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Wind Deficit Model in a Wind Farm Using Finite Volume Method

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Abstract— A wind deficit model for wind farms is developed in this work using finite volume method. The main question addressed here is to calculate approximately the wind speed in the vicinity of each wind turbine of a farm. The procedure followed is to solve the governing equations of flow for the whole wind farm. Spatial discretization is performed on a computational domain, using finite volume techniques. In ideal circumstances, wind speed will calculate on a fine mesh for the whole wind farm, where the effect of wind turbines is observed by means of their thrust coefficient. The end goal of this method is to assist the development of an analytical expression of an offshore wind farm wake. We focus in this work on wind deficit that can be used for better wind farm control strategies and load optimization.

I. INTRODUCTION

Recently, advanced automatic wind farm controllers has been developed because of the considerable growth of large wind farms. The wind farm controllers should intend to improve the power reference distribution among wind turbines and controls the structural loads and optimize the energy production. To address these goals, There is an compelling need for a wind farm flow model [1], [2]. An approach to the problem of modeling wind farms addressed in [3], assumes that when an area contains a large number of wind turbines, they considered as distributed roughness elements, so the ambient atmospheric flow will modify. One of the main issues in wind farm modeling is the wake interaction and the effect of velocity deficit and turbulence increase in the place of interaction [3]; though in this paper the focus will be only on the velocity deficit. Since the ambient flow of each wind turbine, is to affected by the wakes produced by upstream turbines, a good approach will be considering the flow equations for the whole wind farm instead of its part. Some of the models recently used to estimate production losses due to wake of the neighboring wind turbines are based on local momentum equations, neglecting the atmospheric interaction. Another way of wind farm modeling is to provide a detail model which considers each wind turbine dynamic and factors such as the coherence and correlation of turbulence as presented in [4]. In these models computational resources and the approximation of the behavior of the wind farm is not good enough [5]. Another option is to apply computational fluid dynamics (CFD) schemes, which used to be computational demanding; but it will be shown in this work that the elapsed time

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for computation using this method will be smaller. There is a far distance between engineering analytical methods and CFD models and a connection is needed between them to provide detailed information for better wind farm and turbine design and for more efficient control strategies [6]. One version of the wind flow model in a farm was based on quasi-steady wake deficits computed in a loop involving a coupling between an external CFD actuator disc model of the rotor-wake interaction and an aeroelastic model. The flow problem solved using a finite difference scheme, with initial conditions defined by the downstream wake deficit [7]. Using finite volume methods instead of finite difference is preferred, as in here. Since, finite volume methods are basically a generalization of the finite difference method but use the integration of the governing equations of flow, instead of their differential form. This provides a better flexibility, especially in modeling complex domains [8].

This paper is one of the few research efforts demonstrating a FVM to simulate the flow over a large wind farm [9]. The basic idea of the method presented here is to calculate approximately the wind speed in the vicinity of each wind turbine in a farm. Spatial discretization is performed on a computational domain, using finite volume techniques. The procedure followed is to divide the wind farm into disjoint cells by lines that define the boundary of each individual cell; the created pattern is called mesh. The key element is to solve flow equation for the whole wind farm, where wind turbines are modeled by means of their thrust coefficient (actuator disk approach). The problem is solved for both steady and unsteady flow, which the major difference is how to specify the wind speed correctly at the computational inlet and boundaries (that are taken as symmetric conditions). The end goal of this method is to assist the development of an analytical expression of an offshore wind farm wake, nevertheless the turbulent effect has not been considered. This work provides almost a full knowledge of what is happening downstream wind turbines, therefore it can be useful for estimation of fatigue loads or annual energy production.

II. FINITE VOLUME METHOD (FVM)

Most studies on wakes have made a division between the near wake and far wake regions. The near wake is taken as the region just behind the rotor, where the effect of the rotor is considerable [10]. In the near wake region there is an intense turbulence generated by the blades, shear, and the decrease of tip vortices. The far wake is the region further than the near wake. One of the basic objectives of modeling in the far wake region is mentioned to be the evaluation of

the wind turbines effect on each other in farms [11]. Our approach is considered to be in this region.

