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Displacement characteristics of coarse fluvial bed sediment

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[1] Previous work highlights the need for data collection to identify appropriate models for temporal evolution of tracer dispersal in rivers. Results of 64 gravel-bed field tracer experiments covering a wide range of flow and sediment supply regimes are compiled here to determine the probabilistic character of gravel transport. We focus on whether particle travel distances and waits are thin- or heavy-tailed. While heavy-tailed travel distance distributions are observed between successive monitoring events in different hydrological and sediment supply regimes, heavy-tailedness does not persist through total travel distance over multiple monitoring events, suggesting that individual monitoring events occur before particle travel distance exceeds the characteristic correlation length for the channel (such that particles that start in fast paths remain in fast paths and particles in slow paths remain in slow paths). After a large number of transport events, super-diffusive spreading was not observed at any of the gravel bed streams. Continuous-time tracking of x, y, z coordinates of tracers in natural streams is necessary to capture exact step and waiting time distributions.

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1. Introduction

[2] Quantification of sediment transport in streams has been dominated by the capacity approach, in which focus is on the overall ability of a stream to transport sediment. This approach was first developed for sand bed rivers, but may not be suitable for gravel bed rivers, where stream capacity can change with sediment supply, sediment composition, or state of bed armoring [e.g., *Hassan et al*., 2008a]. Characterization of individual particle displacement patterns in streams is an alternative approach for estimating sediment transport and channel stability. Since it is not currently possible to measure the deterministic forces driving a particle as it interacts with streamflow, the bed, and other particles, a stochastic approach to movement is used to understand bulk transport properties and their link to channel stability and morphology. Bedload transport in rivers consists of movements of individual particles. The motion of grains is not continuous, but consists of a series of steps and rest periods due to the irregular bed surface boundary and the turbulent nature of flows [e.g., *Einstein*, 1937; *Schick et al*., 1987; *Hassan et al*., 1991; *McEwan et al*., 2004; *Lajeunesse et al*., 2010], either under the condition of partial or full mobility [*Wilcock and McArdell*, 1993].

[3] Probabilistic models of transport generally begin with the concept that individual particle "steps" have random lengths and the sum of these step lengths represents total travel distance. Travel distance for these random walk models is a random variable itself and can be described by a probability density that incorporates our uncertainty in exactly how far a particle will travel through time. Stochastic theory shows that long term transport for particles undergoing random walks is governed by advection-diffusion equations. There are a number of reasons for which deviation from classical diffusive-type transport occurs. First, we consider sources of nondiffusive or "anomalous" transport while particles are in motion. These include particle steps with long-range correlation, particle steps with extreme deviation from "average" transport behavior, and deterministic step components such as harmonics. Even if mobile particle transport is diffusive, the long term bulk motion of, say, a group of tracers, may still be anomalous if the distribution of particle immobile periods is sufficiently wide (or heavy-tailed) such that the slowdown in particle virtual velocities cannot be described by an average value.

[4] A key distinction in applicability of governing partial differential equations for tracer dispersion is that of thin- versus heavy-tailed particle step lengths and rest periods. Specifically, the commonly used advection-diffusion equation (ADE), which describes diffusive change in particle concentration with time as a function of average velocity and spread around that average, is only valid for thin-tailed step and rest distributions [*Ganti et al*., 2010]. Classical diffusive spreading implies that the mean square displacement (MSD) of particle spread is a linear function of time. The standard form of the ADE applies only to this case. Probability density functions (PDFs) for random travel lengths or rest periods with heavy tails decay much more slowly

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with increasing travel length or rest time than thin tails. The dispersion associated with heavy tails can show an MSD of particle spread that deviates from linear, as characterized by either super-diffusive or subdiffusive anomalous behavior. If the MSD is a power-law function of time with an exponent greater than unity, the process is super-diffusive, in which case space-fractional advection-dispersion equations are well suited to describing the temporal evolution of relative tracer concentrations. If, on the other hand, the exponent is less than unity, the process is subdiffusive, in which case time-fractional advection-dispersion equations reproduce relevant advective-dispersive characteristics [*Ganti et al*., 2010]. The important point is that it is only the tail characteristics of the travel distance and waiting time PDFs, and not their overall shape, that determines the long-term scaling behavior of a sum of particle steps with "random" length and waiting time.

[5] Historically, it has been assumed that distributions of sediment step lengths and rest periods, which together affect the overall transport distribution, have well-defined mean values surrounded by a characteristic amount of variability, suggesting that an advection-dispersion (diffusion) equation should reproduce the relevant features of sediment transport. Recent work suggests that this may not be the case. For example, *Ganti et al*. [2010] argued that the evolution of a patch of nonuniform sediment may be super-diffusive as a result of mixtures of step length distributions that arise due to grain size variation. To explore the applicability of heavy-tailed distributed step lengths, *Bradley et al*. [2010] reanalyzed the classic data collected by *Sayre and Hubbell* [1965; see also *Hubbell and Sayre*, 1964] for sand-bed rivers. They developed a model similar to that proposed by *Sayre and Hubbell* [1965] but assumed a heavy-tailed distribution of particle step lengths. To improve the performance of their model, they partitioned the tracers into a detectable mobile phase and an undetectable immobile phase. They concluded that super-diffusive models (in the form of a space-fractional advection dispersion equation) match the observed plume shape and growth rates better than classical step length models. These results contrast with detailed observations in a gravel bed irrigation canal under uniform flow conditions, which suggest that anomalous super-diffusive transport exists only as a preasymptotic "local" range within which correlated particle motions dominate transport [*Nikora et al*., 2002]. This is followed by an intermediate period during which tracer dispersion becomes subdiffusive as the effects of particle immobilization dominate [*Bradley et al*., 2010, *Nikora et al*., 2012]. There is evidence suggesting that wide distributions of particle immobile periods slow down the virtual velocity of coarse sediment transport [*Ferguson et al*., 2002; *Nikora et al*., 2002; *Martin et al*., 2012]. Recent laboratory flume studies designed to study particle step and rest characteristics [*Martin et al*., 2012] have yielded super-diffusive step length distributions that persisted after sediment was well mixed across the flume. More detailed studies of particle dispersion suggest, on the other hand, that anomalous transport behavior may be a result of periodicities in particle motion rather than super-diffusive step lengths [*Furbish et al*., 2012], and that particle velocities, which incorporate effects of both step time and length, are exponential-like and do not have heavy tails [*Roseberry et al*., 2012].

[6] The central question addressed by this paper may be posed as follows. What happens to coarse particles subject to intermittent flow capacity in natural streams? Older, field-based research shows that the distribution of step lengths of individual particles follows the Einstein-Hubbell-Sayre compound Poisson model or a simple Gamma, or exponential distribution for small displacements [e.g., *Hassan et al*., 1991; *Ferguson and Wathen*, 1998; *Ferguson et al*., 2002; *Pyrce and Ashmore*, 2003, 2005]. These studies, however, did not consider the possibility of power-law distributed.

