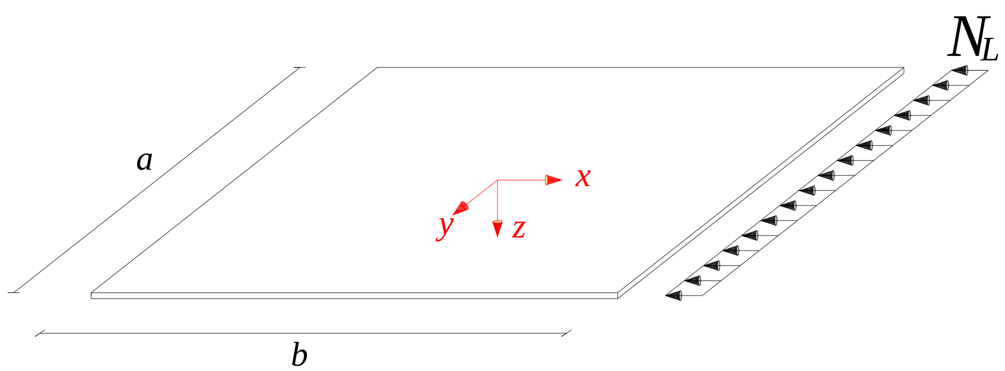

Figure 1. Thin rectangular plate under in-plane load: dimensions and reference frame

Under the classical thin plate hypothesis (Kirchhoff-Love's theory and von Kàrman strain-displacement relationship), the partial differential equation of equilibrium of the pate, in terms of the out-of-plane displacement *w*, is:

$$D\nabla^4 w + N_L w,_{xx} = 0 \tag{1}$$

where ∇^4 is the bi-harmonic differential operator ($i.e., w,_{xxxx} + 2w,_{xxyy} + w,_{yyyy}$ in rectangular coordinates), $D = \frac{Et^3}{12(1-\nu^2)}$ is the flexural rigidity of the plate. Here $(\bullet)_{,i}$ represents $\frac{\partial(\bullet)}{\partial x_i}$.

Eq. (1) does not allow a closed form solution to be sought, but under more restrictive hypothesis on the displacement field, it is possible to reduce Eq. (1) to a set of one-dimensional ordinary differential equations suitable for analytical solution. Classical approaches, based on closed-form solution, are *Semi-analytical approach* (SAA), where the displacements are represented by harmonic functions in the *y* direction and *beam-like theory* (BLT), where the displacements are represented by constant functions in the *x* direction. The above quoted methods are numerically implemented using the analytical solution instead of the conventional shape functions into a FEM-based approach. Due to the analytical nature of such functions, the errors introduced by discretization vanish, and the mesh size depends only on the geometry description requirement. In Fig. 2, comparison is made between typical discretizations used in the FEM, FSM, and analytical approach with the geometrical and mechanical description of the plates highlighted.