In far wake region, mostly the ambient shear flow is being neglected; and the profile of velocity deficit is assumed to be axis-symmetric. The only characteristic of the turbines that is being applied in this profile as a parameter is the thrust coefficient of the turbine [3], [12]. Hence, in the flow equation for the whole wind farm, which is explained with Navier-Stokes equations some terms can be neglected. At first, the flow is considered to be steady axis-symmetric and gradients of the standard deviation, turbulent and viscous terms are neglected. Afterwards, the steady assumption will revoke and the time dependent term will be added to the flow equation.

In the first case flow can be described with the two dimensional Reynolds equation in the thin shear layer approximation without viscous terms in Cartesian co-ordinates. This equation is combined with the continuity equation to form the differential equation system, which is solved by FVM to model the wake flow.

$$\frac{\partial}{\partial x}(\Gamma \frac{\partial U}{\partial x}) + \frac{\partial}{\partial y}(\Gamma \frac{\partial U}{\partial y}) + S = 0 \qquad (1)$$

Where U is the mean wind speed, Γ is the diffusion coefficient, and S is the source term. We seek the solution U(x, y) subject to appropriate conditions on the boundary.

In the second case, the acceleration term is added to the flow equation [13].

$$\frac{\partial}{\partial x}(\Gamma \frac{\partial U}{\partial x}) + \frac{\partial}{\partial y}(\Gamma \frac{\partial U}{\partial y}) + S = \rho \frac{\partial U}{\partial t} \qquad (2)$$

Where ρ is the air density and we seek the solution U(x, y, t).

In order to solve the problem in each case, first step is to define a finite volume grid on a rectangular domain. Wind farm is divided into non-overlapping control volumes by the lines that define the boundaries of the individual control volumes. The pattern created by the lines is called the computational grid or mesh. In Fig. 1 and 2 the control volumes are square, but this is not a requirement. Two sets of grid lines can be identified: the grid lines that define the control volume faces, and the grid lines (dashed line) that define the locations of the nodes. The finite volume method is used to transform (1) and (2) to a system of discrete equations for the nodal values of U. First, (1) and (2) are integrated over the typical control volume depicted in Fig. 2. This reduces each equation to one involving only first derivatives in space. Then these first derivatives are replaced with central difference approximations [13].
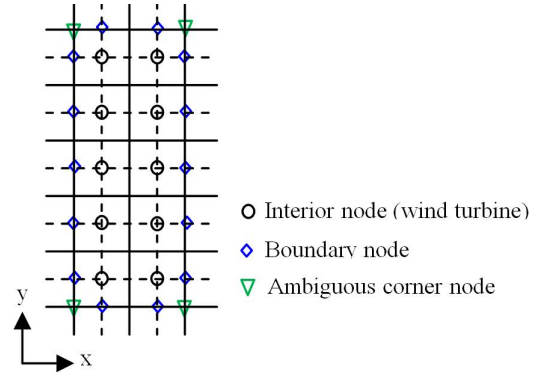

Fig. 1. Computational grid

○ Interior node (wind turbine)
◇ Boundary node
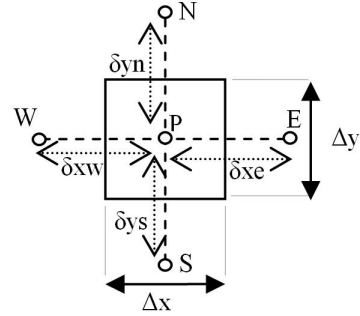▽ Ambiguous corner node

Fig. 2. A control volume

The control volume integration forms the key step of the finite volume method, for each case yields the equations (3) and (4) respectively.

$$\int_{cv} \left[\frac{\partial}{\partial x}(\Gamma \frac{\partial U}{\partial x}) + \frac{\partial}{\partial y}(\Gamma \frac{\partial U}{\partial y}) \right] dv + \int_{cv} S dv = 0 \qquad (3)$$

$$\int_{\Delta t} \int_{cv} \left[\frac{\partial}{\partial x}(\Gamma \frac{\partial U}{\partial x}) + \frac{\partial}{\partial y}(\Gamma \frac{\partial U}{\partial y}) \right] dv dt + \int_{\Delta t} \int_{cv} S dv dt = \int_{\Delta t} \frac{\partial}{\partial t} \left(\int_{cv} \rho U dv \right) dt \qquad (4)$$

