Measurement of the Gravitational Time Delay Using Drag-Free Spacecraft and an Optical Clock

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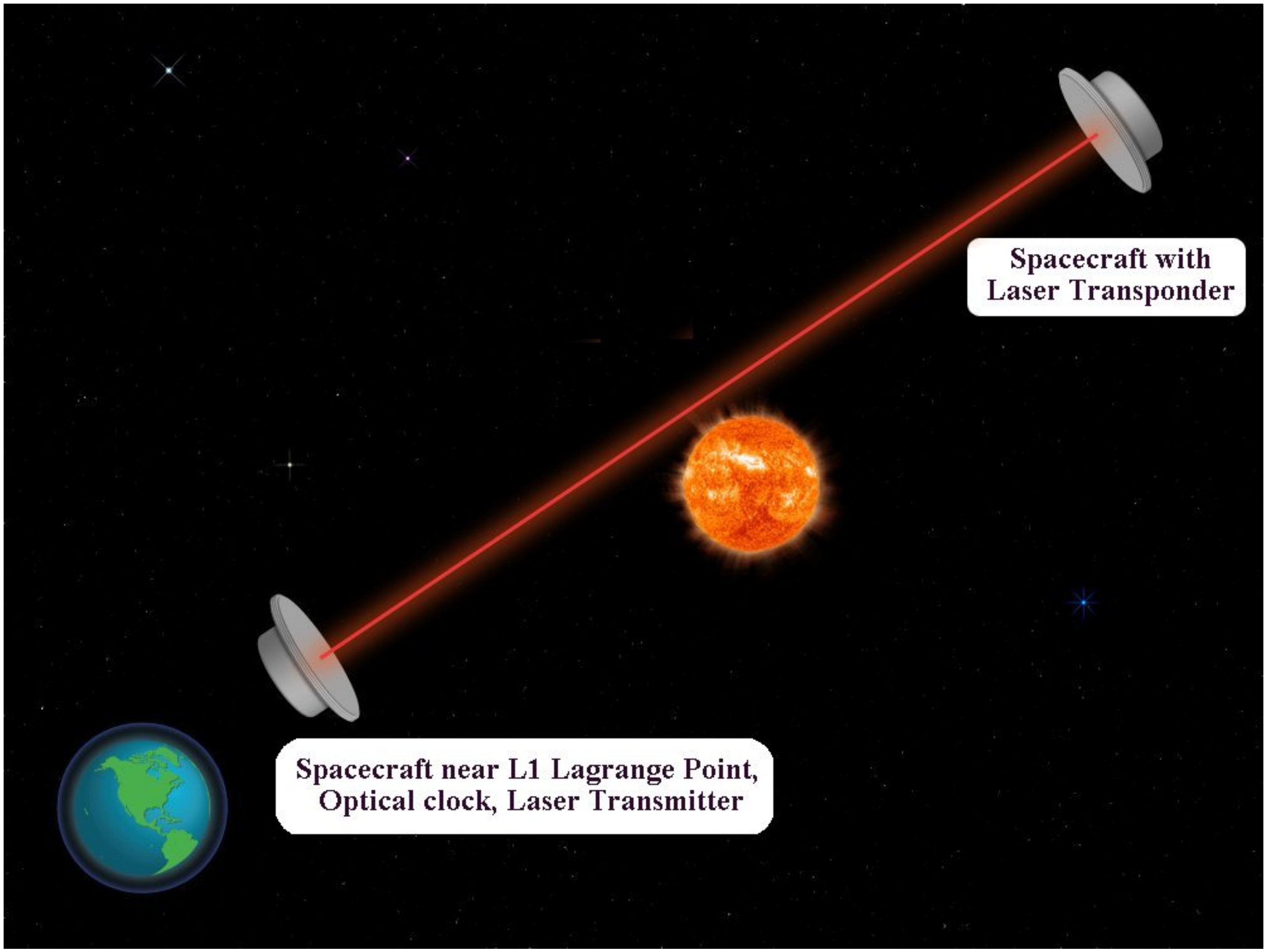
Tests of Gravity Workshop
Cleveland, OH May 19-21, 2009

