Essays on the Theory of Production

An Algorithmic and Empirical Approach to Classical Economics

Fredholm, Thomas

Publication date:
2010

Document Version
Accepted author manuscript, peer reviewed version

ESSAYS ON THE THEORY OF PRODUCTION

AN ALGORITHMIC AND EMPIRICAL APPROACH TO CLASSICAL ECONOMICS

BY
THOMAS FREDHOLM

ESSAYS ON THE THEORY OF PRODUCTION

An algorithmic and empirical approach to classical economics

PhD Thesis submitted to the Department of Economics, Politics and Public Administration, University of Aalborg, September 2009

by

Thomas Fredholm
Department of Economics, Politics
and Public Administration
University of Aalborg
Fibigerstræde 3
9220 Aalborg Ø, Denmark
thomasfredholm@gmail.com

Principal supervisor:
Stefano Zambelli
Professor of Economics
Department of Economics - CIFREM
University of Trento
Via Inama, 5
38100 Trento, Italy

Project supervisor:
Charlotte Bruun
Associate Professor
Department of Economics, Politics
and Public Administration
University of Aalborg
Fibigerstræde 3
9220 Aalborg Ø, Denmark

Preface

This is a work of a specialist character, addressed to those interested in practical applications of classical economics. Its approach builds on the theoretical framework found in Sraffa's 'Production of Commodities by Means of Commodities – Prelude to a Critique of Economic Theory'. Sraffa's work is however not used as a 'prelude to a critique', but provides for this thesis a consistent theoretical foundation for a range of constructive empirical applications. The main objective is however not *per se* the empirical results, but more the procedures by which they can be obtained.

I am grateful for the teaching, guidance, encouragement, and inspiration provided by Stefano Zambelli during both my years as an undergraduate and the gestation of this thesis. I thank Stefano, not only for teaching me mathematics, programming, and Sraffian economics, but also for awakening an abiding interest in computational economics and the mathematical foundations of economic theory. In short, Stefano has directly or indirectly contributed at each vital step leading to the realization of this thesis. Also Charlotte Bruun, Carsten Heyn-Johnsen, and K. Vela Velupillai deserve special gratitude for their perpetual support and encouragement shown to me in an always friendly and candid fashion. Matteo Degasperi, friend and co-author, also deserves special thanks for the huge effort he put in our joint paper. Finally, I would like to thank the Department of Economics, Politics and Public Administration at Aalborg University together with CIFREM at the University of Trento for providing good environments to write a thesis and to SAMF-IT for their generous support, help, and access to computer power.

It goes without saying, that I am solely responsible for the content, and errors remain my own.

Thomas Fredholm
Aalborg, September 14, 2009

Contents

List of Tables

List of Figures

Introduction

This PhD Thesis consists of four interrelated chapters on the theory of production. The first three chapters are collected in Part I, entitled 'Empirical Studies on Technological Progress, Structural Change, and Convergence', which is considered the core of the project. Common for these three chapters is the application of Sraffian economics to real input–output tables in a quest to provide indices for economic phenomena such as technological progress, structural change, and convergence; and procedures applicable for economic policy making.

The orthodox approach to study these problems often include the use of (aggregate) production functions and is for that reason subject to a wide range of innate problems. Some of these issues are introduced and analysed in Part II entitled 'A Small Contribution to the Critique of the Neoclassical Theory of Growth and Distribution'. The approach set forth in Chapter 1 to 3 considers production as a circular process where all inputs, except labour, are produced within the system. As oppose to a process beginning with *factors of production* and ending with *consumption goods*. Consequently, the evolution of the economic system is studied focusing on changes in the entire inter-industrial flow of commodities. Another important feature of this approach is an intertwined relationship between the theory of value and technological progress.

Each chapter is self-contained, i.e., can be read independently, but there is a logical progression from Chapter 1 to 3, both in the problems addressed and the complexity of the analysis. Notation occasionally differs among chapters, since a universal style of notation would be exceedingly cumbersome.

As regards the subtitle 'An algorithmic and empirical approach to classical economics', the term *algorithmic* requires an explanation. Algorithmic is a constructive (not necessarily in the sense of *constructive mathematics*) way of stating and dealing with economic problems. All mathematically formulated economic problems confronted have been solved by providing algorithmic procedures on how to actually compute a solution. As oppose to 'simply' showing that a solution exists. In my opinion, an algorithmic formalized economic theory provides a more direct transition from theory to empirical applications, and *vice versa*.[1] A direct consequence of the al-

[1]The inspiration of this approach is found in Velupillai (2000): 'Computable Eco-

gorithmic approach is that all theoretical statements implicitly or explicitly are restricted to the set of rational numbers.

Another advantage of an algorithmic approach, is that it so to speak unbinds economic reasoning from *classical mathematical*. In others words, it provides the ability of bending the mathematics towards the economic problem, as oppose to much of the mainstream economics which seems to be constructed such as to fit the well-known theorems of classical mathematics (by assuming compact sets, twice differentiable functions etc.).

Furthermore, attention should be drawn to the distinction between *computability* and *complexity*. Computability implies that there exists an algorithmic procedure (equivalent a Turing Machine) that in a finite number of steps computes a given problem. We will encounter at least one problem that is computable, but the (time) complexity of the problem is such that the finite number of steps, needed to compute the solution, exceeds what even today's supercomputers are able to solve within any reasonable time frame.

The thesis can, however, be read without prior knowledge of computability and complexity theory.[2] To tell the truth, the author has only scratched the surface of this fascinating world of logic. Consequently, this is not a formal exercise in the theory of computation or *constructive* mathematical reasoning. It is alone an attempt to combine empirical and computational methods to classical economics. Hence, the more loose term *algorithmic*.

In Chapter 1, entitled 'Productivity Accounting Based on Production Prices',[3] an alternative method of productivity accounting is proposed. By using input–output tables from four major OECD countries between 1970 and 2005, we compute the associated wage-profit frontiers and the net national products, and from these derive two measures of productivity growth based on production prices and a chosen *numéraire*. Our findings support the general conclusions in the existing literature on the productivity slowdown and later rebound, and supply new important insights to the extent and timing of these events.

Chapter 2 is entitled 'Measuring Structural and Technological Change from Technically Autarkic Subsystems – A Study of Danish Industries, 1966–2005'.[4] The main objectives of which are to study procedures to decompose

---

nomics', Oxford University Press, Oxford. And in particular, Velupillai (2008): 'Sraffa's Mathematical Economics - A Constructive Interpretation', Journal of Economic Methodology, 15, pp. 325–342.

[2]For an introduction, see Sipser (2005): 'Introduction to the Theory of Computation', Course Technology, Boston (MA).

[3]This chapter is a slightly updated version of a paper forthcoming in Metroeconomica, 2010, written jointly with Matteo Degasperi from the Interdepartmental Centre for Research Training in Economics and Management (CIFREM), University of Trento; via Rosmini 70, 38100 Trento, Italy; E-mail: matteo.degasperi_1@amm.unitn.it.

[4]This paper, that is now Chapter 2, is written jointly with Stefano Zambelli. A pre-

the aggregate technological progress down to the industry level, explicitly taking into account both the direct effects from within the industry as well as the indirect effects from the supporting industries, and to apply these procedures to Danish industry-level data. Among the results are evidence of a strong convergence in the industry-level productivity and the identification of, for economic policy making, important structural characteristics of the Danish economy. This includes a subsystem based CO_2 accounting that identifies the origin of the specific demand that eventually led to the production that caused the emission, e.g., it is shown that the public sector is responsible for four times the CO_2 emission reported in the official (direct) statistics.

Chapter 3 is entitled 'The Technological Frontier – An International and Inter-industrial Empirical Investigation of Efficiency, Technological Change, and Convergence'.[5] The approach taken in Chapter 3, to capture the international state of technological progress, goes through the so-called *technological frontier*. The technological frontier shows, for a given distribution of the net national product, the combination of production activities that would yield the highest wage income; equivalently the cost minimising choice of techniques. Different versions of the technological frontier are computed for a selection of OECD countries using input–output data 1970–2005. From these frontiers a set of indices is extracted to provide global as well as country-specific condensed measures of technological progress. Among the results are evidence of a global development that shows a more moderate rate of productivity growth compared with the conventional stylized facts of a 1.5–2.0 percent year-to-year increase and evidence of an economic development in the US driven at least as much by an increased work effort as by an increased productivity. The technological frontiers are intrinsically difficult to compute, but by applying two theoretical properties associated with the switch points between techniques of production, an algorithm is developed and invoked to efficiently compute the frontiers.

The empirical results associated with Chapter 2 and 3 far exceed what is convenient to present within the two chapters. Therefore, a large amount of the empirical results are collected in a statistical companion located in Part III.

Part II consists of only one chapter entitled 'Production Functions Behaving Badly – Reconsidering Fisher and Shaikh'. This chapter reconsiders Anwar Shaikh's critique of the neoclassical theory of growth and distribution based on its use of aggregate production functions. This is done by reconstructing and extending Franklin M. Fisher's 1971 computer simulations, which Shaikh used to support his critique. Together with other recent

liminary version of the paper was presented at the Second Schumpeter Summer School in Graz 2009.

[5]This Chapter is also written jointly with Stefano Zambelli.

extensions to Shaikh's seminal work, the results support and strengthen the evidence against the use of aggregate production functions.

Where Part I is considered a constructive prelude to empirical applications of Sraffian economics, Part II deals with a well-known internal critique of the neoclassical theory of growth and distribution. Max Planck said that "science advances one funeral at a time", unfortunately it is hard to observe this in Economics. When dealing with the existence of an aggregate production function, it is not the lack of sound arguments against the use of this (HUMBUG) device that is keeping it alive. Nevertheless, some *faith* based powers continue to postpone the fall of the aggregate production function.[6]

In my opinion, the way to proceed is therefore not to continue supplying good arguments against the orthodox parable, but to provide alternatives. In particular, unambiguous applicable empirical tools for the study of core economic phenomena. This is the main aim of this thesis.

[6] With an implicit reference to Ferguson (1969, p. 269): 'The Neoclassical Theory of Production and Distribution', Cambridge University Press.

Part I

Empirical Studies on Technological Progress, Structural Change, and Convergence

Chapter 1

Productivity Accounting Based on Production Prices

Written jointly with Matteo Degasperi

1.1 Introduction

The main aim of this chapter is to introduce an alternative method of productivity analysis using input–output tables and production prices, and to use this method to study productivity growth in four major OECD countries from 1970 to 2005.

This method has several appealing properties, the most important of which is its ability to take into account — for the economy as a whole — the interdependent relationships among industries as a consequence of technological innovations in the single industries.

The method adopted is based on the scheme of production developed by von Neumann (1945–46), Leontief (1941), and Sraffa (1960), while the algorithms employed were first proposed by Velupillai and Zambelli (1993) and Zambelli (2004).

By doing this we show how productivity accounting can be accomplished without utilising an aggregate production function, which suffers from several serious drawbacks (see Pasinetti, 2000; Cohen and Harcourt, 2003; Felipe and Fisher, 2003; Felipe and McCombie, 2007).

The chapter is structured as follows: Sections 1.2, 1.3, and 1.4 present the theory and the algorithms adopted for the productivity accounting. Sections 1.5 and 1.6 present and analyses the data, and Section 1.7 concludes the chapter.

1.2 The Theoretical Model

Following the tradition of von Neumann, Leontief, and Sraffa, production, growth, and distribution are described in terms of a multi-sector input–output system, where production is described as an interdependent circular process.[1]

The economic system consists of m industries producing n commodities by means of some combination of the n commodities and labour. Let $\boldsymbol{A}$ be a $m \times n$ quadratic non-singular matrix of interindustry inputs, where the ijth entry represents the ith industry's use of the jth commodity in the production of the ith commodity. Likewise, $\boldsymbol{L}$ is a $n \times 1$ vector of labour inputs and $\boldsymbol{B}$ is a $m \times n$ positive definite diagonal matrix of outputs, where the ith diagonal entry is the gross output of the ith industry. As usual these elements can be collected in the following long-run equilibrium relationship that captures the distribution of the total production among wages, profits, and means of production, where the wage and profit rates are assumed to

[1] This section and the next are inspired by Sraffa (1960), Pasinetti (1977), Zambelli (2004), and Velupillai and Zambelli (1993).

be uniform.[2]

$$\boldsymbol{Ap}(1+r)+\boldsymbol{L}w=\boldsymbol{Bp} \tag{1.2.1}$$

System (1.2.1) consists of n linear independent equations and $n+2$ variables, i.e., the system has initially two degrees of freedom. Choosing a *numéraire* $\boldsymbol{\eta}$, for which it holds that $\boldsymbol{\eta}'\boldsymbol{p}=1$, the degrees of freedom reduces to one.

Given the profit rate, it is straightforward to calculate the wage rate and the relative prices that solve system (1.2.1). Isolate $\boldsymbol{p}$, $\boldsymbol{p}=\big(\boldsymbol{B}-\boldsymbol{A}(1+r)\big)^{-1}\boldsymbol{L}w$, premultiply with the *numéraire*, and rearrange to obtain the wage-profit frontier function and the associated production prices, *viz.*

$$w=\Big(\boldsymbol{\eta}'\big(\boldsymbol{B}-\boldsymbol{A}(1+r)\big)^{-1}\boldsymbol{L}\Big)^{-1} \tag{1.2.2}$$

$$\boldsymbol{p}=\frac{\big(\boldsymbol{B}-\boldsymbol{A}(1+r)\big)^{-1}\boldsymbol{L}}{\boldsymbol{\eta}'\big(\boldsymbol{B}-\boldsymbol{A}(1+r)\big)^{-1}\boldsymbol{L}} \tag{1.2.3}$$

The production prices give a measure of the cost of production for the n commodities — in terms of a given *numéraire* and as a function of the rate of profit. Using these production prices, the value of the NNP is obtained by the following accounting identity, where $\boldsymbol{e}$ is a $n\times 1$ unit vector.

$$\text{NNP}=\boldsymbol{e}'(\boldsymbol{B}-\boldsymbol{A})\boldsymbol{p} \tag{1.2.4}$$

The following section provides an intuitive description of how this theoretical framework can be employed to study technological change.

1.3 Productivity Accounting

This section consists of three parts. The first two parts define and describe what we will call *labour productivity* and *technological progress* based on production prices. The third part describes the major differences between these two interrelated measures and emphasises the main strengths of this method as a whole.

As usual the NNP is the value added in the given accounting period, hence NNP divided by the total use of labour is a measure of labour productivity. Note here that the NNP is a function of the price vector, which again is a function of the rate of profit. As a distribution free measure of labour productivity, we propose to use the area under the NNP per unit of labour curves, i.e., integrate with respect to the rate of profit from zero to maximum rate of profit. Furthermore, to obtain an index, which is comparable across countries and over time, we divide this area with the maximum profit rate.

[2]The mathematical notation in this chapter is kept as parsimonious as possible, e.g., no indexes are used, but everything should be clear from the context.

Given the complex interdependent structure of the input–output system, changes in labour productivity are not only due to *ceteris paribus* changes in the quality of labour or innovations that make labour more productive in the single industries. It is also influenced by the effect of a change in the scale of production in the single industries and depends on how the relative prices changes with the profit rate and the relative sizes of the physical net products for the different sectors. A simple example will clarify this point.

Assume that there is an increase in the scale of production in a given sector, without this changing the applied technology. The value of the NNP per unit of labour will change and the change will vary with the profit rate depending on the relative labour intensity in the chosen industry and the industry's weight in the physical NNP. Consequently, if the scale of production is increased in a sector for which the relative price increases with the profit rate, then the difference between the *ex ante* and *ex post* NNP per unit of labour will increase with the rate of profit.

As a supplement to the above measure of labour productivity, we propose to use the area under the wage-profit frontiers as a measure of what we call *technological progress*. If a wage-profit frontier dominates another frontier and hence we have (production) prices allowing in principle the system to reproduce, we would have a higher wage rate measured in the terms of the same *numéraire* associated with the same profit rate. The main advantage of this measures of *technological progress* over our measure of *labour productivity*, is that it will not change as a consequence of simple changes in the scale of production in single industries, but only if real technological innovations are observed in one or more industries. By real technological innovations we mean change in the matrix of technological coefficients and/or in the corresponding (normalised) vector of labour inputs.[3]

One of the main strengths of productivity accounting based on production prices is that it takes into account the effects of technological change in the single industries for the economy as a whole. A way to see this, is to think of the production prices as weights in the process of aggregation (into for example the NNP) together with the fact that the production prices change with and only with real technological innovations. The fact that the weights/prices only change as a consequence of technological innovations is appealing, because it circumvents the traditional problem of delineating the effects from changes in market prices and that of real technological innovations.

It is important to note that technological changes in the single industries has an effect on all the relative prices (intuitive since this alters the relative scarcity of all commodities in the system), i.e., the total effect on

[3]The matrix of technical coefficients is a normalised form of $\boldsymbol{A}$, where the ij entry represents the ith industry's use of the jth commodity in the production of one unit of the ith commodity, see Appendix 1.B for further details.

our measures of productivity from technological change in a single industry is not simply the local effect multiplied by some *ex ante* given weight.

1.4 Algorithms and the Choice of *Numéraire*

Given the input–output tables from a given country for a given year and an appropriate *numéraire*, it is straightforward computations to calculate the wage rate, the production prices, and the NNP for any given profit rate using (1.2.2)–(1.2.4). After this point it is a simple programming task to compute areas and to collect and organise the results.

The critical step is to choose an appropriate *numéraire*, because all the subsequent results are influenced by this choice. How to construct or select the *numéraire* is a classical problem in economics, because the value of the *numéraire* should be invariant of other economic factors, such as the distribution between wage and profit. This problem, which was posed by Ricardo, was to some extent solved by Sraffa, since his Standard Commodity gives a distribution free measure of value given the set of techniques represented by matrix $\boldsymbol{A}$, $\boldsymbol{L}$, and $\boldsymbol{B}$.

However, since the purpose of our work is to be able to study technological progress over time and across countries, the standard commodity is no longer an invariant measure of value. We choose the vector of physical sectoral net products (total supply of the ith commodity minus the sum of the ith column in $\boldsymbol{A}$) in the US in 2000 calculated from a standard system with a zero profit rate and normalised with the hours worked. This is not a perfect *numéraire* — if such a thing exists — but in our opinion the interpretation of this *numéraire* is fairly intuitive and has a number of convenient properties, which will be clear in the following. Still, the consequences of the choice among many possible *numéraires* call for further research. In Appendix 1.C the robustness of our results has been assessed by repeating our analysis with different *numéraires*.

The standard system can be constructed from any viable system,[4] by reproportioning the system, such that the ratios between the final demand and the sum of intermediate goods are the same for all commodities. The multiplier $\boldsymbol{q}$ used to reproportion the system into a standard system is the (unique) non-trivial solution of the following homogeneous system of equations.

$$\big(\boldsymbol{B} - \boldsymbol{A}'(1+R)\big)\boldsymbol{q} = \boldsymbol{0} \tag{1.4.1}$$

[4] The system is said to be viable, if and only if $\lambda \leq 1$, such that the maximum rate of profit will be positive, for further details see Pasinetti (1977, p. 78) and Appendix 1.B.

hence the *numéraire* is given by:[5]

$$\boldsymbol{\eta}' = \frac{\boldsymbol{e}'\big((\boldsymbol{B}-\boldsymbol{A})\otimes\boldsymbol{q}\boldsymbol{e}'\big)}{\boldsymbol{L}'\boldsymbol{q}} \tag{1.4.2}$$

This has the appealing property to normalise the maximum wage rate in 2000 to one, i.e., the wage rate by which the workers can buy all the NNP in 2000 given a zero profit rate. Furthermore, the use of the standard system guarantees a strictly non-negative *numéraire*, which is not *a priori* given.[6]

1.5 Data

We use OECD input–output tables that belong to three different datasets for the US, the UK, Germany, and France. All containing matrices in current prices and domestic currency. The first covers roughly five year intervals from around 1970 to 1990 and follows the system of industrial classification 'ISIC Revision 2' (35 sectors) and the System of national accounts 'SNA 68'.[7]

The second dataset includes 42-by-42 sector matrices covering one year in the mid-1990s. The matrices follow the new system of industrial classification 'ISIC Revision 3'.

The third dataset has been recently published by the OECD in 2009. What is new with respect to the older editions is the high degree of comparability among countries, because the tables are constructed according to the standard industry list based on ISIC Revision 3. The 2009 edition consists of matrices for 28 member countries and 9 non member countries covering 1995 and 2005. Each matrix describes the inter-industrial relationships for 48 sectors that cover both the industrial part of the economy and services.[8]

The data have been adjusted in order to have matrices that can be adopted within a Sraffian model. In fact, in order to find an inverse matrix, the original matrix must be non-singular. That is, no linear combination of rows and columns and no zero rows and columns. Consequently, the original tables have been modified to satisfy these requirements. The aggregation cancels out the rows and columns with all zero values minimizing the loss of information due to the merge. As a consequence of the need to both aggregate some sectors in order to clear the null vectors and preserve comparability, the number of sectors is reduced to 23.[9] Each column of the

[5]See Appendix 1.B for details and a numerical example.

[6]The psychical net product can be negative, because imports enable the system to reproduce itself. This is not an uncommon observation in the actual OECD tables.

[7]See Appendix 1.A for details.

[8]For further information see: www.OECD.org and OECD (2001a, 2001b).

[9]A detailed description of the database and the method of harmonization used is found in Appendix 1.A.

table describes the nominal value of an industry's inputs and each row reports the nominal value of an industry's output used as means of production; therefore, we take the transpose matrix.

As previously said, the tables are in current prices and domestic currency. Although previous studies treat the nominal coefficients as physical (see for instance Petrovic, 1991 and Han and Schefold, 2006), we decide to follow an alternative procedure for two reasons. First, from that time on, the OECD website improves the availability of data, and second because, although experimental, an empirical work on productivity growth cannot treat nominal values as physical quantities. The best way is to use the respective deflator for each sector. Unfortunately, the OECD statistics on national accounts are highly aggregated and captures only six macro-sectors. Consequently, the ratio previously reported has been calculated for the six sectors available and it has been used on the corresponding micro branches. At the end, we have a set of tables that report the quantity of commodities used and produced with respect to the reference year 2000, the coefficients are expressed in constant Purchasing Power Parities, and the change in relative prices is preserved, although roughly, by using different PPPs.

Finally, the physical quantity of labour is given by the number of hours worked. In default of detailed information for the number of hours worked in each sector we decide to attribute in proportion to the compensation of employees. In any case, further improvements would be achievable when data on labour quality will be available.

1.6 Analysing the Data

The aim of this section is to describe the rate of change in productivity over time in the US, the UK, Germany, and France. However, it should be noted that the data are not perfectly commensurable across countries and over time. For instance, we dispose of eight input–output matrices for the US, seven for France and the UK, and only five for Germany. Furthermore, the years do not always coincide; for example, we have the 1984 table for the UK, the 1986 table for Germany, and the 1985 tables for France and the US. Nevertheless, in the comparative analysis we use fixed five year intervals from 1970–2005.

Figure 1.1 shows the wage-profit frontiers for the four countries in the period considered. The frontiers move outwards over time but with temporary country-specific fall downs, implying a non-smooth technological progress. In particular, the 1977 and 1982 frontiers for the US and to a less extent the 1984 and 2000 frontiers for the UK and the 2005 frontier for France move back towards the origin. Figure 1.1 also shows that, from the beginning of the period considered the US was the dominating economy in terms of technological progress, a position the US maintains, although the gap tightens

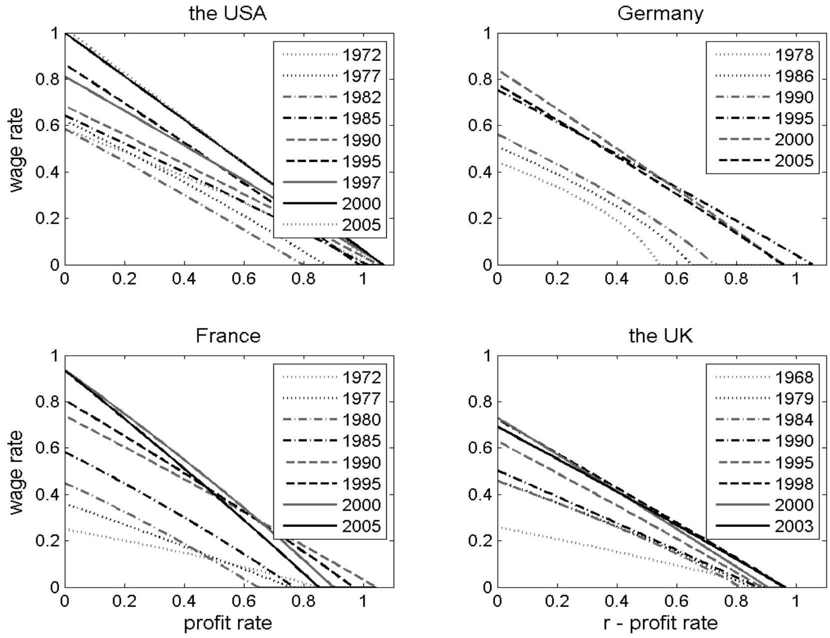

Fig. 1.1: *The wage-profit frontiers for the US, the UK, Germany, and France*

over time. Moreover, we see evidence of a convergence for all the examined countries towards a maximum profit rate close to unity.

As explained in Section 1.3, if one wage-profit frontier entirely dominates another the technological progress occurs in an unambiguous way. However, we found a few cases where the frontiers cross and thus it is not always possible, in an objective way, to clearly assess whether or not we have technological progress. If we were to consider this problem one needs to take

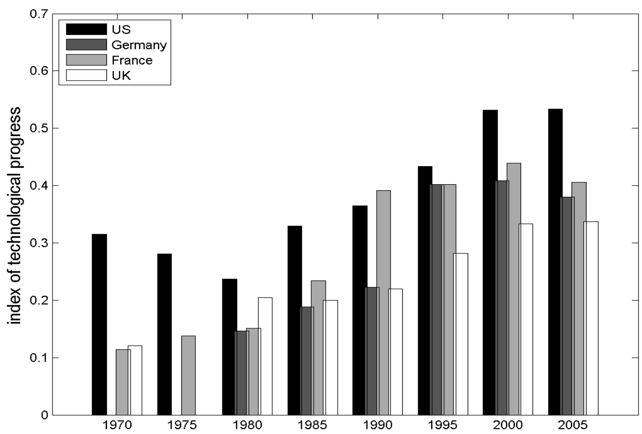

Fig. 1.2: *Technological progress*

into account the actual distribution of income. This could be a matter for further research.

Figure 1.2 reports the areas under the wage-profit frontiers. It can clearly be seen that a serious slowdown hit the US economy in the 1970s, and to some extent also the UK economy in the 1980s. Conversely, France and Germany were characterized by a more steady technological progress (catching up).

In looking for an explanation for this behaviour, one should consider the already established literature on the so-called *productivity slowdown* during the 1970s, cf. Nordhaus (2004). However, it is worth noting that our results show not only a slowdown, but a clear decline in the level of US productivity.

The 1990s is another decade that deserves special attention. During this period, commonly known as the *new technology era*, the UK and especially the US productivity grew faster than in the other two countries. As a consequence, the level of productivity in the US at the end of the 20th century was much higher than elsewhere.

To sum up, the technological development over the thirty years examined in the UK and in particular in the US exhibits a cyclical pattern. At the beginning of the 1970s the US were the leading economy, during the economic slowdown from the early 1970s to the mid-1980s, the level of technological progress in Germany and France converged slowly towards the US level and overcame the UK level (or rather the US 'convergence down'!). In the 1990s the US and the UK productivity growth was faster than in the EU's two biggest economies. As a result, the US became again in the 1990s the leading economy.

Thus, our findings support only partially the existing literature and the empirical evidences of the pattern of productivity growth. Notwithstanding, the results are similar to those reported by other studies, see for instance Nordhaus (2004), but the magnitudes are not the same. In particular the US productivity slowdown of the 1970s is more prominent in our case, because not only the rate of growth, but even the level of productivity declines. In addition the US and UK productivity boom begins in the early 1990s, five years before the OECD estimate.[10]

Figure 1.3 and 1.4 show the NNP curves and the areas under these curves, respectively. The histograms in Figure 1.2 and 1.4 sometimes differ in term of the magnitude of the change, but with few exceptions the countries' order of rank is the same, and thus supports the general story told above. In particular, here we do not observe so clear a decline in the levels of US productivity.

Before concluding this section, it is worth to mention that the interpretation of the wage-profit frontiers behaviour deserve further investigation. In particular, it is important to identify which sectors are mainly responsible for productivity changes in each period.

[10]See the OECD data on labour and multi-factor productivity, www.OECD.org

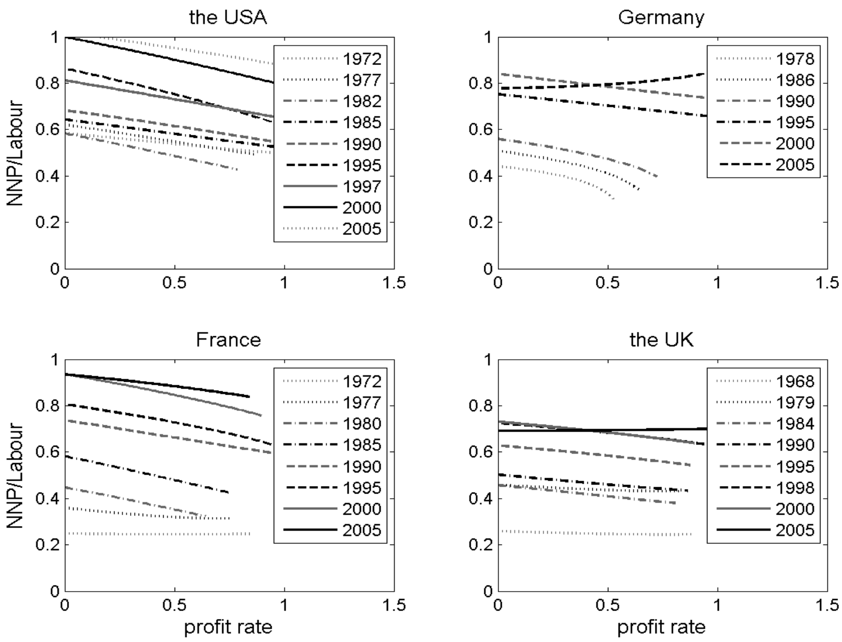

Fig. 1.3: *The NNP curves for the US, the UK, Germany, and France*

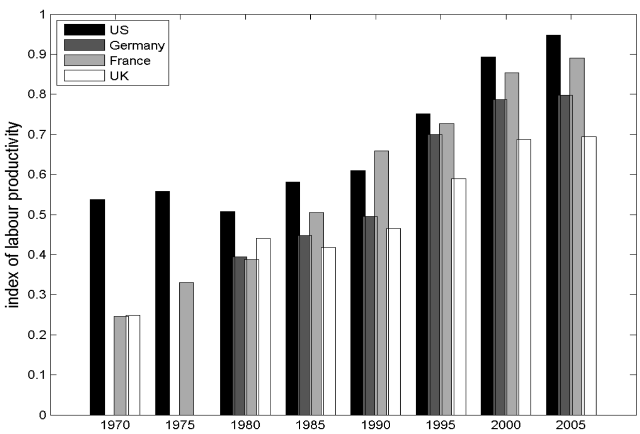

Fig. 1.4: *Labour productivity*

1.7 Conclusion

In this chapter we have described an alternative way of productivity accounting based on the work of von Neumann, Leontief, and Sraffa on production systems. We have proposed to use the areas under the net national product and wage-profit curves to construct indices of labour productivity and technological progress, respectively. Then, we have applied this method to the US, Germany, France, and the UK.

The main difference between our method and the orthodox way of measuring productivity consists in the use of industry-level input–output data and the associated production prices. We think the use of production prices in the process of aggregation has at least two appealing properties; production prices change only as a consequence of real technological innovations, and take into account the complex interdependencies among industries in the economic system as a whole.

We have found that the path of technological progress and the growth rates in labour productivity differ substantially between the US and the UK on the one hand and France and Germany on the other. In particular, the US and the UK show a decrease in productivity levels during the 1970s and the early 1980s while France and Germany exhibit more steady technological progress during the same period. Conversely, from 1990 to 2005 the rate of productivity growth was again higher for the US and the UK than for France and Germany. Our findings show both similarities and differences compared to the results based on the traditional ways of productivity accounting. For instance, the well-known literature on the US productivity slowdown identifies a decrease in the rate of growth in productivity, while our results show not only a slowdown, but a clear decline in the level of US productivity in the 1970s.

Acknowledgements

We are grateful for the inspiration and guidance provided by Stefano Zambelli; if it were not for him, we would never have initiated this project. Also K. Vela Velupillai and two anonymous referees deserve special gratitude for the support they provided in the gestation of this project. However, it goes without saying that we are solely responsible for the contents of this chapter.

References

Cohen A. J., Harcourt G. C. (2003): 'Retrospectives: whatever happened to the Cambridge capital theory controversies?', *Journal of Economic Perspectives*, 17, pp. 199–214.

Felipe J., Fisher F. M. (2003): 'Aggregation in production functions: what applied economists should know', *Metroeconomica*, 54, pp. 208–262.

Felipe J., McCombie J.S.L (2007): 'Is a theory of total factor productivity really needed?', *Metroeconomica*, 58, pp. 195–229.

Han Z., Schefold B. (2006): 'An empirical investigation of paradoxes: reswitching and reverse capital deepening in capital theory', *Cambridge Journal of Economics*, 30, pp. 737–765.

Leontief W. (1941): *The Structure of American Economy, 1919–1929*, Harvard University Press, Cambridge, MA.

Neumann J. v. (1945–46): 'A model of general economic equilibrium', *The Review of Economic Studies*, 13, pp. 1–9.

Nordhaus W. (2004): 'Retrospective on the 1970s productivity slowdown', *NBER Working Paper*, No. 10950.

OECD (2001a): *Measurement of Aggregate and Industry-level Productivity*, OECD Manual, www.OECD.org

OECD (2001b): *Measuring Capital*, OECD Manual, www.OECD.org

Pasinetti L. L. (1977): *Lectures on the Theory on Production*, Macmillian Press, London.

Pasinetti L. L. (2000): 'Critique of the neoclassical theory of growth and distribution', *BNL Quarterly Review*, no. 215, pp. 383–431.

Petrovic, P. (1991): 'Shape of a wage-profit curve, some methodology and empirical evidence', Metroeconomica, 42, pp. 93112.

Sraffa P. (1960): *Production of Commodities by Means of Commodities*, Cambridge University Press, Cambridge.

Velupillai K., Zambelli S. (1993): 'The international comparisons programme and production based indices of Economic Performances', *mimeo*, World Bank, Washington.

Zambelli S. (2004): 'The 40% neoclassical aggregate theory of production', *Cambridge Journal of Economics*, 28, pp. 99–120.

Appendices

1.A The Datasets

This appendix describes the national input-output tables used and the procedure adopted for making these tables suitable for the computation of wage-profit frontiers and NNP curves. This procedure has two stages: aggregation and statistical error distribution.

The input–output tables are made available by the OECD. They refer to three different time periods and are inconsistent with respect to the number of sectors and the order in which sectors are listed. Therefore, some sectors have been merged and re-ordered in order to harmonize the data.

The first set of tables refers to the period 1970–1990 (ISIC rev.2). The tables are available for the following years:

- US: 1972, 1977, 1982, 1985, 1990
- Germany: 1978, 1986, 1990
- UK: 1968, 1979, 1984, 1990
- France: 1972, 1977, 1980, 1985, 1990

The 1970 table for Germany, although available, is not used because it gives unreliable results. The following list describes in detail which sectors were combined:

- Chemicals and Pharmaceuticals
- Iron, Steel, and Non-Ferrous Metals
- Electrical machinery and apparatus nec; Radio, Television, and Communication Equipment; Office and Computing Machinery; Professional Goods
- Shipbuilding and Repairing; Other Transport; Motor Vehicles; Aircraft
- Restaurant and Hotels; Transport and Storage
- Finance and Insurance; Real Estate and Business Services
- Community, Social and Personal Services; Producers of Government Services; Other Producers

As a result, the original 35-by-35 sector tables have been reduced to 23-by-23 sector.

The second set of tables (ISIC rev.3) is smaller and refers only to one year: 1997 or 1998. Unfortunately, this dataset does not include data for France and Germany. The original 41-by-41 sector tables have been reduced to 23-by-23 sector and these sectors coincide with those in the set of tables from 1970 to 1990. The following sectors were combined:

- Chemicals and Pharmaceuticals
- Iron, Steel and Non-Ferrous Metals
- Office Accounting and Computing Machinery, Electrical Machinery and Apparatus nec; Radio, Television and Communication Equipment; Medical Precision and Optical Instruments
- Motor Vehicles, Trailers and Semi-trailers; Building and Repairing of Ship and Boats; Aircraft and Spacecraft; Railroad Equipment and Transport Equipment nec

- Hotels and Restaurant; Transport and Storage
- Financial, Insurance; Real Estate Activities; Renting of Machinery and Equipment; Computer and Related Activities; Research and Development; Other Business Activities
- Public Administration, Defence, Compulsory and Social Security; Education; Health and Social Work; Other Community Social and Personal Services; Private Household with Employed Persons

Finally, the third set of tables has been made accessible by the OECD in 2009. It is the most recent available and refers to two years: 1995 and 2005. The original 48-by-48 sector tables (ISIC rev.3) have been reduced to 23-by-23 sector and these sectors again coincide with those in the set of tables from 1970 to 1990. Accordingly, the following sectors were combined:

- Mining and quarrying (energy); Mining and quarrying (non-energy)
- Chemicals excluding pharmaceuticals; Pharmaceuticals
- Iron and steel; Non-ferrous metals
- Office, accounting and computing machinery; Electrical machinery and apparatus, nec; Radio, television and communication equipment; Medical, precision and optical instruments
- Motor vehicles, trailers and semi-trailers; Building and repairing of ships and boats; Aircraft and spacecraft; Railroad equipment and transport equip nec
- Production, collection and distribution of electricity; Manufacture of gas; distribution of gaseous fuels through mains; Steam and hot water supply; Collection, purification and distribution of water
- Hotels and restaurants; Land transport; transport via pipelines; Water transport; Air transport; Supporting and auxiliary transport activities; activities of travel agencies
- Finance and insurance; Real estate activities; Renting of machinery and equipment; Computer and related activities; Research and development; Other Business Activities
- Public admin. and defence; compulsory social security; Education; Health and social work; Other community, social and personal services; Private households with employed persons and extra-territorial organisations and bodies

In many cases the tables have a residual sector that is the statistical error and/or the non-comparable import. The values included in the residual sector are distributed in proportion to the ratio between the sum of values of intermediate inputs in that sector and the total value of intermediate goods for the entire economy.

1.B A Note on the *Numéraire*

The following is a numerical example of how the *numéraire* is constructed.

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 2 & 4 \end{pmatrix} \quad \boldsymbol{B} = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 18 \end{pmatrix} \quad \boldsymbol{L} = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}$$

To calculate the maximum rate of profit, we need the matrix of technical coefficients $\boldsymbol{A^*}$, which is a normalised form of $\boldsymbol{A}$, where the ijth entry represents the ith industry's use of the jth commodity in the production of one unit of the ith commodity, *viz.*

$$\boldsymbol{A^*} = \frac{\boldsymbol{A}}{diag(\boldsymbol{B})e'} = \begin{pmatrix} 0.200 & 0.100 & 0.300 \\ 0.083 & 0.167 & 0.083 \\ 0.167 & 0.111 & 0.222 \end{pmatrix} \tag{1.B.1}$$

From this it is straightforward to calculate the maximum eigenvalue of $\boldsymbol{A^*}$ denoted by λ and the maximum rate of profit, R. Here $\lambda = 0.4907$ and hence $R = \lambda^{-1} - 1 = 1.04 = 104\%$. Next, we determine the multiplier $\boldsymbol{q}$ that allows us to construct the standard system, i.e., the non-trivial solution of the following homogeneous system:

$$\big(\boldsymbol{B} - \boldsymbol{A}'(1+R)\big)\boldsymbol{q} = \boldsymbol{0} \tag{1.B.2}$$

The unique solution of this example is $q = [0.582\ 0.533\ 0.614]'$, which gives the following standard system.

$$\bar{\boldsymbol{A}} = \boldsymbol{A} \otimes \boldsymbol{q}\boldsymbol{e}' = \begin{pmatrix} 1.73 & 2.30 & 2.30 \\ 1.55 & 0.52 & 1.55 \\ 1.24 & 1.24 & 3.10 \end{pmatrix} \tag{1.B.3}$$

$$\bar{\boldsymbol{B}} = \boldsymbol{B} \otimes \boldsymbol{q}\boldsymbol{e}' = \begin{pmatrix} 8.06 & 0 & 0 \\ 0 & 7.25 & 0 \\ 0 & 0 & 12.4 \end{pmatrix} \tag{1.B.4}$$

$$\bar{\boldsymbol{L}} = \boldsymbol{L} \otimes \boldsymbol{q} = \begin{pmatrix} 2.26 \\ 2.07 \\ 1.29 \end{pmatrix} \tag{1.B.5}$$

Hence the vector of sectoral net products we use as the *numéraire* is given by:

$$\boldsymbol{\eta}' = \frac{\boldsymbol{e}'(\bar{\boldsymbol{B}} - \bar{\boldsymbol{A}})}{\boldsymbol{e}'\bar{\boldsymbol{L}}} = \frac{\boldsymbol{e}'\big((\boldsymbol{B} - \boldsymbol{A}) \otimes \boldsymbol{q}\boldsymbol{e}'\big)}{\boldsymbol{L}'\boldsymbol{q}} = [0.641\ 0.576\ 0.987] \tag{1.B.6}$$

1.C A Robustness Check

As stated in Section 1.4 the production prices and the wage rate are measured in term of a chosen *numéraire*. Consequently, all successive results depend on this choice, which naturally leads to the following research question, how robust are our final results to changes in the *numéraire*?

Two sub-questions immediately emerge; what do we exactly mean by changes in the *numéraire*, and how can the consequences of these changes be measured? By changes in the *numéraire* we will here limit our study to other directly available and easily interpretable *numéraires*, i.e., *numéraires* which we just as well could have chosen for our analysis. To recollect, we choose the vector of sectoral net products from the US in year 2000 calculated from a standard system. Consequently, analogue *numéraires* can be constructed from any country in any time period in our sample.

How to evaluate the robustness of a given *numéraire*, has like the previous questions no clear-cut answer, but we first of all want to know if the general patterns in the evolution in our indices remain unaffected if another *numéraire* is chosen, e.g., does the evidence of the productivity slowdown remain roughly unaffected and what about the precise timing, absolute levels, and location of these events?

The following is a concise description of the procedure we have employed in our essay to answer the posed research questions. Naturally, this is one of many possible procedures among which no universal best procedure exist.

We restrict the experiment to the analogue *numéraires* from the four countries and the seven distinct time periods. Let $x^i_{j,t}$ be a given measure (e.g., our measure of labour productivity), where the superscript $i = 1, 2, ..., \kappa$ identifie the *numéraire* used to calculate x, and the subscripts j, t the country and time period in which we are measuring, e.g., $x^1_{2,3}$ is the measure of labour productivity in the second country (the UK) in the third time period 1980 calculated using the first *numéraire* from our list. For simplicity we define $\overline{x}_{j,t}$ as the index produced in our study, i.e., based on the US 2000 *numéraire*.

$$S = \frac{1}{\kappa^2}\sum_{i=1}^{\kappa} \max\left\{\#(x^i_{j,t} - \overline{x}_{j,t}) > 0, \#(x^i_{j,t} - \overline{x}_{j,t}) < 0\right\} \tag{1.C.1}$$

$$j = 1, 2, ..., N \quad t = 1, 2, ..., T$$

$$\sigma_i = \text{std}\left(\frac{x^i_{j,t} - \overline{x}_{j,t}}{\overline{x}_{j,t}}\right) \tag{1.C.2}$$

$$i = 1, 2, ..., \kappa \quad j = 1, 2, ..., N \quad t = 1, 2, ..., T$$

A robust *numéraire* should have a S value close to unity meaning that changes in the *numéraire* results in a symmetric movement in our index.

Furthermore, the σ^i should be as small as possible implying an overall low volatility.

We find that $S = 0.947$ for the index of technological progress and $S = 0.950$ for labour productivity, i.e., changes in the *numéraire* does only to a very small extent produce asymmetric shifts in the indices.

Figure 1.5 shows low but quiet different values of σ. It is interesting,

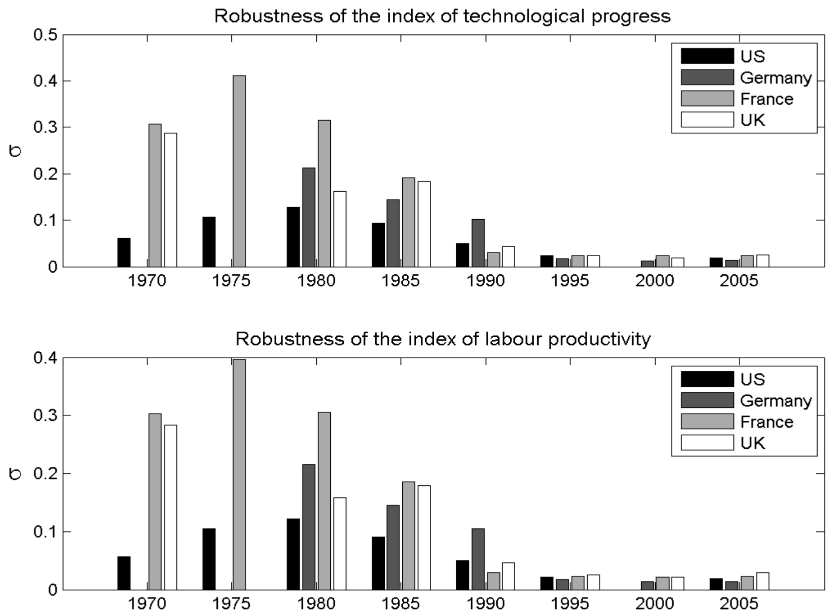

Fig. 1.5: *The sensitivity of the two indices to changes in the numéraire*

but maybe not surprising, to see that the lowest effects from changing the *numéraire* are found when changing to a contemporary *numéraire* or another US *numéraire*. Equivalent, we observe relatively clear increases in the volatility when *numé-raires* from the 70s and 80s are selected. The natural explanation of this is of course that the vector of sectoral net products changes over time and are different from country to country.

As a supplement to this mechanical procedure, we have made a visual inspection of the indices and found that the fundamental conclusion form in our previous study remain unaffected by changes in the *numéraire*, e.g., the productivity slowdown.

All in all, this small exercise confirms the obvious fact that our results change when we change the standard of value, but also suggests that these changes are relatively inconsequential for the general conclusion from our previous study.

Chapter 2

Measuring Structural and Technological Change from Technically Autarkic Subsystems

A Study of Danish Industries, 1966–2005

Written jointly with Stefano Zambelli

2.1 Introduction

The purpose of this chapter is to study structural and technological changes in and among Danish industries, and its effects on the economy as a whole. Furthermore, we show how the approach can be utilised to evaluate economic policy. This is accomplished by applying the subsystem approach introduced by Sraffa (1960) to the unique Danish input–output tables 1966–2005. The subsystem approach allows us to decompose the effects from industry-level technological change to the overall industry interdependences.

Structural change and the link between aggregate and industry-level productivity can be measured in different ways, but one method has since its introduction by Domar (1961) and generalization by Hulten (1978) dominated the productivity accounting carried out by major statistical bureaus. This method uses *Domar weights* to—supposedly—capture the combined effect of productivity growth within the individual industries and indirect effects through the supporting industries.[1]

From a fundamentally different methodological point of view we have the subsystem approach introduced by Sraffa (1960) and further developed by Gossling (1972), Pasinetti (1973), and others. The idea behind the notion of subsystems is to construct technically autarkic subsystems that as a final (net) output produce only one industry's output. This enables us to compute all the intermediate goods and labour directly and indirectly needed to produce this single commodity. A subsystem can be thought of as a isolated complex supply chain, including all commodities, producing only one final product.

One advantage of this approach is that changes in methods of production, interdependences, and structural change can be detected by the study of subsystems alone. Hence relative changes in the importance of the different industries can be detected. Another important property of the individual subsystems is that they are additive, i.e., the individual subsystems can be combined into meso level groups of autarkic subsystems producing as a final output any subset of the basket of final products from the entire system.

The notion of subsystems was introduced by Sraffa (1960, p. 89) in a typically concise three-quarter page appendix that is worth quoting in full:[2]

> Consider a system of industries (each producing a different commodity) which is in a self-replacing state. The commodities forming the gross product can be unambiguously distinguished as those which

[1] Domar weights are computed as the ratio of industry gross output to total deliveries to final demand. For further information see OECD (2001, 2008) and Hulten et. al. (2001).

[2] See Velupillai (2008) for a discussion on the intrinsic algorithmic content of Sraffa's arguments, including Sraffa's description of subsystems. This fundamentally algorithmic way of posing the problem, and the procedures by which they can be solved, is instrumental in our quest to utilise this powerful tool in an unambiguous fashion.