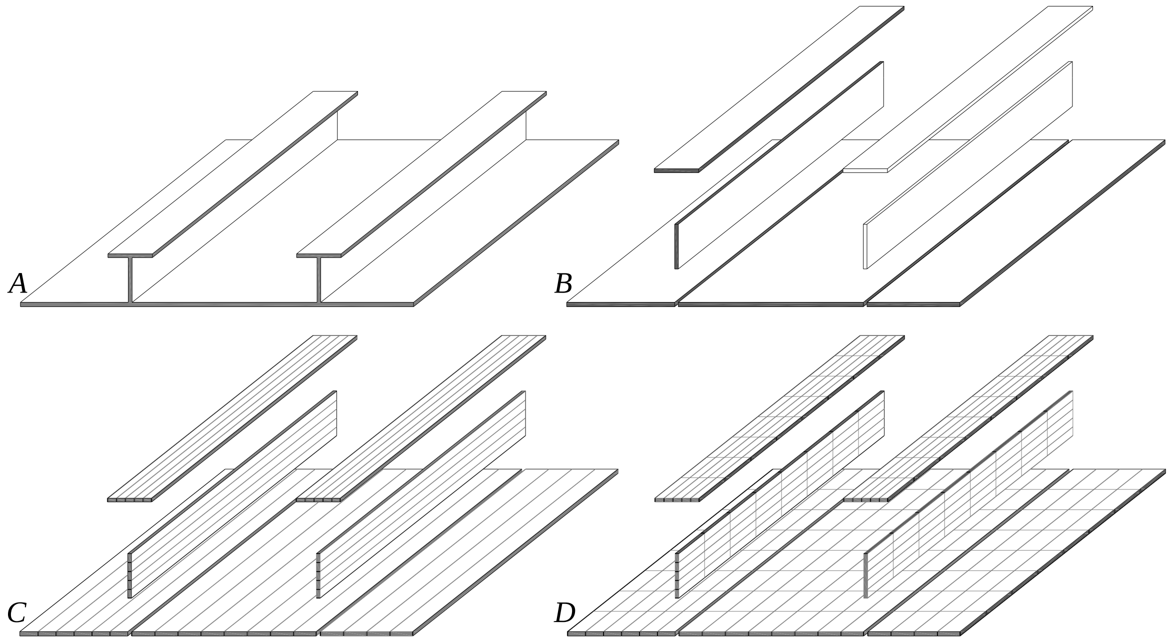

Figure 2. Comparison of discretizations used: (A) Typical stiffened plate, (B) analytical, (C) FSM, (D) FEM.

The basic idea of the proposed model is to couple two classical 1D models, in order to remove the restriction of both procedures, while obtaining a proper two-dimensional model. Let us assume the displacement function in Eq. (1) to be

$$w(x,y) = \mathbf{n}(x)\mathbf{w}(y) \tag{2}$$

where separation of variables is done as usual, the x representation of the displacement is assumed to be a linear combination of trigonometric, linear and constant functions, henceforth

$$\mathbf{n}(x) = \left[\sin\frac{m\pi x}{a} \quad \cos\frac{m\pi x}{a} \quad \frac{x}{a} \quad 1 \right]^T \tag{3}$$

The combination functions are collected into the vector $\mathbf{w}(y)$ whose components have to be determined.

$$\mathbf{w}(y) = [w_A(y) \quad w_B(y) \quad w_C(y) \quad w_D(y)]^T \tag{4}$$

In order to get the solution, firstly Eq. (2) is substituted into Eq. (1) to yield

$$\begin{aligned}&\sin\frac{m\pi x}{a}\left[w_{A},_{yyyy} - 2\frac{m^2\pi^2}{a^2}w_{A},_{yy} + \left(\frac{m^4\pi^4}{a^4} - \frac{N_L}{D}\frac{m^2\pi^2}{a^2}\right)w_A\right] + \\ &\cos\frac{m\pi x}{a}\left[w_{B},_{yyyy} - 2\frac{m^2\pi^2}{a^2}w_{B},_{yy} + \left(\frac{m^4\pi^4}{a^4} - \frac{N_L}{D}\frac{m^2\pi^2}{a^2}\right)w_B\right] + \\ &\frac{x}{a}\left[w_{C},_{yyyy}\right] + \left[w_{D},_{yyyy}\right] = 0\end{aligned} \tag{5}$$

Eq. (5) admits solution for any x only if each expression inside the square brackets is equal to zero. The resulting equations can be solved analytically, and the result can be collected as follows:

$$\mathbf{w}(y) = \mathbf{F}(y)\cdot\mathbf{a} \tag{6}$$

where

$$\mathbf{F}(y) = \begin{bmatrix} e^{-\alpha y} & e^{\alpha y} & \cos\beta y & \sin\beta y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-\alpha y} & e^{\alpha y} & \cos\beta y & \sin\beta y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & y^3 & y^2 & y & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & y^3 & y^2 & y & 1 \end{bmatrix} \tag{7}$$

contains the analytical solutions of Eq. (5). Moreover $\mathbf{a}$ contains 16 constants to be determined by imposing the boundary conditions on one point on any side of the elements of the structure, namely

$$\mathbf{a} = [a_1 \quad a_2 \quad a_3 \quad a_4 \quad b_1 \quad b_2 \quad b_3 \quad b_4 \quad c_1 \quad c_2 \quad c_3 \quad c_4 \quad d_1 \quad d_2 \quad d_3 \quad d_4]^T \tag{8}$$

Equation (7) depends on the applied axial load N_L which via the two parameters α and β:

$$\alpha = \sqrt{\frac{m^2\pi^2}{b^2} + \sqrt{\frac{N_L}{D}\frac{m^2\pi^2}{b^2}}} \qquad \beta = \sqrt{-\frac{m^2\pi^2}{b^2} + \sqrt{\frac{N_L}{D}\frac{m^2\pi^2}{b^2}}} \tag{9}$$

Substituting Eq. (6) into Eq. (2), we obtain

$$w(x,y) = \mathbf{n}(x)\cdot\mathbf{F}(y)\cdot\mathbf{a} \tag{10}$$

Using Eq. (10) as a shape function in a FEM procedure, it is possible to obtain the critical load and mode shape of the 2D structure by solving the corresponding eigenvector and eigenvalue problem.