Mission Concept

OBJECTIVE: measure γ to 10^{-8}



Outline

- Motivation for proposed mission
- Description of mission
    - Orbits
    - optical clocks
    - drag-free capabilities
    - uncertainty analysis
    - results of analysis

Motivation

- It is important to test Gravitational Theories—General Relativity in particular—wherever they can be tested.
- Accurate measurement of time delay is a promising method for detecting possible deviations from General Relativity (GR).
- Some scalar-tensor extensions of GR predict deviations from general relativity in the range

$$|\gamma - 1| \approx 10^{-5} - 10^{-8}$$

- The best measurement of γ came from the Cassini mission:

$$\sigma_{|\gamma-1|} \approx 2.3 \times 10^{-5}$$

(Bertotti, Iess, Tortora, Nature **425**, 374 (2003).

Proposed mission design

- Two-way measurements between a “near” spacecraft orbiting at the L-1 point, 1.5 million km from earth, and a distant spacecraft in the ecliptic plane, with orbital period 2 years and eccentricity 0.37.

- Solar conjunctions occur 1, 3, 5 years after launch.

- The most accurate time delay measurements are assumed to be made over roughly 8 day time periods around each solar conjunction.

- Nearly continuous measurements over longer periods are expected to improve the accuracy for γ by improving its separation from orbit parameters.

- The rate of change of the impact parameter is 1.9 solar radii per day.

Requirements:
- Low-noise space borne optical clock
- Drag-free spacecraft
- Laser transponder on distant spacecraft, transmitter on nearby spacecraft

Spacecraft clock noise

- Laser-cooled atoms in optical lattices and other schemes appear capable of providing highly accurate optical clocks that can be flown in spacecraft, with suitable development.

  For example: a power spectral density amplitude $2\times10^{-15}/\sqrt{Hz}$ has been demonstrated in the lab based on the 698 nm line of Sr^{87} (Ludlow *et al.,* Science, 2008) The 435 nm line of Yb^{+} may be a better possibility for early space qualification.

- Our simulations assume a power spectral density amplitude $5\times10^{-15}/\sqrt{Hz}$ down to 1 microhertz. Only white frequency noise is considered.

- For measurements extending over 3 hours, short-term clock jitter will limit the time delay precision to about 0.02 picoseconds.

- The main accuracy limitation for determining γ comes from clock frequency variations over the entire measurement time.

Drag-free spacecraft

- The required performance builds on that planned for the LISA gravitational wave mission: the power spectral density amplitude of the acceleration noise assumed for LISA is

