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A fixed point theorem for a Meir-Keeler type contraction through rational expression

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Abstract

In this paper, we establish a new fixed point theorem for a Meir-Keeler type contraction through rational expression. The presented theorem is an extension of the result of Dass and Gupta (1975). Some applications to contractions of integral type are given.

Keywords: Fixed point, Meir-Keeler type contraction, Rational expression, Contraction of integral type.
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1. Introduction

The Banach contraction principle [4] is the most celebrated fixed point theorem. It is a very useful, simple, and classical tool in nonlinear analysis. Moreover, this principle has many generalizations; see ([1]-[30]) and others. For example, Meir and Keeler [20] proved the following fixed point theorem.

Theorem 1.1. *Let (X, d) be a complete metric space and let T be a mapping from X into itself satisfying the following condition:*

$$\forall \varepsilon > 0, \exists \delta(\varepsilon) > 0 \text{ such that } \varepsilon \leq d(x, y) < \varepsilon + \delta(\varepsilon) \Rightarrow d(Tx, Ty) < \varepsilon.$$

Then T has a unique fixed point $\xi \in X$. Moreover, for all $x \in X$, the sequence $\{T^n x\}$ converges to ξ.

It is clear that Theorem 1.1 is a generalization of the Banach contraction principle. Some generalizations of Theorem 1.1 exist in literature; see [10, 15, 19] and others.

Dass and Gupta [11] proved the following fixed point theorem.

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Theorem 1.2. *Let (X,d) be a complete metric space and let T be a mapping from X into itself satisfying:*

$$d(Tx,Ty) \leq \alpha d(y,Ty)\,\frac{1+d(x,Tx)}{1+d(x,y)} + \beta d(x,y)$$

for all $x,y \in X$, where α,β are constants with $\alpha,\beta > 0$ and $\alpha+\beta < 1$. Then T has a unique fixed point $\xi \in X$. Moreover, for all $x \in X$, the sequence $\{T^n x\}$ converges to ξ.

Some generalizations of Theorem 1.2 exist in literature; see [8, 27] and others.

In this paper, we derive a new fixed point theorem of Meir-Keeler type that generalizes Theorem 1.2 of Dass and Gupta in the case $\alpha,\beta \in (0,1/2)$. Our main result is given in Section 2. In Section 3, following the ideas of Branciari [7] and Suzuki [28], some applications to contractions of integral type are given.

2. Main result

Our main result is the following.

Theorem 2.1. *Let (X,d) be a complete metric space and T be a mapping from X into itself. We assume that the following hypothesis holds:*
given $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that

$$2\varepsilon \leq d(y,Ty)\,\frac{1+d(x,Tx)}{1+d(x,y)} + d(x,y) < 2\varepsilon + \delta(\varepsilon) \Rightarrow d(Tx,Ty) < \varepsilon. \tag{2.1}$$

Then T has a unique fixed point $\xi \in X$. Moreover, for any $x \in X$, the sequence $\{T^n x\}$ converges to ξ.

Proof. We first observe that (2.1) trivially implies that T satisfies:

$$x \neq y \text{ or } y \neq Ty \quad \text{implies} \quad d(Tx,Ty) < \frac{1}{2}d(y,Ty)\,\frac{1+d(x,Tx)}{1+d(x,y)} + \frac{1}{2}d(x,y). \tag{2.2}$$

Now, let $x \in X$ and consider the sequence $\{x_n\} = \{T^n x\}$. We will show that $\{x_n\}$ is a Cauchy sequence in X.

If there exists $p \in \mathbb{N}$ such that $x_p = x_{p+1}$, then x_p is a fixed point of T. For this reason, we will assume that $x_p \neq x_{p+1}$ for all $p \in \mathbb{N}$. Let

$$c_n = d(x_n, x_{n+1}),\ \forall\, n \in \mathbb{N}.$$

From (2.2), we have:

$$c_n = d(Tx_{n-1},Tx_n) < \frac{1}{2}d(x_n,x_{n+1})\,\frac{1+d(x_{n-1},x_n)}{1+d(x_{n-1},x_n)} + \frac{1}{2}d(x_{n-1},x_n) = \frac{1}{2}c_n + \frac{1}{2}c_{n-1}.$$

