LAPORAN TAHUN TERAKHIR
PENELITIAN DASAR UNGGULAN PERGURUAN TINGGI (PDUPT)

PENGEMBANGAN MODEL MATEMATIKA TIPE LOGISTIK DAN PERBANDINGANNYA DENGAN MODEL EMPIRIS: STUDI KASUS DINAMIKA BERAT UNGGAS

TAHUN KE-2 (DUA) DARI RENCANA 2 (DUA) TAHUN

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SESUAI DENGAN PERJANJIAN PENDANAAN PENELITIAN DAN PENGABDIAN KEPADA MASYARAKAT
NOMOR: 122/SP2H/PTNBH/DPRM/2018

UNIVERSITAS AIRLANGGA
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RINGKASAN

Jumlah penduduk Indonesia mengalami peningkatan dari tahun ke tahun. Hal ini menyebabkan kebutuhan daging, termasuk daging ayam juga mengalami peningkatan. Sektor peternakan dituntut untuk menyediakan daging (termasuk daging unggas) untuk mencukupi kebutuhan masyarakat terhadap protein hewani. Diperlukan berbagai upaya strategis dalam bidang peternakan agar kebutuhan masyarakat terhadap daging unggas dapat terpenuhi. Salah satu upaya strategis dalam bidang peternakan adalah peningkatan jumlah populasi ternak.

Selain peningkatan jumlah ternak, upaya strategis lain yang dapat dilakukan untuk strategi pemberian pakan ternak secara optimal. Strategi pemberian pakan ternak optimal perlu mempertimbangkan fase pertumbuhan ternak, yaitu dinamika pertumbuhan ternak terhadap waktu. Dalam hal ini, model matematika yang menggambarkan kurva pertumbuhan ternak memegang peranan penting. Model matematika kurva pertumbuhan dapat digunakan untuk menentukan pemilihan bahan pakan yang optimal dan pemberian waktu pakan yang cocok untuk pengembangan ternak. Model matematika kurva pertumbuhan juga dapat digunakan untuk menentukan umur pemotongan ternak agar diperoleh berat daging optimal. Model matematika pertumbuhan ternak juga dapat digunakan untuk menganalisa efisiensi produksi ternak selama masa hidup ternak tersebut (*lifetime production efficiency*).

Penelitian ini merupakan penelitian non eksperimental dan merupakan penelitian tahun kedua dari rencana dua tahun peneltian. Pada penelitian tahun pertama, Tim peneliti telah mengkonstruksi modifikasi model pertumbuhan logistik yang diturunkan dari modifikasi persamaan diferensial logistic. Modifikasi model pertumbuhan logistik memuat empat parameter yaitu laju pertumbuhan efektif, ukuran maksimum populasi (*mature weight parameter*), waktu kritis dan parameter yang berhubungan dengan berat awal unggas. modifikasi model pertumbuhan logistic dapat digunakan sebagai model alternatif untuk menggambarkan pertumbuhan suatu populasi makhluk hidup. Pada penelitian tahun kedua, tim Peneliti juga mengkaji implementasi model pertumbuhan logistik orde fraksional untuk mendeskripsikan dinamika berat unggas. Tim Peneliti juga mengkaji pengaruh pemotongan data (*trimmed data*) pada hasil estimasi parameter pada beberapa model pertumbuhan populasi. Tim Peneliti juga mengembangkan model pertumbuhan von Bertalanffy orde fraksional, sebagai pengembangan dari model pertumbuhan von Bertalanffy standar.

Hasil penelitian pada tahun pertama dan tahun kedua sudah dipublikasikan. Satu artikel yang berjudul "*On Implementation of Fractional Logistic Growth Model in Describing Rooster Growth*" telah diterima untuk publikasi (*accepted for publication*) pada penerbit "ScitePress". ScitePress merupakan penerbit artikel konferensi terindeks Scopus. Artikel ilmiah berjudul "*A new modified logistic growth model for empirical use*" terkirim ke jurnal internasional "Communication in Biomathematical Sciences" dengan status *Accepted after minor revision*. Artikel ilmiah berjudul "*On Trimmed Data Effect in Parameter Estimation of Some Population Growth Models*" terkirim ke konferensi ICMIs 2018 (International Conference on Mathematics and Islam 2018) dan akan diterbitkan oleh penerbit "ScitePress" *after minor revision*. Artikel ilmiah berjudul "*A Comparison of Continuous Genetic Algorithm and Particle Swarm Optimization in Parameter Estimation of Gompertz Growth Model*" terkirim ke konferensi Symposium on Biomathematics 2018 dengan status *accepted after minor revision*, dan akan diterbitkan pada AIP Conference Proceeding (terindeks Scopus). Satu manuskrip berjudul "*On the fractional order of von Bertalanffy growth model*" terkirim ke konferensi University of Malaya – Indonesian Universities Symposium 2018.

Hasil penelitian tahun kedua sudah dipresentasikan pada seminar ICPS 2018 (The 2nd International Conference Postgraduate School Universitas Airlangga) yang diselenggarakan di Surabaya pada tanggal 10 – 11 Juli 2018 di Surabaya. Hasil penelitian juga sudah dipresentasikan pada seminar ICMIs 2018 yang diselenggarakan di Mataram pada tanggal 3 – 5 Agustus 2018. Hasil penelitian juga sudah dipresentasikan pada seminar Symomath 2018 (Symposium on Biomathematics 2018) yang diselenggarakan di Depok pada tanggal 31 Agustus – 2 September 2018. Hasil penelitian tahun kedua juga dipresentasikan pada University of Malaya – Indonesian Universities Symposium 2018 yang diselenggarakan di Kuala Lumpur pada tanggal 8 – 9 November 2018.

Kata Kunci: pertumbuhan unggas, model matematika, modifikasi model pertumbuhan logistik.

PRAKATA

Tim Peneliti memanjatkan puji syukur kepada *Alloh Subhanahu wa Ta'ala*. Atas rahmat-Nya semata, Laporan Akhir PDUPT 2018 ini dapat terselesaikan dengan baik. Penelitian ini mengambil topik "Pengembangan Model Matematika Tipe Logistik Dan Perbandingannya dengan Model Empiris: Studi Kasus Dinamika Berat Unggas". Penelitian ini bertujuan membangun model matematika alternatif yang dapat digunakan untuk menggambarkan dinamika pertumbuhan unggas dan dinamika pertumbuhan suatu populasi makhluk hidup.

Tim Peneliti menyampaikan terima kasih kepada Ketua dan staf Lembaga Penelitian dan Inovasi (LPI) Universitas Airlangga atas segala dukungan yang diberikan, sehingga penelitian ini dapat berjalan dengan baik. Sampai saat ini, telah dihasilkan lima artikel ilmiah dengan satu artikel berstatus *accepted*, tiga artikel berstatus "*accepted after minor revision*" dan satu artikel berstatus *submitted*. Tim Peneliti berharap bahwa hasil penelitian ini dapat meningkatkan publikasi ilmiah dan sitasi artikel ilmiah staf dosen dan peneliti dari Universitas Airlangga pada jurnal ilmiah internasional bereputasi dan prosiding terindeks Scopus.

Semoga hasil penelitian ini memberikan manfaat bagi para pembaca dan memberikan sumbangan positif dalam peningkatan publikasi ilmiah di Universitas Airlangga. Semoga *Alloh Yang Maha Kuasa* senantiasa memberikan pertolongan kepada Tim Peneliti dalam mempublikasikan hasil – hasil penelitian. Amin.

Surabaya, 14 November 2018
Ketua Tim Peneliti,

Dr. Windarto, M.Si.

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BAB I

PENDAHULUAN

Dalam kehidupan sehari-hari, manusia memanfaatkan unggas untuk memenuhi kebutuhan daging dan telur. Daging unggas merupakan salah satu makanan yang banyak mengandung protein, salah satu zat yang sangat dibutuhkan oleh manusia. Harga daging unggas (misalkan daging ayam) relatif lebih murah dibandingkan dengan daging sapi dan daging kerbau. Akibatnya, di Indonesia, konsumsi daging ayam lebih tinggi dari pada konsumsi daging sapi atau daging kerbau. Data Badan Pusat Statistik menunjukkan bahwa pada tahun 2014, konsumsi perkapita mingguan terhadap daging ayam kampung/ras sebesar 0,086 kg perminggu/penduduk atau 4,472 kg/tahun/penduduk. Konsumsi perkapita daging ayam tersebut jauh lebih tinggi dari konsumsi perkapita daging sapi/kerbau sebesar 0,005 kg perminggu/penduduk atau 0,26 kg/tahun/penduduk (Badan Pusat Statistik, 2017).

Di sisi lain, jumlah penduduk Indonesia mengalami peningkatan dari tahun ke tahun. Hal ini menyebabkan kebutuhan daging, termasuk daging ayam juga mengalami peningkatan. Sektor peternakan dituntut untuk menyediakan daging (termasuk daging unggas) untuk mencukupi kebutuhan masyarakat terhadap protein hewani. Diperlukan berbagai upaya strategis dalam bidang peternakan agar kebutuhan masyarakat terhadap daging unggas dapat terpenuhi. Salah satu upaya strategis dalam bidang peternakan adalah peningkatan jumlah populasi ternak. Dengan meningkatnya jumlah populasi ternak, produksi daging dari hasil peternakan juga akan mengalami peningkatan.

Selain peningkatan jumlah ternak, upaya strategis lain yang dapat dilakukan untuk meningkatkan produksi daging adalah strategi pemberian pakan ternak secara optimal. Strategi pemberian pakan ternak optimal perlu mempertimbangkan fase pertumbuhan ternak, yaitu dinamika pertumbuhan ternak terhadap waktu. Dalam hal ini, model matematika yang menggambarkan kurva pertumbuhan ternak memegang peranan penting. Model matematika kurva pertumbuhan dapat digunakan untuk menentukan pemilihan bahan pakan yang optimal dan pemberian waktu pakan yang cocok untuk pengembangan ternak (Selvaggi dkk, 2015). Model matematika kurva pertumbuhan juga dapat digunakan untuk menentukan umur pemotongan ternak agar diperoleh berat daging optimal. Selain itu, model kurva pertumbuhan juga dapat digunakan sebagai parameter dalam metode seleksi pada waktu pra sapih pada hewan ternak besar misalnya sapi, kerbau, kambing dan domba. Model matematika pertumbuhan ternak juga dapat digunakan untuk menganalisa efisiensi produksi ternak selama masa hidup ternak tersebut (*lifetime production efficiency*) (Inounu dkk, 2007).

Proses pertumbuhan suatu ternak, termasuk unggas dapat diukur dari profil massa (berat) ternak tersebut terhadap waktu (Aggrey, 2002). Model matematika tentang pertumbuhan hewan ternak berbentuk suatu ekspresi matematis yang menggambarkan dinamika berat ternak terhadap waktu. Secara umum, model matematika yang menggambarkan dinamika berat ternak terhadap waktu dapat dibedakan menjadi beberapa pendekatan, yaitu:

(a) Model yang diturunkan dari persamaan diferensial.
(b) Model yang diturunkan dari persamaan empiris.
(c) Model yang diturunkan dari persamaan regresi.
(d) Model yang diturunkan menggunakan pendekatan jaringan syaraf tiruan (*artificial neural networks*).

Model matematika berbentuk persamaan diferensial yang pertama digunakan untuk memodelkan pertumbuhan suatu populasi adalah model eksponesial atau model Malthus (Stewart, 2012). Model eksponensial jarang digunakan untuk memodelkan pertumbuhan populasi, karena model eksponensial menghasilkan populasi tumbuh tanpa batas. Salah satu modifikasi dari model eksponensial adalah model logistik atau model Verhulst. Pada model logistik, populasi tumbuh secara terbatas (Stewart, 2012). Model matematika pertumbuhan populasi yang diturunkan dari suatu persamaan diferensial mempunyai kelebihan dibandingkan dengan model-model lainnya, yaitu bahwa model matematika tersebut mempunyai **interpretasi biologi secara jelas**.

Model matematika pertumbuhan berat ternak yang berbentuk persamaan empiris antara lain adalah model eksponensial (Raji dkk, 2014) dan model logistik (Aggrey, 2002; Roush dan Branton, 2005; Koncagul dan Cadici, 2009; Raji dkk, 2014). Model pertumbuhan eksponensial dan model pertumbuhan logistik berturut-turut diperoleh dari penyelesian persamaan diferensial eksponensial dan persaman diferensial logistik. Model persamaan empiris lainnya adalah model Richard, model Gompertz (Duan-yai dkk, 1999; Aggrey, 2002; Roush dan Branton, 2005; Koncagul dan Cadici, 2009; Eleroglu dkk, 2014; Raji dkk, 2014; Al-Samarai, F.R., 2015), model asimtot, model monomolekuler, model Weibull (Raji dkk, 2014) dan model Bertalanffy (Mohammed, 2015). Ketika diterapkan pada data real, sebagian model empiris menghasilkan galat (kesalahan) model yang cukup signifikan. Selain itu, **parameter-parameter pada sebagian model empiris sulit diinterpretasikan secara jelas**.