> go to replace the means of production and those which together form the net product of the system.
>
> Such a system can be subdivided into as many parts as there are commodities in its net product, in such a way that each part form a smaller self-replacing system the net product of which consists of only one kind of commodity. These parts we shall call 'subsystems'.
>
> This involves subdividing each of the industries of the original system (namely, the means of production, the labour and the product of each) into parts of such size as will ensure self-replacements for each subsystem.
>
> Although only a fraction of the labour of a subsystem is employed in the industry which directly produces the commodity forming the net product, yet, since all other industries merely provide replacement of the means of production used up, the whole of the labour employed can be regarded as directly or indirectly going to produce that commodity.
>
> Thus in the subsystem we see at a glance, as an aggregate, the same quantity of labour that we obtain as the sum of the series of terms when we trace back the successive stages of the production of the commodity.

Sraffa is here pointing to the possibility of using subsystems as units of measurement in a way which is both theoretically relevant and useful for empirical analysis.

Empirical applications of the subsystem approach were originally developed by Gossling (1972) to study the American agricultural industry. Other empirical applications of the subsystem approach, on the measurement of productivity and on the relation among market prices, production prices, and labour values, include: Juan and Febrero (2000), Dietzenbacher et. al. (2000), Miller and Gowdy (1998), Tsoulfidis and Mariolis (2007), Tsoulfidis (2008), and Alcántara and Padilla (2009). This chapter is a contribution to the literature on empirical subsystem analysis, that both empirically and computationally will go deeper into the practical applications of this powerful tool.

A major advantage, of all the indices that will be presented, is that they do not change as a consequence of changes in the scale of production alone, even if it is asymmetric across industries. Hence, the indices will only change, when real technological innovations take place. It is exactly the consequences of such changes we want to capture in the indices, since they can influence both the structural relationship among industries and the productivity level in the single industries. It cannot be stressed enough, that this is not based on an assumption of *constant returns to scale*. If a changes occur in the scale of production in one or more industries, without changing the proportional use of the means of production (including labour), the indices remain unaffected. If on the other hand the relative proportions of the means of production are affected, then it will influence both the decomposition into subsystems and the vector of production prices.

A general property of this approach is that it circumvents many of the theoretical problems, innate to neoclassical studies of structural change and technological progress. In particular, thus related to the use of aggregate production function, see Pasinetti (2000); Cohen and Harcourt (2003); and Felipe and Fisher (2003).

Consequently, this approach has huge potentials, not only, as an analytical and descriptive tool, but also to provide procedures to evaluate economic policy. To accomplice this, we combine algorithmic reasoning with a naturalistic approach to the theory of production. We consider only practically observable phenomena, and from this work our way through the problems applying only mathematical statements, for which we can actually provide procedures to compute.

This chapter is structured as follows: Section 2.2 presents the theoretical framework, Section 2.3 the data, Section 2.4 the results associated with structural and technological change, and Section 2.5 the applications to economic policy. Section 2.6 concludes the chapter. Appendix 2.A and 2.B contain a both technical and practical introduction to the construction of subsystems and Appendix 2.C and 2.D respectively a list of symbols and details on the data used. Appendix 5 in the statistical companion contains a comprehensive collection of the empirical results obtained.

2.2 On Subsystems

Let $[\boldsymbol{A}_t, \boldsymbol{l}_t, \boldsymbol{B}_t]$ be a set of data variables measured in physical quantities. The entries are respectively, the non-singular indecomposable semi-positive $n \times n$ *input-matrix*, the $n \times 1$ column *vector of labour inputs*, and the $n \times n$ semi-positive diagonal *gross output-matrix.* $\boldsymbol{A}_t$ is composed of row vectors of intraindustry inputs and column vectors of interindustry flows. Furthermore, let $\boldsymbol{e}$ be a $n \times 1$ unit vector. It is necessary to introduce a rather cumbersome mathematical notation, *viz.*[3]

$a_{(i,j,t)}$	the ijth entry of $\boldsymbol{A}$ at time t
$\boldsymbol{a}_{(i,:,t)}$	the ith row of $\boldsymbol{A}$ at time t
$\boldsymbol{A}_{(\neg i,t)}$	$\boldsymbol{A}$ at time t, but without its ith row and column
$\boldsymbol{a}_{(i,\neg j,t)}$	the ith row of $\boldsymbol{A}$ at time t, but without its jth entry

Only single production systems will be considered, but most—if not all—results are valid in the more general case of *joint production.* Furthermore, only circulating and not fixed capital will be considered. At this point it is not only a matter of convenience, but also one of deep theoretical and empirical considerations. Both the standard way of approximated fixed capital

[3]Matrices, vectors, and scalars are respectively represented by bold capital letters, bold non-capital letters, and non-bold non-capital letters. Furthermore, single entries and vectors from a given matrix are represented by the corresponding non-capital letter.

in value terms by invoking the *ad hoc* and partly stochastic *perpetual inventory method*, and the theoretical sound, but empirical intractable, method of treating fixed capital in the framework of joint production, are so problematic that we *pro tempore* have choosing to abstract from fixed capital.[4]

2.2.1 The subsystem multipliers

Two additional assumptions are necessary and sufficient for the following results to hold; i) fixed production techniques, over the accounting period and ii) viable economic systems in a self-replacing state, i.e., systems capable of and actually producing at least the commodities required to replace the circulating capital goods.

To construct the technically autarkic *gross output subsystem* associated with the ith industry at time t from the parent system $[\boldsymbol{A}_t, \boldsymbol{l}_t, \boldsymbol{B}_t]$, the system must be rescaled such that the entire subsystem as a gross output produce the gross output of the ith industry in the original system, while the final output in all supporting industries are zero. This can be done, applying the following intuitive and computational direct procedure, that as an auxiliary tool use multipliers to decompose the parent system into subsystems.[5]

To obtain the ith *gross output subsystem multiplier* at time t, first compute the non-trivial strictly positive unique solution, $\overline{\boldsymbol{q}}_t^i$, of the following system of equations:[6]

$$\left[\boldsymbol{B}_{(\neg i,t)} - \boldsymbol{A}'_{(\neg i,t)}\right]\overline{\boldsymbol{q}}_t^i = \boldsymbol{a}'_{(i,\neg i,t)} \qquad i = 1,2,...,n \quad t = 1,2,...,T \qquad (2.2.1)$$

Second, on the ith entry of the vector $\overline{\boldsymbol{q}}_t^i$ *squeeze in* a single "1". The intuition behind this procedure is that the final output from the supporting industries (the LHS of 2.2.1) must be equal to the interindustry flow into the industry associated with the subsystem (the RHS of 2.2.1).

From this it is straightforward to compute the *final output subsystem multipliers*, henceforth called the *subsystem multipliers*, $\tilde{\boldsymbol{q}}_t^i$. Rescale the gross subsystem multipliers, such that the net products of the individual subsystems are equal to the corresponding sectoral net products in the par-

[4]For a discussion on the consequence of not including fixed capital, see Han and Schefold (2006, p. 752).

[5]Appendix 2.A contains an introduction to numerical equivalent, but conceptually different, procedures to construct subsystems.

[6]A unique non-trivial solution requires that $\left[\boldsymbol{B}_{(\neg i,t)} - \boldsymbol{A}'_{(\neg i,t)}\right]$ is non-singular for all $t = 1,2,....,T$ and $i = 1,2,...,n$. The strictly positive solution with $0 < \overline{\boldsymbol{q}}_t^i \leq 1$ is guaranteed if the original system is in a self-replacing state, since the supporting industries, in the ith gross output subsystem, as a final output only produce what is needed to produce the gross output in the ith industry, which is necessarily less than the same plus a 'non-negative' surplus.

ent system, *viz.*

$$\tilde{\boldsymbol{q}}_t^i = \overline{\boldsymbol{q}}_t^i \frac{b_{(i,i,t)} - \boldsymbol{e}'\boldsymbol{a}_{(:,i,t)}}{b_{(i,i,t)} - \overline{\boldsymbol{q}}_t^{i\prime}\boldsymbol{a}_{(:,i,t)}} \tag{2.2.2}$$

Using the subsystem multipliers obtained above, the matrices forming the ith final output subsystem at time t measured in physical quantities are given by $[\tilde{\boldsymbol{A}}_t^i, \tilde{\boldsymbol{l}}_t^i, \tilde{\boldsymbol{B}}_t^i] = [\boldsymbol{A}_t \otimes \tilde{\boldsymbol{q}}_t^i \boldsymbol{e}', \boldsymbol{l}_t \otimes \tilde{\boldsymbol{q}}_t^i, \boldsymbol{B}_t \otimes \tilde{\boldsymbol{q}}_t^i \boldsymbol{e}']$.

The following two subsections are respectively devoted to indices based on physical quantities and prices of production. Moreover, a distinction is made between *productivity indices*, i.e., a measure of output divided by a measure of inputs, and *structural change indices* that try to capture changing interdependence among industries.

Appendix 2.B contains detailed numerical examples on how to compute the indices presented in the following.

2.2.2 Indices based on physical quantities

Two simple and intuitive measures of productivity, derived from the final output subsystems, are the following σ- and ξ-indices:

$$\sigma_t^i = \frac{\tilde{b}_{(i,i,t)}^i - \tilde{a}_{(i,i,t)}^i}{\boldsymbol{e}'\tilde{\boldsymbol{l}}_t^i} = \frac{\text{external output}}{\text{direct + indirect labour}} \tag{2.2.3}$$

$$\xi_t^i = \frac{\tilde{b}_{(i,i,t)}^i - \boldsymbol{e}'\tilde{\boldsymbol{a}}_{(:,i,t)}^i}{\boldsymbol{e}'\tilde{\boldsymbol{l}}_t^i} = \frac{\text{final output}}{\text{direct + indirect labour}} \tag{2.2.4}$$

The ξ-index is also known as the *Gossling I* index, see Gossling (1972, p. 45). It is numerical equal to reciprocal of Pasinetti's vertically integrated labour coefficients. Itself a measure of labour productivity, *viz.*

$$\boldsymbol{v}_t = (\boldsymbol{B}_t - \boldsymbol{A}_t)^{-1}\boldsymbol{l}_t \tag{2.2.5}$$

The vertically integrated labour coefficients provide the units of direct and indirect labour needed to produce one unit of the ith industry's final output. See also Appendix 2.A.1 and 2.B.2.

In Equation 2.2.3 and 2.2.4 the external and final output in the numerators refer to the external and final output in the single subsystems. The external output is by definition the gross output minus the industry's sales of its own output to itself. The denominator consists of the total labour employed in the ith subsystem, i.e., the direct labour employed in the ith industry and the indirect employed in the supporting industries.

To be precise, a proportion of the labour employed in the ith industry of the ith subsystem, should be accounted as indirect, since a proportion of the industry's output in the subsystem is sold as means of production and hence enters indirect into the industry's own production. In most cases this complication is not important, since we are mainly interested in the total of the direct and indirect labour, but in one of the following indices the distinction is explicitly taken into account.

The α-, β-, and ρ-indices below are measures of structural change, the last two of which are derived from the subsystems. They all provide measures of the integration of the single industries with the system as a whole. The three indices are bounded within the unit interval and the closer the indices are to unity the more isolated is the industry.

$$\alpha_t^i = \frac{b_{(i,i,t)} - \boldsymbol{e}'\boldsymbol{a}_{(:,i,t)}}{b_{(i,i,t)}} = \frac{\text{final output*}}{\text{gross output*}} \tag{2.2.6}$$

$$\beta_t^i = \frac{\tilde{b}^i_{(i,i,t)} - \boldsymbol{e}'\tilde{\boldsymbol{a}}^i_{(:,i,t)}}{\tilde{b}^i_{(i,i,t)}} = \frac{\text{final output}}{\text{gross output}} \tag{2.2.7}$$

$$\rho_t^i = \frac{\beta_t^i \tilde{l}^i_{(i,t)}}{\boldsymbol{e}'\tilde{\boldsymbol{l}}_t^i} = \frac{\text{direct labour}}{\text{direct + indirect labour}} \tag{2.2.8}$$

The asterisk '*' here denotes 'for the system as a whole'. The β-index is computed as the ratio of final to gross output for the ith industry in the ith subsystem. The analogue α-index is computed from the system as a whole and is a common measure for the integration of the industry with the system as a whole. If there is a large difference between an industry's final and gross output (for the whole system) it implies that a large amount of the industry's output is sold as means of production, and *vice versa*.

The interpretation of the β-index is different. It provides a measure of the importance of the ith commodity within the ith subsystems, i.e., the importance of the single commodities within their own supply chain. This should be interesting for detailed inter- and intraindustry studies.

The β-index can be used to compute the intraindustry direct and indirect labour discussed above. This is done in the numerator of the ρ-index which is computed as the ratio of direct to direct and indirect labour. Think of $(1-\beta_t^i)\tilde{l}^i_{(i,t)}$ as the amount of labour employed in the ith industry in the ith subsystem, that is producing commodities that are eventually used as means of production within the subsystem.

The ρ-index is therefore a properly generated measure of the ratio between direct labour and the total amount of labour employed throughout the supply chain.

2.2.3 Indices based on production prices

A physical production system $[\boldsymbol{A}_t, \boldsymbol{l}_t, \boldsymbol{B}_t]$ has, for a given distribution of the Net National Product (NNP) between wages and profits, a unique vector of Sraffian *production prices*, $\boldsymbol{p}_t(r)$, measured in terms of a given *numéraire*.

Following the *Non-substitution theorem* these production prices are unaffected by any rescaling of the system.[7] Consequently, also of the transformation into subsystems. As usual production prices and the associated *wage-profit frontier* are for $t = 1, 2, ..., T$ given by:

$$\boldsymbol{p}_t(r) = \frac{\big(\boldsymbol{B}_t - \boldsymbol{A}_t(1+r)\big)^{-1}\boldsymbol{l}_t}{\boldsymbol{\eta}'\big(\boldsymbol{B}_t - \boldsymbol{A}_t(1+r)\big)^{-1}\boldsymbol{l}_t} \qquad r = \{r \in \mathbb{Q} : 0 \le r \le R_t\} \tag{2.2.9}$$

$$w_t(r) = \Big[\boldsymbol{\eta}'\big(\boldsymbol{B}_t - \boldsymbol{A}_t(1+r)\big)^{-1}\boldsymbol{l}_t\Big]^{-1} \tag{2.2.10}$$

Where $\boldsymbol{\eta}$ is a *pro tempore* unspecified *numéraire* and R_t the maximum rate of profit.[8]

Consequently, the value in terms of production prices of the net products for the system as a whole and the final output subsystems, are given by the following accounting identities:

$$\boldsymbol{\zeta}_t(r) = \big(\boldsymbol{B}_t - \boldsymbol{A}_t\big)\boldsymbol{p}_t(r) \tag{2.2.11}$$

$$\tilde{\zeta}_t^i(r) = \boldsymbol{e}'\big(\tilde{\boldsymbol{B}}_t^i - \tilde{\boldsymbol{A}}_t^i\big)\boldsymbol{p}_t(r) \tag{2.2.12}$$

An obvious property, following the additivity of the final output subsystems, is that $\mathrm{NNP}_t(r) = \boldsymbol{e}'\boldsymbol{\zeta}_t(r) = \sum_{i=1}^n \tilde{\zeta}_t^i(r)$.

Following Degasperi and Fredholm (2010) a procedure to construct a distribution free measure of labour productivity from the above net products is to compute the following definite integrals (by means of computational methods):

$$\mu_t^i = \frac{1}{l_{(i,t)}R_t}\int_0^{R_t} \zeta_{(i,t)}(r)\,dr \tag{2.2.13}$$

$$\psi_t^i = \frac{1}{\boldsymbol{e}'\tilde{\boldsymbol{l}}_t^i R_t}\int_0^{R_t} \tilde{\zeta}_t^i(r)\,dr \tag{2.2.14}$$

The maximum rate of profit, R_t, associated with the system, which like the production prices is unaffected by the rescaling into subsystems, is used to normalise the indices, such that systems with different maximum profit rates better can be compared. The ψ-index takes into account the effect from the supporting industries, while the μ-index does not.

[7] See Kurz and Salvadori (1995, p. 26–28) for discussion of the origin and implications of this peculiar result.

[8] The maximum rate of profit can be computed as $R_t = \lambda_t^{-1} - 1$, where λ_t is the maximum eigenvalue of the matrix of interindustry coefficients, $\boldsymbol{B}_t^{-1}\boldsymbol{A}_t$, at time t, see Pasinetti (1977, p. 76).

Two measures which can be used to study structural change are:

$$\gamma_t^i = \frac{1}{R_t} \int_0^{R_t} \frac{\big(\tilde{b}^i_{(i,i,t)} - \tilde{a}^i_{(i,i,t)}\big) p_{(i,t)}(r)}{\boldsymbol{e}'\tilde{\boldsymbol{A}}^i_t \boldsymbol{p}_t(r) + \boldsymbol{e}'\tilde{\boldsymbol{l}}^i_t w_t(r)}\, dr \tag{2.2.15}$$

$$\delta_t^i = \frac{1}{R_t} \int_0^{R_t} \frac{\big(\tilde{b}^i_{(i,i,t)} - \tilde{a}^i_{(i,i,t)}\big) p_{(i,t)}(r)}{\tilde{\boldsymbol{a}}^i_{(i,:,t)} \boldsymbol{p}_t(r) + \tilde{l}^i_{(i,t)} w_t(r)}\, dr \tag{2.2.16}$$

The γ-index is based on the ratio of the 'value of external output' over the 'social costs', where the social costs are the total costs for the subsystem as a whole in terms of capital goods and wages. The δ-index is based on the ratio of the 'value of external output' over the 'local costs', where the local costs are the total costs, in terms of capital goods and wages, for the ith industry in the ith final output subsystem. The γ-index takes into account the effect from the supporting industries, while the δ-index does not.

A major advantage of all the indices presented, is that they do not change alone as a consequence of changes in the scale of production in the original system, even if it is asymmetric across the single industries. Hence, the indices will only change when real technological innovations take place. It is exactly the consequence of such changes we want to capture in the indices, since they can influence both the structural relationship among industries and the productivity level in the single industries. This very convenient property follows from the Non-substitution theorem (the invariance of the production prices) and the fact that the reproportioning into subsystems, likewise independently of the vector of final consumption, determines the relative proportions of inputs to outputs.

2.2.4 Policy implications

The subsystem approach has several useful features for both for ex ante and ex post evaluations of economic policy. Ex post, the indices presented here can be use to better separate structural, technological, and scale effects from a given economic policy. Of course with the usual reservations about the *ceteris paribus* assumption in such analysis.

Ex ante, it is possible to provide a first approximation of the total (direct plus indirect) effect on, e.g., labour demand, emission of greenhouse gasses, or the balance of trade following a change in the scale of production in a single or group of industries. Not only is it possible to provide an estimate on the aggregate effect, but also how these effects are distributed across industries. Remember that the final output subsystems are additive such that it is possible to move freely between local and social effects.

Here the subsystem multipliers, obtained in Section 2.2.1, emerge as a very convenient auxiliary tool. Why this is so, is shown together with a few example in Section 2.5.

2.3 Data and the Choice of *Numéraire*

The Danish input–output tables cover the entire period 1966–2005 for 130 industries following international standards of national accounting and include the flow among Danish industries as well as industry/commodity specific imports. The data are available in both current and fixed prices with base-period 2000.[9]

The fixed base-period denominated tables are used as a proxy for the physical inter-industrial flow of commodities. It must be stressed that by so doing we only see the "shadow" of the physical flow, but if empirical studies on this are to be carried out, this is as good as it gets.[10] In effect, what we are using is data measured in *Leontief Units*, i.e., a volume of physical goods worth one Danish krone, but the data is treated as if it were heterogeneous physical quantities, e.g., we do not sum distinct commodities.

Furthermore, detailed employment data are used on the total hours worked in each industry in each accounting period. Note, that labour is treated as a homogeneous input, both over time and across industries. This is likewise a very strong assumption, but again necessary given the data availability.

The 130 industries must be aggregated down to 123, to ensure non-singular matrices for all periods. For convenience in presenting the results—and only for that reason—the tables are aggregated into 52 industries. The full list of industries and details on the aggregation are found in Appendix 2.D.

As we will see in the results, there are cases where it is seems more plausible to be residuals from monetary shocks left in the data, rather than real technological phenomena, that are causing the dynamics observed in the computed indices. Examples of this are the oil price shocks in 1970s, the financial turmoil in 1987, and a breakdown of an international monetary system.

A choice has been made to exclude the 1970 and 1971 tables from the dataset. The 1970 and 1971 tables can be seen as outliers, especially the 1971 table is extreme, in the sense that the system has a maximum interest rate very close to zero.[11] Hence, the economic system is close to *non-viable*, see Section 2.3.1. Excluding tables from the dataset does not in any way influence the other results, since the production prices, wage-profit frontiers, and the decomposition into subsystems are fully determined within each period. This is another practical feature of this approach compared with

[9] Statistics Denmark, www.dst.dk/inputoutput

[10] For a discussion on monetary vs. physical denominated input–output data, see Han and Schefold (2006, p. 750).

[11] An possible explanation of this phenomenon, is the economic turmoil around the collapse of the Bretton Woods system of monetary management, i.e., monetary and not technological phenomena that are not properly deflated from the data.

standard econometric exercises.

As a *numéraire*, for the computation of production prices, we choose the vector of domestic net products from the base year 2000 normalised with the total hours worked. We use the domestic net product, because if imported means of production were subtracted we would not necessarily obtain a vector of non-negative entries. This naturally leads to the next section.

2.3.1 Viability, self-replacing, and imports

All the systems 1966–2005 are viable, i.e., there exist the possibility (by rescaling) for the system to reproduce itself, but not all systems are in a self-replacing state.[12] Not being in a self-replacing state implies that the system in its current state does not produce a strictly positive vector of final (net) products.

In the Danish data, this is mainly a consequence of the inclusion of imported capital goods. The input-matrix, $\boldsymbol{A}_t$, is the sum of the matrix of domestic interindustry flow and the matrix of industry specific imports. In hindsight, this is not surprising for a small open economy, as the Danish, where exports account for roughly half of the NNP.

This constitutes a problem, since the subsystems per construction are set to produce and only produce the final output of the given industry. Nevertheless, the practical implications of this problem do not seem critical. All the $52 \cdot 40 = 2080$ (52 industries in 40 time periods) gross output subsystem multipliers (Equation 2.2.1) are strictly positive. The final output subsystem multipliers (Equation 2.2.2) on the other hand are strictly negative for the subsystems associated with commodities for which the system as a whole produces a negative final output.

This is still conceptually a problem, but since all the subsystem based indices are scale-independent, also negative numerators and denominators will cancel out. Hence, the interpretation of the subsystem based indices are not affected by non-self-replacement. The full extend of the non-self-replacement can be seen in the α-index (Figure 5.34–5.41 in the statistical companion), i.e., the ratio of final to gross output for the system as a whole.

2.4 On Structural and Technological Change

Computing all the presented indices produces a huge number of time-series (52 series for each index). Therefore, only a small subset of the results is presented and analysed. The full set of results based on the Danish input–output tables is collected in the statistical companion.

[12]See Chiodi (1998) for a theoretical discussion on the notion of viability and non-self-replacing states.

Since the final output subsystems are additive, the 52 industries can be grouped in seven meso-sectors, as summarised in Table 2.1.

Meso-sector	industry classification	industry #
1	Agriculture, fishing, and quarrying	1–7
2	Manufacturing	8–20
3	Electricity, gas, and water supply; and Construction	21–22
4	Wholesale-, retail trade, hotels, restaurants	23–30
5	Transport, storage, and communication	31–35
6	Financial intermediation, business activities	36–43
7	Public and personal services	44–52

Table 2.1: *Grouping the 52 industries into seven meso-sectors*

2.4.1 Technological progress

Grouping the industries allows us to construct figures such as Figure 2.1 for the σ-index, which can be used to illustrate the decomposition of the aggregate technological change. The σ-index shows a clear positive but cyclical trend in the (aggregate) technological progress in Denmark since the 1960s. Figure 2.1 also shows that the primary forces behind this technological progress is to be found within the meso-sectors of 'Agriculture, fishing, and quarrying' and 'Manufacturing'.

Figure 5.1–5.18 in the statistical companion show the industry-level technological progress within the single meso groups measured by the σ- and ξ-index. From this it is clear that the main sources of the progress within 'Agriculture, fishing, and quarrying' and 'Manufacturing' are respectively 'Extr. of crude petroleum, natural gas etc.' and 'Mfr. of refined petroleum products etc.'. These two industries are so important that, if their subsystems were removed from the productivity accounting the level of technological progress for 2005 would be similar to that of the mid 1990s, see Figure 5.10 in Appendix 5.A in the statistical companion.

The same pattern is found using the μ- and ψ-index, see Appendix 5.G and 5.H. As noted in Section 2.3, it is possible that some of these effects are caused by monetary phenomena (change in the price of oil etc.) which are not sufficiently deflated from the interindustrial flow. See also Figure 5.41 for the α-index which show for the aggregate economy a drastic development in the ratio of final to gross output in 'Extr. of crude petroleum, natural gas etc.'.

Figure 5.4 in the statistical companion, the σ-index for 'Wholesale-, retail trade, hotels, restaurants', shows that the σ-index for 'Retail trade of food etc.' increased steadily from the 1960s until 1994/95 where after it decreased for almost a decade returning to the level of 1985. From around 2003 it again increased. Unfortunately, since the food crisis of 2007-2008 would be interesting to follow, the series ends in 2005. Nevertheless, these

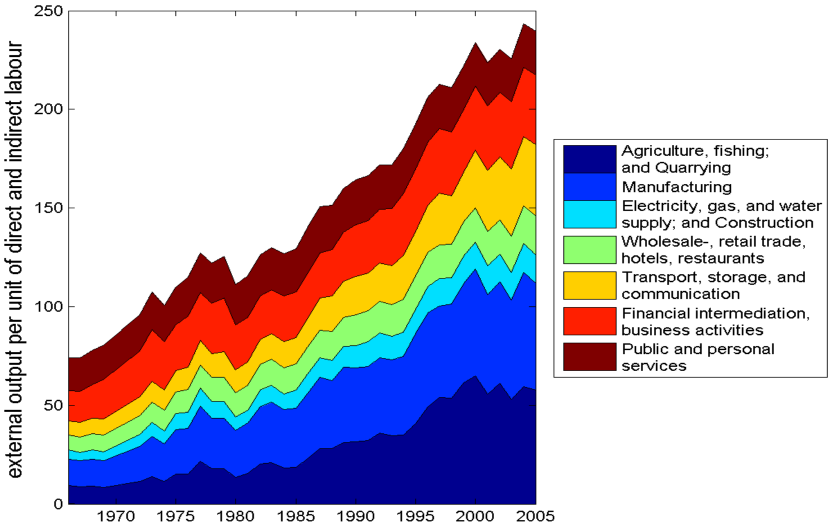

Fig. 2.1: *The σ-index for the seven meso-sectors*

results could, supported from the other indices and the evolution in similar industries, such as 'Agriculture' and 'Mfr. of food, beverages and tobacco', be interesting in the quest of understanding the local effects from apparent global phenomena. Moreover, the study could be extended to a industry-level comparative study among countries. Future studies should also try to go deeper into the identification of possible 'monetary residue' in the fixed base-period denominated tables, because it seems unlikely that such monetary phenomenon, as a global food crises, could be fully deflated in fixed base-period inter-industrial data.

Figure 5.6 in the statistical companion shows that the σ-index for 'Financial intermediation' increased slowly until around 1982 where after it accelerated to a higher rate of growth which was maintained until around 2000, where the rate of growth further increased. The series ends in 2005. Without going into detail the *structural break* around 1982 corresponds with a strong deregulations of the Danish financial sector, a trend which continued until the outbreak of the present financial crisis. The increased growth rate from around 2000 coincides with initiatives for further European integration of financial markets, e.g., the 'Financial Services Action Plan' of 1999, see Kurek (2004). Whether or not there is an actual causal relationship between the institutional changes and the observed development, calls for further research, again together with the search for monetary residue in the data.

2.4.2 Direct and indirect labour

The ρ-index (direct over direct plus indirect labour), Figure 5.27–5.33, for the primary and secondary industries: 'Agriculture, fishing, and quarrying' and 'Manufacturing' in general fall between two-third and one, while the tertiary industries (the industries in Meso-sector 3–7) between 0.95 and 1. This implies that the tertiary industries are the more isolated, in the sense that they use relatively less amounts of indirect labour compared with the primary and secondary industries.

The results also show the relative use of direct and indirect labour is fairly stable over time.

2.4.3 The Great Convergence

The evolution of the vertically integrated labour coefficients are collected in Figure 2.2 for the industries collected in the meso groups of 'Agriculture, fishing, and quarrying'; 'Manufacturing'; and 'Wholesale-, retail trade, hotels, restaurants'. From this a peculiar result emerges; the vertically integrated labour coefficients tend to converge to a common factor around one-half.

This might be a consequence of investment allocation towards low productive industries/subsystems with the expectations of higher returns, i.e., the usual explanation. Independently of the causes behind this convergence,

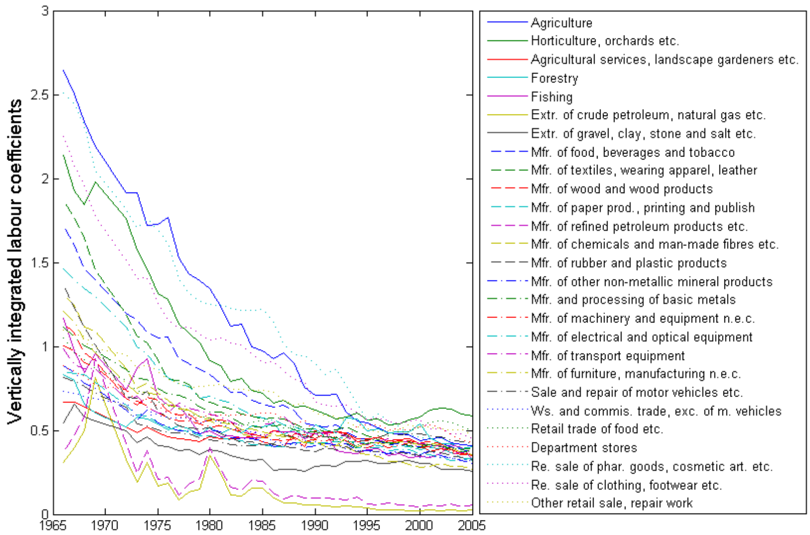

Fig. 2.2: *The v-index, Agriculture, fishing, and quarrying; Manufacturing; and Wholesale-, retail trade, hotels, restaurants*

it is a very clear and strong empirical observation that calls for further research to establish the causes and consequences of this economic phenomenon.

The only two industries in Figure 2.2 that do not converge to a value close to one-half are *the usual suspects* 'Extr. of crude petroleum, natural gas etc.' and 'Mfr. of refined petroleum products etc.' that steadily approach zero. For the industries in the other meso-sectors this convergence is less clear, but still a tendency is observed for most industries, see Figure 5.20–5.26.

2.4.4 Comparing the different indices

Two different measures are used to access the quantitative differences among the indices. The first is the correlation coefficients for simple linear regression, where to obtain one measure for each par of indices, a simple average is computed across the 52 time series.

The second measure is a modified Mean Absolute Percentage Error (MAPE), computed as follows: First, normalise all indices, such that the index for the base-period 2000 is equal to unity. This is done to better abstract from differences in levels. Second, compute the following mean of the modified MAPE, *viz.*

$$m_{(k,\kappa)} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{T}\sum_{t=1}^{T}\left|\frac{x_t^{i,k} - x_t^{i,\kappa}}{\max\left\{x_t^{i,k}, x_t^{i,\kappa}\right\}}\right|\right) \quad \forall\ k < \kappa \qquad (2.4.1)$$

Where $x_t^{i,k}$ is the kth index for the ith subsystem at time t. As with the correlation coefficients a simple average is computed across the 52 time series (the first summation). The denominator is chosen as the maximum of the two entries in the numerator, because when more than two time series are compared it can make a huge difference which is chosen as the base. A value of $m_{(k,\kappa)} = 0.10$ should be read as an average absolute deviation of 10 percent.

These measures are collected in Table 2.2 where the upper triangles show the means of the correlation coefficients and the lower triangles the means of the modified MAPEs. The productivity indices are collected in the left

Indices of productivity						Indices of structural change					
	σ	ξ	υ	μ	ψ		α	β	ρ	γ	δ
σ		0.99	0.68	0.32	0.30	α		0.50	0.50	0.52	0.45
ξ	0.01		0.69	0.32	0.30	β	1.40		0.61	0.63	0.48
υ	0.48	0.48		0.49	0.58	ρ	0.91	0.15		0.88	0.73
μ	0.27	0.27	0.45		0.93	γ	1.30	0.06	0.11		0.80
ψ	0.26	0.26	0.46	0.07		δ	1.50	0.05	0.18	0.10	

Table 2.2: *Comparing the different indices – correlation coefficients and mean modified MAPEs*

hand side matrix of Table 2.2 and the indices of structural change in the right hand side matrix.

The σ-index (external output over direct and indirect labour) and the ξ-index (final output over direct and indirect labour) are almost identical. The difference between the two measures, is the amount of the ith commodities in the ith subsystem used in the supporting industries, which consequently must be relatively small. Hence, choosing either two does not seem to make much of an difference.

The two measures of productivity based on production prices are also closely correlated, i.e., the μ-index (not subsystem based) and the ψ-index (subsystem based).

The measures based on production prices and those based on physical quantities are also correlated, but not to the same extent as indices based on production prices and physical quantities, respectively.

What cannot be seen in the table is however that the v-index (the vertically integrated labour coefficients) and the ξ-index are perfectly correlated, but not linear so. The ξ-index is equal to the reciprocal of the v-index, see Appendix 2.A and 2.B.

For the indices of structural change, in the right hand side matrix of Table 2.2, the correlation coefficients and the modified MAPEs report evidence of some co-evolution among the indices. The α-index, which is not based on the subsystems, is compared with the other indices clearly the most distinct.

This exercise should be taken into consideration, when one index is chosen over another.

2.5 On Economic Policy

2.5.1 A subsystem based CO_2 accounting

Table 2.3 shows a subsystem based CO_2 accounting for Denmark 2005. For practical reasons the results are presented only for the seven meso-sectors, but could easily be constructed at any level of aggregating. Table 2.3 is constructed by multiplying element-by-element the vector of industry specific emission of CO_2 found in The Danish Air Emissions Accounts[13] with the subsystem multipliers for 2005. The outcome is the matrix below which decompose the total CO_2 emission for 2005, 48,731 units, into not only where the pollution occurred, but also to the industries in which the demand that let to this pollution was created. In a sense this is a distinction between cause and effect, i.e., what drives the production (cause) and where the production/pollution take place (effect). The columns represent cause and the rows effect, e.g., the sum of the second row, 7,833 units of CO_2, is the total

[13] www.dst.dk/inputoutput, which includes data on the eight groups of greenhouse gasses; Carbon dioxide (CO_2), Sulphur dioxide (SO_2), Nitrogen oxides (NO_x), Carbon oxide (CO), Laughing gas (N_2O), Ammonia (NH_3), Methane (CH_4), and Non methane volatile organic compounds (NMVOC) all covering 130 industries 1990–2006.

emission occurring in Meso-sector 2 'Manufacturing' and the single entries in the second row show where the demand which let to this pollution was created, e.g., the 736 units in last entry of the second row is the demand created in Meso-sector 7 'Public and personal services'. Consequently, the

demand from/emission in	1	2	3	4	5	6	7	totals
1. Agriculture, fishing, and quarrying	1444	2166	286	497	360	95	382	5222
2. Manufacturing	76	2877	74	3144	406	532	736	7833
3. Electricity, gas, and water supply; and Construction	243	3522	13699	3666	972	799	3100	25996
4. Wholesale-, retail trade, hotels, restaurants	9	113	17	2055	42	183	135	2555
5. Transport, storage, and communication	32	455	19	1188	2811	167	659	5322
6. Financial intermediation, business activities	1	20	2	44	10	152	48	277
7. Public and personal services	4	69	6	153	24	59	1222	1533
totals	1800	9211	14102	10732	4622	1999	6277	48731

Table 2.3: *The decomposition of the total Danish CO_2 emission for 2005, all measured in units of 1000 tonnes of CO_2*

entries on the main diagonal show the emission caused by and occurred in the single industries.

An interesting observation is the differences between the totals found in last row and column, e.g., the emission directly created in the public sector is 1,533 units, but the indirect from the supporting industries, necessary to maintain the activity level in the public sector, is 6,277 units, i.e., a difference of a factor four.

This observation is in line with a similar empirical study of the Spanish economy by Alcántara and Padilla (2009). Based on the evidence from the Danish economy, we support both the general conclusion and policy recommendation stated by Alcántara and Padilla (2009, p. 913):

> The results of our work refute the idea that a services economy is necessarily a less polluting economy. Although industrial productive processes are more directly linked to energy consumption, the final responsibility of their emissions rests on the industries that demand their production. [...]
>
> A policy designed to control and mitigate emissions should consider the importance of the consumption of energy, and the emissions needed to facilitate these industries' production.

2.5.2 Direct and indirect employment effects

Imaging that the government was to implement a policy that would increase construction activities by 5 percent. What is the effect on aggregate employment and how is the increased employment distributed across industries? Assuming constant returns to scale, we can use the subsystem approach to provide an answer which takes into account the fact that employment must increase in the supporting industries as well as the supporting industries of the supporting industries and so on and so forth.

This way of reasoning is closely related to the seminal work by Kahn (1931) on 'The relation of home investment to unemployment', upon which Keynes based the theory of his famous multiplier (Keynes, 1936 ch. 10). We compute what Kahn called the *primary employment*, i.e., the sum of direct and indirect employment.[14]

Table 2.4 shows the decomposition of the aggregate Danish employment for 2005, 430 million hours, into where they are employed (the rows) and where the demand which let to this employment originated (the columns). Consequently, to see the direct and indirect effect of an increased activity

demand from/employment in	1	2	3	4	5	6	7	totals
1. Agriculture, fishing, and quarrying	307	851	17	86	100	19	109	1490
2. Manufacturing	55	2963	48	1652	290	370	765	6144
3. Electricity, gas, and water supply; and Construction	2	24	122	27	7	6	25	213
4. Wholesale-, retail trade, hotels, restaurants	31	416	46	7884	163	480	467	9487
5. Transport, storage, and communication	20	298	12	779	2025	116	420	3670
6. Financial intermediation, business activities	21	290	29	549	137	2058	618	3701
7. Public and personal services	37	661	50	1500	219	566	15313	18347
totals	474	5504	324	12477	2941	3615	17717	43052

Table 2.4: *The decomposition of the total Danish employment 2005, all measured in 10000 hours*

in 'Manufacturing', simply increase all entries in the second column by the desired proportion and compute the new sums to see, in the rightmost column, how much the employment *ceteris paribus* will increase in the different industries. Hence, we obtain not only the total effect on employment, but also how the employment is distributed across industries. This is convenient, if for example the economy is close to full employment for groups primarily employment in specific industries.

To make the computation for the construction sector all you need to do is to go to the disaggregated table (available upon request) and follow the procedure presented above.

2.6 Concluding Remarks

One of the main advantages of the subsystem approach over conventional methods is according to Gossling (1972, pp. 40–41) that

[14]The *secondary employment*, *viz.* the convergent infinite series of effects caused by the initial increase in wages and profits, lies outside what can be explained in this theoretical framework.

> partial productivity measures [...] are always bedevilled by the reservation that the interdependence of the industry with the economy is changing from year to year. It occurs to us that productivity indexes for the sub-system are always free from this reservation because its interdependence with the economy is permanently nought.

It is for that reason, we in this chapter have tried to collect, organize, define, compute, and assess indices based on the subsystem approach. In our opinion this approach has huge potentials not only as a descriptive tool, but also since it provides procedures to evaluate economic policy. In particular, considering that the subsystem multiplier approach is easy to implement from both a computational and intuitive point of view. By following this approach we have been able to study the evolution in, and integration among, Danish industries.

First, we observe very strong evidence of a convergence in the levels of productivity as measured by Pasinetti's vertically integrated labour coefficients, i.e., a convergence in the amount of direct and indirect labour needed to produce one unit of the final output.

Second, by inspecting the industry-specific contributions to the aggregate technological progress, we observe that two industries: 'Extr. of crude petroleum, natural gas etc.' and 'Mfr. of refined petroleum products etc.' have a surprisingly high impact on the overall development. Without these two industries in the productivity accounting the level of technological progress for 2005 would be similar to that of the mid 1990s.

Third, the subsystem approach has been shown to provide important insight for policy making. A fairly simple method have been presented, to measure direct and indirect emission of CO_2 which enables us to identify the origin of the specific demand that causes the emission. Such a method could prove instrumental, in the current discussion among world leaders on how to reduce the emission of greenhouse gasses. If focus is kept on only the direct (observable) emission of greenhouse gasses, the treatment of the problem can never be on more than on the symptoms.

Likewise, the direct and indirect employment effects from industry specific expansions should prove useful when policy makers are faced with different initiatives to increase (industry-specific) employment.

To sum up, the subsystem approach can both be used as a powerful descriptive tool and as a complexity-reducing tool for policy making. We can only endorse the following statement by Gossling (1972, p. 28):

> It is now to be hoped that whenever the reader sees a tableau of interdependent single-product activities, whether for a firm or an economy, he may also visualize the corresponding sets of independent isolated sectors (or sub-systems), and thereby may *abstract* from interdependence.

Acknowledgements

We would like to thank Charlotte Bruun, Carsten Heyn-Johnsen, Matteo Degasperi, and K. Vela Velupillai for their much appreciated help and support. Also the participants at the Second Schumpeter Summer School in Graz 2009 deserve our gratitude for suggestions, well placed critique, and words of encouragement.

References

Alcántara V., Padilla E. (2009): 'Input–output subsystems and pollution: An application to the service sector and CO_2 emission in Spain', *Ecological Economics*, 68, pp. 905–914.

Cohen A. J., Harcourt G. C. (2003): 'Retrospectives: whatever happened to the cambridge capital theory controversies?', *Journal of Economic Perspectives*, 17, pp. 199–214.

Chiodi G. (1998): 'On non-self-replacing states', *Metroeconomica*, 49, pp. 97–107.

Degasperi M., Fredholm T. (2010): 'Productivity accounting based on production prices', *Metroeconomica*, forthcoming.

De Juan O., Febrero E. (2000): 'Measuring productivity from vertically integrated sectors', *Economic Systems Research*,12, pp. 65–82.

Dietzenbacher E., et. al. (2000): 'Labour productivity in Western Europe 1975–1985: An intercountry, interindustry analysis', *Journal of Regional Science*, 40, pp. 425–452.

Domar E.D. (1961): 'On the measurement of technological change', *The Economic Journal*, 71, pp. 709–729.

Felipe J., Fisher F. M. (2003): 'Aggregation in production functions: what applied economists should know', *Metroeconomica*, 54, pp. 208–262.

Goodwin R. M. (1976): 'The use of normalized general coordinates in linear value and distribution theory', in Polenske K. R. and Skolka J. V., *Advances in Imput–Output Analysis*, Ballinger Publishing Company, Cambridge, MA.

Gossling W.F., Dovring F. (1966): 'Labor productivity measurement: the use of subsystems in the interindustry approach, and some approximating alternatives', *Journal of Farm Economics*, 48, pp. 369–377.

Gossling W.F. (1972): *Productivity Trends in a Sectoral Macro-economic Model*, Input-Output Publishing CO., London.

Han Z., Schefold B. (2006): 'An empirical investigation of paradoxes: reswitching and reverse capital deepening in capital theory', *Cambridge Journal of Economics*, 30, pp. 737–765.

Hulten C.R. (1978): 'Growth accounting with intermediate inputs', *The Review of Economic Studies*, 45, pp. 511–518.

Hulten C.R., et. al. (2001): *New Development in Productivity Analysis*, The University of Chicago Press, Chicago.

Kahn R.F. (1931): 'The relation of home investment to unemployment', *The Economic Journal*, 41, pp. 173–198.

Keynes J.M. (1936): *The General Theory of Employment, Interest, and Money*, Macmillan, London.

Kurek D. (2004): 'EUs handlingsplan for finansielle tjenesteydelser', *Kvartalsoversigt – 1. kvartal*, Danmarks Nationalbank

Kurz H. D., Salvadori N. (1995): *Theory of Production: A Long-Period Analysis*, Cambridge University Press, Cambridge.

Miller J.L., Gowdy J. (1998): 'Vertically integrated measures of the rate profit in the United States 1950–90', *Review of Income and Wealth*, 44, pp. 555–563.

OECD (2001): *Measurement of Aggregate and Industry-level Productivity*, OECD Manual, www.OECD.org

OECD (2008): *Productivity Indicators 2008*, OECD Compendium, www.OECD.org

Pasinetti L. L. (1973): 'The notion of vertical integration in economic analysis', *Metroeconomica*, 25, pp. 1–29;

Pasinetti L. L. (1977): *Lectures on the Theory on Production*, Macmillian Press, London.

Pasinetti L. L. (1988): 'Growing subsystems, vertically hyper-integrated sectors and the labour theory of value ', *Cambridge Journal of Economies*, 12, pp. 125–134.

Pasinetti L. L. (2000): 'Critique of the neoclassical theory of growth and distribution',

BNL Quarterly Review, no. 215, pp. 383–431.
Sraffa P. (1960): *Production of Commodities by Means of Commodities*, Cambridge University Press, Cambridge.
Tsoulfidis L. (2008): 'Price-value deviations: further evidence from input–output data of Japan', *International Review of Applied Economics*, 22, pp. 707–724.
Tsoulfidis L., Mariolis T. (2007): 'Labour values, prices of production and the effects of income distribution: Evidence from the Greek Economy', *Economic Systems Research*, 19, pp. 425–437.
Velupillai K., (2008): 'Sraffa's mathematical economics: a constructive interpretation', *Journal of Economic Methodology*, 15, pp. 325–342.

Appendices

2.A On the Construction of Subsystems

There are at least four conceptually different procedures to construct subsystems, Gossling's (1972) iterative method, Pasinetti's (1973) notion of vertically integrated sectors, Sraffa's reduction to dated quantities of labour, and the direct multiplier method presented in this chapter.[15] They all yield quantitatively identical results, but some applications follow more direct from one procedure than another.[16]

Let $[\boldsymbol{A}_t,\boldsymbol{B}_t,\boldsymbol{l}_t,\boldsymbol{O}_t,\boldsymbol{z}_t,\boldsymbol{U}_t,\boldsymbol{\tau}_t]$ be a set of data variables measured in physical quantities. The entries are respectively, the non-singular indecomposable semi-positive $n \times n$ *input-matrix*, the diagonal *gross output-matrix*, the column *vector of labour inputs*, the diagonal *final output-matrix*, the row *vector of the industries' total sales to means of production*, the *matrix of market share coefficients to other activities*, and the *vector of market share coefficients to final buyer*. $\boldsymbol{A}_t$ is composed of row vectors of intraindustry inputs and column vectors of interindustry flow. Furthermore, let $[\check{\boldsymbol{A}}_t,\check{\boldsymbol{l}}_t,\boldsymbol{I}]$ be the associated matrix of interindustry coefficients, the vector of direct labour coefficients, and the identity matrix, respectively.

Connected with this we have the following accounting identities, where $\boldsymbol{e}$ is a $n \times 1$ unit vector and $\widehat{\boldsymbol{b}}$ the diagonal from the corresponding $\boldsymbol{B}$ matrix, now a $n \times 1$ vector.

$$\boldsymbol{z}_t = \boldsymbol{e}'\boldsymbol{A}_t \tag{2.A.1}$$

$$\widehat{\boldsymbol{o}}_t = \boldsymbol{B}_t\boldsymbol{e} - \boldsymbol{z}_t' = \big(\boldsymbol{B}_t - \boldsymbol{A}_t'\big)\boldsymbol{e} \tag{2.A.2}$$

$$\boldsymbol{U}_t = \boldsymbol{A}_t\boldsymbol{B}_t^{-1} \tag{2.A.3}$$

$$\boldsymbol{\tau}_t = \widehat{\boldsymbol{o}}_t\big/\widehat{\boldsymbol{b}}_t \tag{2.A.4}$$

$$\boldsymbol{e} = (\boldsymbol{e}'\boldsymbol{U}_t)' + \boldsymbol{\tau}_t \tag{2.A.5}$$

$$\check{\boldsymbol{A}}_t = \boldsymbol{B}_t^{-1}\boldsymbol{A}_t \tag{2.A.6}$$

$$\check{\boldsymbol{l}}_t = \boldsymbol{l}_t\big/\widehat{\boldsymbol{b}}_t \tag{2.A.7}$$

In a paper by Gossling and Dovring (1966) and a book by Gossling (1972), both based on Gossling's unpublished PhD thesis from 1964, several years before Pasinetti's (1973) paper, it is shown how to construct *gross* and *final output subsystems* from input–output data measured in physical quantities as well as market prices. As will be seen later in this appendix Gossling's

[15] The fifth guise of the subsystems is found in Goodwin's Normalized General Coordinates approach, see e.g. Goodwin (1976). Conceptually, this approach seems very different and it should be checked whether or not there is a numerical difference.

[16] A note on terminology; the terms *incorporated* labour will be used when referring to the reduction to dated quantities of labour (equivalent to *embodied* labour), i.e., the classical notion used by Ricardo and Marx. The terms *direct* and *indirect* (e.g. labour) refer to the contemporary use of that input from respectively the industry producing the output and the supporting industries.

final output subsystems based on physical input–output data and Pasinetti's vertically integrated sectors yield quantitatively identical results.[17]

The following three sections provide an introduction to Pasinetti's vertically integrated sectors, Gossling's gross and final output subsystems, and on the reduction to dated quantities of labour.