[7] Our objective is to identify the nature of downstream dispersion of tracer particles at long timescales. Many studies focus on transport characteristics while particles are moving. However, long-time transport properties of gravel in streams are a function of both mobile transport characteristics and the often long periods during which particles are immobilized at the surface or through burial. Although we are interested in the characteristics of gravel transport when gravel is actually moving, and also the long-term bulk transport characteristics that include effects of both motion and nonmotion, we can realistically only measure the latter in the field. Laboratory experiments of particle transport allow for real-time measurement of particle velocities, entrainment and rest times, but these measurements are not currently feasible in long-term field studies. Tracer location is typically surveyed at convenient intervals, and 100% recovery of tracer stones is rarely achieved during a monitoring event. Thus, the number of transport events for individual particles is known precisely only if it is zero. Particles that have moved since previous surveys may have taken one or more steps during one or more flow events. When particles are not observed during a monitoring event, they may be buried deeply within the surveyed reach or be located beyond the survey area. This can only be resolved if the tracer is identified in subsequent monitoring events.

[8] The ultimate purpose of characterizing the statistical nature of steps and rests in streams is the identification of, and elucidation of the underlying physics contained in the governing equation for coarse sediment transport. This requires distinguishing between step lengths or travel distances that follow distributions such that $P(X > x) \sim Cx^{-\alpha}$, where $P(\cdot)$ denotes probability, X is the travel distance, C is a proportionality constant, and the tail parameter α is less than two for heavy-tailed steps and greater than or equal to two for thin-tailed steps.

[9] We now refine the question posed above. When coarse particles move as bedload in rivers with all the complexities associated with a natural setting, do their travel distances follow thin- or heavy-tailed distributions? To achieve our goal, we used magnetically and passive integrated transponders (PIT) tagged particle field data collected under a range of bed states and bed morphologies.

2. Data and Study Sites

[10] In this study, we analyze travel distances of individual coarse particles under a range of sediment transport rates. Travel distances were derived from observations of painted, magnetically and PIT tagged particles. We analyzed published field data supplemented with new studies covering a wide range of flow and sediment supply regimes and channel morphologies (Table 1). We considered both single and multiple events. In arid lands, single events are isolated because the bed is dry between sequential events. In humid areas, single events are distinguished by a return to baseflow

Table 1. Data of Particle Travel Measurements on Gravel Bed Streams and Flumes

Stream	Grain Size (mm)			Number of Events/ Surveys	Percent Recovered	Flow Regime	Tracing Method	Channel Morphology	Source
	Surface[a]	Subsurface[a]	Tracers						
Nahal Hebron	70	35	45–180	3	90–93	Arid	Magnetic	Riffle-pool	*Hassan et al.* [1991]
Nahal Og	35	15	45–180	2	55–56	Arid	Magnetic	Riffle-pool-bar	*Hassan et al.* [1991]
Allt Dubhaig (reaches 1,2,3)	39–60	15–31	22–185	13	64–81	Humid	Magnetic	Riffle-pool-bar	*Drew* [1991]; *Ferguson et al.* [2002]
Monachyle Burn	75	49.5	30–140	9	73–99	Humid	Magnetic	Riffle-pool-bar	*Drew* [1991]
Lainbach		50–65	30–170	18	80	Humid	Magnetic	Step-pool	*Gintz et al.* [1996]
Harris Creek	60	20	06–512	6	70	Snowmelt	Magnetic	Riffle-pool	*Hassan et al.*, [1992]
East Creek	40–57	20–31	08–128	5[b]	65–95	Humid	Magnetic	Riffle-pool-rapid	This study
Forfar	40–50	29	32–172	15	60–100	Snowmelt	Magnetic	Riffle-pool-bar	*Gottesfeld et al.* [2004]; *Hassan et al.*, [2008]
O'Ne-ell	40–50	31	32–176	13	60–100	Snow melt	Magnetic	Riffle-pool-bar	*Gottesfeld et al.* [2004]; *Hassan et al.* [2008]
Bouinence Torrent	15–29		23–520	3[b]	25–78	Humid	PIT	Riffle-pool-bar	*Liebault et al.* [2012]

[a]Median size.
[b]Annual survey.

for a few days, while in snowmelt-dominated streams single events cover an entire snowmelt season. Multiple events here refer to the cumulative movement over more than one event.

[11] Both Nahal Hebron (Israel) and Nahal Og (Palestinian Territories) are ephemeral desert streams with flash floods usually of short duration and steep, rising hydrographs that mobilize large quantities of sediment over relatively short periods of time. It has been assumed that during a short event with a single peak discharge, the number of steps that particles take is approximately the same for all particles [*Hassan et al*., 1991]. We suppose that Nahal Hebron and Nahal Og data represent a very dynamic environment and are relatively simple field cases due to short flow duration and single peak flows. We analyzed single and multiple events for both streams. The Lainbach (Germany) is a small stream in the Bavarian Alps with a steep channel dominated by step-pool morphology. Floods occur after snowmelt and heavy rain, and therefore represent a mixed hydrological regime [*Gintz et al*., 1996]. Cobbles, boulders, and large pebbles armor the bed surface. Data are available for 19 single events covering a wide range of flow magnitudes and durations (for details see Table 1, *Gitnz et al*. [1996]). The Bouinenc Torrent (France) is a wandering gravel bed stream that drains a 38.9 km^2 mountainous catchment [*Liebault et al*., 2012]. Floods are mostly induced by heavy rainfall in spring and autumn, and summer convective storms. For the Bouinenc Torrent, we have tracer data from annual surveys for the years 2008–2010 representing multiple events [*Liebault et al*., 2012]. Harris Creek, Forfar, and O'Ne-ell (all in Canada) are strongly seasonal streams dominated by melting snow with relatively long events (up to 3 weeks). Sediment transport in these creeks is low due to low sediment supply [*Hassan and Church*, 2001; *Gottesfeld et al*., 2004; *Hassan et al*., 2008b] and the well armored-structured bed surface of Harris Creek [*Hassan and Church*, 2001]. These creeks (e.g., Harris Creek, Forfar and O'Ne ell) represent a snowmelt system with gradual variation in flow with a low sediment transport regime. Allt Dubhaig, Monachyle Burn (both in the UK) and East Creek (Canada), represent humid environments with a relatively large number of sediment mobilizing events, mostly of small and medium magnitudes. For the Allt Dubhaig and Monachyle Burn, we have tracer data for single and multiple events that were collected over 2 years [*Drew*, 1991]. Data from multiple events for Allt Dubhaig are also available for the period 1993 and 1999 [see *Ferguson et al*., 2002]. Due to low recovery rates, we used 8 out of the 11 surveys in Allt Dubhaig [*Drew*, 1991]. East Creek is a small stream in the Malcolm Knapp Research Forest near Vancouver, British Columbia, Canada. The creek experiences between 8 and 12 mobilizing events annually, mostly of small to medium magnitude (e.g., < 50% of the particles moved). The study channel consists of three distinct morphologies, namely rapids, riffle-pool and step-pool. In 2004, 1400 magnetically tagged particles were seeded in the rapid and riffle-pool reaches. In this study we report results from annual surveys for the years 2004–2011, representing multiple events.