Persamaan regresi juga telah digunakan untuk memodelkan pertumbuhan populasi ternak. Aggrey (2002) telah menggunakan regresi spline linear untuk memodelkan pertumbuhan berat unggas. Selain itu, pendekatan model jaringan syarat tiruan juga telah digunakan untuk memodelkan pertumbuhan berat unggas (Ahmad, 2009). Salah satu kekurangan pendekatan model regresi statistik dan pendekatan jaringan saraf tiruan adalah bahwa **nilai parameter yang diperoleh** dari kedua model tersebut **sulit untuk diinterpretasikan secara jelas.**

Berdasarkan uraian tersebut, pengembangan model matematika pertumbuhan berat hewan ternak (termasuk unggas) yang dibangun dari suatu persamaan diferensial mutlak diperlukan. Diperlukan model matematika berbentuk persamaan diferensial yang akurat untuk menjelaskan pertumbuhan berat hewan ternak. Model matematika yang akurat dapat digunakan untuk menentukan pemilihan bahan pakan yang optimal dan pemberian waktu pakan yang cocok untuk pengembangan ternak.

BAB II

TINJAUAN PUSTAKA

Ternak unggas, sebagaimana makhluk hidup halnya makhluk hidup lainnya mengalami pertumbuhan. Pertumbuhan ternak dapat dilihat dari pertambahan berat badan atau ukuran tubuh sesuai dengan umur ternak. Dalam upaya pemenuhan kebutuhan daging melalui bidang peternakan, pengetahuan tentang pola pertumbuhan ternak sangat penting. Model matematika yang menggambarkan kurva pertumbuhan ternak memegang peranan penting. Model matematika kurva pertumbuhan dapat digunakan untuk menentukan pemilihan bahan pakan yang optimal dan pemberian waktu pakan yang cocok untuk pengembangan ternak (Selvaggi dkk, 2015). Model matematika kurva pertumbuhan juga apat digunakan untuk menentukan umur pemotongan ternak agar diperoleh berat daging optimal. Selain itu, model kurva pertumbuhan juga dapat digunakan sebagai parameter dalam metode seleksi pada waktu pra sapih pada hewan ternak besar misalnya sapi, kerbau, kambing dan domba. Model matematika pertumbuhan ternak juga dapat digunakan untuk menganalisa efisiensi produksi ternak selama masa hidup ternak tersebut (*lifetime production efficiency*) (Inounu dkk, 2007).

Proses pertumbuhan suatu ternak, termasuk unggas dapat diukur dari profil massa (berat) ternak tersebut terhadap waktu (Aggrey, 2002). Model matematika tentang pertumbuhan hewan ternak berbentuk suatu ekspresi matematis yang menggambarkan dinamika berat ternak terhadap waktu. Secara umum, model matematika yang menggambarkan dinamika berat ternak terhadap waktu dapat dibedakan menjadi beberapa pendekatan, yaitu model yang diturunkan dari persamaan diferensial, model yang diturunkan dari persamaan empiris, model yang diturunkan dari persamaan regresi dan model yang diturunkan menggunakan pendekatan jaringan syaraf tiruan (*artificial neural networks*).

Model matematika berbentuk persamaan diferensial yang pertama kali digunakan digunakan untuk memodelkan pertumbuhan eksponesial atau model Malthus (Stewart, 2012). Model pertumbuhan eksponensial untuk berat ternak berbentuk

$$\frac{dy}{dt} = ry \tag{2.1}$$

dengan $y(t)$ adalah berat ternak pada saat t dan r adalah laju pertambahan ternak tiap satuan waktu. Penyelesaian model pertumbuhan eksponensial pada persamaan (2.1) diberikan oleh

$$y(t) = y_0 e^{rt} \tag{2.2}$$

dengan y_0 adalah berat ternak pada saat awal $(t = 0)$. Model eksponensial jarang digunakan untuk memodelkan pertumbuhan populasi, karena model eksponensial menghasilkan populasi tumbuh tanpa batas.

Model pertumbuhan logistik atau model Verhulst merupakan pengembangan model pertumbuhan eksponensial. Pada model logistik, populasi tumbuh secara terbatas (Stewart, 2012). Model pertumbuhan logistik diberikan oleh

$$\frac{dy}{dt} = ry\left(1 - \frac{y}{K}\right). \tag{2.3}$$

Pada persamaan (2.3), K menyatakan berat maksimal yang dapat dicapai oleh hewan ternak. Penyelesaian persamaan (2.3) diberikan oleh

$$y(t) = \frac{K}{1 + \exp(-rt)\left(\frac{K}{y_0} - 1\right)}. \tag{2.4}$$

Dengan mendefinisikan

$$t_{inf} = \frac{1}{r}\ln\left(\frac{K}{y_0} - 1\right) \tag{2.5}$$

maka persamaan logistik pada (2.4) dapat dituliskan ke dalam bentuk

$$y(t) = \frac{K}{1 + \exp[-r(t - t_{inf})]}. \tag{2.6}$$

Pada persamaan (2.5) dan (2.6) t_{inf} merupakan *waktu kritis*, yaitu waktu pertumbuhan unggas paling optimal. Model matematika pertumbuhan populasi yang diturunkan dari suatu persamaan diferensial mempunyai kelebihan dibandingkan dengan model-model lainnya, yaitu bahwa model matematika tersebut mempunyai **interpretasi biologi secara jelas**.

Model matematika pertumbuhan berat ternak yang berbentuk persamaan empiris antara lain adalah model eksponensial (Raji dkk, 2014) dan model logistik (Aggrey, 2002; Roush dan Branton, 2005; Koncagul dan Cadici, 2009; Raji dkk, 2014). Model pertumbuhan eksponensial dan model pertumbuhan logistik berturut-turut diperoleh dari penyelesian persamaan diferensial eksponensial dan persaman diferensial logistik. Model pertumbuhan eksponensial dan model pertumbuhan logistik berturut-turut disajikan pada persamaan (2.2) dan persamaan (2.5).

Model persamaan empiris lainnya adalah model Richard, model Gompertz (Duan-yai dkk, 1999; Aggrey, 2002; Roush dan Branton, 2005; Koncagul dan Cadici, 2009; Eleroglu dkk, 2014; Raji dkk, 2014; Al-Samarai, F.R., 2015), model asimtot, model monomolekuler, model Weibull (Raji dkk, 2014) dan model Bertalanffy (Mohammed, 2015). Model Richard disajikan pada persamaan (2.7) berikut (Aggrey, 2002)

$$y(t) = K\left[1 - (1 - m)\, exp\left(-r\,(t - t_{inf})\right)\right]^{1/(1-m)}. \tag{2.7}$$

Pada persamaan (2.6), r_{max} adalah nilai maksimum pertumbuhan relative dan m adalah parameter yang menentukan bentuk kurva pertumbuhan. Model Gompertz disajikan pada persamaan (2.8) berikut (Aggrey, 2002)

$$y(t) = Y_0 exp\left[\frac{A}{B}(1 - \exp(-Bt))\right]. \tag{2.8}$$

dengan Y_0, A dan B merupakan parameter-parameter yang harus ditentukan pada model Gompertz. Model persamaan asimtot, model monomolekuler dan model Weibull berturut-turut disajikan pada persamaan (2.9)-(2.12) berikut:

$$y(t) = A - BC^t, \qquad (2.9)$$

$$y(t) = A[1 - exp(-Bt + C)], \qquad (2.10)$$

$$y(t) = A - B\, exp[-C\, t^D], \qquad (2.11)$$

dan

$$y(t) = A[1 - B \exp(-Ct)]. \qquad (2.12)$$

Pada persamaan (2.9)-(2.12), A, B, C, D merupakan parameter empiris pada model. Ketika diterapkan pada data real, sebagian model empiris menghasilkan galat (kesalahan) model yang cukup signifikan. Kecuali model eksponesial dan model logistik, **parameter-parameter pada sebagian model empiris sulit diinterpretasikan secara jelas.**

Persamaan regresi juga telah digunakan untuk memodelkan pertumbuhan populasi ternak. Aggrey (2002) telah menggunakan regresi spline linear untuk memodelkan pertumbuhan berat unggas. Selain itu, pendekatan model jaringan syarat tiruan juga telah digunakan untuk memodelkan pertumbuhan berat unggas (Ahmad, 2009). Salah satu kekurangan pendekatan model regresi statistik dan pendekatan jaringan saraf tiruan adalah bahwa **nilai parameter yang diperoleh** dari kedua model tersebut **sulit untuk diinterpretasikan secara jelas.**

Berdasarkan uraian tersebut, pengembangan model matematika pertumbuhan berat hewan ternak (termasuk unggas) yang dibangun dari suatu persamaan diferensial mutlak diperlukan. Diperlukan model matematika berbentuk persamaan diferensial yang akurat untuk menjelaskan pertumbuhan berat hewan ternak. Model matematika yang akurat dapat digunakan untuk menentukan pemilihan bahan pakan yang optimal dan pemberian waktu pakan yang cocok untuk pengembangan ternak.

Pada dasarnya, penelitian ini merupakan penelitan lanjutan dari penelitian yang telah dilakukan oleh Tim Peneliti pada waktu-waktu sebelumnya. Peta jalan penelitian Tim Peneliti dalam topik pemodelan matematika dapat bidang kehayatan (*life science*) dapat dilihat pada Tabel 2.1 berikut. Peta jalan riset dari Tim Pengusul sejalan dengan RIP (Rencana Induk Penelitian) Universitas Airlangga, terutama dengan topik penelitian unggulan "Pemodelan dan Desain Sistem di bidang *Life Science*, Ekonomi dan Industri berbasis ICT".

Tabel 2.1. Peta jalan riset Tim Peneliti

Waktu	Tahun 2011 dan sebelumnya	2012-2016	2017 dan tahun-tahun berikutnya
Tahap Lanjut			Pemodelan matematika untuk pada bidang kehayatan lainnya.
Tahap Pengembangan		Pemodelan Matematika Pengendalian Penyebaran HIV Dalam Komunitas Pengguna Narkotika Jarum Suntik (2014-2016) Simulasi model kebocoran plasma akibat infeksi DBD (2014) Konstruksi model pertumbuhan dan pemanenan rumput gajah (2013)	
Tahap Awal (Inisiasi)	Publikasi model matematika penyebaran HIV pada komunitas PSK (2007).		

Pada penelitian tahun pertama, tim peneliti telah mengembangkan modifikasi model pertumbuhan logistik yang berbentuk (Windarto dkk., 2018)

$$y(t) = \frac{K - A\,exp(-\alpha t)}{1+exp(-\alpha(t-t_{inf}))}. \quad (2.13)$$

Model matematika pada persamaan (2.13) tersebut merupakan solusi dari modifikasi persamaan diferensial logistik yang berbentuk

$$\frac{dy}{dt} = (a + ry)\left(1 - \frac{y}{K}\right), y(0) = y_0 > 0 \tag{2.14}$$

dengan $y(t)$ menyatakan berat ternak pada waktu t. Pada persamaan (2.13), K adalah parameter berat maksimal ternak (*mature weight parameter*), α adalah parameter pertumbuhan efektif ternak, t_{inf} adalah waktu kritis (waktu pertumbuhan ternak paling cepat) dan A adalah parameter yang berhubungan dengan berat awal ternak.

Pada penelitian tahun kedua, tim Peneliti mengkaji modifikasi model pertumbuhan logistic untuk mendeskripsikan dinamika berat hewan ternak terhadap waktu. Dari model matematika tersebut, dapat ditentukan waktu kritis pertumbuhan ternak, yaitu waktu dengan pertumbuhan ternak paling cepat. Penentuan waktu kritis tersebut dapat digunakan untuk menentukan "masa panen" atau waktu optimal pemotongan ternak.

BAB III
TUJUAN DAN MANFAAT PENELITIAN

3.1. Tujuan Penelitian

Penelitian ini bertujuan untuk mengkonstruksi suatu model matematika berbentuk persamaan diferensial yang menggambarkan pertumbuhan berat ternak. Dengan pendekatan pemodelan matematika, dapat ditentukan waktu kritis pertumbuhan ternak, yaitu waktu dengan pertumbuhan ternak paling cepat. Penentuan waktu kritis tersebut dapat digunakan untuk menentukan "masa panen" atau waktu optimal pemotongan ternak.

Penelitian ini merupakan penelitian tahun kedua dari rencana dua tahun penelitian. Pada tahun pertama penelitian, Tim peneliti telah mengkonstruksi modifikasi model pertumbuhan logistik yang diturunkan dari modifikasi persamaan diferensial logistic. Modifikasi model pertumbuhan logistik memuat empat parameter yaitu laju pertumbuhan efektif, ukuran maksimum populasi (*mature weight parameter*), waktu kritis dan parameter yang berhubungan dengan berat awal unggas. modifikasi model pertumbuhan logistic dapat digunakan sebagai model alternatif untuk menggambarkan pertumbuhan suatu populasi makhluk hidup.