2.A.1 Pasinetti's vertically integrated sectors

In the case of no joint production and excluding fixed capital. The *vector of vertically integrated labour coefficients* is given by:

$$\boldsymbol{v}_t = \left(\boldsymbol{I} - \check{\boldsymbol{A}}_t\right)^{-1} \check{\boldsymbol{l}}_t \tag{2.A.8}$$

Where $\left(\boldsymbol{I} - \check{\boldsymbol{A}}_t\right)^{-1}$ is the well known Leontief inverse. The ith entry of $\boldsymbol{v}_t$ constitutes the direct and indirect labour needed to produce one unit of the ith final output. Consequently, the aggregated labour directly and indirectly required to produce the final output of the n commodities is given by:

$$\boldsymbol{\iota}_t = \boldsymbol{O}_t \left(\boldsymbol{I} - \check{\boldsymbol{A}}_t\right)^{-1} \check{\boldsymbol{l}}_t = \boldsymbol{O}_t \boldsymbol{v}_t \tag{2.A.9}$$

Furthermore, the total quantities of the n commodities as respectively gross output and total outlays in each vertically integrated sector are given by:

$$\boldsymbol{\Pi}_t = \boldsymbol{O}_t \left(\boldsymbol{I} - \check{\boldsymbol{A}}_t\right)^{-1} \tag{2.A.10}$$

$$\boldsymbol{\Upsilon}_t = \boldsymbol{O}_t \check{\boldsymbol{A}}_t \left(\boldsymbol{I} - \check{\boldsymbol{A}}_t\right)^{-1} = \boldsymbol{O}_t \boldsymbol{H}_t \tag{2.A.11}$$

Where $\boldsymbol{H}_t = \check{\boldsymbol{A}}_t \left(\boldsymbol{I} - \check{\boldsymbol{A}}_t\right)^{-1}$ is the so-called *vertically integrated technical coefficient matrix.* For further details on vertically integrated sectors see Pasinetti (1973).

[17] The more historical oriented reader might add that the basic concepts behind the subsystems can be traced back to Petty, Smith, Ricardo, and more recently Hicks. See Pasinetti (1973), Scazzieri (1990), and Kurz and Salvadori (1995, pp. 175–80) on the origin of subsystems. However, the point we want to stress is that the procedure by which the subsystems can be constructed, first appears in Gossling's writings, but is always credited to Pasinetti. If this is because Gossling's work is unknown to most economists or that it is hitherto unknown that the two procedures yield identical results, is not clear to us.

This is not meant to discredit Pasinetti's work on vertically integrated sectors. The procedure presented in Pasinetti (1973) is much easier to apply than Gossling's iterative method, and the subsequent work on *dynamic subsystem* based on Pasinetti (1988) has developed much further the theoretical models.

2.A.2 Gossling's gross and final output subsystems

Following Gossling (1972) there are two distinct, but closely related subsystems, *viz.* the gross and the final output subsystem. The ith *gross output subsystem* is the system that produces the gross output associated with the ith industry from the original system and no final output in all other industries than the ith. The ith *final output subsystem* is the system that produces (and only produces) the final output associated with the ith industry from the original system. Only the final output subsystems are additive.

The matrix used by Gossling to transform the original system into gross output subsystems is the $[\boldsymbol{I} + \boldsymbol{P}_t]$ matrix, where $\boldsymbol{P}_t$ is the following matrix sum of an infinite series of restricted matrix products, where $\boldsymbol{U}_t$ (Equation 2.A.3) is the matrix of market share coefficients to other activities, common for both physical and value denominated systems, *viz.*

$$\boldsymbol{P}_t = \boldsymbol{U}'_{t\boldsymbol{B}}\boldsymbol{I} + \boldsymbol{U}'_{t\boldsymbol{\Gamma}}\boldsymbol{U}'_t + \boldsymbol{U}'_{t\boldsymbol{\Gamma}}[\boldsymbol{U}'_{t\boldsymbol{\Gamma}}\boldsymbol{U}'_t] + \boldsymbol{U}'_{t\boldsymbol{\Gamma}}\big[\boldsymbol{U}'_{t\boldsymbol{\Gamma}}[\boldsymbol{U}'_{t\boldsymbol{\Gamma}}\boldsymbol{U}'_t]\big] + \cdots \tag{2.A.12}$$

The B-operation is the usual matrix multiplication for square matrices, but with the main diagonal entries replaced with zeros. The Γ-operation is like the B-operation except that, in forming the scalar products associated with the ijth entry, the jth element from the jth column vector is replaced with a zero.[18]

The jth column in the $[\boldsymbol{I}+\boldsymbol{P}_t]$ is equal to the ith gross output subsystem multiplier $\overline{\boldsymbol{q}}^i_t$ presented in Section 2.2.1.

This procedure to construct subsystems, i.e., also the subsystem multiplier approach, is independent of the units of which the input–output data is denominated (physical quantities, current, or fixed market prices) as long as each industry's output has its own unique price.[19] The reason for this peculiar property is that Gossling's iterative procedure is based on the price independent market shares to other activities, and not the matrix of interindustry coefficients as in the Pasinetti's representation. Consequently, the procedure above is not only applicable on data measured in physical quantities as well as (constant) market prices, but yields identical multipliers.

[18]This iterative procedure is similar to Sraffa's construction of the *standard commodity.* See Velupillai (2008) for a discussion about the *constructive* mathematical logic behind this intrinsically algorithmic approach to the *mathematics of economics.* More general this is an excellent example of how mathematics more often should be applied in economics. As Gossling (1972, p. 29) writes "These matrices have been produced—some as 'by-products' in the quest for sub-systems' definition—using a direct method that bends the mathematics to the economic logic rather than *vice versa.*" Too much economics is bending to fit into conventional mathematical logic, in particular *classical real analysis.*

[19]In real input–output tables each industry's output is itself a composite commodity. Consequently, each entry possesses innate index number problems.

2.A.3 Reduction to dated quantities of labour

Closely related to the concept of subsystems we have the reduction to dated quantities of labour (Sraffa 1960, ch. 6).[20] The quantities of labour needed to produce one unit of final output of the individual commodities can be traced back in time as:

$$\check{\boldsymbol{l}} + \check{\boldsymbol{A}}\check{\boldsymbol{l}} + \check{\boldsymbol{A}}^2\check{\boldsymbol{l}} + \check{\boldsymbol{A}}^3\check{\boldsymbol{l}} + \cdots = \left(\boldsymbol{I} - \check{\boldsymbol{A}}\right)^{-1}\check{\boldsymbol{l}} \tag{2.A.13}$$

This is exactly Pasinetti's vector of vertically integrated labour coefficients.

Consequently, the total labour incorporated in each final and gross output is respectively given by:

$$\boldsymbol{\Pi} = \boldsymbol{O}\left(\boldsymbol{I} - \check{\boldsymbol{A}}\right)^{-1}\check{\boldsymbol{l}} \tag{2.A.14}$$

$$\boldsymbol{\Theta} = \boldsymbol{B}\left(\boldsymbol{I} - \check{\boldsymbol{A}}\right)^{-1}\check{\boldsymbol{l}} \tag{2.A.15}$$

Hence, the total non-discounted labour incorporated is equal to the total of the contemporary direct and indirect labour computed using the subsystem approach. It is a strange property that two conceptually so different procedures yield identical results. This Sraffa (1960, p. 89) could "see at a glance"!

This is however only the pure flow of physical quantities of labour. The discounted value of the dated labour is derived from the following identity.

$$\boldsymbol{p}(r) = w\check{\boldsymbol{l}} + (1+r)\check{\boldsymbol{A}}\boldsymbol{p}(r) \tag{2.A.16}$$

By recursively substituting the right-hand side $\boldsymbol{p}(r)$ with the right-hand side of the equation. The value in labour terms as a function of the distribution of the net nation product is given by:

$$\begin{aligned}\boldsymbol{p}(r) &= w\check{\boldsymbol{l}} + (1+r)\check{\boldsymbol{A}}\big(w\check{\boldsymbol{l}} + (1+r)\check{\boldsymbol{A}}\boldsymbol{p}(r)\big) = w\check{\boldsymbol{l}} + w(1+r)\check{\boldsymbol{A}}\check{\boldsymbol{l}} + w(1+r)^2\check{\boldsymbol{A}}^2\check{\boldsymbol{l}}\boldsymbol{p}(r)\\ &= w\big[\check{\boldsymbol{l}} + (1+r)\check{\boldsymbol{A}}\check{\boldsymbol{l}} + w(1+r)^2\check{\boldsymbol{A}}^2\check{\boldsymbol{l}} + \cdots + w(1+r)^t\check{\boldsymbol{A}}^t\check{\boldsymbol{l}} + \cdots\big]\\ &= w\big[\boldsymbol{I} + (1+r)\check{\boldsymbol{A}} + w(1+r)^2\check{\boldsymbol{A}}^2 + \cdots + w(1+r)^t\check{\boldsymbol{A}}^t + \cdots\big]\check{\boldsymbol{l}}\\ &= w\big[\boldsymbol{I} - (1+r)\check{\boldsymbol{A}}\big]^{-1}\check{\boldsymbol{l}} \qquad \forall\ \ 0 \le r < R\end{aligned} \tag{2.A.17}$$

Note that in the special case where $r = 0$ and the wage rate is chosen as the *numéraire*, the two measures (2.A.13) and (2.A.17) coincide.[21]

[20] See also Pasinetti (1977, ch. 4) and Kurz and Salvadori (1995, ch. 6).

[21] Two indices based on the reduction to dated quantities of labour seems natural to consider.

$$\varrho_t = \check{\boldsymbol{l}}_t + \check{\boldsymbol{A}}_t\check{\boldsymbol{l}}_t + \check{\boldsymbol{A}}^2_{t-1}\check{\boldsymbol{l}}_{t-1} + \check{\boldsymbol{A}}^3_{t-2}\check{\boldsymbol{l}}_{t-2} + \cdots + \check{\boldsymbol{A}}^k_{t-k-1}\check{\boldsymbol{l}}_{t-k-1} \tag{2.A.18}$$

$$\varphi_t = \frac{1}{R_t}\int_0^{R_t} \boldsymbol{p}(r)\,dr \tag{2.A.19}$$

The ϱ-index is an approximation of 2.A.13, but instead of using the same techniques of production when the inputs are traced back in time, the actual techniques used in each of the $k-1$ preceding periods are used. The φ-index is an attempt extract on scalar for each industry the time t.

2.B Subsystems Explained Using Examples

If otherwise not explicitly stated, the entries in the following examples are considered as physical quantities and all non-natural numbers in the following are rounded, i.e., minor discrepancies must be expected and "=" should be read as "≈".

	input-matrix				labour	gross	final
Industry 1	120	80	240	45	160	520	90
Industry 2	40	210	330	100	250	670	105
Industry 3	160	225	75	125	80	900	80
Industry 4	110	50	175	25	350	510	215
total use	430	565	820	295	840		

Table 2.5: *Original system*

	input-matrix				labour	gross	final
Industry 1	120	80	240	45	160	520	255
Industry 2	19	101	159	48	120	323	0
Industry 3	89	125	42	69	44	499	0
Industry 4	37	17	59	8	117	171	0
total use	265	323	499	171	441		

Table 2.6: *Gross output subsystem for Industry 1*

	input-matrix				labour	gross	final
Industry 1	42	28	85	16	56	183	90
Industry 2	7	36	56	17	42	114	0
Industry 3	31	44	15	24	16	176	0
Industry 4	13	6	21	3	41	60	0
total use	93	114	176	60	155		

Table 2.7: *Final output subsystem for Industry 1*

	input-matrix				labour	gross	final
Industry 1	55	37	111	21	74	240	0
Industry 2	40	210	330	100	250	670	263
Industry 3	99	140	47	78	50	559	0
Industry 4	45	20	72	10	143	209	0
total use	240	407	559	209	517		

Table 2.8: *Gross output subsystem for Industry 2*

	input-matrix				labour	gross	final
Industry 1	22	15	44	8	29	96	0
Industry 2	16	84	132	40	100	267	105
Industry 3	40	56	19	31	20	223	0
Industry 4	18	8	29	4	57	83	0
total use	96	162	223	83	206		

Table 2.9: *Final output subsystem for Industry 2*

	input-matrix				labour	gross	final
Industry 1	70	47	141	26	94	305	0
Industry 2	26	134	211	64	160	428	0
Industry 3	160	225	75	125	80	900	396
Industry 4	49	22	78	11	155	226	0
total use	305	428	504	226	489		

Table 2.10: *Gross output subsystem for Industry 3*

	input-matrix				labour	gross	final
Industry 1	14	9	28	5	19	62	0
Industry 2	5	27	43	13	32	87	0
Industry 3	32	45	15	25	16	182	80
Industry 4	10	4	16	2	31	46	0
total use	62	87	102	46	98		

Table 2.11: *Final output subsystem for Industry 3*

	input-matrix				labour	gross	final
Industry 1	66	44	132	25	88	285	0
Industry 2	19	101	158	48	120	321	0
Industry 3	90	127	42	70	45	507	0
Industry 4	110	50	175	25	350	510	342
total use	285	321	507	168	603		

Table 2.12: *Gross output subsystem for Industry 4*

	input-matrix				labour	gross	final
Industry 1	41	28	83	16	55	179	0
Industry 2	12	63	100	30	75	202	0
Industry 3	57	80	27	44	28	319	0
Industry 4	69	31	110	16	220	321	215
total use	179	202	319	106	378		

Table 2.13: *Final output subsystem for Industry 4*

2.B.1 The subsystem multiplier method

Using equation 2.2.1 the gross output multiplier for the first gross output subsystem can be computed as the non-trivial solution of the following system.

$$\left(\begin{bmatrix} 670 & 0 & 0 \\ 0 & 900 & 0 \\ 0 & 0 & 510 \end{bmatrix} - \begin{bmatrix} 210 & 225 & 50 \\ 330 & 75 & 175 \\ 100 & 125 & 25 \end{bmatrix} \right) \bar{\boldsymbol{q}}^1 = \begin{bmatrix} 80 \\ 240 \\ 45 \end{bmatrix}$$

The solution of which is $[\ 0.482 \quad 0.555 \quad 0.335\ ]'$. Next *squeeze in* a single "1" on the ith entry. Following this procedure the four gross output multipliers can be computed as.

$$\bar{\boldsymbol{q}}^1 = \begin{bmatrix} 1.000 \\ 0.482 \\ 0.555 \\ 0.335 \end{bmatrix}, \bar{\boldsymbol{q}}^2 = \begin{bmatrix} 0.461 \\ 1.000 \\ 0.621 \\ 0.409 \end{bmatrix}, \bar{\boldsymbol{q}}^3 = \begin{bmatrix} 0.586 \\ 0.639 \\ 1.000 \\ 0.444 \end{bmatrix}, \bar{\boldsymbol{q}}^4 = \begin{bmatrix} 0.548 \\ 0.480 \\ 0.564 \\ 1.000 \end{bmatrix}$$

Using equation 2.2.2 the final output multiplier for the first subsystem can be computed as.

$$\tilde{\boldsymbol{q}}^1 = \begin{bmatrix} 1.000 \\ 0.482 \\ 0.555 \\ 0.335 \end{bmatrix} \frac{520 - (120 + 40 + 160 + 110)}{520 - (120 + 0.482 \cdot 40 + 0.555 \cdot 160 + 0.335 \cdot 110)} = \begin{bmatrix} 0.353 \\ 0.170 \\ 0.200 \\ 0.118 \end{bmatrix}$$

The full set of final output multipliers is given by.

$$\tilde{\boldsymbol{q}}^1 = \begin{bmatrix} 0.353 \\ 0.170 \\ 0.120 \\ 0.118 \end{bmatrix}, \tilde{\boldsymbol{q}}^2 = \begin{bmatrix} 0.184 \\ 0.399 \\ 0.248 \\ 0.163 \end{bmatrix}, \tilde{\boldsymbol{q}}^3 = \begin{bmatrix} 0.119 \\ 0.130 \\ 0.202 \\ 0.090 \end{bmatrix}, \tilde{\boldsymbol{q}}^4 = \begin{bmatrix} 0.345 \\ 0.302 \\ 0.354 \\ 0.629 \end{bmatrix}$$

Now it is straightforward to compute the σ-, ξ-, α-, β-, and ρ-indices for the final output subsystems. Here computed for the first industry, Table 2.7:

$$\sigma^1 = \frac{\tilde{b}_{(1,1)} - \tilde{a}_{(1,1)}}{\boldsymbol{e}'\tilde{\boldsymbol{l}}^1} = \frac{\text{external output}}{\text{direct + indirect labour}} = \frac{183 - 42}{155} = 0.91$$

$$\xi^1 = \frac{\tilde{b}_{(1,1)} - \boldsymbol{e}'\tilde{\boldsymbol{a}}^1_{(:,1)}}{\boldsymbol{e}'\tilde{\boldsymbol{l}}^1} = \frac{\text{final output}}{\text{direct + indirect labour}} = \frac{90}{155} = 0.58$$

$$\alpha^1 = \frac{b_{(1,1)} - \boldsymbol{e}'\boldsymbol{a}_{(:,1)}}{b_{(1,1)}} = \frac{\text{final output*}}{\text{gross output*}} = \frac{90}{520} = 0.17$$

$$\beta^1 = \frac{\tilde{b}_{(1,1)} - \boldsymbol{e}'\tilde{\boldsymbol{a}}^1_{(:,1)}}{\tilde{b}_{(1,1)}} = \frac{\text{final output}}{\text{gross output}} = \frac{90}{183} = 0.49$$

$$\rho^1 = \frac{\beta^1 \tilde{l}_1}{\boldsymbol{e}'\tilde{\boldsymbol{l}}^1} = \frac{\text{direct labour}}{\text{direct + indirect labour}} = \frac{0.49 \cdot 56}{155} = 0.18$$

The asterisk '*' here denotes for the system as a whole, i.e., Table 2.5.

	σ^i	ξ^i	α^i	β^i	ρ^i
subsystem 1	0.91	0.58	0.17	0.18	0.49
subsystem 2	0.89	0.51	0.16	0.19	0.39
subsystem 3	1.69	0.81	0.09	0.07	0.44
subsystem 4	0.80	0.57	0.42	0.39	0.67

Table 2.14: *Collection of indices based on physical quantities*

The indices based on physical quantities and production prices are collected in Table 2.14 and 2.15, respectively. The indices based production prices are all computed with the first commodity as a *numéraire*.

	μ^i	ψ^i	γ^i	δ^i
subsystem 1	0.58	0.59	0.32	0.85
subsystem 2	0.57	0.56	0.36	0.76
subsystem 3	0.60	1.09	0.41	1.02
subsystem 4	0.50	0.38	0.32	1.04

Table 2.15: *Collection of indices based on production prices*

2.B.2 Pasinetti's vertically integrated sectors

The Leontief inverse associated with the system is given by:

$$\left(\boldsymbol{I}-\breve{\boldsymbol{A}}\right)^{-1} = \left(\boldsymbol{I}-\begin{bmatrix} 0.23 & 0.15 & 0.46 & 0.09 \\ 0.06 & 0.31 & 0.49 & 0.15 \\ 0.18 & 0.25 & 0.08 & 0.14 \\ 0.22 & 0.10 & 0.34 & 0.05 \end{bmatrix}\right)^{-1} = \begin{bmatrix} 2.04 & 1.26 & 1.96 & 0.67 \\ 0.91 & 2.55 & 2.12 & 0.79 \\ 0.77 & 1.08 & 2.27 & 0.57 \\ 0.93 & 0.94 & 1.48 & 1.49 \end{bmatrix}$$

The vector of vertically integrated labour coefficients is consequently given by:

$$\boldsymbol{v} = \left(\boldsymbol{I}-\breve{\boldsymbol{A}}\right)^{-1}\breve{\boldsymbol{l}} = \begin{bmatrix} 2.04 & 1.26 & 1.96 & 0.67 \\ 0.91 & 2.55 & 2.12 & 0.79 \\ 0.77 & 1.08 & 2.27 & 0.57 \\ 0.93 & 0.94 & 1.48 & 1.49 \end{bmatrix}\begin{bmatrix} 0.31 \\ 0.37 \\ 0.09 \\ 0.69 \end{bmatrix} = \begin{bmatrix} 1.73 \\ 1.96 \\ 1.24 \\ 1.76 \end{bmatrix}$$

Furthermore, the labour directly and indirectly required to produce the final output of the ith commodity is given by:

$$\boldsymbol{\iota} = \boldsymbol{O}\left(\boldsymbol{I}-\breve{\boldsymbol{A}}\right)^{-1}\breve{\boldsymbol{l}} = \boldsymbol{O}\boldsymbol{v} \tag{2.B.1}$$

Where $\boldsymbol{O}$ is a diagonal matrix of final outputs.

$$\boldsymbol{\iota} = \boldsymbol{O}\boldsymbol{v} = \begin{bmatrix} 90 & 0 & 0 & 0 \\ 0 & 105 & 0 & 0 \\ 0 & 0 & 80 & 0 \\ 0 & 0 & 0 & 215 \end{bmatrix}\begin{bmatrix} 1.73 \\ 1.96 \\ 1.24 \\ 1.76 \end{bmatrix} = \begin{bmatrix} 155 \\ 206 \\ 98 \\ 378 \end{bmatrix}$$

Note, that this is exactly the total use of labour in the final output subsystems found in Table A.3, A.5, A.7, and A.9. Furthermore, the total quantities of the n commodities as respectively gross output and total outlays in each vertically integrated sector are given by:

$$\begin{aligned}
\boldsymbol{\Pi} = \boldsymbol{O}\left(\boldsymbol{I} - \breve{\boldsymbol{A}}\right)^{-1} &= \begin{bmatrix} 90 & 0 & 0 & 0 \\ 0 & 105 & 0 & 0 \\ 0 & 0 & 80 & 0 \\ 0 & 0 & 0 & 215 \end{bmatrix} \begin{bmatrix} 2.04 & 1.26 & 1.96 & 0.67 \\ 0.91 & 2.55 & 2.12 & 0.79 \\ 0.77 & 1.08 & 2.27 & 0.57 \\ 0.93 & 0.94 & 1.48 & 1.49 \end{bmatrix} \\
&= \begin{bmatrix} 183 & 114 & 176 & 60 \\ 96 & 267 & 223 & 83 \\ 62 & 87 & 182 & 46 \\ 179 & 202 & 319 & 321 \end{bmatrix} \\
\boldsymbol{\Upsilon} = \boldsymbol{O}\boldsymbol{H} &= \begin{bmatrix} 90 & 0 & 0 & 0 \\ 0 & 105 & 0 & 0 \\ 0 & 0 & 80 & 0 \\ 0 & 0 & 0 & 215 \end{bmatrix} \begin{bmatrix} 1.04 & 1.26 & 1.96 & 0.67 \\ 0.91 & 1.55 & 2.12 & 0.79 \\ 0.77 & 1.08 & 1.27 & 0.57 \\ 0.93 & 0.94 & 1.48 & 0.49 \end{bmatrix} \\
&= \begin{bmatrix} 93 & 114 & 176 & 60 \\ 96 & 162 & 223 & 83 \\ 62 & 87 & 102 & 46 \\ 179 & 202 & 319 & 106 \end{bmatrix}
\end{aligned}$$

Compare with the gross output listed in Table A.3, A.5, A.7, and A.9 to see that these are the same.

2.B.3 Gossling's iterative method

The matrix of market share coefficients to other activities associated with the system presented in Table 2.5 is given by:

$$\boldsymbol{U} = \boldsymbol{A}\boldsymbol{B}^{-1} = \begin{bmatrix} 0.231 & 0.119 & 0.267 & 0.088 \\ 0.077 & 0.313 & 0.367 & 0.196 \\ 0.308 & 0.336 & 0.083 & 0.245 \\ 0.212 & 0.0746 & 0.194 & 0.049 \end{bmatrix}$$

From this the $\boldsymbol{P}$ and $[\boldsymbol{I}+\boldsymbol{P}]$ matrices are computed as:

$$\begin{aligned}
\boldsymbol{P} =& \boldsymbol{U}'_{\boldsymbol{B}}\boldsymbol{I} + \boldsymbol{U}'_{\Gamma}\boldsymbol{U}' + \boldsymbol{U}'_{\Gamma}[\boldsymbol{U}'_{\Gamma}\boldsymbol{U}'] + \boldsymbol{U}'_{\Gamma}[\boldsymbol{U}'_{\Gamma}[\boldsymbol{U}'_{\Gamma}\boldsymbol{U}']] + \cdots \\
=& \begin{bmatrix} 0 & 0.077 & 0.308 & 0.212 \\ 0.119 & 0 & 0.336 & 0.075 \\ 0.267 & 0.367 & 0 & 0.194 \\ 0.088 & 0.196 & 0.245 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0.172 & 0.149 & 0.114 \\ 0.134 & 0 & 0.160 & 0.114 \\ 0.083 & 0.089 & 0 & 0.100 \\ 0.093 & 0.106 & 0.105 & 0 \end{bmatrix} + \\
& \begin{bmatrix} 0 & 0.090 & 0.069 & 0.066 \\ 0.077 & 0 & 0.076 & 0.083 \\ 0.074 & 0.074 & 0 & 0.081 \\ 0.051 & 0.042 & 0.050 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0.052 & 0.032 & 0.046 \\ 0.053 & 0 & 0.036 & 0.061 \\ 0.044 & 0.038 & 0 & 0.055 \\ 0.036 & 0.028 & 0.023 & 0 \end{bmatrix} + \\
& \begin{bmatrix} 0 & 0.030 & 0.015 & 0.032 \\ 0.034 & 0 & 0.017 & 0.043 \\ 0.030 & 0.023 & 0 & 0.039 \\ 0.023 & 0.015 & 0.011 & 0 \end{bmatrix} + \ldots = \begin{bmatrix} 0 & 0.461 & 0.586 & 0.548 \\ 0.482 & 0 & 0.639 & 0.480 \\ 0.555 & 0.621 & 0 & 0.564 \\ 0.335 & 0.409 & 0.444 & 0 \end{bmatrix} \\
[\boldsymbol{I}+\boldsymbol{P}] =& \begin{bmatrix} 1 & 0.461 & 0.586 & 0.548 \\ 0.482 & 1 & 0.639 & 0.480 \\ 0.555 & 0.621 & 1 & 0.564 \\ 0.335 & 0.409 & 0.444 & 1 \end{bmatrix}
\end{aligned}$$

Compare with the gross subsystem multipliers $\overline{\boldsymbol{q}}^1$, ..., $\overline{\boldsymbol{q}}^4$ to see that these are identical to the ith columns in the $[\boldsymbol{I}+\boldsymbol{P}]$ matrix. The Γ-operation is here presented as a directly applicable M-code.

```
function [Z]=gossling_gamma(X,Y);
n=size(X,1); e=ones(n,1);
for i=1:1:n;
    for j=1:1:n;
        a=e; a(j)=0;
        Z(i,j)=X(i,:)*(Y(:,j).*a);
        if i==j;
            Z(i,j)=0;
        end;
    end;
end;
```

For a full description including proofs see Gossling (1972).

2.C Core Variables

$\boldsymbol{A}$	input-matrix	$n \times n$	physical quantities
$\check{\boldsymbol{A}}$	matrix of interindustry coefficients	$n \times n$	
$\boldsymbol{B}$	output-matrix	$n \times n$	physical quantities
$\boldsymbol{O}$	final output	$n \times n$	physical quantities
$\boldsymbol{H}$	vertically integrated technical coefficient matrix	$n \times n$	
$\boldsymbol{I}$	identity matrix	$n \times n$	
$\boldsymbol{P}$	Gossling's P	$n \times n$	
$\boldsymbol{U}$	market shares coefficients to other activities	$n \times n$	
$\boldsymbol{e}$	unit vector	$n \times 1$	
$\boldsymbol{l}$	direct labour inputs	$n \times 1$	physical quantities
$\check{\boldsymbol{l}}$	direct labour input coefficients	$n \times 1$	
$\boldsymbol{v}$	vertically integrated labour coefficients	$n \times 1$	
$\boldsymbol{p}$	production prices	$n \times 1$	
$\boldsymbol{q}$	subsystem multiplier	$n \times 1$	
$\boldsymbol{\tau}$	market share coefficients to final buyer	$n \times 1$	
$\boldsymbol{z}$	industries total sales to means of production	$1 \times n$	physical quantities
w	uniform wage-rate	1×1	
R	maximum rate of profit	1×1	

2.D Data

The industry classification used in this study is as follows, where the numbers in the brackets refer to their original classification used by Statistics Denmark, www.dst.dk/inputoutput.

(1) Agriculture {1}
(2) Horticulture, orchards etc. {2}
(3) Agricultural services, landscape gardeners etc. {3}
(4) Forestry {4}
(5) Fishing {5}
(6) Extr. of crude petroleum, natural gas etc. {6}
(7) Extr. of gravel, clay, stone and salt etc. {7}
(8) Mfr. of food, beverages and tobacco {8–18}
(9) Mfr. of textiles, wearing apparel, leather {19–21}
(10) Mfr. of wood and wood products {22}
(11) Mfr. of paper prod., printing and publish {23–26}
(12) Mfr. of refined petroleum products etc. {27}
(13) Mfr. of chemicals and man-made fibres etc. {28–35}
(14) Mfr. of rubber and plastic products {36–38}
(15) Mfr. of other non-metallic mineral products {39–41}
(16) Mfr. and processing of basic metals {42–47}
(17) Mfr. of machinery and equipment n.e.c. {48–52}
(18) Mfr. of electrical and optical equipment {53–56}
(19) Mfr. of transport equipment {57–59}
(20) Mfr. of furniture, manufacturing n.e.c. {60–62}
(21) Electricity supply {63}
(22) Gas and water supply {64–66}
(23) Construction {67–70}
(24) Sale and repair of motor vehicles etc. {71–73}
(25) Ws. and commis. trade, exc. of m. vehicles {74}
(26) Retail trade of food etc. {75}
(27) Department stores {76}
(28) Re. sale of phar. goods, cosmetic art. etc. {77}
(29) Re. sale of clothing, footwear etc. {78}
(30) Other retail sale, repair work {79}
(31) Hotels and restaurants {80–81}
(32) Land transport, transport via pipelines {82–85}
(33) Water transport {86}
(34) Air transport {87}
(35) Support. trans. activities, travel agencies {88–89}
(36) Post and telecommunications {90}
(37) Financial intermediation {91–92}
(38) Insurance and pension funding {93–94}
(39) Activities auxiliary to finan. intermediat. {95}
(40) Real estate activities {96–98}
(41) Renting of machinery and equipment etc. {99}
(42) Computer and related activities {100–101}
(43) Research and development {102–103}
(44) Consultancy etc. and cleaning activities {104–109}
(45) Public administration etc. {110–113}
(46) Education {114–118}
(47) Health care activities {119–120}
(48) Social institutions etc. {121–122}
(49) Sewage and refuse disp. and similar act. {123–125}
(50) Activities of membership organiza. n.e.c. {126}
(51) Recreational, cultural, sporting activities {127–128}
(52) Other service activities {129–130}

Chapter 3

The Technological Frontier

An International and Inter-industrial Empirical Investigation of Efficiency, Technological Change, and Convergence

Written jointly with Stefano Zambelli

3.1 Introduction

The concept of technological progress is not a simple or straightforward one. Intratemporal and intertemporal comparison of technological possibilities is problematic, because also in its ideal type conceptualization the intrinsic nature of most commodities changes according to time and space. New commodities are introduced and either substitute old ones or coexist; new methods of production and new markets emerge so as to influence the prices and trade; exhaustible resources are depleted cutting off access to old production techniques; and so on and so forth. Normally the process of technological innovation is studied focusing on the creation of new products or on new ways to produce the same product. But for the whole system the detection of technological progress is problematic.

The problem of measuring technological progress is related to aggregation and hence to some form of indexation. Normally comparison between bundles of different types of commodities is in the literature made following two methods or a combination of them: the *value method* and the *index number method*. In the value method the different commodities are assigned a value in terms of the value of a *numéraire* (which is either one single commodity or a bundle of commodities), while in the index number method the heterogeneous physical commodities are transformed into an index number. Both methods are problematic and have been widely discussed in the literature. The value method has been studied since Smith and Ricardo's labour theory of value and the index number problem is still centred around Irving Fisher's 1922 study on 'The Making of Index Numbers'.

The literature on index number is enormous and here we will not make a review. What are sufficient to point out are two things. Firstly, that the function of an index number is to transform something which is intrinsically heterogeneous into a homogeneous (scalar) magnitude.

Second, that there is broad consensus that the ideal index number, in the sense of Fisher (1922), does not exist and hence cannot be constructed. See among many Leontief (1936), Afriat (1977), Samuelson and Swamy (1974), and Velupillai and Zambelli (1993). Samuelson and Swamy (1974, p. 568) in their survey on invariant economic index numbers summarize and declare at the outset that

> we cannot hope for one ideal formula for the index number: if it works for the tastes of Jack Spratt, it won't work for his wife's tastes; if, say, a Cobb-Douglas function can be found that works for him with one set of parameters and for her with another set, their daughter will in general require a non-Cobb-Douglas formula! Just as there is an uncountable infinity of different indifference contours—there is no counting tastes—there is an uncountable infinity of different index number formulas, which dooms Fisher's search for the ideal one. It does not exist even in Plato's heaven.

The approach taken in this chapter, to capture the state of technological progress, goes through the so-called *technological frontier*. The technological frontier shows, for a given uniform rate of profit, what combination of production activities that would yield the highest wage income, i.e., the envelope of all the possible wage-profit frontiers that can be constructed from a given set of production techniques; equivalent to the cost minimising choice of production activities.[1] One advantage of using wage-profit frontiers is that it takes into account the fact that new techniques generate a different demand vector of the factors of production and eventually different prices and hence different costs and revenues.

From this approach we will construct indices that in several ways differ from the orthodox indices referred to by Samuelson and Swamy in the quotation above. The orthodox indices take as given (homothetic) preference and use these as weights in the process of aggregation. As pointed out, Jack Spratt might possess a particular Cobb-Douglas utility function, but his family might not.[2]

The method we propose use on the other hand production costs, measured in prices of production, as weights in the process of aggregation. The prices of production have the advantage of taking into account both the production techniques and the demand for means of production. Hence, where neoclassical indices are generated using axiomatic preferences, we use uniquely determined (endogenous) prices of production.[3] The prices of production do not however take into account the demand for final consumption, but following the *Non-substitution theorem*, the production prices are independent of the composition of the vector of final consumption.[4] It is a major advantage that the wage-profit frontiers are independent of simple differences in the scale of production, even if the changes are asymmetric across industries.

Take note, this does not imply that we assume *constant returns to scale.* If changes occur in the scale of production in one or more industries, without changing the proportional use of the means of production (including labour), then the wage-profit frontiers, together with the indices based hereupon, remain unaffected. If on the other hand the relative proportions of the means

[1]This problem could just as well have been stated as the highest level of consumption for a given uniform growth rate. For more information on this duality see Pasinetti (1977, ch. 7), Bruno (1969), and Burmeister and Kuga (1970).

[2]For those, as we, who wonder whom this mysteries Jack Spratt might be. He, spelled as Jack Sprat, appears in a fitting nursery rhyme, (thanks to Wikipedia):

> Jack Sprat could eat no fat.
> His wife could eat no lean.
> And so between them both, you see,
> They licked the platter clean.

[3]For an introduction to the neoclassical theory of index numbers and productivity measurement, see Coelli et. al. (2001).

[4]Explained in Section 3.2.

of production are affected, then it will influence the vector of production prices, which subsequently together with the new production techniques are used to assess the consequences of the technical innovations for the economy as a whole.

The technological frontier is found in both 'smooth' neoclassical economics and 'discrete' technologies models. As observed by Bruno (1969, p. 51) "any neo-classical technology could be simulated by a 'very dense' spectrum of discrete techniques", hence the approach taken in this chapter is not necessarily restricted to the unorthodox setting in which it is framed.

We compute and study three different versions of the so-called technological frontier using input–output data from eight OECD countries from 1970–2005 with five year intervals. The three frontiers we shall call respectively; the *contemporary*, the *rolling*, and the *intertemporal* technological frontier—all theoretical constructs with their own set of useful applications.

The *contemporary technological frontiers* are constructed from all the production techniques extracted from the OECD input–output tables available for a given year, i.e., from a range of countries at a given point in time (one accounting period). Comparing the contemporary technological frontiers with the actual wage-profit frontiers for the individual countries can be used to study efficiency since the contemporary technological frontier, at a given point in time, provides a measure of the maximum potentials for international trade and/or gains by exchange of production techniques. Furthermore, the evolution of the contemporary technological frontiers provides a measure of global technological progress.

The *rolling technological frontiers* are the envelopes formed by the production techniques available in 1970; 1970,1975; ...; 1970,1975,...,2005, i.e., a backward looking set of production techniques, which together with the contemporary technological frontiers are used to study which countries' industry-level production techniques that are the most effective, and how this displacement of production techniques evolves over time.

The *intertemporal technological frontier*, equivalent to the last of the rolling technological frontiers, is computed from the full set of techniques available over time and across countries. The intertemporal technological frontier provides a theoretical global and intertemporal measure of the maximum economic potentials for the 'world economy' as a whole.[5] The intertemporal technological frontier is also useful in the study of convergence. In particular, as an alternative to the usual approach of using the US as a reference point, this is problematic because the reference point itself change over time.

The country specific wage-profit frontiers, the contemporary technolog-

[5]By the world economy we here mean the eight OECD countries. Contingent on data availability, this analysis could and should of course be extended to include additional countries.

ical frontiers, and the intertemporal technological frontier are combined to construct indices for country specific technological progress and global convergence towards the maximum theoretical technical potentials. These indices provide both empirically and conceptually new insight to the well-known *catching up hypothesis*.[6] Among the problems confronted, is the fundamental question; is the US (the leader) catching up?

Common for the different versions of the technological frontiers is that they can be seen as an empirical proxy for what is known in the literature on economic development as the *access to technology constraint*, i.e., the situation in which a country is approaching the technological frontier and consequently *ceteris paribus* finds it more difficult to substitute currently used techniques of productions with more efficient ones.[7]

To actual obtain these results it has been necessary to develop an algorithm that in an effective way computes these frontiers. The mathematical notion of an envelope is conceptual straightforward, but the natural *brute force* algorithm associated with the computation of such an envelope is for every single point computational infeasible. This problem and the algorithm developed to solve it is fully described in Section 3.5, but can be skipped by readers more interested in the empirical results.

Section 3.2 presents the theoretical framework, Section 3.3 the indices used, Section 3.4 the data, and Section 3.6 the empirical results. Section 3.7 concludes the chapter.

3.2 The Technological Frontier

The economic system consists of n industries each producing one unique commodity by means of some combination of the n commodities and labour.[8]

Let $\boldsymbol{A}$ be a $n \times n$ indecomposable semi-positive non-singular matrix of interindustry coefficients, where the ijth entry represents the ith industry's use of the jth commodity in the production of one unit of the industry's output. Likewise, $\boldsymbol{l}$ is a $n \times 1$ vector of labour input coefficients where the ith entry represents the ith industry's use of labour in the production of one unit of output. As usual these elements can be collected in the following long-run equilibrium relationship that captures the distribution of the total production among wages, profits, and means of production, where the wage and profit rates are assumed to be uniform:

$$\boldsymbol{A}\boldsymbol{p}(1+r) + \boldsymbol{l}w = \boldsymbol{p} \tag{3.2.1}$$

[6] See among many Abramovitz (1986).

[7] See Ernst et. al. (1998, p. 15–16), where the the access to technology constraint is discussed in relation to Korea and Taiwan in the 1980s and 1990s.

[8] This theoretical part is based on the seminal work by von Neumann (1945–46), Leontief (1941), and Sraffa (1960), and subsequent work found in Pasinetti (1977), Velupillai and Zambelli (1993), and Zambelli (2004).

Choosing a *numéraire* $\boldsymbol{\eta}$, for which it holds that $\boldsymbol{\eta}'\boldsymbol{p} = 1$, the degrees of freedom reduces from two to one, such that for a given rate of profit, the wage rate can be computed by isolating $\boldsymbol{p}(r, \boldsymbol{A}, \boldsymbol{l}) = \big(\boldsymbol{I} - \boldsymbol{A}(1+r)\big)^{-1}\boldsymbol{l}w$, premultiplying with the *numéraire*, and rearranging, *viz.*

$$w(r, \boldsymbol{A}, \boldsymbol{l}) = \Big(\boldsymbol{\eta}'\big(\boldsymbol{I} - \boldsymbol{A}(1+r)\big)^{-1}\boldsymbol{l}\Big)^{-1} \qquad r = \{r \in \mathbb{Q} : 0 \leq r \leq R\} \quad (3.2.2)$$

Where R, the maximum rate of profit, can be computed as $R = \lambda^{-1} - 1$, where again λ is the maximum eigenvalue of $\boldsymbol{A}$.

It must be stressed that the production prices $\boldsymbol{p}(r, \boldsymbol{A}, \boldsymbol{l})$ and the wage rate $w(r, \boldsymbol{A}, \boldsymbol{l})$ are scale-independent, not only of the scale of the economy

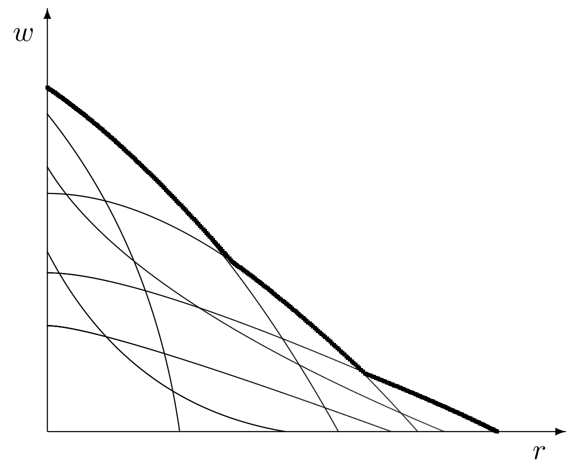

Fig. 3.1: *The technological frontier*

as a whole, but also the scale of production in the single industries. This property is known as the Non-Substitution Theorem.[9]

For each unique set of techniques $\{E^{(k)}\} = \{\boldsymbol{A}^{(k)}, \boldsymbol{l}^{(k)}\}, k = 1, 2, ..., m$, from the set of systems $\boldsymbol{E} = \{E^{(1)}, E^{(2)}, ..., E^{(m)}\}$ there is a unique wage-profit frontier. The envelope of these frontiers, illustrated in Figure 3.1, is the technological frontier, *viz.*

$$w^{\mathrm{TF}}(r, \boldsymbol{E}) = \max\left\{w(r, E^{(1)}), w(r, E^{(2)}), ..., w(r, E^{(m)})\right\} \quad (3.2.3)$$

As defined in the introduction we study three versions of the technological frontier; the contemporary $w_t^{\mathrm{CTF}}(r, \boldsymbol{E}_t)$, the rolling $w_t^{\mathrm{RTF}}(r, \boldsymbol{E}_1, \boldsymbol{E}_2, ..., \boldsymbol{E}_t)$, and the intertemporal $w^{\mathrm{ITF}}(r, \boldsymbol{E})$, where $\boldsymbol{E}_t$ denotes the set of techniques used at time t and $\boldsymbol{E}$ the total set of techniques. An obvious analytical property of these three versions of the technological frontier is that:[10]

$$w_t^{\mathrm{CTF}}(r, \boldsymbol{E}_t) \leq w_t^{\mathrm{RTF}}(r, \boldsymbol{E}_1, \boldsymbol{E}_2, ..., \boldsymbol{E}_t) \leq w^{\mathrm{ITF}}(r, \boldsymbol{E}) \ \forall\ t = 1, 2, ..., T \quad (3.2.4)$$

[9] See Kurz and Salvadori (1995, p. 26–28) for a discussion of the origin and implications of this peculiar result.

[10] Since $\{\boldsymbol{E}_t\} \subseteq \{\boldsymbol{E}_1, \boldsymbol{E}_2, ..., \boldsymbol{E}_t\} \subseteq \{\boldsymbol{E}\}\ \forall\ t = 1, 2, ..., T$

For convenience and completeness we restate the analytical properties associated with the technological frontier. For proofs and further discussion of these properties, see Pasinetti (1977 p. 158–59).

1. At the switch point between two techniques, each commodity has the same price.
2. If, for a given rate of profit, one technique dominates another, then it will yield prices, in terms of the wage rate, that are strictly lower than those yielded by the other technique.
3. The switch points are independent of the choice of *numéraire*.
4. The technological frontier is strictly decreasing as the rate of profit increases.
5. (Corollary) At the switch points between two techniques, the change will occur in one, and only one, industry, i.e., piecemeal.

For the purpose of computing and interpreting the technological frontier, property number three is very convenient, since it implies that the set of technologies forming the technological frontier is independent of the choice of *numéraire*. Since an objective of this study is to construct indices for comparative studies it is imperative that these indices are (more or less) independent of the *numéraire* chosen to compute it. However, while the set of techniques constituting the technological frontier is independent of the choice of *numéraire*, the shape of the frontier is not. Consequently, the choice of *numéraire* will to some extent influence our results, but the influence will be suppressed by the stability of the switch points. In the analysis, the number of switch points on the envelopes will be reported to provide a first approximation of the robustness of our results.

Furthermore, as will be clear later, property number five turns out, from a computational point of view, to be extremely convenient.

3.3 The Velupillai-Fredholm-Zambelli Index

The technological frontier can be interpreted as an *access to technology constraint*, since it provides a proxy for the maximum potential level of productivity. The technological frontier allows us to reformulated, in a more general terms, the well known *catching up hypothesis*, i.e., that the growth rate in productivity varies inversely with the productivity level. Replacing the US (the leader) with the intertemporal technological frontier, allows us to study the same problems, but with a benchmark extracted from the entire sample. Furthermore, we are now able to address the question 'is the US catching up?' Why should the US not be able to catch up to something more efficient already potentially available in the system, defined by the technological frontier? Naturally, this also provides a convenient framework in which to study *overtaking*, i.e., if one country should overtake the leader.

To study this and more, we construct what we shall call the country specific *Velupillai-Fredholm-Zambelli* (VFZ) index that provides a measure of the average efficiency relative to the intertemporal technological frontier. For the jth country at time t the VFZ-index is computed as.

$$VFZ_{j,t} = 1 - \frac{1}{R_{j,t}} \sum_{r=0}^{R_{j,t}} \Big[w^{\mathrm{ITF}}\big(r, \boldsymbol{E}\big) - w\big(r, \boldsymbol{A}_{j,t}, \boldsymbol{l}_{j,t}\big) \Big] \tag{3.3.1}$$
$$j = 1, 2, ..., N, \quad t = 1, 2, ..., T$$

In words, the VFZ-index is computed as one minus the average vertical distance between the individual countries wage-profit frontiers and the intertemporal 1970–2005 technological frontier. The range of the index is between zero and one. The closer the index is to unity the more efficient is the technology used in the single country relative to the theoretical maximum computed from the entire set of production activities.[11]

An analogue global version of the VFZ-index is computed from the vertical distances between the contemporary technological frontiers and the intertemporal technological frontier, *viz.*

$$VFZ_t^{\mathrm{global}} = 1 - \frac{1}{R_t^{\mathrm{CTF}}} \sum_{r=0}^{R_t^{\mathrm{CTF}}} \Big[w^{\mathrm{ITF}}\big(r, \boldsymbol{E}\big) - w_t^{\mathrm{CTF}}\big(r, \boldsymbol{E}_t\big) \Big] \tag{3.3.2}$$
$$t = 1, 2, ..., T \tag{3.3.3}$$

Where R_t^{CTF} is the maximum profit rate associated with the contemporary technological frontier at time t. The global VFZ-index provides a measure of the technological progress for the global economy as a whole.

The advantages of the VFZ-indices over conventional ones are:

1. The method is non-parametric and non-stochastic.
2. Technology, value, and aggregation are fully integrated through the prices of production, hence to some extend circumvents standard index number and value problems.
3. The indices are time-invariant, i.e., they are fully determined within single accounting period.[12]
4. The stability of the switch points greatly limits the sensitivity of changes in the *numéraire.*
5. The interdependence among industries is endogenously captured by changes in the prices of production.

[11] An alternative index could be computed using some proxy for the actual distribution among wages and profits. Hence, using only one point on (or a segment of) each frontier.

[12] However, updating the entire dataset with new data, say the 2010 OECD tables, will almost certainly change the intertemporal technological frontier, but the within-period ranking will remain unaffected.

6. The indices will not change as a consequence of simple changes in the scale of production in the single industries, but only if real technological innovations are observed in one or more industries.[13]
7. In the study of convergence, the benchmark/reference point is determined from the system as a whole and not simple a 'leading country'.

3.4 Data and the Choice of *Numéraire*

For the actual computation of the technological frontiers we have chosen the OECD 1970–2005 input–output tables from the US, Germany, the UK, France, Canada, Denmark, Japan, and Australia. All based on the the ISIC 2 or ISIC 3 classifications with respectively 35 and 48 industries.[14] The tables contain both the domestic interindustrial flow and industry-specific imports of capital goods.

Some problems of comparability exist between the two methods of classification, but steps have been taken to minimize these problems. The initial 48 and 35 industries have been aggregated into 23 industries following standards of national accounting. The main reason for so doing is that it ensures comparability over time and non-singular matrices for the whole dataset.

Unfortunately, tables are not available for all countries for all time periods. To further increase comparability we have chosen to substitute the missing tables with the most commensurable table, typically the table from the previous accounting period in the same country. For details, see Table 3.2 in Appendix 3.A.

