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1. Introduction

Snow on roofs gives many practical problems as extra load on the roof, sliding of the snow, icing on the roof in gutters, and generation of icicles. Sliding of snow and ice from roofs can in worst case kill people and damage property. A better understanding of the physics of snow and ice on roof can help in reducing the risk of damages. This research is supported by the research foundation at Länsförsäkringar, which is a Swedish banking and insurance alliance company.

A typical winter problem is snow and ice on roofs. This includes a number of problems that are related to building physics and heating of the house. Sloped roofs with external gutters can give problems. An example is melting of the snow on the roof and the freezing of the water on the overhang. The result is generation of an ice layer along the eaves. The ice layer can result in ice dams, so that melting water is collected behind. As the water does not freeze on the roof it will give a water pressure on the lower part of the roof. This gives a risk for water leakage into the building if the roof is not watertight. Another example is icing and generation of icicles on the roof edges.

Icicles hanging from the eaves are a serious problem as they can fall down and hit people walking beneath. The impact of a falling icicle or ice from ice dams can in the worst case kill people. Such incidents have happened in Sweden and Norway. According to Swedish law, it is the owner of the building who is responsible for prevention of sliding of snow and ice from the building. The Swedish Association of Buildings Owners (Fastighetbranchens Utvik-lingsforum) has made a report (Snö och is på tak 2004) about the problems of snow and ice on roofs. It describes some law cases and examples of contracts with a firm to remove the ice and icicles, when they form in the winter. It is very helpful for the building owner as a basis for reducing the risk of snow and ice problems but it only sketches the physics behind the problem. A better solution is to prevent or at least reduce the risk by a better knowledge of snow melting, freezing and icicles generation on roofs. The problem with icing and icicles on roof is a complex problem involving architecture, meteorology, glaciology and building physics.

We can divide roofs in two types: cold (ventilated) roofs and warm (non-ventilated) roofs. In warm roofs, it is normal to have internal drainage with downpipes in the building. This solution has no or very little risk for icicles. Freezing of the melting water on the roof can still be a problem. Ventilated roofs introduce a ventilated gap or roof space to prevent moisture problems and to keep the surface of the roof cold. These roofs are in most cases sloped. The drainage is external to gutters along the eaves and to downpipes. The result is a high risk of ice formation on the overhang at the eave and icicles formation if the melting water freezes for instance in the gutter.

In this report we present calculations for ventilated roofs with a known inside temperature. The inside temperature can be defined in 2 cases:

1. The inside temperature is the same as the indoor temperature. We use the indoor temperature of the building and the U-value from the interior of the building to outside roof surface. This is used if we have no ventilated airspaces in the construction or the ventilation with outdoor air is rather low.

2. The inside temperature is the same as the attic temperature.

We use the attic temperature of the building and the U-value from the attic to the outside surface of the roof. This must be cases, where the attic temperature is influenced by air flows or heat sources. If we have heat sources as heat pipes, ventilation duct or ventilation systems, then this will increase the temperature in the attic and give a higher risk of icicles. If the construction between the building and the attic is not airtight then we will have an air flow from the house to the attic that will increase the attic temperature. If the attic is ventilated with outdoor air, the attic temperature and the risk of icicles will decrease. If the attic temperature is around -2 °C or lower in freezing periods, there is no melting and no icicles.

If we use the calculation on existing buildings, is it important to decide which case is most relevant for the building. As mentioned, the attic temperature can in second case be higher or lower than in the first case.

2 Problem

Figure 1 shows the considered roof. The inside temperature below the roof is T_r (around 20°C or any lower attic temperature) and the exterior or outdoor temperature T_e. The width of the roof is L (m) from roof-ridge to overhang. The U-value or thermal conductance of the roof (between T_r and the upper, outer side of the roof) is U_r (W/m^2,K). The thickness of the snow layer, $D(t)$, decreases with time t, if the snow melts due to sufficient heating from the indoor temperature T_r. The width of the overhang is L_{oh} and the U-value U_{oh} (W/m^2,K). The thermal conductivity of the snow on the roof and overhang is λ_s (W/K,m), and the density of the snow is ρ_s (kg/m^3). Changes over time of these snow parameters are neglected in this study.

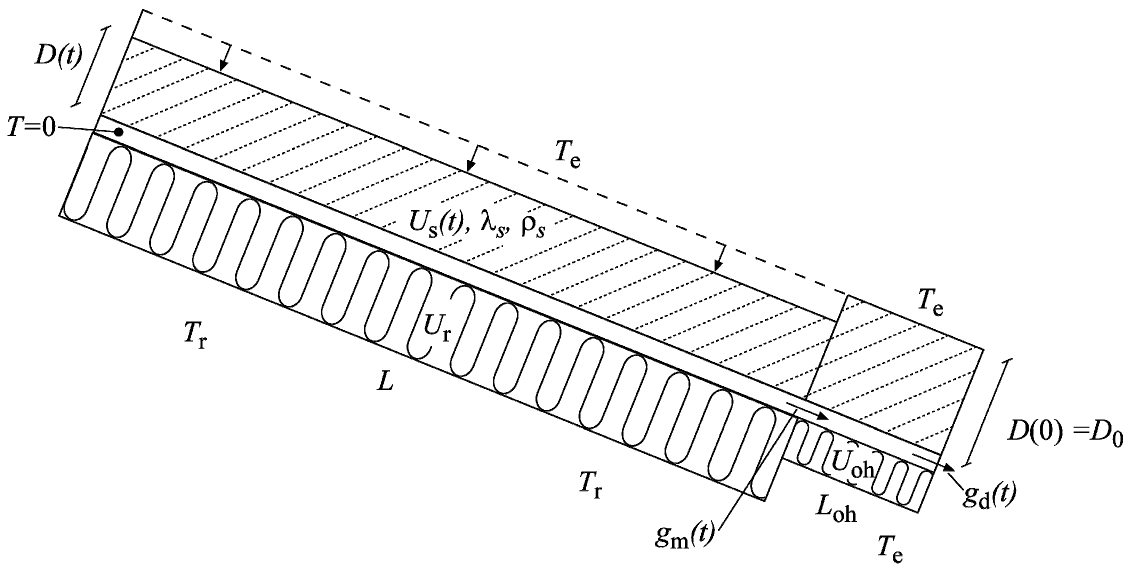


Figure 1. Snow on a roof with overhang. The task is to calculate of the melting of snow on the roof, and the ensuing ice and icicle formation at the overhang.

The outdoor temperature is below zero and, in this analysis, constant. By assumption there is no melting of snow from above. The U-value of the snow on the roof, $U_s(t)$, is varying with the snow depth $D(t)$. The initial snow depth is D_0. The snow on the overhang does not melt, which means the U-value of the snow on the overhang is equal to the initial U-value of the snow, λ_s / D_0. We have:

$$T_{\mathrm{r}} > 0, \qquad T_{\mathrm{e}} < 0, \qquad U_{\mathrm{s}}(t) = \frac{\lambda_{\mathrm{s}}}{D(t)}, \qquad D(0) = D_0. \tag{2.1}$$

The aim of this study is to calculate of the melting of snow on the roof. The water will flow to the overhang and freeze to ice under the snow on the overhang. Part of the water may drip or ooze from the lower end of the overhang to form ice and icicles there or leave the overhang as water drops. All melted snow ends up as ice again. Our aim is to quantify as function of time the melted snow, the formation of ice under the snow on the overhang and the amount of dripping water, which gives an upper limit for the ice formation at the outer end of the overhang.

3 Melting of snow on a roof

The snow on the roof will melt from below if the heating from T_{r} is larger than the cooling to T_{e}. Let $g_{\mathrm{m}}(t)$ (kg/s,m) denote the rate of snow melting on the roof (per meter roof width), and $m_{\mathrm{m}}(t)$ (kg/m) the accumulated amount. The melted water from the roof enters the overhang, where it will freeze again due to the cold outdoor temperature that surrounds the overhang. Some of the water may drip from the overhang and form ice and icicles at the outer end of the overhang. Let $g_d(t)$ (kg/s,m) denote the rate of dripping at the outer end of the overhang, and $m_{\mathrm{d}}(t)$ (kg/m) the accumulated amount. We have:

$$m_{\mathrm{m}}(t) = \rho_{\mathrm{s}} L\left[D_0 - D(t)\right] = \int_0^t g_{\mathrm{m}}(t')\,dt', \qquad m_{\mathrm{d}}(t) = \int_0^t g_{\mathrm{d}}(t')\,dt'. \tag{3.1}$$

The time derivative of $m_{\mathrm{m}}(t)$ becomes

$$\frac{dm_{\mathrm{m}}}{dt} = g_{\mathrm{m}}(t) = -\rho_{\mathrm{s}} L \cdot \frac{dD}{dt}. \tag{3.2}$$

3.1 Heat flows and criterion for snow melting

The temperature at the roof below the snow layer is equal to $\left(U_{\mathrm{r}} \cdot T_{\mathrm{r}} + U_{\mathrm{s}}(t) \cdot T_{\mathrm{e}}\right) / \left(U_{\mathrm{r}} + U_{\mathrm{s}}(t)\right)$ provided that this temperature lies below zero. There will be melting when the value is positive. We study the case when this temperature is *positive* at the start $t = 0$ with the snow thickness $D(0) = D_0$:

$$U_{\mathrm{r}} \cdot T_{\mathrm{r}} + U_{\mathrm{s}}(0) \cdot T_{\mathrm{e}} = U_{\mathrm{r}} \cdot T_{\mathrm{r}} + \frac{\lambda_{\mathrm{s}}}{D_0} \cdot T_{\mathrm{e}} > 0 \quad \text{or} \quad D_0 > \frac{\lambda_{\mathrm{s}}\left(-T_{\mathrm{e}}\right)}{U_{\mathrm{r}} T_{\mathrm{r}}}. \tag{3.3}$$

The snow thickness limit D_{m}, above which melting occurs, becomes:

$$D_{\mathrm{m}} = \frac{\lambda_{\mathrm{s}}\left(-T_{\mathrm{e}}\right)}{U_{\mathrm{r}} T_{\mathrm{r}}}, \qquad D_0 = D(0) > D_{\mathrm{m}}. \tag{3.4}$$

The temperature is zero at the roof adjacent to the snow layer, when snow is melting. Let q_r (W/m) denote the heat flux through the roof, and $q_\mathrm{e}(t)$ the heat flux through the snow layer (when the temperature is zero at the boundary between roof and snow):

$$q_\mathrm{r} = LU_\mathrm{r}\left(T_\mathrm{r} - 0\right), \qquad q_\mathrm{e}(t) = \frac{L\lambda_\mathrm{s}\left(0 - T_\mathrm{e}\right)}{D(t)} = q_\mathrm{r} \cdot \frac{D_\mathrm{m}}{D(t)}. \tag{3.5}$$

The melting limit D_m and the net heat flux to melt snow are:

$$D_\mathrm{m} = \frac{L\lambda_\mathrm{s}\left(-T_\mathrm{e}\right)}{q_\mathrm{r}}, \qquad q_\mathrm{r} - q_\mathrm{e}(t) = q_\mathrm{r}\left(1 - \frac{D_\mathrm{m}}{D(t)}\right). \tag{3.6}$$

The snow melts as long as this heat flux is positive:

$$q_\mathrm{r} - q_\mathrm{e}(t) > 0 \quad \Leftrightarrow \quad 1 - \frac{D_\mathrm{m}}{D(t)} > 0, \qquad D(t) > D_\mathrm{m}. \tag{3.7}$$

The limit for snow melting, D_m, must lie below the initial snow depth D_0, if melting is to occur. The criteria for snow melting are then:

$$q_\mathrm{r} > q_\mathrm{e}(0) \;\Leftrightarrow\; D_\mathrm{m} < D_0, \qquad D_\mathrm{m} < D(t) \le D_0. \qquad (3.8).$$

3.2 Differential equation for snow depth $D(t)$

The melting heat flux is equal to the rate of snow melting multiplied by the latent heat of melting for snow h_m (334 kJ/kg):

$$q_\mathrm{r} - q_\mathrm{e}(t) = q_\mathrm{r}\left(1 - \frac{D_\mathrm{m}}{D(t)}\right) = h_\mathrm{m} \cdot g_\mathrm{m}(t), \qquad D_\mathrm{m} < D(t) \le D_0. \tag{3.9}$$

Combining (3.2) and (3.9), we get the differential equation for the snow thickness $D(t)$:

$$1 - \frac{D_\mathrm{m}}{D(t)} = -\frac{h_\mathrm{m}\rho_\mathrm{s} L}{q_\mathrm{r}} \cdot \frac{dD}{dt}, \qquad D_\mathrm{m} < D(t) \le D_0 = D(0). \tag{3.10}$$

or, introducing a time t_r, (3.12):

$$-\frac{t_\mathrm{r}}{D_0} \cdot \frac{dD}{dt} = 1 - \frac{D_\mathrm{m}}{D}, \qquad D(0) = D_0 > D_\mathrm{m}, \qquad 0 \le t < \infty. \tag{3.11}$$

Here, t_r is the time required to melt the snow layer with the initial thickness D_0 for $T_\mathrm{e} = 0$, i.e. for zero heat flux through the snow:

$$t_{\mathrm{r}} = \frac{h_{\mathrm{m}} \rho_{\mathrm{s}} L D_0}{q_{\mathrm{r}}} = \frac{h_{\mathrm{m}} \rho_{\mathrm{s}} D_0}{U_{\mathrm{r}} T_{\mathrm{r}}}, \qquad U_{\mathrm{r}} \left(T_{\mathrm{r}} - 0 \right) \cdot t_{\mathrm{r}} = h_{\mathrm{m}} \cdot \rho_{\mathrm{s}} D_0. \tag{3.12}$$

3.3 Solution for the inverse relation $t = t(D)$

The above differential equation (3.11) may be solved by considering the inverse relation $t = t(D)$. The equation may be written:

$$\frac{dt}{dD} = -\frac{t_{\mathrm{r}}}{D_0} \cdot \frac{D}{D - D_{\mathrm{m}}} = -\frac{t_{\mathrm{r}}}{D_0} \cdot \left(1 + \frac{D_{\mathrm{m}}}{D - D_{\mathrm{m}}} \right), \qquad D_{\mathrm{m}} < D \le D_0 \tag{3.13}$$

The equation is integrated from any D, $D_{\mathrm{m}} < D < D_0$, to D_0:

$$t\left(D_0\right) - t\left(D\right) = -\frac{t_{\mathrm{r}}}{D_0} \cdot \left[D + D_{\mathrm{m}} \cdot \ln\left(D - D_{\mathrm{m}} \right) \right]_D^{D_0} \tag{3.14}$$

Using $t\left(D_0\right) = 0$, we get the basic formula:

$$t\left(D\right) = t_{\mathrm{r}} \cdot \left[1 - \frac{D}{D_0} + \frac{D_{\mathrm{m}}}{D_0} \cdot \ln\left(\frac{D_0 - D_{\mathrm{m}}}{D - D_{\mathrm{m}}} \right) \right], \qquad D_{\mathrm{m}} < D \le D_0 \tag{3.15}$$

The time t increases to infinity when D tends to the lower limit D_{m}, where the melting stops. The snow thickness $D(t)$ is obtained by a numerical inversion of (3.15) for any considered time t.

Formula (3.15) may be written in a dimensionless form using dimensionless time τ, snow depth d, and melting limit d_{m}:

$$\tau = \frac{t}{t_{\mathrm{r}}}, \qquad d = \frac{D}{D_0}, \qquad d_{\mathrm{m}} = \frac{D_{\mathrm{m}}}{D_0}. \tag{3.16}$$

The dimensionless form of relation (3.15) between time and snow depth becomes:

$$\tau = f_t(d, d_{\mathrm{m}}) = 1 - d + d_{\mathrm{m}} \cdot \ln\left(\frac{1 - d_{\mathrm{m}}}{d - d_{\mathrm{m}}} \right), \qquad d_{\mathrm{m}} < d \le 1. \tag{3.17}$$

This function $f_t(d, d_{\mathrm{m}})$ is shown in Figure 2.

