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## Storm tracking based on rain gauges for flooding control in urban areas

M. Carbone<sup>a</sup>, G. Garofalo<sup>a</sup>, G. Tomei<sup>a</sup>, P. Piro<sup>a\*</sup>

<sup>a</sup>University of Calabria, Department of Civil Engineering, Ponte Bucci cubo 42B, Rende 87036, Italy

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### Abstract

The fervent urbanization process coupled with the climate changes has been generating a series of more frequent and intense floods all over the world. Although flood risk can never be completely eradicated, its impacts require to be reduced by either improving the modelling of urban drainage systems and deepening the knowledge of flood produced by extreme storm rainfalls. The detailed study of urban drainage networks plays a fundamental role not only on problems related to flooding phenomena that are repeated with increasing frequency, but also on issue related to water quality of run-off (Piro et al., 2012). The objective of the study is to demonstrate the rainfall tracking prediction can be accurately performed in areas where radar measurements are not available, by using a dense network of rain gauges. The results of numerous storm tracking studies reveal that the choice of the hyetograph feature is a very difficult task (Hindi et al., 1977; Felgate et al., 1975; Shaw, 1983). In this study the storm tracking was studied on the basis of the method proposed by Diskin (1990). The methodology proposed has been applied to data from a network of rain gauges distributed over an area of about 1,600 km<sup>2</sup> around the city of London, UK. The results demonstrate how rain gauges, that are more approachable than radars for either economical and practical reasons, are very useful in forecasting the movements of storm events in the monitored area.

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\* Corresponding author. Tel.: +39-0984-496546; fax: +39-0984-496546.  
E-mail address: [patrizia.piro@unical.it](mailto:patrizia.piro@unical.it)

## 1. Introduction

Nowadays with the expansion of urban environments and the subsequent modification of the territory, the study of urban drainage networks plays a fundamental role in the life of the cities, both on problems related to water quality of run-off (Piro et al., 2010; 2012) and flooding phenomena that are repeated with increasing frequency. Historically the attention of media and institutions has been paid mainly to river or coastal floods, whereas phenomena associated with overloaded sewerage systems or inefficient drainage inlets, also known as sewer flooding, have been neglected until now (Balmforth et al., 2006). However, such an issue can no longer be ignored because of the precarious working conditions characterizing the existing drainage systems. In fact, these structures rapidly reach their maximum capacity and tend to work pressurized even in the case of medium storms (Piro et al., 2011). Moreover, this situation worsens dramatically in developing countries due to the much heavier local rainfall and lower drainage standards.

Although flood risk can never be completely eradicated, its impacts require to be reduced by both improving the modeling of urban drainage systems both deepening the knowledge of flood producing storm rainfalls. In particular this research was focused on this second aspect because many characteristics of heavy rainfalls are still not determined (Shaw, 1983):

- the distribution of the rainfall at ground level, which is the most important factor for the hydrologist;
- the rate and direction of movement of storms, relative to the orientation of the catchment, that are also relevant to flood studies.

All this is vital to enhance decision making by urban planners, engineers and policy makers in view of the rainfall effects in causing regular flash floods (Desa et al., 1997).

Consequently storm movement parameters, velocity and direction, were derived by analyzing rainfall data through available storm tracking procedures. These methods were originally developed in order to take into account the storm kinematics into rainfall input used for calculation of runoff (Niemczynowicz, 1991). Nevertheless, in this case, they were employed in order to demonstrate how the use of a dense network of rain gauges could be considered as a valid alternative in rainfall movement prediction, to be taken into account in areas where radar measurements cannot be obtained yet.

Radar has the advantage that it is sometimes possible to view an entire storm system whereas a rain-gauge network often acts only as a "window" of storm observation (Shaw, 1983). However, currently, rain gauge data are often available whereas radar data are not. Moreover radar instruments enable the investigation of convective cells motion, whereas rain gauges data allow the analysis of the movement of rainfall patterns recorded on the ground, that is more important for hydraulic modeling (Niemczynowicz et al., 1984). In fact it will be also showed how storm tracking method results could be potentially used in connection with hydraulic models, previously calibrated for the same study area, in order to evaluate in advance the possible flood-prone areas so that all the planned security measures could be implemented in time.