3. Methods

[12] Tracer tests are used to obtain information on the fluvial transport of sediment. Ideally, this will include the rate and direction of sediment transport, length of rest periods and mobile periods, step length of individual particles, residence time, flow competence, virtual rate of sediment movement, travel distances, burial depth, sediment sources and depositional area (for detailed information about tracers methods and research questions see *Hassan and Ergenzinger* [2003] and *Bradley and Tucker* [2012]). In the field, the most common measure of sediment dispersion is the travel distance of particles for a given flow event, season, or year. These may reflect a single step or the sum of many steps, making comparisons of various datasets a nontrivial task. Travel distance over single or multiple events is the focus of this paper. In comparison to flume data, the tracers in the field cover a range of sizes which may affect travel distance. *Ganti et al*. [2010] argued that a heavy-tailed, unconditional exceedance probability distribution of travel distance could emerge from a thin-tailed, conditional exceedance probability distribution of travel distance given grain size by marginalizing the conditional distribution over a thin-tailed, grain size density. Due to the lack of a relation between distance of travel of individual grains and particle size [e.g., *Einstein*, 1937; *Hassan et al*., 1991; *Drew*, 1991; *Gottesfeld et al*., 2004] and limited number

of tracers, we did not analyze the travel distance of each size separately.

[13] The most common first step in evaluating the tail character of sample data is the estimation of the rate of decay of the exceedance distribution tail, $P(X > x)$, where X is travel distance. Heavy tails are identified when the log-log slope of the exceedance tail $\alpha < 2$. In this case, heavy tail distributions do not have finite second moments. If the exceedance tail slope is $\alpha \geq 2$, then the distribution does have a finite second moment. Ordinary least squares (OLS) is known to be an imprecise estimator of tail slope because (among other reasons) power laws are not shift-invariant [*Aban and Meerschaert*, 2001], and the identification of the "true" tail in a sample distribution is frequently a judgment call. The distribution property that the tail slope remains constant when the location of the distribution is adjusted by some constant is called shift invariance. Hill's estimator [*Hill*, 1975], a conditional Maximum Likelihood Estimator (MLE) for the tail characteristics of sample data, suffers similar biases. Hill's estimators are a ranking method [*Aban et al*., 2006] given by

$$\hat{\alpha}_H = \left[r^{-1} \sum_{i=1}^{r} \left\{ \ln X_{(i)} - \ln X_{(r+1)} \right\} \right]^{-1}, \hat{C} = \frac{r}{n} \left(X_{(r+1)} \right)^{\hat{\alpha}_H},$$

where $\hat{\alpha}_H$ and $\hat{C}$ are estimators for power and slope in the Pareto distribution $P(X > x) = Cx^{-\alpha}$, $X_{(i)}$ are order statistics such that $X_{(1)} \geq X_{(2)} \geq \cdots \geq X_{(n)}$, and r is the highest index of a straight line segment of tail in a log-log plot of the exceedance distribution [*Aban et al*., 2006]. Although other alternatives to the Hill's estimator have been developed [*Clauset et al*., 2009], the quality of natural data sets are typically a limiting factor in performing rigorous statistical tests for heavy-tailedness [*Stumpf and Porter*, 2012]. We chose to evaluate tail slopes and use the Hill's estimator with the assumption that thin-tailed conclusions are likely accurate, while heavy-tailed conclusions may warrant further statistical confirmation. We look for a major slope-break from the highest values in the exceedance tail from which to estimate tail slope using the methods of OLS and Hill's estimator. Comparing both methods shows the bias of the OLS, however the OLS method also provides the ability to evaluate significance of regression in the tail slope. We cannot evaluate a goodness of fit statistic for the distribution since we are fitting the tail of the distribution rather than the entire distribution. If there are no slope-breaks in the tail, we use Hill plots (paired values of tail slope and number of points used to obtain the slope) to evaluate the minimum number of tail point where tail slope converges.

[14] To illustrate the difficulty in evaluating the rate of tail decay from a sample, we present three examples in Figure 1 (see also Table 2). The estimation of the tail slope with the Hill's estimator for the distribution of travel distances in East Creek survey 4 was based on 2.4% of the data and relied on a major break in the exceedance tail to obtain a Hill's estimate (Figure 1a). A similar argument can be made for O'Ne-ell Reach R1550 Survey 6 (Figure 1b). The only problematic estimate is for Forfar R250 survey 6, where there are only two points included in the tail estimate since they manifest themselves as a severe slope break from the rest of the tail (Figure 1c).

[15] Recovery rates in study site tracer tests range from 25 to 100% (Table 1). Given the difficulty in evaluating the rate of tail decay from a sample (e.g., Figure 1 and Table 2), a question surrounds the fate of the missing particles. If missing particles moved out of the surveyed reach, this would imply that they moved longer distances, and inclusion of missing data would influence the travel distance distribution. This would mean the data are not suitable for determination of thin- versus heavy-tailed travel distributions.