The function $f_t(d, d_{\mathrm{m}})$ decreases from infinity to zero in the interval $d_{\mathrm{m}} < d \le 1$:

$$f_t(d_{\mathrm{m}} + 0, d_{\mathrm{m}}) = \infty, \qquad f_t(1, d_{\mathrm{m}}) = 0; \qquad \frac{\partial}{\partial d}\left[f_t(d, d_{\mathrm{m}}) \right] = -\frac{d}{d - d_{\mathrm{m}}}. \tag{3.18}$$

In the limit $T_{\mathrm{e}} = 0$, D_{m} and d_{m} are zero, and the snow layer decreases linearly with t:

$$f_t(d,0) = 1-d, \qquad 0 < d \le 1; \qquad D(t) = \begin{cases} D_0\left(1 - t/t_\mathrm{r}\right) & t < t_\mathrm{r} \\ 0 & t \ge t_\mathrm{r} \end{cases}. \tag{3.19}$$

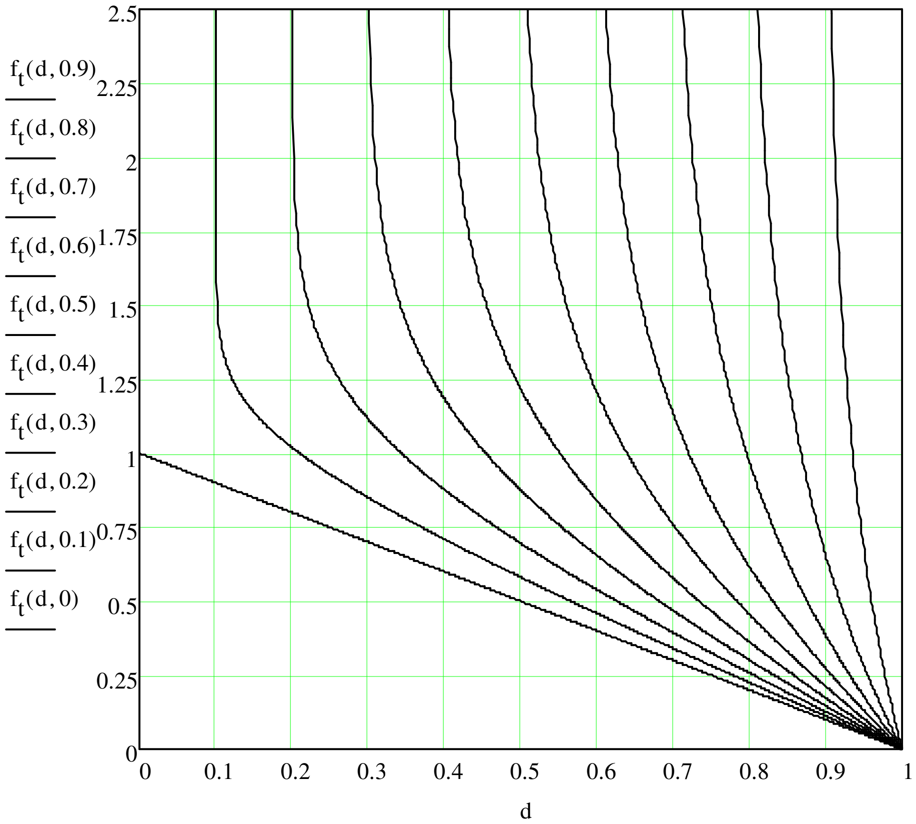


Figure 2. The function $\tau = f_t(d, d_\mathrm{m})$, $\tau = t/t_\mathrm{r}$, $d = D/D_0$, $d_\mathrm{m} = D_\mathrm{m}/D_0$, which gives $t = t(D)$ for $d_\mathrm{m} = 0$ (the lowest straight line), $0.1,\ 0.2, \ldots 0.9$ (the rightmost curve).

Equation (3.17) defines the inverse relation, i.e. the relative snow thickness $d = D/D_0$ as function of $\tau = t/t_\mathrm{r}$ with $d_\mathrm{m} = D_\mathrm{m}/D_0$ as parameter: $d = f_d(\tau, d_\mathrm{m})$. This function is shown in Figure 3. The set of curves is the same as in Figure 2, but the axes are interchanged. For any considered τ and d_m, we have to calculate the root to the equation $f_t(d, d_\mathrm{m}) - \tau = 0$ to determine *d*:

$$\tau = f_t(d, d_\mathrm{m}) \quad \Leftrightarrow \quad d = f_d(\tau, d_\mathrm{m}); \qquad \tau = f_t(\underbrace{f_d(\tau, d_\mathrm{m})}_{d}, d_\mathrm{m}). \tag{3.20}$$

The root may be somewhat difficult to determine numerically for *d* close to d_m. The following approximation may then be used:

$$f_d(\tau, d_\mathrm{m}) = d_\mathrm{m} + (1-d_\mathrm{m}) \cdot e^{(1-d_\mathrm{m}-\tau)/d_\mathrm{m}}, \qquad \tau > 3, \qquad 0 < d_\mathrm{m} < 1. \tag{3.21}$$

This relation is obtained from (3.17) by putting $d = d_\mathrm{m}$ in the second right-hand term. The error is smaller than $0.000\,06$ for $\tau > 3$.

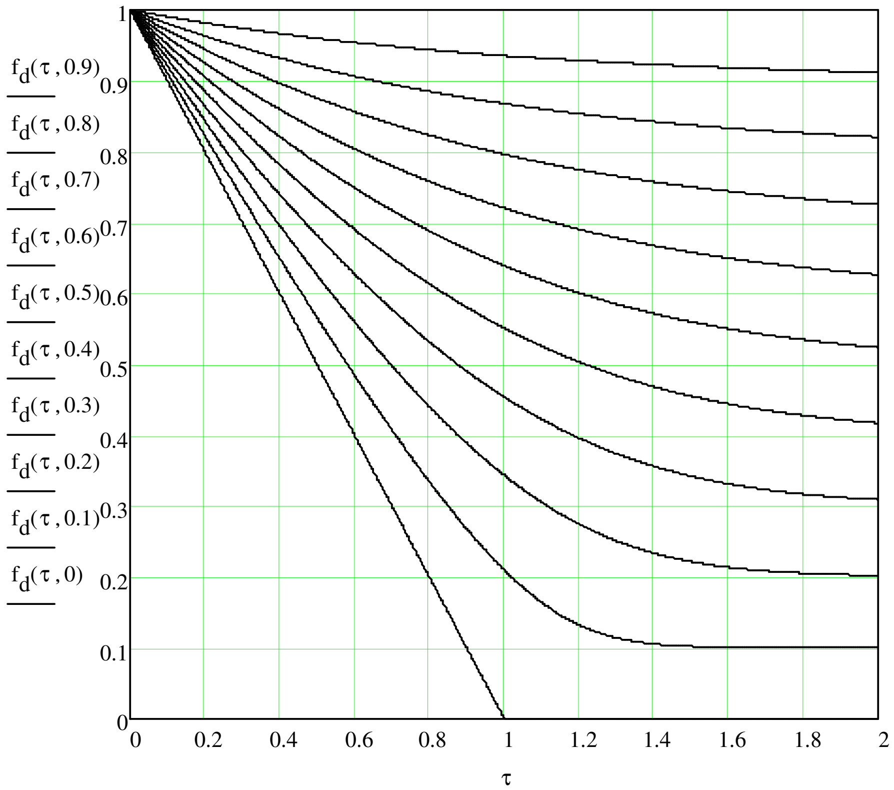


Figure 3. The function $d = f_d(\tau, d_\mathrm{m})$, the snow thickness $d = D / D_0$ as function of $\tau = t / t_\mathrm{r}$ with $d_\mathrm{m} = D_\mathrm{m} / D_0$ as parameter; $d_\mathrm{m} = 0$ (the lowest straight line), 0.1, $0.2, \ldots 0.9$ (top curve).

We will need the derivative of $d = f_d(\tau, d_\mathrm{m})$ with respect to τ. We have from (3.18):

$$\frac{\partial}{\partial \tau} \underbrace{\left[f_d(\tau, d_\mathrm{m}) \right]}_{d} = \frac{1}{\dfrac{\partial}{\partial d} \underbrace{\left[f_t(d, d_\mathrm{m}) \right]}_{\tau}} = -\frac{d - d_\mathrm{m}}{d} = -\left(1 - \frac{d_\mathrm{m}}{f_d(\tau, d_\mathrm{m})} \right). \tag{3.22}$$

3.4 Melted snow $m_\mathrm{m}(t)$

The accumulated amount of melted snow at time t is from (3.1):

$$m_\mathrm{m}(t) = \rho_\mathrm{s} L \big(D_0 - D(t) \big) = m_0 \cdot \big(1 - d(t) \big), \qquad m_0 = \rho_\mathrm{s} L D_0. \tag{3.23}$$

Here, m_0 (kg/m) the initial amount of snow on the roof. The total amount of melted snow M_{m} (kg/m) is obtained for very large t:

$$M_{\mathrm{m}} = \rho_{\mathrm{s}} \cdot L\left(D_0 - D_{\mathrm{m}}\right) = m_0 \cdot \left(1 - d_{\mathrm{m}}\right). \tag{3.24}$$

Equation (3.23) may be written in dimensionless form:

$$\frac{m_{\mathrm{m}}(t)}{m_0} = 1 - d(t) = m'_{\mathrm{m}}(\tau, d_{\mathrm{m}}), \qquad m'_{\mathrm{m}}(\tau, d_{\mathrm{m}}) = 1 - f_d(\tau, d_{\mathrm{m}}). \tag{3.25}$$

The dimensionless snow depth $d(t) = D(t)/D_0$ is shown in Figure 3. We get directly the accumulated amount of melted snow by changing to $1 - d(t) = m'_{\mathrm{m}}(\tau, d_{\mathrm{m}})$ on the vertical axis. See Figure 4.

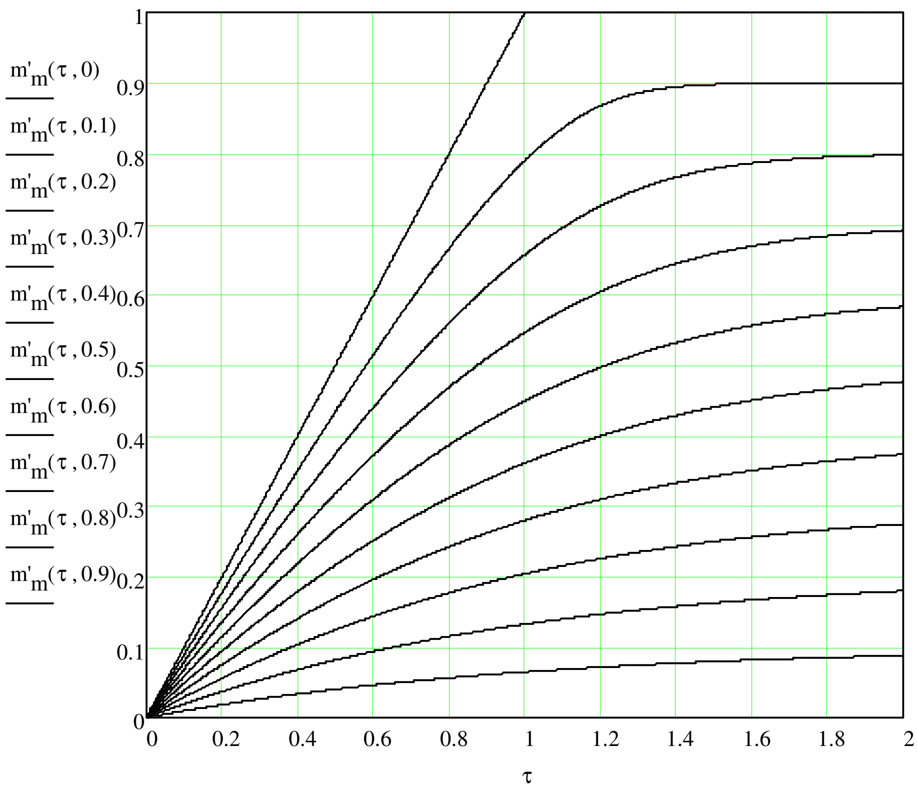


Figure 4. Accumulated amount of melted snow, $m_{\mathrm{m}}(t)/m_0 = m'_{\mathrm{m}}(\tau, d_{\mathrm{m}})$, as function of τ with d_{m} as parameter; $d_{\mathrm{m}} = 0$ (upper straight line), 0.1, 0.2, ... 0.9 (bottom curve).

Combining (3.23) and (3.11), we have:

$$\frac{1}{\rho_s L}\cdot\frac{dm_m}{dt}=-\frac{dD}{dt}=\frac{D_0}{t_r}\cdot\left(1-\frac{D_m}{D(t)}\right) \tag{3.26}$$

Integration over $0\le t'\le t$ gives

$$D_0-D(t)=\frac{D_0}{t_r}\cdot\left(t-D_m\cdot\int_0^t\frac{dt'}{D(t')}\right), \tag{3.27}$$

or, inserting $t=t(D)$ from (3.15):

$$\int_0^t\frac{D_m}{D(t')}dt'=t_r\cdot\frac{D_m}{D_0}\cdot\ln\left(\frac{D_0-D_m}{D-D_m}\right). \tag{3.28}$$

This relation will be used below.

4 Freezing in the overhang and dripping from it

The melted snow $g_m(t)$ will flow down the sloped roof into the overhang, where the surrounding temperature T_e is below zero. See Figure 1. All water will freeze below the snow in the overhang as long as the water influx is small. There is an upper limit above which part of the water freezes and the rest $g_d(t)$ drips from the overhang. This latter part, the dripping flow, may leave the overhang as water drops, or create icicles and ice at the lower end of the overhang.

4.1 Heat balance in overhang. Dripping limit

In the case of dripping, the heat balance for the overhang $\left(g_d(t)>0\right)$ is:

$$\left(g_m(t)-g_d(t)\right)\cdot h_m=q_{oh}, \qquad q_{oh}=K_{oh}\cdot\left(0-T_e\right). \tag{4.1}$$

Here, q_{oh} (W/m) is the heat flux from the ice/water layer of zero temperature under the snow on the overhang through the snow upwards and through the overhang roof downwards. The factor K_{oh} (W/(Km)) is the thermal conductance between the ice/water layer in the overhang and the surrounding air with the temperature T_e. This heat flux gives the freezing capacity of the overhang. Assuming that the thickness of snow on the overhang is equal to the initial value D_0 with the U-value λ_s / D_0, we have:

$$K_{oh}=L_{oh}\cdot\frac{\lambda_s}{D_0}+L_{oh}\cdot U_{oh}. \tag{4.2}$$

From (3.2) and (3.9) we have:

$$h_{\mathrm{m}} \cdot g_{\mathrm{m}}(t) = h_{\mathrm{m}} \cdot \frac{dm_{\mathrm{m}}}{dt} = q_{\mathrm{r}} - q_{\mathrm{e}}(t) \tag{4.3}$$

This expression is inserted in (4.1) and we get

$$h_{\mathrm{m}} \cdot g_{\mathrm{d}}(t) = h_{\mathrm{m}} \cdot \frac{dm_{\mathrm{d}}}{dt} = q_{\mathrm{r}} - q_{\mathrm{e}}(t) - q_{\mathrm{oh}}. \tag{4.4}$$

Dripping occurs when this expression is positive, and the expression becomes zero at the dripping limit $t = t_{\mathrm{d}}$:

$$q_{\mathrm{r}} - q_{\mathrm{e}}(t) - q_{\mathrm{oh}} > 0; \qquad q_{\mathrm{r}} - q_{\mathrm{e}}(t_{\mathrm{d}}) - q_{\mathrm{oh}} = 0. \tag{4.5}$$

Below, we will analyze these conditions for freezing in the overhang and dripping from the overhang in two ways. In the *first analysis*, we use (3.5), right:

$$q_{\mathrm{r}} - q_{\mathrm{e}}(t) - q_{\mathrm{oh}} = q_{\mathrm{r}} \cdot \left[\frac{q_{\mathrm{r}} - q_{\mathrm{oh}}}{q_{\mathrm{r}}} - \frac{D_{\mathrm{m}}}{D(t)} \right] \geq 0. \tag{4.6}$$

The above heat flux is never positive for $q_{\mathrm{r}} \leq q_{\mathrm{oh}}$. The conditions at the dripping limit become:

$$q_{\mathrm{r}} > q_{\mathrm{oh}} \quad \text{and} \quad \frac{q_{\mathrm{r}} - q_{\mathrm{oh}}}{q_{\mathrm{r}}} = \frac{D_{\mathrm{m}}}{D(t_{\mathrm{d}})}. \tag{4.7}$$

The snow thickness $D(t_{\mathrm{d}}) = D_{\mathrm{d}}$ at the dripping limit is now:

$$D_{\mathrm{d}} = D_{\mathrm{m}} \cdot \frac{q_{\mathrm{r}}}{q_{\mathrm{r}} - q_{\mathrm{oh}}}, \qquad q_{\mathrm{r}} > q_{\mathrm{oh}}; \qquad \frac{q_{\mathrm{oh}}}{q_{\mathrm{r}}} = 1 - \frac{D_{\mathrm{m}}}{D_{\mathrm{d}}}. \tag{4.8}$$

We note that the dripping limit is larger than the melting limi: $D_{\mathrm{d}} > D_{\mathrm{m}}$.