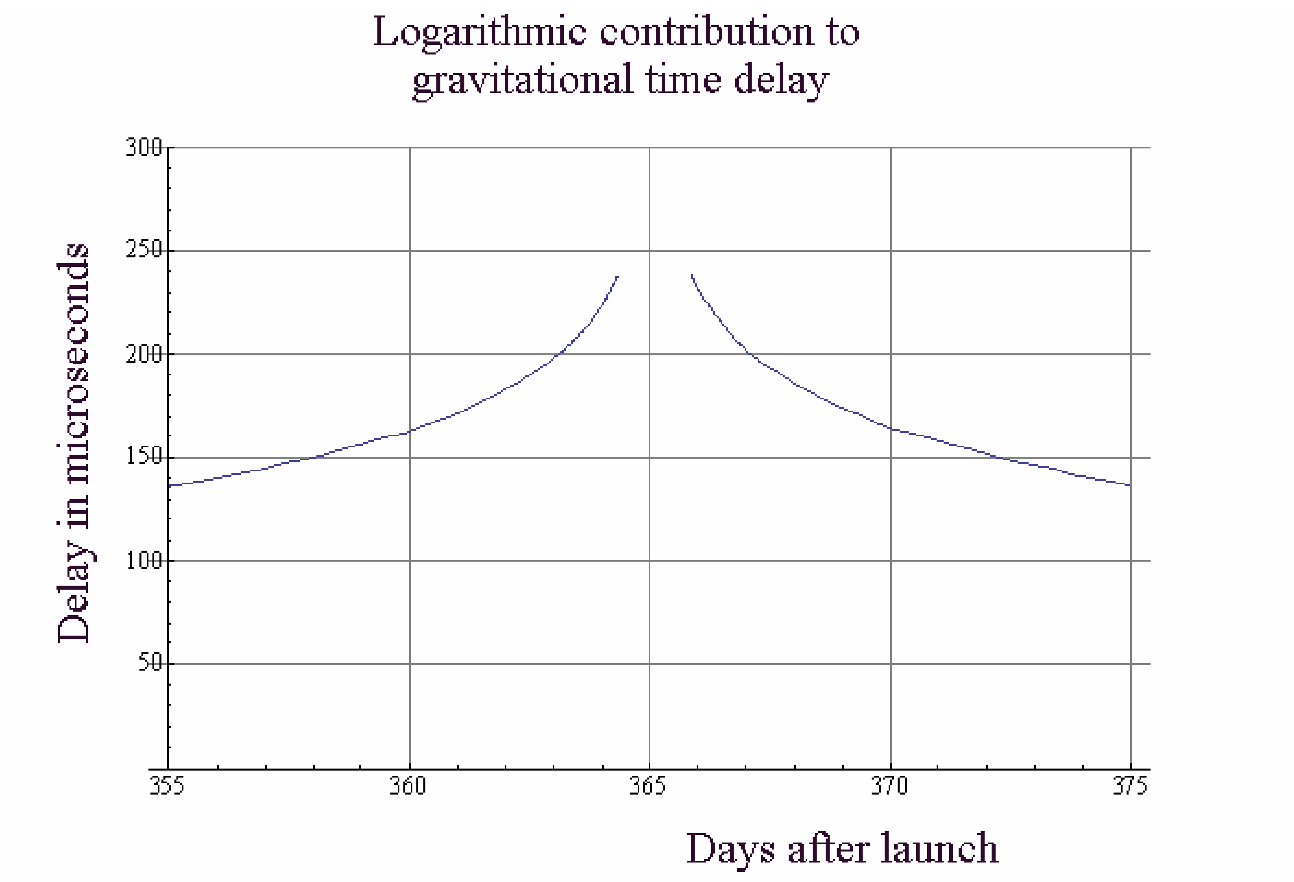
$$0.1\,\mathrm{mHz} < f < 1\,\mathrm{Hz},\ \sqrt{S_a} < 3\times10^{-15}\ \mathrm{m/s^2}/\sqrt{\mathrm{Hz}};$$

(we assume $10^{-13}\ \mathrm{m/s^2}/\sqrt{Hz}$)

- Much of the desired performance for LISA has been demonstrated in the laboratory with torsion pendula, and will be verified on ESA's LISA Pathfinder Mission.

- For a 2-year period orbit for the distant spacecraft, aphelion occurs near conjunction so the spacecraft temperature will not change much during main observing period.

Shapiro time delay (Log term)



The logarithmic time signature is unlike other time signatures that occur in the problem and allows the effect of γ to be picked out.