3. Numerical results and discussion

In this section, the buckling analysis of simple structures is performed, and the results obtained are compared with well established ones in the literature.

3.1. Simply supported plate

As a first example, a simply supported plate is solved using one element. The boundary conditions are enforced at the middle point of any side, whose coordinates in the reference frame shown in Fig. 3, are $A=[a/2,0], B=[-a/2,0], C=[0,b/2], D=[0,-b/2]$. The boundary equations[1] are listed as follows:

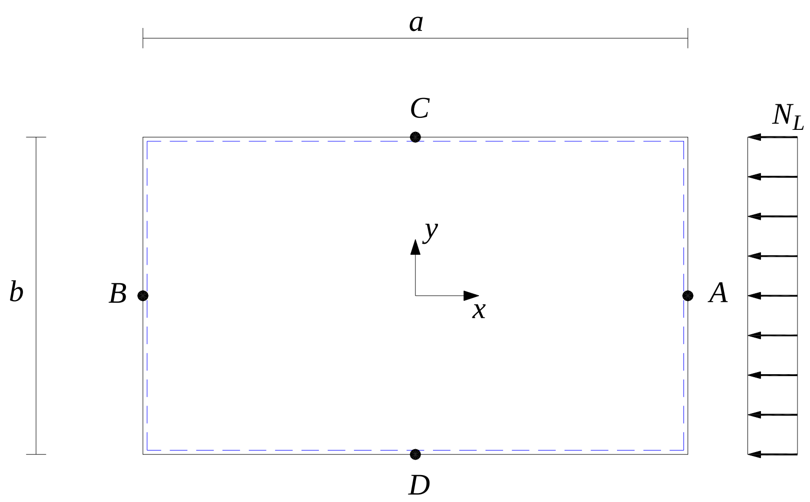

Figure 3: First example, geometrical representation, boundary conditions and representative points.

$$w^{(A,B,C,D)}=0; \quad \Rightarrow \quad \begin{array}{l} \mathbf{n}\left(\frac{a}{2}\right)\cdot\mathbf{f}(0)\cdot\mathbf{a}=0 \\ \mathbf{n}\left(-\frac{a}{2}\right)\cdot\mathbf{f}(0)\cdot\mathbf{a}=0 \\ \mathbf{n}(0)\cdot\mathbf{f}\left(\frac{b}{2}\right)\cdot\mathbf{a}=0 \\ \mathbf{n}(0)\cdot\mathbf{f}\left(-\frac{b}{2}\right)\cdot\mathbf{a}=0 \end{array}$$

$$M_x^{(A,B)}=D\left(w,_{xx}^{(A,B)}+\nu w,_{yy}^{(A,B)}\right)=0 \quad \Rightarrow \quad \begin{array}{l} D\left(\mathbf{n},_{xx}\left(\frac{a}{2}\right)\cdot\mathbf{f}(0)+\nu\,\mathbf{n}\left(\frac{a}{2}\right)\cdot\mathbf{f},_{yy}(0)\right)\cdot\mathbf{a}=0 \\ D\left(\mathbf{n},_{xx}\left(-\frac{a}{2}\right)\cdot\mathbf{f}(0)+\nu\,\mathbf{n}\left(\frac{a}{2}\right)\cdot\mathbf{f},_{yy}(0)\right)\cdot\mathbf{a}=0 \end{array} \tag{11}$$

$$M_y^{(C,D)}=D\left(w,_{yy}^{(C,D)}+\nu w,_{xx}^{(C,D)}\right)=0 \quad \Rightarrow \quad \begin{array}{l} D\left(\mathbf{n}(0)\cdot\mathbf{f},_{yy}\left(\frac{b}{2}\right)+\nu\mathbf{n},_{xx}(0)\cdot\mathbf{f}\left(\frac{b}{2}\right)\right)\cdot\mathbf{a}=0 \\ D\left(\mathbf{n}(0)\cdot\mathbf{f},_{yy}\left(-\frac{b}{2}\right)+\nu\mathbf{n},_{xx}(0)\cdot\mathbf{f}\left(-\frac{b}{2}\right)\right)\cdot\mathbf{a}=0 \end{array}$$