There are now three cases to consider: no melting, melting, and melting and dripping. The possibilities are illustrated in Figure 5. The snow thickness $D = D(t)$ divided by the melting limit D_{m} is given by the horizontal axis, and the vertical axis gives the heat flux ratio $q_{\mathrm{oh}} / q_{\mathrm{r}}$. In the melting region, snow melts on the roof and freezes again to ice in the overhang. The curve for the dripping limit is from (4.8) given by:

$$\frac{q_{\mathrm{oh}}}{q_{\mathrm{r}}} = 1 - \frac{D_{\mathrm{m}}}{D_{\mathrm{d}}} = \frac{D_{\mathrm{d}}/D_{\mathrm{m}} - 1}{D_{\mathrm{d}}/D_{\mathrm{m}}} = f_{\mathrm{d,lim}}\left(D_{\mathrm{d}}/D_{\mathrm{m}}\right), \qquad f_{\mathrm{d,lim}}(1) = 0, \quad f_{\mathrm{d,lim}}(\infty) = 1. \tag{4.9}$$

This curve increases from zero to one as D / D_{m} increases from one to infinity.

We have now three possibilities:

1. No melting: $D / D_{\mathrm{m}} \leq 1$

2. Melting without dripping for $D/D_{\mathrm{m}} > 1$ and $q_{\mathrm{oh}}/q_{\mathrm{r}} > 1$, and for $1 < D/D_{\mathrm{m}} < D_{\mathrm{d}}/D_{\mathrm{m}}$ and $q_{\mathrm{oh}}/q_{\mathrm{r}} < 1$.
3. Melting and dripping: $D_{\mathrm{d}}/D_{\mathrm{m}} < D/D_{\mathrm{m}} < D_0/D_{\mathrm{m}}$ and $q_{\mathrm{oh}}/q_{\mathrm{r}} < 1$.

The initial snow depth is $D(0) = D_0$. There is no melting if $D_0 < D_{\mathrm{m}}$. The lines A, B and C show what happens for a certain $D_0 > D_{\mathrm{m}}$. A and B: melting from D_0 to D_{m}. C: melting and dripping for $D_{\mathrm{d}} < D < D_0$, and melting only with ice accumulation at the overhang for $D_{\mathrm{m}} < D < D_{\mathrm{d}}$.

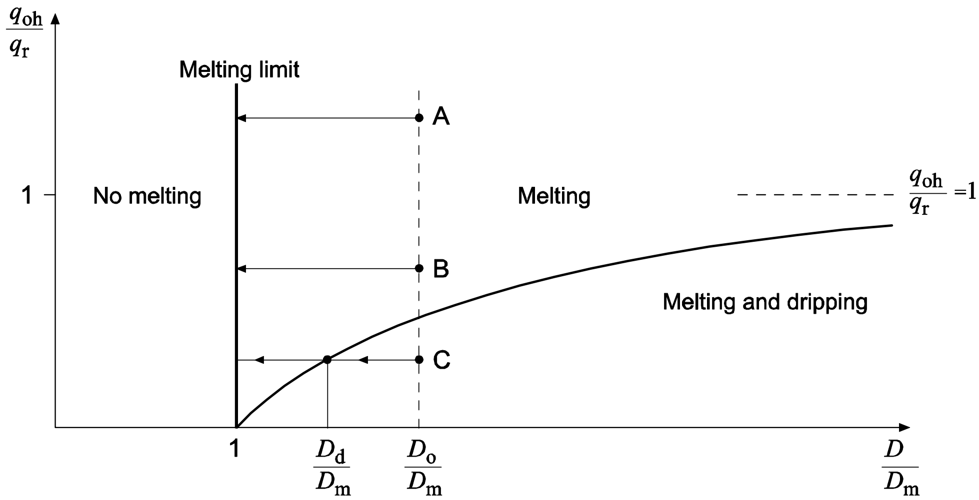


Figure 5. Regions of no melting, melting, melting and dripping.

In the *second analysis* of freezing in the overhang and dripping from overhang, we use the initial heat flux through the snow $q_{\mathrm{e0}} = q_{\mathrm{e}}(0)$ and rewrite $q_{\mathrm{e}}(t)$ in the following way, (3.5):

$$q_{\mathrm{e}}(t) = \frac{q_{\mathrm{e0}}}{d(t)}, \qquad q_{\mathrm{e0}} = q_{\mathrm{e}}(0) = \frac{L\lambda_{\mathrm{s}}\left(-T_{\mathrm{e}}\right)}{D_0} = q_{\mathrm{r}} \cdot d_{\mathrm{m}}. \tag{4.10}$$

The dripping criteria (4.5) are then:

$$q_{\mathrm{r}} - \frac{q_{\mathrm{e0}}}{d(t)} - q_{\mathrm{oh}} > 0; \qquad q_{\mathrm{r}} - \frac{q_{\mathrm{e0}}}{d_{\mathrm{d}}} - q_{\mathrm{oh}} = 0, \qquad d_{\mathrm{d}} = d(t_{\mathrm{d}}). \tag{4.11}$$

The condition for dripping at the initial time is:

$$q_{\mathrm{r}} - \frac{q_{\mathrm{e0}}}{1} - q_{\mathrm{oh}} = 0 \quad \text{or} \quad 1 = \frac{q_{\mathrm{e0}}}{q_{\mathrm{r}}} + \frac{q_{\mathrm{oh}}}{q_{\mathrm{r}}}. \tag{4.12}$$

Figure 6 shows a coordinate system with the axes $q_{\mathrm{e0}}/q_{\mathrm{r}}$. and $q_{\mathrm{e0}}/q_{\mathrm{r}}$. Each point represents a set of heat fluxes q_{r}, q_{e0} and q_{oh}. In the triangular region below the line (4.12), right, there is melting and dripping, and in the region above the triangle there is melting without dripping. To the right of $q_{e0}/q_r = d_m = 1$ no melting takes place.

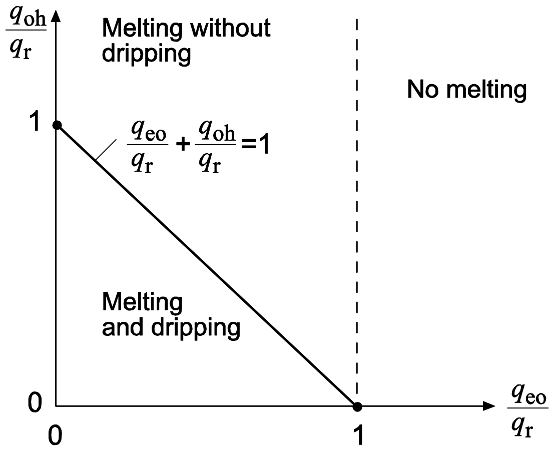


Figure 6. Melting and dripping depending of the heat fluxes q_r, q_{e0} and q_{oh}.

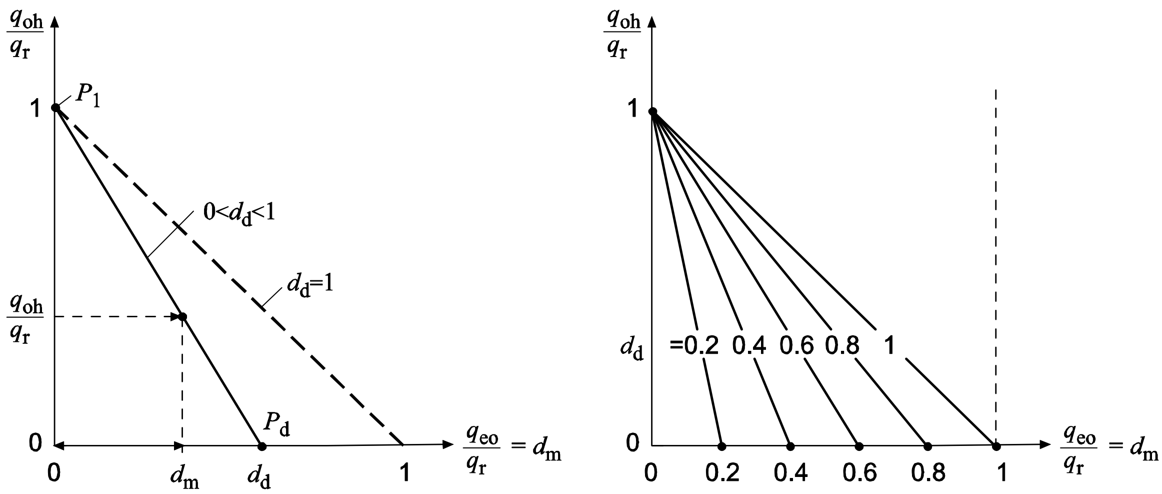


Figure 7. Dripping limit d_d, (4.14), as lines in a plane with the axis $q_{e0}/q_r = d_m$ and q_{oh}/q_r.

The dripping limit (4.11), right, may be written in the following way:

$$1 - \frac{q_{e0}}{q_r d_d} - \frac{q_{oh}}{q_r} = 0 \quad \text{or} \quad \frac{q_{oh}}{q_r} = 1 - \frac{d_m}{d_d}. \tag{4.13}$$

The relation between $d_{\mathrm{m}} = q_{\mathrm{e0}} / q_{\mathrm{r}}$ and $q_{\mathrm{oh}} / q_{\mathrm{r}}$ is, for any constant d_{d}, a straight line. It goes through the point $P_1 = (0,1)$ and has the slope $-1/d_{\mathrm{d}}$. The line cuts the horizontal axis in the point $P_{\mathrm{d}} = (d_{\mathrm{d}}, 0)$. See Figure 7, left. All points along the line have the same dripping limit d_{d}:

$$d_{\mathrm{d}} = \frac{d_{\mathrm{m}}}{1 - q_{\mathrm{oh}}/q_{\mathrm{r}}} = \frac{q_{\mathrm{e0}}}{q_{\mathrm{r}} - q_{\mathrm{oh}}}, \qquad 0 < d_{\mathrm{m}} < d_{\mathrm{d}} < 1. \tag{4.14}$$

It is seen from Figure 7, left, that the lines fall inside the triangle $P_1, (1,0), (0,0)$ for $0 < d_{\mathrm{d}} < 1$. For d_{d} outside this interval, the line lies wholly outside the triangle. Then there is no dripping. Figure 7, right, shows the dripping limit as straight lines through $P_1 = (0,1)$ for $d_{\mathrm{d}} = 0.2,\ 0.4,\ 0.6,\ 0.8,$ and 1. For $d_{\mathrm{d}} = 1$ along the line from P_1 to $(1,0)$, the dripping limit coincides with the initial snow depth: $D_{\mathrm{d}} = D_0$.

4.2 Dripping water

From (4.4)-(4.7), and from (3.12) and (3.23), right, we have:

$$\frac{dm_{\mathrm{d}}}{dt} = \frac{q_{\mathrm{r}}}{h_{\mathrm{m}}} \cdot \left(\frac{D_{\mathrm{m}}}{D_{\mathrm{d}}} - \frac{D_{\mathrm{m}}}{D(t)} \right), \qquad \frac{q_{\mathrm{r}}}{h_{\mathrm{m}}} = \frac{\rho_{\mathrm{s}} L D_0}{t_{\mathrm{r}}} = \frac{m_0}{t_{\mathrm{r}}}. \tag{4.15}$$

We see that dripping occurs for $D_{\mathrm{d}} \le D \le D_0$.

Integration of (4.15) with $m_{\mathrm{d}}(0) = 0$ gives

$$m_{\mathrm{d}}(t) = \frac{m_0}{t_{\mathrm{r}}} \cdot \left(\frac{D_{\mathrm{m}}}{D_{\mathrm{d}}} \cdot t - \int_0^t \frac{D_{\mathrm{m}}}{D(t')} dt' \right) \tag{4.16}$$

Using (3.28) we get:

$$m_{\mathrm{d}}(t) = \frac{m_0}{t_{\mathrm{r}}} \cdot \left[\frac{D_{\mathrm{m}}}{D_{\mathrm{d}}} \cdot t - t_{\mathrm{r}} \cdot \frac{D_{\mathrm{m}}}{D_0} \cdot \ln \left(\frac{D_0 - D_{\mathrm{m}}}{D(t) - D_{\mathrm{m}}} \right) \right], \qquad D_{\mathrm{m}} < D_{\mathrm{d}} \le D \le D_0. \tag{4.17}$$

We may eliminate *t*, (3.15), to get m_{d} as function of $D = D(t)$:

$$m_{\mathrm{d}}(t) = m_0 \cdot \left[\frac{D_{\mathrm{m}}}{D_{\mathrm{d}}} \cdot \left(1 - \frac{D(t)}{D_0} \right) - \left(1 - \frac{D_{\mathrm{m}}}{D_{\mathrm{d}}} \right) \frac{D_{\mathrm{m}}}{D_0} \cdot \ln \left(\frac{D_0 - D_{\mathrm{m}}}{D(t) - D_{\mathrm{m}}} \right) \right] \tag{4.18}$$

The dripping may be expressed in dimensionless from. We use dimensionless quantities for snow depth:

$$d(t) = \frac{D(t)}{D_0}, \qquad d_\mathrm{m} = \frac{D_\mathrm{m}}{D_0}, \qquad d_\mathrm{d} = \frac{D_\mathrm{d}}{D_0}. \tag{4.19}$$

Then we have:

$$m_\mathrm{d}(t) = m_0 \cdot m'_\mathrm{d}(\tau, d_\mathrm{m}, d_\mathrm{d}), \qquad d(t) = f_\mathrm{d}(\tau, d_\mathrm{m}), \qquad d_\mathrm{m} < d_\mathrm{d} \le f_\mathrm{d}(\tau, d_\mathrm{m}) \le 1. \tag{4.20}$$

$$m'_\mathrm{d}(\tau, d_\mathrm{m}, d_\mathrm{d}) = \frac{d_\mathrm{m}}{d_\mathrm{d}} \cdot \left[1 - f_\mathrm{d}(\tau, d_\mathrm{m}) - \left(d_\mathrm{d} - d_\mathrm{m}\right) \cdot \ln\left(\frac{1 - d_\mathrm{m}}{f_\mathrm{d}(\tau, d_\mathrm{m}) - d_\mathrm{m}} \right) \right], \qquad 0 \le \tau \le \tau_\mathrm{d}. \tag{4.21}$$

The dripping stops when $d(t) = f_\mathrm{d}(\tau, d_\mathrm{m}) = d_\mathrm{d}$. The corresponding time $\tau_\mathrm{d} = t_\mathrm{d} / t_\mathrm{r}$ is from (3.16)-(3.17):

$$\tau_\mathrm{d} = f_t(d_\mathrm{d}, d_\mathrm{m}) = 1 - d_\mathrm{d} + d_\mathrm{m} \cdot \ln\left(\frac{1 - d_\mathrm{m}}{d_\mathrm{d} - d_\mathrm{m}} \right). \tag{4.22}$$

The increase of the accumulated melted snow $m_\mathrm{d}(t)$ stops at this time:

$$m_\mathrm{d}(t) = m_\mathrm{d}(t_\mathrm{d}), \qquad t \ge t_\mathrm{d}; \qquad m'_\mathrm{d}(\tau, d_\mathrm{m}, d_\mathrm{d}) = m'_\mathrm{d}(\tau_\mathrm{d}, d_\mathrm{m}, d_\mathrm{d}), \qquad \tau \ge \tau_\mathrm{d}. \tag{4.23}$$

The maximum value $m'_\mathrm{d}(\tau_\mathrm{d}, d_\mathrm{m}, d_\mathrm{d}) = M'_\mathrm{d}(d_\mathrm{m}, d_\mathrm{d})$, (4.31), is discussed further in Section 3.4.

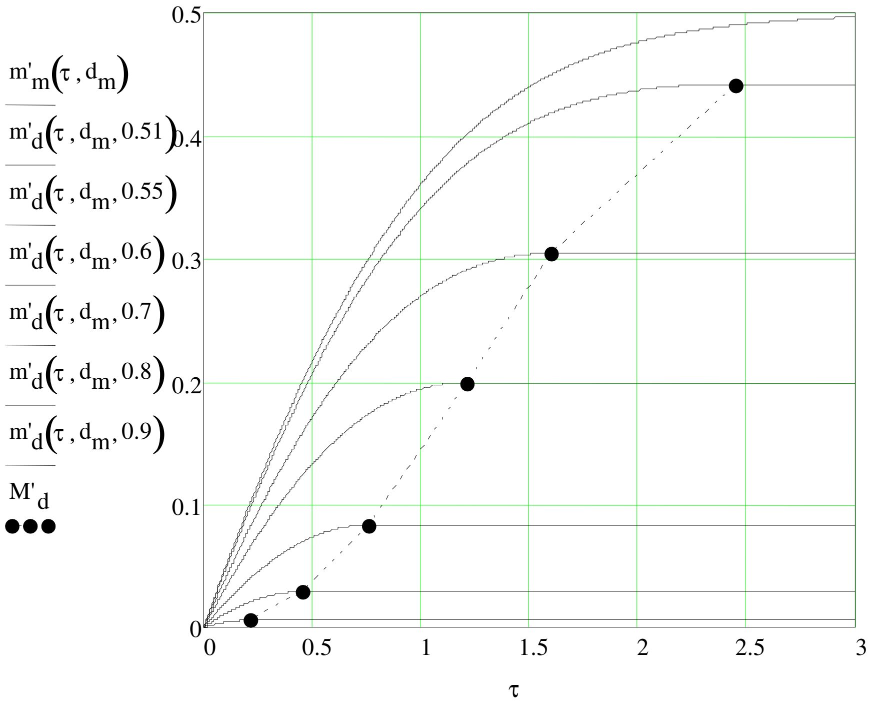


Figure 8. Amount of melting $m'_\mathrm{m}(\tau, 0.5)$ and dripping $m'_\mathrm{d}(\tau, 0.5, d_\mathrm{d})$ for $d_\mathrm{m} = 0.5$ for a few d_d. The dots show the points $(\tau_\mathrm{d}, M'_\mathrm{d})$.