As labour inputs we use data from the OECD on the industry-level 'compensation of employees' and use this to distribute the total employment in hours to the single industries. When available we use detailed industry-level employment data from The Groningen Growth and Development Centre.[15] Note, that both over time and across industries labour is treated as a homogeneous input. This is a very strong assumption, but necessary given the data availability.

There is a fundamental problem related to the units of accounting, since the tables are denominated in current values of the national currency. Macro-industry deflators have been computed as the differences between macro-industry GDP denomination in respectively current and base period prices, and used to deflate the value denominated tables. This is probably the best available proxy for the physical flow among industries found in the OECD input–output tables. For a discussion on monetary vs. physical denominated input–output data, see Han and Schefold (2006, p. 750).

[13] By real technological innovations we mean changes in the matrix of technological coefficients and/or in the corresponding (normalised) vector of labour inputs.

[14] See www.OECD.org.

[15] See www.GGDC.net.

Appendix 3.A contains additional information on the data used.

As a *numéraire* we choose the vector of domestic net products from the base year 2000 normalised with the total hours worked. We use the domestic net products, because if imported means of production were subtracted we would not necessarily obtain a vector of non-negative entries.[16] Section 3.6.4 discusses the relationship between the choice of *numéraire* and the stability of our results based on the number of switch points on the envelopes.

3.5 Algorithms

Formally speaking, the problem of computing a technological frontier is *computable*, i.e., there exist an algorithmic procedure that in a finite number of steps can compute it, or equivalent under the Church-Turing thesis there exists a Turing-machine that always halts.[17] The formal proof of this is given in terms of the following brute-force algorithm.

3.5.1 A brute-force algorithm

1. import data and convert the data into matrices of technical coefficients
2. loop through all possible systems, $k = 1 : 1 : N^n$
   (a) compute the maximum eigenvalue $\lambda(k)$ for the kth system
3. use the minimum $\lambda(k)$ to compute $\overline{R}$ associated with the technological frontier
4. loop through all possible systems
   (a) compute the wage rate for incremental steps of $0 < r < \overline{R}$
5. for each r select the system associated with the maximum wage rate

However, when using the above algorithm the computational complexity of the problem implies that it is practical impossible to compute the technological frontier for even small datasets, since for each rate of profit all possible combinations of techniques must be evaluated. Using the *Big-O notation* the time-complexity is (at least) $O(N^n)$. This implies that no matter how powerful a computer that will be developed within, say the next century, it will always be possible to include addition available data, such that the algorithm will not halt within any reasonable time frame.[18] What makes this

[16] See also the discussion in Subsection 2.3.1 in Chapter 2.

[17] Note that the domain of the technological frontier, see Equation 3.2.2 and 3.2.3, has been defined on the rational line.

[18] A *back-on-the-envelope* (!) estimate of the computer power needed to compute the intertemporal technological frontier based on $(T \cdot N)^n = (8 \cdot 8)^{23} \approx 3.5 \cdot 10^{41}$ unique systems; for just one rate of profit, running a whole year, the computer must evaluate $1.1 \cdot 10^{34}$ systems per second, each including several matrix operations. Not anything near such a computer exists today or will within any reasonable time frame.

problem serious for even rather small values of N and n, is that the algorithmic procedures must contain computations of eigenvalues and inversions of $n \times n$ matrices.

The computational complexity can however be drastically reduced (in the order of N^n to $N \cdot n$) by exploiting property number five listed in Section 3.2. Using any point on any frontier the following procedure, so to say, climbs the individual wage-profit frontiers using the switch points as stepping stones.

3.5.2 The Piecemeal algorithm

1. import data and convert it into matrices of technical coefficients
2. choose an initial point on any frontier
3. from this point, while $r > 0$, lower the profit rate one increment[19] and compute the wage rate without changing the techniques, save this as $\overline{w}$
   (a) one by one, change the techniques (piecemeal), i.e., $n \cdot (N-1)$ times, and for each system
      i. if the profit rate is smaller than the maximum profit rate, compute the wage rates
      ii. if this wage rate is greater than $\overline{w}$, then we have passed a switch point. Fix the new set of techniques and the associated wage rate. Else use $\overline{w}$
4. Now reverse the procedure, while $w > 0$, increase the profit rate one increment and compute the wage rate without changing the techniques, save this as $\overline{w}$
   (a) one by one, change the techniques and for each system:
      i. if the profit rate is smaller than the maximum profit rate, compute the wage rates.
      ii. if this wage rate is greater than $\overline{w}$, then we have passed a switch point. Fix the new set of techniques and the associated wage rate. Else use $\overline{w}$
5. go to point # 3 as long as loop # 3 and 4 do not produce identical results, else terminate and collect the results

Both algorithms can be implemented with no serious demand for the available memory, but unlike the brute-force algorithm the Piecemeal algorithm cannot be run parallel.

An easy way to verify the outcome from the Piecemeal algorithm is to apply the two algorithms on a tractable subset and check that they yield identical results. This has been done with positive results.[20]

[19] In the actual computation the stepsize is fixed at $\frac{1}{1000}$. Between $\frac{1}{500}$ and $\frac{1}{1000}$ the number of switch point increased, which implies that the algorithm missed some switch points. No changes in the results are found when narrowing the stepsize to $\frac{1}{2000}$.

[20] There exist one potential problem; it is theoretical possible, by some fluke, that the envelope is not connected by intersections with the initially chosen frontier. However, the probability of this occurring tends to zero as the number of techniques tends to infinity.

The full set of results based on the eight OECD countries for eight time periods can be computed within a few hours, with the Piecemeal algorithm using a standard desktop computer.

3.6 Efficiency, Technological Change, and Convergence

This analysis is both from a theoretical and empirical point of view 'average', as oppose to 'marginal', since it deals with average costs, returns, revenues, etc. while mainstream (marginal) theory focus on the corresponding marginal magnitudes. Given that marginal magnitudes can never be observed, but average magnitudes can, this is the appropriate approach to empirical studies. However, in one case where the orthodox theory is average, this approach is specific; we do not assume a representative firm.

A general problem associated with the measurement of technological progress is related to the fact that different production activities use different sets of factors of production. For example one can consider the production of energy; nuclear energy, wind mills, hydro-power, solar-energy, oil, coal, gas, etc. It would be rather difficult to asses which production process that is most efficient. Moreover, it is not always the case that the adoption of a new method indicates that the method is superior. There might be other reasons different from technological superiority, and the expected costs could differ from from the actual costs.

The economic system as a whole most likely adopts a combination of the different methods of production. Consequently, the observed output from an industry and the corresponding vectors of industry inputs are not only average over the accounting period (as it should be), but also average across the techniques used. However, this problem should diminish as the number of industries in the national accounting increases.

3.6.1 The empirical technological frontiers

Figure 3.2 shows the complete collection of contemporary and rolling technological frontiers. Analogue to the study of the wage-profit frontiers for the individual countries, an outward shift of the frontier implies unambiguously technological progress. If two frontiers intersect, it cannot unambiguously be determined whether of not a higher level of productivity is reached.

The contemporary technological frontiers show a clockwise and steady shift outwards, while the rolling technological frontiers show a more parallel shift. This difference provides a first-hand insight into the nature of the global technological progress. But however tempting as it might be, it is not unambiguous, to interpret the shifts of the contemporary technological frontiers as a global labour-saving technological progress, since the value of the circulating capital not necessarily changes monotonic with the profit rate.

The problem of intersection(s) between frontiers does however not exist for the rolling technological frontier, since these by construction will never intersect. Consequently, together with the other frontiers, this property makes the rolling technological frontier a strong analytical tool. An observed

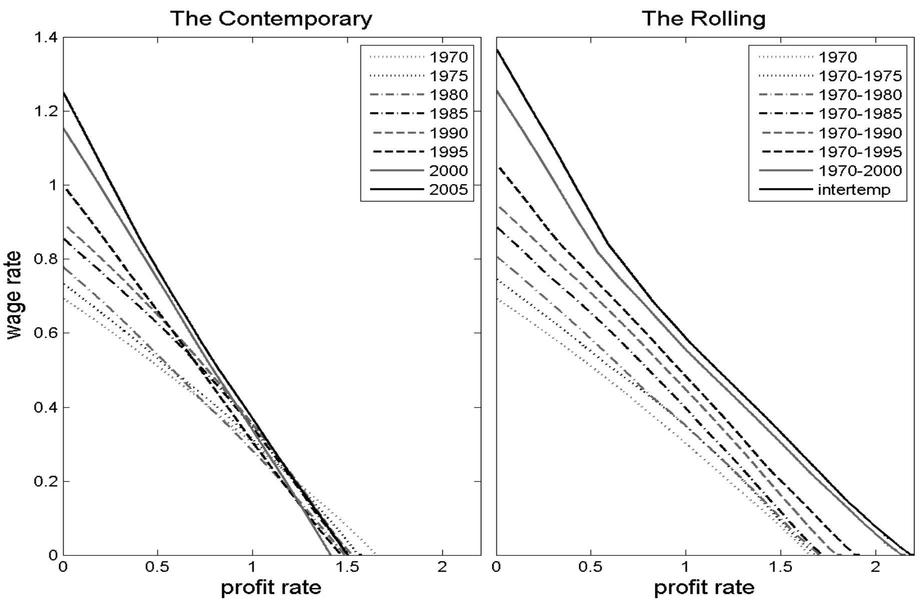

Fig. 3.2: *The contemporary and rolling and technological frontiers*

difference between the contemporary and rolling frontiers implies that there exist some combinations of the old and new production techniques, which are more productive than all combinations of the techniques currently used.

However, it could be argued that some old techniques of production should be discarded from the set of techniques forming the rolling (and intertemporal) technological frontier. This could be techniques that are both (under some circumstances) superior to contemporary techniques, but practical obsolete, e.g., because of severe negative externalities or depletions of raw materials. And hence *de facto* no longer exist in the *book of available blueprints*.

Figure 3.3 and 3.4 show the wage-profit frontiers for the individual countries together with the contemporary and rolling technological frontiers. Figure 3.3 for the period 1970–1985 and Figure 3.4 for 1990–2005. As expected the US is from the 1970s the leading country, but the US wage-profit frontiers do not shift as much as the other countries' frontiers in the 1970s, i.e., evidence of a slowdown in the US and catching up by the other countries.[21]

[21] See Chapter 1 Section 1.6 for a discussion of the US productivity slowdown.

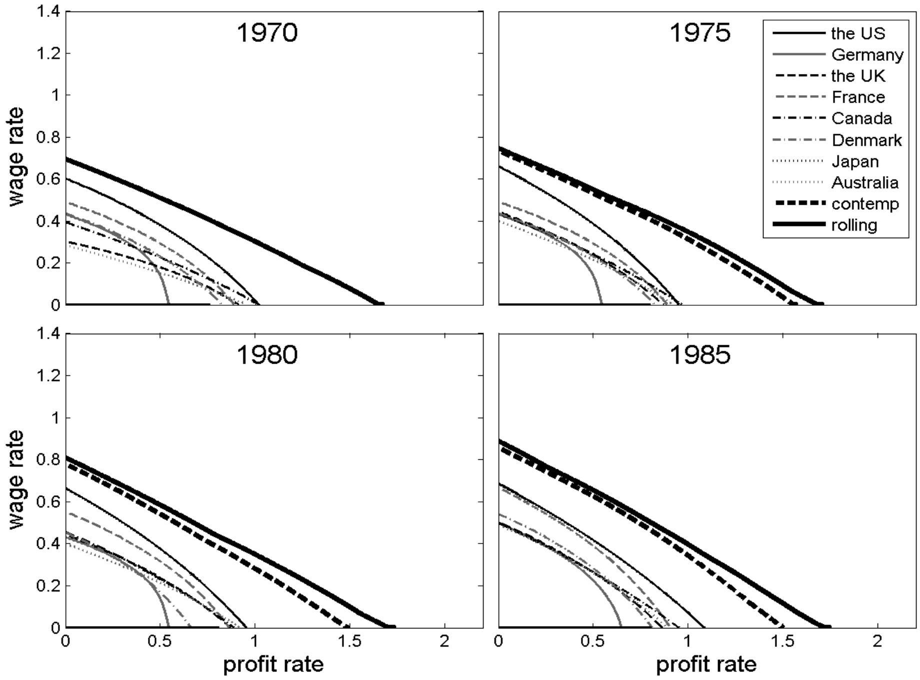

Fig. 3.3: *Wage-profit, contemporary, and intertemporal technological frontiers: 1970–1985*

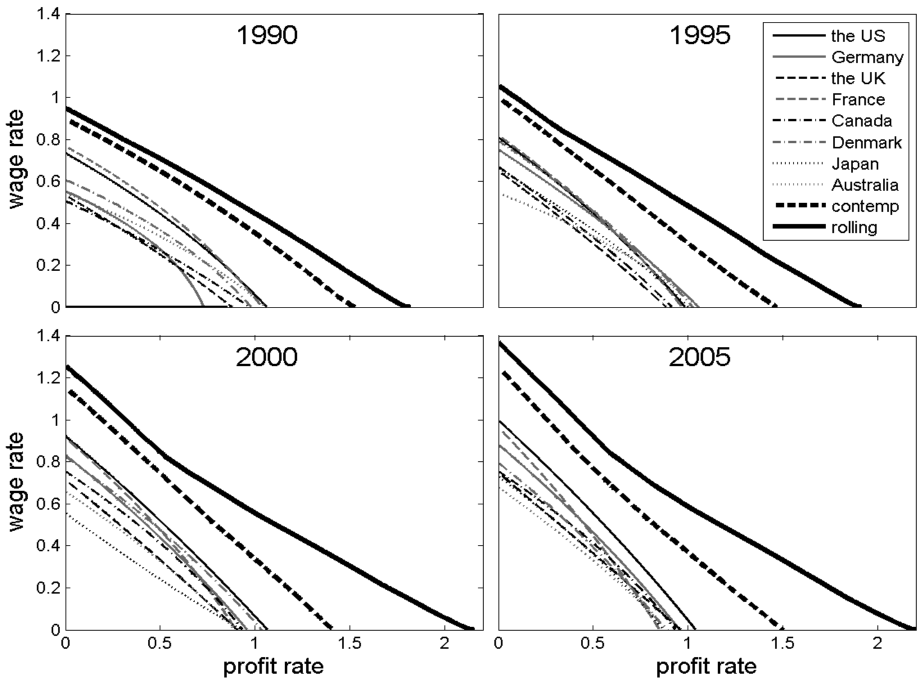

Fig. 3.4: *Wage-profit, contemporary, and intertemporal technological frontiers: 1990–2005*

See also Figure 6.2 and 6.3 in the statistical companion, where the frontiers are presented country-by-country.

3.6.2 The Velupillai-Fredholm-Zambelli index

The *Fredholm-Zambelli-Velupillai* index computed for the eight countries and the economy as a whole is collected in Figure 3.5 and Table 3.1. The global VFZ-index tells a story of stepwise technological development for the economy as a whole. During the 1970s the index stayed at a fairly stable level about 40–45 percent of the intertemporal maximum represented by the intertemporal technological frontier. From the mid 1980s to the mid 1990s

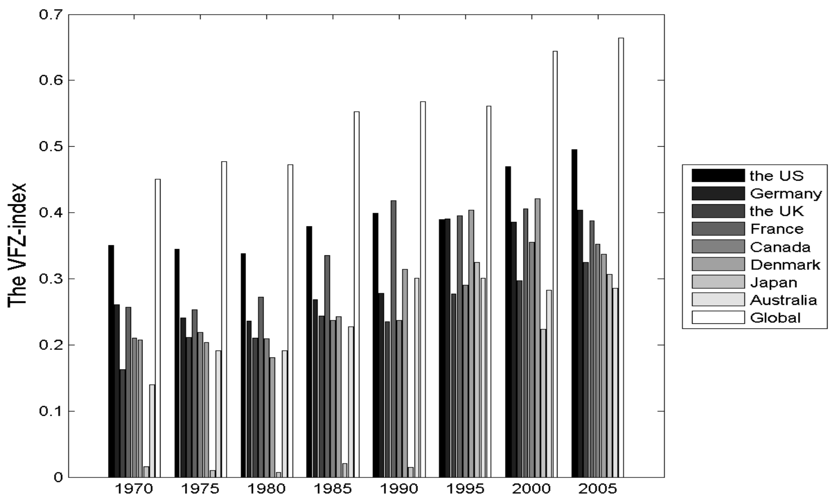

Fig. 3.5: *The Velupillai-Fredholm-Zambelli index*

the global productivity level stabilised at a new level about 55 percent, and finally in 2000 and 2005 reached a level close to 65 percent.[22]

It is surprising that over a period of 35 years the global VFZ-index has only increased from 0.45 to 0.66. This corresponds to a compounded growth rate at 1.1 percent per year, which is far less than the often reported 1.5–2 percent. Over a period of 35 years the difference between a growth rate of 1.1 and 2.0 percent corresponds to an increase of a factor 1.5 and 2, respectively.

For the single countries the difference between the level of 1970 and 2005 corresponds to a compounded growth rate of; the US 1.0, Germany 1.2, the UK 2.0, France 1.2, Canada 1.5, Denmark 1.4, Japan 8.1, and Australia 2.1 percent per year. As with the global index these growth rates are, except

[22]Of course much more could be said, if data had been available at a yearly basis, and the stepwise technological development must to some extent a consequence of the five year intervals.

for Japan, surprisingly small. Especially, the 1.0 percent annual growth for US.

If these results are—as implicitly claimed—more reliable than the usual indices of technological progress, then the results are indeed interesting. In particular, since technological progress determines the limit for a sustainable increase in the standard of living. Sustainable as oppose to the extreme increases in asset prices and consumption preceding the current economic turmoil. Reliable indices of technological change, determined from the physical flow among industries, could prove instrumental to better assess the technological constraints for our long-run standard of living.[23]

	1970	1975	1980	1985	1990	1995	2000	2005
the US	0.35	0.34	0.34	0.38	0.40	0.39	0.40	0.50
Germany	0.26	0.24	0.24	0.27	0.28	0.39	0.39	0.40
the UK	0.16	0.21	0.21	0.24	0.24	0.28	0.30	0.32
France	0.26	0.25	0.27	0.34	0.42	0.40	0.41	0.39
Canada	0.21	0.22	0.21	0.24	0.24	0.29	0.36	0.35
Denmark	0.21	0.20	0.18	0.24	0.31	0.40	0.42	0.34
Japan	0.02	0.01	0.01	0.02	0.02	0.32	0.22	0.31
Australia	0.14	0.19	0.19	0.23	0.30	0.30	0.28	0.29
global	0.45	0.48	0.47	0.55	0.57	0.56	0.64	0.66

Table 3.1: *The Velupillai-Fredholm-Zambelli index*

Apart from the differences in levels, the evolution in the global index is partly mimicked by the US index which also evolves in uneven steps. The difference between the US index and the global index increased slightly over the period considered with a large spike around 2000. It is here worth noting, that the global index from 1995 to 2000 increased from 56 to 64 percent while the US index only increased with one basis point. Hence, it is unlikely that it was the US that was driving the global development in the late 1990s. More will be said on this in Section 3.6.3, where the analysis is carried to the industry level.

Another interesting point is that the US is not facing an impending access to technology constraint. Naturally, this depends on the availability of the foreign production techniques. Some techniques might be country-specific, i.e, cannot be transferred; a great deal will probably not be 'public goods', but internal to multinational corporations, which at least limits the transferability; and some (if not most) production techniques require a great deal of human capital which in one way or another also must be transferred. In any case, we observe that the US from the 1980s has been approaching

[23] As oppose to some, however deflated, market price denominated proxy for (net) output per unit of labour. Not to mention indices build on an *ad hoc* and partly stochastic measure of fixed capital (aka the *perpetual inventory method*).

the intertemporal technological frontier, but also that there still potentially is a long way to go.[24]

The other countries, apart from Japan, show a more steady technological development, with a slow convergence towards both the US and the intertemporal frontier. In 1990, France actually reached the level of the US which it maintained until the US took off between 2000 and 2005. The same

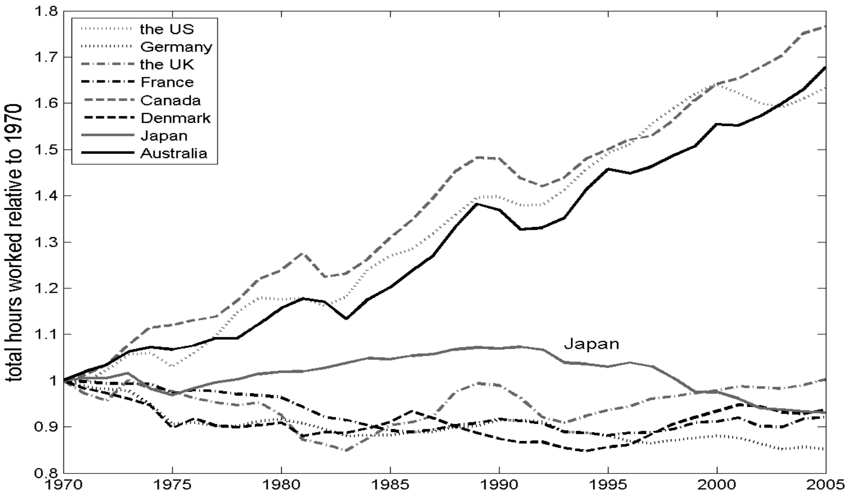

Fig. 3.6: *Index of 'total hours worked', 1970 = index 1. (Source: Official OECD employment data)*

goes for Germany and Denmark from 1995, while the UK and Australia remained behind.

Japan is showing an extraordinary development, until 1990 it is far behind all the other countries, but around 1995 Japan attained the level of the UK, Canada, and Australia. Without going into details, part of the explanation is probably found in the deregulations between 1990 and 1995 and subsequent drastic decrease in 'total hours worked' as shown in Figure 3.6.[25]

Figure 3.6 shows a very uneven development in the total hours worked. The US, Canada, and Australia show steady increases in the total hours worked (much of which is likely a consequence of immigration), while the European countries show a relatively stable or slightly decreasing development. Japan is the odd one out, it increased steadily during the 1970s and 1980s, where after it decreased with an average annual rate corresponding to about 0.8 percent between 1990 and 2005. Furthermore, Japan is the

[24]When the next set of OECD tables are published, will this increase the intertemporal frontier more than the US wage-profit frontier?

[25]For details on the deregulation initiatives in Japan in the 1990, see the homepage of The Japan Institute of Labour and Training, www.jil.go.jp.

only country where there is no apparent periodic business cycle to be found in the employment data.[26]

Combining our surprising result of the 1.0 percent US year-to-year increase in our measure of technological progress between 1970 and 2005 with the aggregate employment data reported in Figure 3.6, points towards an economic development in the US driving more by increased work effort than increased productivity. Between 1970 and 2005 the US employment measured in hours increased by a factor 1.6 corresponding to a yearly increase of about 1.4 percent.

Going into the causes behind the convergence in productivity levels lie outside the scope of the chapter. As concluded by Abramovitz (1986, p. 405):

> differences among countries in productivity levels create a strong potentiality for subsequent convergence of levels, provided that countries have a "social capability" adequate to absorb more advanced technologies. [However,] the institutional and human capital components of social capability develop only slowly as education and organization respond to the requirements of technological opportunity and to experience in exploiting it.

Hence, we might be able to observe the actual effects from the processes of catching up, but the deep causes explaining country-specific differences must be understood in terms of social and institutional characteristics. However, it is possible to venture a step deeper into a descriptive study of the country-specific development, *viz.* the industry-level development.

3.6.3 Industry-level development

Figure 3.7 shows the (unweighted) average industry-level frequency of the single countries contribution to the contemporary and rolling technological frontiers.

Even though the US is considered the leading country, it is only in few cases the country that is contributing most to the technological frontiers (including the intertemporal which is the rightmost of the rolling frontiers). This indicates that the US in a few industries strongly dominates, i.e., all or most segments of the envelope include a particular US technique, and that these industries must play a vital role for the economy as a whole. By inspecting Figures 6.4–6.26 for the single industries in the statistical companion, it is found that the US dominates in 'Construction'; 'Machinery and equipment, nec'; and 'Business activities (finance, real estate, and R&D)'.

[26]However, it must be noted that between 1990 and 1995 the input–output tables change from the ISIC 2 to the present ISIC 3 standard of accounting. Whether or not this greatly influence our results is *pro tempore* unknown.

Together with Germany the US also dominates in 'Manufacturing, nec'. Germany dominates in 'Electrical machinery and apparatus'; 'Transport equipment'; and 'Manufacturing nec; recycling (include Furniture)'. Canada in

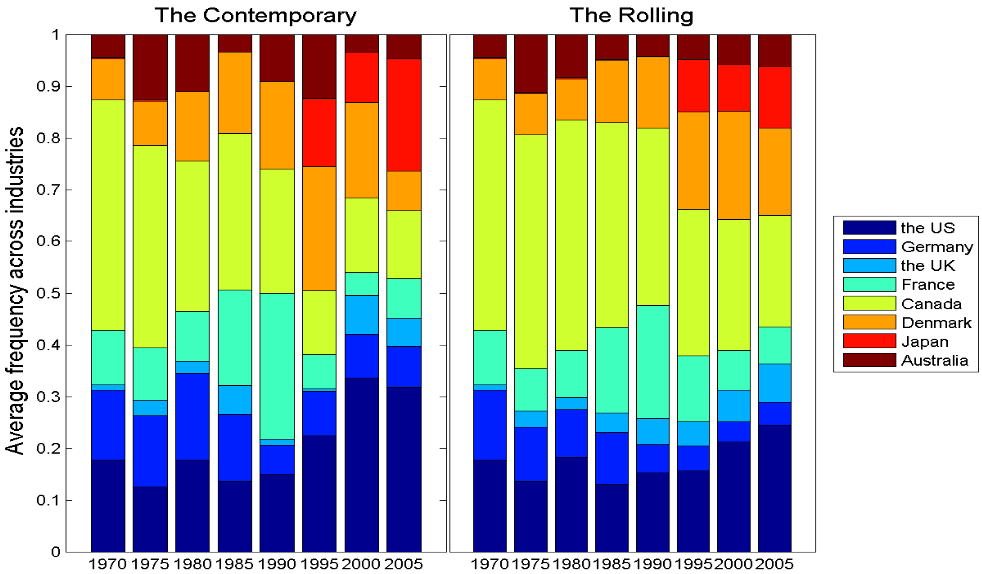

Fig. 3.7: *The composition of the contemporary and rolling technological frontiers*

'Other non-metallic mineral products'; 'Metals'; 'Fabricated metal products, except machinery and equipment'. Denmark, however insignificant on the world marked, dominates in 'Mining and quarrying' and 'Food products, beverages, and tobacco'.

In many cases, about 30–40 percent of the production techniques entering the envelopes are Canadian. This is surprising given that Canada, measured by, e.g., the VFZ-index, does not rank as one of the most productive countries.

An important general point is that no country at a single point in time dominates the entire technological frontier (contemporary or rolling). Hence, all countries could, at any point in time, potentially gain from further global integration, either through increased trade or transfer of production techniques (including human capital).

The statistical companion contains detailed empirical evidence on the country/industry specific contributions to the different technological frontiers. From these results it is possible to go deeper into an analysis of the displacement of the production techniques over time. In particular, it would be interesting to study the difference between the displacement of techniques in the contemporary and rolling technological frontiers, since this can tell us to what extent new more productive techniques have been introduced as oppose to new combinations of old techniques of production.

3.6.4 The number of switch points

Since the switch points on the envelopes are independent of the choice of *numéraire* a large number of switch points would imply that our results would be relatively unaffected by the choice of *numéraire*.

For the contemporary frontiers the number of switch points starting with the 1970 frontier are 25, 22, 22, 21, 22, 25, 26, and 31, and for the rolling frontiers 25, 36, 44, 46, 42, 44, 53, and 49. It is assessed that the number of switch points in general is of a magnitude, that ensures a fairly *numéraire* independent envelope. The intuition behind this conclusion follows directly from the properties listed in Section 3.2. If the *numéraire* is changed the subsequent results must also change, but a strictly decreasing envelope, together with 20 to 50 fixed points, do not leave much room for disturbance.

In hindsight not surprising, is the fact that the number of switch points tends to increase with the number of techniques on which the envelope is computed. For the contemporary frontiers the number of unique systems is 8^{23} and for the rolling it increase as $8^{23}, 16^{23}, 24^{23}, ..., 64^{23}$.

For the record, no *reverse capital deepening* or *reswitching* are found on any of the technological frontiers computed for this study.[27]

3.7 Concluding Remarks

The value-added of this chapter is two-fold. First, an algorithm, the Piecemeal algorithm, has been developed that is capable of computing actual technological frontiers from huge collections of production techniques. The algorithm computes the entire technological frontier, the envelope, without going through partial or stochastic *ad hoc* short cuts.

Second, by exploiting the power the Piecemeal algorithm three different versions of the technological frontier have been computed and analysed; the contemporary, the rolling, and the intertemporal. From these frontiers a set of indices, the VFZ-indices, has been extracted to provide global as well as country-specific condensed measures of technological progress.

The global VFZ-index, computed from the contemporary technological frontiers and intertemporal technological frontier, provides a measure of the technological progress for the global economy as a whole. The index based on the eight OECD countries tells a story of a stepwise technological development, with major jumps in the periods 1980–1985 and 1995–2000. But also a global development that shows a more moderate rate of productivity growth compared with the conventional stylized facts of a 1.5–2.0 percent year-to-year increase. The global VFZ-index increased from 0.45 in 1970 to 0.66 in 2005, i.e., an increase of a factor 1.5 against the factor 2 implied by a 2.0 percent per year compounded growth over 35 years.

[27] See Han and Schefold (2006).

The VFZ-indices for the single countries, constructed from the individual countries' wage-profit frontiers and the intertemporal technological frontier, have provided new insight into the country-specific technological progress and convergence. Among the results are evidence of an economic development in the US driven at least as much by an increased work effort as an increased productivity.

Furthermore, by identifying which production techniques that enter the different technological frontiers, we have carried out a preliminary analysis of the country/industry specific contributions to the overall development. It has been shown that even though the US is the leading country, it is only in few cases the US that is contributing most to the technological frontiers. The few industries where the US strongly dominates on the technological frontiers are 'Construction'; 'Machinery and equipment, nec'; and 'Business activities (finance, real estate, and R&D)'. Moreover, we see that no country at any point in time dominates anything near an entire technological frontier, i.e., the potential gains from further global integration have not been exhausted for any country at any point in time.

Common for these frontiers and the indices based hereupon is a considerable resilience to the theoretical problems that hitherto have haunted the construction of index numbers and thereby any form of productivity accounting. In particular, the technological frontiers and the associated indices are; non-parametric and non-stochastic; the interdependence among industries is endogenously captured by changes in the prices of production; and will not change as a consequence of simple changes in the scale of production in the single industries.

While the envelope is *numéraire* dependent, the stability provided by the 20–50 switch points greatly limits the sensitivity to changes in the *numéraire*.

A huge work remains to be carried out going deeper into an analysis of the displacement of the production techniques on the contemporary and rolling technological frontiers.

Acknowledgements

We would like to thank Charlotte Bruun, Carsten Heyn-Johnsen, and K. Vela Velupillai for their valuable support and encouragement throughout the gestation of this project.

References

Abramovitz M. (1986): 'Catching Up, Forging Ahead, and Falling Behind', *The Journal of Economic History*, 46, pp. 385–406.

Afriat S. N (1977): *The Price Index*, Cambridge University Press, Cambridge.

Bruno M. (1969): 'Fundamental duality relations in the pure theory of capital and growth', *The Review of Economic Studies*, 36, pp. 39–53.

Burmeister E., Kuga K. (1970): 'The factor-price frontier, duality and joint production', *The Review of Economic Studies*, 37, pp. 11–19.

Coelli T. (2001): *An Introduction to Efficiency and Productivity Analysis*, Kluwer Academic Publishers, Fifth Printing, Boston.

Degasperi M., Fredholm T. (2010): 'Productivity accounting based on production prices', *Metroeconomica*, forthcoming.

Ernst D., et. al. (1998): 'Technological capabilities in the context of export-led growth: a conceptual framework', in *Technological Capabilities and Export Success in Asia*, edited by Ernst D., Ganiatsos T., and Mytelka L., Routledge, London.

Fisher I. (1922): *The Making of Index Numbers: A Study of Their Varieties, Tests, and Reliability*, The Riverside Press, Cambridge, MA.

Han Z., Schefold B. (2006): 'An empirical investigation of paradoxes: reswitching and reverse capital deepening in capital theory', *Cambridge Journal of Economics*, 30, pp. 737–765.

Kurz H. D., Salvadori N. (1995): *Theory of Production: A Long-Period Analysis*, Cambridge University Press, Cambridge.

Leontief W. (1936): 'Composite commodities and the problem of index numbers', *Econometrica*, 4, pp. 39–59.

Leontief W. (1941): *The Structure of American Economy, 1919–1929*, Harvard University Press, Cambridge, MA.

Neumann J. v. (1945–46): 'A model of general economic equilibrium', *The Review of Economic Studies*, 13, pp. 1–9.

Pasinetti L. L. (1977): *Lectures on the Theory on Production*, Macmillian Press, London.

Samuelson P. A., Swamy S. (1974): 'Invariant economic index numbers and canonical duality: survey and synthesis', *The American Economic Review*, 64, pp. 566–593.

Sraffa P. (1960): *Production of Commodities by Means of Commodities*, Cambridge University Press, Cambridge.

Velupillai K. (2008): 'Sraffa's mathematical economics: a constructive interpretation', *Journal of Economic Methodology*, 15, 325–342.

Velupillai K., Zambelli S. (1993): 'The international comparisons programme and production based indices of Economic Performances', *mimeo*, World Bank, Washington.

Zambelli S. (2004): 'The 40% neoclassical aggregate theory of production', *Cambridge Journal of Economics*, 28, pp. 99–120.

Appendices

3.A Data Description

Table 3.2 shows which OECD input–output tables that are available from the period 1970–2005. Tables are not necessarily available from the exact five

	1970	1975	1980	1985	1990	1995	2000	2005
the US	×	×	×	×	×	×	×	×
Germany			×	×	×	×	×	×
the UK	×		×	×	×	×	×	×
France		×	×	×	×	×	×	×
Canada	×	×	×	×	×	×	×	
Denmark		×	×	×	×	×	×	×
Japan	×	×	×	×	×	×	×	×
Australia	×	×		×	×		×	×

Table 3.2: *Available input–output tables*

year intervals, e.g., the US tables here labelled 1970 and 1975 are actually the 1972 and 1977 tables, respectively.[28]

The list below shows how the original tables have been aggregated down to the 23×23 used in this study. The numbers in the brackets refer to their respective ISIC 2 and ISIC 3 classification, *viz.* {[ISIC 3],[ISIC 2]}.

1. Agriculture, hunting, forestry, and fishing {[1],[1]}
2. Mining and quarrying {[2–3],[2]}
3. Food products, beverages, and tobacco {[4],[3]}
4. Textiles, textile products, leather, and footwear {[5],[4]}
5. Wood and products of wood and cork {[6],[5]}
6. Pulp, paper, paper products, printing, and publishing {[7],[6]}
7. Coke, refined petroleum products, and nuclear fuel {[8],[9]}
8. Chemicals {[9–10],[7–8]}
9. Rubber and plastics products {[11],[10]}
10. Other non-metallic mineral products {[12],[11]}
11. Metals {[13–14],[12–13]}
12. Fabricated metal products, except machinery and equipment {[15],[14]}
13. Machinery and equipment, nec {[16],[15]}
14. Electrical machinery and apparatus {[17–20],[16–18]}
15. Transport equipment {[21–25],[19–22]}
16. Manufacturing nec; recycling (include furniture) {[25],[23–24]}
17. Production and distribution of electricity, gas, and water {[26–29],[25]}
18. Construction {[30],[26]}
19. Wholesale and retail trade {[31],[27]}
20. Service activities (transport, hotels and restaurants) {[32–36],[28–29]}
21. Post and telecommunications {[37],[30]}
22. Business activities (finance, real estate, and R&D) {[38–43],[31–32]}
23. Public administration, education and health {[44–48],[33–35]}

[28] The full list of available tables are: the US {1972, 1977, 1982, 1985, 1990, 1995, 2000, 2005}, Germany {1978, 1986, 1990, 1995, 2000, 2005}, the UK {1968, 1979, 1984, 1990, 1995, 2000, 2003}, France {1977, 1980, 1985, 1990, 1995, 2000, 2005}, Canada {1971, 1976, 1981, 1986, 1990, 1995, 2000}, Denmark {1977, 1980, 1985, 1990, 1995, 2000, 2004}, Japan {1970, 1975, 1980, 1985, 1990, 1995, 2000, 2005}, and Australia {1968, 1974, 1986, 1989, 1999 ,2005}.

Table 3.3–3.10 show the macro-industry deflators used to convert the tables denominated in current prices (possible domestic currency) into tables denominated in fixed US 2000 prices. The transition to the EURO has been taken into account in the tables below. The deflators are computed as the ratio between GDP in constant prices and GDP in current prices and when necessary also divided by the dollar-domestic currency exchange rate (www.sourceoecd.org). The missing values marked with a '–' correspond with the unavailable OECD tables.

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	1.23	0.84	0.71	0.74	0.65	0.67	1.00	0.91
II Industry including energy	2.83	1.89	1.14	1.09	1.02	0.96	1.00	0.93
III Construction	4.97	3.28	2.03	1.79	1.43	1.26	1.00	0.71
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	2.13	1.53	1.12	1.05	0.97	0.87	1.00	0.98
V Financial intermediation; real estate, business activities	4.79	3.45	2.34	1.88	1.47	1.23	1.00	0.88
VI Other service activities	5.15	3.57	2.37	1.98	1.53	1.24	1.00	0.82

Table 3.3: *Macro-industry deflators for the US 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	–	–	0.53	0.60	0.64	1.00	1.03	1.39
II Industry including energy	–	–	0.81	0.60	0.57	1.02	1.03	1.01
III Construction	–	–	1.15	0.84	0.71	1.02	1.03	0.98
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	–	–	0.78	0.63	0.60	1.01	1.03	1.03
V Financial intermediation; real estate, business activities	–	–	1.01	0.69	0.61	1.02	1.03	0.94
VI Other service activities	–	–	0.98	0.75	0.67	1.08	1.03	0.97

Table 3.4: *Macro-industry deflators for Germany 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	5.97	–	2.26	2.43	1.40	1.06	1.57	1.37
II Industry including energy	11.3	–	3.65	2.21	1.86	1.64	1.57	1.57
III Construction	18.4	–	4.44	3.14	2.15	2.00	1.57	1.35
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	13.2	–	4.41	2.87	1.91	1.72	1.57	1.51
V Financial intermediation; real estate, business activities	14.8	–	4.50	2.87	2.01	1.71	1.57	1.34
VI Other service activities	21.5	–	5.81	3.84	2.33	1.94	1.57	1.38

Table 3.5: *Macro-industry deflators for the UK 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	–	0.25	0.23	0.17	0.15	0.98	1.06	1.09
II Industry including energy	–	0.34	0.26	0.17	0.16	1.04	1.06	1.12
III Construction	–	0.56	0.38	0.26	0.21	1.19	1.06	0.86
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	–	0.39	0.30	0.20	0.17	1.05	1.06	0.95
V Financial intermediation; real estate, business activities	–	0.53	0.42	0.29	0.22	1.22	1.06	0.94
VI Other service activities	–	0.58	0.41	0.26	0.21	1.18	1.06	0.91

Table 3.6: *Macro-industry deflators for France 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	3.53	1.63	1.00	0.96	0.95	0.78	0.81	–
II Industry including energy	3.95	2.37	1.38	1.18	1.04	0.94	0.81	–
III Construction	3.33	1.89	1.31	1.18	0.92	0.87	0.81	–
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	2.63	1.76	1.20	0.96	0.85	0.82	0.81	–
V Financial intermediation; real estate, business activities	4.26	2.50	1.58	1.17	0.94	0.87	0.81	–
VI Other service activities	4.40	2.58	1.71	1.29	1.02	0.90	0.81	–

Table 3.7: *Macro-industry deflators for Canada 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	–	.104	.094	.074	.078	.101	.119	.143
II Industry including energy	–	.305	.246	.178	.149	.140	.119	.113
III Construction	–	.360	.290	.222	.173	.147	.119	.104
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	–	.244	.210	.134	.120	.117	.119	.109
V Financial intermediation; real estate, business activities	–	.329	.253	.179	.150	.128	.119	.109
VI Other service activities	–	.360	.285	.197	.151	.136	.119	.103

Table 3.8: *Macro-industry deflators for Denmark 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	.0135	.0083	.0067	.0063	.0060	.0056	.0065	.0071
II Industry including energy	.0111	.0078	.0063	.0058	.0058	.0059	.0065	.0074
III Construction	.0313	.0152	.0104	.0087	.0073	.0066	.0065	.0066
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	.0123	.0082	.0067	.0062	.0062	.0061	.0065	.0068
V Financial intermediation; real estate, business activities	.0177	.0119	.0091	.0078	.0069	.0064	.0065	.0068
VI Other service activities	.0288	.0137	.0104	.0086	.0075	.0066	.0065	.0067

Table 3.9: *Macro-industry deflators for Japan 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	3.32	2.09	–	0.98	0.73	–	0.88	0.75
II Industry including energy	3.78	2.76	–	0.94	0.88	–	0.80	0.55
III Construction	4.45	2.72	–	1.05	0.88	–	0.77	0.65
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	5.35	3.42	–	1.02	0.84	–	0.77	0.69
V Financial intermediation; real estate, business activities	7.30	4.98	–	1.34	0.99	–	0.80	0.65
VI Other service activities	5.04	2.96	–	1.20	1.01	–	0.79	0.61

Table 3.10: *Macro-industry deflators for Australia 1970–2005*

Part II

A Small Contribution to the Critique of the Neoclassical Theory of Growth and Distribution

Chapter 4

Production Functions Behaving Badly

Reconsidering Fisher and Shaikh

4.1 Introduction

The notion of aggregate production functions has long been a widely used theoretical concept in economics and remains among the fundamental concepts presented in almost every course in micro- and macroeconomics. Furthermore, aggregate production functions constitute the core of the supply side in most modern econometric as well as theoretical models, e.g., *CGE* models.

In 1974 Anwar Shaikh proposed a serious critique of the neoclassical theory of growth and distribution based on its use of aggregate production functions. Empirical studies had hitherto shown that aggregate production functions of the Cobb-Douglas type usually fit the data well, and that the estimated coefficients typically coincide with observed wage and profit shares of income. These empirical findings were used not only to support the neoclassical theory of growth and distribution, but also to contest non-micro founded theory, because of its lack of this kind of "indisputable" support.

However, as Shaikh (1986, p. 191) claims the 'apparent empirical strength of aggregate production functions is often interpreted as support for neoclassical theory. But there is neither theoretical nor empirical basis for this conclusion.'

The purpose of this chapter is to reconsider Shaikh's critique (Shaikh, 1974; see also Shaikh, 1980, 1986, 2005) and parts of the subsequent work on the subject. This is done by reconstructing and extending the original computer simulations by Fisher (1971), which Shaikh used to support his thesis. I extend Fisher's simulations by introducing CES production functions at the industry level, but continue to estimate a simple Cobb-Douglas production function from the aggregated data. This, I claim, provides further insight into the extent and implications of Shaikh's critique.

I will show that Fisher's simulation experiment can be reconstructed and by doing this I can also confirm Fisher's 1971 findings, which in itself is of interest because these results have been widely used, but to the best of my knowledge never verified. Furthermore, by inspecting the goodness of fit in Fisher's almost 1000 experiments, I will show that Shaikh's interpretation of Fisher's work is correct. Finally, I compare my results with those obtained by McCombie and Dixon (1991), Felipe and Holz (2001), and Shaikh (2005). In line with these researchers, I find evidence to support a more general version of Shaikh's original critique.

Section 4.2 and 4.3 present Shaikh's original critique and subsequent extensions, Section 4.4 and 4.5 Fisher's original model and the reconstruction, and Section 4.6 presents the extension of Fisher's model. Section 4.7 concludes the chapter with a discussion on the consequences of this critique for the neoclassical theory of growth and distribution.

4.2 Laws of Algebra

Shaikh claims and proves that whenever input–output data exhibit constant income shares, there is a very good chance that regardless of the true nature of the data, an aggregate production function of the Cobb-Douglas type will fit the data very well. Therefore, Shaikh concludes that when one estimates a Cobb-Douglas production function on input–output data, there is a good chance that one only observes *laws of algebra* and not *laws of production.*

The following is a concise version of Shaikh's proof. It starts with the universal income accounting identity, *viz.*

$$Y = wL + rK \tag{4.2.1}$$

Let $y = Y/L,\ k = K/L,\ \alpha = rK/Y,\ 1 - \alpha = wL/Y$, and assumes that labour's share of income is constant over time. Now (4.2.1) can be written as $y = w + rk$.

$$\begin{aligned} y = w + rk \Rightarrow \dot{y} = \dot{w} + \dot{r}k + r\dot{k} \Leftrightarrow \frac{\dot{y}}{y} &= \frac{w}{y}\frac{\dot{w}}{w} + \frac{rk}{y}\frac{\dot{r}}{r} + \frac{rk}{y}\frac{\dot{k}}{k} \\ \Rightarrow \frac{\dot{y}}{y} &= (1-\alpha)\frac{\dot{w}}{w} + \alpha\frac{\dot{r}}{r} + \alpha\frac{\dot{k}}{k} \\ \Rightarrow \ln y &= (1-\alpha)\ln w + \alpha \ln r + \alpha \ln k + \ln c_0 \\ \Rightarrow y &= C_1 k^{\alpha} \Leftrightarrow Y = C_1 K^{\alpha} L^{1-\alpha} \end{aligned} \tag{4.2.2}$$

Where the shift term C_1 is given by:

$$C_1 = c_0 \cdot r^{\alpha} w^{1-\alpha} \tag{4.2.3}$$

To sum up, from a tautology of input–output data and an assumption of constant input shares (plus an implicit assumption of differentiable functions), a function of the Cobb-Douglas type follows directly through basic applications of the laws of algebra! This is an important result, since it implies that regressions of a Cobb-Douglas production function, given that the data exhibit constant input shares, are predetermined to give high correlation coefficients, and are thereby meaningless.

Because of this Shaikh named the Cobb-Douglas production function the "HUMBUG" production function, and emphasised the message by showing that the coordinates in the Cartesian plane spelling the word "HUMBUG" together with profit shares from the US (Solow's 1957 data) could be fitted almost perfectly by a Cobb-Douglas production function (Shaikh, 1974).[1]

[1] Shaikh's results have been challenged by Solow (1974), but subsequently defended by Shaikh (1980), after which the discussion, to the best of my knowledge, seems to have gone quiet.

4.3 Related Work

The use of aggregate production functions has long been a subject of serious discussion, and no consensus has yet been reached. The debate can be divided into two major parts: the so-called *index number problem* and *value problem*, which respectively refer to the problems of aggregation and the logical problem in determining the value of capital independently of the profit rate.[2] This chapter only deals with the index number problem, or to be more specific, the issues of interpreting aggregated empirical results from technologically diverse economies.[3]

Following the first paper by Shaikh on the HUMBUG production function, a number of theoretical and empirical studies have been published on the subject with J. Felipe and J.S.L. McCombie as the main contributors. I will not provide a full survey of these results, but refer to Felipe and Fisher (2003) for an extensive and excellent survey.

However, the papers by McCombie and Dixon (1991), Felipe and Holz (2001), and Shaikh (2005) are of special interest for the present chapter. McCombie and Dixon (1991) proves that Shaikh's critique also stands when factor shares are not constant as long as the shift term grows with a constant rate. Furthermore, they show that even if the shift term does not grow with a constant rate, it is possible 'with sufficient ingenuity, to find a functional form which will produce a very good fit to the underlying identity' McCombie and Dixon (1991, p. 40), and they refer to the CES and the translog production function as potential candidates.[4]

The paper by Felipe and Holz (2001, p. 281) presents an interesting Monte Carlo simulation that shows

> that the Cobb-Douglas form is robust to relatively large variations in the factor shares. However, what makes this form quite often fail are the variations in the growth rates of the wage and profit rates. The weighted average of these two growth rates has been shown to be the coefficient of the time trend. This implies that, in most applied work, a Cobb-Douglas form (i.e. approximation to the income accounting identity) should work. We just have to find *which* Cobb-Douglas form with a dose of patience in front of the computer.

Moreover, they show that spurious regression cannot explain the systematic (near) perfect fit of the Cobb-Douglas function.

[2] See Pasinetti (2000) for an excellent discussion of the neoclassical theory of growth and distribution, where the notion of aggregate production functions stands as a central element.

[3] See Cohen and Harcourt (2003) for a extensive survey and Zambelli (2004) for a more concise survey and interesting computer simulations on the value problem.

[4] See also Felipe and McCombie (2001) for a very interesting study of the CES production function's ability to fit input–output data, where they reconsider Arrow *et. al.* (1961) seminal work on the CES function.

In a recent paper, Shaikh (2005) presents a more general version of his original results; the so-called *Perfect Fit Theorem.* This theorem states that, given a stable labour share, it is always possible to construct a time function $F(t)$, 'that will always make fitted production functions work "perfectly" in the sense of Solow: that is, make them yield perfect econometric fits with partial derivatives that closely approximate observed factor prices' Shaikh (2005, p. 457). The time function must merely be constructed in the following way:

$$SR_t = \alpha_{t-1}\Delta \log r + (1 - \alpha_{t-1})\Delta \log w \tag{4.3.1}$$

$$F_t = \beta + h\left(SR_t - \tfrac{1}{t}\sum SR_t\right) \tag{4.3.2}$$

Note that (4.3.1) resembles the shift term (4.2.3) and that an affine function of the Solow Residual SR_t yields an affine time function F_t.