Tujuan khusus penelitian pada tahun kedua adalah:

1. Melakukan kajian implementasi model pertumbuhan logistik orde fraksional untuk mendeskripsikan dinamika berat unggas.
2. Melakukan kajian matematis sensitivitas hasil estimasi terhadap data-terpotong (*trimmed data*) pada modifikasi model logistik (model yang disajikan pada penelitian tahun pertama), model logistic, model Gompertz, model Richards, model Weibull dan model Bertalanffy.
3. Mengembangkan model model matematika pertumubuhan populasi orde fraksional sebagai pengembangan model matematika yang diperoleh pada tahun pertama dan membandingkan ketelitian hasil prediksi yang diperoleh dari model dengan hasil yang diperoleh dari model empiris yang sudah ada.

3.2. Manfaat Penelitian

Hasil penelitian berupa model matematika yang dapat digunakan untuk **mendeskripsikan pertumbuhan berat hewan ternak (termasuk unggas) secara kualitatif dan secara kuantitatif.** Dengan pendekatan pemodelan matematika, dapat ditentukan waktu kritis pertumbuhan ternak, yaitu waktu dengan pertumbuhan ternak paling cepat. Penentuan waktu kritis tersebut dapat digunakan untuk menentukan "masa panen" atau waktu optimal pemotongan ternak. Hasil penelitian juga diharapkan dapat meningkatkan jumlah publikasi peneliti Indonesia pada jurnal ilmiah internasional bereputasi.

BAB IV
METODE PENELITIAN

4.1. Tempat dan Waktu Penelitian

Tempat penelitian adalah Laboratorium Komputer sebagai Laboratorium Penelitian di Departemen Matematika, Fakultas Sains dan Teknologi Universitas Airlangga. Penelitian ini merupakan penelitian tahun kedua dari rencana penelitian selama dua tahun. Pada tahun kedua ini, penelitian dilaksanakan secara intensif selama 10 bulan, yaitu pada bulan Februari – November 2018.

4.2. Prosedur Penelitian

Penelitian yang diusulkan merupakan penelitian non eksperimental. Fenomena pertumbuhan ternak unggas yang direpresentasikan dengan pertambahan berat hewan ternak akan "dipotret" (dideskripsikan) melalui suatu model matematika. Selanjutnya dilakukan simulasi atas model matematika tersebut. Model yang dikembangkan berbentuk persamaan diferensial dan merupakan modifikasi dari model persamaan logistik. Oleh karena itu, alat terpenting dari penelititian ini adalah komputer dan perangkat lunak (program) komputer untuk mendukung simulasi model matematika.

Prosedur penelitian pada tahun pertama ini adalah sebagai berikut:

(1) Kajian implementasi model pertumbuhan logistic orde fraksional untuk mendeskripsikan dinamika berat unggas, dengan prosedur sebagai berikut:

(a) Memilih model matematika pertumbuhan logistik orde fraksional berbentuk

$$\frac{d^\alpha y}{dt} = ry\left(1 - \frac{y}{K}\right), y(0) = y_0. \quad (4.1)$$

Pada persamaan (4.1), $y(t)$ menyatakan berat unggas pada saat t, r dan K adalah parameter laju pertumbuhan (growth rate) dan berat maksimum unggas (mature weight parameter). Parameter α pada persamaan (4.1) adalah orde turunan fraksional, dengan $0 < \alpha \leq 1$.

(b) Menentukan nilai parameter model pada persamaan (4.1) dengan menggunakan data sekunder pertumbuhan unggas yang dirujuk dari literature (Aggrey, 2002).

(c) Membandingkan ketelitian hasil prediksi yang diperoleh dari model dengan hasil yang diperoleh dari model pertumbuhan logistik.

(2) Melakukan kajian matematis sensitivitas hasil estimasi terhadap data-terpotong (*trimmed data*) pada beberapa model pertumbuhan populasi dengan prosedur sebagai berikut:

(a) Memilih model pertumbuhan logistik, model Gompertz, model Richards, model Weibull, model Morgan-Mercer-Flodin dan modifikasi model pertumbuhan logistik.

(b) Menentukan nilai parameter model pada persamaan prosedur (2a) dengan menggunakan data sekunder pertumbuhan unggas yang dirujuk dari literature (Aggrey, 2002).

(c) Menentukan nilai parameter model pada persamaan prosedur (2a) dengan menggunakan data terpotong berupa data sekunder pertumbuhan unggas yang dirujuk dari literature (Aggrey, 2002).

(d) Menentukan sensitivitas parameter model logistik, model Gompertz, model Richards, model Weibull, model Morgan-Mercer-Flodin dan modifikasi model pertumbuhan logistik terhadap pengaruh data terpotong.

(3) Mengembangkan model model matematika pertumubuhan populasi orde fraksional sebagai pengembangan model matematika yang diperoleh pada tahun pertama, dengan prosedur sebagai berikut:

(a) Memilih persamaan differensial von Bertalanffy orde fraksional yang berbentuk

$$\frac{d^{\alpha}y}{dt} = ay^{2/3} - by = ay^{2/3}\left(1 - \left(\frac{y}{K}\right)^{1/3}\right), y(0) = y_0. \quad (4.2)$$

(b) Menentukan nilai parameter model pada persamaan (4.2) dengan menggunakan data sekunder pertumbuhan unggas yang dirujuk dari literature (Aggrey, 2002).

(c) Membandingkan ketelitian hasil prediksi yang diperoleh dari model dengan hasil yang diperoleh dari model pertumbuhan lain, yaitu model pertumbuhan logistic dan model Gompertz.

4.3. Target Capaian Penelitian

Output yang ditargetkan pada penelitian ini adalah satu artikel terpublikasi pada jurnal internasional/prosiding internasional terindeks Scopus untuk setiap tahun penelitian. Rencana target capaian penelitian disajikan secara rinci pada Tabel 4.1.

Tabel 4.1. Rencana Target Capaian Penelitian

No	Jenis luaran		Indikator Capaian	
			2017	2018
1	Publikasi ilmiah	Internasional	Reviewed	Accepted
		Nasional terakreditasi	Tidak ada	Tidak ada
2	Pemakalah dalam temu Ilmiah	Internasional	Sudah dilaksanakan	Sudah dilaksanakan
		Nasional	Tidak ada	Tidak ada
3	*Inivited speaker* dalam temu Ilmiah	Internasional	Tidak ada	Tidak ada
		Nasional	Tidak ada	Tidak ada
4	*Visiting Lecturer*	Internasional	Tidak ada	Tidak ada
5	Hak Kekayaan Intelektual (HKI)	Paten Paten sederhana Hak cipta Merk dagang Rahasia dagang Desain produk industri Indikasi geografis Perlindungan varietas tanaman Perlindungan topografi sirkuit terpadu	Tidak ada	Tidak ada
6	Teknologi Tepat Guna		Tidak ada	Tidak ada
7	Model/Purwarupa/Desain/Karya seni/ Rekayasa Sosial		Tidak ada	Tidak ada
8	Buku Ajar (ISBN)		Tidak ada	Tidak ada
9	Tingkat Kesiapan Teknologi		1	2

BAB V
HASIL DAN LUARAN YANG DICAPAI

5.1. Implementasi Model Pertumbuhan Logistik Fraksional

Model pertumuban logistic telah digunakan secara luas untuk mendeskripsikan pertumbuhan makhluk hidup. Model pertumbuhan logistic suatu spesies diturunkan dari penyelesaian persamaan diferensial

$$\frac{dy}{dt} = ry\left(1 - \frac{y}{K}\right), y(0) = y_0 \geq 0. \quad (5.1)$$

Pada persamaan (5.1) $y(t)$ menyatakan banykanya populasi spesies tersebut pada saat t. Parameter r dan K adalah laju pertumbuhan perkapita populasi dan maksimum banyaknya populasi yang dapat didukung oleh lingkungan (parameter *carrying capacity*). Jika nilai awal y_0 bernilai positif, maka solusi eksak persamaan diferensial logistic pada persamaan (5.1) diberikan oleh (Aggrey, 2002; Windarto et al., 2014)

$$y(t) = \frac{K}{1+exp\left(-r(t-t_{inf})\right)} \quad (5.2)$$

dengan $t_{inf} = \frac{1}{r}\ln\left(\frac{K}{y_0}\right)$.

Persamaan diferensial logistic pada (5.1) telah diperumum ke dalam suatu persamaan diferensial logistic orde fraksional yang disajikan pada persamaan berikut (El-Sayed et al., 2007)

$$\frac{d^{\alpha}y}{dt^{\alpha}} = ry\left(1 - \frac{y}{K}\right), y(0) = y_0 \geq 0. \quad (5.3)$$

Parameter α merupakan parameter orde fraksional dengan $0 < \alpha \leq 1$. Untuk setiap nilai awal positif y_0, solusi eksak persamaan diferensial logistic orde fraksional belum dapat/tidak dapat ditentukan. Pada situasi ini, metode heuristic seperti algoritma genetika atau particle swarm optimization dapat diterapakan untuk mengesimasi nilai parameter dari persamaan diferensial logistic orde fraksional tersebut (Windarto, 2016).

Particle swarm optimization merupakan metode optimasi yang menggunakan proses pencarian stokastik (probabilistic) berbasis suatu populasi (Eberhart R. & Kennedy, 1995; Kuo et al., 2011). Particle swarm optimization telah digunakan secara luas dalam berbagai bidang, termasuk peningkatan performansi *Artificial Neural Network* (Salerno, 1997; Zhang et al., 2000), masalah penjadwalan (Koay and Srinivasan, 2003; Weijun et al., 2004), masalah perjalanan sales/*traveling salesman problems* (Wang et al, 2003), masalah rute kendaraan pengirim barang/*vehicle routing problems* (Wu et al., 2004) dan analisis kelompok/*clustering analysis* (Kuo et al., 2011).

Particle swarm optimization dimulai dengan pemililah partikel awal (solusi awal) secara acak dalam ruang solusi. Fungsi fitness dari setiap populasi perlu ditentutkan. Jika nilai fitness suatu partikel lebih baik dari nilai terbaik sebelumnya, maka posisi terbaik loka suatu partikel akan diupdate. Partikel terbaik global diupdate berdasarkan nilai fitness terbaik yang diperoleh dari seluruh partikel yang sudah ditemukan.

Berikut adalah prosedur particle swarm optimization. Prosedur berikut diulang sampai ditemukan kondisi terimasi (Kuo et al., 2011; Rini et al., 2011):

1. Hitunglah nilai fitness setiap partikel (solusi). Untuk masalah penentuan nilai maksimum, jika semakin besar nilai fungsi tujuan, maka nilai fungsi fitness juga akan semakin besar. Sebaliknya untuk masalah penentuan nilai minimum, semakin kecil nilai fungsi tujuan, maka akan semakin besar nilai fungsi fitness.
2. Update posisi partikel terbaik local dan posisi partikel terbaik global.
3. Update kecepatan partikel dengan menggunakan persamaan berikut

$$v_i(t+1) = wv_i(t) + c_1r_1\big(lbest(t) - x_i(t)\big) + c_2r_2\big(gbest(t) - x_i(t)\big), \quad (5.4)$$

dengan $v_i(t)$ dan $x_i(t)$ adalah kecepatan partikel i dan posisi partikel i pada waktu diskrit t, $lbest(t)$ dan $gbest(t)$ adalah posisi terbaik local dan posisi terbaik globat pada saat t, r_1 dan r_2 adalah bilangan acak berdistribusi seragam antara nol dan satu.

4. Update posisi setiap partikel menggunakan persamaan berikut

$$x_i(t+1) = x_i(t) + v_i(t+1). \quad (5.5)$$

Pada persamaan (5.4), w adalah bobot inersia, c_1 dan c_2 berturut-turut adalah koefisien kognitif dan koefisien social. Nilai koefisien inersia biasanya antara 0.8 dan 1.2, sedangkan nilai koefisien kognitif dan koefisien social biasanya dekat dengan 2.

Pada bagian ini, model logistik orde fraksional pada persamaan (5.3) digunakan untuk mendeskripsikan pertumbuhan ayam jantan. Nilai parameter pada model logistic orde fraksional akan diestimasi menggunakan data pertumbuhan ayam jantan yang dirujuk dari literature. Data berat ayam jantan (y) pada hari (t) disajikan pada Tabel 5.1 (Aggrey, 2002; Windarto et al., 2014).

Dari Table 5.1, diperoleh bahwa berat awal ayam jantan adalah $y(0) = 37$ gram. Nilai parameter α (orde fraksional), r (laju pertumbuhan) dan K (bobot maksimal ayam jantan dewasa). Nilai parameter bobot inersia yang digunakan adalah $w = 1$, nilai parameter koefisien kognitif $c_1 = 2$ dan nilai parameter koefisien social $c_2 = 2$. Algoritma particle swarm optimization diimplementasikan hingga 100 iterasi.