## 2. Methodology

### 2.1. Description of the study area

Thirty-eight automatic tipping-bucket rain gauges are placed fairly evenly over the city of London:

- six have been operating since 2006 (Atomwide, Carshalton, Harringay, Hayes, Kensington, New Malden);
- seven since 2007 (Belmont, Catford, Eltham, Hillingdon, Hornchurch N, Rainham, Southwark);
- six since 2008 (Clapton, Colindale, Greenwich, Hornchurch W, Newham, Walthamstow);
- nineteen since 2009 (Bedfont, Bow, Brent, Camden, Dagenham, Enfield, Islington, Merton, Mill-Hill, Richmond S, Southall, Stanmore, Streatham, Thornton, Twickenham, Wandsworth S, Wandsworth SW, Welling, Westminster).

The network covers an area of about 1,600 km<sup>2</sup>, consequently there is one gauge per 42 km<sup>2</sup> (fig. 1). The registration is governed by the same clock assuring absolute time synchronization and the time resolution of the registration is fifteen minutes.

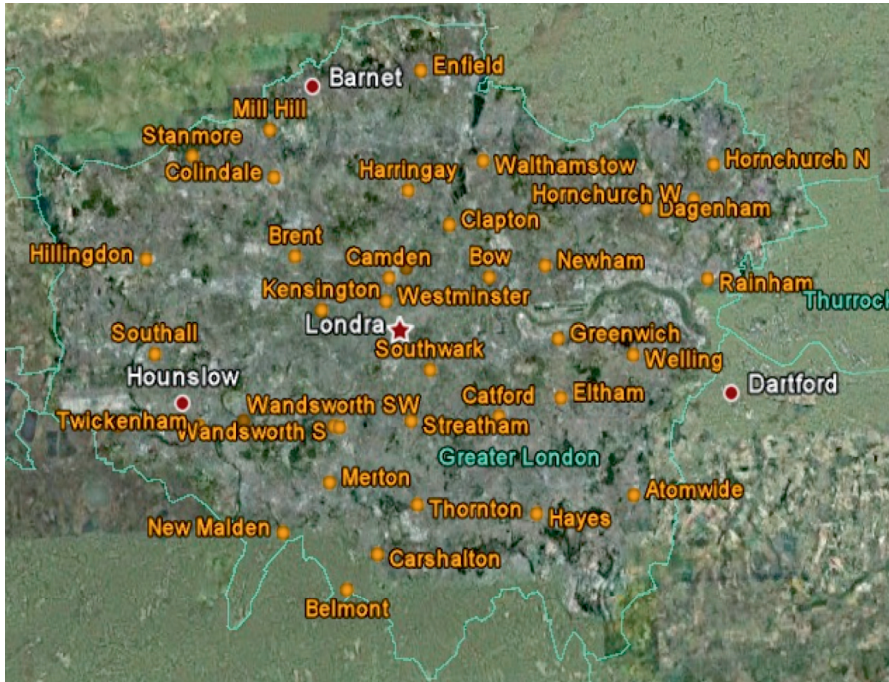


Fig. 1. Location map of rain gauges in London.

Therefore it is evident that it was not possible to respect the maximal rainfall data collection requirements recommended for such elaborations: i.e. one gauge per 1 km<sup>2</sup>, 1 min time resolution and 0.1 mm volume resolution of registration (“1-1-0.1” rule reported in Krejci et al., 1989 and Niemczynowicz, 1989). However the results obtained were in agreement with findings from other previous studies performed in other parts of the world.

Further information concerning the gauging system are available from the web site <http://weather.lgfl.org.uk/map.aspx>.

## 2.2. Storms selection

On the average, from three to twenty-nine stations successfully recorded the rainfall data via data logger system set in event mode. Unfortunately a part of these data could not be analyzed because periodic malfunctioning of the gauges and data loggers were not avoided totally.

Only storms, recorded by at least three rain gauges and with a duration at least of five hours, were considered. In particular, initially, the storm rainfalls were recognized from the most operating rain gauge, i.e. Atomwide. Later the durations of all the storms identified were assumed as reference for determining the storms for the other rain gauges. Altogether thirty-nine rainfall events were found appropriate for the successive elaborations.