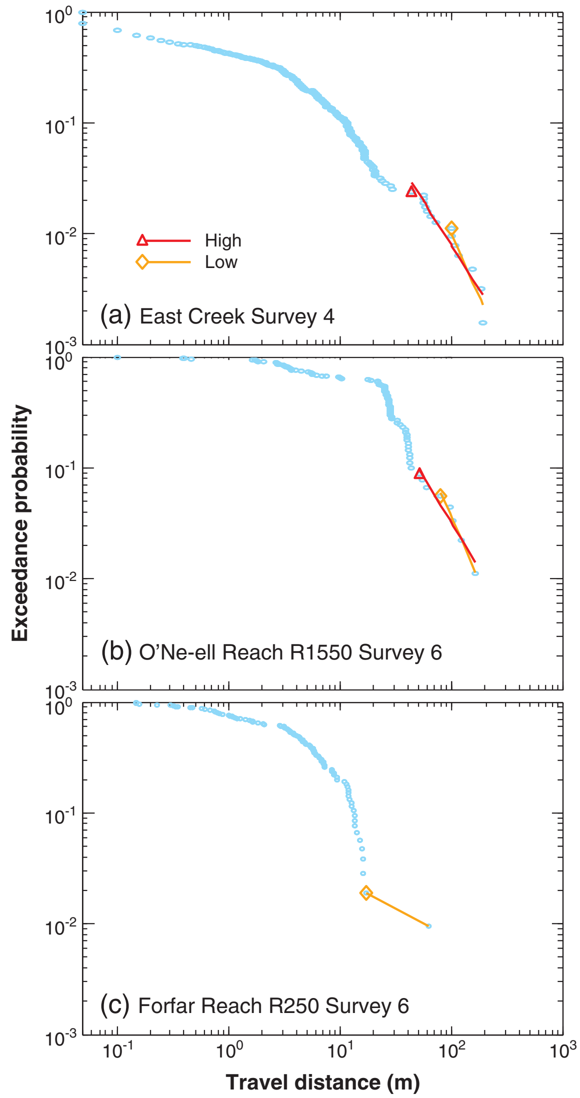

Figure 1. Cumulative exceedance distributions of downstream travel distance of three examples of events with low and high exceedance tail points used in Hill's estimates, for a few field cases with possibly heavy-tailed distributions. For details see Table 2.

[16] To explore the fate of missing particles, we plot the spatial distribution of moved, stationary, and missing particles along two reaches in East Creek for two seasons (Figure 2). Many of the missing particles in 2007 were found unmoved within the same reach in subsequent surveys (i.e., 2008–2010). It could be that some small stones left the study reach, but we have no evidence of long travel distances, since no stones were found beyond the downstream end of the reach after an extensive search. Furthermore, a few small tracer stones have been known to disintegrate.

Table 2. Characteristics of Particle Movement and Tail Slope Estimates on Gravel Bed Streams[a]

Stream	Reach	Survey	Number of Tracers	Percent Recovery	Percent Moved	Percent Buried	Low	High	Cum
Allt Dubhaig		2	600	81.3	33.0	35.1	1.82	3.15	5.33
		5	600	78.8	6.3	72.3	1.33		7.08
East Creek		4	1465	82.6	52.3	72.6	1.29	3.56	3.51
	RP	5	1465	73.8	79.3	76.3	1.86		1.97
Forfar	1545	2	165	98.0	80.0	2.5	1.26	2.38	1.26
	250	1	236	99.0	43.0	59.2	1.73	3.24	1.73
	250	2	203	99.1	35.6	6.8	1.61	3.25	1.61
	250	6	203	99.0	100.0	23.9	0.67		0.67
Monachyle Burn		4	200	89.0	47.8	33.1	1.57	2.49	4.17
		7	200	93.0	39.2	36.0	1.57		3.70
Nahal Og		2	252	56.0	21.3	63.1	1.11		2.31
O'Ne-ell	1550	1	262	86.0	40.0	8.9	1.02	3.21	1.02
	1550	6	90	67	100.0	59.7	1.52	2.07	1.52

[a]Only cases with heavy-tailed travel distribution are reported. For details see text.

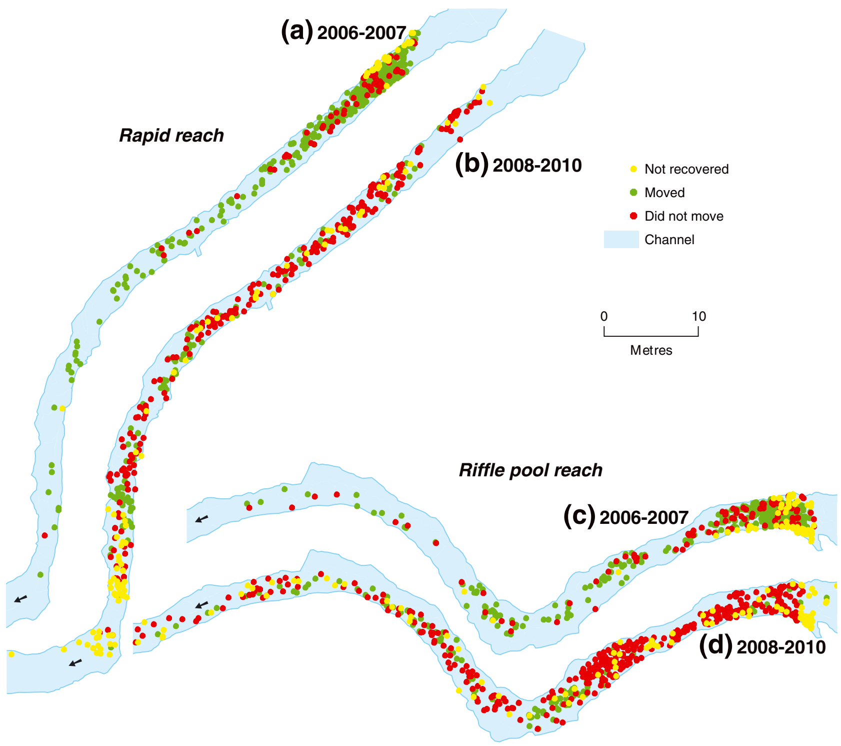

Figure 2. East Creek—spatial distribution of moved, stationary, and missing particles for rapid and riffle-pool reaches for a year with high flows (2006–2007) and for 2 years with small flows (2008–2010). (a) Rapids reach in 2006–2007, (b) rapids reach in 2008–2010, (c) riffle-pool reach in 2006–2007, and (d) riffle-pool reach 2008–2010.

[17] To further explore this point, we conducted numerical simulations to observe the effect of truncation on heavy-tailed data (Figure 3). Two scenarios are presented: (1) truncation of extreme values (i.e., particles that travel out of the surveyed area or a flume (e.g., *Wong et al.* [2007] experiments)) and (2) random truncation. Truncation of extreme values of about 1% of the tracer population resulted in the transformation of a heavy-tailed distribution to a thin-tailed distribution, changing estimates of the tail slope from $\alpha < 2$ to $\alpha \geq 2$ (Figure 3a). This is particularly true for $1.5 < \alpha < 2$. On the other hand, random truncation does not affect the travel distance distribution; that is, the heavy-tailed distribution is maintained (Figure 3b). However, our field data (Figure 2) support the hypothesis that unidentified stones reflect shortcomings in measurement techniques rather than transport beyond the surveyed area. Therefore, we proceed under the assumption that the missing particles in our field sites have little impact on the inference as to whether transport characteristics are heavy-tailed or thin-tailed.