where **a** is an unknown vector, as defined in Eq. (6), which can be extracted to highlight the functional matrix K. Hence Eq. (11) can be rewritten as follows:

$$\mathbf{Ka}=\mathbf{0} \tag{12}$$

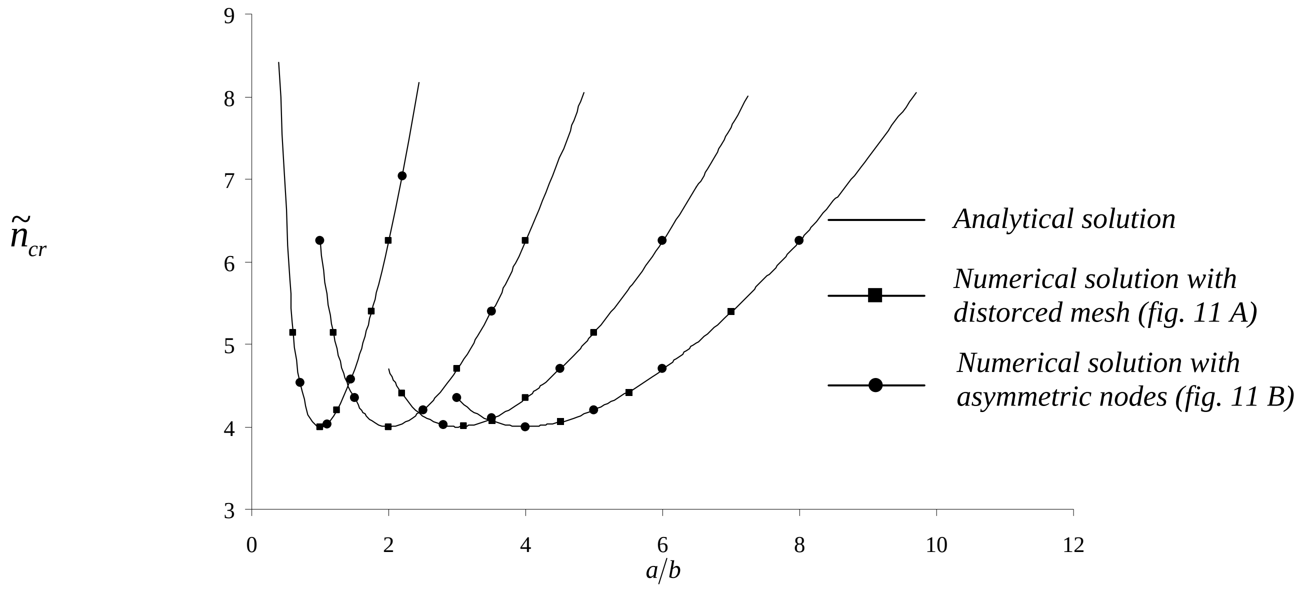

Figure 4: First example, comparison of analytical results with numerical ones for eigenvalue problem, obtained with distorted mesh.

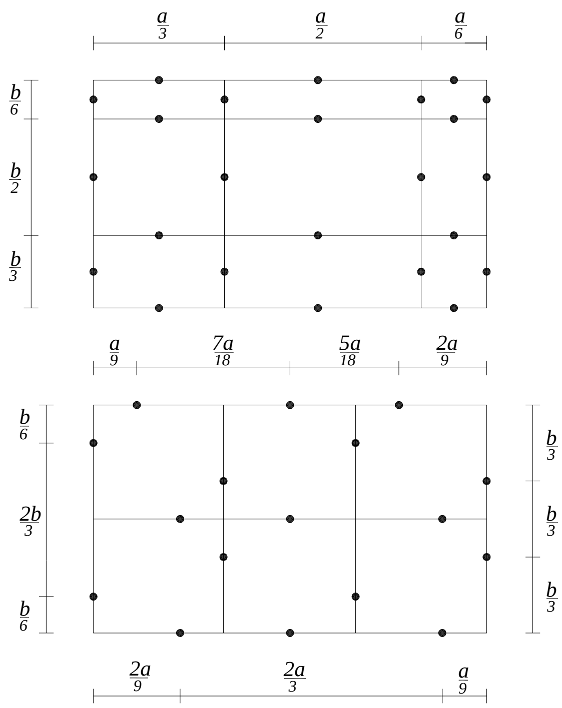

Figure 5: First example, distorted mesh and asymmetric nodes adopted for numerical analysis

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In Fig. 4, in order to show that the solution obtained does not depend either on the position of the nodes where the boundary conditions are enforced along the elements side, or on the mesh size, a comparison is presented between analytical solution given for a single element plate [1] and the numerical solution obtained by the proposed procedure where the structure is divided into several elements as shown in Fig. 5, in which different boundary nodes and different element shape are shown. The results are presented in Fig. 4 as a function of the aspect ratio, a/b, of the plate.