## 2.3. Method of analysis

Several methods of different complexities have appeared in numerous literatures trying to quantify storm dynamical properties (Felgate et al., 1975; Fooks, 1965; Hindi et al., 1977; Jinno et al., 1993; Marshall, 1980,

Marshall, 1983; Niemczynowicz et al., 1984; Niemczynowicz, 1984, 1987; Zawadzki, 1973; Shaw, 1983; Shearman, 1977; Kottegoda et al., 1991). The basic assumption, common to all these procedures, is that the computations produce single values of storm speed and direction of movement for each storm event. In other words, the possibility of changes in the speed or direction as the storm sweeps over the network is ignored (Diskin, 1987). This simplification is justified since convective storms move at a reasonably steady velocity, in fact changes occur only slowly over distances of perhaps hundreds of miles (Hindi et al., 1977).

In particular the motion is expressed by the fact that rainfall hyetographs recorded at the various gauges are displaced relative to one another along the time axis. These relative displacements depend obviously on the speed and direction of storm movement. Consequently all the storm tracking methods start with the identification of some recognizable feature of the hyetograph, that is then followed as it moves in space across the gauge network. Examples of such reference points are the peak, the centroid of the hyetograph, the maximum of the cross-correlation function, the maximum of the lag-correlation structure, etc. The main difference between these methods can be found just in the different choice of the reference point.

In this case the method proposed by Diskin (1990) was tested because it is practically sound and straightforward. Moreover it enables to overcome the main limitation characterizing most of available storm tracking methods: i.e. the rejection of some results based on threshold values set arbitrary in the elaborations, such as the correlation coefficient in the “lag-correlation” and “full correlation” analysis.

In particular the method consists in determining the equation of the inclined plane that best fits to a given set of  $n$  points in the  $x, y, t$  space (fig. 2), where  $n$  is the number of stations for which data are available.

$$t = a \cdot x + b \cdot y + c \quad (1)$$

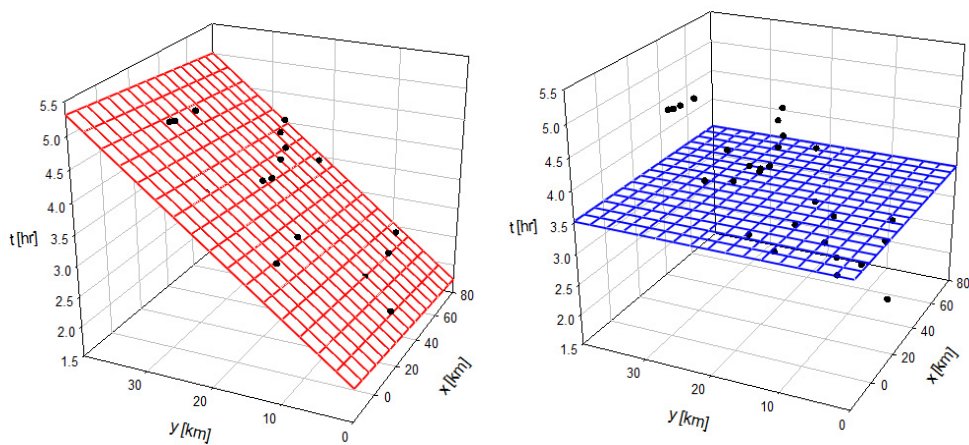


Fig. 2. Geometrical interpretation of the movement of storms through the hypothesis of the inclined plane and the horizontal plane.

The coordinates of each point are the location of the station, relative to a pair of (arbitrary)  $x, y$  axes, and the time of arrival of a specified feature at the station.

The values of the parameters,  $a$ ,  $b$  and  $c$ , can be obtained from data sets by minimizing the sum of the squared deviations between the observed arrival times ( $T_i$ ) and the predicted values ( $t_i$ ) computed by eq. (1) for all stations. In particular the speed of movement can be later calculated by the inverse of the maximum slope of the plane

$$V = \frac{1}{\sqrt{a^2 + b^2}} \quad (2)$$

whereas the direction, relatively to the x-axis, is parallel to that of the maximum slope

$$\Theta = \arctan\left(\frac{b}{a}\right) \quad (3)$$

It is clear that the units of measurement of the computed parameters depend on the units of measurement assumed for the input data.

Nevertheless eq. (1) is not the only possible model to represent the data. In fact the observed different arrival times could be also random fluctuations from an equal arrival time for all stations, that could be the average of the observed arrival times themselves. Reverting to the geometrical interpretation, an equal arrival time is represented by a horizontal plane (parallel to the x,y axes) at the level of the average time of arrival (fig. 2).