Wind turbines are modeled as momentum absorbers by means of their thrust coefficient (actuator disk approach). An algorithm for velocity updating is applied, which maintains the compatibility of the velocity and thrust coefficient based on following assumption and equations. With respect to wind speed in any second, which is constant just for the first case, an average constant thrust coefficient, C_T, can be obtained using thrust curve for each wind turbine. The source term is proportional to wind pressure, that should be updated in

computation loop for each turbine in the farm:

$$a = \frac{1}{2}(1 + \sqrt{1 - C_T}) \qquad (5)$$

$$V_{-\infty} = (1 - 2a)V \qquad (6)$$

$$P_D^+ - P_D^- = \frac{1}{2}\rho(V^2 - V_{-\infty}^2) \qquad (7)$$

In which a is defined as the axial flow interference factor, P_D^+ and P_D^- are the air pressure immediately before and after the disc. As it has been shown in Fig. 3 the air that passes through the disc undergoes a speed drop $V^2 - V_{-\infty}^2$ [14].
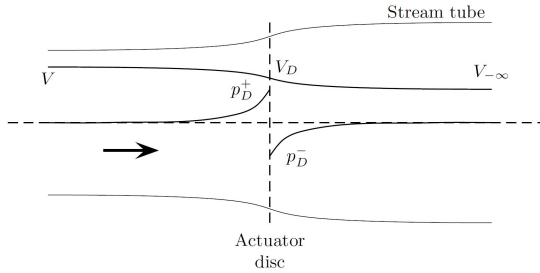

Fig. 3.   Wind speed and pressure in actuator disc model [14]

III. SIMULATION AND VALIDATION

Validation investigates if the conceptual and computational models as applied into the CFD code and calculative simulations concur with real world observations. The purpose is to error and uncertainty identification by comparing the simulation results with measurement data [15]. In order to validate the method for both cases, simulation has been done for a wind farm with five wind turbines that are located in a line. The situations of wind turbines in the farm and wind direction are shown in Fig. 4, which is one of the cases discussed in [16]. In [16] the wind speed is calculated by an analytical method and the calculation results data and also the measurement data for that situation are available; and are compared to simulation results of the first case FVM in Fig. 6.
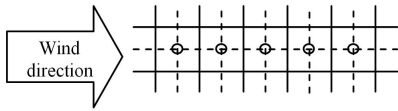

Fig. 4.   Simple farm simulation regime

As in Fig. 6 the simulation results based on current method is in an acceptable range, but in comparison to measurement
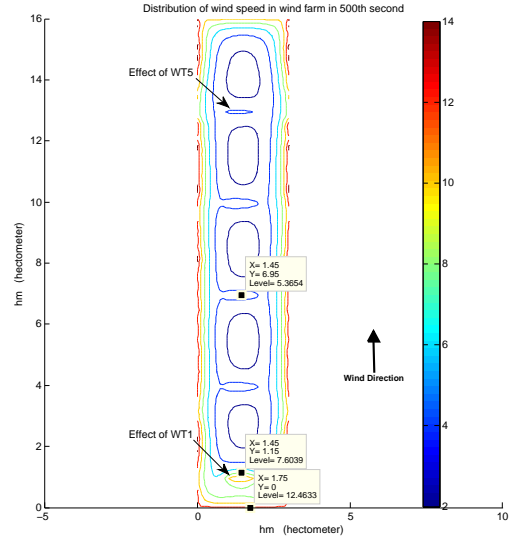

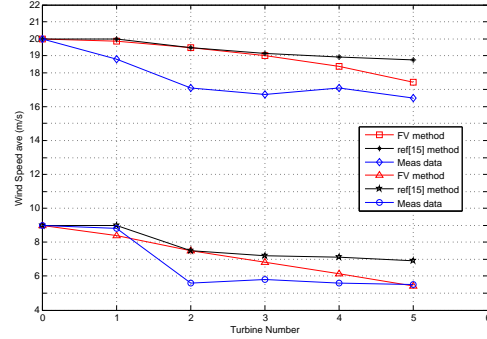Fig. 5.   Small-scale wind farm simulation

Fig. 6.   FVM simulation results in compare with [16] calculation results and measurement data

data there are some uncertainties. The second case is in fact a dynamic model, which is derived by adding the time dependent part to the flow equation. Simulation is performed for a time interval using a meteorological mast measurements as input in every second.

