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Teacher Training at Math Club

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Teacher Training at Math Club

Maria Marta da Silva [α] & Lukas Adriel Francisco Alves [σ]

Abstract- **The article aims to investigate how the planning of teaching activities based on the historical movement of the process of emergence and development of mathematical concepts by a collective of Mathematics teachers in initial training contributed to the understanding that these concepts are a product of the humanitie needs and that the organization of Mathematics teaching in schools must address this understanding. Related to this objective, we sought to answer the following question: What contributions can a training space called the Mathematics Club bring to the initial training of Mathematics teachers? In the search for answers to this problem and to achieve the objective, the formative experiment was chosen as research methodology, happening from 2020 to 2023. The data analysis structure chosen was: unit, episodes and flashes. The results shows that teachers were able to establish a historical understanding of the concepts that were approached.**

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I. Introduction

Conflict is fertile soil for the generation of new knowledge (Rubstov, 1986). It only takes hold when there are different subjects and, obviously, different opinions. It has been installed when the issue at hand is the training of Mathematics teachers.

How can we move forward and find answers to the challenges surrounding the training of these teachers? How have Mathematics Degrees dealt with these issues? Are there other spaces capable of training this teacher from a perspective other than the currently dominant one?

As D'Ambrósio (2005, p. 23) highlights, in order for the training of these teachers to break away from existing models, we need to organize other spaces that are willing to break the "absolutist vision of mathematics towards an understanding of mathematics as a constructed and negotiated discipline within a participatory community" that has developed throughout the history of humanity.

Therefore, in accordance with Siegel and Borasi (1994), there is a need for Mathematics teacher training that challenges perspectives regarding teaching learning that is based on cumulative Mathematics learning and full of rules and procedures. Model repeated exhaustively also in Mathematics degrees that exist in Brazil (Gatti & Nunes, 2009). This training model for these teachers prevents them from having a deep understanding of the mathematical concepts that are part of the school curriculum (Ma, 1999). This intense apprehension would be the essence of these concepts, and it can only be found throughout the knowledge of the process of emergence and historical development of these concepts (Kopnin, 1978; Sousa, 2018).

Given this scenario, there is a great need to modify the training processes for these teachers. After all, it is unlikely that a Mathematics teacher trained in a traditional program will be prepared to face the challenges of promoting student learning in a way other than what he has experienced, whether it works or not. Research on teacher action shows that in general the teacher teaches the way he was taught (Cooney & Hirsch, 1990).

For these reasons, teaching predominates in which the teacher explains the content, shows how to solve some examples and asks students to solve numerous similar problems. In this view of teaching, the student receives instruction passively and imitates the teacher's steps in solving problems slightly different from the examples. Success through memory and repetition predominates. Rarely do these students generate needs to solve the problems offered.

Faced with this reality, the question arises: How can the emergence of training spaces for Mathematics teachers, which differ from teaching degrees, allow the planning of teaching activities[1] that alleviate the problems surrounding learning to teach Mathematics?

This permission can only occur if training spaces are built where the absolutist view of Mathematics is broken and can generate a context in which subjects should not only accumulate knowledge. This model considers mathematical knowledge to be absolute and unquestionable and therefore individuals should simply accumulate their knowledge. This model has been guiding our teacher training and our teaching of Mathematics for several centuries (D'Ambrósio, 1993).

Contrary to this model, a teacher training process is proposed combined with an organization of

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[1] Teaching activities are activities that are planned based on a process of meaning of the content to be taught. To achieve this, these activities require attention to the genesis of concepts, the historical processes of their development in humanity, their interdependence with the formation of other scientific concepts in the historical moment of their emergence, their presence as a symbolic instrument in the current context and the relevance of their learning to develop the subjects' potential (Moura, 2017).

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Mathematics teaching where teachers and students have legitimate mathematical experiences, that is, experiences similar to those actually experienced by men during their historicity, when they had to create concepts mathematicians. These experiences must be characterized by the planning, development and re-elaboration of activities capable of triggering the learning of these concepts, which reach schools as school content (Silva & Cedro, 2021).

Such activities must be capable of proposing problems, and their solutions through collective negotiation for the legitimacy of the proposed responses. This negotiation process will lead those involved (teachers in training and students from schools where these activities are developed) to discuss the nature of the process of creation, development, formalization and symbolization of mathematical content.

A formative space based on such ideas will lead subjects to understand that mathematical concepts are responses to human needs throughout the history of man on our planet. They are the product of historical-social problems, which man has faced (Radford, 2011; Moretti & Radford, 2015; Sousa, 2018) and, therefore, should not be discussed in the classroom in an arbitrary way and disconnected from their history as Borasi (1992) points out. Because if this is done, the understanding of the social decision-making process that human communities had to make for such concepts to emerge will be disregarded (Arcavi, 1991).

In line with the discussions made, the main objective is to investigate how the planning of teaching activities based on the historical movement of the process of emergence and development of mathematical concepts by a collective of Mathematics teachers in initial training contributed to the understanding that these concepts are a product of human needs and the organization of Mathematics teaching in schools must address this understanding. Interconnected with this objective is the problematizing research question: What contributions can a training space called the Mathematics Club bring to the initial training of Mathematics teachers?

But what would the Math Club be?

The Mathematics Club (MathClub) is a training-collaborative space that emerged in 1999 as a supervised internship project linked to the Faculty of Education of the University of São Paulo (FE-USP). In this sense, the concepts defended within this space were spreading throughout the Brazilian territory, in several Higher Education Institutions. At this juncture, specifically, in 2017, MathClub was created within the scope of the State University of Goiás, Campus Sudoeste - Headquarters: Quirinópolis – Brazil and continues to this day, however, the period of this research ended in 2023.

The training process that takes place at MathClub also aims to conduct teacher training so that the teacher can find a general way of organizing their main activity, teaching. Such training takes place in a way that highlights the importance of the interaction between history and the society that produced the concepts to be studied. This is a prominent element within the Club so that teachers in training can be led to "understand conceptual developments" placing "the knowledgeable subject and the entire mathematical activity under study within their cultural conception" (Radford, 2011, p. 82).