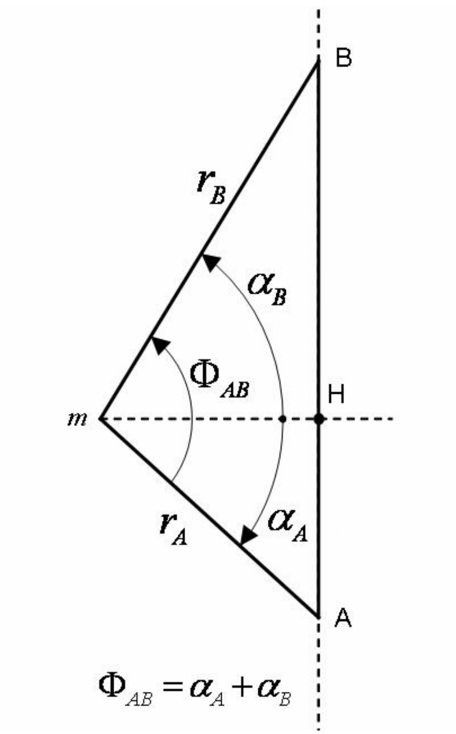
Gravitational Time Delay

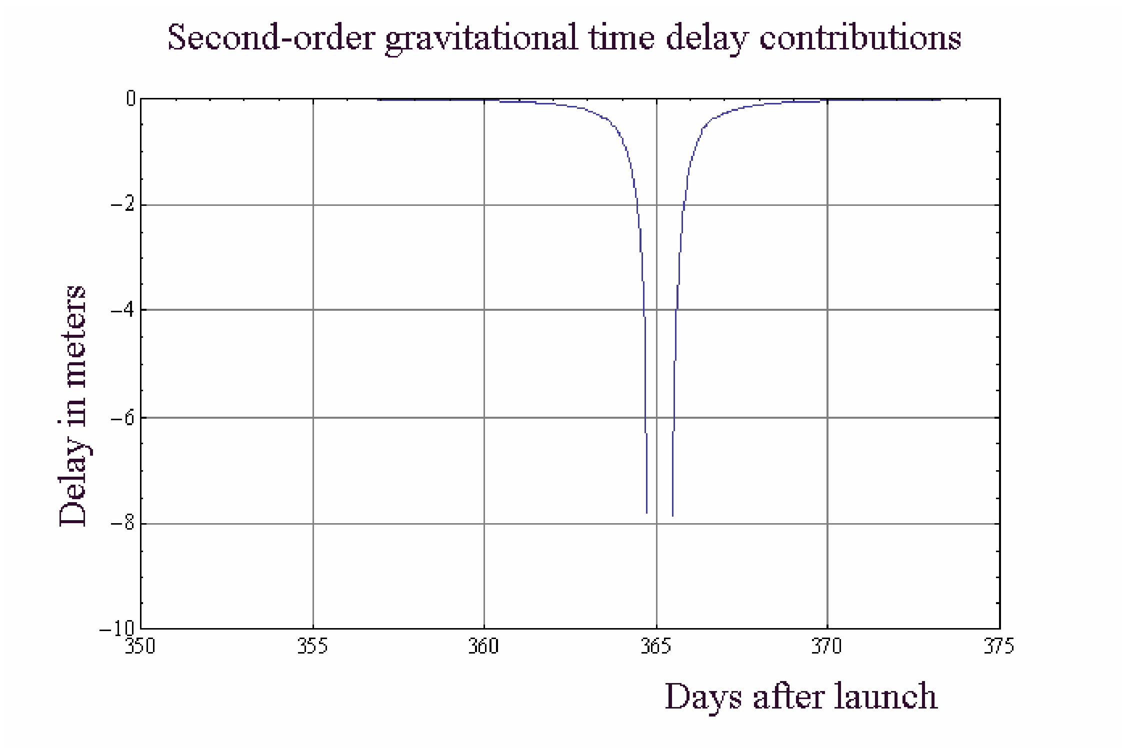
Gravitational time delay to second order in $\mu = GM_{\odot} / c^2$:

$$c\Delta t = \mu(\gamma+1)\log\left[\frac{r_A + r_B + r_{AB}}{r_A + r_B - r_{AB}}\right] +$$

$$\mu^2 \frac{r_{AB}}{r_A r_B}\left(\frac{8+8\gamma-4\beta+3\varepsilon}{4}\frac{\Phi_{AB}}{\sin\Phi_{AB}} - \frac{(\gamma+1)^2}{1+\cos\Phi_{AB}}\right)$$

(If Φ_{AB} is not too close to π.)