4. Results

4.1. Travel Distance Distribution

[18] We first describe the single-event data sets collected from streams covering a wide range of hydrological regimes. Second, we examine cumulative travel distance for multiple

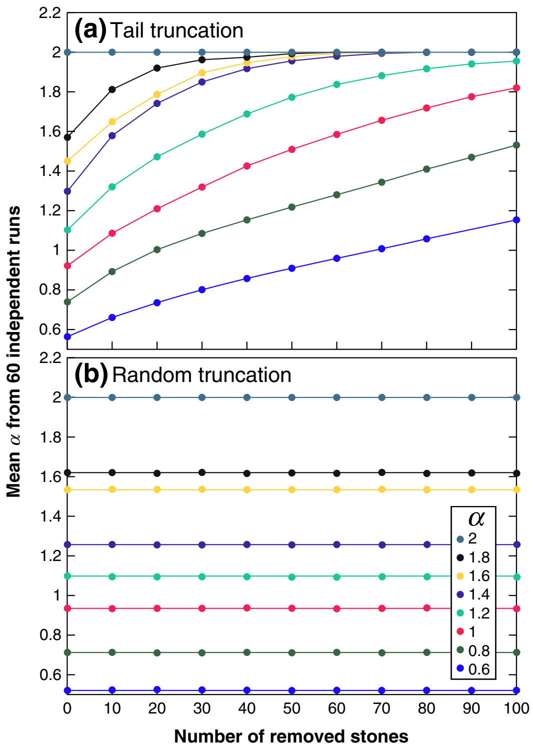

Figure 3. Truncation effect on the value of the tail estimate for (a) truncation of extreme values and (b) random truncation due to non-detection of particles. Ten thousand heavy-tailed densities were randomly generated using a stable distribution with zero skew, unit scale and zero location parameters. The slope of the straight section of the exceedance tail is fitted by ordinary least squares and Hill's estimators, and the slope is recorded. This is followed by trimming the last ten values from the vector before fitting the tail slope again to obtain a new slope value. This is repeated until one hundred elements of the end of the random vector have been removed and the final slope value is obtained. In the case of random truncation, the ten values removed from the sample density were randomly selected from the population. For the purposes of visualization, only values for $\alpha < 2$ are presented. For ADE models, values do not go higher than 2.

events and years. Finally, we discuss the impact of burial on waiting time and sediment advection/dispersion in gravel bed streams.

[19] To examine travel distance distributions, we analyzed field data from a range of environments, over both single and multiple flow events. It is important to note that a single survey may include one or more flow events, and likely reflects more than one particle step representing motion between consecutive immobile periods. Thus, from a statistical point of view, we use the term "step" loosely. However, the tail thickness for a sum of particle steps reflects the tails of individual steps. If the tail thickness for a sum of random particle steps has a heavy tail, then the tail for one or more of the individual steps must be heavy-tailed.

[20] In total we considered 79 travel distances for individual surveys but only used 64 due to a low recovery rate. At two sites, Allt Dubhaig and East Creek, cumulative travel distances (obtained by either adding travel distance by single events or by annual measurements) ranging from a few events to up to a few tens of events were analyzed. One thin-tailed and three heavy-tailed cases are presented in Figure 4. Out of 64 surveys, 51 displayed a tail parameter estimate suggesting a thin-tailed distribution. *Liebault et al.* [2012] reported heavy-tailed distributions for two of the three Bouinenc Torrent surveys. However, our analysis of the tail of the data resulted in a thin-tailed distribution for all three Bouinenc Torrent cases. For eight of the remaining 13 surveys, the distinction between heavy- and thin-tailed distributions was dependent upon the number of points used to define the "tail." For example, the tail parameter for travel distance obtained from data analysis of the first Forfar 250 survey changes from $\alpha = 1.73$ (heavy) to $\alpha = 3.24$ (thin) depending on whether 18 or 13 points are chosen to define the tail (Table 2).

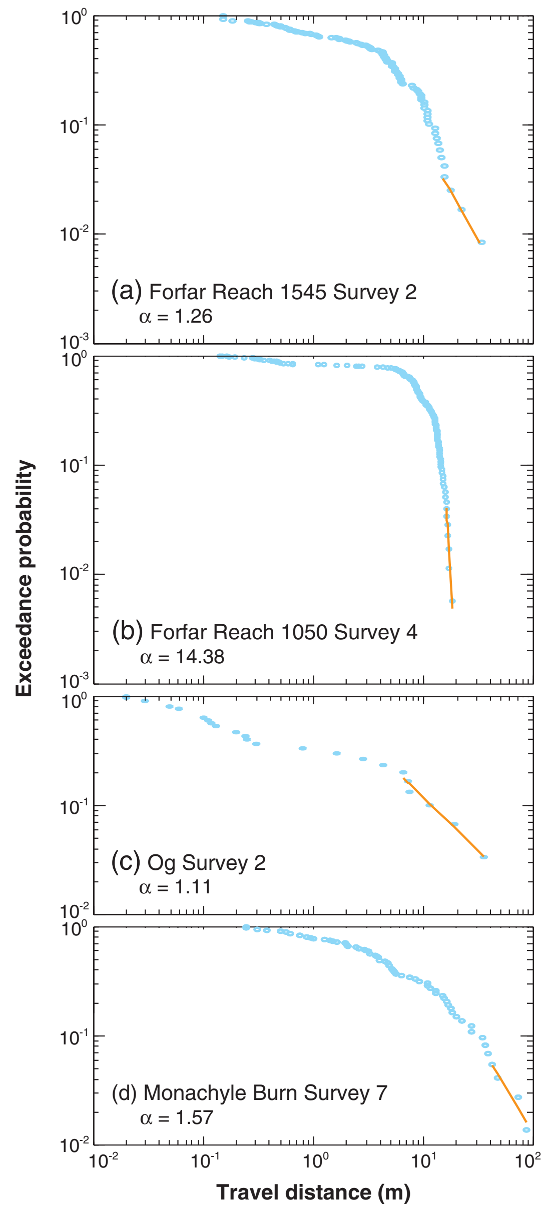

Figure 4. Examples of cumulative exceedance distributions of downstream travel of thin- and heavy-tailed distributions.

[21] Table 2 lists the cases for which there may be heavy-tailed distributions of travel distances, as well as the range of possible tail parameters. Since we are fitting tail data only, there is no applicable distributional goodness-of-fit test. However, the largest *p*-value for heavy-tailed fits using OLS was 0.013, indicating statistical significance of power law fits. Also shown in Table 2 is the percentage of all recovered tracers found during the survey, the number of tracers used at the site, the percentage of recovered tracers mobilized since previous surveys, and the percentage of recovered tracers found buried during the survey. Hill's estimates ranged from low (LowEstm) to high (HighEstm) where subjective selection as to the number of inclusion points was considered. The final column shows Hill's cumulative estimate for the surveys at a site. Recovery percentage of tracer stones for each of the heavy-tail distributions was moderately high (>60%) to very high (>90%) with the exception of Nahal Og survey 2, where it was less than 60%. Burial percentage ranged from 2.5% to 89%, with a mean of 41.3%, suggesting that burial is independent of travel distance tail shape. This is due to the fact that in gravel bed rivers, burial occurs after particles stop moving [e.g., *Hassan and Church*, 1994].