Table 5.1. Rata-rata berat ayam jantan pada saat t

t (hari)	y (gram)	t (hari)	y (gram)
0	37	42	519.72
3	41.74	45	577.27
6	59.19	48	633.59
9	79.94	51	667.18
12	102.96	54	717.17
15	132.13	57	786.35
18	170.18	71	1069.28
21	206.56	85	1326.49
24	250.71	99	1589.71
27	285.27	113	1859.26
30	324.92	127	2015.44
33	372.83	141	2142.31
36	417.41	155	2220.54
39	469.13	170	2262.63

Nilai parameter pada model logistic orde fraksional (α, r, K) diestimasi sedemikian hingga nilai Rata-rata Jumlah Kuadrat Galat/Mean Squaed Error (MSE) yang diberikan oleh

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \qquad (5.6)$$

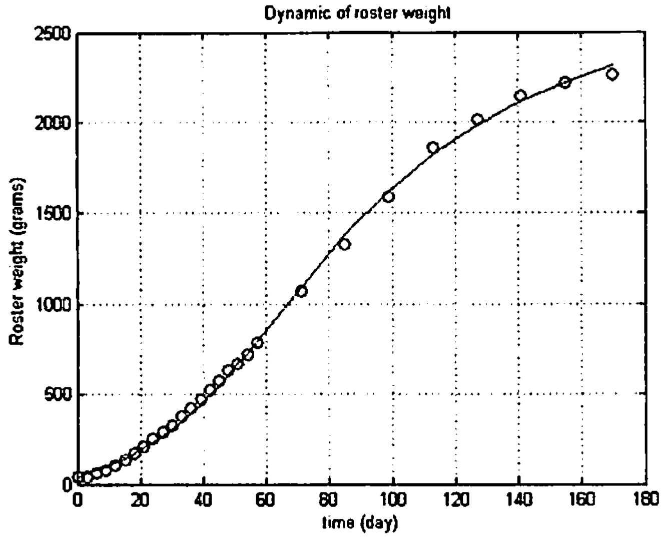
bernilai minimum. is minimum. Pada persamaan (5.6), y_i dan $\hat{y}_i$ adalah data berat ayam jantan dan berat ayam jantan yang dipediksi model pada hari ke-i, dan n adalah banyaknya data pengamatn. re rooster weight data and predicted rooster weight at the i-th day, while n is number of observation data. Hasil estimasi parameter model logistic orde fraksional, disajikan pada Tabel 5.2.

Table 5.2. Nilai parameter hasil estimasi menggunakan metode particle swarm optimization

α	R	K	MSE
0.3999	0.3018	4000.00	714.93
0.4753	0.2461	3491.86	772.41
0.5242	0.2182	3152.67	1001.66
0.4080	0.2946	3898.96	872.93
0.4620	0.2524	3529.82	1248.19
0.4678	0.2500	3565.34	807.87
0.4722	0.2466	3500.00	803.93
0.4395	0.2738	3630.19	710.35
0.4695	0.2500	3500.00	680.08
0.3621	0.3319	4500.00	996.41
0.4705	0.2498	3500.00	711.06

Dari Table 5.2, diperoleh parameter terbaik adalah $\alpha = 0.4695, \mathrm{r} = 0.2500, \mathrm{K} = 3500.00$ dengan nilai MSE = 680.0 8. Di sisi lain, nilai parameter terbaik pada model pertumbuhan logistic adalah $r = 0.0403, t_{inf} = 74.68, K = 2279.90$ dengan nilai MSE = 1887.46. Dari hasil ini, dapat disimpulkan bahwa model logistic orde fraksional adalah lebih akurat dari model pertumbuhan logistic klasik.

Telah diketahui bahwa solusi analitik model logistic orde fraksional akan konvergen ke nilai parameter K. Dalam permasalahan ini, bobot ayam jantan akan menuju nilai maksimum bobot ayam jantan dewasa. Dinamika berat ayam jantan dengan menggunakan nilai parameter terbaik mendukung hasil analitik tersebut. Berat ayam jantan juga menuju ke nilai parameter K. Perbandingan antara data berat ayam jantan dan berat ayam jantan yang diprediksi oleh model, disajikan pada Gambar 5.1. Dari Gambar 5.1, berat ayam jantan yang diprediksi ole model tidak berbeda secara signifikan dengan data. Hal ini menunjukkan bahwa model pertumbuhan logistic orde fraksional dapat digunakan untuk mendeskripsikan dinamika berat ayam jantan dengan baik.



Gambar 5.1. Perbandingan data berat ayam jantan dan berat ayam yang diprediksi dari model logitik fraksional.

5.2. Pengaruh Data Terpotong terhadap Sensitivitas Hasil Estimasi

Model matematika pertumbuhan makhluk hidup dapat diklasifikan menjadi dua kelompok, yaitu model pertumbuhan empiris dan model pertumbuhan dinamik (model pertumbuhan yang diturunkan dari suatu persamaan diferensial). Model pertumbuhan empiris meliputi model pertumbuhan Weibull dan model MMF (Morgan-Mercer-Flodin) growth model. Model Weibull dan model MMF telah digunakan untuk mendeskripsikan dinamika pertumbuhan ayam (Topal dan Bolukbasi, 2008). Model pertumbuhan dinamik meliputi model pertumbuhan logistic, model pertumbuhan Gompertz dan model pertumbuhan Richards. Model pertumbuhan dinamik tersebut telah digunakan untuk memodelkan pertumbuhan berbagai hewan termasuk ayam (Aggrey, 2002), mammalia (Franco dkk., 2011), ikan (Santos dkk., 2013), reptil (Bradsley dkk., 1995) dan amphibi (Mansano dkk., 2013).

Topal dan Bolukbasi melaporkan bahwa model MMF, Weibull dan Gompertz dapat digunakan untuk mendeskripsikan pertumbuhan ayam, mengingat ketiga model pertumbuhan tersebut merupakan model yang paling bersesuaian dengan data pengamatan and Gompertz the MMF, Weibull and Gompertz growth model can be useful for describing chicken growth performance, since these models were the best fitted models (Topal dan Bolukbasi, 2008). Aggrey menemukan bahwa model Richards dan model Gompertz merupakan model yang paling sesuai untuk mendeskripsikan dinamika pertumbuhan ayam jantan dan ayam betina (Aggrey, 2002). Zadeh dan Gooshani melaporkan bahwa model Richards juga merupakan model yang paling sesuai dalam mendeskripsikan pertumbuhan domba Iranian Gulian (Zadeh dan Gholsani, 2016).

Suatu model pertumbuhan dapat dikatakan sebagai model yang baik jika model tersebut memberiakan hasil prediksi yang akurat dan model tersebut *robust* terhadap pemotongan data. Pada bagian ini, akan disajikan hasil kajian efek pemotongan data pada beberapa model pertumbuhan.

Misalkan $y(t)$ adalah berat ayam pada saat t. Model Weibull dan MMF diberikan oleh

$$y(t) = K - (K - A)\exp(-Bt^D), \tag{5.7}$$

dan

$$y(t) = \frac{AB+Ct^D}{B+t^D}, \tag{5.8}$$

Parameter A, B, C, D merupakan parameter empiris (Topal dan Bolukbasi, 2008).

Model pertumbuhan Gompertz diturunkan dari persamaan diferensial Gompertz berikut

$$\frac{dy}{dt} = ry\ln\left(\frac{K}{y}\right), y(0) = y_0 > 0. \tag{5.9}$$

Solusi eksak persamaan diferensial Gompertz pada persamaan (5.9) diberikan oleh

$$y(t) = \frac{K}{exp\left(\exp\left(-r(t-t_{inf})\right)\right)} \tag{5.10}$$

where $t_{inf} = \frac{1}{r}\ln\left(\ln\left(\frac{K}{y_0}\right)\right)$.

Model Richards diturunkan dari persamaan diferensial Richads sebagai berikut

$$\frac{dy}{dt} = ry\left(1 - \left(\frac{y}{K}\right)^\beta\right), y(0) = y_0 > 0. \tag{5.11}$$

Parameter β merupakan "*shape parameter*" pada persamaan diferensial Richards. Untuk β=1, maka persamaan diferensial Richards dapat disederhankan menjadi persamaan diferensial logistic. Oleh karena itu, persamaan diferensial Richards dapat dipandang sebai perluasan persamaan diferensial logistic. Solusi eksak persamaan diferensial Richards pada persamaan (5.11) diberikan oleh

$$y(t) = \frac{K}{\left[1+\beta \exp\left(-r^*(t-t_{inf})\right)\right]^{1/\beta}} \tag{5.12}$$

dengan $r^* = r\beta$, $t_{inf} = \frac{1}{r\beta}\ln\left(\frac{\left(\frac{K}{y_0}\right)^{\beta}-1}{\beta}\right)$.

Pada penelitian tahun pertama, tim Peneliti telah mengembangkan modifikasi model pertumbuhan logistik (dinamakan dengan WEP-logistik) yang berbentuk (Windarto dkk., 2018)

$$y(t) = \frac{K - A\,exp(-\alpha t)}{1+exp(-\alpha(t-t_{inf}))}, 0 < A < K. \tag{5.13}$$

Pada bagian ini, efek data terpotong pada model pertumbuhan tersebut akan dipelajari. Nilai parameter pada model tersebut akan diestimasi menggunakan data pertumbuhan ayam jantan yang dirujuk dari literature. Data berat ayam jantan (y) pada hari (t) disajikan pada Tabel 5.1 (Aggrey, 2002; Windarto et al., 2014). Pada tahap awal, nilai parameter akan diestimasi diestimasi sebelum adanya pemotongan data. Parameter-parameter model akan diestimasi sedemikian hingga MSE yang disajikan pada persamaan (5.6), bernilai minimum.

Metode Lavenberg-Marquardt dapat digunakan untuk menentukan parameter-parameter model sehingga MSE bernilai minimum. Hasil estimasi parameter pada model Weibull, MMF, logistic, Gompertz, Richards dan WEP-logistik, disajikan pada Tabel 5.3. Dari Tabel 5.3 dapat dilihat bahwa model WEP-logistik merupakan model terbaik dan model logistic merupakan model terburuk.

Untuk mengkaji pengaruh efek pemotongan dat, parameter-parameter model juga diestimasi untuk data terpotong (data dari t = 0 sampai 127 hari). Hasil estimasi parameter pada data terpotong, disajikan pada Tabel 5.4. Dari Tabel 5.4 dapat dilihat bahwa model WEP-logistik merupakan model terbaik dan model logistic merupakan model terburuk.

Table 5.3. Hasil estimasi parameter untuk data lengkap

Growth Model	Parameters	Estimated value	MSE
Weibull	K	2426.1709	347.743
	A	58.2211	
	B	0.000197	
	D	1.8699	
MMF	A	67.7095	793.779
	B	14411.3917	
	C	2996.0317	
	D	2.1030	
Logistic	K	2279.9041	1887.461
	R	0.0403	
	t_{inf}	74.6775	
Gompertz	K	2539.6505	384.666
	R	0.0220	
	t_{inf}	63.4975	
Richards	K	2512.9724	376.277
	r*	0.0230	
	t_{inf}	64.3072	
	β	0.0541	
WEP-logistik	K	2399.7491	164.172
	A	2233.4563	
	α	0.0313	
	t_{inf}	71.5844	

Table 5.4. Hasil estimasi parameter pada data terpotong

Growth Model	Parameters	Estimated value	MSE
Weibull	K	2992.8983	111.903
	A	41.9675	
	B	0.000334	
	D	1.6772	
MMF	A	43.5080	139.359
	B	5355.9663	
	C	4540.7213	
	D	1.7266	
Logistic	K	2132.0511	1708.691
	R	0.0433	
	t_{inf}	70.3077	
Gompertz	K	2694.6160	230.084
	R	0.0206	
	t_{inf}	66.8981	
Richards	K	2694.3571	230.053
	r*	0.0206	
	t_{inf}	66.8987	
	β	0.0002	
WEP-logistik	K	2512.7735	128.699
	A	2421.5756	
	α	0.0290	
	t_{inf}	73.6291	

Untuk mengukur efek pemotongan data, didefinisikan indeks sensitivitas parameter untuk seluruh parameter. Untuk sebarang parameter λ, didefinisikan indeks sensitivitas sebagai berikut

$$SI_\alpha = \left|\frac{\lambda-\lambda_{trim}}{\lambda}\right|, \alpha \neq 0. \tag{5.14}$$

Parameter λ_{trim} merupakan nilai parameter setelah proses pemotongan data. Indeks sensitivitas parameter disajikan pada Tabel 5.5.