The Mathematical Club highlights the importance of concepts being studied in their production process with the historical meanings intrinsic to the historical period in which they emerged and developed (Radford, 2006). According to the author, [...] human activity is the generator of conceptual objects, which become the root of changes in the activities themselves" (Radford, 2006, p. 112). Still according to Radford (2006), attention must be paid to the explanation of how the acquisition of knowledge deposited in culture is carried out, which is commonly known as concepts, which are arranged in school as content: this is a fundamental problem in the formation of students. Mathematics teachers in particular, and learning in general.

The subjects[2] participating in the research that supported this article were Mathematics teachers in initial training (ranging from 25 to 30 in each year). The activities planned by the Club are carried out at the University that hosts the project. The development of activities takes place in public schools in the city where the Club's headquarters are located. The activities are developed with the final grades of Elementary School I and all of Elementary School II, the number of students varies on average from 28 to 35 students per class. All moments of the training experiment were recorded in audiovisual form, later, these recordings were transcribed and became the universe of data for subsequent analysis.

However, in order to understand the actions that preceded the preparation of this article, so that the objective was achieved and answers were found, initially, we sought to understand the global panorama regarding teacher training and how the Mathematics Club stands in this scenario, as well as its theoretical stance is explained. Next, the formative experiment is presented as a methodological option for developing actions. Afterwards, data analysis is discussed based

[2] During the years the research was carried out, the number of teachers in training (subjects of this research) who participated was approximately 90 (the vast majority remained in the project for the entire duration of the degree – 4 years). The number of students from the schools that participated in the project was approximately 600 students (these are not the subjects of the selection made for writing this article).

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on a structure that privileges the movement of the phenomenon and to this end, the following will be used: unity, episodes and flashes. Finally, some considerations about the research carried out

II. Global Panorama about the Training of Mathematics Teachers and the Particular Case of the Mathematics Club

The topic of Mathematics teacher training is a topic that always deserves urgency. After all, national and international researchers reveal a large number of difficulties in training these teachers (D'Ambrosio, 2007; Silva, Oliveira, 2021; Borasi, 1992; D'Ambrosio, Campos, 1992; Fiorentini, Nacarato, 2005; Cochran-Smith, Lytle, 1999, Hargreaves 2002).

Most of these difficulties are a product of the rapid transformations in teaching work, the teacher's understanding of his social function and his lack of understanding of the processes of genesis and development of mathematical concepts (Sousa, 2018; Radford, 2011). Such processes can be found in the History of Mathematics and are rarely addressed in initial training processes, which, subsequently, are not taught in Mathematics classes at school (Miguel & Brito, 1996; Moretti, 2004). These difficulties are also based on the emergence and expansion of information and communication technologies that place this subject immersed in a computerized and globalized world (Silva, Nery & Nogueira, 2020), among others.

With the aim of offering answers to these questions, the field of research into Mathematics teacher training has been increasingly expanding in our country and around the world (D'Ambrosio, 1993; Lopes, Traldi & Ferreira, 2015; Kamii & Declark, 1985; Steffe & Cobb, 1988; Radford, 2011). This may be due to the fact that teachers are at the center of discussions about educational reforms. Never before has so much data about education, teachers and student performance been commented on and published in the media. External assessments, whether international, national, state or municipal, always bring bad data about learning in Mathematics and this boosts discussions about the training of these teachers.

Faced with this scenario, the question arises: How is the training of Mathematics teachers? Are other training proposals necessary? In response to these issues Ponte (1996), Cury (2001), Fiorentini (2004), Furlong, Cochran-Smith and Brenann (2009) and Moura (2021) highlight that teacher training has become increasingly controlled by the State, with the aim of achieving political goals. These authors also argue that, in this conception, the emphasis is placed on knowledge in action, which is expressed and conveyed in practice, based on the teacher's reflections about this practice and about it, in his teaching.

Still on the way to constructing answers to these questions, it is argued that the training of Mathematics teachers is a topic whose research possibilities have not yet been exhausted. Researchers such as Thompson (1992), Furlong, Cochran-Smith and Brenann (2009), Cooney, Hirsch (1990), Cochran-Smith, Lytle, (1999), Bicudo (1999), D'Ambrósio (2007), Fiorentini and Lorenzato (2009), Libâneo (2004), Silva (2018), Moura et al. (2010), among others, have made public the studies carried out regarding how the training of Mathematics teachers is taking place in universities.

The publications resulting from these studies have sparked discussions regarding the training of these teachers. These researchers highlight that part of the problems regarding the teaching of Mathematics is related to the initial teacher training process. They start from the idea that the training of Mathematics teachers based on the reproduction of content that does not transform the objective reality we have nor contribute to the emergence of a new reality, it only quantitatively multiplies qualitative changes already produced previously. Thus, "[...] for years, students enter and leave Mathematics classes with the feeling that mathematical concepts are fragmented and have no history. When many of these students become Mathematics teachers, they claim, with some reason, that theory and practice are not related" (Sousa, 2018, p. 41).

In this sense, the cited authors agree that the training process of these teachers should challenge the models that, according to Silva (2018) and Fiorentini and Nacarato (2005), are based on versions that sometimes prioritize only specific knowledge of mathematics, sometimes focus only on methodological aspects of them. For this reality to change, it is necessary to allow the emergence and maintenance of spaces that allow for another formation, different from what we currently have.

Thus, it is proposed to question the training purposes and the search for spaces that include the dynamic relationship of the development of subjects in training based on their participation in teaching and learning activities that begin and are directed towards the movement of understanding history the emergence and development of mathematical concepts from the study of the history of Mathematics (Sousa, 2018; Sousa & Moura, 2016; Silva & Silvestre, 2022).

The justification for this path of organizing the teaching of mathematical concepts and their discussion in initial training meets Thompson's (1992) defense that it is necessary to dispel the idea of many individuals who consider that "Mathematics is a discipline with precise results and procedures infallible, whose fundamental elements are arithmetic operations, algebraic procedures and geometric definitions and theorems" (p. 127). In this way, the organization of teaching its contents would be fixed due to the belief in

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a ready and finished state of mathematical knowledge, leaving very little space for proposals for organizing activities that could break with this conception.

But what would be the questions that would raise the proposal for teacher training that confronts the established model? There would be many, including: What type of teachers should this process train for what type of students?