3.2. Simply supported in $x = \pm a/2$, free in $y = \pm b/2$.

As example, the buckling analysis of the plate shown in Fig. 6 is performed. The prescribed constraints of the plate fulfill that implicitly considered in SAA and BLT, respectively, on $x = \pm \frac{a}{2}$ and $y = \pm \frac{b}{2}$. As in the previous example, a closed form solution is obtained, and compared with the analytical one. The boundary constraints are enforced on the following nodes: $A = [a/2, 0], B = [-a/2, 0], C = [0, b/2], D = [0, -b/2]$, which gives the following equations:

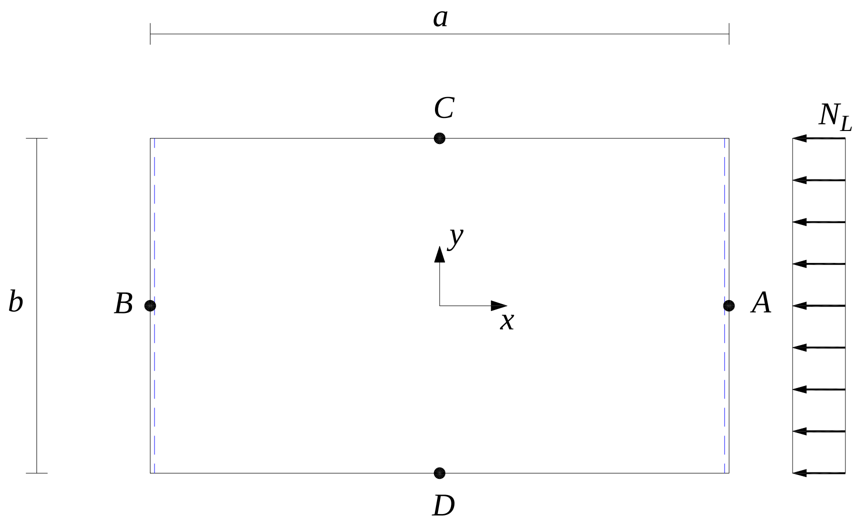

Figure 6: Second example, geometrical representation, boundary conditions and representative points.

$$w^{(A,B)} = 0 \quad \Rightarrow \quad \begin{aligned} &\mathbf{n}\left(\frac{a}{2}\right)\cdot \mathbf{f}(0)\cdot \mathbf{a} = 0 \\ &\mathbf{n}\left(-\frac{a}{2}\right)\cdot \mathbf{f}(0)\cdot \mathbf{a} = 0 \end{aligned}$$

$$M_x^{(A,B)} = D\left(w,_{xx}^{(A,B)} + \nu w,_{yy}^{(A,B)}\right) = 0 \quad \Rightarrow \quad \begin{aligned} &D\left(\mathbf{n},_{xx}\left(\frac{a}{2}\right)\cdot \mathbf{f}(0) + \nu\,\mathbf{n}\left(\frac{a}{2}\right)\cdot \mathbf{f},_{yy}(0)\right)\cdot \mathbf{a} = 0 \\ &D\left(\mathbf{n},_{xx}\left(-\frac{a}{2}\right)\cdot \mathbf{f}(0) + \nu\,\mathbf{n}\left(-\frac{a}{2}\right)\cdot \mathbf{f},_{yy}(0)\right)\cdot \mathbf{a} = 0 \end{aligned}$$

$$M_y^{(C,D)} = D\left(w,_{yy}^{(C,D)} + \nu w,_{xx}^{(C,D)}\right) = 0 \quad \Rightarrow \quad \begin{aligned} &D\left(\mathbf{n}(0)\cdot \mathbf{f},_{yy}\left(\frac{b}{2}\right) + \nu\,\mathbf{n},_{xx}(0)\cdot \mathbf{f}\left(\frac{b}{2}\right)\right)\cdot \mathbf{a} = 0 \\ &D\left(\mathbf{n}(0)\cdot \mathbf{f},_{yy}\left(-\frac{b}{2}\right) + \nu\,\mathbf{n},_{xx}(0)\cdot \mathbf{f}\left(-\frac{b}{2}\right)\right)\cdot \mathbf{a} = 0 \end{aligned}$$