[22] Six of the sites with heavy-tailed travel distance distributions have nival hydrologic regimes (Forfar and O'Ne-ell), five of them have rain-driven hydrologic regimes (Allt Dubhaig, Monachyle Burn and East Creek), and one is an ephemeral desert flash-flood hydrologic regime (Nahal Og). Allt Dubhaig and Monachyle Burn, rain-dominated streams in the Highlands of Scotland, each had two heavy-tailed estimates from surveys having high recovery rates (>78%), each of which has one subjective estimate of tail thickness depending upon how the tail is defined. Of the four East Creek surveys, only one case yielded a heavy-tailed distribution. East Creek was also analyzed for endpoint morphology dependence by partitioning tracers into two dominant morphologies: riffle-pool and rapids (Table 2). The riffle-pool reach is located downstream from the rapids reach; both had about the same width (~2.5 m) and length (~100 m). The median size of the bed surface material in the rapids ($D_{50} = 50$ mm) is coarser than that of the riffle-pool ($D_{50} = 40$ mm) and relatively well structured. The two reaches experience the same floods and sediment supply regimes. The riffle-pool (RP in Table 2) section was heavy-tailed while the rapids section was not. Forfar and O'Ne-ell, snowmelt dominated systems, yielded both heavy- and thin-tailed step distributions. Four out of 15 surveys in Forfar and two out of 13 surveys in O'Ne-ell yielded heavy-tailed travel distance distributions. Finally, Nahal Og, an ephemeral stream, had one heavy-tailed parameter estimate.

[23] To examine the long-term behavior of tracers seeded on the bed surface that have been vertically well-mixed within the bed layer and spatially distributed between morphologies and along the channel, we examined the distribution of cumulative travel distances. Where possible (7 of 13 sites with "heavy-tailed" events), we evaluated total particle travel distances (sum of individual surveyed measurements). Only the East Creek riffle-pool morphology had heavy-tailed total travel distances (Table 2 –CumEstm column). The Forfar and O'Ne-ell Creek cumulative distributions are the same as their single-survey distributions, since tracers were reseeded upon completion of each survey.

[24] Our analysis yielded a wide range of tail slopes estimates for field site particle travel distance distributions. Do channel morphology, sediment supply or flow regime, therefore, impact step length tail characteristics? In Figure 5a, we plot the range of tail slopes sorted by channel morphology. Overall, step-pool and rapids morphologies yielded thinner tailed travel distances with a wider range of values than riffle-pool bar morphologies (Figure 5a). Of particular interest are results for the East Creek riffle-pool (East Cr RP1) and rapids (East Cr RAP) reaches. The two reaches experience the same floods and sediment supply regimes, and therefore a direct comparison to assess the influence of channel morphology on the tail estimates is possible. Travel distance distributions in the rapids reach had thinner tails and a wider range of tail slopes than the riffle-pool reach (Figure 5a). This implies greater particle retention in the

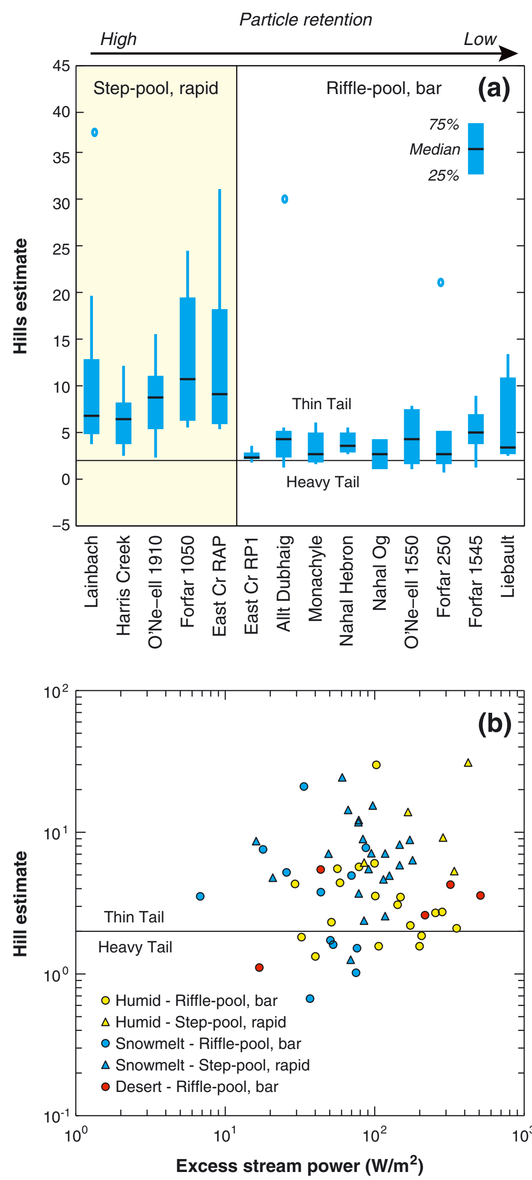

Figure 5. (a) Hill's estimates of tail parameter for travel distance distributions sorted based on channel morphology. (b) Hill's estimates versus excess stream power sorted by hydrological regime and channel morphology.

rapids than the riffle-pool. Similar results were observed in Forfar and O'Ne-ell Creeks (Figure 5a).

[25] No correlation was observed between tail slope of the travel distance distribution and flow magnitude, flow duration, total excess stream power, or stream power associated with peak flow. An example of the relation between tail slope and stream power associated with the peak flow is presented in Figure 5b. The lack of correlation with flow characteristics at East Creek, Forfar and O'Ne-ell is evident in Figure 5a. For example, the East Creek reaches (East Cr RP1 and East Cr RAP) experience the same flow characteristics but the rapids reach yielded thinner tailed travel distances than the riffle-pool reach (Figure 5a). However, most of the heavy-tailed travel distance distributions developed during relatively small to medium events. Figure 2 demonstrates the episodic nature of sediment entrainment in both morphologies in East Creek; some particles move while adjacent particles remain stationary. During small events in both rapids and riffle-pool reaches in East Creek (2008–2010 maps) the sediment mobility was spatial variable and limited to small areas within the channel (Figure 2). This implies that large areas within the channel remained static during these events. Furthermore, most tracers did not change their vertical position within the bed, implying little vertical mixing of sediment. Most tracers moved during the large events of 2006–2007 (Figure 2), and a large proportion of the particles became buried within the sediment layer, implying intense vertical mixing of the bed.