Then

$$c_n < c_{n-1},\ \forall\, n \in \mathbb{N}^*$$

and the sequence $\{c_n\}$ is decreasing with n. Suppose now that $c_n \downarrow \varepsilon > 0$ as $n \to +\infty$. Then $c_n + c_{n-1} \downarrow 2\varepsilon$ as $n \to +\infty$. This implies that there exists $N \in \mathbb{N}^*$ such that

$$2\varepsilon \leq c_N + c_{N-1} < 2\varepsilon + \delta(\varepsilon).$$

We get:

$$2\varepsilon \leq d(x_N,Tx_N)\,\frac{1+d(x_{N-1},Tx_{N-1})}{1+d(x_{N-1},x_N)} + d(x_{N-1},x_N) < 2\varepsilon + \delta(\varepsilon).$$

From (2.1), we obtain:

$$d(Tx_{N-1},Tx_N) = d(x_N,x_{N+1}) = c_N < \varepsilon,$$

that is a contradiction. Then we deduce that

$$c_n \downarrow 0 \text{ as } n \to +\infty. \tag{2.3}$$

Let $\varepsilon > 0$. Condition (2.1) will remain true with $\delta(\varepsilon)$ replaced by $\delta'(\varepsilon) = \min(\delta(\varepsilon), \varepsilon, 1)$. From (2.3), there exists $k \in \mathbb{N}$ such that

$$d(x_m, x_{m+1}) < \frac{\delta'(\varepsilon)}{4}, \ \forall\, m \geq k. \tag{2.4}$$

Now, we introduce the set $\Lambda \subset X$ defined by

$$\Lambda := \left\{ x_p \,|\, p \geq k,\ d(x_p, x_k) < 2\varepsilon + \frac{\delta'(\varepsilon)}{2} \right\}.$$

Let us prove that

$$T(\Lambda) \subset \Lambda. \tag{2.5}$$

Let $\lambda \in \Lambda$. There exists $p \geq k$ such that $\lambda = x_p$ and $d(x_p, x_k) < 2\varepsilon + \dfrac{\delta'(\varepsilon)}{2}$.
If $p = k$, we have $T(\lambda) = x_{k+1} \in \Lambda$ (by (2.4)). Then we will assume that $p > k$. We distinguish two cases.
• First case:

$$2\varepsilon \leq d(x_p, x_k) < 2\varepsilon + \frac{\delta'(\varepsilon)}{2}. \tag{2.6}$$

First, let us prove that

$$\varepsilon \leq \frac{1}{2} d(x_k, x_{k+1}) \frac{1 + d(x_p, x_{p+1})}{1 + d(x_p, x_k)} + \frac{1}{2} d(x_p, x_k) < \varepsilon + \frac{\delta'(\varepsilon)}{2}. \tag{2.7}$$

From (2.6), we have:

$$\varepsilon \leq \frac{1}{2} d(x_p, x_k) \leq \frac{1}{2} d(x_k, x_{k+1}) \frac{1 + d(x_p, x_{p+1})}{1 + d(x_p, x_k)} + \frac{1}{2} d(x_p, x_k). \tag{2.8}$$

On the other hand, we have:

$$\begin{aligned}
\frac{1}{2} d(x_k, x_{k+1}) \frac{1 + d(x_p, x_{p+1})}{1 + d(x_p, x_k)} + \frac{1}{2} d(x_p, x_k) \quad &\leq \quad \frac{1}{2} d(x_k, x_{k+1}) + \frac{1}{2} d(x_k, x_{k+1}) \frac{d(x_p, x_{p+1})}{d(x_p, x_k)} + \frac{1}{2} d(x_p, x_k) \\
\text{by (2.4)} \quad &< \quad \frac{\delta'(\varepsilon)}{8} + \frac{1}{2} \frac{d(x_k, x_{k+1})}{d(x_p, x_k)} d(x_p, x_{p+1}) + \frac{1}{2} d(x_p, x_k) \\
\text{by (2.4) and (2.6)} \quad &< \quad \frac{\delta'(\varepsilon)}{8} + \frac{1}{2} d(x_p, x_{p+1}) + \frac{1}{2} d(x_p, x_k) \\
\text{by (2.4)} \quad &< \quad \frac{\delta'(\varepsilon)}{8} + \frac{\delta'(\varepsilon)}{8} + \frac{1}{2} d(x_p, x_k) \\
\text{by (2.6)} \quad &< \quad \frac{\delta'(\varepsilon)}{4} + \frac{1}{2} \left(2\varepsilon + \frac{\delta'(\varepsilon)}{2} \right) \\
&= \quad \varepsilon + \frac{\delta'(\varepsilon)}{2}.
\end{aligned}$$