Following McCombie and Dixon (1991) and Felipe and Holz (2001), I agree that the assumption of constant input shares is not needed for the main results to hold. However, I claim that the Cobb-Douglas production function still does a very good job in fitting the data, even when shares are not stable **and** the shift term (4.2.3) fails in a test of trend stationarity.

As for Shaikh's Perfect Fit Theorem, I acknowledge the power of the theorem in its ability to ensure a perfect fit, but I also underline that constant input–shares are still a required assumption.

It is important to note that my results are not conditioned on *a perfect fit*, but on *a good fit*, by which I mean a fit that would make most econometricians, given the usual reservations, accept the model as a good description of the data. In other words, I am not *per se* interested in the theoretical — but very possible — possibility of making a neoclassical production function fit the data perfectly, with the help from cleverly constructed trend terms or more flexible functional forms such as the CES or translog. I am merely interested in the basic method of regressing a log-linearized Cobb-Douglas function with a simple (affine) trend term on input–output data, and will show that this method often is sufficient to ensure a good fit, even when the underlying data should not be explainable with such a simple model. I believe this is an interesting approach, because this method is extensively used by not only students of economics, but also established researchers. Showing that these claims hold will be the main quest in the following.

4.4 Fisher's Model

The purpose of Fisher's 1971 paper was to study the conditions under which the production possibilities of a technologically diverse economy can be represented by an aggregate production function.

The work consists of a huge simulation experiment, where production is simulated at the micro level in a neoclassical model with n heterogeneous firms — all possessing Cobb-Douglas technology. Labour is assumed to be perfectly mobile, but capital and technology are bound to the respective firms. Wage and profit are as usual given by the marginal productivity of labour and capital, respectively. Furthermore, it is assumed that through perfect competition the labour inputs in each period are distributed such that wages would be uniform.

The experiments are divided into two major groups: the so-called *Capital experiments* in which economic development is based on the evolution in the stock of capital, and the *Hicks experiments* in which development is based on changes in a Hicks neutral technology. The experiments were divided into a total of five subgroups depending on the underlying pattern of technological progress. The experiments ran over 20 periods with two, four, or eight firms, and for each experiment three different initial capital or technology endowments, two choices of weights in the production function, and eleven different growth rates in capital or technology were chosen. This gives a total of 990 ($5 \times 3 \times 3 \times 2 \times 11$) unique experiments. See Appendix 4.A for further details.

The experiments were constructed in order to systematically violate the conditions for a theoretically consistent aggregation; see Fisher (1969) for a discussion of these conditions. Capital is aggregated using the profit rates, *viz.*

$$J_t = \sum_{i=1}^{n} \left(\frac{\sum_{t=1}^{20} r_{i,t} K_{i,t}}{\sum_{t=1}^{20} K_{i,t}} \right) K_{i,t} \qquad i = 1, 2, ..., n \quad t = 1, 2, ..., 20 \tag{4.4.1}$$

The aggregate Cobb-Douglas production function is given by

$$Y_t = A_t J_t^{\alpha} L_t^{1-\alpha} \tag{4.4.2}$$

4.4.1 Evaluation of the Model

The primary measurement of performance is the relative root-mean-square error together with the standard deviation of labour's share, *viz.*

$$S = \frac{\sqrt{\frac{1}{20} \sum_{t=1}^{20} (w_t - \hat{w}_t)^2}}{\frac{1}{20} \sum_{t=1}^{20} w_t} \tag{4.4.3}$$

$$\sigma_\alpha = \sqrt{\frac{1}{20-1} \sum_{t=1}^{20} \left(\hat{\alpha}_t - \frac{1}{20} \sum_{t=1}^{20} \hat{\alpha}_t \right)^2} \tag{4.4.4}$$

Where $\hat{\alpha}$ and $\hat{w}$ denotes estimated values.

In relation to the analysis of Shaikh's thesis, the standard deviation of labour's share σ_α is important, because of the assumption of a constant labour share.

The parameter α in the aggregate production function is estimated from the following simple log-linearized model:

$$\ln \tfrac{Y_t}{L_t} = \beta_1 + \beta_2 t + \alpha \ln \tfrac{J_t}{L_t} + \epsilon_t \tag{4.4.5}$$

The work presented in Section 4.3 would predict that the correlation coefficients from the above regression will be equal to or very close to one, whenever the input–output data exhibits either (A) constant factor shares or (B) factor shares that change so that the shift term, see equation (4.2.3), grows at a constant rate. It is these conditions, I investigate below.

The trend term $\beta_2 t$ is included to capture what can be characterised as a constant growth in the (aggregated) Hicks neutral technology. Following Fisher (1971, p. 313) this trend term is only included in the Hicks experiments.

To check whether or not assumption A and/or B are satisfied in the experiments, the following methods are used. Constant factor shares are checked by the standard deviation of labour's share σ_α; if this is sufficiently small, it would seem reasonable to accept assumption A. As for assumption B, equation (4.2.3) states that the shift term is given by a weighted average of the wage and the profit rate; i.e., testing assumption B is equivalent to testing whether or not the following variable is a trend stationary time series.

$$C_t = r_t^{\alpha_t} w_t^{1-\alpha_t} \tag{4.4.6}$$

Where $1 - \alpha_t = \frac{L_t w_t}{Y_t}$ and $r_t = \frac{\sum r_{i,t}}{J_t}$. As usual this is done by including a trend term in the ADF test. Details will be given in the following sections.

4.4.2 A note on the simulations

It is fairly easy to describe the simulations, because it is simply a reconstruction based on the thorough documentation in Fisher (1971). I have used MATLAB to write three small programs, which are available upon request.

These programs consist of a master m-file, which basically is Fisher's model as described in his paper plus the extensions I employ. This program also contains algorithms performing different methods for evaluation, e.g., a set of loops that automatically perform standard ADF tests for stationarity by calculating test statistics and comparing these with the appropriate table values. The significance level for all tests is 5 percent.

The set-up of the experiments is programmed in another m-file, e.g., the different combinations of exogenously given parameter values. Furthermore, this program collects and organises the output.

The last m-file is a wage-equilibrating-algorithm, which is used because in every period in every experiment the wage rates must be uniform among the n firms; see Fisher (1971, p. 308) or Appendix 4.A for further details. The wage-equilibrating-algorithm is extremely time-consuming due to inefficient programming and computational complexity.

4.5 The Reconstruction

It cannot be expected that the reconstruction yields a perfect replication of Fisher's work, because his model is not deterministic. The stochastic shocks are however relatively inconsequential and can for that reason only justify a rather small deviation from the original results.

For reasons of comparability, the original and the reconstructed data are presented in the same type of matrices as Fisher used. These matrices sum

S/σ_α	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	> 3.0
0.0-0.5	296	1	1	0	0	0	0
0.5-1.0	104	91	2	0	0	0	0
1.0-1.5	26	31	41	3	0	0	0
1.5-2.0	13	7	26	20	5	0	0
2.0-3.0	13	11	12	18	19	14	5
3.0-4.0	6	11	4	6	5	2	16
4.0-5.0	3	2	6	3	2	0	13
5.0-10.0	5	14	6	9	7	8	23
10.0-20.0	0	5	6	5	0	4	13
>20.0	1	1	8	5	2	3	26

Table 4.1: *Summary of the Capital and Hicks experiments (original data)*

S/σ_α	0.0–0.5	0.5–1.0	1.0–1.5	1.5–2.0	2.0–2.5	2.5–3.0	> 3.0
0.0–0.5	290	0	0	0	0	0	0
0.5–1.0	122	40	0	0	0	0	0
1.0–1.5	32	50	14	0	0	0	0
1.5–2.0	7	21	37	4	0	0	0
2.0–3.0	24	16	10	17	14	0	0
3.0–4.0	5	10	7	3	6	11	1
4.0–5.0	5	6	12	2	1	2	9
5.0–10.0	14	22	12	9	7	5	15
10.0–20.0	2	11	7	6	8	8	8
>20.0	1	4	10	6	5	7	46

Table 4.2: *Summary of the Capital and Hicks experiments (reconstructed data)*

up the frequency of observations with a given combination of σ_α and S.

There are some deviations, but these deviations can be justified by the stochastic elements in the model. In any case, Fisher's basic observation is confirmed, i.e., an aggregate production function often provides a good explanation of wages, provided that the input weights are relatively stable over time. Given Fisher's earlier work on the subject (Fisher, 1969), these results must have been surprising, as the following quote also suggests:

> The point of our results, however, is not that an aggregate Cobb-Douglas fails to work well when labor's share ceases to be roughly constant, it is that an aggregate Cobb-Douglas will continue to work well so long as labor's share continues to be roughly constant ... (Fisher 1971, p. 307)

This reconstruction of Fisher's work allows us to examine the goodness of fit of the underlying regressions. Note that confirming Fisher's original results is *per se* useful, since several authors over the years have referred to these results.

Inspecting the correlation coefficients from Equation 4.4.5 and standard deviations of labour's share from the 990 unique experiments show that almost all correlation coefficients are very close to 1; 98 percent are greater than 0.90 and 85 percent are greater than 0.99. Moreover, the correlation does not seem to decrease as σ_α increases. Even more interesting, the 96 series with non-trend stationary shift terms continue to give high correlation coefficients: 95 percent of all correlation coefficients are greater than 0.90 and 78 percent are greater than 0.99.

These observations imply that Shaikh's *law of algebra* may very well be more general than formally constrained by assumption A, constant labour shares, and assumption B, a constant growth rate in the shift term.

Note however, that (near) perfect correlation is not always observed, but $R^2 > 0.90$ would lead most researchers, given the usual reservations, to (in this case wrongly) conclude that the estimated model is a good explanation of the underlying system.

To avoid any misconceptions, these results do not contradict those of Shaikh or the subsequent work presented in Section 4.3, they show that in applied work the risks of making wrong conclusions are not restricted to the cases where assumptions A and B are satisfied.

To ensure that these high correlations are not observations of spurious regressions, the explanatory and the dependent variables in equation (4.4.5) are checked for possible unit roots by a simple ADF tests. From this it is inferred whether or not there is a potential risk for spurious regression, i.e., if both the dependent and explanatory variables have a unit root. These tests shows that there is only a potential risk for spurious regression in 5.2 percent of the 990 regressions, i.e., the high correlation coefficients cannot be explained by spurious regression. This result is consistent with Felipe and Holz (2001), who also conclude that spurious regression cannot explain the uniformly high fit.

4.6 The Extended Model

In the following an extension of Fisher's model is employed to further investigate the generality of Shaikh's critique. The model is changed by replacing the micro Cobb-Douglas production functions with CES production functions, but still estimating an aggregate Cobb-Douglas production function after the conditions for a theoretically consistent aggregation are violated as in the original model.[5] The CES production function is of the following form, where ν is the reciprocal of the elasticity of substitution between capital and labour σ_{KL}, *viz.*

$$y_{i,t} = A_{i,t}\left(\alpha_i K_{i,t}^{1-\nu} + (1-\alpha_i)L_{i,t}^{1-\nu}\right)^{\frac{1}{1-\nu}} \tag{4.6.1}$$

The elasticity of substitution is chosen to be 0.20, 0.40, 0.60 or 0.80. The experiments are in every other way identical to Fisher's, i.e., a total of 3960 (4×990) unique experiments.

However a minor problem emerges: in 760 experiments it was not possible to ensure uniform wages in every period through the redistribution of labour between the n firms. This is a consequence of an obvious mathematical property of the CES function, when capital and technology are *ex ante* given. To circumvent this problem, all of these experimental sessions, in which it was not possible to determine a set of uniform wage rates in one or more periods, have been removed. Consequently, the following results are based on 3200 ($3960 - 760$) experiments.

Inspecting the correlation coefficients and standard deviations of labour's share from these 3200 unique experiments show that 81 percent of the correlation coefficients are greater than 0.90 and 59 percent are greater than 0.99. In the 595 time series with a non-trend stationary shift term, 80 percent are greater than 0.90 and 44 percent are greater than 0.99. Moreover, there is not a clear connection between the standard deviation of labour's share and the correlation coefficients, i.e., again it is shown that under very general circumstances, there is a high risk that this kind of empirical work will result in fundamentally misleading conclusions about the underlying technology.

That *only* 44 percent of the series with a non-trend stationary shift are greater than 0.99 emphasises; that these findings do not generalise Shaikh's result that guaranties a perfect fit under the more restrictive conditions, but simply imply that it is very likely to obtain a very good fit under very general circumstances.

Again the series are checked for potential spurious regressions. The tests show that there is potential risk of spurious regression in 7 percent of

[5] Fisher et. al. (1977) analysed wage explanation in simulations with CES micro production functions. In this study aggregate Cobb-Douglas as well as CES production functions were estimated and in general both types fit the data well, as long as the shares were stable.

the 3200 regressions, i.e., the high correlation coefficients can again not be explained by spurious regression.

To sum up, the results from the extended model also support a more general version of Shaikh's critique, because even though the likelihood of observing near perfect correlation drops, when assumptions A and B are violated, it is still very likely to obtain correlation coefficients that most researchers would (wrongly) interpret as support for the estimated functional form.

4.7 Concluding Remarks

Fisher's 1971 computer experiment has been reconstructed and his results verified. Strengthened by the extensions I have employed, Shaikh's original findings have been confirmed along with the extensions presented in Dixon and McCombie (1991) and Felipe and Holz (2001). The main contribution of this chapter has been to show that even under the very general circumstances where neither Shaikh's Perfect Fit Theorem nor the results presented in Dixon and McCombie (1991) and Felipe and Holz (2001) would predict a (near) perfect fit, the Cobb-Douglas production function still shows an "impressive" ability to mimic the data, even with the most simple and popular econometric method.

The implications of these cumulative results are important because they imply that empirical studies, in which a Cobb-Douglas production function is estimated, are necessarily inconclusive. This undermines empirical support for the neoclassical theory of growth and distribution, because that support — to a wide extent — is based on the Cobb-Douglas production function. Moreover, it is a serious warning against using *AS IF* justifications for economic theory.

The lesson should be extreme caution is necessary when applying aggregate production functions; indeed, I would propose instead of aggregate production functions, the implementation of (physical) multi-sector input–output systems in general macroeconomic models, because there is neither theoretical nor empirical support for the use of aggregate production functions. In my opinion, an aggregate production function is simply a notion used for mathematical convenience and elegance.

Some might argue that a more "realistic" production function like the (nested) CES or translog would evade these problems, but the Cobb-Douglas function's ability to fit (plausible and implausible) data are of course fully embedded in the more flexible functional forms.

Acknowledgements

I am grateful for the inspiration and guidance provided by Stefano Zambelli; if it were not for him, I would never had initiated this project. After finishing the draft of this chapter, I sent it to Anwar Shaikh, Franklin M. Fisher, and Jesus Felipe who all replied with precise and critical comments as well as words of encouragement, for which I am very grateful. Also Charlotte Bruun and K. Vela Velupillai deserve special gratitude for the support they provided in the gestation of this project. However, it goes without saying that I am solely responsible for the contents of this chapter.

References

Arrow K. J., Chenery H. B., Minhas B. S., Solow R. M. (1961): 'Capital–labour substitution and economic efficiency', *The Review of Economics and Statistics*, 43, pp. 225–250.

Cohen A. J., Harcourt G. C. (2003): 'Retrospectives: whatever happened to the Cambridge capital theory controversies?', *Journal of Economic Perspectives*, 17, pp. 199–214.

Felipe J., Fisher F. M. (2003): 'Aggregation in production functions: what applied economists should know', *Metroeconomica*, 54, pp. 208–262.

Felipe J., Holz C. A. (2001): 'Why do aggregate production functions works? Fisher's simulations, Shaikh's identity and some new results', *International Review of Applied Economics*, 15, pp. 261–285.

Felipe J., McCombie J. S. L. (2001): 'The CES production function, the accounting identity, and Occam's Razor', *Applied Economics*, 33, pp. 1221–1232.

Felipe J., McCombie J. S. L. (2007): 'On the rental price of capital and the profit rate: the perils and pitfalls of total factor productivity growth', *Review of Political Economy*, 19, pp. 317–345.

Fisher F. M. (1969): 'The existence of aggregate production functions', *Econometrica*, 37, pp. 553–577.

Fisher F. M. (1971): 'Aggregate production functions and the explanation of wages: a simulation experiment', *The Review of Economics and Statistics*, 53, pp. 305–325.

Fisher F. M., Solow R. M., Kearl J. M. (1977): 'aggregate production functions: some CES experiment', *The Review of Economics Studies*, 44, pp. 305–320.

McCombie J. S. L., Dixon R. (1991): 'Estimating technical change in aggregate production functions: a critique', *International Review of Applied Economics*, 5, pp. 24–46.

Pasinetti L. L. (2000): 'Critique of the neoclassical theory of growth and distribution', *BNL Quarterly Review*, 215, pp. 383–431.

Shaikh A. (1974): 'Laws of production and laws of algebra: the Humbug production function', *The Review of Economics and Statistics*, 56, pp. 115–120.

Shaikh A. (1980): 'Laws of production and laws of algebra: the Humbug II', in *Growth, Profit, and Property: Essays in the Revival of Political Economy*, edited by Edward J. Nell. Cambridge: Cambridge University Press, pp. 80–95.

Shaikh A. (1986): 'Humbug production function', in *The New Palgrave: A Dictionary of Economic Theory and Doctrine*, edited by J. Eatwell, M. Milgate and P. Newman. London: Macmillan.

Shaikh A. (2005): 'Nonlinear dynamics and pseudo-production functions', *Eastern Economic Journal*, 31, pp. 447–466.

Solow R. M. (1957): 'Technical change and the aggregate production function', *The Review of Economics and Statistics*, 39, pp. 312–320.

Solow R. M. (1974): 'Laws of production and laws of algebra: the Humbug production function: a comment', *The Review of Economics and Statistics*, 56, pp. 121.

Zambelli S. (2004). 'The 40 % neoclassical aggregate theory of production', *Cambridge Journal of Economics*, 28, pp. 99–120.

Appendices

4.A The Experiments

Fisher (1971) presents a neoclassical model comprising of n firms each producing one homogeneous output. The inputs consist of homogeneous and perfectly mobile labour and heterogeneous capital and technology that are bound to the individual firms. The experiments run for 20 periods, $t = 1, 2, ..., 20$.

Production at the ith firm is either modelled by a Cobb-Douglas or CES production function, *viz.*

$$y_{i,t} = A_{i,t} L_{i,t}^{\alpha_i} K_{i,t}^{1-\alpha_i} \tag{4.A.1}$$

$$y_{i,t} = A_{i,t} \Big[\alpha_i L_{i,t}^{1-\nu} + (1-\alpha_i) K_{i,t}^{1-\nu} \Big]^{\frac{1}{1-\nu}} \tag{4.A.2}$$

Where ν is the reciprocal of the elasticity of substitution between labour and capital. The aggregate production function and the associated aggregate capital stock is given by:

$$Y_t = A_t L_t^{\alpha} J_t^{1-\alpha} \tag{4.A.3}$$

$$J_t = \sum_{i=1}^{n} \left(\frac{\sum_{t=1}^{20} r_{i,t} K_{i,t}}{\sum_{t=1}^{20} K_{i,t}} \right) K_{i,t} \tag{4.A.4}$$

Wages and profits are paid there marginal products and it is assumed that labour is distributed such that the wage level coincide across the n firms. The algorithm applied to ensure distribution of labour is presented.

In all experiments the evolution of the total supply of labour, Hicks neutral technology, and capital endowments are exogenously given by:

$$L_t = \exp(0.3t + 0.02\varepsilon_t) \qquad \varepsilon_t \sim \mathrm{N}(0,1) \tag{4.A.5}$$

$$A_{i,t} = \exp(\gamma_{i,1} t) \qquad v_{i,t} \sim \mathrm{N}(0,1) \tag{4.A.6}$$

$$K_{i,t} = \exp(\beta_{i,0} + \beta_{i,1} t + 0.0001\eta_{i,t}) \qquad \eta_{i,t} \sim \mathrm{N}(0,1) \tag{4.A.7}$$

The experiments include two, four, or eight firms. Depending on this, the parameter, α_i, from the production function can take the following values:

$$n = 2: \quad (\alpha_1, \alpha_2) \in \big\{(0.7, 0.8), (0.6, 0.9)\big\}$$

$$n = 4: \quad (\alpha_1, ..., \alpha_4) \in \big\{(0.6, 0.7, ..., 0.9), (0.7, 0.725, ..., 0.8)\big\}$$

$$n = 8: \quad (\alpha_1, ..., \alpha_8) \in \big\{(0.6, 0.6 + \tfrac{1}{7}0.3, ..., 0.9), (0.7, 0.7 + \tfrac{1}{7}0.1, ..., 0.8)\big\}$$

The initial capital endowments can be distributed in three different ways, *viz.*

1. $\beta_{i,0} = 0 \ \forall \ i = 1, 2, ..., n$

2. $\beta_{i,0} = 0 \ \forall \ i = 1, 2, ..., \frac{n}{2}$
   $\beta_{i,0} = 2 \ \forall \ i = \frac{n}{2}, \frac{n}{2} + 1, ..., n$

3. $$\beta_{i,0} = 2 \ \ \forall \ \ i = 1, 2, ..., \tfrac{n}{2}$$
$$\beta_{i,0} = 0 \ \ \forall \ \ i = \tfrac{n}{2}, \tfrac{n}{2} + 1, ..., n$$

Finally, the experiments fall into the following five groups:

1. Two group capital
$$\begin{aligned} &\beta_{i,1} \in \{-0.05, -0.04, ..., 0.05\} && \forall \ \ i = 1, 2, ..., \tfrac{n}{2} \\ &\beta_{i,1} = 0 && \forall \ \ i = \tfrac{n}{2}, \tfrac{n}{2} + 1, ..., n \\ &\gamma_{i,1} = 0 && \forall \ \ i = 1, 2, ..., n \end{aligned}$$
2. Two group Hicks preliminary
$$\begin{aligned} &\gamma_{i,1} \in \{-0.05, -0.04, ..., 0.05\} && \forall \ \ i = 1, 2, ..., \tfrac{n}{2} \\ &\gamma_{i,1} = 0 && \forall \ \ i = \tfrac{n}{2}, \tfrac{n}{2} + 1, ..., n \\ &\beta_{i,1} = 0 && \forall \ \ i = 1, 2, ..., n \end{aligned}$$
3. Two group Hicks
$$\begin{aligned} &\gamma_{i,1} \in \{(\alpha_i - 1)0.05, (\alpha_i - 1)0.04, ..., (1 - \alpha_i)0.05\} && \forall \ \ i = 1, 2, ..., \tfrac{n}{2} \\ &\gamma_{i,1} = 0 && \forall \ \ i = \tfrac{n}{2}, \tfrac{n}{2} + 1, ..., n \\ &\beta_{i,1} = 0 && \forall \ \ i = 1, 2, ..., n \end{aligned}$$
4. The fanning capital
$$\begin{aligned} &\beta_{i,1} \in \{(i - 1)0.05, (i - 1)0.04, ..., (i - 1)0.05\} && \forall \ \ i = 1, 2, ..., n \\ &\gamma_{i,1} = 0 && \forall \ \ i = 1, 2, ..., n \end{aligned}$$
5. The fanning Hicks
$$\begin{aligned} &\gamma_{i,1} \in \{(\alpha_i - 1)(i - 1)0.05, (\alpha_i - 1)(i - 1)0.04, ..., (1 - \alpha_i)(i - 1)0.05\} \\ & \qquad\qquad\qquad\qquad\qquad \forall \ \ i = 1, 2, ..., n \\ &\beta_{i,1} = 0 \qquad\qquad\qquad\qquad\quad \forall \ \ i = 1, 2, ..., n \end{aligned}$$

This concludes the description of the experiments which by simple combinatorial calculates amount to 990 unique settings.

The algorithm, used to distribute labour among the n firms such that the wage levels are approximate equal across the firms, has the following structure.

1. Distribute the initial endowments of capital and technology.
2. Uniformly distribute the total labour supply across the n firms and compute the n wage levels.
3. Allocate a given amount of labour from the firms with a low wage level to the the firms with high wage level.
4. Repeat step three until the maximums deviation among the wage levels are less than 1 percent.

Computational this is however not always so straightforward, because the production functions satisfy the *Inada Conditions*, i.e., when labour inputs are close to zero small changes have large effect on the marginal products. The solution is to dynamical reducing the allocation of labour as the wage levels converge.

Part III

Statistical Companion

Chapter 5

Additional Results from Chapter 2 'Measuring Structural and Technological Change from Technically Autarkic Subsystems'

5.A The σ-index (Productivity Index, Physical Quantities)

For some of the indices, extreme cases have been collected in separate figures to better be able to study the evolution in the single industries.

$$\sigma_t^i = \frac{\tilde{b}^i_{(i,i,t)} - \tilde{a}^i_{(i,i,t)}}{\boldsymbol{e}'\tilde{\boldsymbol{l}}^i_t} = \frac{\text{external output}}{\text{direct + indirect labour}}$$

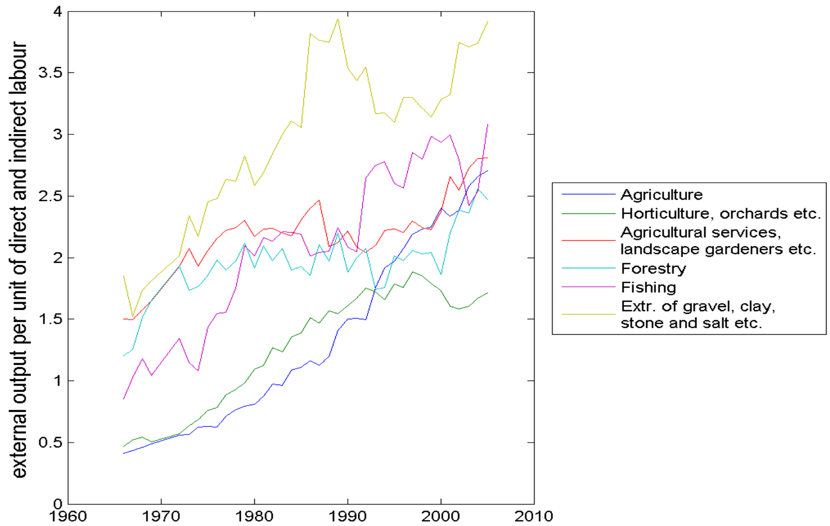

Fig. 5.1: *The σ-index, Agriculture, fishing, and quarrying*

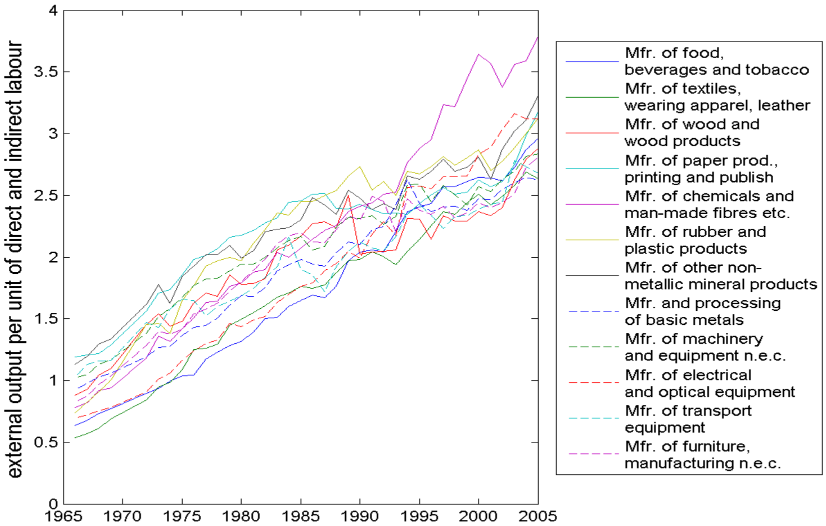

Fig. 5.2: *The σ-index, Manufacturing*

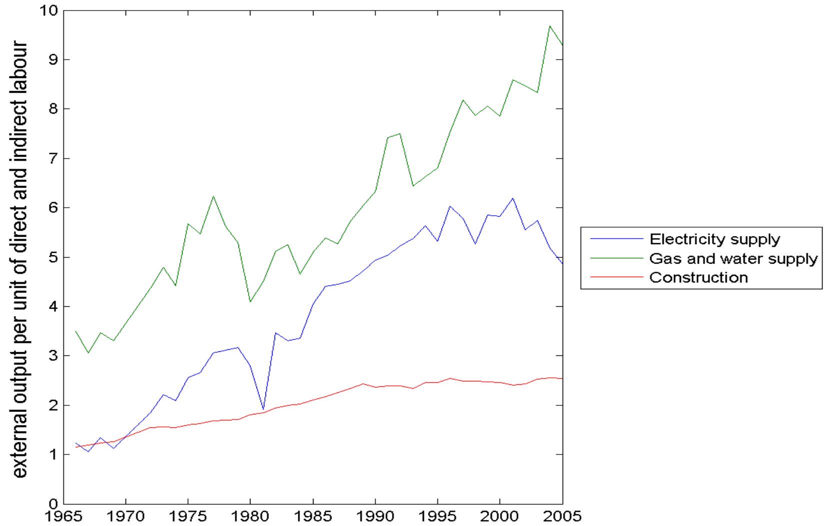

Fig. 5.3: *The σ-index, Electricity, gas, and water supply; and Construction*

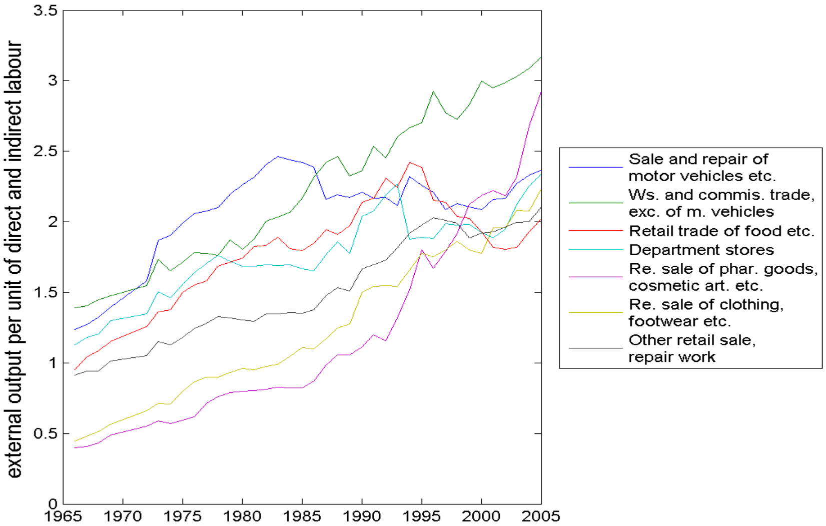

Fig. 5.4: *The σ-index, Wholesale-, retail trade, hotels, restaurants*

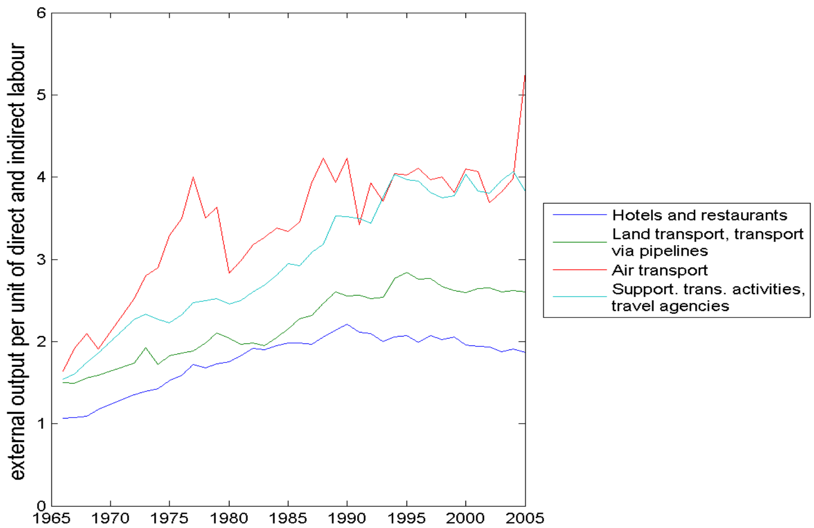

Fig. 5.5: *The σ-index, Transport, storage, and communication*

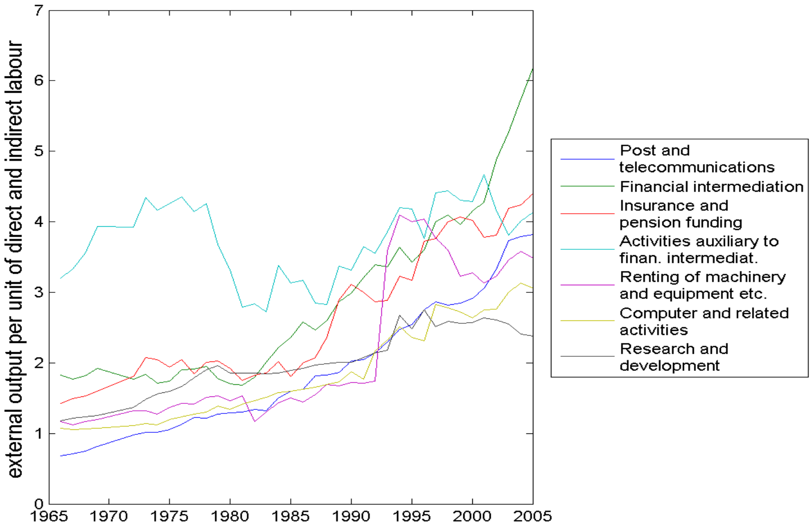

Fig. 5.6: *The σ-index, Financial intermediation, business activities*

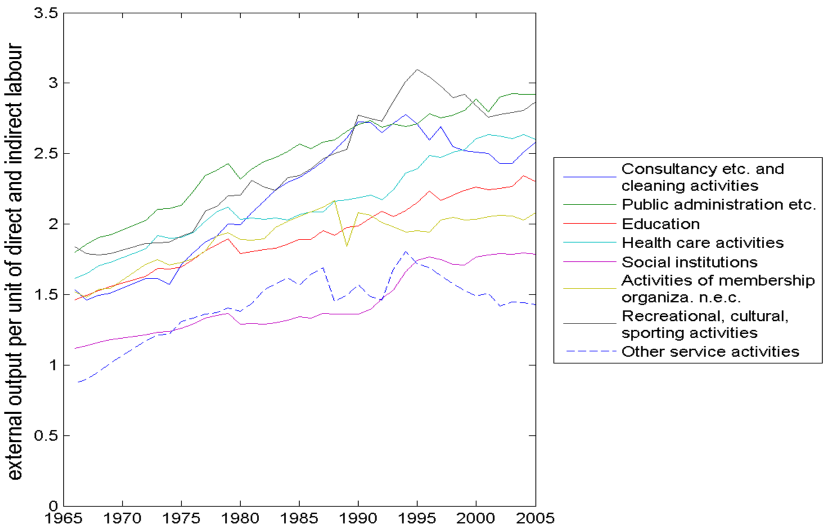

Fig. 5.7: *The σ-index, Public and personal services*

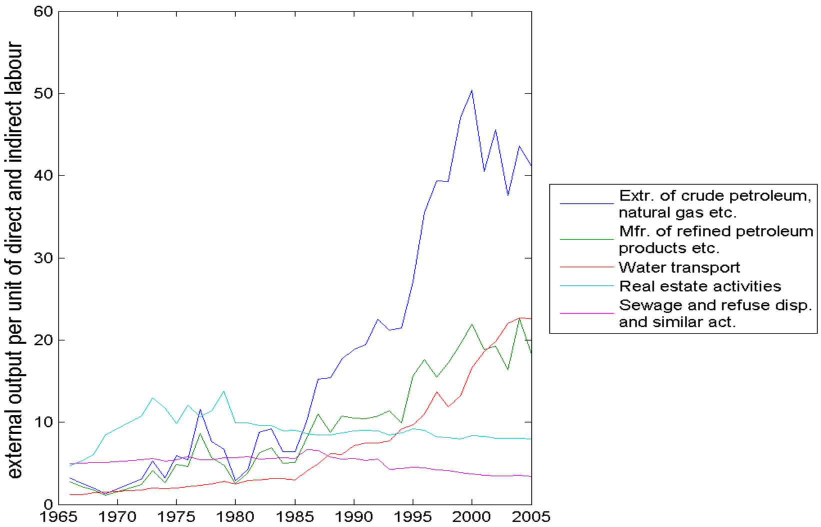

Fig. 5.8: *The σ-index, extreme cases*

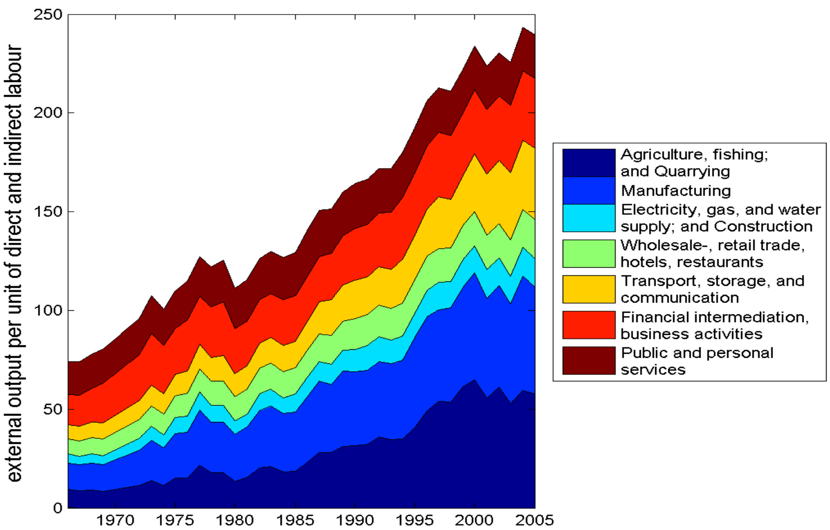

Fig. 5.9: *The σ-index for the meso-sectors*

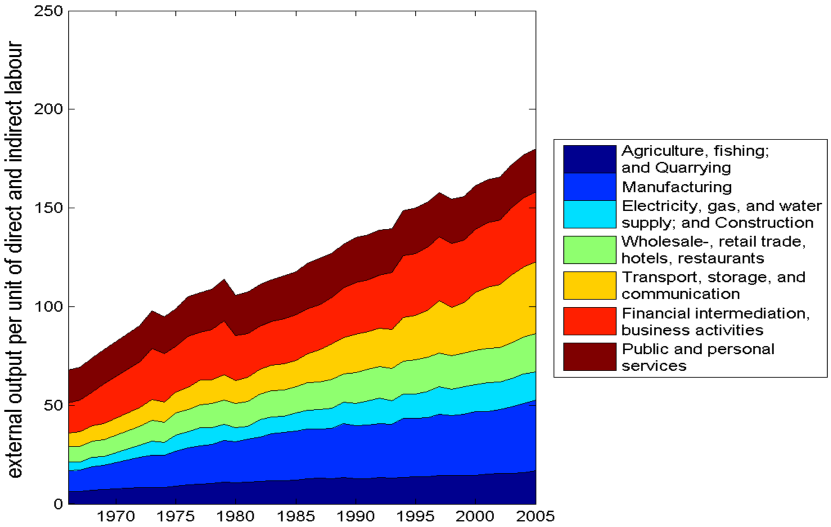

Fig. 5.10: *The σ-index for the meso-sectors, excluding 'Extr. of crude petroleum, natural gas etc.' and 'Mfr. of refined petroleum products etc.'*

5.B The ξ-index (Productivity Index, Physical Quantities)

$$\xi_t^i = \frac{\tilde{b}^i_{(i,i,t)} - \boldsymbol{e}'\tilde{\boldsymbol{a}}^i_{(:,i,t)}}{\boldsymbol{e}'\tilde{\boldsymbol{l}}^i_t} = \frac{\text{final output}}{\text{direct + indirect labour}}$$

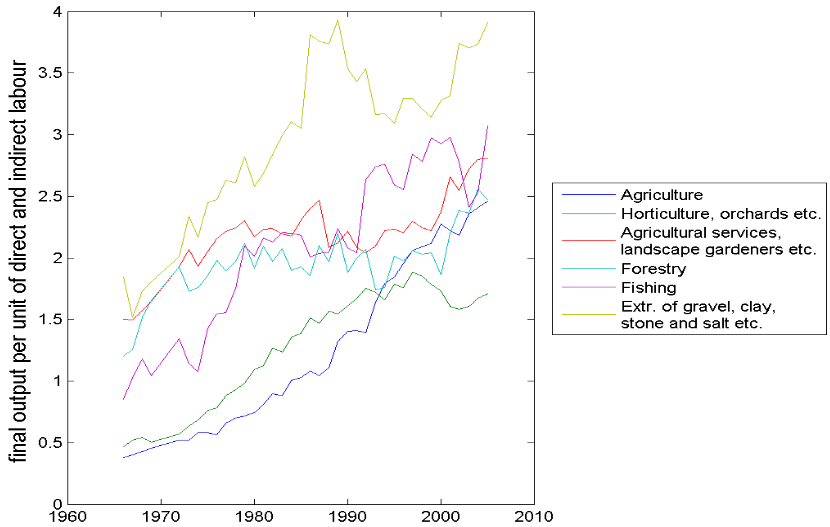

Fig. 5.11: *The ξ-index, Agriculture, fishing, and quarrying*

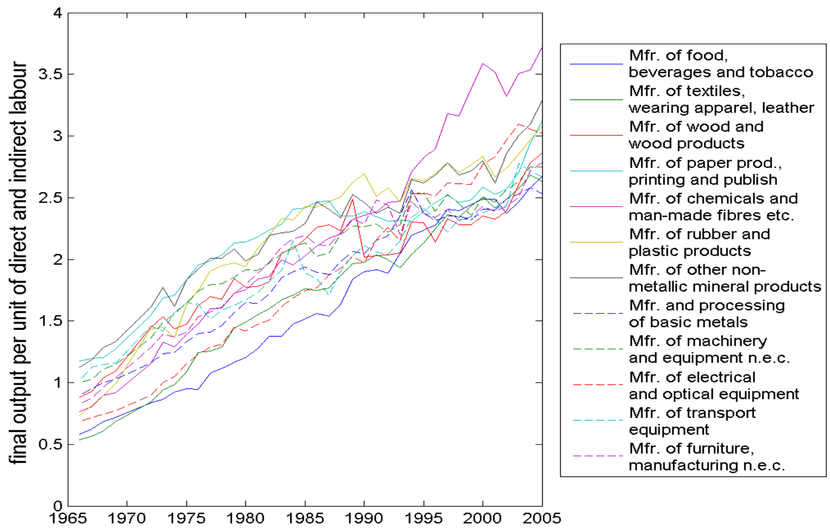

Fig. 5.12: *The ξ-index, Manufacturing*

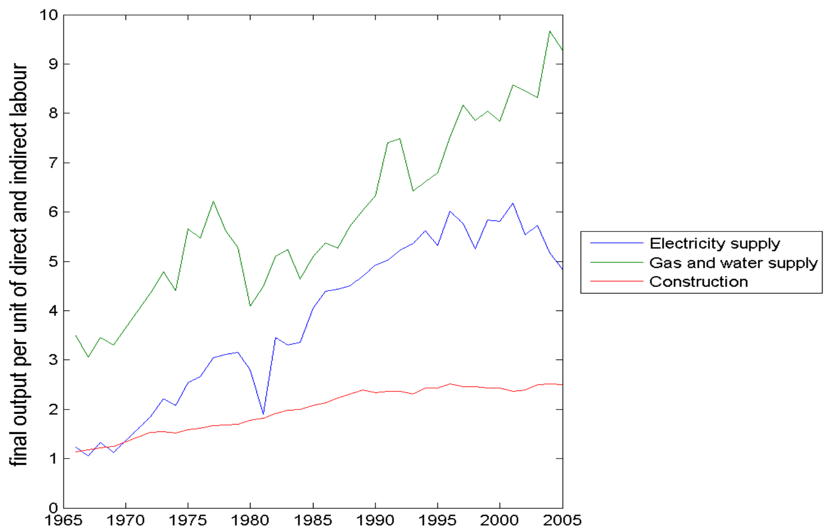

Fig. 5.13: *The ξ-index, Electricity, gas, and water supply; and Construction*

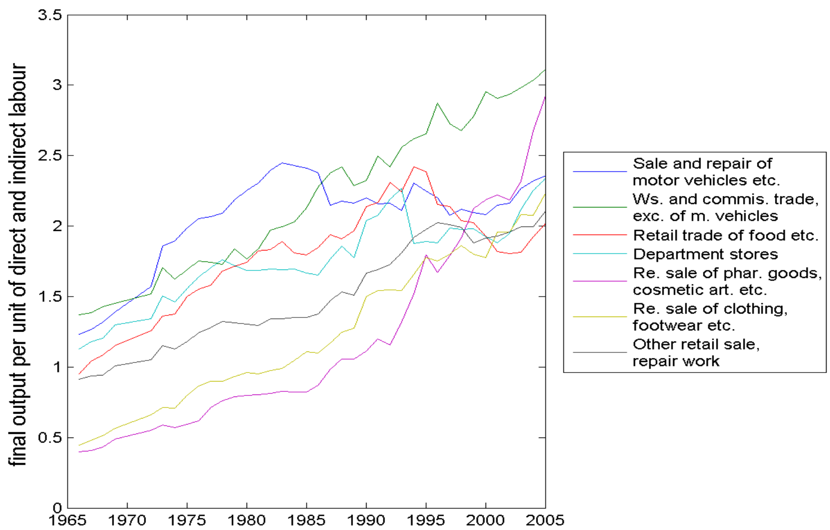

Fig. 5.14: *The ξ-index, Wholesale-, retail trade, hotels, restaurants*

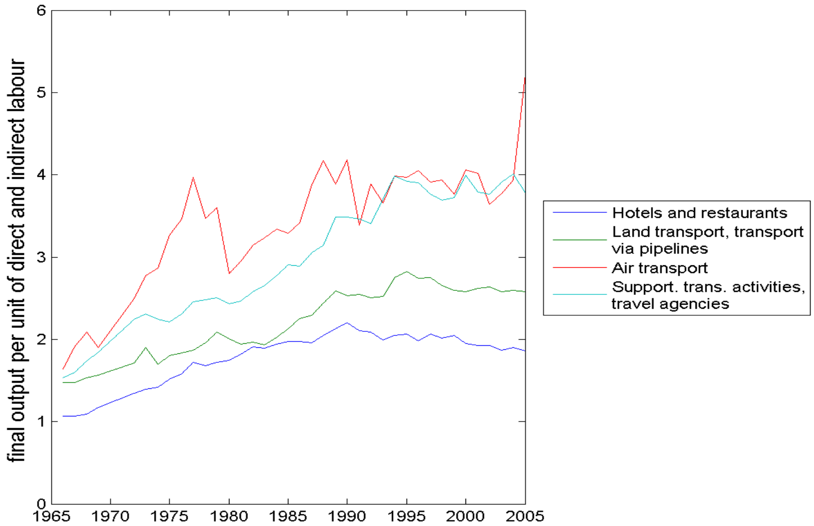

Fig. 5.15: *The ξ-index, Transport, storage, and communication*

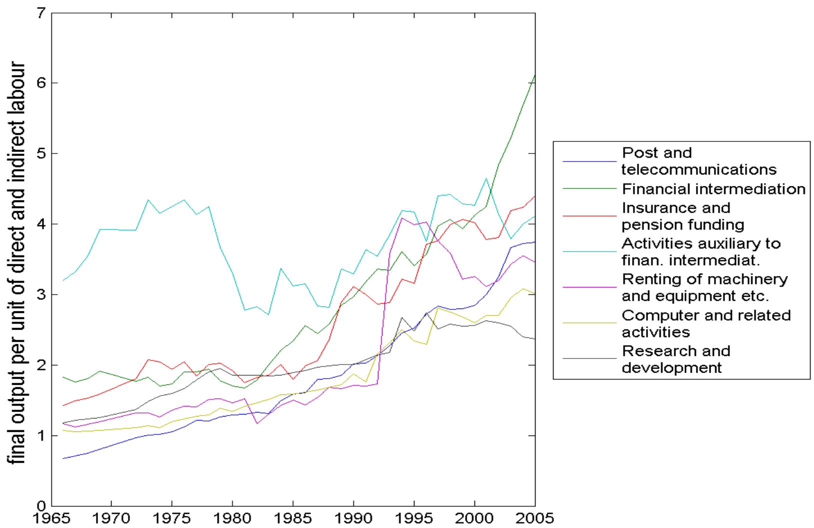

Fig. 5.16: *The ξ-index, Financial intermediation, business activities*

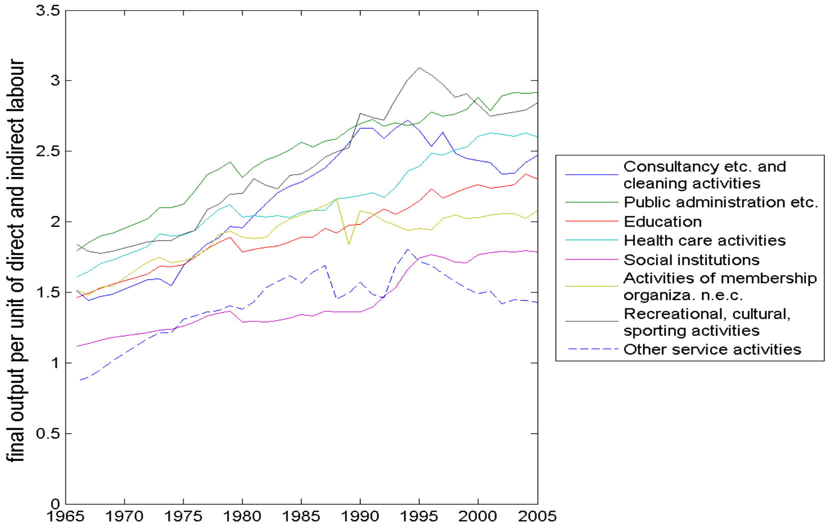

Fig. 5.17: *The ξ-index, Public and personal services*

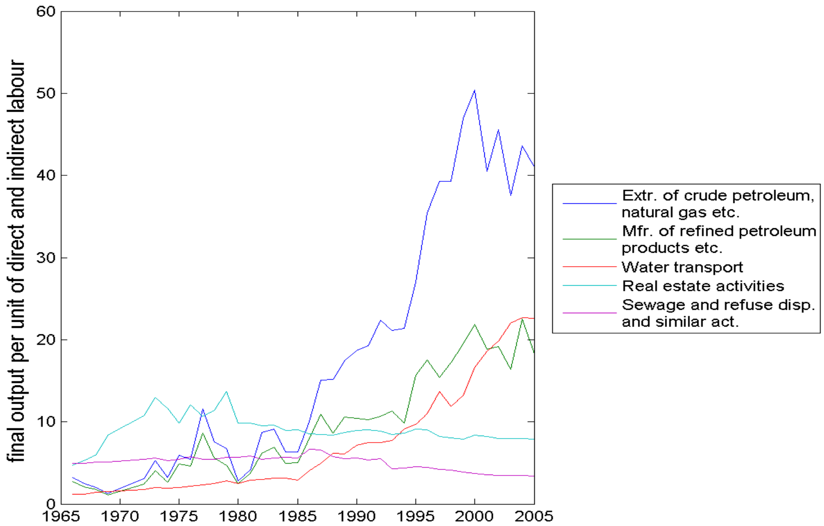

Fig. 5.18: *The ξ-index, extreme cases*

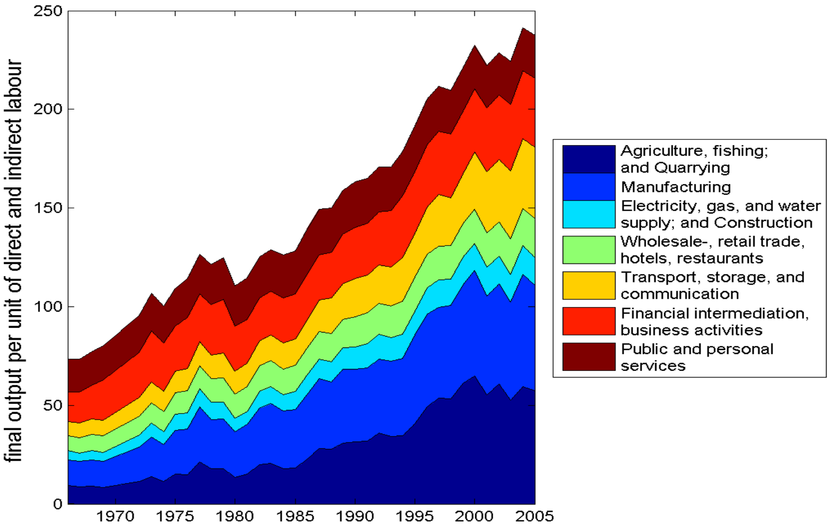

Fig. 5.19: *The ξ-index for the meso-sectors*

5.C The v-index (Productivity Index, Physical Quantities)

$$\boldsymbol{v}_t = (\boldsymbol{B}_t - \boldsymbol{A}_t)^{-1} \boldsymbol{l}_t$$