Table 5.5. Indeks sensitivitas seluruh parameter

Growth Model	Parameters	Sensitivity index	Average value
Weibull	K	0.2336	0.3278
	A	0.2792	
	B	0.6954	
	D	0.1031	
MMF	A	0.3574	0.4201
	B	0.6284	
	C	0.5156	
	D	0.1790	
Logistic	K	0.0649	0.0659
	R	0.0744	
	t_{inf}	0.0585	
Gompertz	K	0.0610	0.0594
	R	0.0636	
	t_{inf}	0.0536	
Richards	K	0.0722	0.3034
	r*	0.1049	
	t_{inf}	0.0403	
	β	0.9961	
WEP-logistik	K	0.0471	0.0581
	A	0.0843	
	α	0.0722	
	t_{inf}	0.0286	

Dari Tabel 5.3 dan 5.4, diperoleh bahwa MSE model Weibull dan model MMF meningkat secara drastic akibat penambahan sedikit dari. Dari Tabel 5.5, nilai rata-rata indeks sensitivitas parameter bervariasi dari 5.81% sampai 42.01%. selain itu, model WEP-logistik merupakan model yang paling robust. Model pertumbuhan empiris lebih sensitive terhadap pemotongan data dibandingkan dengan model pertumbuhan dinamik. Untuk keperluan praktis, model pertumbuhan WEP-logistik dan model Gompertz dapat diaplikasikan untuk mendeskripsikan dinamika pertumbuhan populasi makhluk hidup.

5.3. Model Pertumbuhan von Bertalanffy Orde Fraksional

Model pertumbuhan von Bertalanffy telah digunakan untuk mendeskripsikan pertumbuhan ikan dan banteng. Model pertumbuhan von Bertalanffy dapat diturunkan dari persamaan diferensial von Bertalanffy yang berbentuk (Ohnishi et al., 2012)

$$\frac{dy}{dt} = ay^{2/3} - by, y(0) = y_0. \tag{5.15}$$

Pada persamaan (5.15) a dan b merupakan laju anabolisme dan laju katabolisme. Persamaan diferensial von Bertalanffy tersebut dapat dituliskan ke dalam bentuk

$$\frac{dy}{dt} = ay^{2/3}\left(1 - \left(\frac{y}{K}\right)^{1/3}\right), y(0) = y_0 \tag{5.16}$$

dengan $K = \left(\frac{a}{b}\right)^3$. Solusi analitik persamaan (5.16) berbentuk

$$w(t) = K\left(1 - \left(1 - \left(\frac{w_0}{K}\right)^{1/3}\right)exp(-rt)\right)^3, \tag{5.17}$$

dengan $r = \frac{1}{3}aK^{-1/3}$. Model pertumbuhan von Bertalanffy pada persamaan (5.17) dapat disederhanakan ke dalam bentuk

$$w(t) = K\left(1 - c\ exp(-rt)\right)^3 \tag{5.18}$$

dengan

$$c = 1 - \left(\frac{w_0}{K}\right)^{1/3}. \tag{5.19}$$

Parameter r dan K pada persamaan (5.18) dapat diinterpretasikan sebagai parameter pertumbuhan efektif dan berat maksimal hewan ternak.

Termotivasi oleh artikel El-Sayed dkk. (2007), tim Peneliti mengembangkan model persamaan diferensial von Bertalanffy pada persamaan (5.16) menjadi persamaan diferensial von Bertalanffy orde fraksional yang berbentuk

$$\frac{d^\alpha y}{dt^\alpha} = ay^{2/3}\left(1 - \left(\frac{y}{K}\right)^{1/3}\right), y(0) = y_0 \geq 0. \tag{5.20}$$

dengan $y \in \Omega = \{u \in R: u \geq 0\}$. Pada persamaan (5.20) α merupakan orde turunan fraksional dengan $0 < \alpha \leq 1$. Untuk sebarang nilai awal positif y_0 dan $0 < \alpha < 1$, penyelesaian eksak persamaan diferensial von Bertalanffy orde fraksional pada persamaan (5.20), tidak dapat ditentukan.

Persamaan diferensial von Bertalanffy orde fraksional pada persamaan (5.20) mempunyai dua titik setimbang, yaitu $y = 0$ dan $y = K$. Perlu diperhatikan bahwa titik setimbang $y = 0$ merupakan suatu titik singular, karena $f'(0)$ tidak terdefinisi, dengan $f(y) = ay^{2/3}\left(1 - \left(\frac{y}{K}\right)^{1/3}\right)$. Pada teorema berikut, akan ditunjukkan bahwa titik setimbang $y = 0$ is bersifat tidak stabil.

Teorema 5.1. Titik setimbang $y = 0$ bersifat tidak stabil.

Bukti. Didefinisikan fungsi Lyapunov $U: \Omega \to R$ dengan $U(y) = y$. Fungsi U merupakan fungsi definit positif pada pada domain Ω. Diperoleh

$$ {}_0^C D_t^\alpha y(t) = a y^{2/3} \left(1 - \left(\frac{y}{K}\right)^{1/3}\right). $$

Untuk sebarang y yang cukup dekat dengan 0, diperoleh ${}_0^C D_t^\alpha y(t) > 0$. Akibatnya titik setimbang $y = 0$ bersifat tidak stabil. ■

Pada teorema berikutnya, ditunjukkan bahwa titik setimbang $y = K$ bersifat stabil asimtotis.

Teorema 5.2. Titik setimbang $w = K$ bersifat stabil asimtotis pada domain $\Omega \backslash \{0\} = R^+$.

Bukti. Didefinisikan fungsi Lyapunov $V: \Omega \to R$ dengan $V(y) = \frac{1}{2}(y - K)^2$. Fungsi V merupakan fungsi definit positif pada domain R^+. Turunan Caputo fungsi V adalah

$$ {}_0^C D_t^\alpha V = {}_0^C D_t^\alpha \frac{1}{2}(y - K)^2 = \frac{1}{2} {}_0^C D_t^\alpha (y - K)^2. $$

Dengan menggunakan sifat ${}_0^C D_t^\alpha (y - K)^2 \leq (y - K) {}_0^C D_t^\alpha y$ maka diperoleh

$$ {}_0^C D_t^\alpha V \leq (y - K) {}_0^C D_t^\alpha y(t) = a y^{\frac{2}{3}} (y - K) \left(1 - \left(\frac{y}{K}\right)^{\frac{1}{3}}\right). $$

Cukup jelas bahwa untuk setiap $0 < y < K$, $(y - K)\left(1 - \left(\frac{y}{K}\right)^{\frac{1}{3}}\right) < 0$. Selain itu cukup jelas bahwa untuk sebarang $y > K$, $(y - K)\left(1 - \left(\frac{y}{K}\right)^{\frac{1}{3}}\right) < 0$. Akibatnya untukk sebarang $R^+ \backslash \{K\}$, ${}_0^C D_t^\alpha V < 0$. Dengan menggunakan relasi fungsi definit positif and fungsi dalam kelas-K (*class-K function*) (Slotine and Li, 1999; Aguila-Camacho et al., 2014), dapat disimpulkan bahwa titik setimbang $w = K$ bersifat stabil asimtotis pada domain R^+. ■

Pada bagian ini, model pertumbuhan von Bertalanffy orde fraksional pada persamaan (5.20) digunakan untuk mendeskripsikan pertumbuhan ayam jantan. Nilai parameter pada model pertumbuhan von Bertalanffy orde fraksional akan diestimasi menggunakan data pertumbuhan ayam jantan yang dirujuk dari literature. Data berat ayam jantan (y) pada hari (t) telah disajikan pada Tabel 5.1.

Pada bagian ini digunakan metode *particle swarm optimization* dengan parameter bobot inersia $w = 1$, parameter koefisien kognitif $c_1 = 2$ dan parameter koefisien social c_2. Banyaknya partikel adalah 50 partikel. Metode *particle swarm optimization* diaplikasikan sampai 100 iterasi. Parameters pada model von Bertalanffy orde fraksional (α, r, K, y_0) diestimasi sedemikian hingga nilai *mean square error* (MSE)

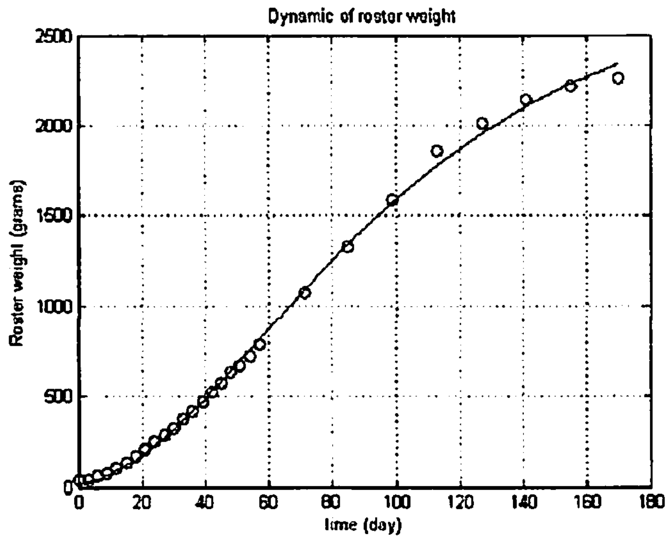
$$ MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 $$

bernilai minimum. Variabel y_i dan $\hat{y}_i$ merupakan data berat ayam jantan dan prediksi berat ayam jantan pada hari ke-i, dengan n adalah banyaknya data pengamatan. Hasil estimasi model von Bertalanffy orde fraksional disajikan pada Tabel 5.6. Hasil estimasi pada Tabel 5.6 tersebut merupakan hasil terbaik dari 20 aplikasi metode particle swarm optimization. Hasil estimasi tersebut tidak seragam, karena metode particle swarm optimization menggunakan bilangan pseudo-random. Hasil estimasi berada pada persekitaran nilai optimal.

Table 5.6. Hasil estimasi model pertumbuhan von Bertalanffy orde fraksional

α	r	K	w_0	MSE
1	0.01489	2939.11	18.70	1200.53
1	0.01631	2683.90	11.34	1407.57
1	0.01633	2722.71	10.00	1084.73
1	0.01651	2734.73	10.00	956.22
1	0.01630	2728.94	10.75	994.43
1	0.01582	2769.14	14.92	921.35
1	0.01593	2798.74	14.64	1016.96
1	0.01636	2750.97	10.07	954.32
1	0.01663	2700.48	10.00	983.22
1	0.01416	2975.68	25.86	1206.97

Dari Tabel 5.6 tersebut, diperoleh bahwa nilai parameter orde fraksional (α) adalah sama dengan satu. Pada kasus ini, model orde fraksional dapat disederhanakan menjadi model pertumbuhan von Bertalanffy standar. Solusi eksak model von Bertalanffy standar disajikan pada persamaan (5.17) dan persamaan (5.18). Karena solusi eksak model telah diketahui, maka metode deterministic seperti metode Newton, Nelder-Mead, atau metode Lavenberg-Marquardt untuk mengestimasi parameter pada model pertumbuhan von Bertalanffy standar. Dengan memanfaatkan metode Lavenberg-Marquardt, diperoleh nilai parameter r = 0.0158, K = 2787.48, w_0 = 14.54, dengan nilai *mean squared error* MSE = 892.65. Dari Gambar 5.2, berat ayam jantan yang diprediksi ole model tidak berbeda secara signifikan dengan data. Hal ini menunjukkan bahwa model pertumbuhan von Bertalanffy dapat digunakan untuk mendeskripsikan dinamika berat ayam jantan dengan baik.



Gambar 5.2. Perbandingan data berat ayam jantan dan berat ayam yang diprediksi dari model von Bertalanffy.

5.4. Luaran Hasil Penelitian

Sampai saat ini, penelitian ini telah menghasilkan luaran penelitian sebagai berikut:

1. Satu manuskrip dengan judul "*On Implementation of Fractional Logistic Growth Model in Describing Rooster Growth*" telah diterima untuk publikasi (*accepted for publication*) pada penerbit "ScitePress". Surat penerimaan (acceptance for publication) dan manuskrip disajikan pada Lampiran 1 dan Lampiran 2.

2. Satu manuskrip berjudul "A new modified logistic growth model for empirical use" terkirim ke jurnal internasional "Communication in Biomathematical Sciences" dengan status *Accepted after Minor Review*. Manuskrip ilmiah disajikan pada Lampiran 3.

3. Satu artikel yang berjudul "*On Trimmed Data Effect in Parameter Estimation of Some Population Growth Models*" terkirim ke konferensi ICMIs 2018 (International Conference on Mathematics and Islam 2018) dengan status *Accepted after minor revision*. Manuskrip ilmiah disajikan pada Lampiran 4.

4. Satu manuskrip berjudul "*A Comparison of Continuous Genetic Algorithm and Particle Swarm Optimization in Parameter Estimation of Gompertz Growth Model*" terkirim ke konferensi Symposium on Biomathematics 2018 dengan status Accepted after minor revision. Manuskrip ilmiah disajikan pada Lampiran 5.

5. Satu manuskrip berjudul "*On the fractional order of von Bertalanffy growth model*" terkirim ke konferensi University of Malaya – Indonesian Universities Symposium 2018. Manuskrip ilmiah disajikan pada Lampiran 6.