Therefore it is necessary to verify which of the planes represents better the data set: the choice can be based on the comparison between the root mean square (RMS) deviation obtained by adopting the two models ( $\sigma_1$ ,  $\sigma_0$ )

$$\sigma_I = \sqrt{\frac{\sum_{i=1}^n (T_i - t_i)^2}{n-1}} \quad (4)$$

$$\sigma_O = \sqrt{\frac{\sum_{i=1}^n (T_i - \bar{T}_i)^2}{n-1}} \quad (5)$$

is the average arrival time.

where

1.  $\bar{T}_i$  is the average arrival time
2.  $T_i$  are the observed times of arrival
3.  $t_i$  are the predicted times of arrival computed by eq. (1);

The ratio of the two values can be assumed as a measure of the significance of the results (*Significance Ratio*):

$$\text{Significance Ratio (S.R.)} = \frac{\sigma_I}{\sigma_O} = \sqrt{\frac{\sum_{i=1}^n (T_i - t_i)^2}{\sum_{i=1}^n (T_i - \bar{T}_i)^2}} \quad (6)$$

The value of  $\sigma_1$  is always smaller than  $\sigma_0$ , therefore their ratio is expected to be comprised between zero and one. Values close to zero indicate high significance of the inclined plane hypothesis whereas values close to one poor performance of the proposed model. In this case a threshold value was adopted for this parameter: in

particular all the elaborations, characterized by a significance ratio bigger than 0.85, were rejected because it meant that the hypothesis of equal arrival times was the best one, and consequently no storm movement could be considered.

Finally the accuracy of all the results produced, including the values of significance ratio, was estimated by repeating computations of the speed and direction for subsets of the data, obtained by omitting each time one of the recording station (Diskin, 1990). The average values of these results were considered for the further analysis, whereas the standard deviations were assumed as an indication of their accuracy. Actually, this accuracy is derived from  $(n-1)$  stations, however, if  $n$  is large, the value obtained can also be taken as a measure of the accuracy of the results obtained with all the data.

### 3. Results and discussion

The results of numerous storm tracking studies reveal that the choice of the hyetograph feature is a very difficult task (Hindi and Kelway, 1977; Felgate and Read, 1975; Shaw 1983). Consequently, in this case, the elaborations were carried out by considering three different features in order to verify which one gave the most reliable results:  $t_{cent}$ , the time to the centroid of the hyetograph,  $t_{max}$ , the time of highest rainfall intensity and  $t_{onset}$ , the time of start of rainfall.

As it was expected, different results were obtained depending on the feature selected (tab. 1): the average velocity was in fact 20.87 km/hr (5.80 m/s) for the elaborations based on the centroid feature, 16.29 km/hr (4.52 m/s) for the elaborations based on the peak feature, 24.61 km/hr (6.84 m/s) for the elaborations based on the onset feature

Table 1. Range variation, average value, standard deviation and coefficient of variation resulted from the elaborations carried out.

	Centroid			Peak			Onset		
	V [km/hr]	$\Theta$ [°]	S. R.	V [km/hr]	$\Theta$ [°]	S. R.	V [km/hr]	$\Theta$ [°]	S. R.
<b>Min.</b>	5.27	55.55	0.04	3.70	6.03	0.02	4.25	7.82	0.22
<b>Max.</b>	55.06	177.89	0.79	34.73	176.89	0.84	78.75	175.64	0.85
<b>Mean</b>	20.87	139.35	0.52	16.29	116.33	0.62	24.61	103.07	0.58
<b>S.D.</b>	14.31	35.92	0.17	10.82	57.46	0.20	20.78	57.40	0.18
<b>C.V.</b>	0.69	0.26	0.33	0.66	0.49	0.33	0.84	0.56	0.31

Instead the average storm direction was 139.35 degrees counted clockwise from the N for the elaborations based on the centroid feature, 116.33 degrees for the elaborations based on the peak feature, 103.07 degrees for the elaborations based on the onset feature (tab. 1).