$$T_y^{(C,D)} = D\left(w,_{yyy}^{(C,D)} + (2-\nu) w,_{xxy}^{(C,D)}\right) = 0 \quad \Rightarrow \quad \begin{aligned} &D\left(\mathbf{n}(0)\cdot \mathbf{f},_{yyy}\left(\frac{b}{2}\right) + (2-\nu)\mathbf{n},_{xx}(0)\cdot \mathbf{f},_{y}\left(\frac{b}{2}\right)\right)\cdot \mathbf{a} = 0 \\ &D\left(\mathbf{n}(0)\cdot \mathbf{f},_{yyy}\left(-\frac{b}{2}\right) + (2-\nu)\mathbf{n},_{xx}(0)\cdot \mathbf{f},_{y}\left(-\frac{b}{2}\right)\right)\cdot \mathbf{a} = 0 \end{aligned} \qquad (13)$$

The preceding equation can be rewritten in a compact form

$$\tilde{\mathbf{K}} \cdot \tilde{\mathbf{a}} = \mathbf{0}$$

where $\tilde{\mathbf{K}}$ is a square matrix containing eight linearly independent equations, as given below:

$$\tilde{\mathbf{K}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 1 \\ \frac{D\pi\alpha^3 \cosh \frac{b\alpha}{2}}{a} & \frac{D(2-\nu)\pi^2\alpha}{a^2}\sinh\frac{b\alpha}{2} & D\left(\frac{-(2-\nu)\pi^2\beta}{a^2}\cos\frac{b\beta}{2} - \frac{\alpha^3}{a}\sinh\frac{b\alpha}{2}\right) & -\frac{D(2-\nu)\pi^2\beta}{a^2}\sin\frac{b\beta}{2} & 0 & 0 & 0 & 0 \\ \frac{D\pi\alpha^3 \cosh \frac{b\alpha}{2}}{a} & \frac{D(2-\nu)\pi^2\alpha}{a^2}\sinh\frac{b\alpha}{2} & D\left(\frac{-(2-\nu)\pi^2\beta}{a^2}\cos\frac{b\beta}{2} + \frac{\alpha^3}{a}\sinh\frac{b\alpha}{2}\right) & \frac{D(2-\nu)\pi^2\beta}{a^2}\sin\frac{b\beta}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -D\frac{\nu}{a^2} & 0 & \frac{2D\nu}{a^2} & 0 \\ 0 & 0 & 0 & 0 & D\frac{\nu}{a^2} & 0 & \frac{2D\nu}{a^2} & 0 \\ 0 & D\left(\alpha^2 - \frac{\nu\pi^2}{a^2}\right)\cosh\frac{b\alpha}{2} & D\left(\beta^2 + \frac{\nu\pi^2}{a^2}\right)\sin\frac{b\beta}{2} & -D\left(\beta^2 + \frac{\nu\pi^2}{a^2}\right)\cos\frac{b\beta}{2} & 0 & 0 & \frac{2D}{a^2} & 0 \\ 0 & D\left(\alpha^2 - \frac{\nu\pi^2}{a^2}\right)\cosh\frac{b\alpha}{2} & -D\left(\beta^2 + \frac{\nu\pi^2}{a^2}\right)\sin\frac{b\beta}{2} & -D\left(\beta^2 + \frac{\nu\pi^2}{a^2}\right)\cos\frac{b\beta}{2} & 0 & 0 & \frac{2D}{a^2} & 0 \end{bmatrix} \qquad (14)$$

And $\tilde{\mathbf{a}}$ is a vector containing the corresponding unknowns:

$$\mathbf{a} = \begin{bmatrix} a_3 & b_2 & b_3 & b_4 & c_2 & c_4 & d_2 & d_4 \end{bmatrix}^T \qquad (15)$$