Then

$$\frac{1}{2} d(x_k, x_{k+1}) \frac{1 + d(x_p, x_{p+1})}{1 + d(x_p, x_k)} + \frac{1}{2} d(x_p, x_k) < \varepsilon + \frac{\delta'(\varepsilon)}{2}. \tag{2.9}$$

It follows from (2.8)-(2.9) that (2.7) holds. Then

$$2\varepsilon \leq d(x_k, Tx_k) \frac{1 + d(x_p, Tx_p)}{1 + d(x_p, x_k)} + d(x_p, x_k) < 2\varepsilon + \delta'(\varepsilon),$$

which implies by (2.1) that

$$d(Tx_p, Tx_k) < \varepsilon. \tag{2.10}$$

Now, we have:

$$\begin{aligned} d(Tx_p, x_k) &\leq d(Tx_p, Tx_k) + d(Tx_k, x_k) \\ \text{by (2.10) and (2.4)} \quad &< \varepsilon + \frac{\delta'(\varepsilon)}{4} \\ &< 2\varepsilon + \frac{\delta'(\varepsilon)}{2}. \end{aligned}$$

This implies that $T\lambda = Tx_p = x_{p+1} \in \Lambda$.

• Second case:

$$d(x_p, x_k) < 2\varepsilon. \tag{2.11}$$

From (2.2), we have:

$$\begin{aligned} d(Tx_p, x_k) &\leq d(Tx_p, Tx_k) + d(Tx_k, x_k) \\ &< \frac{1}{2}d(x_k, x_{k+1})\,\frac{1 + d(x_p, x_{p+1})}{1 + d(x_p, x_k)} + \frac{1}{2}d(x_p, x_k) + d(x_{k+1}, x_k) \\ &\leq \frac{1}{2}d(x_k, x_{k+1}) + \frac{1}{2}\,\frac{d(x_k, x_{k+1})d(x_p, x_{p+1})}{1 + d(x_p, x_k)} + \frac{1}{2}d(x_p, x_k) + d(x_{k+1}, x_k) \\ &= \frac{3}{2}d(x_k, x_{k+1}) + \frac{1}{2}\,\frac{d(x_k, x_{k+1})d(x_p, x_{p+1})}{1 + d(x_p, x_k)} + \frac{1}{2}d(x_p, x_k). \end{aligned}$$

On the other hand, from (2.4), we have:

$$\frac{d(x_k, x_{k+1})}{1 + d(x_p, x_k)} \leq d(x_k, x_{k+1}) < \frac{\delta'(\varepsilon)}{4} < 1.$$

Then

$$\begin{aligned} d(Tx_p, x_k) &< \frac{3}{2}d(x_k, x_{k+1}) + \frac{1}{2}d(x_p, x_{p+1}) + \frac{1}{2}d(x_p, x_k) \\ \text{by (2.4) and (2.11)} \quad &< \frac{3\delta'(\varepsilon)}{8} + \frac{\delta'(\varepsilon)}{8} + \varepsilon \\ &= \frac{\delta'(\varepsilon)}{2} + \varepsilon \\ &< \frac{\delta'(\varepsilon)}{2} + 2\varepsilon. \end{aligned}$$

This implies that $T\lambda = Tx_p = x_{p+1} \in \Lambda$. Hence, (2.5) holds and

$$d(x_m, x_k) < 2\varepsilon + \frac{\delta'(\varepsilon)}{2},\ \forall\, m > k. \tag{2.12}$$

Now, for all $(m, n) \in \mathbb{N}^2$ such that $m > n > k$, by (2.12), we get:

$$d(x_m, x_n) \leq d(x_m, x_k) + d(x_n, x_k) < 4\varepsilon + \delta'(\varepsilon) < 5\varepsilon.$$

This implies that $\{x_n\}$ is a Cauchy sequence in X.