Figure 8 shows a few curves $m'_{\mathrm{d}}(\tau, d_{\mathrm{m}}, d_{\mathrm{d}})$ for $d_{\mathrm{m}} = 0.5$. The top curve shows the melted snow $m'_{\mathrm{m}}(\tau, 0.5)$, which increases from zero to $1 - d_{\mathrm{m}} = 0.5$. The other curves show $m'_{\mathrm{d}}(\tau, 0.5, d_{\mathrm{d}})$ from (4.21) for $d_{\mathrm{d}} = 0.51$ (top curve), 0.55, 0.6, 0.7, 0.8, 0.9 (bottom curve). The dots show the total dripping M'_{d}, (4.31), that occurs at the time τ_{d}, (4.22). The curves for dripping are horizontal after that time.

4.3 Freezing in overhang

The difference between melted snow and dripping water is accumulated as ice in the overhang:

$$m_{\mathrm{oh}}(t) = m_{\mathrm{m}}(t) - m_{\mathrm{d}}(t). \tag{4.24}$$

We have from (4.3) and (4.4):

$$h_{\mathrm{m}} \cdot \frac{dm_{\mathrm{oh}}}{dt} = q_{\mathrm{r}} - q_{\mathrm{e}}(t) - \left(q_{\mathrm{r}} - q_{\mathrm{e}}(t) - q_{\mathrm{oh}}\right) \quad \Rightarrow \quad \frac{dm_{\mathrm{oh}}}{dt} = \frac{q_{\mathrm{oh}}}{h_{\mathrm{m}}}, \qquad 0 \le t \le t_{\mathrm{d}} \tag{4.25}$$

The accumulated ice at the overhang increases linearly as long as water is dripping:

$$m_{\mathrm{oh}}(t) = \frac{q_{\mathrm{oh}}}{h_{\mathrm{m}}} \cdot t = \frac{q_{\mathrm{oh}}}{q_{\mathrm{r}}} \cdot \frac{m_0}{t_{\mathrm{r}}} \cdot t = m_0 \cdot \left(1 - \frac{D_{\mathrm{m}}}{D_{\mathrm{d}}}\right) \cdot \frac{t}{t_{\mathrm{r}}}, \qquad 0 \le t \le t_{\mathrm{d}}. \tag{4.26}$$

Here, (4.15), right, and(4.8), right, is used. This linear increase means that the full heat flux q_{oh} is used to freeze melting snow. After the time when dripping has stopped only a fraction of this heat flux is needed to the freeze the melted water in an upper part of the overhang. The temperature under the snow in the outer part of the overhang will fall below zero. In dimensionless form (4.26) becomes:

$$m'_{\mathrm{oh}}(\tau) = \frac{m_{\mathrm{oh}}(t)}{m_0} = \left(1 - \frac{d_{\mathrm{m}}}{d_{\mathrm{d}}}\right) \cdot \tau, \qquad 0 \le \tau \le \tau_{\mathrm{d}}. \tag{4.27}$$

Figure 9 shows as an example the case $d_{\mathrm{m}} = 0.4$ and $d_{\mathrm{d}} = 0.6$. The top curve shows the melted snow, (3.25), which increases from zero to $1 - 0.4 = 0.6$. The middle curve shows the ice in the overhang, (4.24), and the bottom curve the dripping from the overhang, (4.21). The dripping increases to the maximum $M'_{\mathrm{d}} = 0.12$ given by (4.30)-(4.31). The maximum is attained at $\tau_{\mathrm{d}} = 0.84$ given by (4.22).

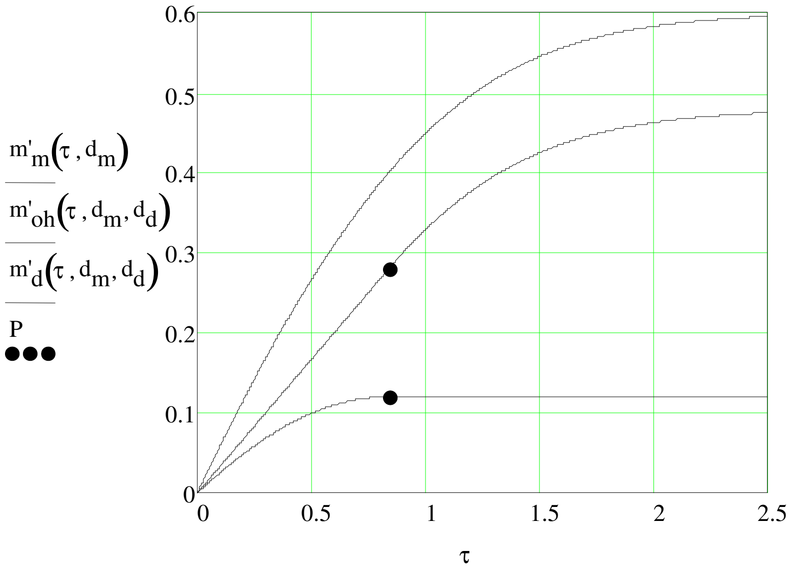


Figure 9. Amount of melting, $m'_{\mathrm{m}}(\tau, d_{\mathrm{m}})$, freezing in the overhang, $m'_{\mathrm{oh}}(\tau)$, and dripping, $m'_{\mathrm{d}}(\tau)$, for the case $d_{\mathrm{m}} = 0.4$ and $d_{\mathrm{d}} = 0.6$. Dots (P): $\tau = \tau_{\mathrm{d}}$ and $m'_{\mathrm{d}} = M'_{\mathrm{d}}$, $m'_{\mathrm{oh}} = M'_{\mathrm{oh}}$.

4.4 Total amounts of melting, freezing and dripping

The total amount of melted snow (kg/m) is from (3.24):

$$M_{\mathrm{m}} = m_0 \cdot M'_{\mathrm{d}}, \qquad M'_{\mathrm{d}} = 1 - d_{\mathrm{m}}. \tag{4.28}$$

In the case without dripping, we have

$$M_{\mathrm{d}} = 0, \qquad M_{\mathrm{oh}} = M_{\mathrm{m}} \quad \text{for } q_{\mathrm{oh}} > q_{\mathrm{r}}, \text{ and for } D_0 < D_{\mathrm{d}}, \quad q_{\mathrm{r}} < q_{\mathrm{oh}}. \tag{4.29}$$

The total amount of dripping water is given by:

$$M_{\mathrm{d}} = m_{\mathrm{d}}(t_{\mathrm{d}}) = m_0 \cdot M'_{\mathrm{d}}(d_{\mathrm{m}}, d_{\mathrm{d}}), \qquad 0 < d_{\mathrm{m}} < d_{\mathrm{d}} < 1. \tag{4.30}$$

The function $M'_{\mathrm{d}}(d_{\mathrm{m}}, d_{\mathrm{d}})$, which gives the dimensionless total amount of dripping water, is obtained from (4.21) for $\tau = \tau_{\mathrm{d}}$ and $f_{\mathrm{d}}(\tau_{\mathrm{d}}, d_{\mathrm{m}}) = d_{\mathrm{d}}$:

$$M'_{\mathrm{d}}(d_{\mathrm{m}}, d_{\mathrm{d}}) = \frac{d_{\mathrm{m}}}{d_{\mathrm{d}}} \cdot \left[1 - d_{\mathrm{d}} - \left(d_{\mathrm{d}} - d_{\mathrm{m}} \right) \cdot \ln\left(\frac{1 - d_{\mathrm{m}}}{d_{\mathrm{d}} - d_{\mathrm{m}}} \right) \right], \qquad 0 < d_{\mathrm{m}} < d_{\mathrm{d}} < 1. \tag{4.31}$$

The total amount of ice in the overhang is:

$$M_{\mathrm{oh}} = M_{\mathrm{m}} - M_{\mathrm{d}} = m_0 \cdot M'_{\mathrm{oh}}(d_{\mathrm{m}}, d_{\mathrm{d}}), \qquad d_{\mathrm{m}} \le d_{\mathrm{d}}. \tag{4.32}$$

$$M'_{\mathrm{oh}}(d_{\mathrm{m}},d_{\mathrm{d}}) = 1 - d_{\mathrm{m}} - M'_{\mathrm{d}}(d_{\mathrm{m}},d_{\mathrm{d}}) = \left(1 - \frac{d_{\mathrm{m}}}{d_{\mathrm{d}}}\right)\left[1 + d_{\mathrm{m}} \cdot \ln\left(\frac{1-d_{\mathrm{m}}}{d_{\mathrm{d}}-d_{\mathrm{m}}}\right)\right]. \tag{4.33}$$

The functions $M'_{\mathrm{d}}(d_{\mathrm{m}},d_{\mathrm{d}})$ and $M'_{\mathrm{oh}}(d_{\mathrm{m}},d_{\mathrm{d}})$ are defined in a triangular region, and we have:

$$M'_{\mathrm{d}}(d_{\mathrm{m}},d_{\mathrm{d}}) + M'_{\mathrm{oh}}(d_{\mathrm{m}},d_{\mathrm{d}}) = M'_{\mathrm{d}} = 1 - d_{\mathrm{m}}, \qquad 0 < d_{\mathrm{m}} < d_{\mathrm{d}} < 1. \tag{4.34}$$

Figures 10 and 11 show these two functions for $d_{\mathrm{d}} = 0.1$ (leftmost curve), $0.2,\ldots,0.9,\ 0.95$ (rightmost curve). The dashed line shows the limit $1 - d_{\mathrm{m}}$. On the boundaries of the triangular region we have in accordance with the sum (4.34):

$$\begin{aligned}
&M'_{\mathrm{d}}(0,d_{\mathrm{d}}) = 0, && M'_{\mathrm{oh}}(0,d_{\mathrm{d}}) = 1, && 0 < d_{\mathrm{d}} < 1;\\
&M'_{\mathrm{d}}(d_{\mathrm{m}},1) = 0, && M'_{\mathrm{oh}}(d_{\mathrm{m}},1) = 1 - d_{\mathrm{m}}, && 0 < d_{\mathrm{m}} < 1;\\
&M'_{\mathrm{d}}(d_{\mathrm{m}},d_{\mathrm{m}}) = 1 - d_{\mathrm{m}}, && M'_{\mathrm{oh}}(d_{\mathrm{m}},d_{\mathrm{m}}) = 0, && 0 < d_{\mathrm{m}} < 1.
\end{aligned} \tag{4.35}$$

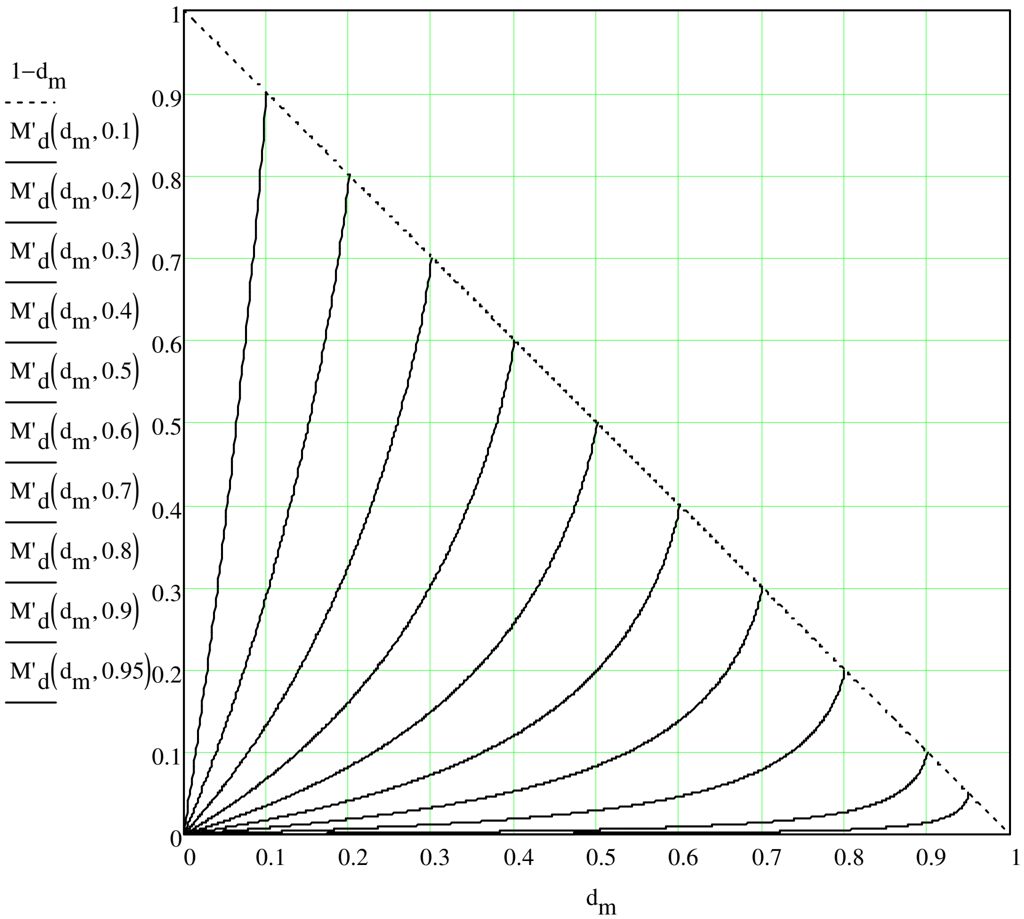


Figure 10. The function $M'_{\mathrm{d}}(d_{\mathrm{m}},d_{\mathrm{d}})$ for the total amount of dripping from the overhang.

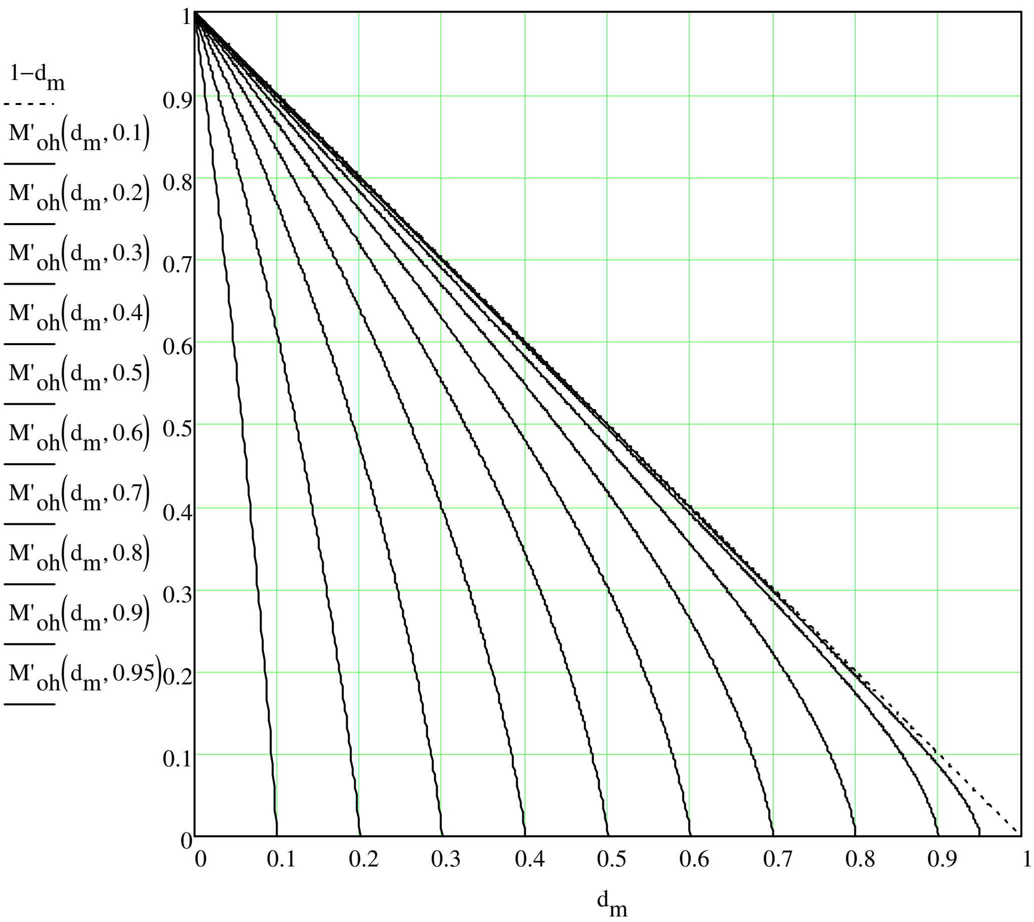


Figure 11. The function $M'_{\mathrm{oh}}(d_{\mathrm{m}}, d_{\mathrm{d}})$ for the total amount of ice in the overhang.