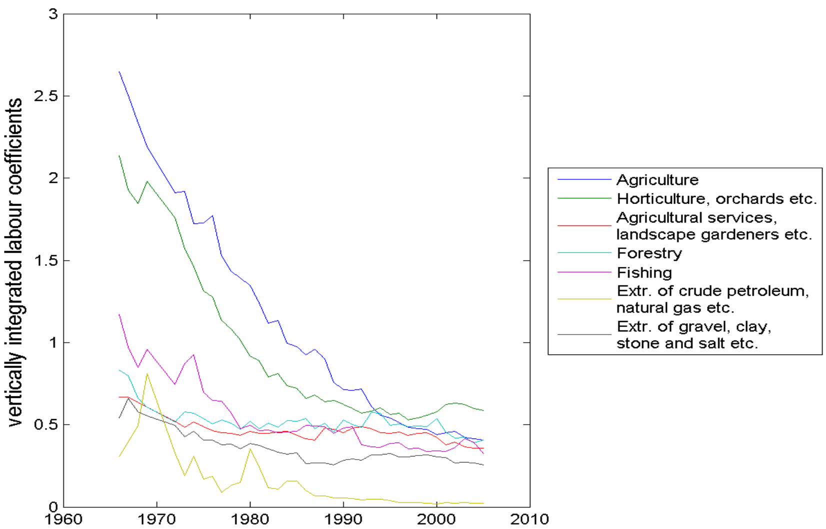

Fig. 5.20: *The v-index, Agriculture, fishing, and quarrying*

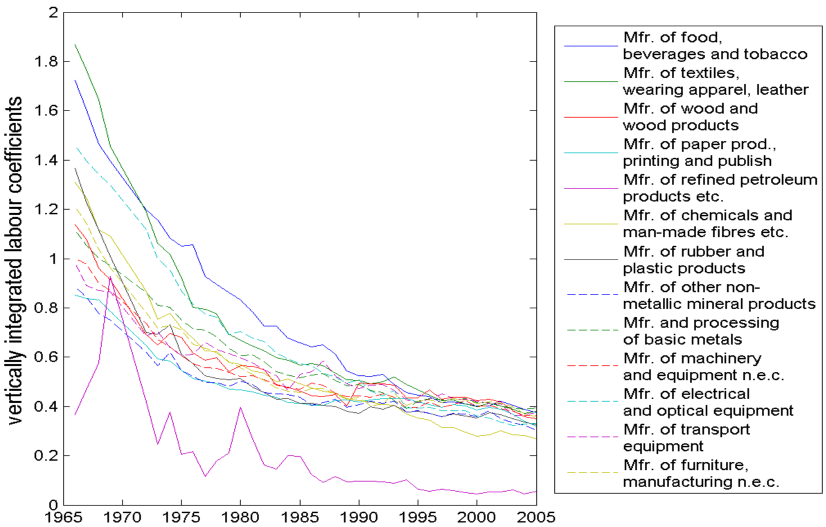

Fig. 5.21: *The v-index, Manufacturing*

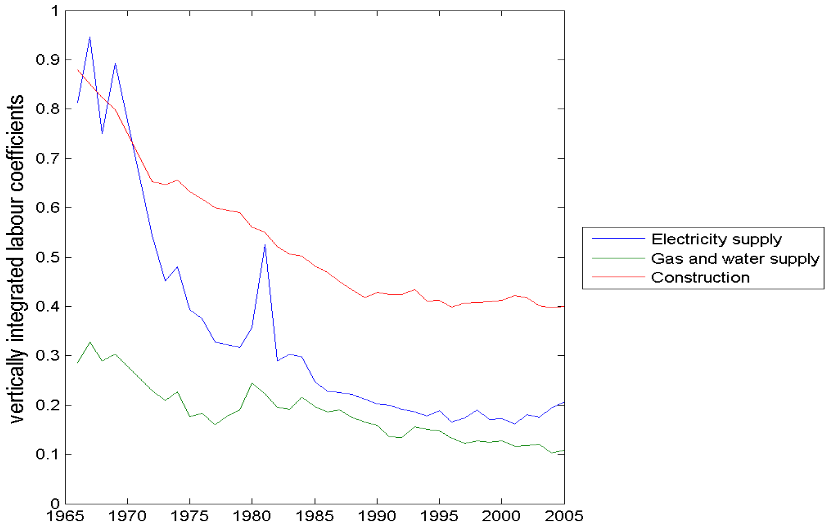

Fig. 5.22: *The v-index, Electricity, gas, and water supply; and Construction*

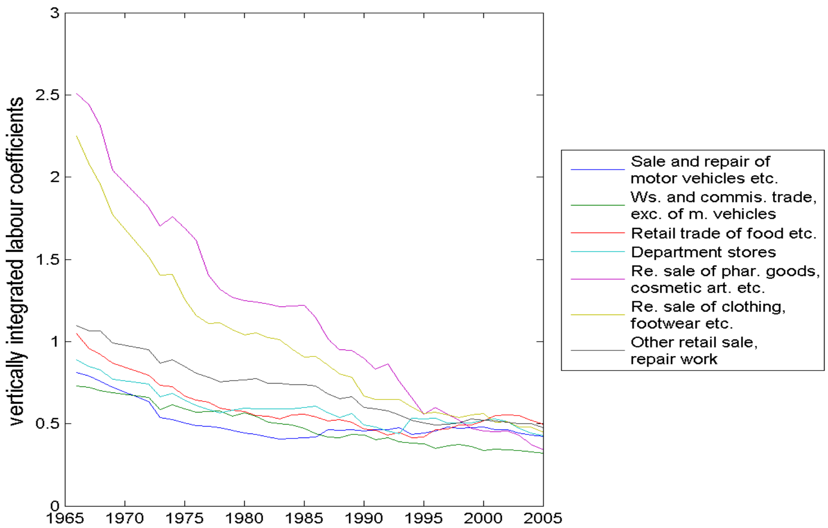

Fig. 5.23: *The v-index, Wholesale-, retail trade, hotels, restaurants*

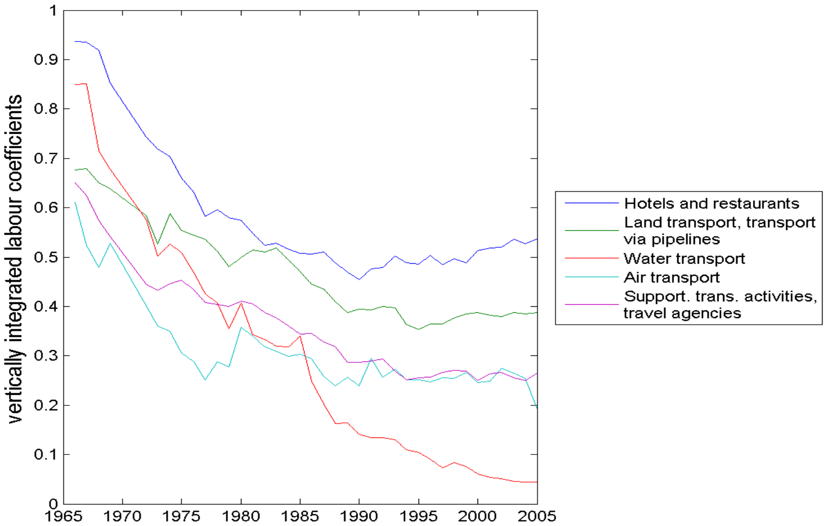

Fig. 5.24: *The v-index, Transport, storage, and communication*

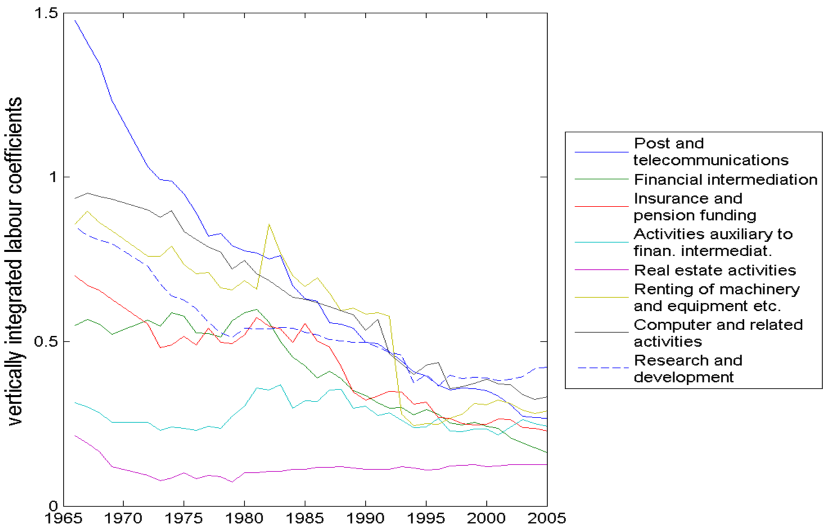

Fig. 5.25: *The v-index, Financial intermediation, business activities*

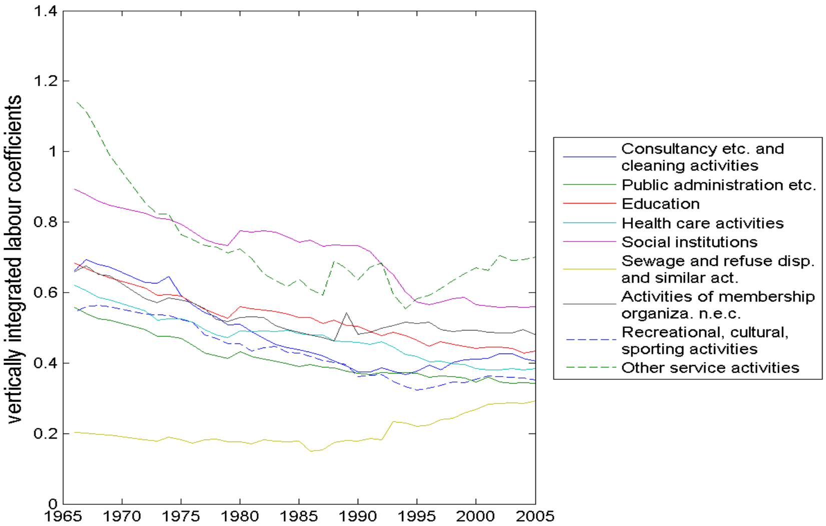

Fig. 5.26: *The v-index, Public and personal services*

5.D The ρ-index (Index of Structural Change, Physical Quantities)

$$\rho_t^i = \frac{\beta_t^i \tilde{l}^i_{(i,t)}}{\boldsymbol{e}'\tilde{\boldsymbol{l}}^i_t} = \frac{\text{direct labour}}{\text{direct + indirect labour}}$$

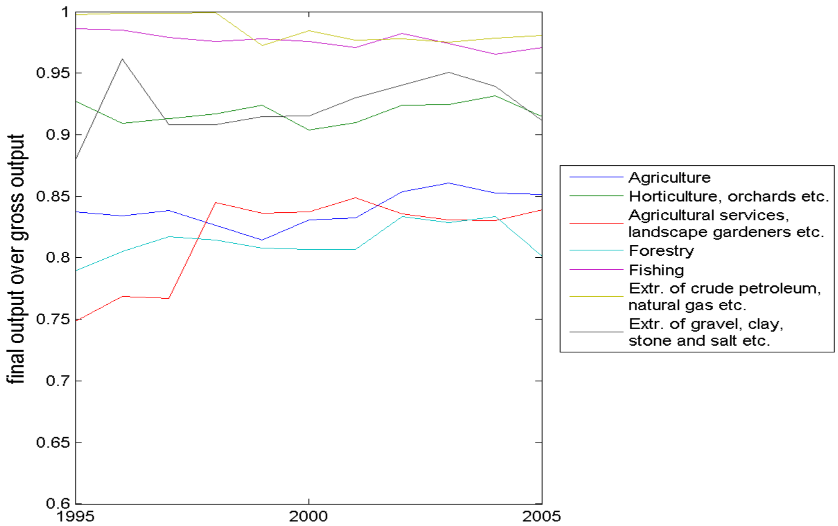

Fig. 5.27: *The ρ-index, Agriculture, fishing, and quarrying*

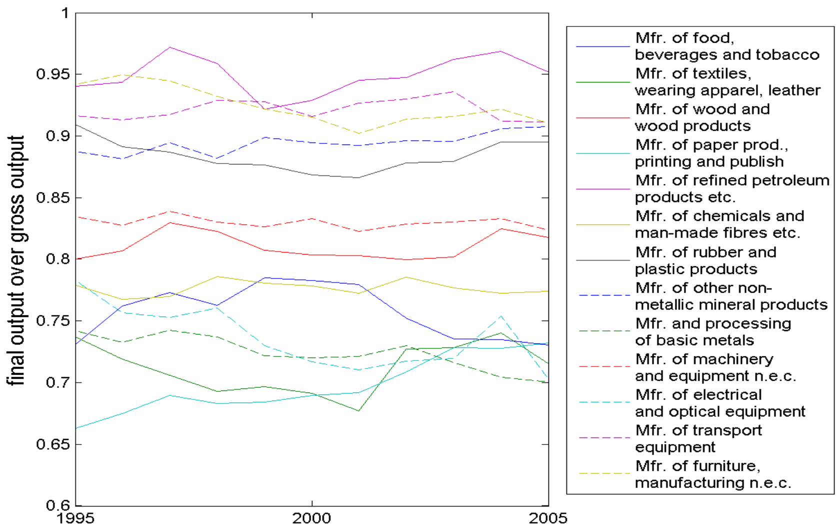

Fig. 5.28: *The ρ-index, Manufacturing*

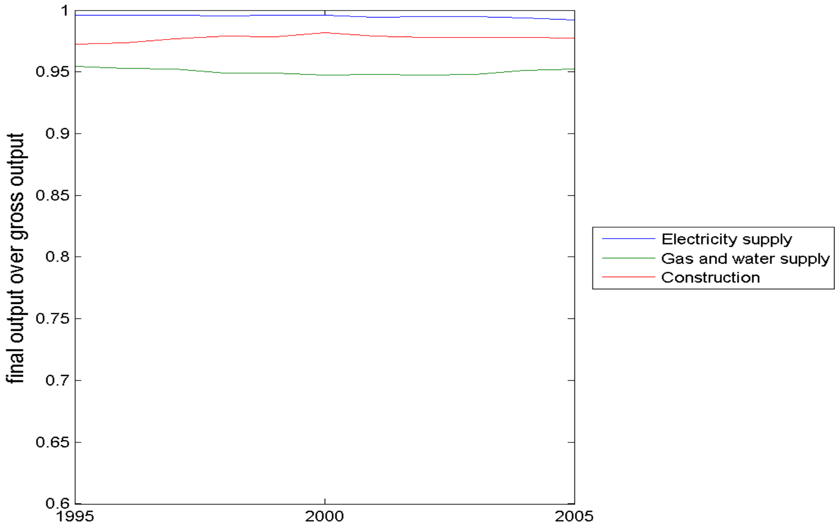

Fig. 5.29: *The ρ-index, Electricity, gas, and water supply; and Construction*

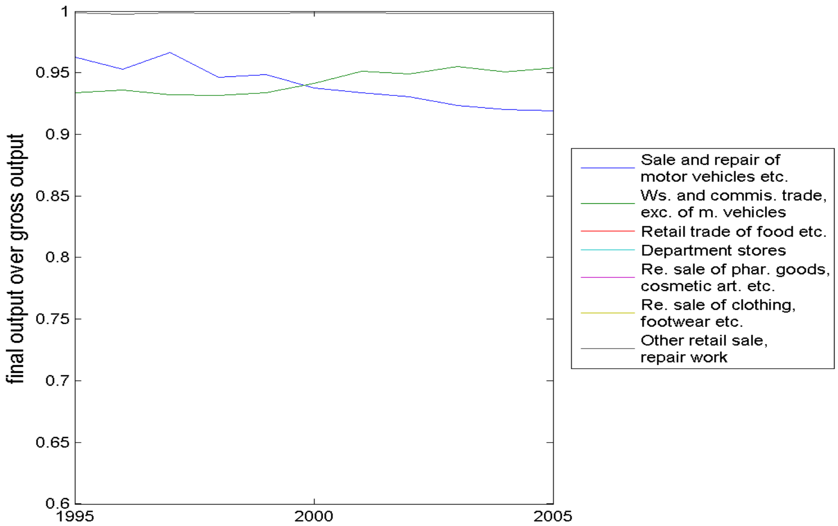

Fig. 5.30: *The ρ-index, Wholesale-, retail trade, hotels, restaurants*

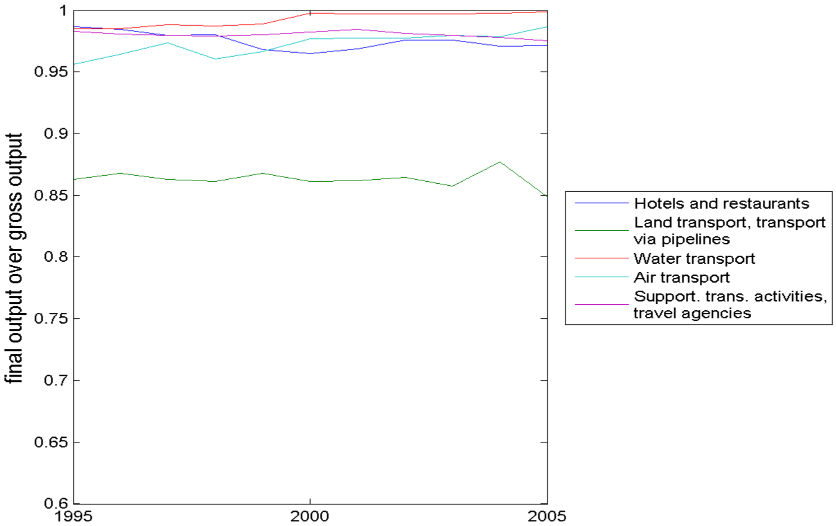

Fig. 5.31: *The ρ-index, Transport, storage, and communication*

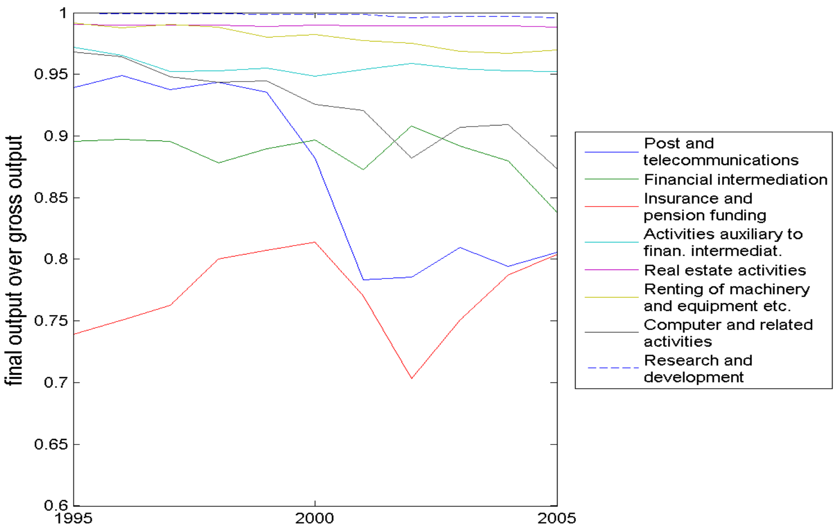

Fig. 5.32: *The ρ-index, Financial intermediation, business activities*

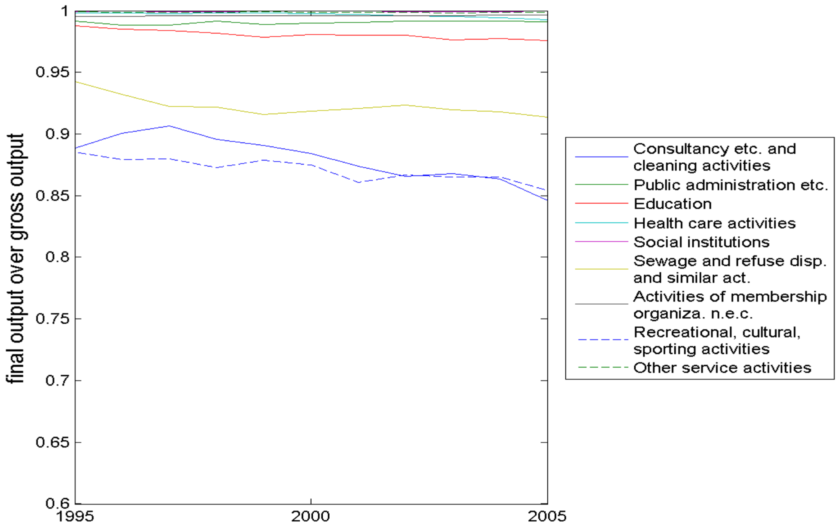

Fig. 5.33: *The ρ-index, Public and personal services*

5.E The α-index (Index of Structural Change, Physical Quantities)

$$\alpha_t^i = \frac{b_{(i,i,t)} - \boldsymbol{e}'\boldsymbol{a}_{(:,i,t)}}{b_{(i,i,t)}} = \frac{\text{final output*}}{\text{gross output*}}$$

Fig. 5.34: *The α-index, Agriculture, fishing, and quarrying*

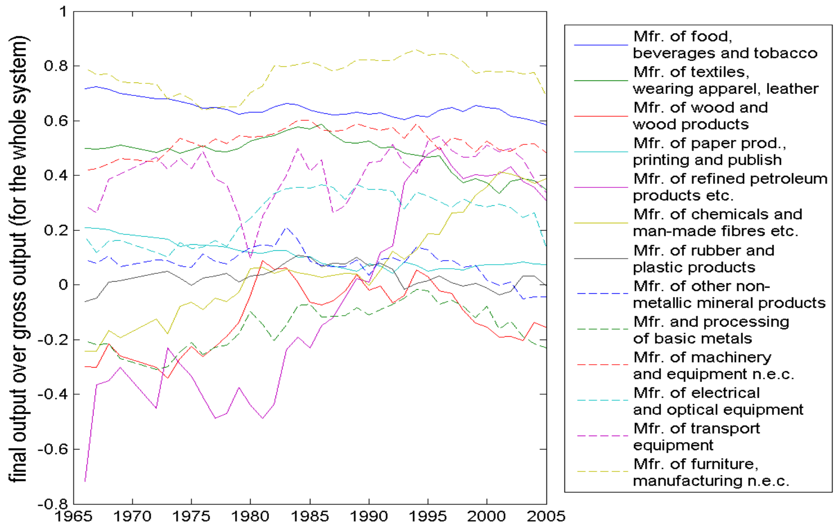

Fig. 5.35: *The α-index, Manufacturing*

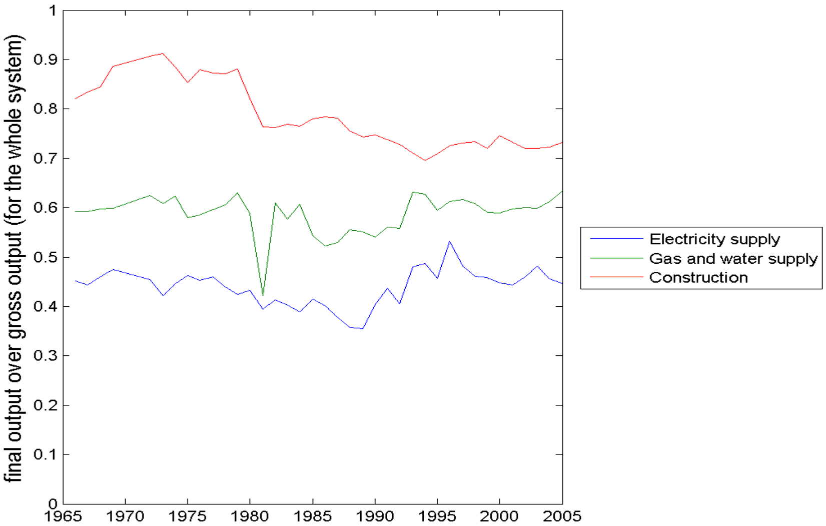

Fig. 5.36: *The α-index, Electricity, gas, and water supply; and Construction*

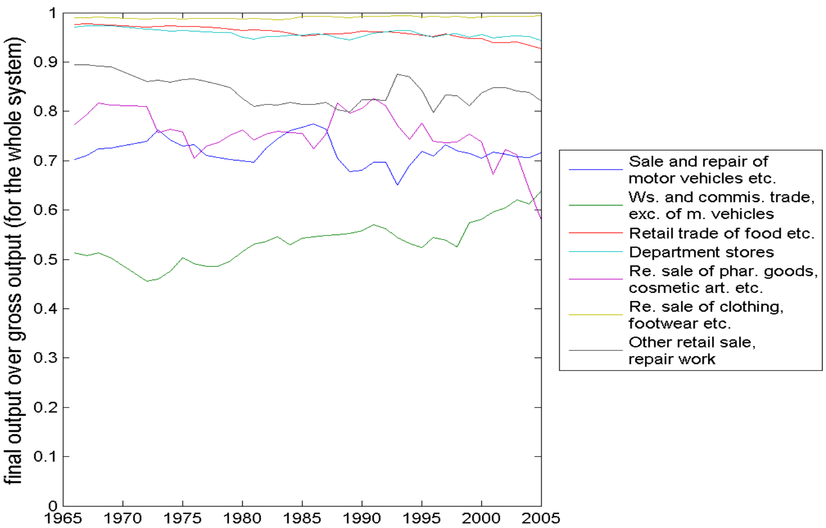

Fig. 5.37: *The α-index, Wholesale-, retail trade, hotels, restaurants*

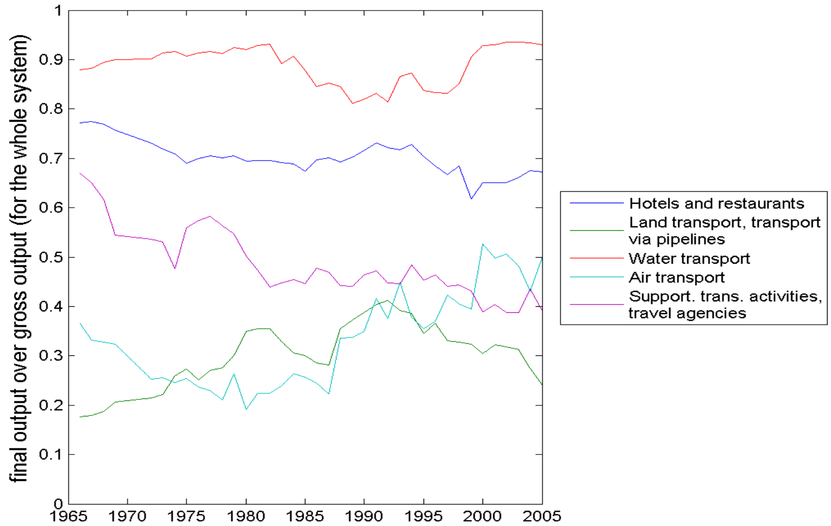

Fig. 5.38: *The α-index, Transport, storage, and communication*

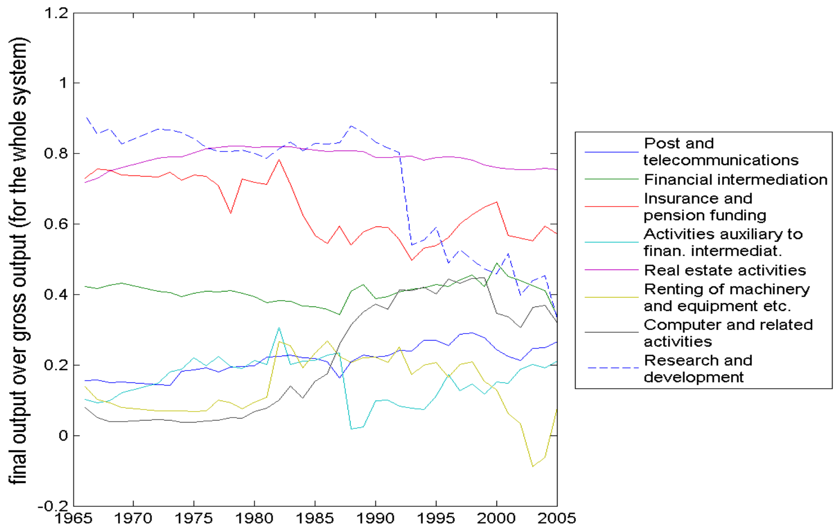

Fig. 5.39: *The α-index, Financial intermediation, business activities*

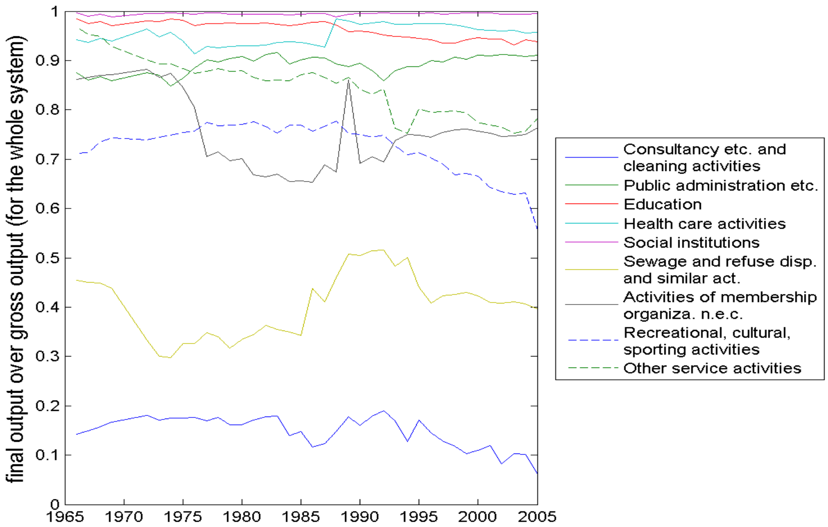

Fig. 5.40: *The α-index, Public and personal services*

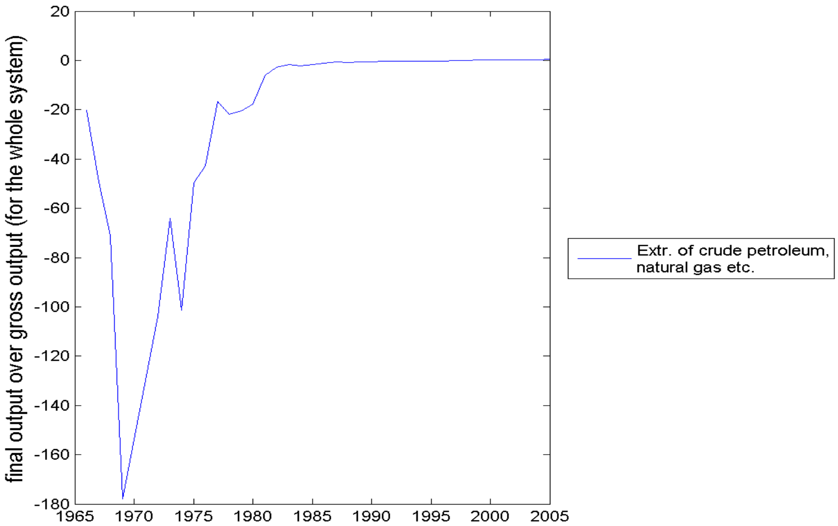

Fig. 5.41: *The α-index, extreme cases*

5.F The β-index (Index of Structural Change, Physical Quantities)

$$\beta_t^i = \frac{\tilde{b}^i_{(i,i,t)} - \boldsymbol{e}'\tilde{\boldsymbol{a}}^i_{(:,i,t)}}{\tilde{b}^i_{(i,i,t)}} = \frac{\text{final output}}{\text{gross output}}$$

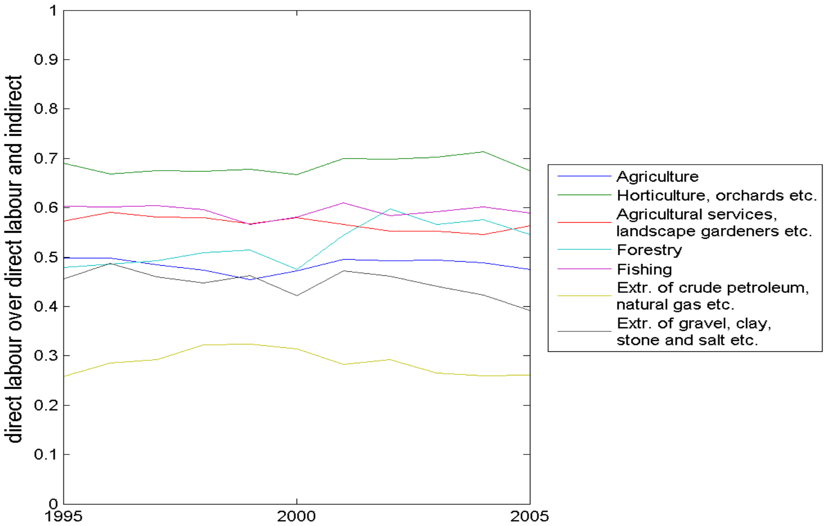

Fig. 5.42: *The β-index, Agriculture, fishing, and quarrying*

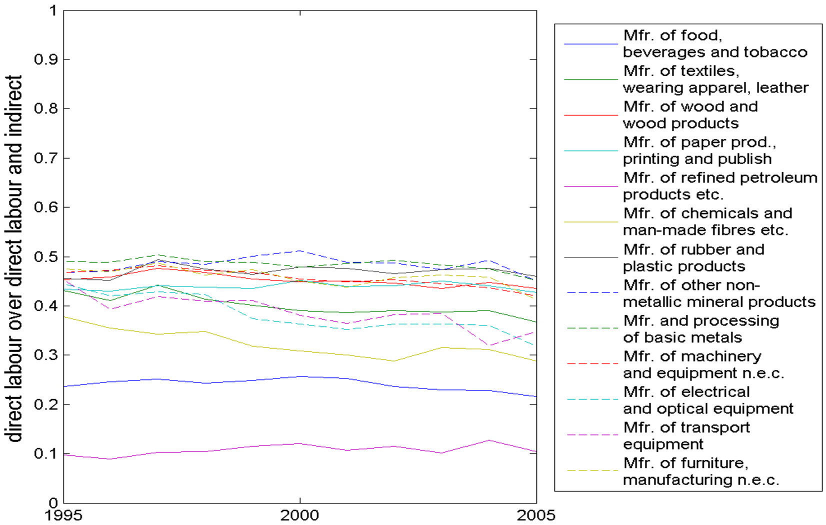

Fig. 5.43: *The β-index, Manufacturing*

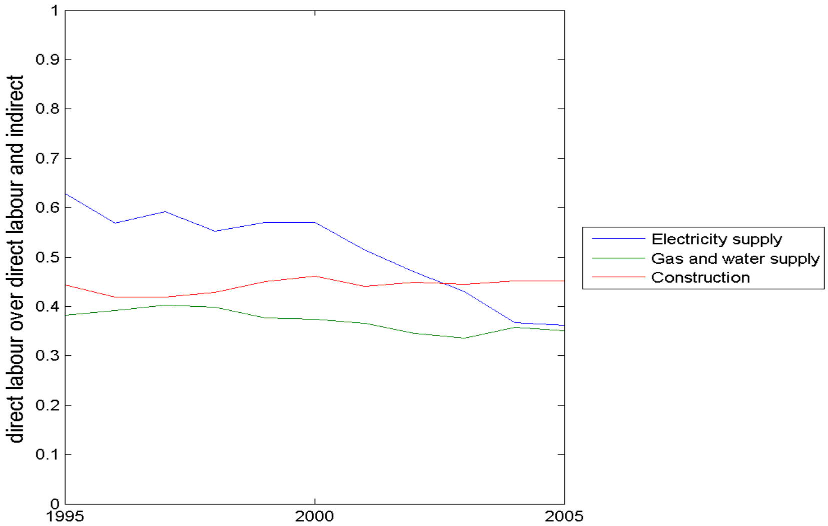

Fig. 5.44: *The β-index, Electricity, gas, and water supply; and Construction*

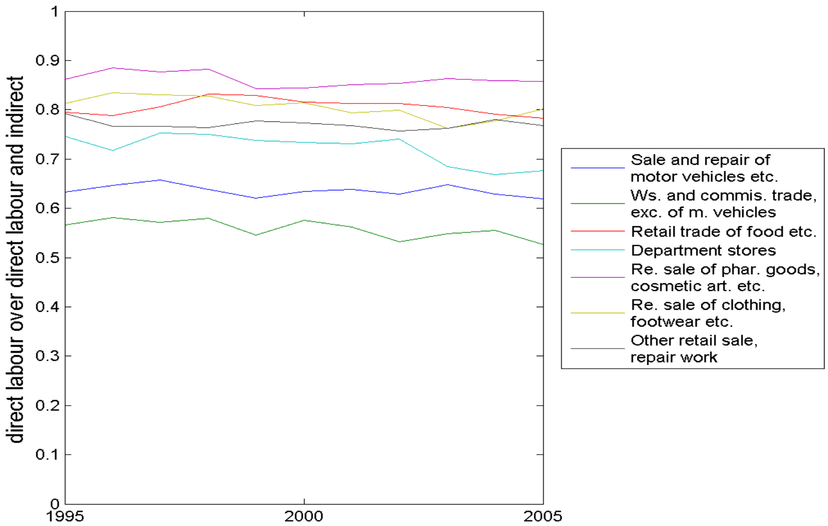

Fig. 5.45: *The β-index, Wholesale-, retail trade, hotels, restaurants*

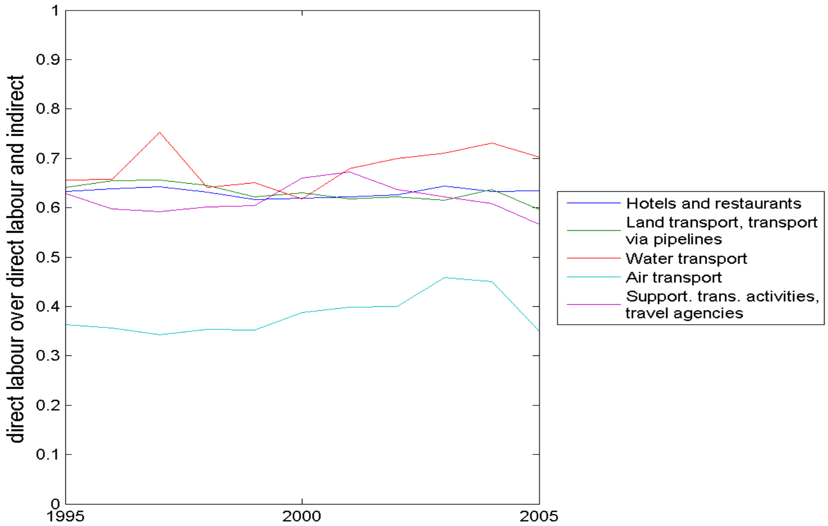

Fig. 5.46: *The β-index, Transport, storage, and communication*

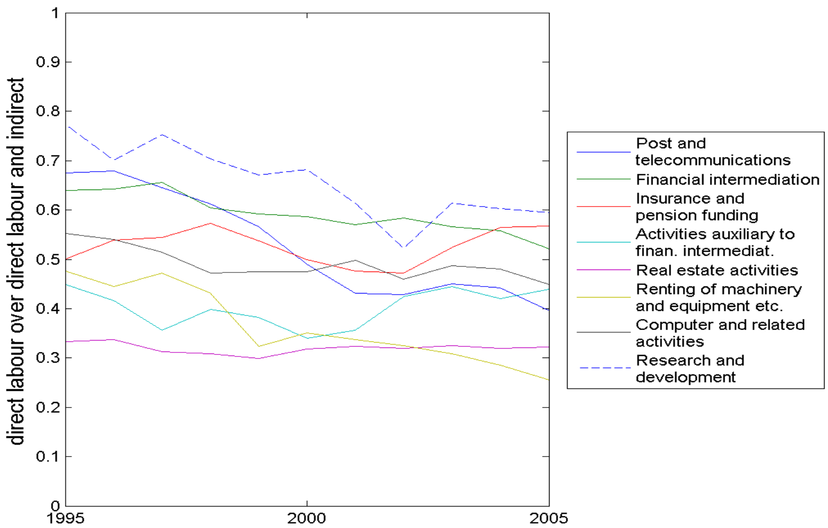

Fig. 5.47: *The β-index, Financial intermediation, business activities*

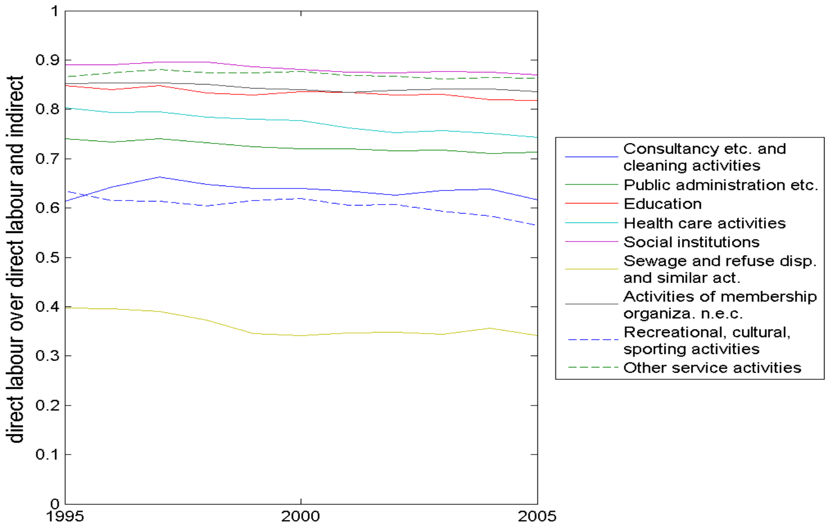

Fig. 5.48: *The β-index, Public and personal services*

5.G The μ-index (Productivity Index, Production Prices)