Since (X, d) is complete, there exists $\xi \in X$ such that $\{x_n\}$ converges to ξ. From (2.2), we have:

$$\begin{aligned} d(T\xi, \xi) &\leq d(T\xi, Tx_n) + d(x_{n+1}, \xi) \\ &< \frac{1}{2}d(x_n, x_{n+1})\,\frac{1 + d(\xi, T\xi)}{1 + d(\xi, x_n)} + \frac{1}{2}d(\xi, x_n) + d(x_{n+1}, \xi). \end{aligned}$$

Now, let $n \to +\infty$, we get:

$$d(T\xi, \xi) \leq 0,$$

which implies that $\xi = T\xi$, i.e, ξ is a fixed point of T.

Suppose now that η is another fixed point of T. From (2.2), we get:

$$d(\xi, \eta) = d(T\xi, T\eta) < \frac{1}{2} d(\eta, \eta) \frac{1 + d(\xi, \xi)}{1 + d(\xi, \eta)} + \frac{1}{2} d(\xi, \eta) = \frac{1}{2} d(\xi, \eta),$$

which is a contradiction. Then the uniqueness of the fixed point is proved. This makes end to the proof. □

Now, we will show that the result of Dass and Gupta [11] (when $\alpha, \beta \in (0, 1/2)$) is a particular case of Theorem 2.1.

Corollary 2.2. *(Dass-Gupta [11])*
Let (X, d) be a complete metric space and T be a mapping from X into itself. We assume that the mapping T satisfies:
for all $x, y \in X$,

$$d(Tx, Ty) \leq k \left(d(y, Ty) \frac{1 + d(x, Tx)}{1 + d(x, y)} + d(x, y) \right), \tag{2.13}$$

where $k \in (0, 1/2)$ is a constant. Then T has a unique fixed point $\xi \in X$. Moreover, for any $x \in X$, the sequence $\{T^n x\}$ converges to ξ.

Proof. Fix $\varepsilon > 0$. We take :

$$\delta(\varepsilon) = \varepsilon \left(\frac{1}{k} - 2 \right).$$

Assume that

$$2\varepsilon \leq d(y, Ty) \frac{1 + d(x, Tx)}{1 + d(x, y)} + d(x, y) < 2\varepsilon + \delta(\varepsilon).$$

From (2.13), we have:

$$\begin{aligned} d(Tx, Ty) &\leq k \left(d(y, Ty) \frac{1 + d(x, Tx)}{1 + d(x, y)} + d(x, y) \right) \\ &< k(2\varepsilon + \delta(\varepsilon)) \\ &= 2\varepsilon k + k\varepsilon \left(\frac{1}{k} - 2 \right) \\ &= \varepsilon. \end{aligned}$$

Then condition (2.1) of Theorem 2.1 is satisfied. This makes end to the proof. □

3. Applications to contractions of integral type

In recent years, Branciari [7] initiated a study of contractive condition of integral type, giving an integral version of the Banach contraction principle, that could be extended to more general contractive conditions. More precisely, he established the following result.

Theorem 3.1. *(Branciari [7])*
Let (X, d) be a complete metric space, $k \in (0, 1)$, and let T be a mapping from X into itself such that for each $x, y \in X$,

$$\int_0^{d(Tx, Ty)} \varphi(t)\ dt \leq k \int_0^{d(x, y)} \varphi(t)\ dt, \tag{3.1}$$

where φ is a locally integrable function from $[0,+\infty)$ into itself and such that for all $\varepsilon > 0$,

$$\int_0^{\varepsilon} \varphi(t)\ dt > 0.$$

Then T admits a unique fixed point $\xi \in X$ such that for each $x \in X$, the sequence $\{T^n x\}$ converges to ξ.

Putting $\varphi \equiv 1$ in the previous theorem , we retrieve the Banach fixed point theorem.

Later on, the authors in [2, 12, 23, 27, 30] established fixed point theorems involving more general contractive conditions.

Suzuki [28] showed that Meir-Keeler contractions of integral type are still Meir-Keeler contractions and so proved that Theorem 3.1 of Branciari is a particular case of the Meir-Keeler fixed point theorem [20]. In this section, following the idea of Suzuki [28], we will show that Theorem 2.1 allows us to obtain an integral version of Corollary 2.2.