However, the answers to these questions will never be unique and definitive. After all, as Vygotsky (1997, p. 350) states: "Questions about education will be resolved when life issues are resolved". Thus, like human reality, the educational system is full of uncertainties.

In this scenario, we see the emergence of teacher training towards external assessments, which are closely linked to economic competitiveness (Cochran-Smith, Feiman-Nemser & Mcintyre, 2008). According to Hargreaves (2002), the systems attributed by these evaluations impose monitoring that rewards successful schools by granting them support, while threatening those that insist on failing with closure.

However, in facing such issues, we persist in discussing how to have teacher training, in this specific case of Mathematics teachers, that meets transformations in relationships between learners and the social situation of their development, that is, a formation that is the result of a process between the internalization and externalization relationships so valued by Vygotski (1997) and Davidov (1988).

Training designed in this way takes into account that such relationships change as teachers assimilate what has historical, social and cultural value, consequently interpreting their social worlds differently and, therefore, act on them in different ways which, in turn, they impact the dynamic relationship between the development of subjects and the historical-social situations they live and experience.

In this sense, the subjects who participate in a training space based on these premises are not, therefore, mere passive recipients of culturally valued concepts, but are actors in and about their cultures, being both humanized and humanizing the places they are part of (Moura, 2017).

Among the possible spaces to allow and encourage such interrelations, the Mathematics Club [3]stands out as an environment capable of maintaining the interface between the University and the School, while the teacher in the process of learning to teach learns to teach, this phenomenon being a process of making sense and meaning as one engages with the world of which it is part (Moura, 2021; Silva & Cedro, 2022; Alves & Silva, 2023).

The Club is a space that allows the understanding that the teacher in initial training, in this process of dealing with change, does not do so as a simple response to his behavior, which was that of a student and now becomes a teacher, but also involves a a process full of contradictions that takes on new forms as it develops and generates its relationships with the historical-social situations of its development.

However, training spaces, such as the Club, must also focus on the role of the training activities they propose.

According to Leontiev (1978) and Moura (2017), activities are also part of a systematized formation that gains existence and form, manifesting itself only through actions carried out by individuals and groups.

In the meantime, the defining characteristic of any activity would be its object, in which the motivating object is incorporated - which elicits a response from the subjects in activity.

In this way, environments that intend to develop training based on these theoretical defenses must get involved with sets of activities that only exist based on a collective movement that highlights the dynamic relationship that can exist between individuals and the historical-social situations of the emergence and development of concepts so that they are able to promote the ability to see them as a result of the historical-cultural needs experienced by humanity.

To this end, it is necessary to understand the process of emergence and historical development of mathematical concepts and how this proposal can be used to organize the teaching of mathematical concepts in Basic Education.

But what can the dynamics of this historical movement of birth and development of concepts mean for teacher training?

To find answers to this question, another question will be needed: What type of teachers and students do we want and/or need to train?

On the way to finding answers to these demands, researchers such as Sousa (2018), Panossian, Moretti, Souza (2017), Alves and Silva (2023), Moura (2017), Radford (2011), D'Ambrósio (1993), Silva and Silvestre (2022), among others, raise questions in their studies about the roles of the historicity of mathematical concepts in the training of Mathematics teachers.

To this defense, we add the perception that such a proposal could lead to another organization of pedagogical activity[4]. For this, it was necessary to have

[3] The MathClub proposal was created in 1999 within the scope of the Universidade São Paulo, USP-SP-Brazil and is currently also found in the following Brazilian Higher Education Institutions: Federal University of Santa Maria (UFSM); Federal University of Goiás (UFG); State University of Goiás (UEG); Federal University of Rio Grande do Norte (UFRN); Federal Institute of Education, Science and Technology of Espírito Santo (IFES); Federal University of Uberlândia (UFU); Federal University of São Paulo (Unifesp). All of these are Brazilian public higher education institutions.

[4] The teaching action that unifies teaching and learning (Moura, 2017).

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training contexts that are based on the idea that mathematical concepts are historically produced by humanity and are, in most cases, the product of the explanation of human life.

In this way, they can be understood based on the constitution of\nman's social practice while life itself develops and becomes more complex, allowing subjects to acquire social and cultural conditions of thinking, in addition to theorizing about this social practice, from its constituent objects and phenomena.

From this perspective, valuing the historical process of genesis and development of mathematical concepts conceives them as part of the product of the work of subjects who are historically constituted, no longer seeing them as contemplative decoding of concrete reality.

This condition highlights another postulate: the understanding and use of such a historical movement of elaboration and subsequent increase of mathematical concepts has shown to be a potential possibility for the development of teacher training activities in a special context: the Mathematics Club[5].

Thus, this context aims to overcome the training of Mathematics teachers in training, a training that has had its focus, in recent decades, exclusively on the contents of the curriculum and on the assessment model that is a copy of exhaustively repeated exercises (Cochran-Smith, Feiman-Nemser & Mcintyre, 2008; Nacarato & Fiorentini, 2005). Formed based on this model, the teacher will hardly understand the Vygotskinian premise that it is not important to teach a quantity of knowledge as it is to instill the ability to acquire such knowledge and make use of it (Vygotski, 1997).

At the Club, the search for understanding the process of emergence and development of mathematical concepts is valued. Such action is based on man's quest to understand and explain the reality that surrounds him, and in the meantime, he creates and acquires knowledge and, therefore, develops.

To this end, thought follows from what already exists, but without being limited to it, to arrive at something that did not exist (Kopnin, 1978). Thus, the path requires a creative activity from the subject, and also rules and laws that govern objective reality.

In this sense, man lacks new concepts that expand his possibilities of mastering the reality that surrounds him. Only in this way, by going beyond the limits of current schemes, does humanity create concepts that allow it to make choices to solve its problems and, thus, thought will not be directed in a rigid manner, but with a certain freedom (Kopnin, 1978).

From this perspective, when understanding mathematical concepts from the historical movement of their emergence and establishment until today, we seek to understand them as a form of human thought that reflects the essence of human needs experienced throughout life (Kopnin, 1978).