Precisely the velocity outcomes demonstrate the quality of the elaborations carried out: in fact, these values were quite in agreement with the results obtained in several studies performed in other regions. Hobs and Locatelli (1978) reported that storm velocity ranged between 2 and 25 m/s. Chaudhry et al. (1994) gave an average storm velocity of 11.67 m/s using data from radar. Marshall (1980) computed 11.4 m/s, Niemczynowicz and Dahlblom (1984) found an average velocity of storms 10.35 m/s while Shearman (1977) obtained about 15 m/s for 60% of the storms analyzed. Finally Eagleson (1970) mentioned the velocity of convective cells to be in the range between 8.94 and 13.41 m/s.

Nevertheless, also some differences emerged: in this case, in fact, computed velocity data set resulted well fitted by a normal distribution for all the three features, whereas Niemczynowicz (1987) observed that the relative frequency of storm velocities was well fitted by a two-parameter lognormal distribution.

Since the influence of storm movement on runoff depends on joint effects of storm velocity and direction, the relative frequencies of storm velocity and direction in velocity and direction classes were also studied. Especially it



was noticed (fig. 3) (1) a distinct maximum of relative frequency around storm velocities 16.88–22.50 km/hr and storm direction towards S-E and S-S-E for the elaborations based on the centroid feature; (2) a distinct maximum of relative frequency around storm velocities 5.63–11.25 km/hr and storm direction towards S-Et and S-S-E for the elaborations based on the peak feature; (3) a distinct maximum of relative frequency around storm velocities 5.63–11.25 km/hr and storm direction towards S-E and S-S-E for the elaborations based on the onset feature.

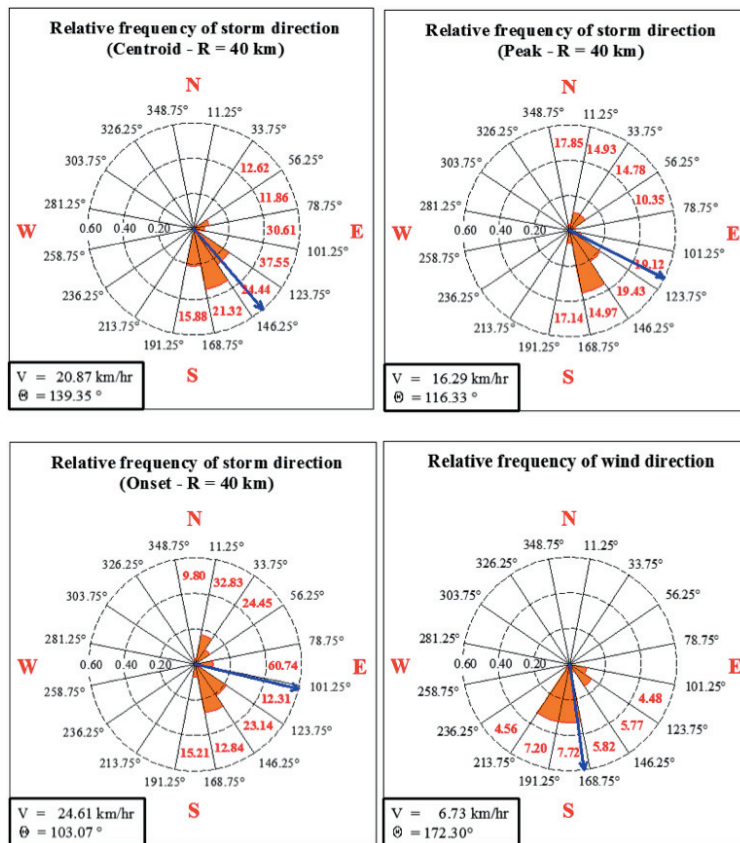


Fig. 3. Relative frequency of storm direction for the three features considered and for the wind data..

The average wind velocity was 6.73 km/hr (1.87 m/s) whereas the average wind direction was 172.30 degrees counted clockwise from the N. Moreover it was observed a distinct maximum of relative frequency around storm velocities 5.63 – 11.25 km/hr and storm direction towards S and S-S-W (fig. 3).

By the comparison carried out it was observed that storm velocities exceeded wind velocities in almost all the cases. Such result was in contrast to the findings by Niemczynowicz and Dahlblom (1984), however, it can be explained because the wind data are recorded on the ground level, therefore smaller values of this parameter should be expected because of the presence of obstacles, such as buildings. About the relations between the variables used (wind velocity and rainfall velocity) the results confirm that there are virtually no relations between the variables, as observed by other several researchers (Shearman, 1977; Marshall, 1980; Niemczynowicz and Dahlblom, 1984). Also the correlation coefficients between wind and storm directions were not significant, although a better agreement was observed between the two data sets.