The simulation is performed for a bounded area at hub height, which the boundary conditions are assumed to be symmetric and their updating is with respect to a mast measurements. Fig. 7 shows wind speed in any second before the wind farm in comparison to wind speed calculated behind the second turbine, for instance. As it can be seen in this picture, the shape of the calculated wind speed behind the second turbine is almost similar to measurement wind speed; though behind next turbines the variations in the shape are

larger. The time shift between the calculated wind speed and measurement data is because of the distance between them (The distance between each wind turbine is about 300m). In order to validate the second case calculations, they
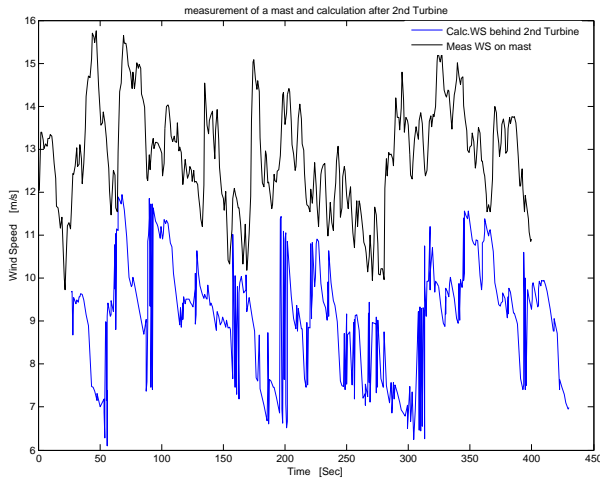

Fig. 7.   Wind speed measured on the mast before wind farm, and calculated wind speed behind the second turbine

are compared to some measurements available by mounted instruments on each wind turbine nacelle. Some results are shown in Fig. 8 and 9; in each figure the red plot shows the absolute error, omitting a part of the bias error. As it can be seen in most of the time the absolute error is less than 2m/s, which is still a considerable error. Some of the sources of uncertainties and errors are due to:

1) The measurements are referenced to nacelle of the turbine but calculations are supposed to be in the far wake region, which there is no measurement available in that region. The model will be evaluated and adjusted with far wake region references in the near future.
2) Thrust curves and thrust coefficients are not accurate.
3) Turbulent flow model: turbulence characteristics of the wind flow are not considered in this work.
4) Selected mesh grid is one of the most important error sources. The selected mesh in here, to resolve the model, may not be the best selection, and using finer meshes the answer may be improved.

IV. LARGE SCALE SIMULATION

The following section links the small-scale (previous section) and large scale features of the flow in wind farms. With the same assumptions as before, wind flow is introduced by (1). As is often can be seen in large wind farms, the configuration is straight rows of wind turbines and equal
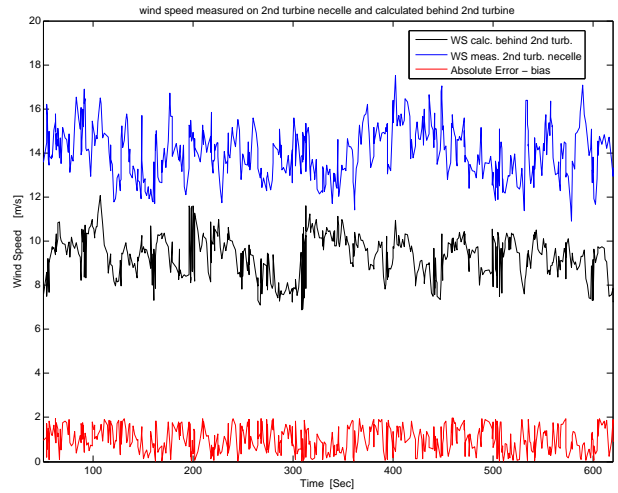