6. **Diseminasi hasil penelitian pada konferensi ICMIs 2018 (International Conference on Mathematics and Islam 2018), Symomath 2018 (Symposium on Biomathematics 2018) dan University of Malaya – Indonesian Universities Symposium 2018.**

BAB VI

KESIMPULAN DAN SARAN

6.1. Kesimpulan

(1) Tim Peneliti telah berhasil mengkonstruksi modifikasi model pertumbuhan logistik yang diturunkan dari modifikasi persamaan diferensial logistik. Modifikasi model pertumbuhan logistik memuat empat parameter yaitu laju pertumbuhan efektif, ukuran maksimum populasi, waktu kritis dan parameter yang berhubungan dengan berat awal unggas.

(2) Modifikasi model pertumbuhan logistik dapat digunakan untuk mendeskripsikan dinamika pertumbuhan unggas. Modifikasi model pertumbuhan logistik dapat digunakan sebagai model alternatif untuk menggambarkan pertumbuhan suatu populasi makhluk hidup.

(3) Model pertumbuhan logistik orde fraksional telah berhasil diimplementasikan untuk mendeskripsikan dinamika pertumbuhan ayam jantan.

(4) Model pertumbuhan dinamik (model yang diturunkan dari persamaan diferensial) lebih robust terhadap efek pemotongan data dibandingkan dengan model pertumbuhan empiris.

(5) Tim Peneliti telah mengembangkan model pertumbuhan von Bertalanffy standar menjadi model pertumbuhan von Bertalanffy orde fraksional. Selain itu, model pertumbuhan von Bertalanffy orde fraksional juga telah berhasil diterapkan untuk mendeskripsikan dinamika pertumbuhan ayam jantan, dengan data pertumbuhan ayam jantan dirujuk dari literatur.

6.2. Saran

Pada penelitian berikutnya, dapat dikaji pengembangan model pertumbuhan Richards orde fraksional dan model pertumbuhan WEP-logistik orde fraksional.

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On Implementation of Fractional Logistic Growth Model in Describing Rooster Growth

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Abstract. Fractional order calculus was used in the study of viscoelastic medium (a medium with viscosity and elasticity properties), image signal processing, and population growth modeling. In this paper, we used the fractional order of the logistic growth model to describe the dynamic of rooster growth, where the rooster growth data was cited from the literature. We also used the particle swarm optimization method to estimate parameters in the fractional order logistic model. We found that the fractional order model is more accurate than the classical logistic growth model in describing the rooster growth..

Keywords: fractional order, logistic growth model, particle swarm optimization method, rooster growth.

1 INTRODUCTION

Logistic growth model is widely used to describe a life organism growth. The logistic growth of a single species is governed by the following differential equation

$$\frac{dy}{dt} = ry\left(1-\frac{y}{K}\right), y(0) = y_0 \geq 0. \quad (1)$$

Here $y(t)$ represents number of population of the species at time t, r and K correspond to per capita growth rate and carrying capacity respectively. If the initial value y_0 is positive, then analytical solution of the logistic growth model in Eq. (1) is given by (Aggrey, 2002; Windarto et al., 2014)

$$y(t) = \frac{K}{1+exp\left(-r(t-t_{inf})\right)} \quad (2)$$

where $t_{inf} = \frac{1}{r}\ln\left(\frac{K}{y_0}\right)$.

The logistic growth ordinary differential equation in Equation (1) has been generalized into the fractional order logistic differential equation which is given by (El-Sayed et al., 2007)

$$\frac{d^\alpha y}{dt^\alpha} = ry\left(1-\frac{y}{K}\right), y(0) = y_0 \geq 0. \quad (3)$$

Here α is fractional order where $0 < \alpha \leq 1$. For any positive initial value y_0, the exact solution of fractional order logistic differential equations can not be determined. In this situation, heuristic method such as simulated annealing, genetic algorithm and particle swarm optimization method can be applied to estimate parameter values from the fractional order logistic differential equation.

Particle swarm optimization is an optimization methods based on a population-based stochastic (probabilistic) search process (Eberhart R. & Kennedy, 1995; Kuo et al., 2011). Particle swarm optimization methods have been widely applied in many areas, including performance improve of Artificial Neural Network (Salerno, 1997; Zhang et al., 2000), scheduling problems (Koay and Srinivasan, 2003; Weijun et al., 2004), traveling salesman problems (Wang et al, 2003), vehicle routing problems (Wu et al., 2004) and clustering analysis (Kuo et al., 2011).

In this paper, we applied particle swarm optimization method for predicting the parameters in fractional logistic growth model. The remainder of this paper is organized as follows. Section 2 briefly presents particle swarm optimization method. Section 3 presents implementation of fractional logistic growth model for describing poultry growth. In addition, parameters in the fractional logistic growth will be estimated by using particle swarm optimization method. Finally, conclusions are presented in Section 4.

2 PARTICLE SWARM OPTIMIZATION METHOD

The particle swarm optimization algorithm was invented by Eberhart and Kennedy in 1995. The algorithm has similarities with evolutionary computation methods such as genetic algorithm. The particle swarm optimization algorithm is initialized with a population of random solutions and searches optimal solution updating generations. But particle swarm optimization algorithm does not have crossover and mutation operators. Potential particles (solutions) in the particle swarm optimization algorithm move through the solution space by following the current optimum particles (Kuo et al., 2011).

The particle swarm optimization algorithm starts by randomly choosing initial (particles) solutions within the search space. Fitness function of the current position of every particle is evaluated. If the fitness value is better than the previous best value, then the local best position of a particle is updated. The global best is updated based on the best fitness value found by any of the neighbour.

The particle swarm optimization algorithm consists of the following steps, which are repeated until some termination condition is met (Kuo et al., 2011; Rini et al., 2011):

1. Evaluate the fitness of every particle (solution). For a maximization problem, the greater the objective function then the greater of the fitness will be. On the other hand, for a minimization problem, the smaller the objective function then the greater of the fitness will be.
2. Update particle best (local best) position and global best position.
3. Update velocity of every particle using the following equation
$$v_i(t+1) = wv_i(t) + c_1 r_1 (lbest(t) - x_i(t)) + c_2 r_2 (gbest(t) - x_i(t)), \quad (4)$$
where $v_i(t)$ and $x_i(t)$ are velocity of particle i and position of particle i at discrete time t, $lbest(t)$ and $gbest(t)$ are local best and global best position at time t, r_1 and r_2 are uniform distributed random number between zero and one.
4. Update position of every particle using the following equation
$$x_i(t+1) = x_i(t) + v_i(t+1). \quad (5)$$

In Equation (4), w is the inertia weight, c_1 and c_2 are cognitive coefficient and social coefficient respectively. The value of the inertial coefficient is typically between 0.8 and 1.2, while the values of cognitive coefficient and social coefficient are typically close to 2.

In order to prevent the particles from moving very far beyond the search space, velocity clamping technique could be applied to limit the maximum velocity of every particle. For a search space bounded by the range $[x_{min}, x_{max}]$, the velocity is limited within the range $[-v_{max}, v_{max}]$ where $v_{max} = m(x_{max} - x_{min})$ for some constant $m, 0.1 \leq m \leq 1$. Some common stopping conditions in particle swarm optimization include a predetermined number of iterations, a number of iterations since the last update of global best solution or a preset target fitness value (Kuo et al., 2011; Rini et al., 2011).

3 IMPLEMENTATION OF FRACTIONAL LOGISTIC GROWTH MODEL

In this section, we apply the fractional order logistic growth in Equation (3) for describing rooster growth. We also estimate parameters in the model from some rooster weight data cite form the literature. The rooster weight data (y) at the day (t) is presented in the Table 1 (Aggrey, 2002; Windarto et al., 2014).

Table 1. Means of the rooster weight data (y)

t (days)	y (grams)	t (days)	y (grams)
0	37	42	519.72
3	41.74	45	577.27
6	59.19	48	633.59
9	79.94	51	667.18
12	102.96	54	717.17
15	132.13	57	(1) 786.35
18	170.18	71	1069.28
21	206.56	85	1326.49
24	250.71	99	1589.71
27	285.27	113	1859.26
30	324.92	127	2015.44
33	372.83	141	2142.31
36	417.41	155 (2)	2220.54
39	469.13	170	2262.63

From the Table 1, we found that initial weight of the rooster is $y(0) = 37$ grams. We estimate parameters α (the fractional order), r (the rooster growth rate) and K (carrying capacity parameter or mature weight of the rooster). We apply particle swarm optimization method described in the Section 2 with the inertia weight parameter $w = 1$, the cognitive coefficient parameter $c_1 = 2$ and the social coefficient parameter $c_2 = 2$ respectively. The particle swarm optimization algorithm is implemented until 100 iterations.

Parameters in the fractional order logistic growth model (α, r, K) are estimated such that the mean square error (MSE) which is given by

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \qquad (6)$$

is minimum. Here, y_i and $\hat{y}_i$ are rooster weight data and predicted rooster weight at the i-th day, while n is number of observation data. The estimation results of fractional order logistic growth are presented in the Table 2.

Table 2. The estimated parameters using particle swarm optimization method

α	r	K	MSE
0.3999	0.3018	4000.00	714.93
0.4753	0.2461	3491.86	772.41
0.5242	0.2182	3152.67	1001.66
0.4080	0.2946	3898.96	872.93
0.4620	0.2524	3529.82	1248.19
0.4678	0.2500	3565.34	807.87
0.4722	0.2466	3500.00	803.93
0.4395	0.2738	3630.19	710.35
0.4695	0.2500	3500.00	680.08
0.3621	0.3319	4500.00	996.41
0.4705	0.2498	3500.00	711.06

It was found form Table 2, we found that the best parameters were $\alpha = 0.4695, r = 0.2500, K = 3500.00$ where the mean square error $MSE = 680.08$. Meanwhile, the best parameters for logistic growth model are $r = 0.0403, t_{inf} = 74.68, K = 2279.90$ where the mean square error $MSE = 1887.46$. Hence, we found that the fractional order logistic model was more accurate than the (classical) logistic growth model.

It was known that the analytical solution of the fractional order logistic growth model converge to the carrying capacity parameter or the mature weight parameter (K). Here, asymptotic rooster weight (y(t)) tends to the mature weight parameter. Dynamic of the rooster weight for the best parameters also confirms the analytical properties. The rooster weight also tends to the mature weight parameter. A comparison between observed and predicted rooster weight is shown in the Figure 1. From the figure, the predicted rooster weight of the fractional order logistic model did not significantly differ from the observed data.

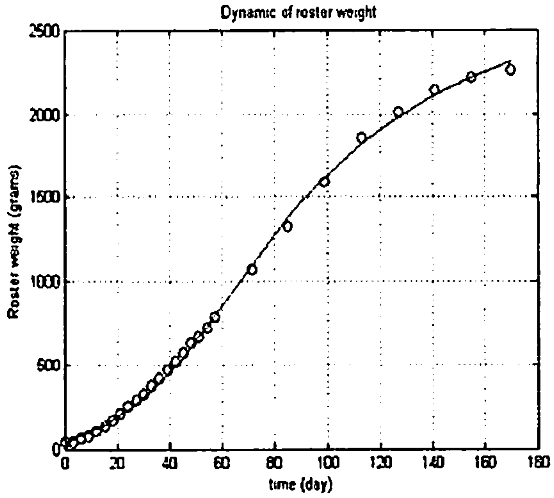

Figure 1: Comparison between observed and predicted rooster weight.

4 CONCLUSIONS

We have applied the fractional order growth model to describe dynamic of rooster weight. We also estimated parameters of the model from secondary data cited from literature. We found that the fractional order logistic model gave more accurate result than the classical logistic growth model.

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A new modified logistic growth model for empirical use

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Abstract

Richards model, Gompertz model, and logistic model are widely used to describe growth model of a population. The Richards growth model is a modification of the logistic growth model. In this paper, we present a new modified logistic growth model. The proposed model was derived from a modification of the classical logistic differential equation. From the solution of the differential equation, we present a new mathematical growth model so called a WEP-modified logistic growth model for describing growth function of a life organism. We also simulated and verified the proposed model by using chicken weight data cited from the literature. It was found that the proposed model gave more accurate predicted results compared to Richard, Gompertz, and logistic model. Therefore the proposed model could be used as an alternative model to describe an individual growth.

Keywords: mathematical model, growth function, modified logistic growth, chicken weight.

1 Introduction

Optimum food utilization strategy is one of the important efforts to increase meat production of a livestock. The dynamics of livestock growth over time is needed to obtain an optimal growth strategy of animal feeds. Mathematical models of the growth curve could be used to determine the selection of suitable feeding materials for livestock development [1]. In addition, the growth curve could also be used to determine the age of livestock slaughter to be optimal. Moreover, the growth curve model could be used as a parameter in pre-harvest methods in large livestock such as cattle, buffalo, goats and sheep. The mathematical model of livestock growth could also be used to analyze the efficiency of livestock production over the lifetime (lifetime production efficiency) [2].

