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Towards a theory of solitary shock waves of fluid pressure and solute density in geologic porous media

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Abstract

In this paper we analyze the presence of "Burgers solitons" in geologic porous rocks. These are quick, strong transients of combined fluid pressure and solute density that are generated from an adjacent matrix, characterized by an intense pressure and contaminant density. The effect that produces such transients is a nonlinear advection process that is not often taken into account in hydrologic science. A brief analysis of solitons in Pierre Shale is also shown in the paper.

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Keywords: Porous media; Burgers solitons; Solitary shock waves.

1. Introduction

Theoretical knowledge gaps in hydrologic research persist even today, in particular about non linear effects as advection in low permeability media. To confront this gap we propose a novel model of solitary shock waves of fluid pressure and solute density in geologic porous media. An example of its application to Pierre Shale, USA, is also shown.

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2. The model

A novel theoretical model on mechanic and chemo-poroelastic coupling of fluids and solute in porous media is here presented. The focus of this model is on advection and the related non linear processes. Analyzing the evolution of fluid pressure p and density of solute ρ in a rock under the action of strong external stress we obtain two coupled non linear equations [1]. Their solutions thus unveil a novel phenomenon: the presence of quick and strong transients of coupled fluid pressure and solute density, known in physics as Burger solitons.

Merlani, Salusti, Violini ([2], *MSV thereafter)* obtained the model equations

$$\frac{\partial p}{\partial t} + E\frac{\partial \rho}{\partial t} + F\frac{\partial^2 p}{\partial x^2} + H\frac{\partial^2 \rho}{\partial x^2} = 0 \quad (1a)$$

$$\frac{\partial \rho}{\partial t} + M\frac{\partial p}{\partial x}\frac{\partial \rho}{\partial x} + N\left(\frac{\partial \rho}{\partial x}\right)^2 + S\frac{\partial^2 \rho}{\partial x^2} + U\frac{\partial^2 p}{\partial x^2} = 0 \quad (1b)$$

(Table 1 and II) where one can see the intrinsic relation $F/H = M/N$.

MSV [2] analysed the non-linear solutions of equations (1), considering a first fluid saturated porous-permeable rock for $x < 0$ (contaminant $\rho_0 + \rho_I$ and pressure $p_0 + p_I$ at $t = 0$) and for $x > 0$ another homogeneous patch of fluid saturated porous-permeable rock (contaminant ρ_0 and pressure p_0 at $t = 0$) (Fig 1). In this way *MSV* obtained a Burgers-like equation

$$\frac{\partial \rho}{\partial t} + A\left(\frac{\partial \rho}{\partial x}\right)^2 - Z\frac{\partial^2 \rho}{\partial x2} = 0 \quad (2)$$

where $A = N - EM$ and $Z = UE - S$. In addition the Darcy fluid velocity is (Fig. 2)

$$u_D = - [k/(\varphi\eta)]\, \partial p/\partial x = Ek/(\varphi\eta)\, \partial \rho/\partial x \quad (3)$$

For an initial solute density ρ^* the solution of (3) is ruled by a Reynolds number [3; 2]

$$\boldsymbol{R} = (2A\rho^*)/Z \quad (4)$$

In more detail if $\boldsymbol{R} > 8$- 10 as for a strong initial jump ρ^*, one has

$$\rho = \rho_0 + \frac{x^2}{2At} \qquad \psi < x < x_B = \sqrt{4A\rho^* t} \qquad t^* < t \quad (5)$$

and $\rho = \rho_0$ otherwise . The corresponding p is

$$p = p_0 - \frac{Ex^2}{2At} + \frac{EF - H}{2A}\ln\left(\frac{t}{t^*}\right) \qquad \psi < x < x_B = \sqrt{4A\rho^* t} \qquad t^* < t \quad (6)$$

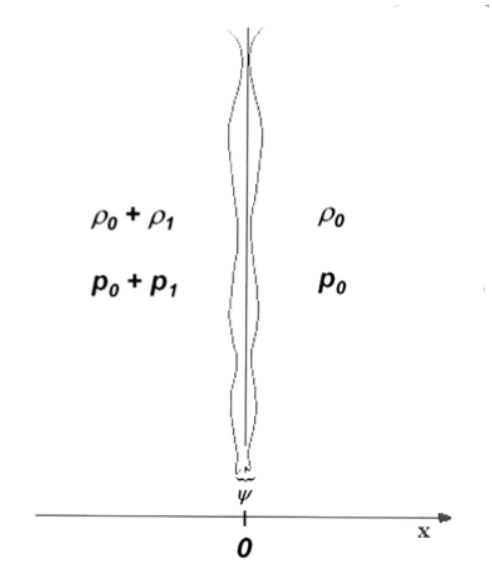

Fig. 1. A sketch of the two rocks system.