$$\mu_t^i = \frac{1}{l_{(i,t)} R_t} \int_0^{R_t} \zeta_{(i,t)}(r)\, dr$$

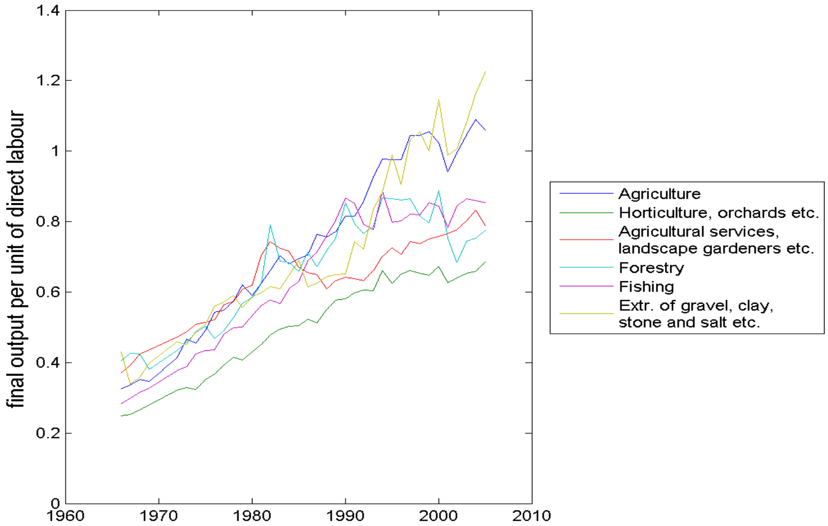

Fig. 5.49: *The μ-index, Agriculture, fishing, and quarrying*

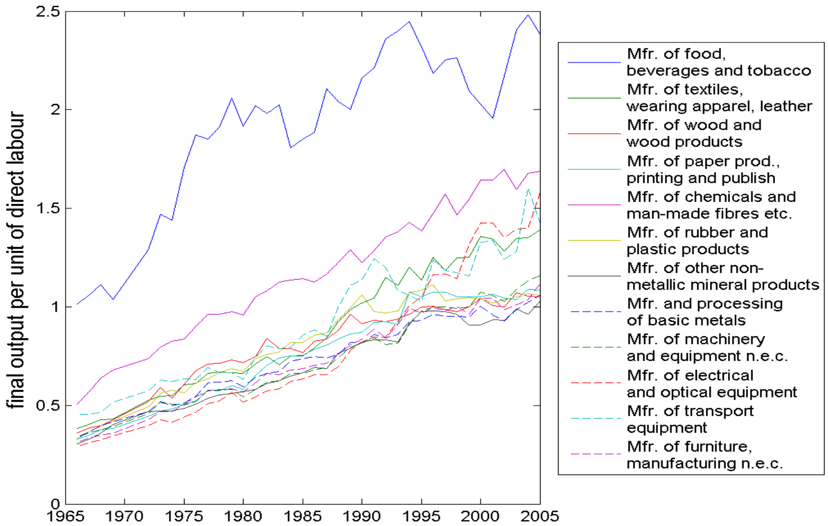

Fig. 5.50: *The μ-index, Manufacturing*

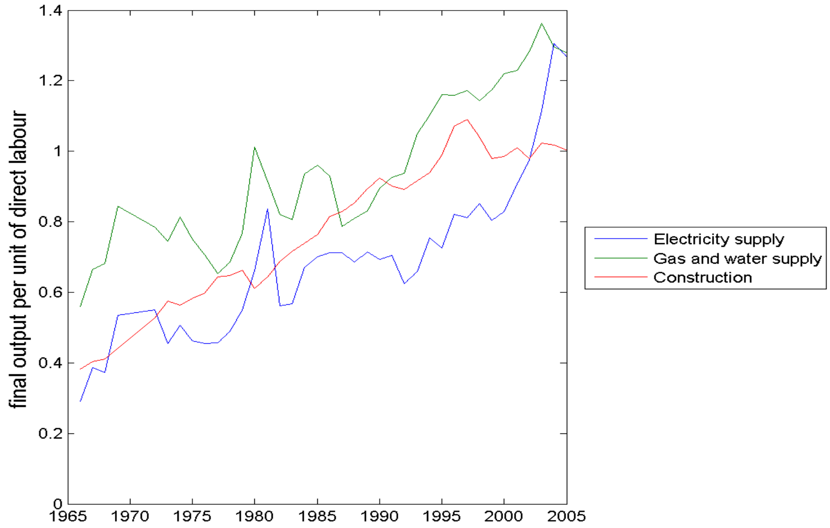

Fig. 5.51: *The μ-index, Electricity, gas, and water supply; and Construction*

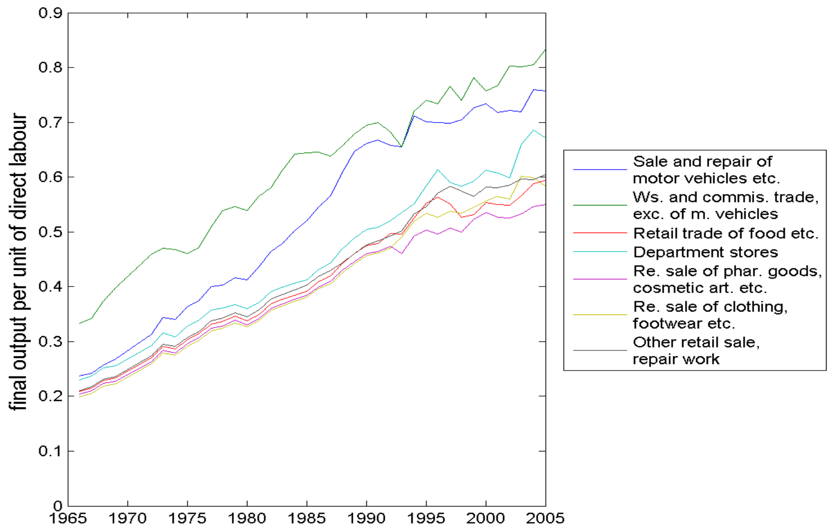

Fig. 5.52: *The μ-index, Wholesale-, retail trade, hotels, restaurants*

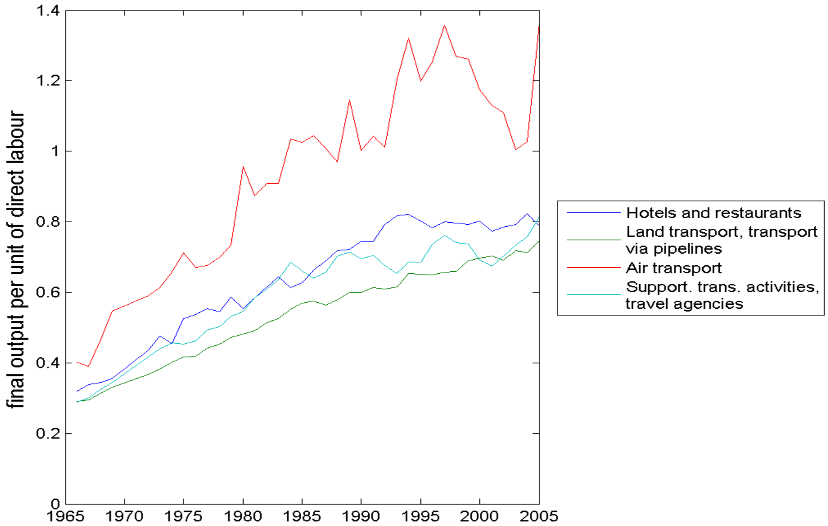

Fig. 5.53: *The μ-index, Transport, storage, and communication*

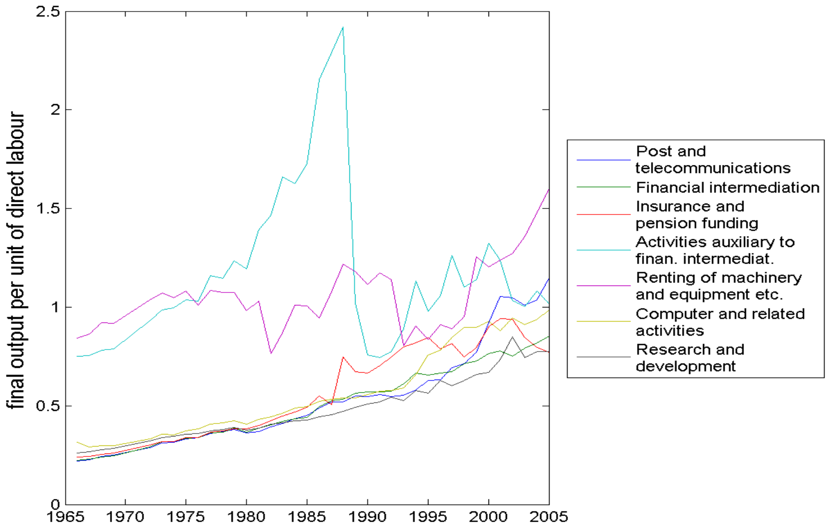

Fig. 5.54: *The μ-index, Financial intermediation, business activities*

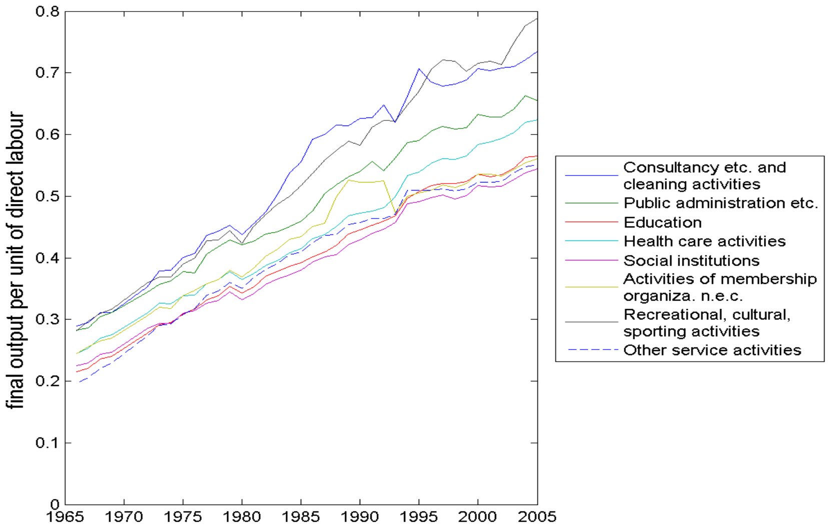

Fig. 5.55: *The μ-index, Public and personal services*

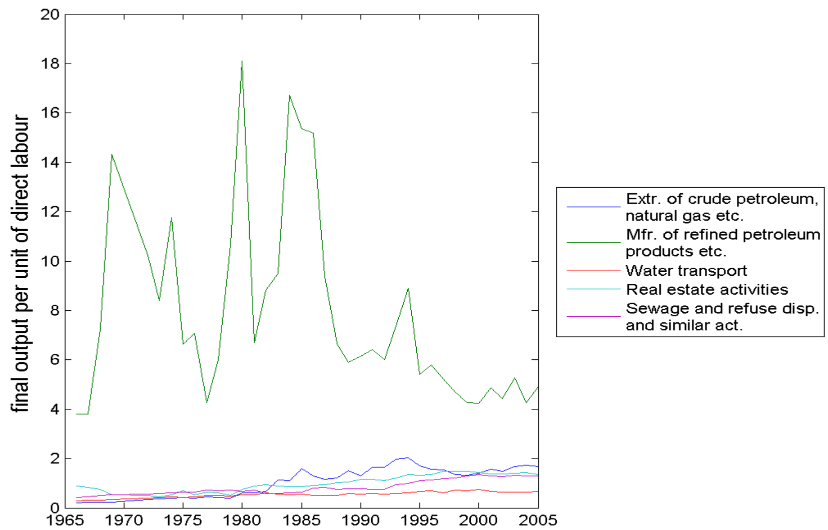

Fig. 5.56: *The μ-index, Extreme cases*

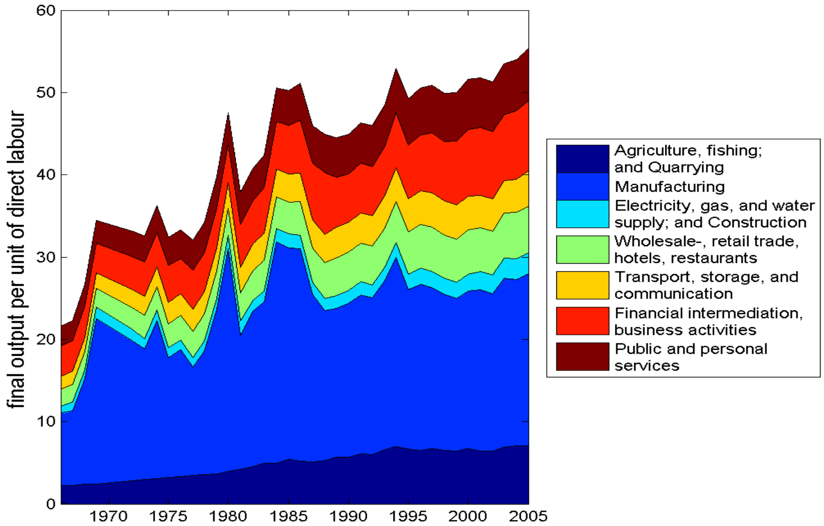

Fig. 5.57: *The μ-index for the meso-sectors*

5.H The ψ-index (Productivity Index, Production Prices)

$$\psi_t^i = \frac{1}{\boldsymbol{e}'\tilde{\boldsymbol{l}}_t^i R_t} \int_0^{R_t} \tilde{\zeta}_t^i(r)\, dr$$

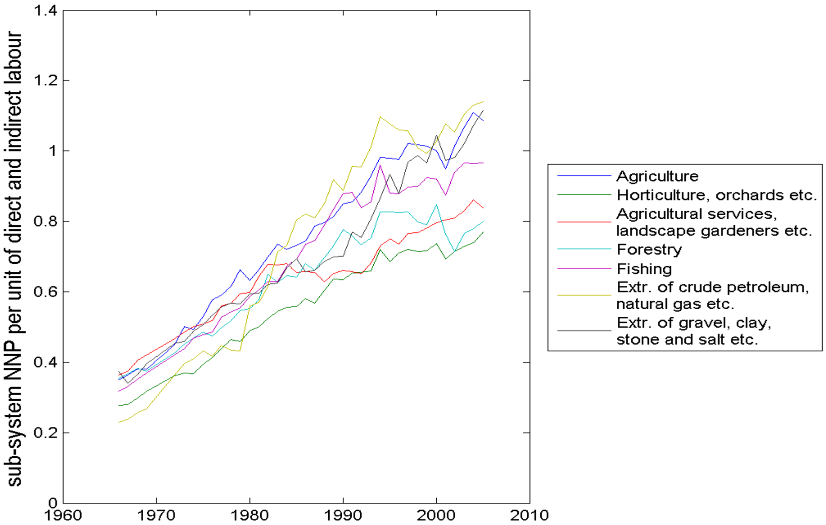

Fig. 5.58: *The ψ-index, Agriculture, fishing, and quarrying*

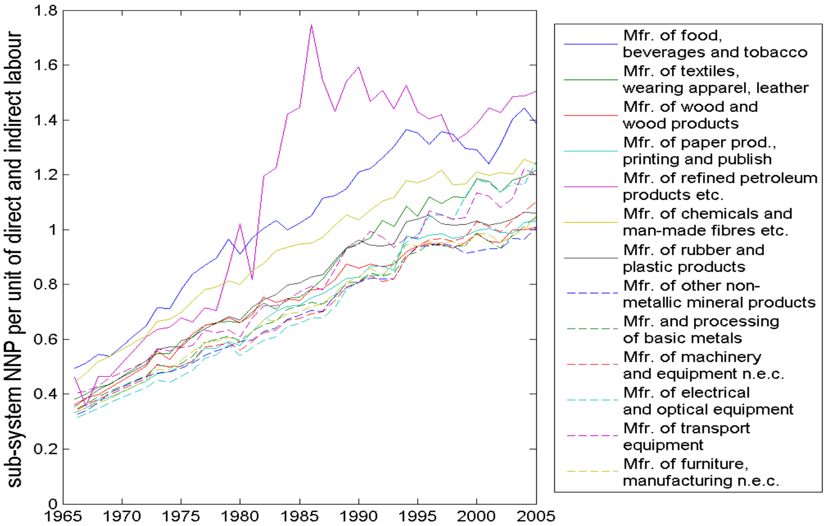

Fig. 5.59: *The ψ-index, Manufacturing*

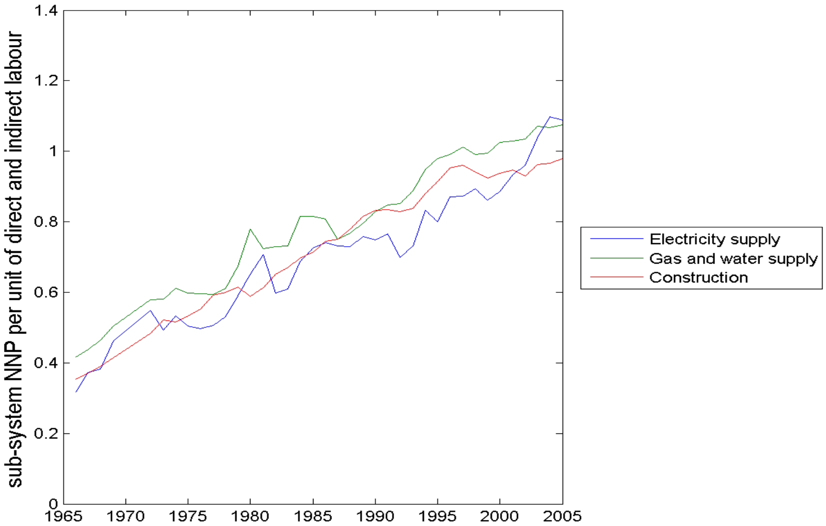

Fig. 5.60: *The ψ-index, Electricity, gas, and water supply; and Construction*

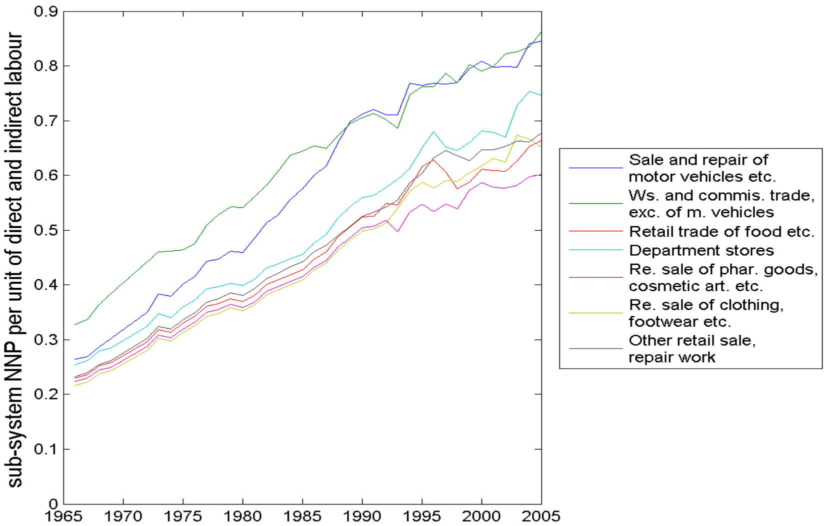

Fig. 5.61: *The ψ-index, Wholesale-, retail trade, hotels, restaurants*

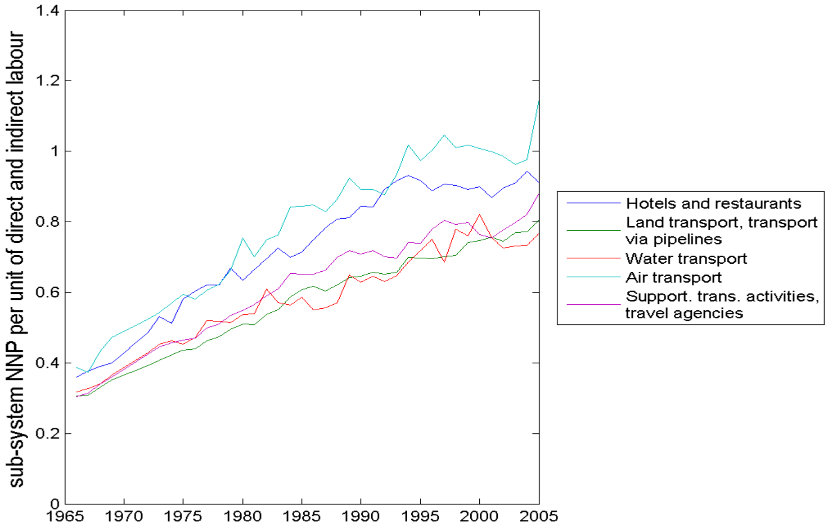

Fig. 5.62: *The ψ-index, Transport, storage, and communication*

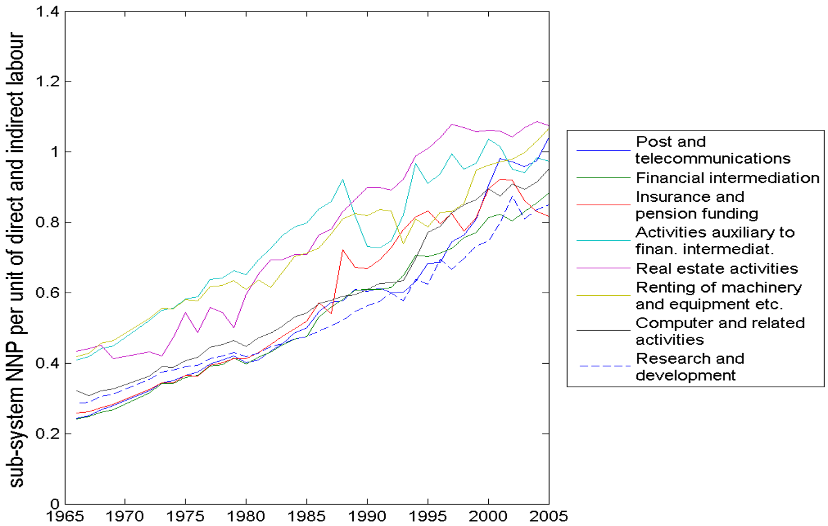

Fig. 5.63: *The ψ-index, Financial intermediation, business activities*

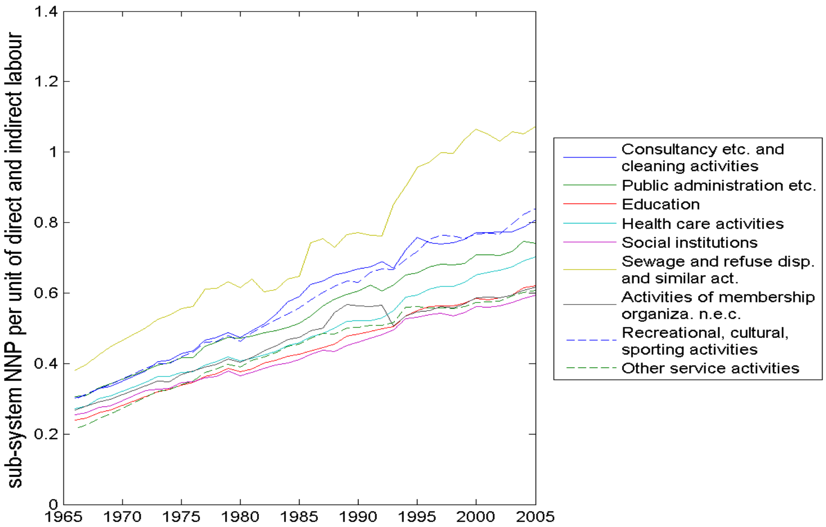

Fig. 5.64: *The ψ-index, Public and personal services*

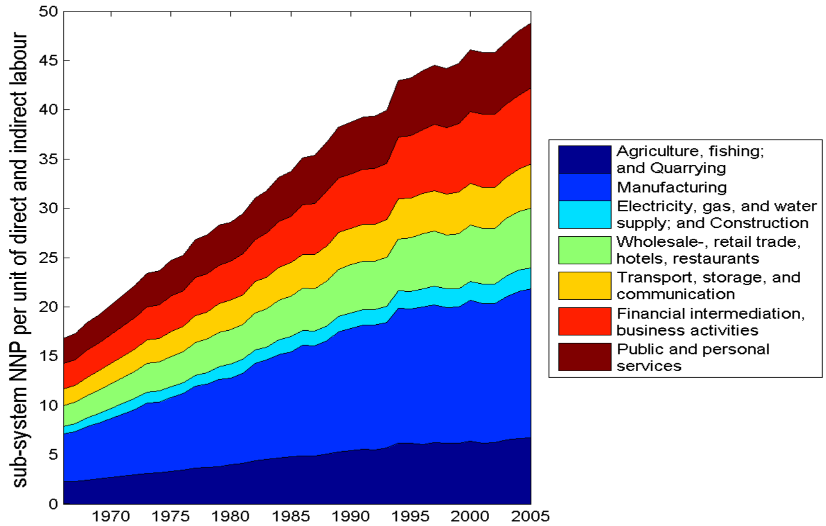

Fig. 5.65: *The ψ-index for the meso-sectors*

5.I The γ-index (Index of Structural Change, Production Prices)

$$\gamma_t^i = \frac{1}{R_t}\int_0^{R_t} \frac{(\tilde{b}^i_{(i,i,t)} - \tilde{a}^i_{(i,i,t)})p_{(i,t)}(r)}{\boldsymbol{e}'\tilde{\boldsymbol{A}}^i_t\boldsymbol{p}_t(r) + \boldsymbol{e}'\tilde{\boldsymbol{l}}^i_t w_t(r)}\,dr = \frac{\text{value of external output}}{\text{social costs}}$$

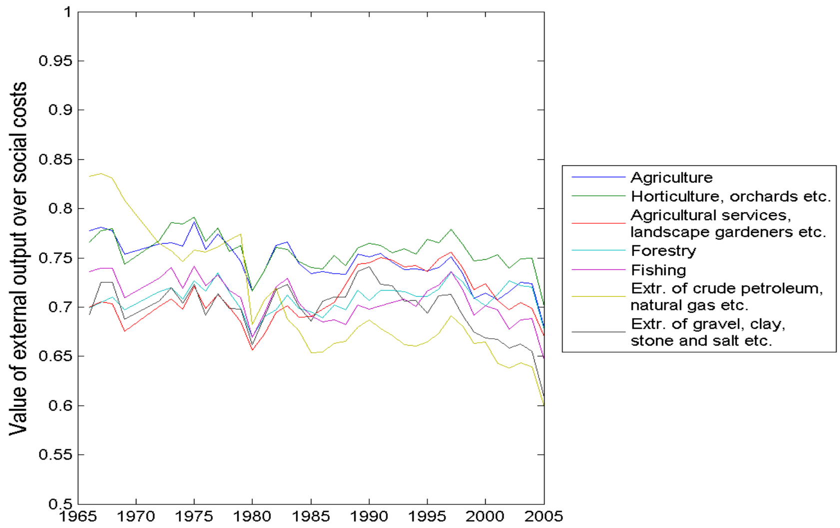

Fig. 5.66: *The γ-index, Agriculture, fishing, and quarrying*

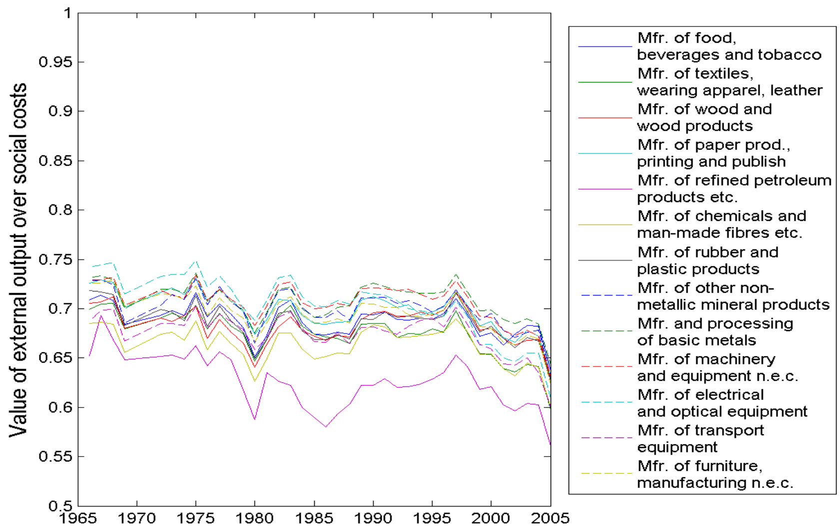

Fig. 5.67: *The γ-index, Manufacturing*

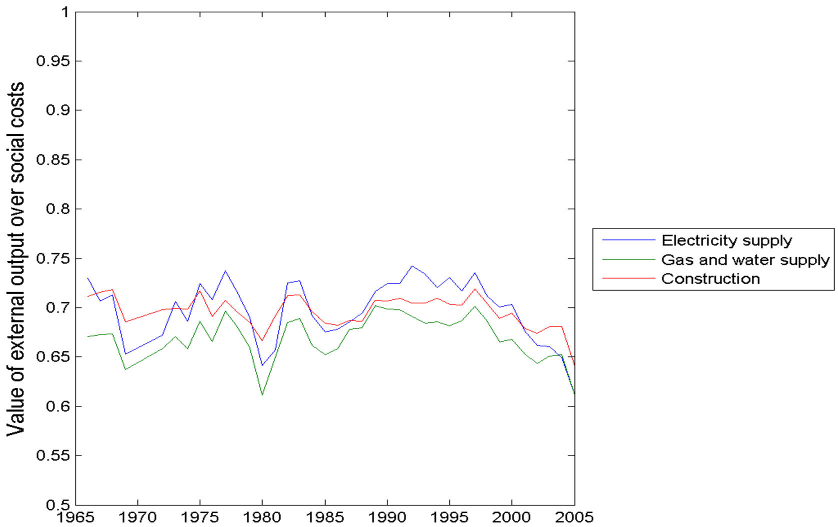

Fig. 5.68: *The γ-index, Electricity, gas, and water supply; and Construction*

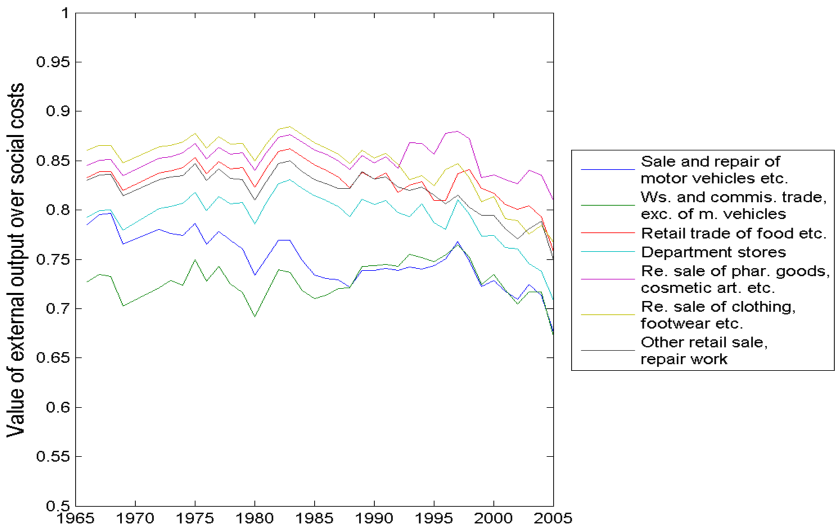

Fig. 5.69: *The γ-index, Wholesale-, retail trade, hotels, restaurants*

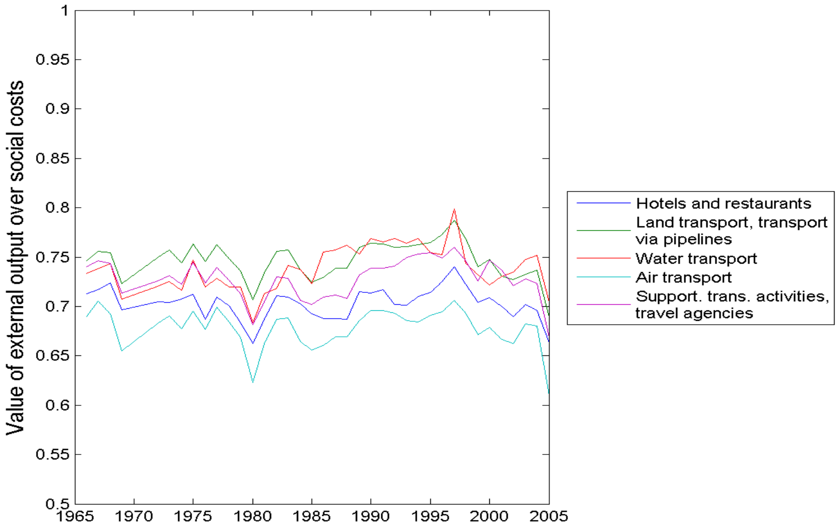

Fig. 5.70: *The γ-index, Transport, storage, and communication*

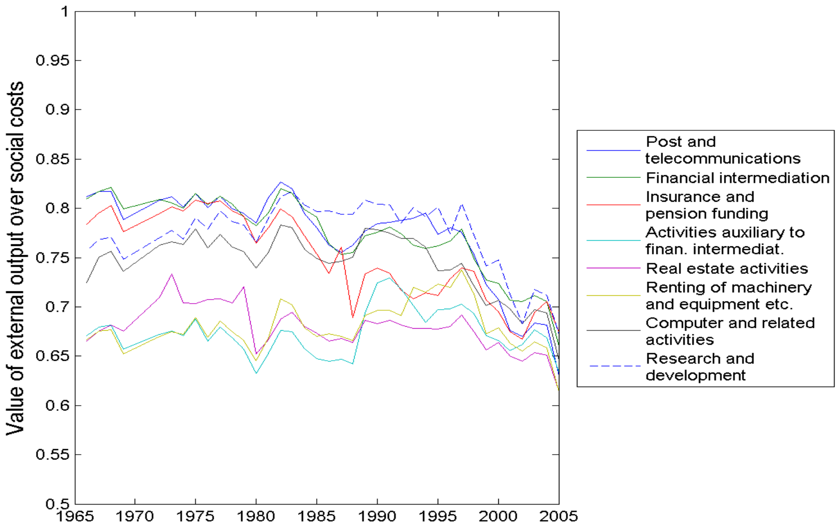

Fig. 5.71: *The γ-index, Financial intermediation, business activities*

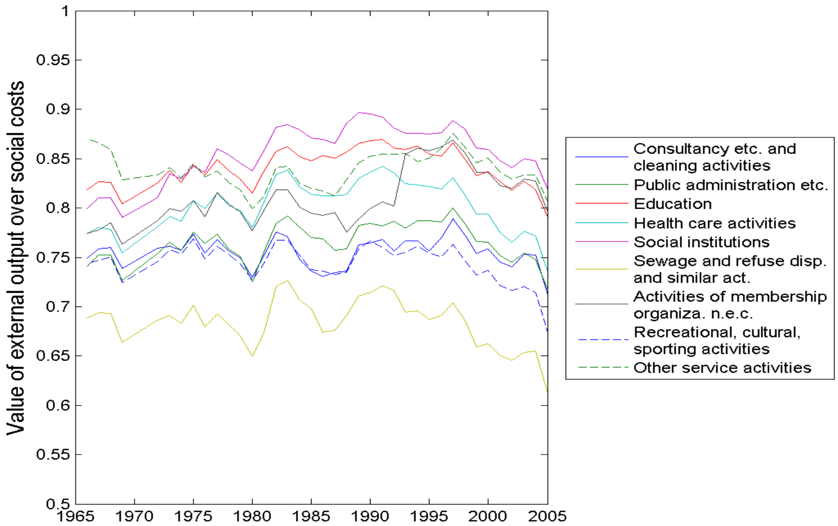

Fig. 5.72: *The γ-index, Public and personal services*

5.J The δ-index (Index of Structural Change, Production Prices)

$$\delta_t^i = \frac{1}{R_t} \int_0^{R_t} \frac{(\tilde{b}^i_{(i,i,t)} - \tilde{a}^i_{(i,i,t)}) p_{(i,t)}(r)}{\tilde{\boldsymbol{a}}^i_{(i,:,t)} \boldsymbol{p}_t(r) + \tilde{l}^i_{(i,t)} w_t(r)} \, dr = \frac{\text{value of external output}}{\text{local costs}}$$

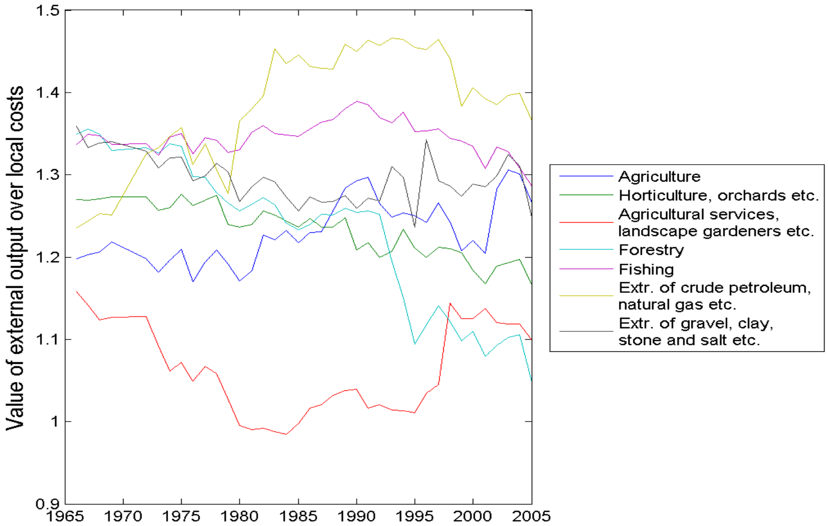

Fig. 5.73: *The δ-index, Agriculture, fishing, and quarrying*

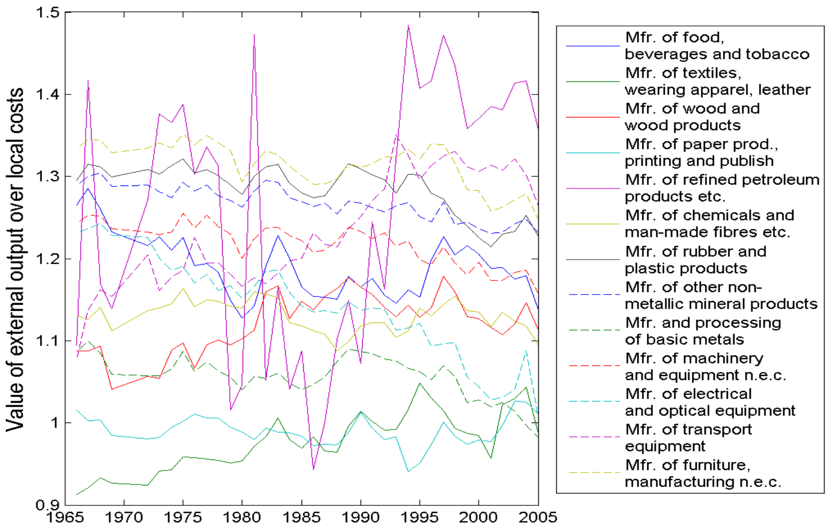

Fig. 5.74: *The δ-index, Manufacturing*

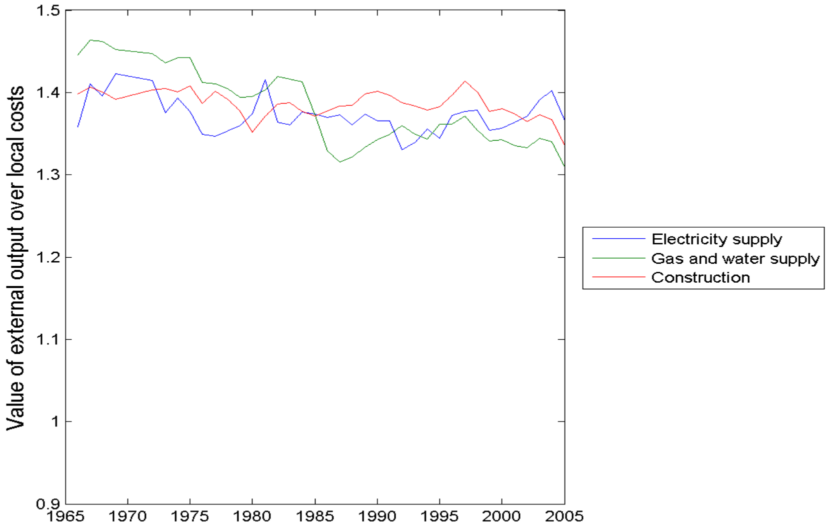

Fig. 5.75: *The δ-index, Electricity, gas, and water supply; and Construction*

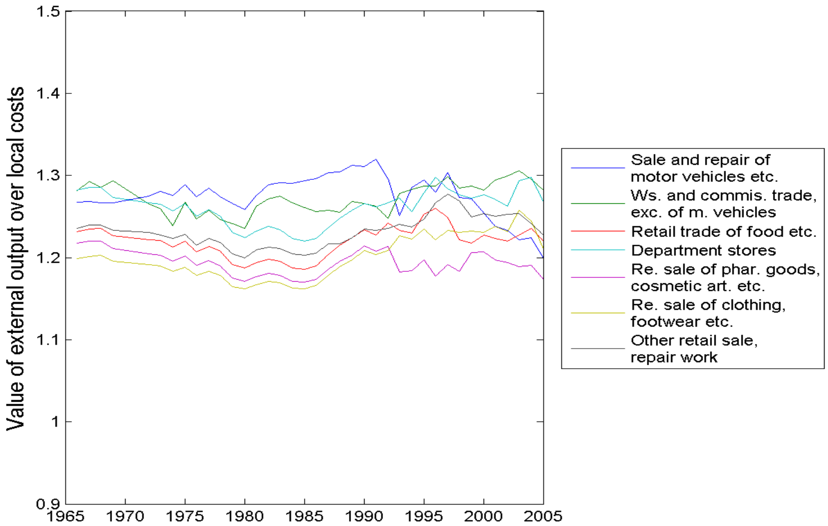

Fig. 5.76: *The δ-index, Wholesale-, retail trade, hotels, restaurants*

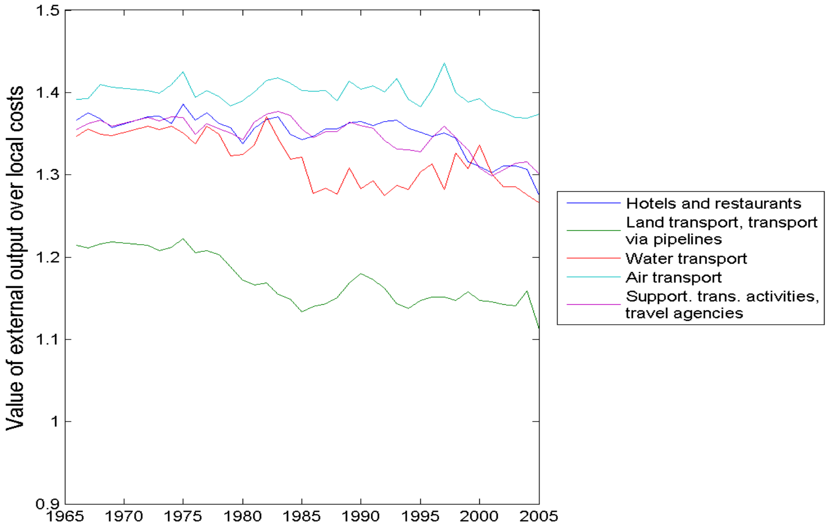

Fig. 5.77: *The δ-index, Transport, storage, and communication*

Fig. 5.78: *The δ-index, Financial intermediation, business activities*

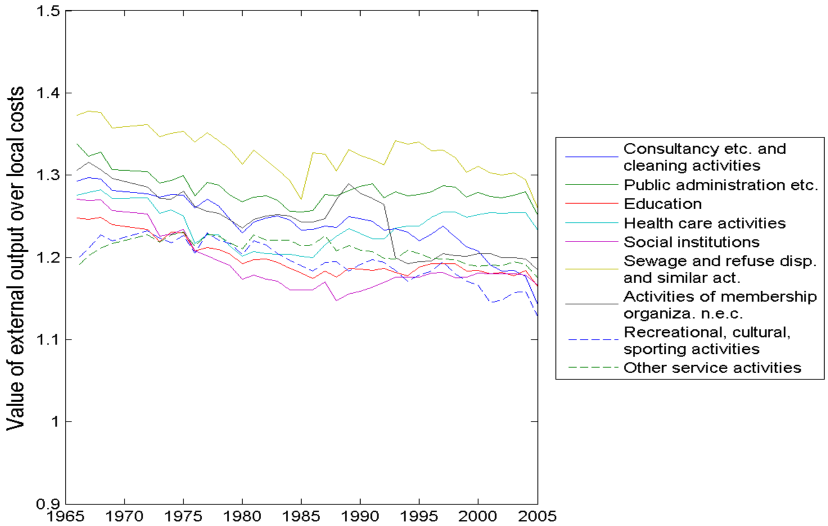

Fig. 5.79: *The δ-index, Public and personal services*

Chapter 6

Additional Results from Chapter 3 'The Technological Frontier'

6.A The Wage-profit and Intertemporal Technological Frontiers

Figure 6.1 shows the wage-profit frontiers forming the intertemporal technological frontier, and Figure 6.2 and 6.3 show the wage-profit frontiers for the individual countries together with the intertemporal technological frontier.

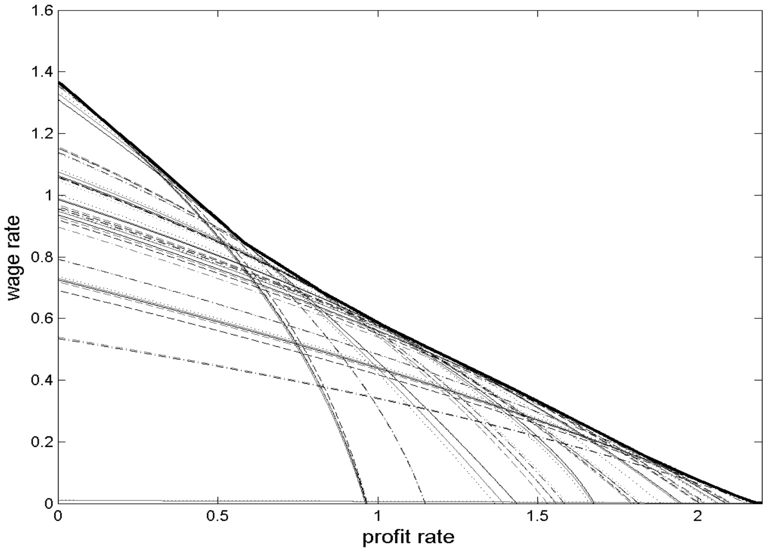

Fig. 6.1: *The intertemporal technological frontier*

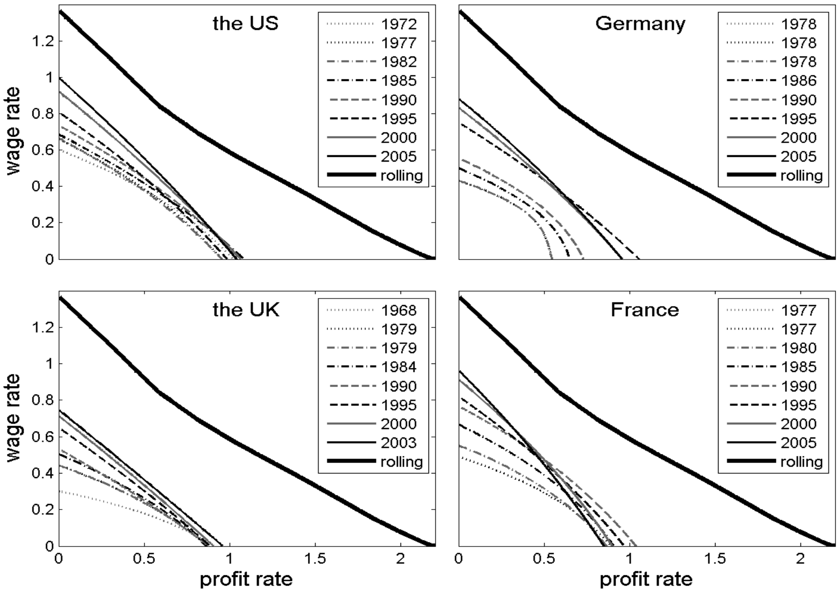

Fig. 6.2: *Wage-profit frontiers and the intertemporal technological frontier: the US, Germany, the UK, and France*

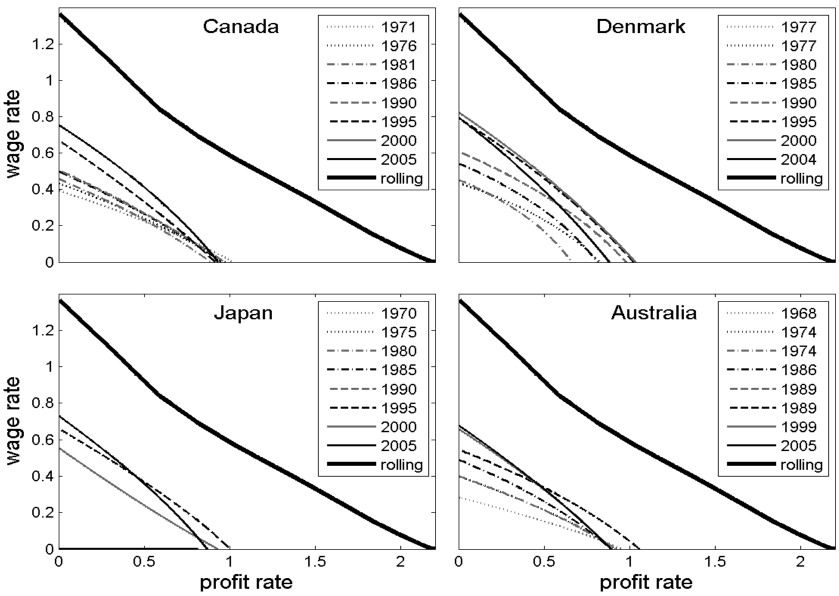

Fig. 6.3: *Wage-profit frontiers and the intertemporal technological frontier: Canada, Denmark, Japan, and Australia*

6.B Industrylevel Frequency of the Single Countries Contribution to the Contemporary Technological Frontiers

The following 23 figures show the industry-level frequency of the single countries contribution to the contemporary and rolling technological frontiers.