By solving $\det\left(\tilde{\mathbf{K}}(\alpha(N), \beta(N))\right) = 0$, we obtain

$$\tilde{N}_{cr}^{(1)} = \frac{Dm^2\pi^2}{a^2}$$
$$\tilde{N}_{cr}^{(2)} = \frac{D\left(m^2 - \nu\right)^2 \pi^2}{a^2 m^2} \qquad (16)$$
$$\tilde{N}_{cr}^{(3)} = \frac{D\left(a^2 + b^2 m^2\right)^2 \pi^2}{a^2 b^4 m^2}$$

Only the minimum value of the critical load has practical meaning. Thus one has to investigate which solution has to be accounted for by comparing the values in Eq. (16). Comparing the values $\tilde{n}_{cr}^{(1)} = \frac{\tilde{N}_{cr}^{(1)}}{D\pi^2}$ and $\tilde{n}_{cr}^{(2)} = \frac{\tilde{N}_{cr}^{(2)}}{D\pi^2}$, for the first half-wave $m = 1$, it is possible to observe from Fig. 7 that $\tilde{n}_{cr}^{(1)} \equiv \tilde{n}_{cr}^{(2)}$, if the Poisson ratio $\nu = 0$, whereas $\tilde{n}_{cr}^{(1)} > \tilde{n}_{cr}^{(2)}$ if $0 < \nu \leq 0.5$.

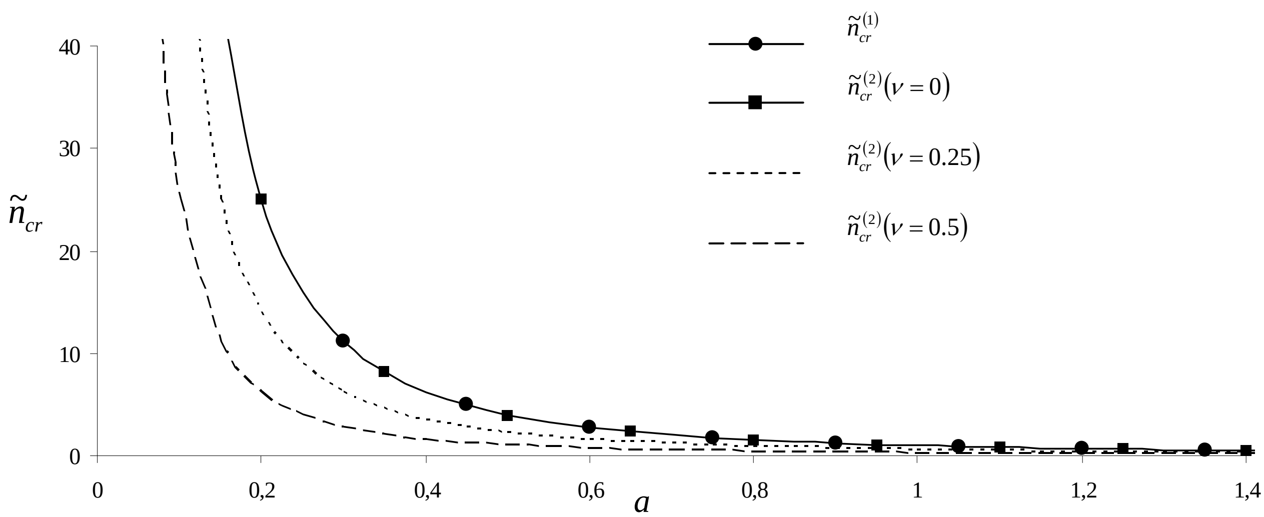

Figure 7: Second example, comparison between first critical load and second one, for different Poisson's ratios.

4. Conclusion

In the presented study, an exact solution procedure for buckling analysis of plates having all possible combinations of boundary conditions was presented. The proposed approach unifies the two one-dimensional models usually adopted in the literature to obtain a full 2-dimensional model that is not restrained by the implicit definition of boundary conditions present in the 1D approaches.

The strategy is to assume a combination of sinusoidal and polynomial displacements in the *x*-direction that enables us to reduce the original problem into four independent ordinary differential equations in terms of the *y variable*. The equations so derived are solved analytically and then the solution is used to solve the buckling problem of the plates assembly. Through comparison with numerical results and analytical ones available elsewhere has shown that the proposed method is capable of obtaining very stable solutions even with very simple representation of the plates assembly. In particular, one has to describe only the variation of geometry or constraints, because any uniform element is solved exactly. The proposed method can be used to enhance a FEM procedure. Namely, by coupling the in-plane and out-of-plane displacements, it can be used to solve the critical response of stiffened plates with general geometric shapes.

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