5 Overview and summary

The above analysis and the most important formulas are summarized in this section.

5.1 Melting of snow on the roof

The primary parameters are shown in Figure 1. The heat flux from the inside of the roof, q_{r}, the initial snow depth, D_0, the initial amount of snow on the roof, m_0 (kg/m), and the time t_{r} to fully melt the initial snow layer with the heat flux q_{r} are:

$$q_{\mathrm{r}} = LU_{\mathrm{r}}T_{\mathrm{r}} \quad \left(T_{\mathrm{r}} > 0\right), \qquad D(0) = D_0, \qquad m_0 = \rho_{\mathrm{s}} LD_0, \qquad t_{\mathrm{r}} = \frac{h_{\mathrm{m}} m_0}{q_{\mathrm{r}}}. \tag{5.1}$$

There is a certain melting limit above which the heat flux from the inside is larger than the heat flux through the snow. The snow on the roof melts when the snow depth lies above the melting limit D_{m} :

$$D(t) > D_{\mathrm{m}} = \frac{L\lambda_{\mathrm{s}}(-T_{\mathrm{e}})}{q_{\mathrm{r}}}, \qquad (T_{\mathrm{e}} < 0). \tag{5.2}$$

There is no melting if the initial snow depth lies below the melting limit. The snow depth $D(t)$ decreases with time as the snow on the roof melts. We have derived an explicit formula for the time as a function of the snow depth, (3.15):

$$t(D) = t_{\mathrm{r}} \cdot \left[1 - \frac{D}{D_0} + \frac{D_{\mathrm{m}}}{D_0} \cdot \ln\left(\frac{D_0 - D_{\mathrm{m}}}{D - D_{\mathrm{m}}} \right) \right], \qquad D_{\mathrm{m}} < D \le D_0. \tag{5.3}$$

The time t increases from zero to infinity as the snow thickness decreases from the initial value D_0 to the melting limit D_{m} (for $D_{\mathrm{m}} < D_0$). This relation and other relations below may be formulated with a few dimensionless variables.

We will use dimensionless time τ, snow depth d, melting limit d_{m}, and dripping limit d_{d}:

$$\tau = \frac{t}{t_{\mathrm{r}}}, \qquad d = \frac{D}{D_0}, \qquad d_{\mathrm{m}} = \frac{D_{\mathrm{m}}}{D_0}, \qquad d_{\mathrm{d}} = \frac{D_{\mathrm{d}}}{D_0}. \tag{5.4}$$

Eq. (5.3) becomes in dimensionless form

$$\tau = f_t(d, d_{\mathrm{m}}) = 1 - d + d_{\mathrm{m}} \cdot \ln\left(\frac{1 - d_{\mathrm{m}}}{d - d_{\mathrm{m}}} \right), \qquad d_{\mathrm{m}} < d \le 1. \tag{5.5}$$

This relation is shown in Figure 2. The inverse relation $d = f_d(\tau, d_{\mathrm{m}})$ is readily plotted by interchanging the axes, Figure 3. In the computer programs, it is obtained by a numerical solution of (5.5) to get $d = f_d(\tau, d_{\mathrm{m}})$ for any τ and d_{m}. Equation (3.21) is used for $\tau > 3$.

The melted snow, $m_{\mathrm{m}}(t)$, is directly obtained from the snow depth $D(t)$:

$$m_{\mathrm{m}}(t) = m_0 \cdot \left(1 - \frac{D(t)}{D_0} \right), \qquad D_{\mathrm{m}} < D \le D_0, \tag{5.6}$$

or, using dimensionless variables,

$$m_{\mathrm{m}}(t) = m_0 \cdot m'_{\mathrm{m}}(\tau, d_{\mathrm{m}}), \qquad m'_{\mathrm{m}}(\tau, d_{\mathrm{m}}) = 1 - f_d(\tau, d_{\mathrm{m}}), \qquad 0 \le \tau < \infty, \qquad d_{\mathrm{m}} < d \le 1. \tag{5.7}$$

The dimensionless relation $m(t)/m_0 = m'_{\mathrm{m}}(\tau, d_{\mathrm{m}})$ for the melted snow is shown in Figure 4. The total amount of melted snow is obtained for $D = D_{\mathrm{m}}$:

$$M_{\mathrm{m}} = m_0 \cdot M'_{\mathrm{m}}, \qquad M'_{\mathrm{m}} = 1 - d_{\mathrm{m}}. \tag{5.8}$$

5.2 Freezing and dripping at the overhang

The water from the melted snow freezes again below the snow on the overhang , which is exposed to the cold outdoor temperature $T_\text{e} < 0$. All melted water freezes if q_r is smaller than the heat flux q_oh from the freezing water in the overhang. For $q_\text{r} > q_\text{oh}$, some of the melted water may drip from the overhang or form ice and icicles when the snow depth lies above the dripping limit D_d. We have from (3.6), right, (4.8) and (4.1):

$$D_\text{m} = \frac{L\lambda_\text{s}(-T_\text{e})}{q_\text{r}}, \qquad D_\text{d} = D_\text{m} \cdot \frac{q_\text{r}}{q_\text{r} - q_\text{oh}} \quad (q_\text{r} > q_\text{oh}), \qquad q_\text{oh} = K_\text{oh} \cdot (-T_\text{e}). \tag{5.9}$$

The thermal conductance of the overhang K_oh is given by (4.2). Dripping with the ensuing ice and icicle formation at the outer end of the overhang will occur if two conditions are fulfilled: $q_\text{r} > q_\text{oh}$ and $D_\text{d} < D_0$. Otherwise there is no dripping.

We consider in this section the case when dripping occurs. Then we have

$$D_\text{m} < D_\text{d} < D_0, \qquad d_\text{m} < d_\text{d} < 1. \tag{5.10}$$

Part of the melted snow, $m_\text{oh}(t)$, freezes in the overhang and the rest, $m_\text{d}(t)$, drips or end up as ice and icicles:

$$m_\text{m}(t) = m_\text{oh}(t) + m_\text{d}(t). \tag{5.11}$$

The dripping stops at the time when the snow thickness is equal to the dripping limit $D(t_\text{d}) = D_\text{d}$:

$$D(t_\text{d}) = D_\text{d} \quad \Leftrightarrow \quad t_\text{d} = t_\text{r} \cdot f_t(d_\text{d}, d_\text{m}), \qquad d_\text{m} < d_\text{d} \le 1. \tag{5.12}$$

The accumulated amount of dripping is from (4.19)-(4.21)

$$m_\text{d}(t) = m_0 \cdot \frac{D_\text{m}}{D_\text{d}} \cdot \left[1 - \frac{D(t)}{D_0} - \frac{D_\text{d} - D_\text{m}}{D_0} \cdot \ln\left(\frac{D_0 - D_\text{m}}{D(t) - D_\text{m}} \right) \right], \qquad 0 \le t \le t_\text{d}. \tag{5.13}$$

The total amount of dripping becomes

$$M_\text{d} = m_\text{d}(t_\text{d}) = m_0 \cdot M'_\text{d}(d_\text{m}, d_\text{d}), \qquad 0 < d_\text{m} < d_\text{d} < 1. \tag{5.14}$$

The dimensionless function $M'_\text{d}(d_\text{m}, d_\text{d})$ for the total dripping becomes:

$$M'_\text{d}(d_\text{m}, d_\text{d}) = \frac{d_\text{m}}{d_\text{d}} \cdot \left[1 - d_\text{d} - (d_\text{d} - d_\text{m}) \cdot \ln\left(\frac{1 - d_\text{m}}{d_\text{d} - d_\text{m}} \right) \right]. \tag{5.15}$$

The accumulated dripping does not change after the time t_d :

$$m_{\mathrm{d}}(t)=m_{\mathrm{d}}(t_{\mathrm{d}})=M_{\mathrm{d}}, \qquad t_{\mathrm{d}}\le t<\infty. \tag{5.16}$$

The accumulated ice in the overhang is obtained from (5.11), (5.13) and (5.7). It increases linearly during the period of dripping:

$$m_{\mathrm{oh}}(t)=m_{\mathrm{m}}(t)-m_{\mathrm{d}}(t); \qquad m_{\mathrm{oh}}(t)=\frac{q_{\mathrm{oh}}}{h_{\mathrm{m}}}\cdot t=m_0\cdot\frac{d_{\mathrm{d}}-d_{\mathrm{m}}}{d_{\mathrm{d}}}\cdot\frac{t}{t_{\mathrm{r}}}, \qquad 0\le t\le t_{\mathrm{d}}. \tag{5.17}$$

The total amount of ice in the overhang is:

$$M_{\mathrm{oh}}=M_{\mathrm{m}}-M_{\mathrm{d}}=m_0\cdot M'_{\mathrm{oh}}(d_{\mathrm{m}},d_{\mathrm{d}}). \tag{5.18}$$

The dimensionless function $M'_{\mathrm{oh}}(d_{\mathrm{m}},d_{\mathrm{d}})$ for the total amount of ice under the snow on the overhang becomes:

$$M'_{\mathrm{oh}}(d_{\mathrm{m}},d_{\mathrm{d}})=1-d_{\mathrm{m}}-M'_{\mathrm{d}}(d_{\mathrm{m}},d_{\mathrm{d}}). \tag{5.19}$$

The functions $M'_{\mathrm{d}}(d_{\mathrm{m}},d_{\mathrm{d}})$ and $M'_{\mathrm{oh}}(d_{\mathrm{m}},d_{\mathrm{d}})$ are shown in Figures 10 and 11.

5.3 A few examples

Let us consider a few examples. We use the following input data:

$$\begin{aligned}&T_{\mathrm{r}}=20\ ^{\circ}\mathrm{C}, \quad T_{\mathrm{e}}=-10\ ^{\circ}\mathrm{C}, \quad L=8\ \mathrm{m}, \quad L_{\mathrm{oh}}=0.4\ \mathrm{m}, \quad D_0=0.2\ \mathrm{m},\\ &\lambda_{\mathrm{s}}=0.0\ \mathrm{W/(m,K)}, \quad \rho_{\mathrm{s}}=200\ \mathrm{kg/m^3}, \quad U_{\mathrm{oh}}=2.0\ \mathrm{W/(m^2,K)}.\end{aligned} \tag{5.20}$$

In the first example we consider a roof with poor thermal insulation or large U-value:

$$\begin{aligned}&U_{\mathrm{r}}=1.0\ \mathrm{W/(m^2,K)} \quad\Rightarrow\\ &D_{\mathrm{m}}=0.030\ \mathrm{m}, \quad m_0=320\ \mathrm{kg/m}, \quad q_{\mathrm{r}}=160\ \mathrm{W/m}, \quad K_{\mathrm{oh}}=0.92\ \mathrm{W/(m,K)},\\ &q_{\mathrm{oh}}=9.2\ \mathrm{W/(m,K)}, \quad t_{\mathrm{r}}=7.7\ \text{days}, \quad D_{\mathrm{d}}=0.032\ \mathrm{m}, \quad t_{\mathrm{d}}=11.8\ \text{days}.\end{aligned} \tag{5.21}$$

Figure 12 shows the decreasing snow depth from 0.02 m to the melting limit D_{m}=0.030 m.

Figure 13 illustrates the melting and dripping in the considered example. We get from our formulas:

$$\begin{aligned}&M_{\mathrm{m}}=272\ \mathrm{kg/m}, \quad M_{\mathrm{oh}}=31\ \mathrm{kg/m}, \quad M_{\mathrm{d}}=241\ \mathrm{kg/m},\\ &t_{\mathrm{d}}=11.8\ \text{days} \quad m_{\mathrm{oh}}(t_{\mathrm{d}})=30\ \mathrm{kg/m}.\end{aligned} \tag{5.22}$$

There are in Figure 13 two horizontal lines, three curves and two points (a circle and a square) in the figure. (The five *t* on the horizontal axis are at the time in days for the top five functions on the vertical axis, while the two t_{dd} give the dripping limit in hours for the two points.) The top horizontal line shows the total amount of melting snow M_{m}=272 kg/m. The first curve (top) shows the accumulated amount of melted snow $m_{\mathrm{m}}(t)$ after *t* days. The second curve from top gives the accumulated amount of dripping water, $m_{\mathrm{d}}(t)$, and the lowest curve the accumulated amount of ice in the overhang, $m_{\mathrm{oh}}(t)$. The curve for dripping water increases up to the dripping time limit t_d=11.8 days. After that, the value is the constant and equal to M_d=241 kg/m. The lower horizontal line shows the total amount of ice in the overhang, M_{oh}=31 kg/m. The second point (a square) shows the accumulated amount of ice in the overhang at the time when dripping stops, $m_{oh}(t_d)$=30 kg/m. The curve $m_{oh}(t)$ is a straight line until the time t_d.

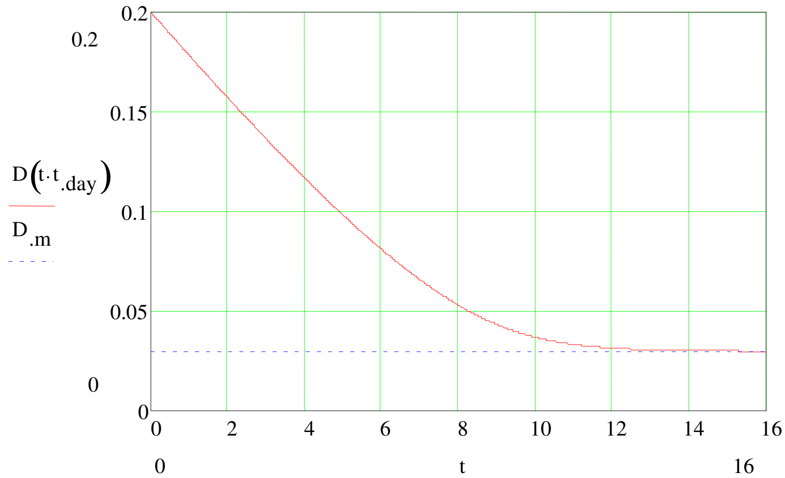


Figure 12. Snow depth as function of time (in days) down to the melting limit D_m=0.030 m

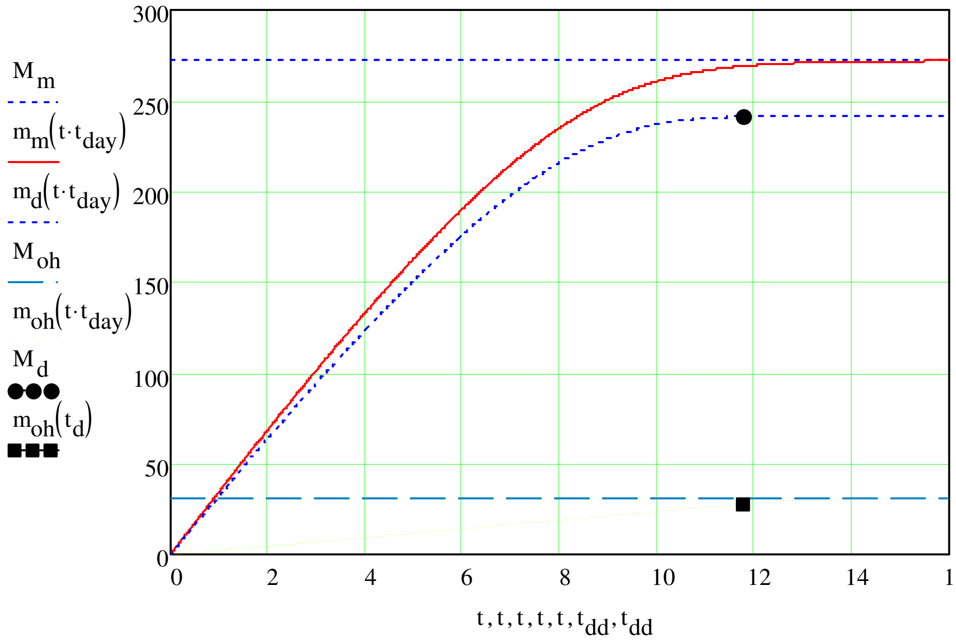


Figure 13. Accumulated melting of snow, ice in overhang and dripping as functions of time (in days).