4.2. Particle Immobile Periods

[26] Since the amount of time particles spend in the subsurface directly impacts how fast and far particles spread in the streamwise direction, we must also consider their residence time in the bed to understand their overall dispersive behavior. Particles are relocated by each transporting event not only over large areas of channel bed, but also vertically within the scour layer. This state of dual intermittency in space and time implies that any theory of bedload distribution needs to address vertical mixing (burial depth) of particles in addition to longitudinal dispersion [*Parker et al.*, 2000]. Buried particles remain stationary in streams subjected to fluvial sediment transport (i.e., no debris flows) until scour events reach the particle position. For a given period of time, the probability of movement for buried particles is smaller than for surface particles, and residence time of buried particles increases with depth in the scour layer. Increased residence time of sediment in the subsurface decreases virtual velocity of sediment resulting in overall shorter travel distances. Overall, residence time distributions are further affected by channel morphology.

[27] The influence of burial depth on the probability of particle movement was examined for the years 2004–2010 in East Creek, as well as two seasons in Allt Dubhaig, i.e., those streams with sufficient data to perform such an analysis. The burial depth of particles was divided into increments of size equal to the median size of the bed surface material, and the probability of movement was calculated for each layer. The probability of movement declines with burial depth (Figure 6), implying that deeply buried particles are likely to have a lower probability of movement than surface or shallowly buried particles. Furthermore, in both cases the top 10 to 15 cm of the bed (~2–3 D_{50} of the bed surface material) is an active bed zone from which particles are frequently entrained, while particles buried >20 cm have a low probability of entrainment.

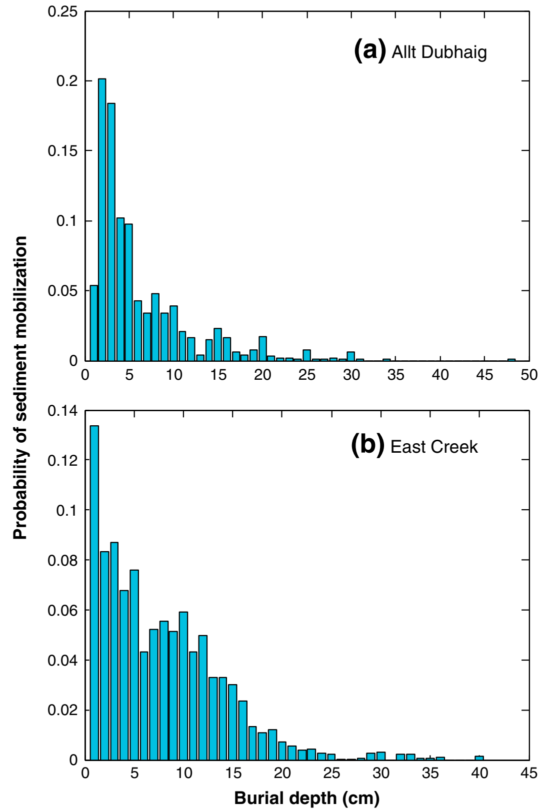

Figure 6. Probability of travel distance versus burial depth in (a) Allt Dubhaig and (b) East Creek.

[28] Time of movement is a key component of overall travel distance distributions through time. It is not possible to track exact waiting time data for our study sites, but we examine a surrogate that allows inferences about the proportion of mobilized sediment during competent flow. We investigated the probability of particle motion given the duration of flow that is competent to move sediment for East Creek and Allt Dubhaig. The total time for which the flow was larger than the critical value needed for entrainment for each event was calculated for each survey. The critical flow needed for particle entrainment was based on field observations of sediment mobility in both creeks. If a particle moved during an event, then it was assigned a zero waiting time (e.g., motion period equal to the length of the event). The waiting time for particles that did not move during flow was taken to be equal to the length of the event. Over the study period, 37 mobilizing events occurred in Allt Dubhaig and 35 in East Creek. For Allt Dubhaig, the immobile time ranged from zero to 600 h (Figure 7a) with a median of about 100 h. The immobile time exceeded 400 h only for 15% of the particles. Here 90% of East Creek particles had immobile periods of 50 h (Figure 7b). Only a few deeply buried stones beyond the frequently scoured layer had waiting times of more than 500 h. The important point is that particles that have moved during previous flow events are more likely to move in the next event.

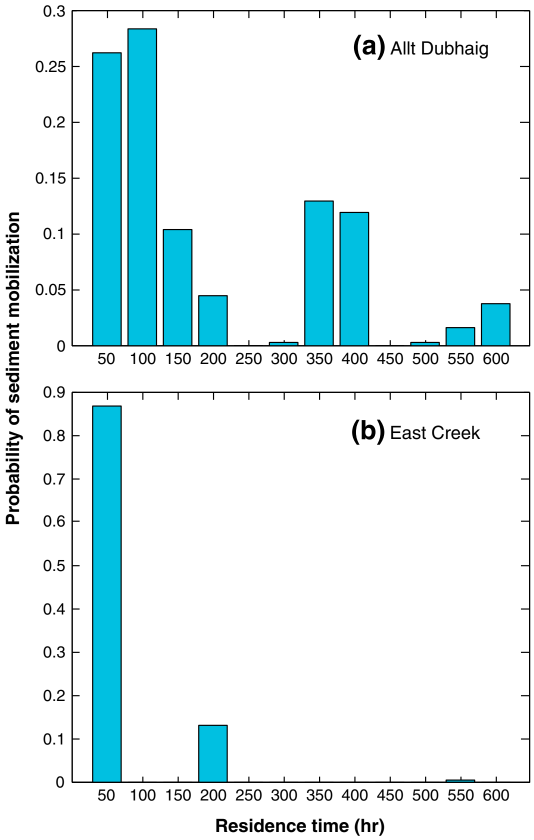

Figure 7. Probability of movement versus waiting time for (a) Allt Dubhaig and (b) East Creek. For details see text.

5. Summary and Discussion

[29] In this paper, we used field observations to describe the travel distance distribution of particles in gravel bed streams. We focused on heavy-tailed versus thin-tailed distributions of particle displacement because these tail characteristics determine whether the appropriate governing equation should permit super-diffusive or subdiffusive spreading. Recent publications on the topic are based on theoretical considerations [e.g., *Stark et al*., 2009; *Ganti et al*., 2010], experimental work [e.g., *Hill et al*., 2010, *Martin et al*., 2012], sand bed rivers [*Bradley et al*., 2010] or analogy from other fields such as groundwater flow [e.g., *Benson*, 1998; *Schumer et al*., 2003, 2009]. Although *Liebault et al*. [2012] reported heavy-tailed distributions for the Bouinenc Torrent, our analysis of the tail of the data resulted in a thin-tailed distribution for all cases.

[30] The displacement of mobile particles from one monitored event to the next was occasionally heavy-tailed, but this characteristic did not persist through consecutive monitored events at our study sites. This is likely because of nonstationarity of transport properties as particles encounter different morphological regimes. Particles in the front of a plume are likely to encounter fast zones first and speed ahead of the group, resulting in a wide overall distribution of travel distances. As the rear of the plume reaches the fast zone, these particles are able to close the gap and the distribution of travel distances no longer appears heavy-tailed.