The growth process of a livestock, including poultry could be measured from mass (weight) profile of the livestock versus time [3, 4]. Livestock and poultry growth generally follows a sigmoidal pattern. Poultry growth usually starts by an accelerating growth phase from hatching. Then, poultry attains the maximum growth rate at a certain time (the inflection time). After that, poultry growth is decelerating. At final phase, poultry weight generally tends to a limiting value (asymptote) mature weight [1, 5].

Many nonlinear growth curves have been developed to describe and fit the sigmoid relationship between poultry weight and time. Logistic model, Gompertz model and Richards model are commonly for describing a relationship between poultry weight and time [1, 3, 5]. Richards and Gompertz models have been shown to give good descriptions of weight growth in many species such as cattle, elks, chicken, ostrich, turkey and emus. Gompertz growth model has been used as the growth model for chicken data based on its overall fit and biological meaning of model parameters [6, 7, 8]. Moreover, the Gompertz model has good fitting for weight information whose inflection points occur, when approximately 35 - 40% of growth have been achieved [5].

Simple and accurate growth models are useful for describing life individual growth. In this paper, we present a new mathematical model for growth function of a life organism. The model was derived from modified logistic differential equation. Then, the model was implemented to describe body weight growth of chicken (rooster and hen), where the growth data cite from literature. Accuracy of predicted results from the model was compared to standard logistic model, Gompertz model and Richards model.

This paper is organized as follows. Section 2 presents some modified logistic growth models. The proposed model and its main property is discussed in the section 3. Implementations of the proposed model, logistic model, Gompertz model and Richard model on chicken (rooster and hen) data cited from literature are presented in Section 4. Conclusions are written in the last section.

2 Modified Logistic Growth Model

The first mathematical model describing population growth is the Malthus model or exponential model [9]. Let $y(t)$ is population size at time t In the exponential model, the growth rate $\frac{dy}{dt}$ is assumed proportional the size of existing population $y(t)$. Hence, the exponential model could be represented by the following differential equation

$$\frac{dy}{dt} = ry, \quad y(0) = Y_0. \tag{1}$$

Here r is the proportional growth rate parameter. The exact solution of the exponential growth model in Eq. (1) is given by

$$y(t) = Y_0 \exp(rt). \tag{2}$$

The exponential growth model in Eq. (2) is rarely used to describe population growth, since it produces an unbounded population growth.

The exponential growth model was improved by logistic growth model. In the logistic model, a population grows until it attains a maximum capacity [9]. The logistic model is based on the assumption that the growth rate $\frac{dy}{dt}$ is proportional to the existing population and the remaining resources available to the existing population. Hence the logistic differential equation could be expressed as

$$\frac{dy}{dt} = ry\left(1 - \frac{y}{K}\right), \quad y(0) = Y_0. \tag{3}$$

When $y(t)$ represents body weight of a livestock at time t, then parameter K in Eq. (3) could be considered as the mature weight (the maximum weight that could be attained by a livestock). The analytical solution of Eq. (3) is given by

$$y(t) = \frac{K}{1 + \exp(-rt)\left(\frac{K}{Y_0} - 1\right)}. \tag{4}$$

By defining

$$t_{\inf} = \frac{1}{r} \ln\left(\frac{K}{Y_0} - 1\right), \tag{5}$$

then the logistic growth model could be presented in the following form

$$y(t) = \frac{K}{1 + \exp[-r(t - t_{\inf})]}. \tag{6}$$

Here $t_{\inf}$ is the inflection time (the optimal time of a population growth).

The logistic growth model has various modifications. One of the modified version is the shifted logistic function. The first version of the shifted logistic function could be presented in the following form [10]

$$y(t) = K\left(\frac{1}{1 + \exp(-r(t - t_{\inf}))} - \frac{1}{1 + \exp(rt_{\inf})}\right). \tag{7}$$

The second version and the third version of the shifted logistic function could be expressed as [11]

$$y(t) = \frac{K}{1 + \exp(-r(t - t_{\inf}))} + L \tag{8}$$

and

$$y(t) = \frac{K + Mt}{1 + \exp(-r(t - t_{\inf}))} + L \tag{9}$$

respectively. Here, L and M are additional parameters. Modification of logistic growth model also occurred in the differential equations model. The logistic differential equation has been modified into von Bertalanffy, Richards, Gompertz, Blumberg, Turner et al. and Tsoularis differential equations. The von Bertalanffy differential equation has the following form [12, 13]

$$\frac{dy}{dt} = ry^{\frac{2}{3}}\left(1 - (\frac{y}{K})^{\frac{1}{3}}\right), \ y(0) = Y_0. \tag{10}$$

Richards (1959) proposed a modified logistic differential equation so-called Richards differential equation. The Richards differential equation has the following form [13, 14]

$$\frac{dy}{dt} = ry\left(1 - (\frac{y}{K})^{\beta}\right), \ y(0) = Y_0. \tag{11}$$

Gompertz differential equation is a limiting case of a modified logistic differential equation. The Gompertz differential equation is derived from

$$\frac{dy}{dt} = \lim_{\beta \to 0} \frac{ry\left(1 - (\frac{y}{K})^{\beta}\right)}{\beta} = ry\ln(\frac{K}{y}),\ y(0) = Y_0. \tag{12}$$

Blumberg (1968) also introduced a modification of logistic differential equation so called the hyper logistic function, accordingly [13, 15]

$$\frac{dy}{dt} = ry^{\alpha}\left(1 - \frac{y}{K}\right)^{\gamma},\ y(0) = Y_0. \tag{13}$$

Turner et al. (1976) proposed a modified logistic differential equation which they named the generic growth function. The modification has the following form [13, 16]

$$\frac{dy}{dt} = ry^{1+\beta(1-\gamma)}\left(1 - (\frac{y}{K})^{\beta}\right)^{\gamma},\ y(0) = Y_0. \tag{14}$$

Tsoularis (2001) proposed a more general modification of logistic differential equation. The Tsoularis differential equation has the form [13]

$$\frac{dy}{dt} = ry^{\alpha}\left(1 - (\frac{y}{K})^{\beta}\right)^{\gamma},\ y(0) = Y_0. \tag{15}$$

In the next section, we propose another version of a modified logistic differential equation.

3 The proposed model

The logistic growth model and the modified logistic growth model presented in the previous section could be represented in the Kolmogorov form

$$\frac{dy}{dt} = yP(y) \tag{16}$$

for some continuous function P. For classical (standard) logistic differential equation, the function P is $P(y) = r(1 - \frac{y}{K})$. In the logistic growth model, it is assumed that the growth rate of a population is proportional to the population number at the current time. Here, we modify the model in Eq. (16) in more general form, namely

$$\frac{dy}{dt} = F(y) \tag{17}$$

for some continuous function F. A simple growth model satisfies Eq. (17) but it does not satisfy the Kolmogorov form in eq. (16), is the monomolecular model. The monomolecular model satisfy the following differential equation [17]

$$\frac{dy}{dt} = q - sy, y(0) = Y_0. \tag{18}$$

Here, q could be considered as constant growth rate while s could be considered as the death rate of a population. In this section, we propose a generalized model of the monomolecular model and the standard logistic growth model. We extend the monomolecular model and the logistic differential equation model into the following differential equation

$$\frac{dy}{dt} = (q + ry)\left(1 - \frac{y}{K}\right), \quad y > 0 \tag{19}$$

and the initial condition $y(0) = Y_0 > 0$. Note that region of biological interest of the model in Eq. (19) is $\mathbf{R}_+ := \{x \in \mathbf{R} : x > 0\}$, since a life organism could not grow from nothing. Here, q and r could be considered as constant growth rate and proportional growth rate respectively.

The modified logistic growth model in Eq. (19) has one equilibrium, namely $y = K$. Global stability of the equilibrium is presented in the following theorem.

Theorem 3.1. *The equilibrium $y = K$ is globally asymptotically stable.*

Proof: We define a Lyapunov function $V : \mathbf{R} \to \mathbf{R}$ by $V(y) = (y - K)^2$. The function V is a $C^\infty(\mathbf{R})$ function. In addition, the equilibrium $y = K$ is the global minimum of V. Moreover, V is a definite positive function around the equilibrium where every $y \in R \setminus \{K\}, V(y) > 0$. The time derivative of V computed along solutions of the mathematical model in Eq. (19) is given by the expression

$$\frac{dV}{dt} = \frac{-2}{K}(q + ry)(y - K)^2.$$

Since all parameters in the model are positive and the variable y is positive, it follows that $\frac{dV}{dt} \leq 0$ for $y > 0$. In addition $\frac{dV}{dt} = 0$ if and only if $y = K$. Therefore the greatest compact invariant set in $\left\{y \in \mathbf{R}_+ : \frac{dV}{dt} = 0\right\}$ is the singleton $\{K\}$. By LaSalle's invariance principle [18], the equilibrium $y = K$ is globally asymptotically stable in $\mathbf{R}_+$.

The population weight at the inflection time (t_{inf}) could be determined as follows. By differentiating both sides of Eq. (19) and setting $\frac{d^2y}{dt^2}(t_{\text{inf}}) = 0$, we find

$$y(t_{\text{inf}}) = \frac{K}{2} - \frac{q}{2r}. \tag{20}$$

Hence, the population weight at the inflection time for this model is smaller than the values obtained from the logistic growth model. Exact values of the inflection time could be obtained whenever the analytical solution of the model in Eq. (19) could be found.

The differential equation in Eq. (19) could be written as

$$\left(\frac{r}{q+ry} + \frac{1}{K-y}\right) dy = \left(\frac{q}{K} + r\right) dt.$$

By integrating the left side with respect to y and the right side with respect to t gives

$$\ln\left(\frac{q+ry}{K-y}\right) = \left(\frac{q}{K} + r\right) t + c_0 \tag{21}$$

for some constant c_0. The mathematical expression in the Eq. (21) could be written as

$$\frac{q+ry}{K-y} = c_1 \exp\left(\left(\frac{q}{K} + r\right) t\right), c_1 = \exp(c_0). \tag{22}$$

By solving Eq. (22) for y, it could be obtained explicit solution of the modified logistic differential equation as

$$y(t) = \frac{c_1 K \exp\left(\frac{qt}{K} + rt\right) - q}{r + c_1 \exp\left(\frac{qt}{K} + rt\right)}. \tag{23}$$

By substituting the initial condition $y(0) = Y_0$, then $c_1 = \frac{qY_0+a}{K-Y_0}$. Hence, the explicit solution in Eq. (23) could be written as

$$y(t) = \frac{K - q\left(\frac{K-Y_0}{rY_0+q}\right)\exp\left(\frac{-qt}{K} - rt\right)}{1 + r\left(\frac{K-Y_0}{rY_0+q}\right)\exp\left(\frac{-qt}{K} - rt\right)}. \tag{24}$$

By defining the following parameters

$$\alpha = \frac{q}{K} + r, A = K - q\left(\frac{K - Y_0}{rY_0 + q}\right), B = r\left(\frac{K - Y_0}{rY_0 + q}\right) \tag{25}$$

then the modified logistic growth model in Eq. (24) could be written as

$$y(t) = \frac{K - (K - A)\exp(-\alpha t)}{1 + B\exp(-\alpha t)}. \tag{26}$$

Here α, A, B, K are positive parameters and $A \leq K$. The parameter α is effective growth rate, K is the maximum capacity (mature weight), while the parameter A, B are corresponding to initial weight and inflection time. The inflection time (t_{inf}) of the model in Eq. (26) is

$$t_{\text{inf}} = \frac{\ln B}{\alpha} = \frac{K}{q + rK}\ln\left(r\left(\frac{K - Y_0}{rY_0 + q}\right)\right). \tag{27}$$

The inflection time in (27) could be determined by evaluating the second derivative of y in Eq. (26) and setting $\frac{d^2y}{dt^2}(t_{\text{inf}}) = 0$. If the constant growth rate parameter (q) is zero, then the inflection time in Eq. (27) could be simplified into Eq. (5). From Eq. (27), the modified logistic growth model in Eq. (26) could be presented in the following form

$$y(t) = \frac{K - (K - A)\exp(-\alpha t)}{1 + \exp(-\alpha(t - t_{\text{inf}}))}. \tag{28}$$

Since there are some well-known modified logistic growth model, then the presented growth model presented in Eq. (29) could be called by a WEP-modified logistic growth model. Here WEP comes from Windarto-Eridani-Purwati.