Noise Analysis

- We model the logarithmic time delay expression that will be observed by

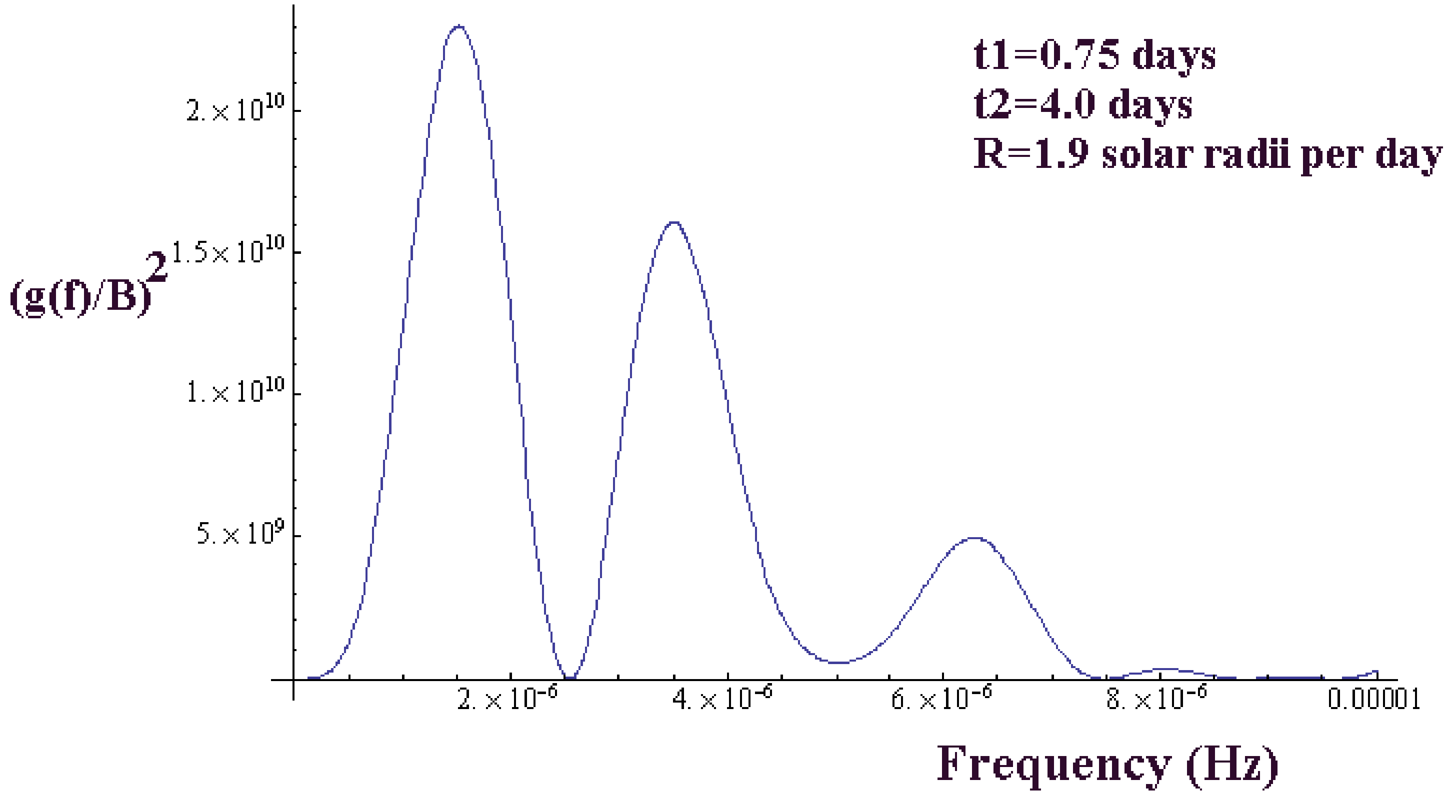
$$g(t) = B(\log|Rt| - M), \; t_1 < |t| < t_2$$

$$B = 3.8\times 10^{-5}\, s;$$
$$R = 1.9\, \text{solar radii/day};$$
$$\text{M} = <\log|Rt|>;$$
$$t_1 = 0.75\, \text{days};\; t_2 = 4\, \text{days}.$$

- We assume optimal Wiener filtering of the data can be used to extract the value of γ:

$$\left(\frac{S}{N}\right)^2 = 2\int_{f_{\min}}^{\infty} \frac{g(f)^2}{n(f)^2} df \simeq \frac{2}{n(f)^2} \int_{f_{\min}}^{\infty} g(f)^2 df$$

The function $g(f)^2$



- Only about 5% of the integral of $g(f)^2$ comes from frequencies below 1 microhertz.

Changes in the round-trip travel time

- The required accuracy is sub-picosecond, so timing with short laser pulses is ruled out.

- With a highly stable laser available, continuous carrier phase measurements with very high accuracy are possible. The observable for travel time variations is the integrated Doppler phase. However, cycle slips must be minimized.