We start by proving the following result.

Theorem 3.2. *Let (X,d) be a metric space and let T be a mapping from X into itself. Assume that there exists a function θ from $[0,+\infty)$ into itself satisfying the following:*

(i) $\theta(0) = 0$ and $\theta(t) > 0$ for every $t > 0$.

(ii) θ is nondecreasing and right continuous.

(iii) For every $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that

$$2\varepsilon \le \theta\left(d(y,Ty)\,\frac{1+d(x,Tx)}{1+d(x,y)} + d(x,y)\right) < 2\varepsilon + \delta(\varepsilon) \Rightarrow \theta(2d(Tx,Ty)) < 2\varepsilon$$

for all $x, y \in X$.

Then (2.1) is satisfied.

Proof. Fix $\varepsilon > 0$. Since $\theta(2\varepsilon) > 0$, by (iii), there exists $\alpha > 0$ such that

$$\theta(2\varepsilon) \le \theta\left(d(y,Ty)\,\frac{1+d(x,Tx)}{1+d(x,y)} + d(x,y)\right) < \theta(2\varepsilon) + \alpha \Rightarrow \theta(2d(Tx,Ty)) < \theta(2\varepsilon). \tag{3.2}$$

From the right continuity of θ, there exists $\delta > 0$ such that $\theta(2\varepsilon + \delta) < \theta(2\varepsilon) + \alpha$. Fix $x, y \in X$ such that

$$2\varepsilon \le d(y,Ty)\,\frac{1+d(x,Tx)}{1+d(x,y)} + d(x,y) < 2\varepsilon + \delta.$$

Since θ is nondecreasing, we get:

$$\theta(2\varepsilon) \le \theta\left(d(y,Ty)\,\frac{1+d(x,Tx)}{1+d(x,y)} + d(x,y)\right) \le \theta(2\varepsilon + \delta) < \theta(2\varepsilon) + \alpha.$$

Then, by (3.2), we have:

$$\theta(2d(Tx,Ty)) < \theta(2\varepsilon),$$

which implies that $d(Tx,Ty) < \varepsilon$. Then (2.1) is satisfied. This completes the proof. □

Since a function $t \mapsto \int_0^t \varphi(s)\ ds$ is absolutely continuous, we obtain the following.

Corollary 3.3. *Let (X,d) be a metric space and let T be a mapping from X into itself. Let φ be a locally integrable function from $[0,+\infty)$ into itself such that $\int_0^t \varphi(s)\ ds > 0$ for all $t > 0$. Assume that for each $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that*

$$2\varepsilon \le \int_0^{d(y,Ty)\,\frac{1+d(x,Tx)}{1+d(x,y)} + d(x,y)} \varphi(t)\ dt < 2\varepsilon + \delta(\varepsilon) \Rightarrow \int_0^{2d(Tx,Ty)} \varphi(t)\ dt < 2\varepsilon. \tag{3.3}$$

Then (2.1) is satisfied.

Now, we are able to obtain an integral version of Corollary 2.2. We have the following result.

Corollary 3.4. *Let (X,d) be a complete metric space and let T be a mapping from X into itself. Let φ be a locally integrable function from $[0,+\infty)$ into itself such that $\int_0^t \varphi(s)\,ds > 0$ for all $t > 0$. We assume that the mapping T satisfies the following condition:*
for all $x,y \in X$,

$$\int_0^{2d(Tx,Ty)} \varphi(t)\,dt \le c\int_0^{d(y,Ty)\frac{1+d(x,Tx)}{1+d(x,y)}+d(x,y)} \varphi(t)\,dt, \tag{3.4}$$

where $c \in (0,1)$ is a constant. Then T has a unique fixed point $\xi \in X$. Moreover, for any $x \in X$, the sequence $\{T^n x\}$ converges to ξ.

Proof. Fix $\varepsilon > 0$. It is easily to check that (3.3) is satisfied with $\delta(\varepsilon) = 2\varepsilon\left(\frac{1}{c} - 1\right)$. Then (2.1) is satisfied and we can apply Theorem 2.1. □

Remark 3.5. Note that the result of Corollary 2.2 can be obtained from Corollary 3.4 by putting $\varphi \equiv 1$ and $c = 2k$, $k \in (0,1/2)$.

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