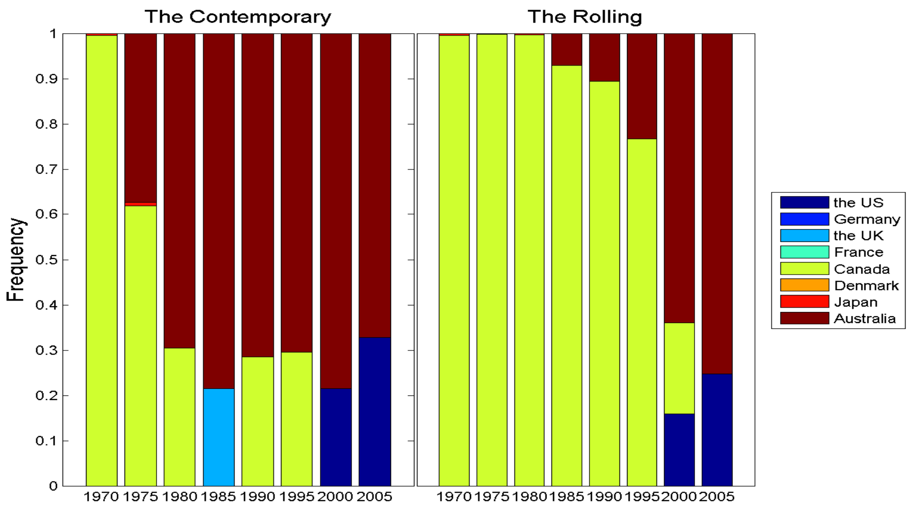

Fig. 6.4: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Agriculture, hunting, forestry, and fishing*

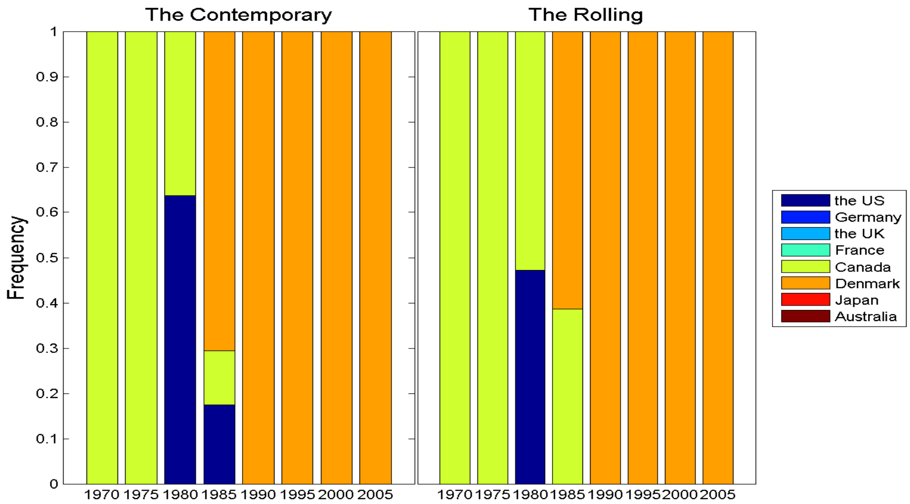

Fig. 6.5: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Mining and quarrying*

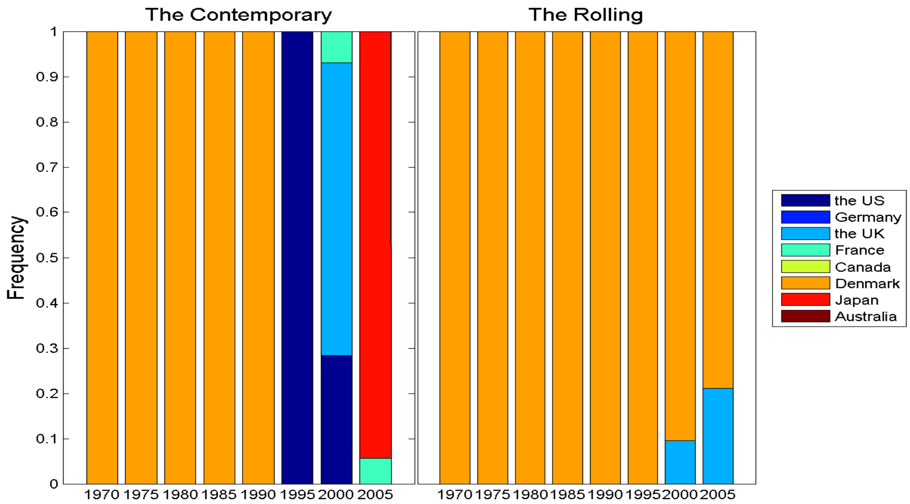

Fig. 6.6: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Food products, beverages, and tobacco*

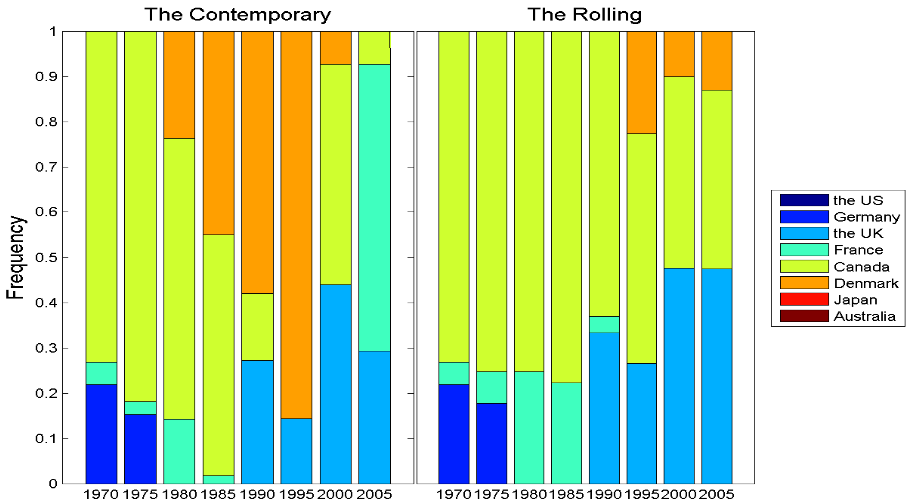

Fig. 6.7: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Textiles, textile products, leather, and footwear*

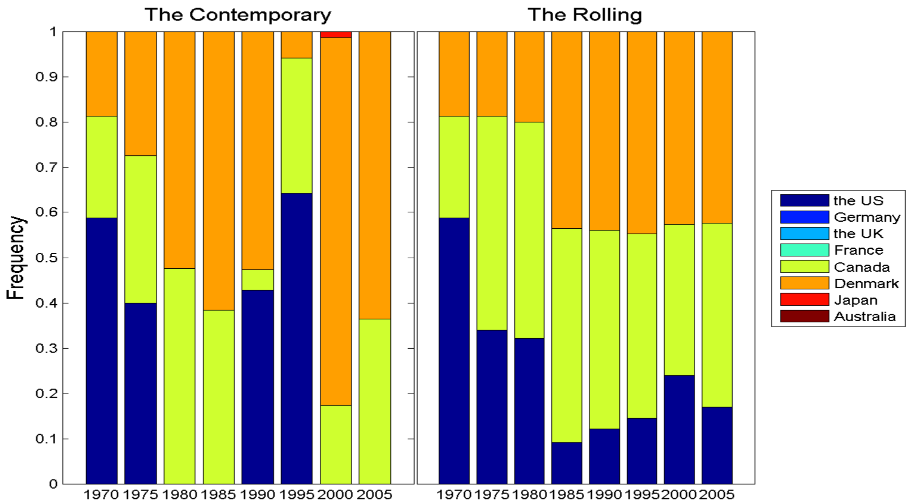

Fig. 6.8: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Wood and products of wood and cork*

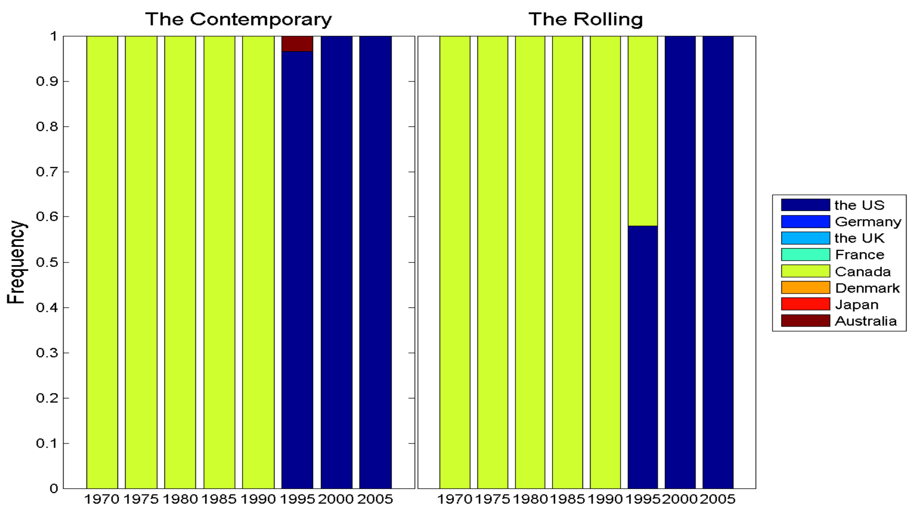

Fig. 6.9: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Pulp, paper, paper products, printing, and publishing*

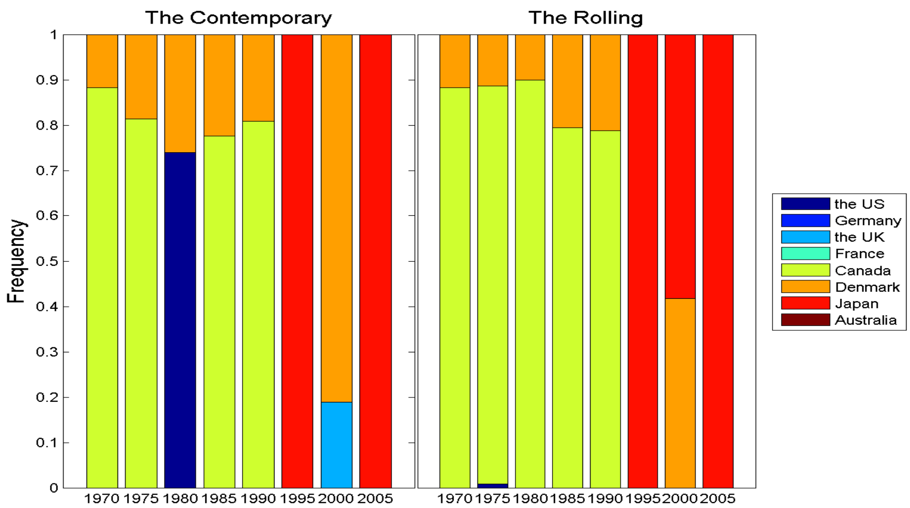

Fig. 6.10: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Coke, refined petroleum products, and nuclear fuel*

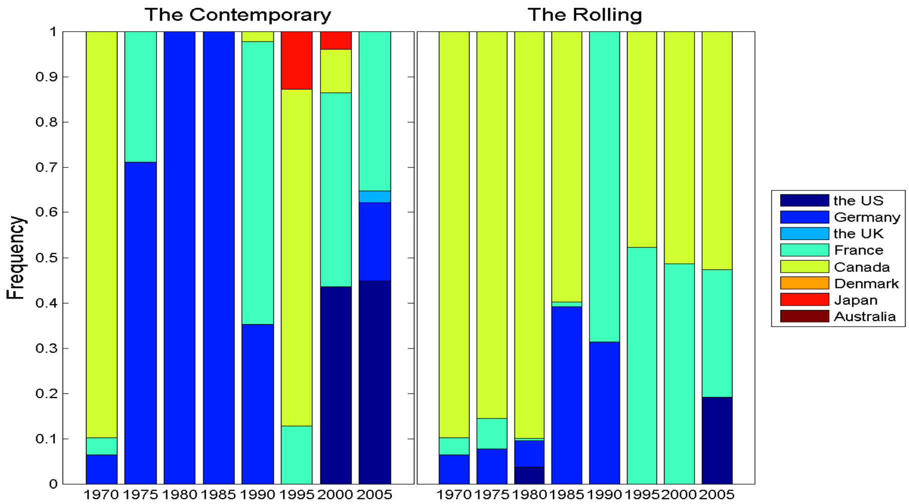

Fig. 6.11: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Chemicals*

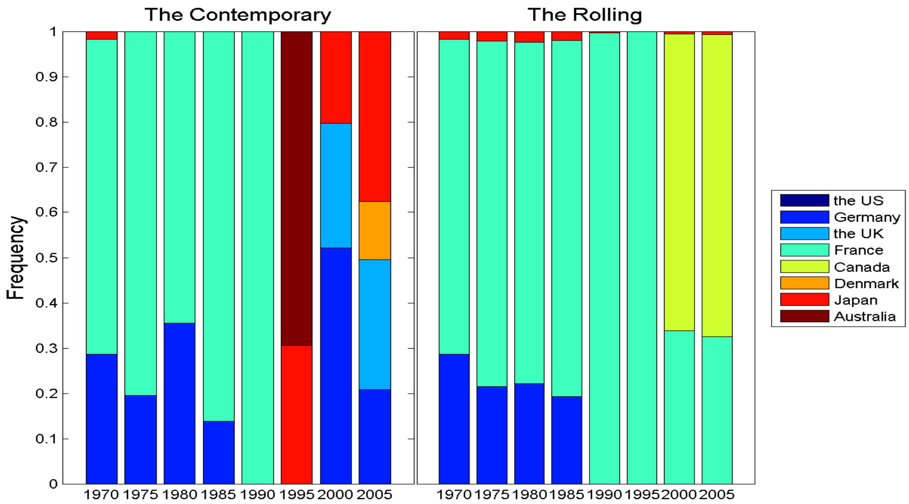

Fig. 6.12: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Rubber and plastics products*

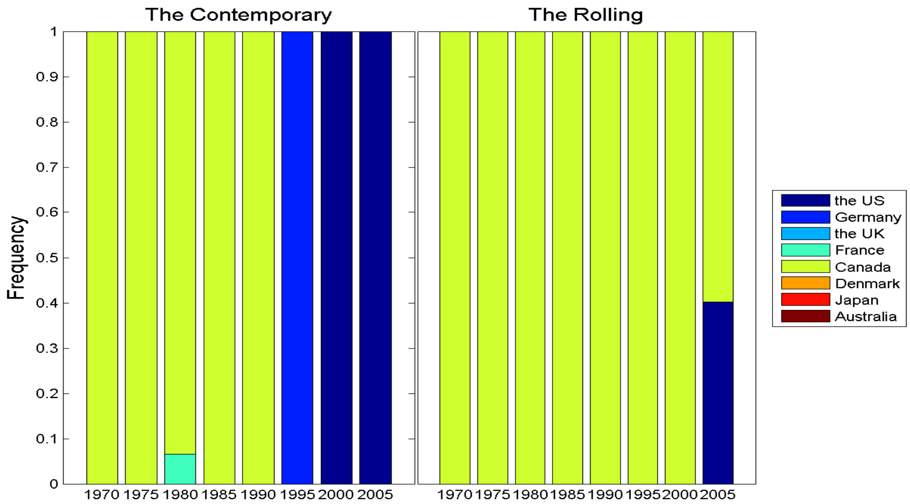

Fig. 6.13: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Other non-metallic mineral products*

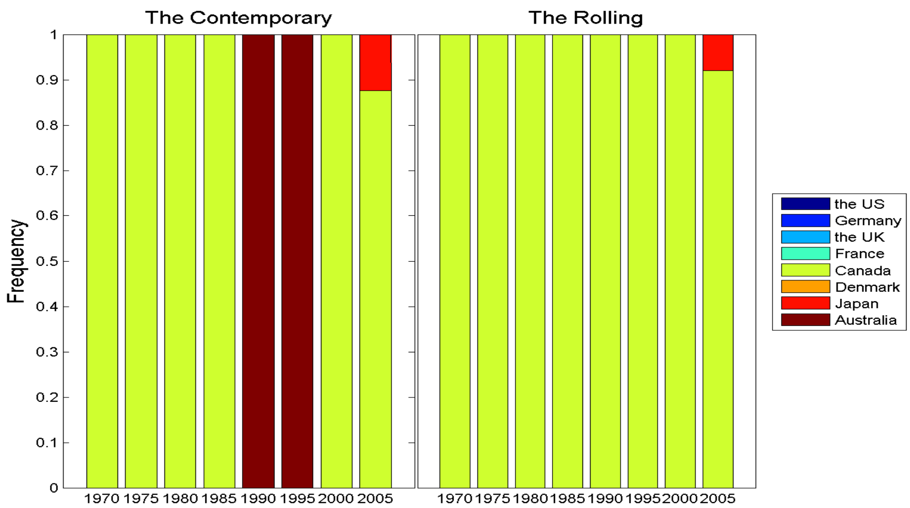

Fig. 6.14: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Metals*

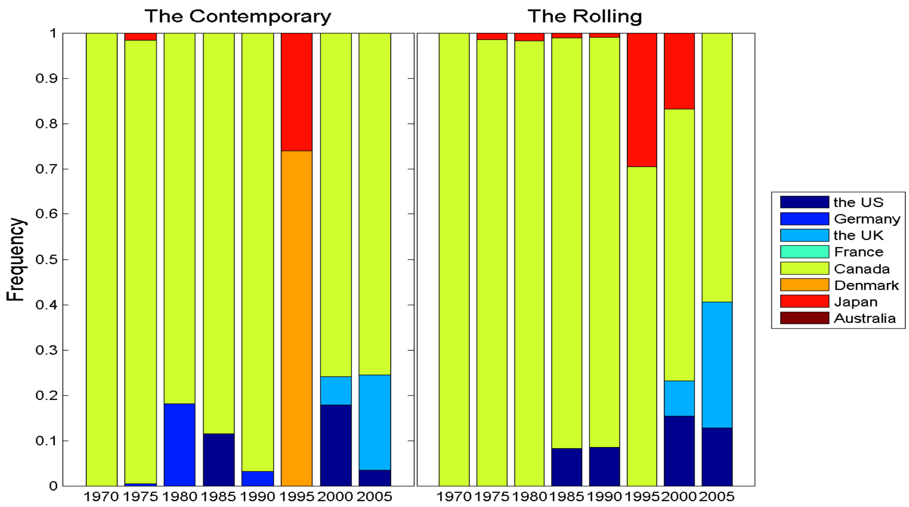

Fig. 6.15: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Fabricated metal products, except machinery and equipment*

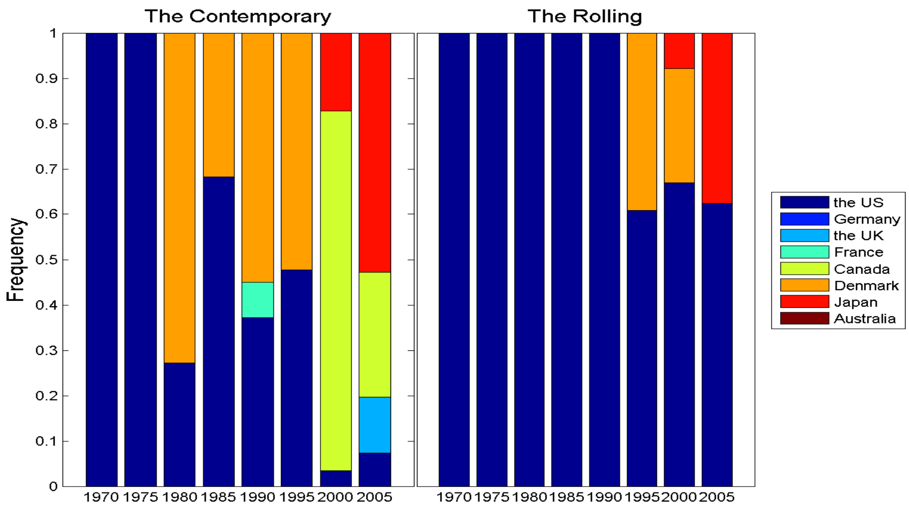

Fig. 6.16: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Machinery and equipment, nec*

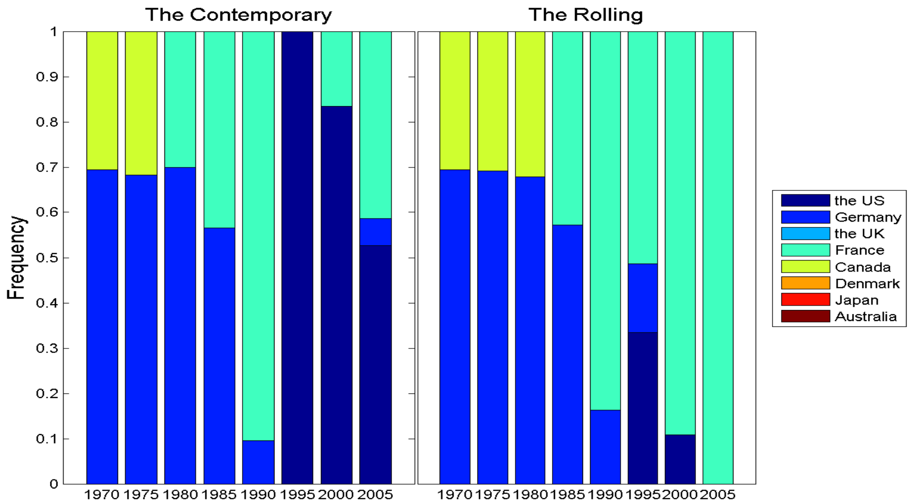

Fig. 6.17: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Electrical machinery and apparatus*

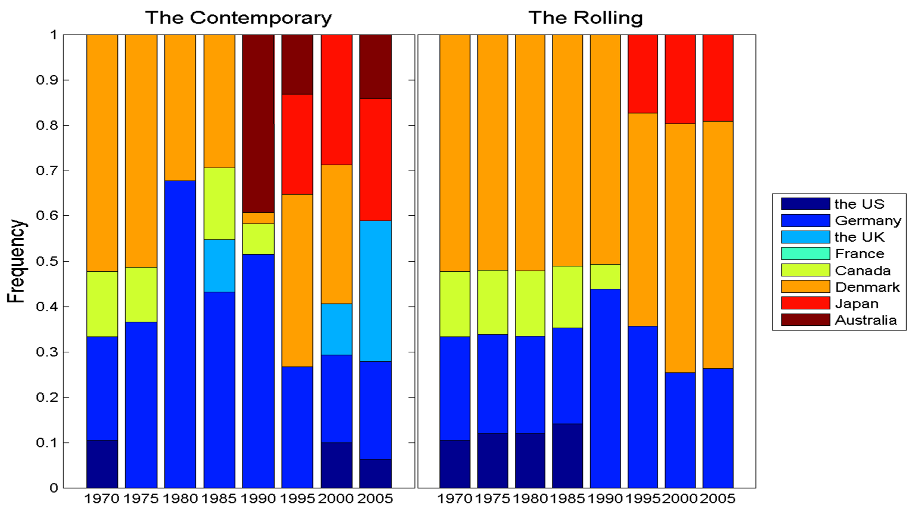

Fig. 6.18: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Transport equipment*

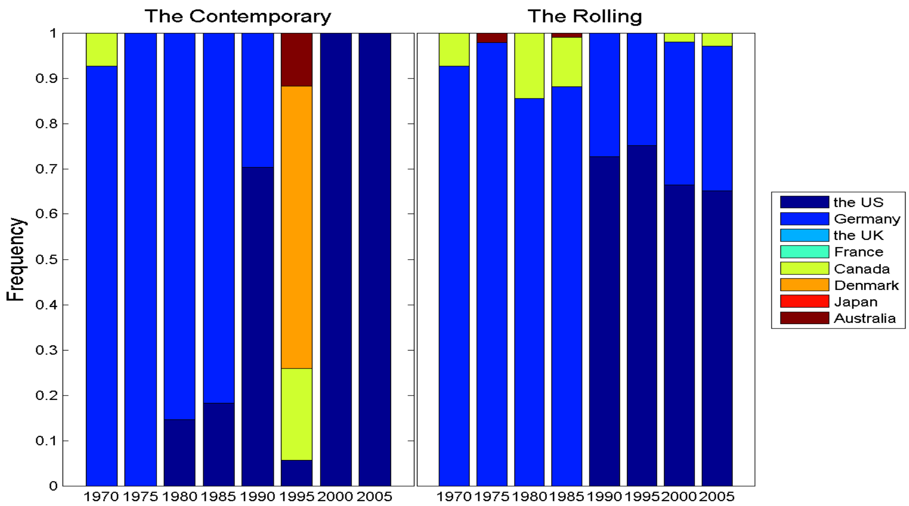

Fig. 6.19: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Manufacturing nec; recycling (include Furniture)*

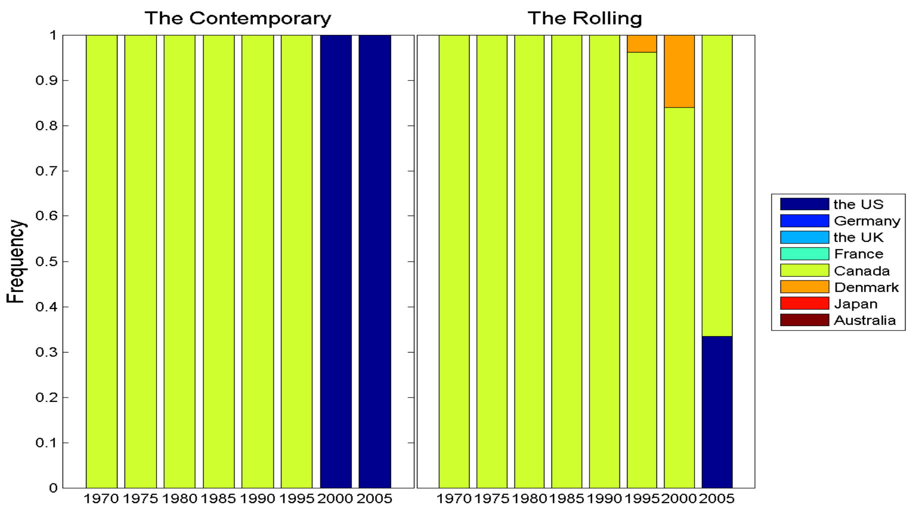

Fig. 6.20: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Production and distribution of electricity, gas, and water*

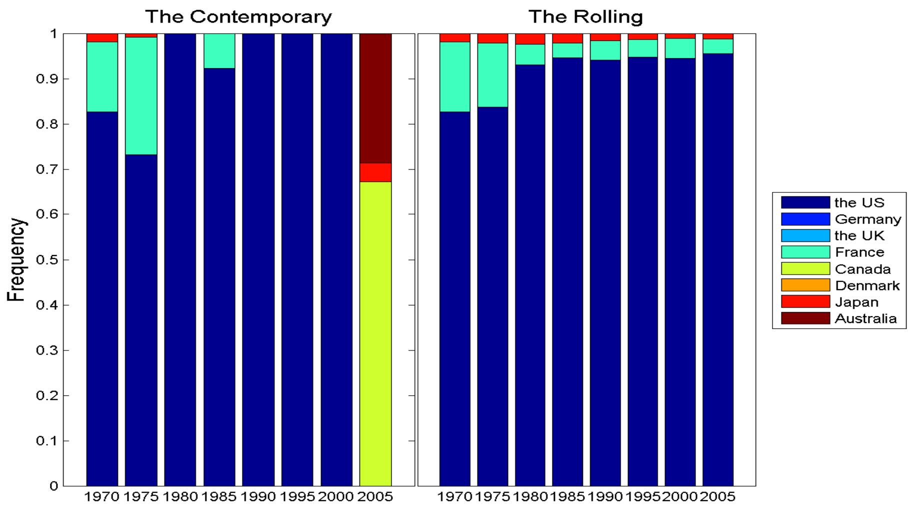

Fig. 6.21: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Construction*

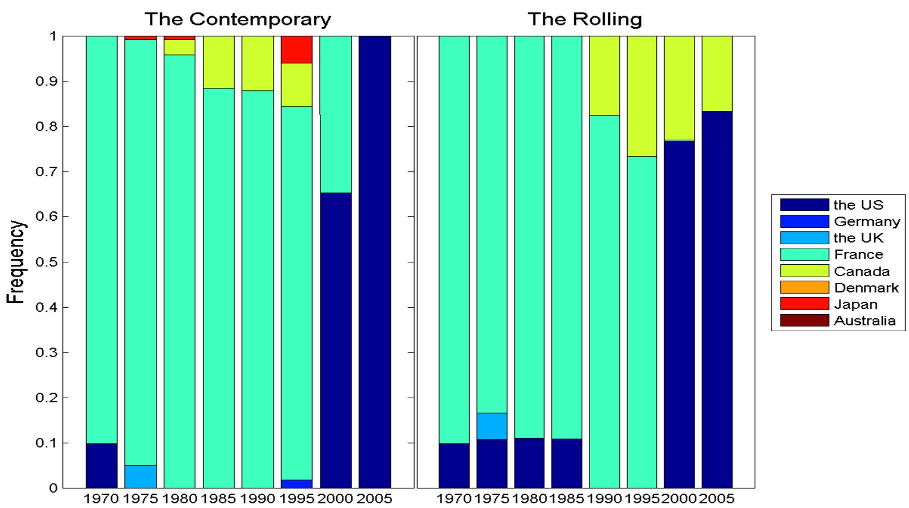

Fig. 6.22: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Wholesale and retail trade*

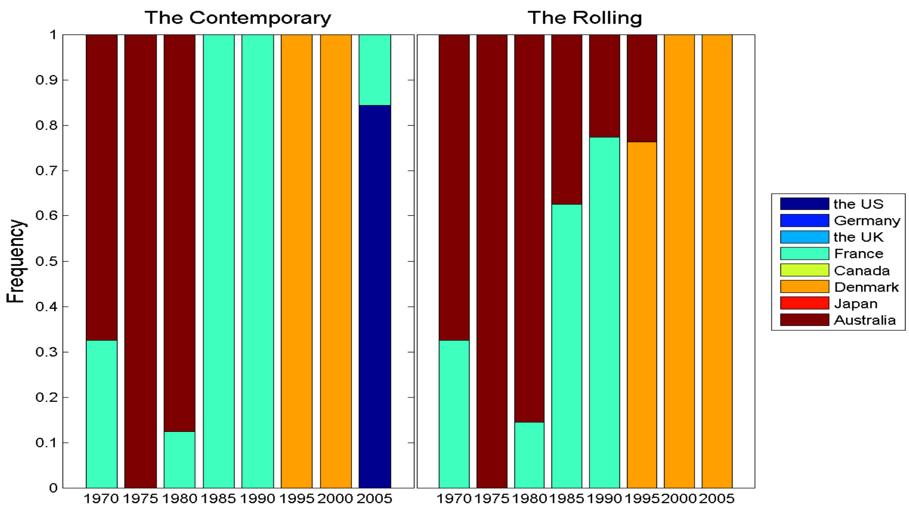

Fig. 6.23: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Service activities (transport, hotels and restaurants)*

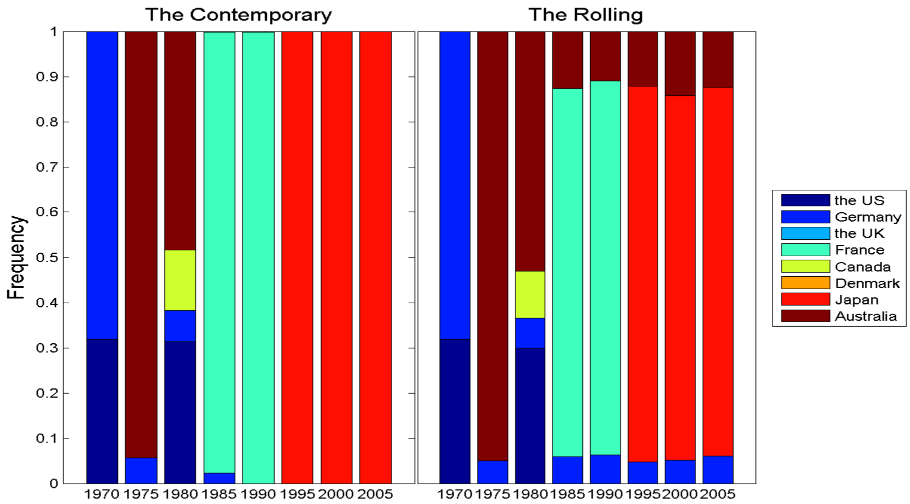

Fig. 6.24: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Post and telecommunications*

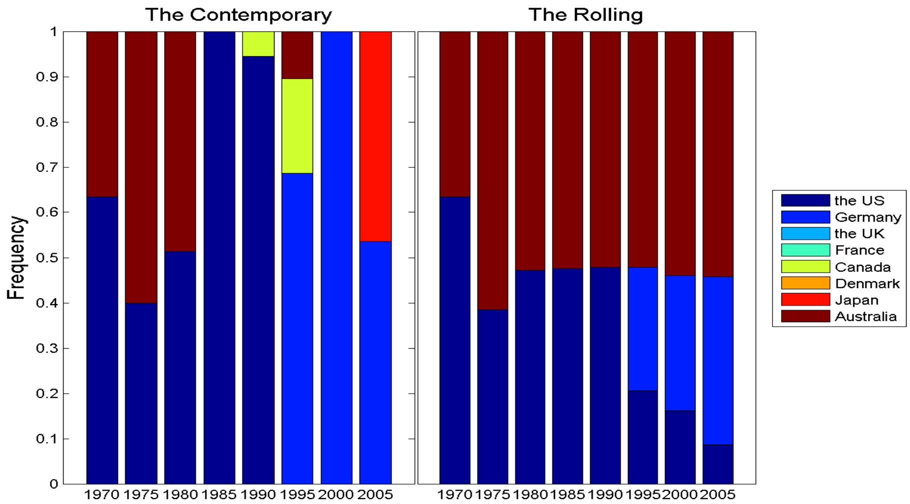

Fig. 6.25: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Business activities (finance, real estate, and R&D)*

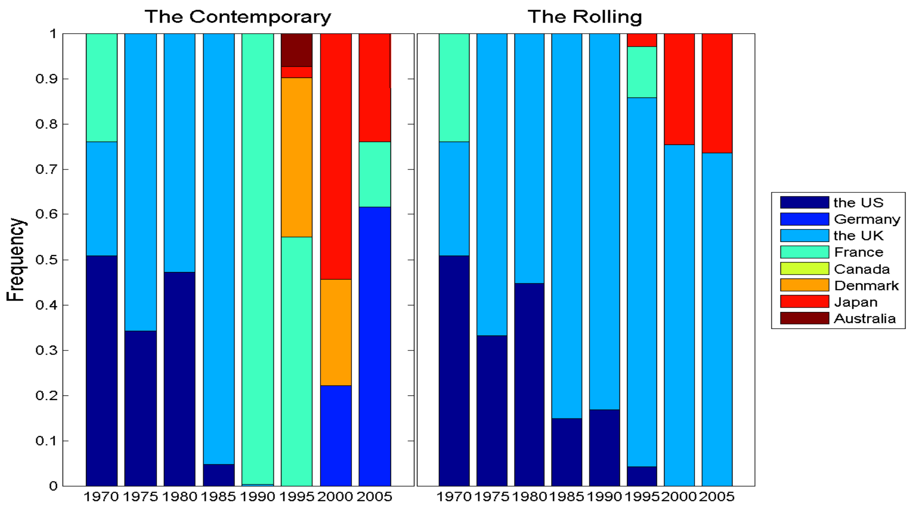

Fig. 6.26: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Public administration, education and health*

6.C Country Specific Contributions to the Contemporary and Rolling Technological Frontiers

The following 8 figures show the country specific contributions to the contemporary and rolling technological frontiers. The maximum value is equal to the number of industries, 23, and would imply that the given country's wage-profit frontier coincided with the technological frontier.

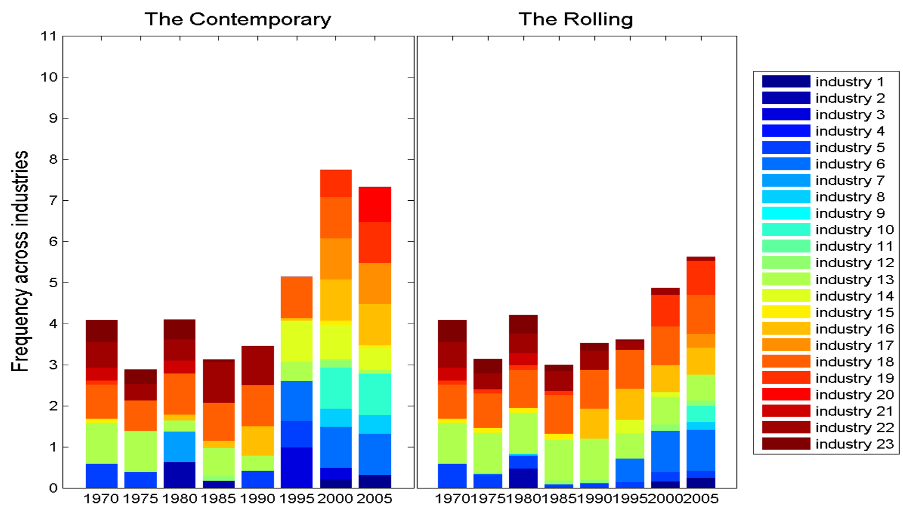

Fig. 6.27: *Country specific contributions to the contemporary technological frontiers, the US*

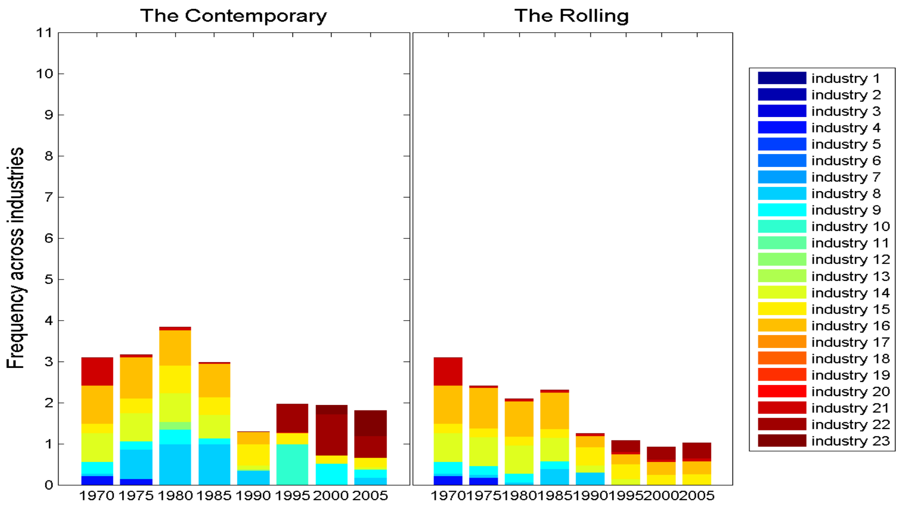

Fig. 6.28: *Country specific contributions to the contemporary technological frontiers, Germany*

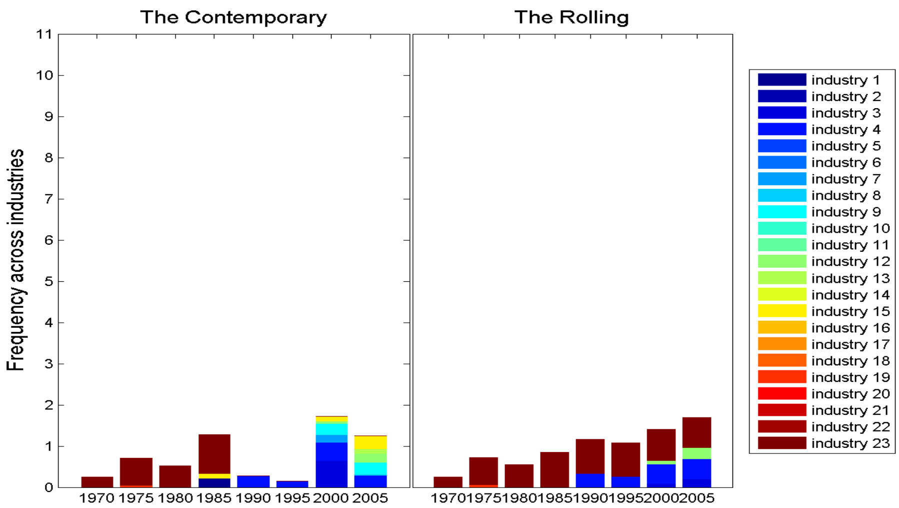

Fig. 6.29: *Country specific contributions to the contemporary technological frontiers, the UK*

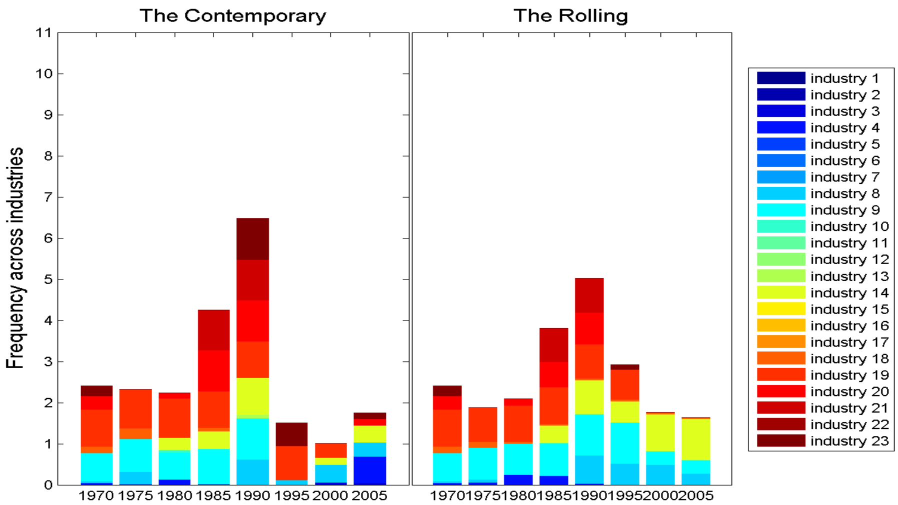

Fig. 6.30: *Country specific contributions to the contemporary technological frontiers, France*

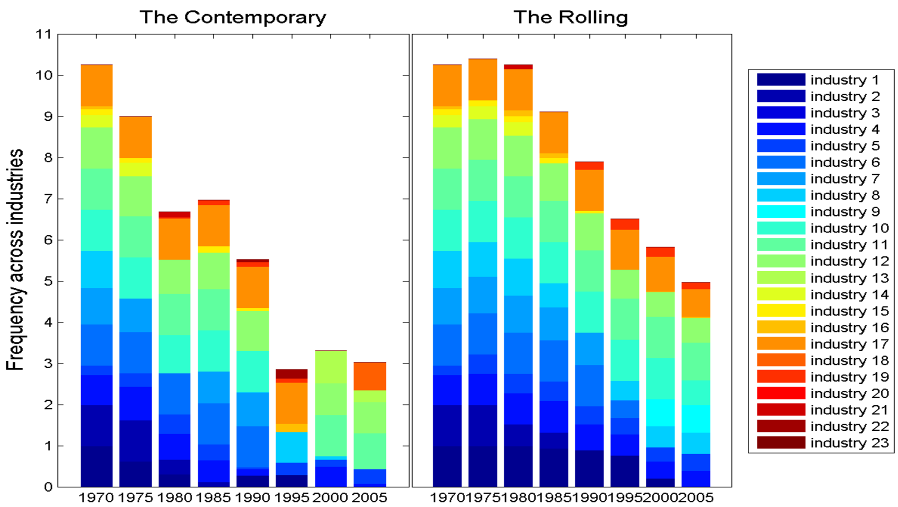

Fig. 6.31: *Country specific contributions to the contemporary technological frontiers, Canada*

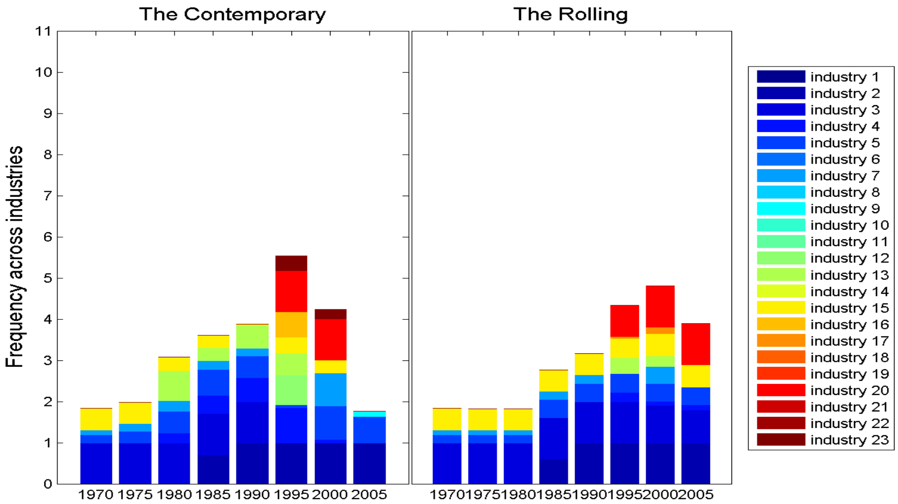

Fig. 6.32: *Country specific contributions to the contemporary technological frontiers, Denmark*

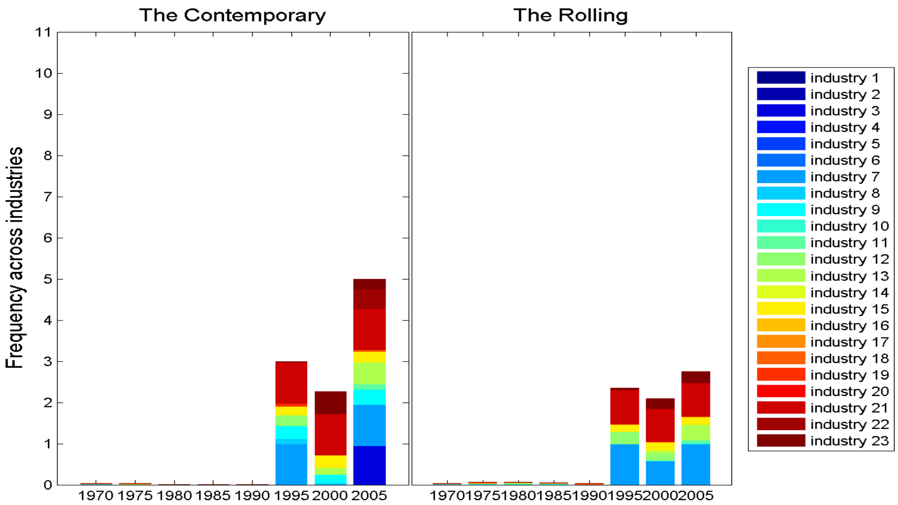

Fig. 6.33: *Country specific contributions to the contemporary technological frontiers, Japan*

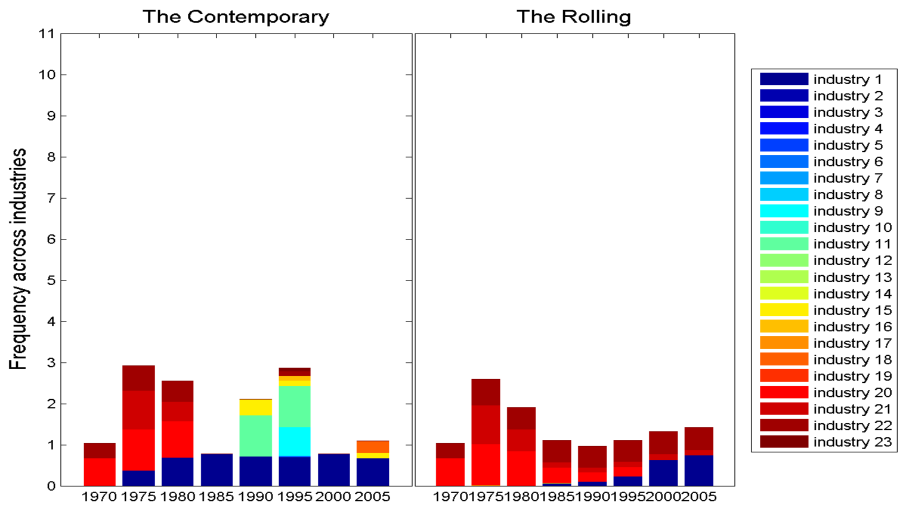

Fig. 6.34: *Country specific contributions to the contemporary technological frontiers, Australia*

Resume (dansk)

Dette ph.d.-projekt består af fire selvstændige kapitler omhandlende klassisk produktionsteori, hvoraf de første tre kapitler betragtes som afhandlingens kerne. Fælles for disse kapitler er sammenkoblingen mellem *Sraffian economics* og empiriske studier af input–output tabeller med det formål at studere og kvantificere økonomiske fænomener såsom teknologisk udvikling, strukturel forandring og konvergens; samt deres anvendelse for økonomisk politik.

Den ortodokse tilgang til disse problemer går typisk gennem aggregerede produktionsfunktioner og indeholder derfor en række iboende teoretiske og empiriske problemer. Nogle af disse problemer behandles i afhandlingens Kapitel 4. Tilgangen, der anvendes i Kapitel 1–3, anskuer—i modsætning til en proces begyndende med produktionsfaktorer og sluttende med forbrugsgoder—produktion som en cirkulær proces, hvor alle inputs, bortset fra arbejdskraft, er produceret i systemet. Evolutionen i det økonomiske system analyseres derfor ved at studere det inter-industrielle flow af varer og tjenesteydelser. Et yderligere centralt karakteristika ved denne tilgang er et sammenbundet forhold mellem værditeori og teknologisk udvikling.

Kapitel 1 introducerer et alternativt aggregeret produktivitetsregnskab. Ud fra faktiske input–output tabeller fra fire OECD lande i perioden 1970–2005 beregnes de empiriske løn-profit- og nettonationalprodukt-kurver, hvorfra to produktivitetsmål konstrueres baseret på produktionspriser og en valgt *numéraire*. Resultaterne støtter den eksisterende litteratur omhandlende *the productivity slowdown* og giver ny viden til timingen og omfanget af disse hændelser.

Hovedformålet med Kapitel 2 er at studere procedurer til at opsplitte den aggregerede teknologiske udvikling ned til industriniveauet, hvor der eksplicit indregnes både de direkte effekter fra de enkelte industrier samt de indirekte effekter fra de støttende industrier, samt at applicere denne metode på faktiske danske forhold 1966–2005. Blandt resultaterne er en tydelig konvergens i industriernes produktivitetsniveauer, samt identifikation af centrale strukturelle karakteristika for den danske økonomi, der bør inddrages i planlægningen af den økonomiske politik. Dette inkluderer et CO_2-regnskab, der identificerer kilden til den efterspørgsel, som i sidste ende ledte til den produktion, hvorfra emissionen faktisk fandt sted. For eksempel viser resultaterne, at den offentlige sektor i Danmark 2005 var direkte og indirekte ansvarlig for fire gange så meget CO_2-emission som den direkte emission, der rapporteres i de officielle statistikker.

Tilgangen, valgt i Kapitel 3 til at studere den globale teknologiske udvikling, går gennem den såkaldte *technological frontier*, der for en given fordeling a nettonationalproduktet, angiver den omkostningsminimerende kombination af produktionsmetoder. Forskellige udgaver af denne *techno-*

logical frontier præsenteres og beregnes ud fra en gruppe af OECD landes input–output tabeller 1970–2005. Blandt resultaterne er en mere moderat teknologisk udvikling end den typisk rapporterede, inklusiv en udvikling i USA drevet mere af stigende arbejdsudbud end teknologisk udvikling. En *technological frontier* er på grund af den kombinatoriske kompleksitet yderst vanskelig at beregne, men ved at udnytte to teoretiske egenskaber, der knytter sig til disse, er der udviklet en algoritme, der drastisk reducerer den kombinatoriske kompleksitet og derved muliggør analysens praktiske udførsel.

Kapitel 4 omhandler Anwar Shaikhs kritik af den neoklassiske vækst og fordelingsteori baseret på dennes anvendelse af aggregerede produktionsfunktioner. Denne analyseres ved at rekonstruere og udvide Franklin M. Fishers 1971 computer simulationer, som Shaikh anvender i sin analyse. Sammenholdt med andre studier baseret på Shaikhs kritik styrker denne analyse argumenterne imod anvendelsen af aggregerede produktionsfunktioner.