- The main challenge is systematic measurement errors that drift with time. The gravitational time delay increases by 64 microseconds from 4 to 0.75 days before conjunction, then decreases by the same amount afterwards.

  A worst case drift of 0.1 ps in the round-trip timing error before conjunction with an opposite drift afterwards would give an error contribution for γ of

$$3 \times 10^{-9}.$$

Simulations including the effects of orbit determination uncertainties

- Simulations of the proposed gravitational time delay measurements have been carried out for the case of both orbits in the ecliptic and only 8 days of measurements.

- Uncorrelated 0.02 ps uncertainties are assumed for the round-trip travel time measurements over 3 hour periods, based on the assumed white clock frequency noise of $5\times10^{-15}/\sqrt{\mathrm{Hz}}$ down to at least 1 microHz.

- Spurious accelerations have not been included yet, but their effect is expected to be small over the assumed 8 day measurement span.

- All of the in-plane orbit parameters, plus γ, are solved for.

- The resulting uncertainty for γ is 1×10^{-8}.

- Simulations over roughly 20 days that include a model for the drag-free system noise down to at least 0.4 microHz are planned.

Additional science measurements- gravitational frequency shifts

A very stable clock at high altitude provides an opportunity for improved measurements of the gravitational redshift. For a clock at L1, there are three main contributions to the fractional frequency shift:

1. Gravitational redshift due to earth:

$$-\frac{GMe}{c^2 r} - \left(-\frac{GMe}{c^2 a_1}(1+\frac{1}{2}J_2) \right) = 6.93\times 10^{-10};$$

2. Second-order Doppler shift difference:

$$-\frac{1}{2}\left(\frac{\Omega r}{c}\right)^2 - \left(-\frac{1}{2}\left(\frac{\omega a_1}{c}\right)^2 \right) = -1.7\times 10^{-12};$$

3. Solar tidal potential difference:

$$-\frac{GM_{\odot} r^2}{c^2 a^3} = -9.9\times 10^{-13};$$

Net: $6.9\times 10^{-10};$

- Could be measured to about 3 parts per million in a few hours, using H maser ensemble referenced to Cs.

Required Technology Improvement

1. Spaceborne clock with $5 \times 10^{-15}/\sqrt{\text{Hz}}$ frequency stability down to 1 microhertz.

2. Drag-free spacecraft with less than 1×10^{-13} $\text{m/s}^2/\sqrt{\text{Hz}}$ spurious acceleration down to 1 microhertz.

3. Measurement of roun stability. el time between distant spacecraft with 0.02 picosecond accuracy.

4. Accurate time delay measurements for lines of sight down to 0.1 degree from the limb of the Sun.

5. High accuracy orbit determination from spacecraft to spacecraft Doppler measurements plus ground tracking.

Conclusions

- A mission to determine the gravitational time delay between drag-free spacecraft to measure γ to 1×10^{-8} or better appears to be feasible.
- However, the orbit determination part of the problem has not yet been investigated for more than the 8 days of high-accuracy measurements.
- Simulations over longer times that fully include spurious acceleration noise and determination of all the spacecraft orbit parameters are under way.
- Other potential science goals that can be achieved with highly stable clocks in space are being investigated.

References

- A. D. Ludlow et al., Science **320,** 1805 (2008)

Epoch determination

$$\sigma_\gamma = \frac{\sqrt{\frac{1}{N}\sum_i \left(\frac{1}{r_1 r_2 (1+\cos\Phi)} (\dot{r}_{12}(r_1+r_2) - r_{12}(\dot{r}_1+\dot{r}_2)) \right)_i^2}}{\left(\sum_i \log\left[\frac{r_1+r_2+r_{12}}{r_1+r_2-r_{12}} \right]_i \right)^2} \times \sigma(t)$$

Summary of contributions to uncertainty in γ

- Epoch determination: negligible
- Spurious accelerations: 2×10^{-9}
- Measurement noise: 6×10^{-10}
- Clock noise: 1×10^{-8}

- Orbit parameter determination is very important, particularly the out-of-ecliptic parameters; this needs to be studied more.

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