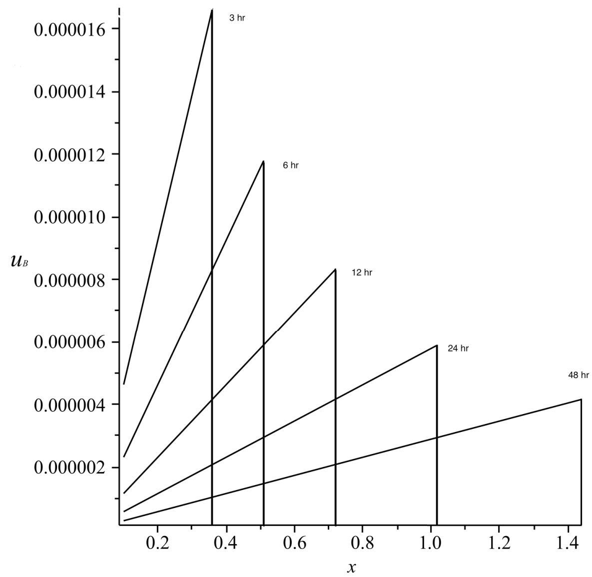

Fig. 2. The fluid velocity at different times

3. Pierre Shale

The Pierre Shale, also called claystone, is a geologic formation, or a series, in the Upper Cretaceous. It is mostly studied in North America, mainly east of the Rocky Mountains in the Great Plains, from North Dakota to New Mexico. This dark-gray shale is remarkably fossiliferous and can reach ~ 210 m in thickness over the Sandstone aquifer systems [4]. It is of marine origin and was deposited in the Western Interior Seaway. Pierre Shale stratigraphy is correlated with other marine shales that occur farther west, such as the Bearpaw Shale in United States and Canada.

The data discussed in Table I were obtained in central South Dakota, USA [5; 6]. Its mineralogy is 70-80% of clay, of which about 80% is a mixed-layer of smectite-illite. The shale, at this site, is saturated at the depth of ~ 75 m.

An estimate of the swelling coefficient ω_0 is questionable. Here we compare our equation on the relation among pressure, contaminant, stress and strain, with the recent study of *Sarout and Detournay* [6]. We call here α^* their parameter α and χ the ratio of moles of solute over that of fluid, while their v_0 is about 1.8 x 10^{-5} m^3. By comparing all this with data for Pierre shales one has

$$\omega_0 \left(\frac{1}{\rho} + \frac{1}{\rho_D}\right) = \frac{\alpha \alpha^* R^* T}{v_0} \tag{7}$$

Sarout and Detournay [6] explained that their parameter α^* is difficult to measure even in a laboratory.

Table 1. Estimated values of relevant parameters of Pierre Shale, in SI.

Parameter	Value in SI	References
ϕ Rock porosity	0.3	
k Intrinsic permeability	10^{-18}	
η Fluid viscosity	3×10^{-4}	
Θ Solute reflection coefficient	0.25	[6]
D Solute diffusion coefficient	10^{-8}	[7]
Ms Solute molar mass (Na Cl)	0.06	
α Biot coefficient	0.7	
ω_0 Swelling coefficient	10^5	
K Bulk modulus	4×10^6	
Ks Bulk modulus of the solid matrix	1.5×10^7	[8]
K_f Fluid bulk modulus	$2\text{x}10^9$	
$\overline{\rho}$ Estimated solute density	1	
$\overline{p}$ Estimated fluid pressure $\approx E\,\overline{\rho}$	10^5	
Formation period	Cretaceous	

They assume that their $\alpha^* = 0.9$. Since $\rho_f \approx \rho_D$ is about 10^3 and $\alpha^* \sim 1$, all this gives an estimate value of $\omega_0 \sim 10^5$. These quantities, from which we compute our Table II, may have uncertainties of about 30-50 % and in the following we will assume that all the considered rocks have comparable uncertainties.

The above data confirm that we have $A = N - M\,E = 10^{-7}$ in the SI and $Z = E\,U - S = 3\text{x}10^{-8}$. This gives $\boldsymbol{R} \approx 7\,\rho^*$ and therefore the presence of solitons can occur if initially the external solute density is $\rho^* > 1.5$, or equivalently the pressure $p^* \approx 10^5\text{-}10^6$.

4. Conclusions

We here describe a novel model that provides a simple analytic technique for estimation of coupled solute and water transports in porous media. The importance of our nonlinear model is that it shows that these transients are much sharper than those predicted by linear models, perturbation theories, or scale analyses presently available in hydrologic literature. Thus, under a strong enough initial impulse the resulting sharp transients can be the origin of many previously unexplained phenomena such as instabilities, or even small earthquakes.

Table 2. Coefficients of equation (1) for Pierre Shale, in SI.

Coefficient	Numerical value in SI
$E = -\frac{\omega_0}{\alpha^2 + KV}\left(\frac{1}{\bar{\rho}_S} - \frac{1}{\bar{\rho}_D}\right)$	$-1.5x10^{5}$
$F = -\left(\frac{k}{\eta}K + \frac{k\Theta K\bar{\rho}_f}{\eta\bar{\rho}_D}\right) /(\alpha^2+VK)$	$-2,5x10^{-8}$
$H = \frac{k\Theta\bar{\rho}_f R^*TK}{\eta M^S}\left(\frac{1}{\bar{\rho}_D} + \frac{1}{\bar{\rho}_S}\right)/ (\alpha^2+KV)$	0.2
$M = -\frac{k}{\phi\eta}\left[\frac{1}{\bar{\rho}_f} + \frac{\Theta}{\bar{\rho}_D}\right]\bar{\rho}_f$	-10^{-14}
$N = \frac{k}{\eta\phi}\Theta\frac{R^*T}{M^S}\left[\frac{1}{\bar{\rho}_D} + \frac{1}{\bar{\rho}_S}\right]\bar{\rho}_f$	10^{-7}
$S = -\frac{D}{\phi}$	$-3x10^{-8}$
$U = \frac{M^S D\bar{\rho}_S}{R^*T\phi \;\; \bar{\rho}_f}$	$7.5x10^{-16}$
$V = \frac{\alpha - \phi}{K_s} + \frac{\phi}{K_f} - \frac{\omega_0}{K}\frac{M^S}{R^*T\,\bar{\rho}_D}$	$2.5 \, x10^{-8}$

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