Therefore, the training proposal offered by the Mathematics Club seeks to plan and develop teaching activities that can capture the process of emergence and development of mathematical concepts in a way that not only "[...] photographs the real historical process with all the its causalities, zigzags and deviations" (Kopnin, 1978, p. 184), but that can reflect the historical in theoretical form and, thus, function as a necessary means to know, as well as interpret the historical process of the genesis of this concept (Kopnin, 1978).

However, in order to understand didactically how such actions could be carried out, a methodological path was necessary to anchor this proposal. In the next topic, this trajectory will be discussed and how the choices were made and materialized.

III. Methodological Path: The Formative Experiment as an Option

The Mathematics teacher training model based on the ideas defended by the Mathematics Club shows that learning is a social process, therefore the interaction between subjects plays a prominent role in its development.

In this way, this space is a training environment where all actions are carried out collaboratively, constituting an environment where teaching learning is internalized in such a way that collective actions precede individual ones, constituting the source of origin of the training process.

To this end, this space was organized at the State University of Goiás – Brazil, (starting in 2017 and continuing until the present year) with the participation of undergraduates from the Mathematics course at the Brazilian Higher Education Institution that houses CluMat, with postgraduate graduates of the Postgraduate Program in Environment and Society at the same University, graduates of that same course and also teachers currently teaching in public schools in the State of Goiás and the Municipality of Quirinópolis.

In this environment, it was decided to develop a formative experiment, which according to Silva (2018, p. 48), supported by Davidov and Markova (1987), is "an investigative structure carried out in several stages,

[5] CluMat -UEG has an agreement (nº 03/2021/SME-UEG) with the Municipal Department of Education of the Brazilian city of Quirinópolis, in the State of Goiás and is part of the research project "Pedagogical Activity in the training of teachers who teach Mathematics from of partnerships between higher education institutions and basic education schools in different Brazilian regions", project financed by the National Council for Scientific and Technological Development via Chama Universal nº 18/2021. The project also receives public funds for its development from FAPEG – Research Support Foundation in the State of Goiás (period: years 2023-2025).

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in which the execution process, as well as Obtaining data, presenting objects to subjects, recording and analyzing the subject's thought movement occur simultaneously".

According to Davydov (1988), among the objectives of the formative experiment is the understanding of the relationships between certain aspects of an individual's learning process, as well as the creation of possibilities to find the reasons that gave the conditions for the need to learn to arise.

In this way, the experiment allows highlighting the laws of the domain of reality of the object being researched. Therefore, for Freitas and Libâneo (2022), the formative experiment would be the "process of identifying, understanding and explaining the historical genesis of human psychic functions in concrete conditions, revealing the movement of their emergence and transformation in social relations", focusing on "individual's activity during the researcher's active intervention focusing on psychic processes in formation" (Freitas & Libâneo, 2022, p. 6).

Didactically, the experiment was organized into planning, re-elaboration and collective evaluation meetings, which took place weekly and lasted 5 hours. Teachers in training who became research subjects ranged from 25 to 30 each year. The development takes place in public schools[6] in the city where the Club's headquarters[7] are located. The activities are developed with students in the final years of Elementary School I and all grades of Elementary School II[8]. The number of students varies on average from 28 to 35 students per class. All moments of the training experiment were recorded in audiovisual form, later, these recordings were transcribed and became the universe of data for subsequent analysis.

Therefore, with the aim of establishing itself as a learning space for Mathematics teaching linked to the teaching of mathematical concepts, the Club's activities have a very peculiar organization which is shown in the table 1 below.

Frame 1: Organizational structure of the Mathematics Club's actions.

Organizational structure of the Mathematics Club's actions			
Collaborative planning	**Shared development with schools**	**The collective assessment**	**Activities that have already been planned and developed**
These moments are intended for the collective organization of pedagogical intervention actions that will make up the teaching activities on the mathematical concepts covered. This action aims to create a collaborative space where activity planning takes place. At this time, historical studies are also carried out to construct the process of emergence and development of each concept based on the history of mathematics.	These actions take place when students go to the Club headquarters at the University. The duration of these moments depends on the need for time to develop the activities and the number of students each room has, and can vary from approximately 2 classes per week for a period of one month to the duration of a four-month period per activity. The development of these activities is carried out by all club members.	Parallel to the development of activities with students, weekly meetings take place where this development is evaluated and, when necessary, some aspects of the activity are reworked. In this way, actions are adjusted along the way and not at the end. These meetings are also moments of collective analysis and synthesis of what has been developed and the understanding of the teacher training process that takes place concomitantly with the learning of the school's students.	5 groups of activities have already been planned and developed: 1- About the arithmetic concept of number in which a comic book called 'the Agnuns[9]' was created. 2- Another on the geometric concept of polygons, using the development of videos[10] and paintings on canvas. 3- The third group of activities addressed the algebraic concept of equations and involved planning a story, which has the title: Mendhi[11], a vizier in Luxiar. This story was presented to the students as a narrative, and the illustration was made by the club members. 4- The fourth activity is about organizing the teaching of the algebraic concept of function and is also presented as a story that has ancient Egypt as its historical setting and as its title: Mitami's journey. 5- Finally, there is a group of activities that focus on teaching the geometric concepts of area and perimeter and use games as a trigger for learning.

Source: Prepared by the authors.

[6] There are 8 municipal schools and 5 state schools.
[7] MathClub has a physical headquarters located within the University Campus. It consists of an auditorium, 3 classrooms, reception, bathrooms and kitchen. The headquarters covers an area of 300m² and is where all actions are carried out with teachers in training and school students, who visit this space weekly.
[8] On June 6, 2006, the President of the Republic of Brazil sanctioned Law No. 11,274, which regulates 9-year elementary education. It is divided into two: Elementary School I which covers the 1st to 5th year and Elementary School II which covers the 6th to 9th year.
[9] The Agnuns (ISBN – 978-65-00-2-312-7); CDD: 741.5; CID: 21.61527.
[10] The videos can be found at the following link: https://clumatuegquirinopolis.com
[11] The full story can be found in the book organized and published by MathClub (Silva, 2022).