In the second example we consider a roof with fair thermal insulation or intermediate U-value:

$$U_r = 0.3 \text{ W/(m}^2\text{,K)} \quad \Rightarrow$$
$$D_m = 0.1 \text{ m}, \qquad m_0 = 320 \text{ kg/m}, \qquad q_r = 48 \text{ W/m}, \qquad K_{oh} = 0.92 \text{ W/(m,K)}, \tag{5.23}$$
$$q_{oh} = 9.2 \text{ W/(m,K)}, \qquad t_r = 26 \text{ days}, \qquad D_d = 0.124 \text{ m}, \qquad t_d = 28 \text{ days}.$$

Figure 14 shows the decreasing snow depth from 0.02 m to the melting limit D_m=0.01 m.

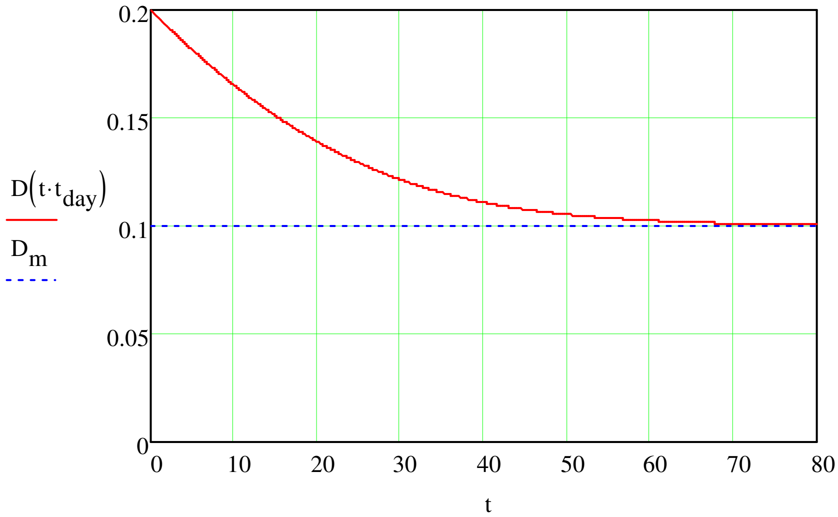


Figure 14. Snow depth as function of time (in days) down to the melting limit D_m=0.1 m

Figure 15 illustrates the melting and dripping in this example. We get from our formulas:

$$M_m = 160 \text{ kg/m}, \qquad M_{oh} = 105 \text{ kg/m}, \qquad M_d = 55 \text{ kg/m}, \tag{5.24}$$
$$t_d = 28 \text{ days} \qquad m_{oh}(t_d) = 68 \text{ kg/m}.$$

There are in Figure 15 two horizontal lines, three curves and two points (a circle and a square) in the figure. The top horizontal line shows the total amount of melting snow M_m=160 kg/m. The first curve (top) shows the accumulated amount of melted snow $m_m(t)$ after t days. The other two curves give the accumulated amount of dripping water, $m_d(t)$, and the accumulated amount of ice in the overhang, $m_{oh}(t)$. The curve for dripping water increases up to the dripping time limit t_d=28 days. After that, the value is the constant and equal to M_d=55 kg/m. The lower horizontal line shows the total amount of ice in the overhang, M_{oh}=105 kg/m. The second point (a square) shows the accumulated amount of ice in the overhang at the time when dripping stops, $m_{oh}(t_d)$=68 kg/m. The curve $m_{oh}(t)$ is a straight line until the time t_d.

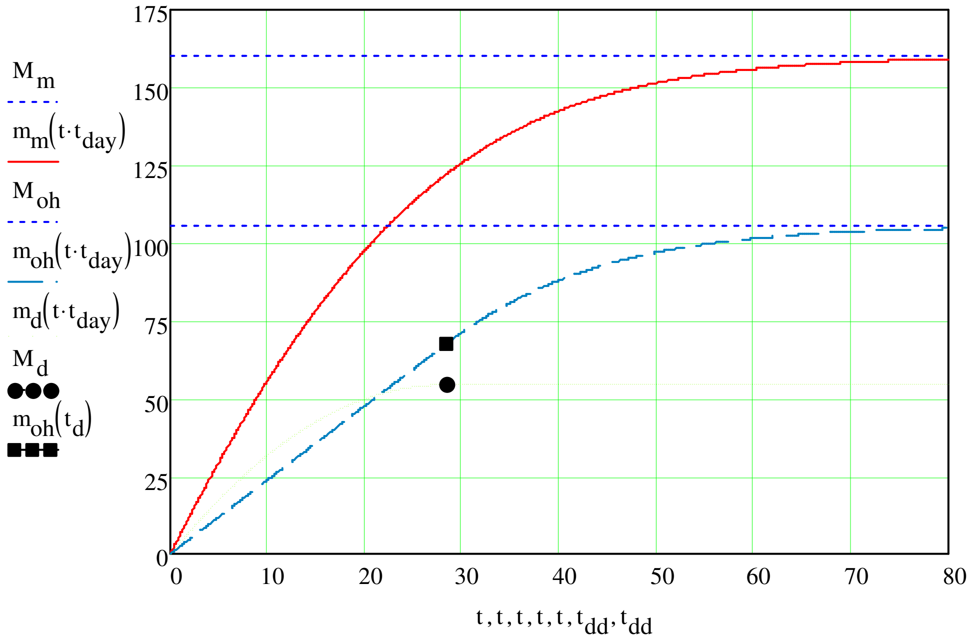


Figure 15. Accumulated melting of snow, ice in overhang and dripping as functions of time (in days).

In the third example we consider a modern roof with good thermal insulation or low U-value:

$$U_r = 0.15 \text{ W}/(\text{m}^2,\text{K}) \quad \Rightarrow \qquad D_m = 0.2 \text{ m} \tag{5.25}$$

This means that the melting limit and the snow depth are equal. There is no melting.

6 Window on the roof

There may be a window on the roof. This is an interesting complication that is studied in this section. The length of the window is L_w, and the length of the remaining roof below and above the window is L_r' and $L_r - L_r'$, respectively. The total roof length from ridge to overhang is $L_r + L_w$, and the roof length excluding the window L_r. See Figure 16.

The notations of Figure 1 for a roof without a window are used. The thickness of the snow on the roof is $D_r(t)$ and on the window $D_w(t)$. The U-value or thermal conductance of the window (between T_r and the upper, outer side of the window) is U_w (W/m^2,K). We assume that $U_w > U_r$, so that the melting of snow is faster on the window: $D_w(t) < D_r(t)$.

The snow melting on the roof above and below the window is identical, and the position of window defined by L_r' does not matter. The analyses and formulas do not depend on L_r'. The window occupies a certain width of the roof. Outside the window area (perpendicular to the cross-section of Figure 12) the previous analyses for a roof is valid. Here we consider a unit width of roof and window.

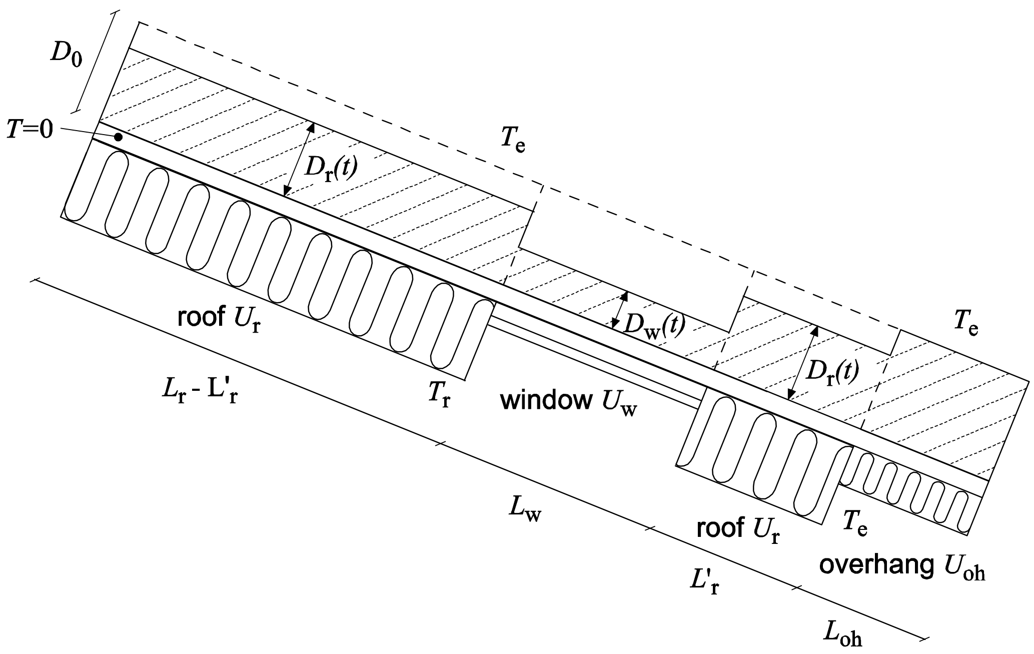


Figure 16. Melting of snow on a roof with a widow.

6.1 Melting of snow on the roof and on the window

The accumulated amounts of melted snow on roof and window are directly obtained from the snow depth. We have as in (3.23):

$$m_{mr}(t) = L_r \rho_s \left(D_0 - D_r(t) \right), \qquad m_{mw}(t) = L_w \rho_s \left(D_0 - D_w(t) \right). \tag{6.1}$$

The initial snow depth on roof and window is D_0. We have for dimensionless snow depth:

$$d_r(t) = \frac{D_r(t)}{D_0}, \qquad d_w(t) = \frac{D_w(t)}{D_0}; \qquad d_r(0) = 1, \qquad d_w(0) = 1. \tag{6.2}$$

$$m_{mr}(t) = L_r \rho_s D_0 \cdot \left(1 - d_r(t)\right), \qquad m_{mw}(t) = L_w \rho_s D_0 \cdot \left(1 - d_w(t)\right). \tag{6.3}$$

The total amount of melted snow becomes:

$$m_m(t) = m_{mr}(t) + m_{mw}(t) = m_0 \cdot \left[1 - L'_r \cdot d_r(t) - L'_w \cdot d_w(t)\right]. \tag{6.4}$$

Here, m_0 (kg/m) is total initial mass of snow on roof and window, and L'_w the relative length of the window:

$$m_0 = \left(L_r + L_w\right) \rho_s D_0, \qquad L'_r = \frac{L_r}{L_r + L_w}, \qquad L'_w = \frac{L_w}{L_r + L_w}. \tag{6.5}$$

The sum of L'_r and L'_w is 1, so L'_r is directly obtained from L'_w :

$$L'_\mathrm{r} + L'_\mathrm{w} = 1, \qquad L'_\mathrm{r} = 1 - L'_\mathrm{w}. \tag{6.6}$$

The relative snow depth for roof and window is obtained from the formulas in Section 2.3 applied for the data of roof and window. We have from (3.20), (3.17), (3.16), (3.12) and (3.4)

$$d_\mathrm{r}(t) = f_d(t/t_\mathrm{r}, d_\mathrm{mr}) \qquad d_\mathrm{w}(t) = f_d(t/t_\mathrm{w}, d_\mathrm{mw}). \tag{6.7}$$

$$t_\mathrm{r} = \frac{h_\mathrm{m}\rho_\mathrm{s}D_0}{U_\mathrm{r}T_\mathrm{r}}, \qquad d_\mathrm{mr} = \frac{\lambda_\mathrm{s}(-T_\mathrm{e})}{U_\mathrm{r}T_\mathrm{r}D_0}, \qquad t_\mathrm{w} = \frac{h_\mathrm{m}\rho_\mathrm{s}D_0}{U_\mathrm{w}T_\mathrm{r}}, \qquad d_\mathrm{mw} = \frac{\lambda_\mathrm{s}(-T_\mathrm{e})}{U_\mathrm{w}T_\mathrm{r}D_0}. \tag{6.8}$$

The explicit formula for the accumulated amount of melted snow is now:

$$m_\mathrm{m}(t) = m_0 \cdot \left[1 - L'_\mathrm{r} \cdot f_d(t/t_\mathrm{r}, d_\mathrm{mr}) - L'_\mathrm{w} \cdot f_d(t/t_\mathrm{w}, d_\mathrm{mw})\right]. \tag{6.9}$$

Using the dimensionless time $\tau = t/t_\mathrm{r}$, we have:

$$m_\mathrm{m}(t) = m_0 \cdot m'_\mathrm{m}(t/t_\mathrm{r}), \qquad m'_\mathrm{m}(\tau) = 1 - L'_\mathrm{r} \cdot f_d(\tau, d_\mathrm{mr}) - L'_\mathrm{w} \cdot f_d(\tau \cdot d_\mathrm{mr}/d_\mathrm{mw}, d_\mathrm{mw}). \tag{6.10}$$

Here, we use the relations:

$$\frac{U_\mathrm{w}}{U_\mathrm{r}} = \frac{t_\mathrm{r}}{t_\mathrm{w}} = \frac{d_\mathrm{mr}}{d_\mathrm{mw}} \;\Rightarrow\; t_\mathrm{r} = t_\mathrm{w} \cdot \frac{d_\mathrm{mr}}{d_\mathrm{mw}}, \qquad \frac{t}{t_\mathrm{w}} = \frac{t}{t_\mathrm{r}} \cdot \frac{t_\mathrm{r}}{t_\mathrm{w}} = \frac{t}{t_\mathrm{r}} \cdot \frac{d_\mathrm{mr}}{d_\mathrm{mw}}. \tag{6.11}$$

The dimensionless amount of melted water, $m_\mathrm{m}(t)/m_0 = m'_\mathrm{m}(\tau)$, becomes a function of the dimensionless time τ with three parameters: d_mr, d_mw and L'_w.

The total amount of melted snow M_m (kg/m) is obtained for very large t:

$$M_\mathrm{m} = m_0 \cdot M'_\mathrm{m}, \qquad M'_\mathrm{m}\left(d_\mathrm{mr}, d_\mathrm{mw}, L'_\mathrm{w}\right) = 1 - \left(1 - L'_\mathrm{w}\right) \cdot d_\mathrm{mr} - L'_\mathrm{w} \cdot d_\mathrm{mw}. \tag{6.12}$$

6.2 Criteria for dripping

The heat flux from the interior minus the heat flux over the snow layer to the exterior melts the snow on roof and window. The heat balance for melting of snow on the roof is from(3.5), (3.9) and (3.2) :

$$h_\mathrm{m} \cdot \frac{dm_\mathrm{mr}}{dt} = L_\mathrm{r}U_\mathrm{r}T_\mathrm{r} - \frac{L_\mathrm{r}\lambda_\mathrm{s}(-T_\mathrm{e})}{D_\mathrm{r}(t)}, \qquad q_\mathrm{r} = L_\mathrm{r}U_\mathrm{r}T_\mathrm{r}. \tag{6.13}$$

Here, q_r is the heat flux through the roof from the interior. The corresponding relations for the window become:

$$h_{\mathrm{m}} \cdot \frac{dm_{\mathrm{mw}}}{dt} = L_{\mathrm{w}} U_{\mathrm{w}} T_{\mathrm{r}} - \frac{L_{\mathrm{w}} \lambda_{\mathrm{s}} (-T_{\mathrm{e}})}{D_{\mathrm{w}}(t)}, \qquad q_{\mathrm{w}} = L_{\mathrm{w}} U_{\mathrm{w}} T_{\mathrm{r}}. \tag{6.14}$$

We introduce special notations for the heat fluxes at the initial time $t = 0$ through the snow on the roof and the window, and their sum:

$$q_{\mathrm{e0r}} = \frac{L_{\mathrm{r}} \lambda_{\mathrm{s}} (-T_{\mathrm{e}})}{D_0}, \qquad q_{\mathrm{e0w}} = \frac{L_{\mathrm{w}} \lambda_{\mathrm{s}} (-T_{\mathrm{e}})}{D_0}, \qquad q_{\mathrm{e0}} = q_{\mathrm{e0r}} + q_{\mathrm{e0w}}. \tag{6.15}$$

We get the relations:

$$q_{\mathrm{e0}} = \frac{\left(L_{\mathrm{r}} + L_{\mathrm{w}}\right) \lambda_{\mathrm{s}} (-T_{\mathrm{e}})}{D_0}, \qquad \frac{q_{\mathrm{e0r}}}{q_{\mathrm{e0}}} = L'_{\mathrm{r}}, \qquad \frac{q_{\mathrm{e0w}}}{q_{\mathrm{e0}}} = L'_{\mathrm{w}}. \tag{6.16}$$