[31] Which conditions will likely promote thin-tailed versus heavy-tailed distributions? Both thin- and heavy-tailed step length distributions were observed in individual streams and for all fluvial environments, from flashy desert streams to snow melt dominated streams. Most of the thin-tailed distributions were obtained during medium to large events. During these events, a high proportion of the tracer population moved, and relatively large areas of the bed were active. Although bed mobility was spatially variable (Figure 2), the magnitude of scour and fill was relatively large. Scour and fill of the channel bed resulted in vertical mixing and progressive burial of tracers at a wide range of depths. Vertical mixing is conceptualized in terms of random exchange between different storage zones [*Schick et al*., 1987; *Hassan*, 1990; *Ferguson and Hoey*, 2002]. Deeply buried particles moved less often and later in the course of a flow event than those originating on or near to the bed surface (Figure 6). Data from East Creek, Alt Dubhaig and other streams have shown that the probability of movement decreases with burial depth. The waiting times and probabilities of movement with burial depth are likely to influence the rate of vertical mixing and the downstream dispersion of the tracers. Therefore, particle burial/vertical mixing causes an overall reduction in the mobility of bed material within the bed layer. Using data from Allt Dubhaig, *Ferguson and Hoey* [2002] (see also *Hassan* [1994] and *Ferguson et al*. [2002]) asserted that slowdown in the downstream dispersion of particles through vertical mixing is much greater than that through advection, an outcome supported by our results for the probability of movement with burial depth and waiting time. *Ferguson and Wathen* [1998] found virtual velocity related to the size fraction of tracers on the Allt Dubhaig. Although we did not consider particle size effects on diffusion rates in this study, future research on travel distance distributions might focus on dispersion rates across particle size classes.

[32] The distribution of tracer particle travel distance is influenced by channel morphology such as bars, pools, riffles, and small structure [e.g., *Sear*, 1996] as well as vertical mixing. This channel morphology is created by spatially differential movement of bed material. Field studies showed that most buried particles were found in bars [e.g., *Drew*, 1991]. In the thalweg and riffles of a channel, tracers remain exposed and have a higher probability of motion. This is due to the heavily armored, nonscouring areas in the bed channel. The stability of the thalweg and riffles is consistent with evidence reported in the literature [e.g., *Muhlhofer*, 1933] indicating that these areas remain relatively intact regardless of the fact that they experience some of the highest shear stress in the channel. Bars are likely to be active during medium to large floods, but exposed and relatively stable during low flow events. It seems that the slowdown of particles due to vertical mixing and local channel morphology are the main reasons for thin-tailed travel distributions during medium to large mobilizing events.

[33] Wide distributions of travel distance were obtained for small events over a range of environments. This outcome is likely related to the spatial dispersion pattern of sediment entrainment, scour, and fill during these events. Floods with a flow slightly higher than the critical value needed to initiate sediment transport resulted in localized scour and fill and a relatively low rate of vertical exchange [e.g., *Schick et al*., 1987; *Hassan and Church*, 1994]. Typically, particles

located on or close to the bed surface moved during flow events. Furthermore, relatively small areas within the channel were mobile. This description of sediment dispersion is supported by field data from East Creek, where particle movement is quite variable both laterally and downstream in the rapids and the riffle-pool reaches (Figure 2). In spite of variable particle mobility and presence of deeply buried particles, a large proportion of those found within the bed surface remained stationary. This indicates that the scouring process is sporadic and highly variable in space. After entrainment, particles move over an almost static bed following preferential paths. In fact, the movement is concentrated in defined lanes which change their location over time and in space. This is likely the reason for the seemingly heavy-tailed distributions for travel distances obtained for few small events, since this condition induces a large separation between extreme distances for a few stones, but much shorter travel distances for a majority of the stones. Super-diffusive motion has been observed and attributed to correlated motions in laboratory flume studies [*Martin et al*., 2012]. Overall transport characteristics in the same study were affected by heavy-tailed residence times. In our field areas, large contributing area, high vertical exchange, large burial depth, and local morphology cause slowdown in particle movement leading to thin-tailed distributions of overall transport.

[34] Heavy-tailed step length distributions reflect an increased probability that a particle will travel well beyond the main observational group. As such, any truncation of observations from the exceedance tail can dramatically impact the interpretation of the distribution type. This is particularly true for cases where stones leave the study reach, or for flume experiments characterizing the travel distance of tracers [e.g., *Wong et al*., 2007]. Finally, the field data reported here has been collected with the most advanced techniques available for the study of particle transport in natural gravel bed streams. Yet many of our conclusions concerning the role of probabilistic modeling of steps and waits in determining tracer transport are to some degree speculative. Development of tracer monitoring techniques that allow tracking of the x, y, z coordinates of gravel tracers in real time will substantially advance this area of inquiry.

6. Conclusions

[35] Of 64 field surveys of gravel tracer movement analyzed, 51 showed thin-tailed step-length distributions, and 13 displayed heavy-tailed step-length distributions. Eight of these 13, however, could be identified as thin-tailed upon appropriate redefinition of the tail of the exceedance distribution.

[36] Particle cumulative travel distance was thin-tailed for all surveys but one (the riffle-pool section of East Creek). While particles may not equally sample slow and fast paths within the channel between individual sampling events, sufficient mixing appears to occur after many sampling events in this study.

[37] Each of the East Creek study reaches experienced the same flow regime, but yielded thinner tails in the riffle-pool regions as compared to the rapids. This indicates that channel morphology influences the tail behavior more strongly than the flow regime. Overall, thin tailed travel distances and higher particle retentions were obtained for step-pool and rapids morphologies than riffle-pool and bar morphologies.

[38] Truncation of extreme values of step length occurs because stones travel beyond the measured bounds of a study reach. This has the effect of making an otherwise heavy-tailed distribution of particle travel appear thin. In this study, missing particles tended not to leave the study reach, but rather failed to be identified due to measurement error or deep burial. Although field surveys frequently fail to obtain full recovery of tracer stones, it appears that missing stones at our study sites were distributed throughout the study reach and so did not cause misinterpretation of tail characteristics.

[39] Bedload transport models should incorporate effects of vertical mixing of sediment [*Parker et al*., 2000]. Deeply buried stones have lower probabilities of being mobilized and longer residence (waiting) times than shallowly buried or surface stones.

[40] Stones that are mobilized late during competent flow periods do not travel as far as those that are entrain early in the same period. This condition has an effect on single-event step length distributions. While step-lengths of particles in gravel-bed streams may occasionally be heavy-tailed due to the extremely long travel paths of a few particles, long waiting times driven by particles that stay deeply buried for extremely long periods appear to have a stronger effect on overall travel distance distributions, accounting for previously observed decreases in particle virtual velocity.

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