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After choosing the mathematical concepts to be taught, the historical synthesis of the concept is carried out, at which point the History of Mathematics seeks to reconstruct the main moments of the genesis and development process up to the current moment of the aforementioned concepts. Based on this synthesis, Learning Triggering[12] Situations (STL) are created, with the aim of allowing subjects to go through the main human actions that led our species to the creation of the mathematical concepts in question. In this way, it is believed to be impossible to reconstruct history, but it is possible to retrace the logical path taken by man and this means, according to Sousa, Panossian and Cedro (2014, p. 11) that "the history of the concept must be seen not as illustrator of what should be taught. It is the true guide of educational activities."

In this sense, the methodological structure chosen by the Club is aimed at the construction of teaching activities that have in their essential features the understanding of thinking in concepts.

This is, in Vygotski's conception, the most appropriate way to imagine "reality because it penetrates the internal essence of objects, since their nature is not revealed in the direct contemplation of one or another object in isolation, but in its development linked to the rest of reality" (Vygotski, 1997, p. 79).

From this perspective, CluMat is configured for teachers in training in the "environment for the development of educational activities that enable the discussion of the most varied aspects within the educational environment" (CEDRO, 2004, p.52) and, also, as a context the realization of learning by subjects guided by the intentional action of those who teach. See in the analysis that is presented later, what were the contributions and understandings of the research subjects.

IV. Data Analysis in Focus: How Data Reveals the Phenomenon Studied

In a scientific investigation, the production of knowledge only makes sense as long as its function is to reveal reality in its contradiction, in counterpoint and in the separation of appearance and essence, of what is not important, of what is fundamental. Therefore, only through this process can their internal connection and, with this, their particular character be shown. "In this process, the secondary is not left aside as unreal or less real, but reveals its phenomenal or secondary character through the demonstration of its truth in the essence of the thing" (Kosik, 1969, p. 18).

In line with this understanding, the concept of unity proposed by Vygotski (1998) was used for the analysis, which states that it is "the result of the analysis which, unlike the elements, enjoys all the fundamental properties characteristic of the set and constitutes a living and indivisible part of the totality (Vygotsky, 1998, p. 20). Likewise, in the necessary search for the singularities that make up the universality of the phenomenon investigated, we looked for episodes, which would be, according to Moura (2004, p. 267), the moment that "can reveal interdependence between the elements of a formative action."

According to this author, episodes would be situations in which regularities in the movement of the analyzed phenomenon can be highlighted, that is, circumstances that can be emphasized in the movement of the investigated process.

In the continuation of the explanation of the proposed data analysis structure, there are flashes. For Silva (2018, p. 150), they would be the parts of the episodes that would configure "the signs of the conscious and internalized reflection of reality", which "have embodied, implicitly, the motives and needs, the meaning and meaning that are they express themselves in language, but, above all, they are not reduced to it".

Therefore, the concept of flashes would be in line with the data analysis structure proposed by Vygotsky and Moura. This proposal was initially made in Silva (2014), with the flashes understood as observable signs that would prove the existence of the process of composing the meaning of the subject.

The search for these signs would not only be to prove facts that demonstrate the existence of this process, but to reveal the dynamics of the movement of its constitution.

The flashes found in the episodes "would not just be a mere definition of the signs, but rather an attempt to find in their plot not only the existence, but also the nature of the meaning process of the subjects involved" (Silva, 2018, p. 150).

From this procedural movement, the following composition of the analysis presented in frame 2 arises.

Frame 2: Composition of data analysis

Unit: The mathematics club as a training space for mathematics teachers	
Episode 1: The particularities of the club and its formative contributions	*Episode 2:* Understanding the importance of understanding the historical process of emergence and development of mathematical concepts

Source: Prepared by the authors.

[12] The STL is a proposal for teaching mathematical concepts. This organization allows the subjects involved to appropriate mathematical concepts based on the understanding of human experiences throughout their history. The teaching activities developed based on the SDA proposition have problem situations that carry within them the essence of the need that led humanity to create the concept to be taught, that is, it has its essence (Moura, 2017).

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In the search for understanding the phenomenon, the analysis unfolds in the episodes that follow and in the flashes that compose them.

Episode 1: The particularities of the club and its formative contributions

The Mathematics Club is thought of as a formative space in which the individual and mathematical knowledge are in uninterrupted and dialectical formation, developing through internal contradiction, not in a deterministic way, but through the sharing of subjects. Thus, "by providing the conditions for students to act collaboratively in conducting collectively planned activities, we are putting into practice a training vision that understands that the development of school education must necessarily be a shared process" (Moura, 2021, p 14).

This means facing conflicts and the courage to complement and be completed in others, giving up the feeling of individuality over appropriate knowledge, as well as allowing oneself to intertwine with the whole (Petrovski, 1986). "In this sense, subjects work together, negotiate and make decisions as a group, invariably dialogue on a basis of coherent equality, in which learning can be accessible to everyone" (Silva & Cedro, 2022, p. 100). The following flashes corroborate the discussion about this characteristic of the Club and how it contributed to the teaching training of its participants: "In my opinion, the biggest difference between the club and the reality we experience in our undergraduate degree is that in the course we are encouraged to be individualists. , doing everything alone and here it is exactly the opposite, we are encouraged to do everything together, we plan and go to school to develop the activities and then come back here to re-elaborate, all together" (Ana[13], Flash 1, Episode 1); "Here at the Club we work together and each person's opinion has value, we are equal when it comes to giving our opinion, each person's opinion is heard, this way learning has another meaning" (Laura, Flash 2, Episode 1).

In this way, the training actions undertaken at CluMat seek to overcome the contradiction between individuality and the condition of being generic, realizing that it becomes a necessity to relate to other individuals, because of what is human about them. "The humanity of the other becomes a necessity of the humanity of each person" (Marx, 1985, p. 142). In spaces with such characteristics, individuals tend to internalize the theoretical and practical proposals experienced, that is, they change their singularities to understand the reality that surrounds them (Yachele, 1990).

Davidov (1998) also suggests that subjects who experience collective spaces tend to share mental models as a product of shared work when they participate in learning spaces in which collaboration is a present element. The author argues that in these places subjects reach the same meanings and representations, which are probably more complex than if they had been formed individually. Thus, "[...] it is not only important how the mathematics club is organized, but rather the possibility that it embodies of being a process of formative intervention capable of making one think about the training of mathematics teachers, the starting from a reference that does not take the individual as the unit of analysis, but rather the process experienced by him" (Silva & Cedro, 2022, p. 110).