We note the further relations:

$$q_{\mathrm{e0}} = q_{\mathrm{r}} \cdot d_{\mathrm{mr}} + q_{\mathrm{w}} \cdot d_{\mathrm{mw}}, \qquad q_{\mathrm{r}} \cdot t_{\mathrm{r}} + q_{\mathrm{w}} \cdot t_{\mathrm{w}} = h_{\mathrm{m}} \cdot m_0. \tag{6.17}$$

The heat balances for melting of snow on roof and window may now be written:

$$h_{\mathrm{m}} \cdot \frac{dm_{\mathrm{mr}}}{dt} = q_{\mathrm{r}} - \frac{q_{\mathrm{e0r}}}{d_{\mathrm{r}}(t)} = q_{\mathrm{r}} \cdot \left(1 - \frac{d_{\mathrm{mr}}}{d_{\mathrm{r}}(t)}\right), \qquad d_{\mathrm{mr}} = \frac{q_{\mathrm{e0r}}}{q_{\mathrm{r}}}. \tag{6.18}$$

$$h_{\mathrm{m}} \cdot \frac{dm_{\mathrm{mw}}}{dt} = q_{\mathrm{w}} - \frac{q_{\mathrm{e0w}}}{d_{\mathrm{w}}(t)} = q_{\mathrm{w}} \cdot \left(1 - \frac{d_{\mathrm{mw}}}{d_{\mathrm{w}}(t)}\right), \qquad d_{\mathrm{mw}} = \frac{q_{\mathrm{e0w}}}{q_{\mathrm{w}}}. \tag{6.19}$$

The total heat balance for snow melting is now:

$$h_{\mathrm{m}} \cdot \frac{dm_{\mathrm{m}}}{dt} = h_{\mathrm{m}} \cdot \frac{dm_{\mathrm{mr}}}{dt} + h_{\mathrm{m}} \cdot \frac{dm_{\mathrm{mw}}}{dt} = q_{\mathrm{r}} + q_{\mathrm{w}} - \frac{q_{\mathrm{e0r}}}{d_{\mathrm{r}}(t)} - \frac{q_{\mathrm{e0w}}}{d_{\mathrm{w}}(t)}, \tag{6.20}$$

or

$$h_{\mathrm{m}} \cdot \frac{dm_{\mathrm{m}}}{dt} = q_{\mathrm{r}} + q_{\mathrm{w}} - q_{\mathrm{e0}} \cdot \left(\frac{L'_{\mathrm{r}}}{d_{\mathrm{r}}(t)} + \frac{L'_{\mathrm{w}}}{d_{\mathrm{w}}(t)}\right). \tag{6.21}$$

Dripping occurs when the net heat flux to melt snow exceeds the cooling heat flux from the overhang:

$$h_{\mathrm{m}} \cdot \frac{dm_{\mathrm{m}}}{dt} \geq q_{\mathrm{oh}} \quad \Leftrightarrow \quad q_{\mathrm{r}} + q_{\mathrm{w}} - q_{\mathrm{e0}} \cdot \left(\frac{L'_{\mathrm{r}}}{d_{\mathrm{r}}(t)} + \frac{L'_{\mathrm{w}}}{d_{\mathrm{w}}(t)}\right) \geq q_{\mathrm{oh}}. \tag{6.22}$$

The factor after q_{e0} is equal to 1 for $t = 0$. The criterion for dripping at the initial time $t = 0$, and the criterion for no dripping are then

$$\text{Dripping at } t = 0: \quad q_{\mathrm{r}} + q_{\mathrm{w}} - q_{\mathrm{e0}} > q_{\mathrm{oh}}; \qquad \text{No dripping}: \quad q_{\mathrm{r}} + q_{\mathrm{w}} < q_{\mathrm{e0}}. \tag{6.23}$$

All melted water from roof and window freezes on the overhang without dripping in the case of no dripping. We introduce a dimensionless dripping parameter:

$$q'_{\mathrm{oh}} = \frac{q_{\mathrm{oh}}}{q_{\mathrm{r}} + q_{\mathrm{w}} - q_{\mathrm{e0}}}. \tag{6.24}$$

The dimensionless heat loss from the overhang as defined above is smaller than one (and larger than zero) when dripping occurs:

$$q_{\mathrm{r}} + q_{\mathrm{w}} - q_{\mathrm{e0}} > q_{\mathrm{oh}} \quad \Leftrightarrow \quad 0 < q'_{\mathrm{oh}} < 1. \tag{6.25}$$

The dripping stops at the time t_{d}:

$$t = t_{\mathrm{d}}: \qquad h_{\mathrm{m}} \cdot \left.\frac{dm_{\mathrm{m}}}{dt}\right|_{t=t_{\mathrm{d}}} = q_{\mathrm{oh}} \quad \text{or} \quad \frac{q_{\mathrm{r}} + q_{\mathrm{w}} - q_{\mathrm{oh}}}{q_{\mathrm{e0}}} = \frac{L'_{\mathrm{r}}}{d_{\mathrm{r}}(t_{\mathrm{d}})} + \frac{L'_{\mathrm{w}}}{d_{\mathrm{w}}(t_{\mathrm{d}})}. \tag{6.26}$$

The left-hand ratio of heat fluxes may be written in the following way:

$$q'_{\mathrm{d}} = \frac{q_{\mathrm{r}} + q_{\mathrm{w}} - q_{\mathrm{oh}}}{q_{\mathrm{e0}}} = 1 + \frac{(q_{\mathrm{r}} + q_{\mathrm{w}} - q_{\mathrm{e0}})(1 - q'_{\mathrm{oh}})}{q_{\mathrm{e0}}}. \tag{6.27}$$

We have from (6.18), (6.19) and (6.16):

$$\frac{q_{\mathrm{r}}}{q_{\mathrm{e0}}} = \frac{q_{\mathrm{e0r}}}{d_{\mathrm{mr}} q_{\mathrm{e0}}} = \frac{L'_{\mathrm{r}}}{d_{\mathrm{mr}}}, \qquad \frac{q_{\mathrm{w}}}{q_{\mathrm{e0}}} = \frac{q_{\mathrm{w}}}{d_{\mathrm{mw}} q_{\mathrm{e0}}} = \frac{L'_{\mathrm{w}}}{d_{\mathrm{mw}}}. \tag{6.28}$$

So we have:

$$q'_{\mathrm{d}} = q'_{\mathrm{oh}} + \left[\frac{L'_{\mathrm{r}}}{d_{\mathrm{mr}}} + \frac{L'_{\mathrm{w}}}{d_{\mathrm{mw}}}\right] \cdot (1 - q'_{\mathrm{oh}}), \qquad L'_{\mathrm{r}} = 1 - L'_{\mathrm{w}}. \tag{6.29}$$

Let τ_{d} denote the dimensionless time when dripping stops:

$$t_{\mathrm{d}} / t_{\mathrm{r}} = \tau_{\mathrm{d}}, \qquad t_{\mathrm{w}} / t_{\mathrm{r}} = \tau_{\mathrm{d}} \cdot d_{\mathrm{mr}} / d_{\mathrm{mw}}. \tag{6.30}$$

The equation to determine τ_{d} is now from (6.26), right:

$$q'_{\mathrm{d}} = \underbrace{\frac{L'_{\mathrm{r}}}{f_d(\tau_{\mathrm{d}}, d_{\mathrm{mr}})} + \frac{L'_{\mathrm{w}}}{f_d(\tau_{\mathrm{d}} \cdot d_{\mathrm{mr}} / d_{\mathrm{mw}}, d_{\mathrm{mw}})}}_{h(\tau_{\mathrm{d}})} \quad \Rightarrow \quad \tau_{\mathrm{d}} = \tau_{\mathrm{d}}(q'_{\mathrm{oh}}, d_{\mathrm{mr}}, d_{\mathrm{mw}}, L'_{\mathrm{w}}). \tag{6.31}$$

Here, q'_{d} is given by (6.29). The time when dripping stops was given explicitly by (4.22) in the case without window. Here, we have to solve the above equation. The dimensionless dripping time $\tau_{\rm d}$ depends on the four parameters $q'_{\rm oh}$, $d_{\rm mr}$, $d_{\rm mw}$ and $L'_{\rm w}$. Dripping occurs in the interval $0 < q'_{\rm oh} < 1$, while $\tau_{\rm d}$ may vary from zero to infinity. At the interval ends we have:

$$\begin{aligned} & q'_{\rm oh} = 1: \quad q'_{\rm d} = 1, \qquad h(0) = \frac{L'_{\rm r}}{1} + \frac{L'_{\rm w}}{1} = 1 \quad \Rightarrow \quad h(0) = q'_{\rm d}; \\ & q'_{\rm oh} = 0: \quad q'_{\rm d} = \frac{L'_{\rm r}}{d_{\rm mr}} + \frac{L'_{\rm w}}{d_{\rm mw}}, \qquad h(\infty) = \frac{L'_{\rm r}}{d_{\rm mr}} + \frac{L'_{\rm w}}{d_{\rm mw}} \quad \Rightarrow \quad h(\infty) = q'_{\rm d}. \end{aligned} \tag{6.32}$$

This means that $\tau_{\rm d}(q'_{\rm oh}, d_{\rm mr}, d_{\rm mw}, L'_{\rm w})$ varies from zero to infinity when $q'_{\rm oh}$ goes from 1 to 0:

$$\tau_{\rm d}(1, d_{\rm mr}, d_{\rm mw}, L'_{\rm w}) = 0, \qquad \tau_{\rm d}(0, d_{\rm mr}, d_{\rm mw}, L'_{\rm w}) = \infty. \tag{6.33}$$

6.3 Freezing in overhang and dripping at the outer end

The accumulated melted snow is equal to the freezing in the overhang and the dripping:

$$m_{\rm m}(t) = m_{\rm oh}(t) + m_{\rm d}(t). \tag{6.34}$$

The corresponding dimensionless quantities are denoted by prime:

$$m_{\rm m}(t) = m_0 \cdot m'_{\rm m}(\tau), \qquad m_{\rm oh}(t) = m_0 \cdot m'_{\rm oh}(\tau), \qquad m_{\rm d}(t) = m_0 \cdot m'_{\rm d}(\tau). \tag{6.35}$$

During the dripping period, there is a constant freezing of melted water in the overhang determined by the overhang heat loss $q_{\rm oh}$:

$$\begin{aligned} & 0 \le t \le t_{\rm d}: \quad h_{\rm m} \cdot \frac{dm_{\rm oh}}{dt} = h_{\rm m} m_0 \cdot \frac{dm'_{\rm oh}}{d\tau} \cdot \frac{1}{t_{\rm r}} = q_{\rm oh} \quad \Rightarrow \\ & m_{\rm oh}(t) = \frac{q_{\rm oh}}{h_{\rm m}} \cdot t, \qquad \frac{dm'_{\rm oh}}{d\tau} = q''_{\rm oh} \cdot \tau, \qquad q''_{\rm oh} = \frac{q_{\rm oh} \cdot t_{\rm r}}{m_0 \cdot h_{\rm m}}. \end{aligned} \tag{6.36}$$

The accumulated amount of ice in the overhang increases linearly in time. The slope for the dimensionless increase becomes:

$$\begin{aligned} & q''_{\rm oh} = \frac{q_{\rm oh} \cdot t_{\rm r}}{m_0 \cdot h_{\rm m}} = \frac{q'_{\rm oh} \cdot (q_{\rm r} + q_{\rm w} - q_{\rm e0})}{(L_{\rm r} + L_{\rm w})\rho_{\rm s} D_0 \cdot h_{\rm m}} \cdot \frac{h_{\rm m} \rho_{\rm s} D_0}{U_{\rm r} T_{\rm r}} = \frac{q'_{\rm oh} \cdot (q_{\rm r} + q_{\rm w} - q_{\rm e0})}{q_{\rm r} + q_{\rm w} \cdot \dfrac{d_{\rm mw}}{d_{\rm mr}}} = \\ & \frac{q'_{\rm oh} \cdot d_{\rm mr} \cdot (q_{\rm r} + q_{\rm w} - q_{\rm e0})}{q_{\rm e0r} + q_{\rm e0w}} = q'_{\rm oh} \cdot d_{\rm mr} \cdot \left(\frac{q_{\rm r}}{q_{\rm e0}} + \frac{q_{\rm w}}{q_{\rm e0}} - 1 \right). \end{aligned} \tag{6.37}$$

In the last expression (6.28) is used. So we have the formula:

$$q''_{\rm oh} = q'_{\rm oh} \cdot d_{\rm mr} \cdot \left(\frac{L'_{\rm r}}{d_{\rm mr}} + \frac{L'_{\rm w}}{d_{\rm mw}} - 1 \right). \tag{6.38}$$

The dimensionless amount of ice in the overhang during the dripping period is now:

$$m'_{\mathrm{oh}}(\tau) = q''_{\mathrm{oh}} \cdot \tau, \qquad 0 \le \tau \le \tau_{\mathrm{d}}. \tag{6.39}$$

The dimensionless amount of dripping is equal to the difference between snow melting and the freezing in the overhang. The dripping stops at the time $\tau = \tau_{\mathrm{d}}$. So we have:

$$m'_{\mathrm{d}}(\tau) = \begin{cases} m'_{\mathrm{m}}(\tau) - q''_{\mathrm{oh}} \cdot \tau & 0 \le \tau \le \tau_{\mathrm{d}} \\ m'_{\mathrm{d}}(\tau_{\mathrm{d}}) & \tau_{\mathrm{d}} < \tau < \infty \end{cases}. \tag{6.40}$$

Here, the constant value from the time τ_{d} and onwards becomes:

$$m'_{\mathrm{d}}(\tau_{\mathrm{d}}) = m'_{\mathrm{m}}(\tau_{\mathrm{d}}) - q''_{\mathrm{oh}} \cdot \tau_{\mathrm{d}} = M'_{\mathrm{d}}. \tag{6.41}$$

The melted snow is given by (6.9)-(6.10) and the dripping by (6.40) and (6.35), right. The difference gives the freezing in the overhang for all times:

$$m'_{\mathrm{oh}}(\tau) = m'_{\mathrm{m}}(\tau) - m'_{\mathrm{d}}(\tau), \qquad 0 \le \tau < \infty. \tag{6.42}$$

For the total amounts we have:

$$M_{\mathrm{m}} = M_{\mathrm{oh}} + M_{\mathrm{d}}, \qquad M_{\mathrm{m}} = m_0 \cdot M'_{\mathrm{m}}, \qquad M_{\mathrm{oh}} = m_0 \cdot M'_{\mathrm{oh}}, \qquad M_{\mathrm{d}} = m_0 \cdot M'_{\mathrm{d}}. \tag{6.43}$$

From (6.41) and (6.10) we get:

$$M'_{\mathrm{d}}\left(q'_{\mathrm{oh}}, d_{\mathrm{mr}}, d_{\mathrm{mw}}, L'_{\mathrm{w}}\right) = 1 - L'_{\mathrm{r}} \cdot f_d(\tau_{\mathrm{d}}, d_{\mathrm{mr}}) - L'_{\mathrm{w}} \cdot f_d(\tau_{\mathrm{d}} \cdot d_{\mathrm{mr}} / d_{\mathrm{mw}}, d_{\mathrm{mw}}) - q''_{\mathrm{oh}} \cdot \tau_{\mathrm{d}}. \tag{6.44}$$

Here, τ_{d} is the solution to (6.31), and q''_{oh} is defined by (6.38). Finally we have from (6.43)

$$M'_{\mathrm{oh}}\left(q'_{\mathrm{oh}}, d_{\mathrm{mr}}, d_{\mathrm{mw}}, L'_{\mathrm{w}}\right) = M'_{\mathrm{m}}\left(d_{\mathrm{mr}}, d_{\mathrm{mw}}, L'_{\mathrm{w}}\right) - M'_{\mathrm{d}}\left(q'_{\mathrm{oh}}, d_{\mathrm{mr}}, d_{\mathrm{mw}}, L'_{\mathrm{w}}\right). \tag{6.45}$$

Here, M'_{m} is given by (6.12).

6.4 Summary of formulas

The following dimensionless quantities are used:

$$\begin{aligned} &\tau = \frac{t}{t_{\mathrm{r}}}, \qquad t_{\mathrm{r}} = \frac{h_{\mathrm{m}} \rho_{\mathrm{s}} D_0}{U_{\mathrm{r}} T_{\mathrm{r}}}, \qquad L'_{\mathrm{r}} = \frac{L_{\mathrm{r}}}{L_{\mathrm{r}} + L_{\mathrm{w}}}, \qquad L'_{\mathrm{w}} = \frac{L_{\mathrm{w}}}{L_{\mathrm{r}} + L_{\mathrm{w}}}, \\ &d_{\mathrm{mr}} = \frac{\lambda_{\mathrm{s}}(-T_{\mathrm{e}})}{U_{\mathrm{r}} T_{\mathrm{r}} D_0}, \qquad d_{\mathrm{mw}} = \frac{\lambda_{\mathrm{s}}(-T_{\mathrm{e}})}{U_{\mathrm{w}} T_{\mathrm{r}} D_0}. \end{aligned} \tag{6.46}$$

The melting of snow on roof and window is:

$$m_{\mathrm{m}}(t)=m_0\cdot m'_{\mathrm{m}}(t/t_{\mathrm{r}}),\qquad m_0=\left(L_{\mathrm{r}}+L_{\mathrm{w}}\right)\rho_{\mathrm{s}}D_0,$$
$$m'_{\mathrm{m}}(\tau)=1-L'_{\mathrm{r}}\cdot f_d(\tau,d_{\mathrm{mr}})-L'_{\mathrm{w}}\cdot f_d(\tau\cdot d_{\mathrm{mr}}/d_{\mathrm{mw}},d_{\mathrm{mw}}).\tag{6.47}$$

Here, $f_d(\tau,d_{\mathrm{m}})$ is given by the inverse to (3.17) in accordance with (3.20).