The most suitable feature was later evaluated by calculating the root mean square deviation between the data sets of wind velocities and storm velocities predicted assuming each time one of the selected features. The values computed are reported in table 2 together with the correlation coefficients.

Table 2. Root mean square deviation and correlation coefficients computed for the three features.

Feature	$\sigma_v$ [km/hr]	Correlation <sub>v</sub>	$\sigma_\theta$ [°]	Correlation <sub><math>\theta</math></sub>	N <sub>r</sub>
Centroid	20.26	0.13	56.50	0.08	34
Peak	14.54	0.03	83.52	0.15	28
Onset	28.41	-0.27	89.10	0.26	29

It emerged that the onset feature was the worst one since it presented the biggest root mean square deviation for both velocity and direction. This outcome was expected because the poor time resolution of the registration would have surely conditioned more the elaborations carried out by considering this feature. In fact, it is evident how a time resolution of fifteen minutes could favour the erroneous individuation of an equal start time of the storms from most of the stations.

Instead, the centroid provided the best approximation of the wind direction, whereas the peak gave the best estimate of the wind velocity. Specifically, wind velocities were overestimated in both cases: it means that the storm will be expected to reach the catchment in less time with respect to reality, therefore, all the safety measures provided will have to be realized in advance.

Consequently, in the end, the centroid was chosen as reference feature because the bigger mistake in the overestimation of the velocity, however advantageous for the safety measures, was compensated by the better evaluation of the direction of movement of the wind. Similar conclusions were also reported by other researchers: in fact, May and Julien (1998) noted that the use of the centroid of the hyetograph gives more reliable results than using time of the onset of the storm.

#### 4. Conclusion

The elaborations carried out demonstrated how rain gauges may be considered as valid alternatives in rainfall movement prediction, to be taken into account in areas where radar measurements cannot be obtained yet. In fact the method of computing storm velocities and directions was sufficiently good since the calculated velocity values were in agreement with the results obtained in several studies performed in other regions. However, also some differences emerged: in this case, in fact, computed velocity and direction data sets resulted well fitted by a normal distribution, whereas other researchers observed that the relative frequency was well fitted by a two-parameter lognormal distribution.

The choice of the best hyetograph feature to be considered in the elaborations was another issue faced. Three features were used for the definition of arrival times:  $t_{cent}$ , the time to the centroid of the hyetograph,  $t_{lmax}$ , the time of highest rainfall intensity and  $t_{onset}$ , the time of start of rainfall. The feature giving the most reliable results was established by comparing, through statistical tests, the results of the elaborations performed with other physical phenomena which are related to the storm movement, such as wind movement.

From the applications carried out it could be noticed that: (1) the time and spatial resolution of the recording stations affect significantly the results of the applications. In fact, in this case, the poor time resolution (15 minutes) determined that the onset was the worst feature since it favoured the erroneous individuation of an equal arrival time of the storms from most of the stations. Instead the centroid feature provided the best approximation of the wind direction, whereas the peak gave the best estimate of the wind velocity. Nevertheless, in the end, the centroid was recommended for future applications because the bigger mistake in the overestimation of the velocity, however advantageous for the safety measures, was compensated by the better evaluation of the direction of movement of the wind; (2) the use of wind data recorded at the ground level should be avoided in similar calculations because they did not enable strong correlations to be found between storm and wind movement parameters. In fact, by comparing the corresponding wind and storm parameters, it emerged that wind velocities were overestimated whereas wind directions were underestimated. This result, already reported in other studies in the literature, was expected because such measurements are affected by the presence of obstacles at ground level,



such as buildings. Consequently, high altitude wind data, available in the airports, will have to be assumed for further applications in order to estimate the quality of the elaborations carried out.

The research work is continuing with the installation of an experimental network of rain gauges distributed over an area of about 500 km<sup>2</sup> around the city of Cosenza, Italy, mainly, to better investigate the site influence and to define a synthetic approach for the identification of the real hyetograph to recognize the most suitable feature to identify and to study the storm tracking.

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