In this context, emphasis is placed on the understanding that teaching and learning actions, which permeate pedagogical activity, are always subject to shared conditions (Jaworski, 2004). Therefore, the planning and development of pedagogical activity requires an environment that can handle such situations. Therefore, training spaces must allow an intentional and collaborative movement, so that conducive means are created to discuss the need for teaching and learning to form a single interdependent process.

Thus, spaces such as the Mathematics Club must exist because there is a need for a training locus in which there are not simple teaching practices, but rather a place in which it can be observed that, in carrying out teaching, there is an interdependence between the subject and the object of knowledge, which in this case is represented by mathematical concepts. Next, there are flashes that show signs of the establishment of such understandings by teachers: "The way we learn to be a teacher here makes it clear that the subject who teaches has to know what he teaches and this is not a simple process, but it's not impossible" (Junior, Flash 3, Episode 1); "It's not really simple, but what a teacher does is not a simple thing, it's complex, which is teaching, teaching someone, so whoever is learning to be a teacher has to understand that this whole process of teaching mathematical concepts has to be learned in a space organized for this and not in any way as happens in most places" (João, Flash 4, Episode 1).

In spaces like CluMat, teacher training is understood as a socio-historical practice, modified by the actions of individuals and determining the changes of those who share it. In this sense, it is essential that the teacher in training is led to distinguish that, alongside the outstanding specialties of the educational phenomenon, there is a process of subjective modification. This process not only changes the reproductions of those involved, but determines a change in the interpretation of it as a whole, which will determine a reorganization in the actions of the pedagogical activity.

Furthermore, the pedagogical activity planned and developed by and in this space is, at every moment, an expression of the moment and current situations and also temporary syntheses that constitute

[13] The names of the subjects are fictitious to protect their identity.

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the process of organizing the teaching and learning of mathematical concepts. Below we have flashes that confirm this discussion: “I believe that many of us only understood what pedagogical activity was after coming to the club, that it is much more than just going to school and teaching” (Joana, Flash 5, Episode 1); “I think we are understanding what it is or reorganizing what we already knew about what pedagogical activity was and as this changes, our ideas and then our actions for when we are teachers will also change” (Pedro, Flash 6, Episode 1).

Most teacher training contexts are constructed as suitable places for the legitimization of dominant socio-political interests. Among these, we have the degrees offered by universities. They are privileged training sites, becoming the result of fractional practice, expressing this fragmentation through their contents, methods and forms of organization of teaching learning and the teaching of mathematical concepts. Following the data analysis, the following flashes shed light on the understanding of the discussions held: “The way in which the course we take is organized completely separates theory from practice and that is why our mathematics classes in high school were the same way, because These teachers were trained in this model that is still valid today and if it weren't for the opportunity experienced at the club, we would only have this model to see and then reproduce, without any opportunity to transform teaching practice” (Maria, Flash 7, Episode 1).

In this sense, the Club materializes the product of the teacher's work as an effective achievement for the subject, who realizes the possibility of transforming his activity into a social object. But the relationship between individuals and the products of human activity cannot be radically transformed if the same transformation does not occur in the relationship between the subject and his own activity, both with the products that already exist in culture and with the products originated by individual activity.

Therefore, the objectification that the individual achieves through his activity comes as a process, in which his individuality becomes a social object, which fulfills the individual and enriches other subjects through the sharing of everything that is achieved. Therefore, in the Club, understood as this training space that allows pedagogical activity to bring together teaching and learning, the aim is to overcome the production relations that direct such activity as alienated, which condemns the individual to see generic life only as means for survival.

This entire process is based on the understanding that pedagogical activity brings in its essence a load of intentionality, which unifies and equips the theory-practice relationship in a dynamic-historical way, in such a way that the characteristics of the sociocultural context, as well as the needs and probabilities of the moment go beyond theoretical perceptions and awareness of habitual acts.

This search leads teachers in training to demonstrate the assumption of the need to seek planning for their pedagogical activity (Antúnez, Imbérnon, Parcerisa & Zabala, 2000). This planning comes to be seen as an interface between theory and practice, which must be permeated by sharing and which not only produces a materialized world, but also allows others to see, understand and appropriate what has been produced. In this way, you will be able to form and transform the material world. The following flashes show signs of the movement of appropriation of the Club's formative contributions to the research subjects: “Another important thing that participating in the club taught us is the role of planning, how important it is that we have done it before going to the classroom and that it can and should also be redone when our objective is not achieved” (Beatriz, Flash 8, Episode 1); “What we normally knew as planning was something very superficial, which had little relation to the theory and practice needed to teach, it was actually a very superficial perception” (Lucas, Flash 9, Episode 1); “Speaking of theory and practice, I have always seen them as completely separate processes because that is how it is shown at school and in the degree course, they are seen as unconnected sides of the pedagogical activity, but here they are seen and taught as linked and with the conditions to ensure that when we are teachers, we can change the way we organize teaching and allow ourselves and the students at school to learn, and in this, planning is fundamental” (Luis, Flash 10, Episode 1).

During the analysis of this first episode, the trainee teachers highlighted the particularities of the Mathematics Club and its training contributions. Among these, the following were highlighted: the sharing of actions, the importance of organizing teaching so that learning occurs, the understanding of what the pedagogical activity would be, the necessary unity between theory and practice and the appreciation of planning teaching actions.

In this way, the analysis demonstrates that the Mathematics Club represents a space endowed with a unique conception of teaching mathematical contents and, at the same time, presents a particular way of organizing the way in which they will be presented to teachers in training and, later, to basic education students, in a connected teaching learning process (Arends, 2007, Chamorro, 2004).

This perspective is supported by Davidov's (1996) defense for establishing the condition that one must organize and produce environments which produce “an adequate activity” so that the individual “is placed in an adequate relationship with reality” (Davidov, 1996, p. 447).

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The Mathematics Club is that space.

On the way to achieving this objective at Clumat, we seek to understand the historical process of emergence and development of mathematical concepts. It is exactly the understanding of the importance of this movement that will be highlighted in the flashes of the next episode.