Fig. 8.   Wind speed measured on the 2nd turbine nacelle and calculated behind the 2nd turbine, and the absolute error
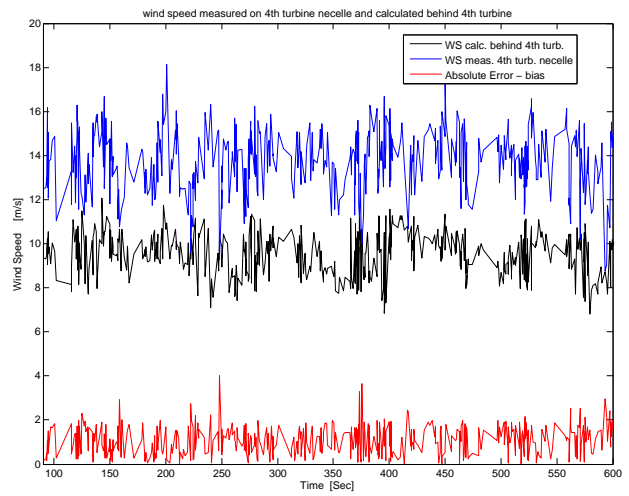

Fig. 9.   Wind speed measured on the 4th turbine nacelle and calculated behind the 4th turbine, and the absolute error

distance between them in each row and equal distance between rows [4]. In the considered regime in here,the flow direction is parallel to rows; considering such a model allows fast computation. Case study includes 15 wind turbines, but for larger farms the procedure would be similar. Simulation has been done for the steady flow (first case) and for the mentioned regime shown in Fig. 10. Simulation results shown in Fig. 11 depict the wind direction propagation.

As it is shown in tables I the computation time is small. In the table "setup" time is to define the mesh, source terms, and

boundary conditions. "solve" time is to compute the finite volume coefficients and "post-process" time is elapsed to update and at the end plot the solution.
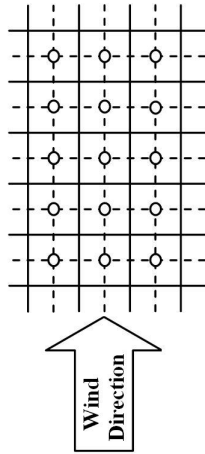

Fig. 10. Larger wind farm simulation in 1 wind direction

For the unsteady flow (the second case), the approach would be exactly the same; but it seems that for this case in large scale simulation, some tendencies can be ascribed to the effect of meandering, which should be considered in the future researches.

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

In this paper a wind deficit model is developed for wind farms using finite volume method considering steady wind flow; then a dynamic model is devised by adding the time dependent part to the flow equation. The core is to calculate approximately the wind speed after each wind turbine of a farm solving the governing equations of flow for the whole wind farm. The scheme is based on finite volume discretization and calculation such that in compare to other CFD methods, the calculation time is small. The present model can be used for the purposes of verification of other analytical models and also for more sophisticated control strategies and load calculations.
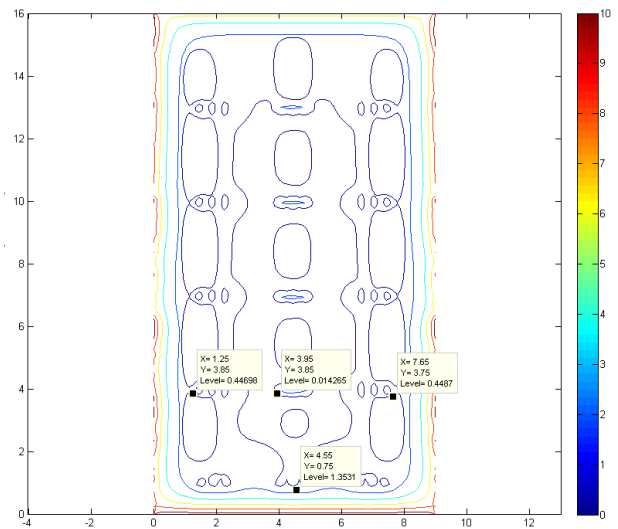

Fig. 11. Larger wind farm simulation (scale and wind direction are the same as Fig. 5)

B. Shortcomings and method limitations

The turbulence effect has not been considered in this paper. In practice, the turbulence in the wind farm can be connected to four sources: (a) The ambient atmospheric turbulence. (b) Production of turbulence by the shear forces. (c) The turbulence generated by the blades. (d) The turbulent kinetic energy due to the temperature variations. In experiments, it is observed that the effect (b) can be neglected in far wake region. So the next work would be adding the turbulence term to flow equation at the first place, equations(1) and (2). Moreover, the meandering effect will be considered in the future research.

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