The criterion for dripping is:

$$q_{\mathrm{r}}+q_{\mathrm{w}}-q_{\mathrm{e0}}>q_{\mathrm{oh}};\qquad q_{\mathrm{oh}}=q'_{\mathrm{oh}}\cdot\left(q_{\mathrm{r}}+q_{\mathrm{w}}-q_{\mathrm{e0}}\right),\qquad 0<q'_{\mathrm{oh}}<1.\tag{6.48}$$

The dimensionless dripping limit τ_{d} is the solution to the equation:

$$\frac{L'_{\mathrm{r}}}{f_{\mathrm{d}}(\tau_{\mathrm{d}},d_{\mathrm{mr}})}+\frac{L'_{\mathrm{w}}}{f_{\mathrm{d}}(\tau_{\mathrm{d}}\cdot d_{\mathrm{mr}}/d_{\mathrm{mw}},d_{\mathrm{mw}})}=q'_{\mathrm{d}}=q'_{\mathrm{oh}}+\left[\frac{L'_{\mathrm{r}}}{d_{\mathrm{mr}}}+\frac{L'_{\mathrm{w}}}{d_{\mathrm{mw}}}\right]\cdot\left(1-q'_{\mathrm{oh}}\right).\tag{6.49}$$

It becomes a function of four parameters:

$$\tau_{\mathrm{d}}=\tau_{\mathrm{d}}(q'_{\mathrm{oh}},d_{\mathrm{mr}},d_{\mathrm{mw}},L'_{\mathrm{w}}),\qquad \left(L'_{\mathrm{r}}=1-L'_{\mathrm{w}}\right).\tag{6.50}$$

Dripping occurs during the time $0\le\tau<\tau_{\mathrm{d}}$.

The dimensionless accumulated masses are given by:

$$m_{\mathrm{m}}(t)=m_0\cdot m'_{\mathrm{m}}(\tau),\qquad m_{\mathrm{oh}}(t)=m_0\cdot m'_{\mathrm{oh}}(\tau),\qquad m_{\mathrm{d}}(t)=m_0\cdot m'_{\mathrm{d}}(\tau).\tag{6.51}$$

The accumulated dripping is given by:

$$m'_{\mathrm{d}}(\tau)=\begin{cases}m'_{\mathrm{m}}(\tau)-q''_{\mathrm{oh}}\cdot\tau & 0\le\tau\le\tau_{\mathrm{d}}\\ M'_{\mathrm{d}} & \tau_{\mathrm{d}}<\tau<\infty\end{cases},\qquad q''_{\mathrm{oh}}=q'_{\mathrm{oh}}\cdot d_{\mathrm{mr}}\cdot\left(\frac{L'_{\mathrm{r}}}{d_{\mathrm{mr}}}+\frac{L'_{\mathrm{w}}}{d_{\mathrm{mw}}}-1\right).\tag{6.52}$$

Here, M'_{d} is defined in (6.54). The accumulated ice in the overhang is given by:

$$m'_{\mathrm{oh}}(\tau)=m'_{\mathrm{m}}(\tau)-m'_{\mathrm{d}}(\tau),\qquad 0\le\tau<\infty.\tag{6.53}$$

The total amounts of melted snow, ice in the overhang and dripping are given by:

$$M_{\mathrm{m}}=m_0\cdot M'_{\mathrm{m}},\qquad M_{\mathrm{oh}}=m_0\cdot M'_{\mathrm{oh}},\qquad M_{\mathrm{d}}=m_0\cdot M'_{\mathrm{d}},$$
$$M_{\mathrm{m}}=M_{\mathrm{oh}}+M_{\mathrm{d}},\qquad M'_{\mathrm{m}}=1-L'_{\mathrm{r}}\cdot d_{\mathrm{mr}}-L'_{\mathrm{w}}\cdot d_{\mathrm{mw}},\qquad M'_{\mathrm{oh}}=M'_{\mathrm{m}}-M'_{\mathrm{d}},\tag{6.54}$$
$$M'_{\mathrm{d}}=1-L'_{\mathrm{r}}\cdot f_d(\tau_{\mathrm{d}},d_{\mathrm{mr}})-L'_{\mathrm{w}}\cdot f_d(\tau_{\mathrm{d}}\cdot d_{\mathrm{mr}}/d_{\mathrm{mw}},d_{\mathrm{mw}})-q''_{\mathrm{oh}}\cdot\tau_{\mathrm{d}}.$$

6.5 An example

We consider an example with a window on the roof with the following input data:

$$T_{\mathrm{r}}=20\ ^{\mathrm{o}}\mathrm{C},\quad T_{\mathrm{e}}=-10\ ^{\mathrm{o}}\mathrm{C},\quad L_{\mathrm{r}}=6.5\ \mathrm{m},\quad L_{\mathrm{w}}=1.5\ \mathrm{m},\quad L_{\mathrm{oh}}=0.6\ \mathrm{m},\quad D_0=0.2\ \mathrm{m},$$
$$\lambda_{\mathrm{s}}=0.0\ \mathrm{W/(m,K)},\quad \rho_{\mathrm{s}}=200\ \mathrm{kg/m^3},\quad U_{\mathrm{r}}=0.2,\quad U_{\mathrm{w}}=3.0,\quad U_{\mathrm{oh}}=2.0\ \mathrm{W/(m^2,K)}.\tag{6.55}$$

Figure 17 shows the decreasing snow depth from 0.2 m to the melting limits D_{mr}=0.12 m for the roof and D_{mw}=0.01 m for the window. The time scales are in days and days times 30, respectively. We see that the melting on the roof has a much longer time scale due to the better insulation.

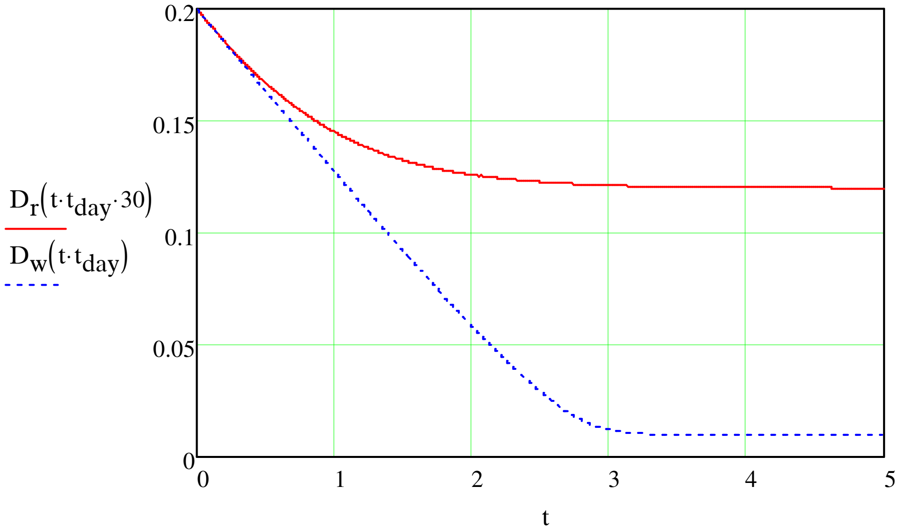


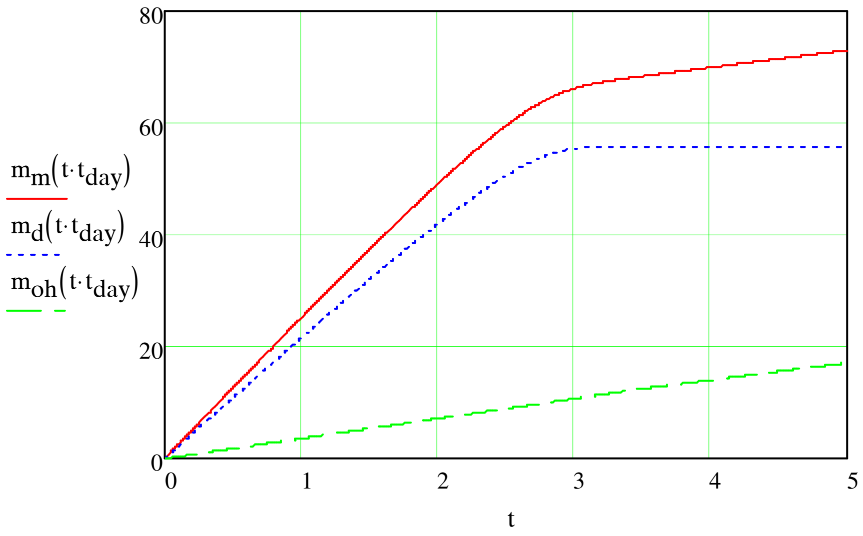
Figure 17. Snow depth as function of time down to the melting limit D_{mr}=0.12 m for the roof in days and down to the melting limit D_{mw}=0.01 m for the window in days times 30.

Figure 18 illustrates the melting and dripping in the considered example. We get from our formulas:

$$M_m = 161\,\text{kg/m}, \quad M_{oh} = 105\,\text{kg/m}, \quad M_d = 56\,\text{kg/m}, \quad t_d = 3.3\,\text{days}. \tag{6.56}$$

The upper graph shows the accumulated amount of melted snow $m_m(t)$ (top curve), the accumulated amount of dripping water and the accumulated amount of ice in the overhang, $m_{oh}(t)$. (lowest curve) during the first 5 days. The increase of dripping water $m_d(t)$ is constant up to t=t_d.

The lower graph shows these curves during the first 100 days. The curves for $m_d(t)$ and $m_{oh}(t)$ cross each other at t=20 days. The horizontal dashed lines show the total amounts M_m=161and M_{oh}=105.



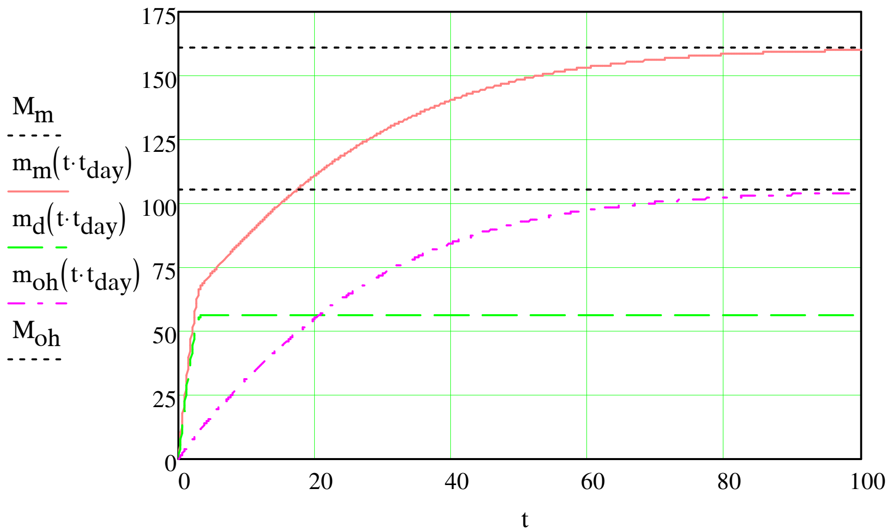


Figure 18. Accumulated melting of snow, ice in overhang and dripping as functions of time (in days).

7 Concluding remarks

The mathematical models in this paper provide a calculation method for melting of snow, freezing on the overhang and dripping from the roof. The diagrams make it possible to compare different roof solutions and see the effect of changing some of the parameters. This is useful in evaluation of risk for icicles on roofs. It is seen that thermal insulation of the roof and ventilation of an open attic is very important to avoid problems with icicles. Roofs with a window (skylight) in the roof will always give more melting water than roofs without and give a higher risk of icicles. Examples on the use of the method will be presented at the 9th Nordic Symposium of Building Physics in Tampere 2011.

Nomenclature

d	dimensionless thickness of the snow layer, $d = D / D_0$	-
d_d	dimensionless limit for dripping, $d_\mathrm{d} = D_\mathrm{d} / D_0$	-
d_m	dimensionless snow thickness limit, $d_\mathrm{m} = D_\mathrm{m} / D_0$	-
$D(t)$	thickness of snow on the roof at time t	m
D_0	initial thickness of snow on the roof at time $t = 0$	m
D_m	snow thickness limit above which melting occurs, (3.4)	m
D_d	snow thickness limit above which dripping occurs, (4.8), left	m
$f_t(d, d_\mathrm{m})$	dimensionless function for time as function of snow depth, (3.16)-(3.17)	-
$f_d(\tau, d_\mathrm{m})$	dimensionless snow thickness, (3.20); inverse to $\tau = f_t(d, d_\mathrm{m})$	-
$g_\mathrm{m}(t)$	rate of snow melting	kg/(s,m)
$g_\mathrm{d}(t)$	rate of water dripping from overhang to form ice, icicles or drops	kg/(s,m)
h_m	latent heat of melting the snow, $h_\mathrm{m} = 334\ 000$	J/kg
K_oh	thermal conductance in the overhang with its snow cover from the ice under the snow to the outside air, (4.2)	W/(Km)
L	roof length (from ridge to overhang)	m
L_oh	length of overhang	m
L_r	length of the roof (above and below the window)	m
L_w	length of window on the roof	m
$m_\mathrm{d}(t)$	accumulated dripping at time t	kg/m
$m'_\mathrm{d}(\tau)$	dimensionless accumulated dripping, (4.21)	-
$m_\mathrm{m}(t)$	accumulated melted snow at time t	kg/m
$m'_\mathrm{m}(\tau)$	dimensionless amount of melted snow, (3.25)	-
$m_\mathrm{oh}(t)$	accumulated ice at the overhang from melted snow at time t	kg/m
$m'_\mathrm{oh}(\tau)$	dimensionless accumulated ice at the overhang	-
m_0	initial mass of snow on the roof, (3.23), right	kg/m
M_d	total amount of melted snow that drips from overhang	kg/m
M'_d	dimensionless total amount of dripping from overhang	-
M_m	total amount of melted snow on the roof	kg/m
M'_m	dimensionless total amount of melted snow on the roof	-
M_oh	total amount of ice on the overhang	kg/m
M'_oh	dimensionless total amount of ice on the overhang,	-
q'_d	the heat flux ratio (6.27) and (6.29) for the dripping limit (6.31)	-
$q_\mathrm{e}(t)$	heat flux from the melting zone the through the snow, (3.5), right	W/m
q_e0	heat flux the through the snow with the initial thickness D_0	W/m
q_i	heat flux from the interior to melt the snow on the roof, (3.5), left	W/m

q_{oh}	heat flux to freeze water under the snow on the overhang	W/m
q'_{oh}	relative heat flux to freeze water on the overhang, (6.24)	W/m
q''_{oh}	slope for dimensionless freezing on the overhang, (6.38)	W/m
t	time	s
t_{r}	time to melt all snow with the thickness D_0 for $T_{\mathrm{e}} = 0$, (3.12)	s
t_{d}	time when dripping stops, (4.22)	s
T_{r}	interior temperature (below roof insulation)	$^{\mathrm{o}}$C
T_{e}	exterior temperature	$^{\mathrm{o}}$C
U_{r}	U-value of the roof	W/(m^2,K)
U_{oh}	U-value of the overhang	W/(m^2,K)
$U_{\mathrm{s}}(t)$	U-value of the snow on the roof at time *t*	W/(m^2,K)

λ_{s}	thermal conductivity of snow on roof	W/(m,K)
ρ_{s}	density of snow on roof	kg/m^3
τ	dimensionless time, t / t_{r}	-
τ_{d}	dimensionless dripping limit, (4.22)	-

The *subscripts* d, e, m, oh, r, s, w and 0 refer to dripping, exterior temperature, melting, overhang, roof, snow, window and initial time, respectively. The subscript *d* (in italics) in $f_d(\tau, d_{\mathrm{m}})$ refers dimensionless snow depth *d*, and not to dripping. A prime is often used to denote the dimensionless form of a quantity. In the case with a window on the roof, a second r or w is added in the subscript whenever appropriate.