Episode 2: Understanding the importance of understanding the historical process of emergence and development of mathematical concepts

Arcavi (1991), D'Ambrósio (1992), Sousa (2018) and Radford (2011) highlight that understanding the process of emergence and development of mathematical concepts from the history of Mathematics can enable improvements in the training process of Mathematics teachers and, consequently, for teaching mathematical concepts in schools. It can also serve as an instrument for demystifying the process of preparing them until the current moment in which they are presented as school content.

The proposal for organizing the teaching of mathematical concepts offered during the teaching learning process that takes place at the Mathematics Club has made it possible to understand this movement where mathematical concepts concomitantly with the process of humanization of man are being constituted. This conception goes against the idea of seeing them as a body of knowledge given a priori to humanity, as a simple set of techniques for solving random problems.

Thus, as the training experiment developed, the teachers in training showed signs of understanding that throughout their lives as students at school they were deprived of Mathematics teaching that linked the historiographies of the contents and their teaching. They also came to understand that this could have allowed them a different line of interpretation, allowing them to approach the same mathematical object from another perspective and, thus, contributing to their better understanding. See the flashes: "During all the time I was a student, never in math classes did any teacher tell me how that content had come about" (Liz, Flash 1, Episode 2); I always had the impression that they created mathematics just to complicate life and not the other way around, that it was to solve life's problems, that these concepts were responses to human needs" (Antônio, Flash 2, Episode 2); "Having the opportunity to know this here at the club means knowing that when I become a teacher I will be able to offer this to my students, that they will be able to have an opportunity that I didn't have, which was knowing why and for what mathematical concepts were really created" (Cris, Flash 3, Episode 2).

Throughout the training experiment, where the planning, development and re-elaboration of activities for teaching mathematical concepts took place, the teachers in training began to perceive the emergence of mathematical concepts from the movement of appropriation of the very human cultural production that is offered to them. by the historiographies present in the history of Mathematics and which are studied by them. In this process, they began to perceive historical man as an active subject who organized his actions and selected instruments that allowed him to objectify the reasons for his activities so that they could provide answers to the most pressing needs and, little by little, in the course of his history, he gave rise to mathematical concepts.

In this movement, teachers move towards the importance of understanding the historical process of emergence and development of mathematical concepts for their teaching training, from a training space that allows the occurrence of "a process of internalization of social meanings in activity and with attribution of personal meaning, which characterizes the dialectical unity between sense and meaning in the constitution of the psyche" (Moretti, 2014, p. 33).

Understanding this process impacted the training process of these Mathematics teachers, since learning causes the appropriation and attribution of personal meanings (Vygotski, 2003). Among these is the particular meaning that pedagogical activity is taking, as one capable and responsible for transfiguring concepts into school content and, in the case of the Mathematics Club proposal, showing them as human products, responses to their needs.

Therefore, organizing a training space like CluMat, within a degree in Mathematics at a public University, had among many contributions the possibility of creating learning conditions for the subjects by proposing them teaching activities that put them in front of the knowledge of human need that generated a given mathematical concept.

This proposed structure is not understood as an exercise in applying mathematical concepts previously presented by the teacher based on clippings of historical facts. This organization presupposed a first approach for the teacher in training with the theoretical basis that would support teaching activities, so that he could appropriate these mathematical concepts as a product of the historical construction of human culture.

They were intentionally led to develop the proposed actions collectively in order to manifest the essence of the concepts that had been previously chosen, that is, they would have to be able to highlight their internal structure and, thus, be impregnated with the need that led humanity to the construction of same. In the next flashes we see signs of these understandings: "What makes the big difference here at the Club is that we can know, learn and then teach, that the mathematical concepts, which we call content at school, were actually constructed by people throughout of the history of humanity on the planet, they were not invented by geniuses in castles" (Isabela, Flash 4,

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Episode 2); "How many people don't think that, for example, the concept of equation, function or area, were not created just to complicate our lives and now we discover when we study the history of the emergence of these concepts so that we can put them into activities that in fact is exactly what On the contrary, these concepts were invented to provide answers to problems, to some needs that existed and that we still use them today to solve problems" (Paulo, Flash 5, Episode 2).

When justifying the relevance of the training space called CluMat, a structure is defended that is capable of allowing subjects involved in the teaching and learning processes of mathematical concepts to appropriate the essence of mathematical concepts. Kopnin (1978, p. 161) defines this essence as being "[...] clues and relationships that surpass the sensorially perceptible through authentic abstraction that generalizes not only the form, but also the content of the object". Easy task? No. After all, establishing the principle of organizing training processes for Mathematics teachers that consider the essence of the mathematical concept may be more difficult than one might imagine.

However, it is not impossible. After all, it is necessary to highlight in this process the possibility that subjects become aware of the relationship between the human production of mathematical concepts and cultural needs such as controlling variations in quantities, discrete or continuous quantities, interdependence between quantities, movement of variable quantities and its regularities, among others (Ifrah, 2005; Kamii, 1989; Sousa, 2018).

In this way, during the development of the formative experiment, mathematical concepts were understood as living productions in direct relationship with human needs and historical times that produced them.

Therefore, appropriating a concept in this theoretical perspective suggests understanding it as a historical and cultural production, implying appropriating not only its formal structure, but also its mechanisms and its historical constitution, the essence of human needs that moved the human species in the trajectory of the socio-historical construction of mathematical concepts, that is, it involves seeing and apprehending it in the movement of its history, from how it emerged until it reaches the present day.

Therefore, understanding the process of producing the concept is an element of the movement of appropriating the concept itself, as shown in the subsequent flashes: "The formative process that I experience here at the Club is very different from what I experienced in the supervised internship of the previous semester, because there there was no concern that the planned activities were capable of teaching the essence of the concept, which goes beyond ready-made formulas and follow-the-model exercises" (André, Flash 6, Episode 2); "All mathematical content has a formal structure, what remains outside and is visible to the eyes, and it is not wrong to teach this, it just cannot be taught only that, teaching mathematics has to go beyond the appearance of the content and to I think that knowing how these contents emerged and developed until arriving here in the books that we will use to teach classes is necessary, without this it is not possible to understand the historical relationship between humanity and the creation of mathematics concepts" (Mel, Flash 7, Episode 2); "When we study the history of how these concepts were created and changed until they are as they are today, we come to understand that there is a connection between their emergence and the way they currently are and the way mathematics is normally learned and taught is very wrong, because it skips all of this, and then the richness of the process is lost, and it becomes meaningless for those who teach and also for those who learn" (Daniel, Flash 8, Episode 2).

In these flashes, the subjects show that in the case of training Mathematics teachers, the historical aspect of the concept has the possibility of revealing itself in the essence of the need for its human production. Moretti (2014, p. 38) highlights that by "articulating the historical aspect in the process of analysis and synthesis that aims to solve this need for the subject, one learns by internalizing the interpsychic movement of collective production of a solution to the need posed". Knowledge of this essence of the concept – in school prefigured in content – is only possible when knowledge of the historicity of the concept is valued.

According to Kopnin (1978), this only takes place with the necessary articulation between the historical and current aspects of the object of knowledge, which allows for a movement of conceptual appropriation that is established in the unity between the essence and the current form of materialization of the object.

Therefore, the "study of the history of the development of the object creates, in turn, the indispensable premises for a deeper understanding of its essence" which is why "enriched with the history of the object, we must once again return to the definition of its essence, correct, complete and develop the concepts that express it"; after all, only in this way "the theory of the object provides the key to the study of its history, while the study of history enriches the theory, correcting it, completing it and developing it (Kopnin, 1978, p. 186).

However, this does not mean that one should "teach Mathematics through history, nor repeat the historical path in the formation of a mathematical concept, but seek in the historical process the movement of thought in the context of the formation of this concept" (Silvestre & Silva, 2019, p. 4).

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The authors warn, anchored in Arcavi (1991) and Sousa (2004), that it is not about reinforcing the prevalence of history, nor the foundations of Mathematics over Mathematics itself and its applications, but rather privileging the elements that enable the construction of mathematical concept to be taught. Therefore, we are looking for a way to help outline a path that leads to the appropriation of the concept by the subject, as done in this special space for training Mathematics teachers.

V. Final Considerations

The proposal for teaching mathematics teachers presented in this article was only realized because it started from the understanding of mathematical concepts in a historical dimension.

In the Mathematics Club, concepts are assumed to be social productions that respond, in a given historical time, to human needs. Such theoretical and methodological structure is anchored among other authors in the defense made by Kopnin (1978), highlighting that appropriating a certain concept involves understanding it in its historical movement, its production and its appropriation by man, and in this movement it involves by understanding its essence.

In this sense, we can state that the resolution of triggering problems, which belong to situations that trigger learning, was taken, in this MathClub proposal, as a teacher training methodology. When proposing that teachers in training plan and then develop these activities with school students, it was hoped that the proposition and resolution of the problems that made up these activities would manifest the teaching and learning processes: in the actions of the subjects in training and in the training actions.

In the first, focusing on the Mathematics teacher in training in this special training space. In the activities, the problem situation or situations were taken as triggers for learning the mathematical concept in question (numbers, equation, function, area and perimeter) in a historical movement, which had as its objective learning about the organization of teaching.

If, on the one hand, the proposition of these triggering situations allowed teachers to come across the essence of the concept, on the other, it was the proposition of the problems that were within the SDA that were to be solved in a collective and collaborative way that allowed teachers to recognize shared ways of carrying out pedagogical activity as the most efficient human activity created to meet the needs of teaching.

Therefore, it was the movement of planning, developing and re-elaborating these teaching activities, which aimed to find the solution to the problems presented in each activity that allowed teachers to attribute new meanings to their actions during the teaching activity and, as consequently, reorganize them.

In a second dimension, focusing on the training perspective, the teaching learning process that took place in the Mathematics Club itself was taken as the problem that triggered its activity by motivating training actions. These actions aimed to create conditions for teachers to analyze and reevaluate their practices with a view to establishing a new teaching practice. These teachers found themselves searching for actions that responded to the needs of contributing to the school, with the teaching of Mathematics.

At the same time, this training space contributed to the construction of the autonomy of teachers in training by offering them challenges in teaching and teaching professionalism. Thus, although the training proposed in MathClub was based on the planning and development of SDAs, it is important to make it clear that the learning-triggering problems that appeared in these SDAs were not all defined a priori.

The principle of placing teachers in the face of challenges that met their needs remained throughout the years of research development. However, the problems were elaborated and re-elaborated based on the needs of the collective in learning to teach mathematical concepts based on the historical movement of their emergence and development.

In this process, the motives of the teachers in training were transformed as the actions they developed gained new meaning, and began to constitute themselves as a teaching activity for teachers. The process of re-elaborating teaching activities constitutes the problem of training action. This way of understanding the problem, in line with the needs of the subjects and their training movement in a collaborative space such as the Club, at the same time that it reinforces the importance and potential of proposing problems in teaching as a triggering element of learning, also indicates that the teaching learning evidenced in the research was not constituted from any problem, not even from the unique relationship of the subject who learns from the proposed problem.

The teaching learning that took place at the Club, manifested in the training movement analyzed, took place in the social relationship made possible by the collaborative space of shared teaching work, intentionally constituted from the training proposal based on the perspective of the importance of organizing the teaching of concepts mathematicians from their historicity. It is believed that such a structure embeds within itself the essence of the needs of the concepts to be addressed.

In this process, as important as the problems proposed in the STLs, were the mediations established by the different subjects participating in the training space in question. Everything they experienced at the Club pushed them towards training activities, giving new meanings to their actions in organizing the teaching of mathematical concepts.

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As a consequence, as the experiment unfolded, they allowed themselves to develop a new organization of teaching mathematical concepts. Thus, this movement of learning to teach Mathematics, defended by the Mathematics Club, results, on the one hand, from the planning and development of SDAs as a methodology for training and teaching mathematical concepts at school. On the other, no less important, understanding the teacher training process itself as a moving problem.

Thus, MathClub requires that the training actions developed are capable of provoking, in future Mathematics teachers, the need and desire to produce new practices and, at the same time, be capable of favoring and offering new proposals for the appropriation and organization of mathematical concepts that allow the possibility of another training space for these teachers.

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