4 Extension of the proposed model

It is well known that length and weight of fish species will grow until they attain some maximum values. By applying the presented model in previous section, the dynamics of fish weight and fish length could be modelled by following differential equations

$$\frac{dW}{dt} = (q_w + r_w W)\left(1 - \frac{W}{K_w}\right), \quad W(0) = w_0, \tag{29}$$

and

$$\frac{dL}{dt} = (q_l + r_l L)\left(1 - \frac{L}{K_l}\right), \quad L(0) = l_0, \tag{30}$$

respectively. Here, $W(t)$ and $L(t)$ are fish weight and fish length at time t respectively. In Eq. (29)-(30), q_w, q_l are constant growth rate of fish weight and fish length, while r_w, r_l are proportional growth rate of fish weight and fish length respectively. By applying analytical solution of the previous section, we found dynamic of fish weight and fish length could be described by

$$W(t) = \frac{K_w - q_w\left(\frac{K_w - w_0}{r_w w_0 + q_w}\right)\exp\left(\frac{-q_w t}{K_w} - r_w t\right)}{1 + r_w\left(\frac{K_w - w_0}{r_w Y_0 + q_w}\right)\exp\left(\frac{-q_w t}{K_w} - r_w t\right)} \tag{31}$$

and

$$L(t) = \frac{K_l - q_l\left(\frac{K_l - l_0}{r_l l_0 + q_l}\right)\exp\left(\frac{-q_l t}{K_l} - r_l t\right)}{1 + r_l\left(\frac{K_l - l_0}{r_l l_0 + q_l}\right)\exp\left(\frac{-q_l t}{K} - r_l t\right)} \tag{32}$$

respectively.

It is also well known that there are length-weight relationship (LWR) of fish species. A mathematical equation was used to show relationships between the average weight of fish at a given length [19, 20]. The length-weight relationship is given by

$$W(t) = aL(t)^b. \tag{33}$$

Here, a and b are empirical parameters. Typically, the b parameters ranges from 2 to 4. Fish can attain either isometric or allometric growth.

Isometric growth indicates that both fish length and fish weight are increasing at the same rate [20]. In order to estimate parameters in Eq. (31) and (32), we need fish weight and fish length data over time. In the next section, we apply the proposed model (WEP-modified logistic growth model) to some secondary data cited from literature.

5 Application of the proposed model

In this section, the proposed model is implemented to describe chicken body weight (rooster and hen) growth, where the data are cited form literature [3, 21]. Rooster (x) and hen (y) body weight at different age (t) are presented in Table 1. In addition, accuracy result of the proposed model will be compared to logistic model, Gompertz model, and Richards model. The logistic model was presented in Eq. (6), while Richards and Gompertz differential equations were presented in Eq. (11) and (12) respectively. Analytical solution of the Richards differential equation in Eq. (11) was given by

$$y_R(t) = \frac{K}{\left[1 + \beta \exp(-r\beta(t - t_{\text{inf}}))\right]^{\frac{1}{\beta}}}, \tag{34}$$

where the inflection time $t_{\text{inf}} = \frac{1}{r\beta} \ln\left(\frac{(\frac{K}{Y_0})^{\beta} - 1}{\beta}\right)$. By defining $m = \beta + 1$, $r^* = r\beta$, then the Richards growth model in Eq. (34) could be expressed as

$$y_R = K\left[1 - (1 - m)\exp\left(- r^*(t - t_{\text{inf}})\right)\right]^{\frac{1}{(1-m)}}. \tag{35}$$

Exact solution of the Gompertz differential equation in Eq. (12) was given by

$$y_G(t) = \frac{K}{\exp\left(\exp(-r(t - t_{\text{inf}}))\right)} \tag{36}$$

where $t_{\text{inf}} = \frac{1}{r} \ln\left(\ln(\frac{K}{Y_0})\right)$. Some authors used the following Gompertz-

Laid growth model [3]

$$.y_G(t) = W_0 \exp\left(\exp(rt_{\text{inf}})\left(1 - \exp(-rt)\right)\right). \tag{37}$$

Here, W_0 is initial chicken weight in the Gompertz model and m is the shape parameter in Richards model. For $m = 2$, then the Richards model could be simplified into logistic model. For m tends to one, then the Richards model could be simplified into the Gompertz model.

Table 1: Means of rooster and hen chicken weight data

t (days)	x (grams)	y(grams)	t (days)	x (grams)	y (grams)
0	37	36.68	42	519.72	436.51
3	41.74	40.8	45	577.27	480.31
6	59.19	57.33	48	633.59	522.91
9	79.94	77.24	51	667.18	547.23
12	102.96	97.96	54	717.17	583.56
15	132.13	121.92	57	786.35	631.77
18	170.18	155.08	71	1069.28	832.57
21	206.56	184.24	85	1326.49	1009.48
24	250.71	218.37	99	1589.71	1183.8
27	285.27	247.12	113	1859.26	1440.18
30	324.92	279.58	127	2015.44	1561.89
33	372.83	319.55	141	2142.31	1619.34
36	417.41	355.13	155	2220.54	1680.29
39	469.13	396.32	170	2262.63	1717.78

There are four parameters in the model should be estimated, namely parameter α (effective growth rate), K (maximum weight/ mature weight of chicken), the inflection time t_{inf} and parameter A (correspond to the initial chicken weight). Since growth function of the model is explicitly presented in the Eq. (28), then nonlinear regression procedures could be applied to estimate the parameters.

The parameters α, K, t_{inf} and A are estimated such that the normalized residual sum of squares (NRSS)

$$NRSS = \sum_i \frac{(z_i - \hat{z}_i)^2}{(z_i - \overline{z})^2}, z = x \vee z = y \tag{38}$$

is minimum. In Eq. (38), $\bar{z}$ is the average of z and $\widehat{z}_i$ is chicken weight at time i predicted from the model. The normalized residual sum of square corresponds to the determination coefficient via the following relation

$$R^2 = 1 - NRSS. \tag{39}$$

Parameters in the logistics, Gompertz, and Richards model also be estimated with similar manner. Accuracy of the predicted results could also be measured by evaluation of Mean Absolute Percentage Error (MAPE), which is given by the following formula

$$MAPE = \sum_i \frac{1}{n}\left|\frac{z_i - \widehat{z}_i}{z_i}\right| 100\%. \tag{40}$$

Here n is the number of observational data. The nonlinear least square (nls) procedure of R open source software is used to estimate parameters of the proposed model, logistic, Gompertz, and Richards model. R open source software was built by the R Foundation for Statistical Computing. Estimation results of the proposed model, logistic, Gompertz, and Richards model for rooster and hen weight, the determination coefficient (R^2) and Mean Absolute Percentage Error (MAPE) for the models are presented in Table 2, while the dynamics of rooster weight and hen weight are shown in Fig. 1 and Fig. 2 respectively.

It could be seen from Fig. 1 and Fig. 2 that rooster growth and hen growth follow sigmoidal patterns. Rooster growth and hen growth stars by an accelerating growth phase from hatching. Then, the chicken attains a maximum growth rate at the inflection time. At final phase, the chicken weight tends to a mature weight. Qualitatively, all of the models, describe the chicken growth well, as seen in figures. But, if we compare its MAPE, as seen in Table 2, we see that logistic model have the biggest MAPE, and it mean that its accuration is poorer than the other models. This apparently due to the logistic model is not accurate in predicting the dynamics of rooster and hen weight at the early times (Fig. 1 and Fig. 2). By adding one additional parameter (q) to the presented model, the dynamics of rooster and hen weight could be better estimated by using the presented model.

From the Table 2, it was found that the growth rate (the effective growth rate) or the maturation rate (α in the proposed model, r in the logistic and Gompertz model and r^* in the Richards model) was higher in rooster than in hen. This result is consistent with the result from Aggrey (2002) [3]. It

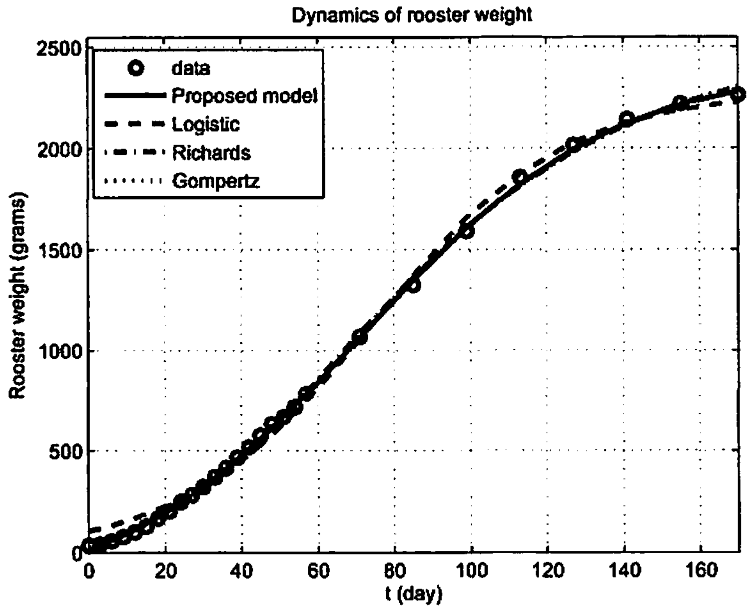

Figure 1: Dynamic of rooster weight.

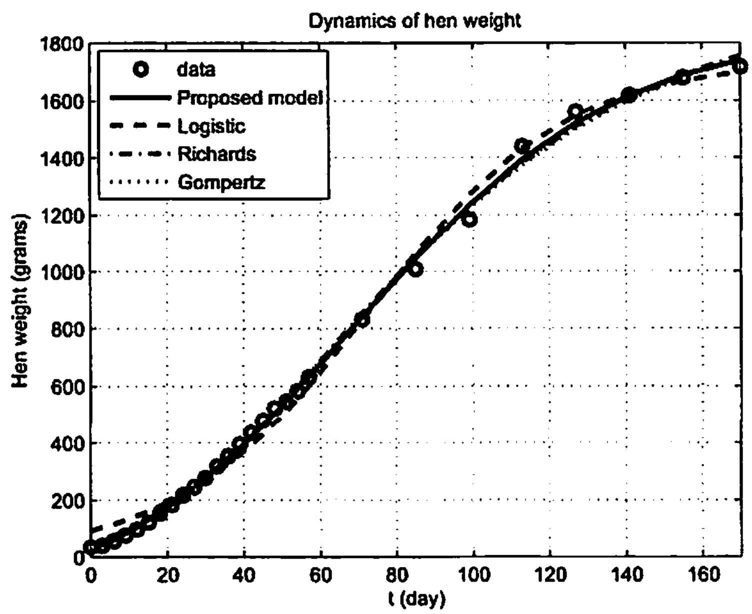

Figure 2: Dynamic of hen weight.

Table 2: Estimated parameters for the proposed model, logistic, Richards and Gompertz growth model

Model	Parameters	Rooster	Hen
The proposed model (WEP-modified logistic growth model)	Mature weight (K)	2399.749	1847.162
	Effective growth rate (α)	0.031	0.029
	Inflection time (t_{inf})	71.584	69.015
	A	166.323	183.061
	NRSS	0.00031	0.00119
	R^2	0.99969	0.99881
	MAPE	0.04754	0.04267
Logistic model	Mature weight (K)	2279.904	1739.652
	Growth rate (r)	0.040	0.039
	Inflection time (t_{inf})	74.677	73.331
	NRSS	0.00357	0.00501
	R^2	0.99643	0.99499
	MAPE	0.299927	0.25398
Richards model	Mature weight (K)	2512.972	1945.342
	Growth rate (r*)	0.023	0.021
	Inflection time (t_{inf})	64.307	61.344
	Shape parameter (m)	1.054	0.978
	NRSS	0.00071	0.00175
	R^2	0.99929	0.99825
	MAPE	0.07373	0.06552
Gompertz model	Growth rate (r)	0.022	0.021
	Inflection time (t_{inf})	63.498	61.704
	Mature weight (K)	2539.651	1936.385
	NRSS	0.00073	0.00175
	R^2	0.99927	0.99825
	MAPE	0.06007	0.07031

also could be found that inflection time of the proposed model is relatively close to inflection time of the logistic model. In addition, inflection time of Richards model is relatively close to the Gompertz model. It is apparently due to the shape parameter m in the Richards model is close to one. Moreover, it was found that the proposed model, logistic model, Richards model and Gompertz model produced a high determination coefficient (R^2 is greater than 0.99). Although the determination coefficients of the four models did not differ significantly, Mean Absolute Percentage Error (MAPE) of the models considerably varied. It was found the proposed model has the smallest MAPE, which is 4.754% in rooster and 4.267% in hen. This indicates that the proposed model could be used as an alternative model to describe poultry growth curve or an individual growth.

6 Conclusion

A new growth model was presented in this paper. The model was derived from modification of logistic differential equation. The proposed model also was simulated and verified using rooster and hen weight data cited from the literature. The estimation results from the model ware compared to the logistic model, Richards, and Gompertz growth model. It was found that the model gave better results compared to the logistic model, Richards, and Gompertz growth model. It indicates that model could be used as an alternative model to describe poultry growth curve or an individual growth.

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Conflict of interest

The authors do not have conflict of interest in regard to this research or its funding.

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On Trimmed Data Effect in Parameter Estimation of Some Population Growth Models

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***Abstract*- Logistic model, Gompertz model, Richard model, Weibull model and Morgan-Mercer-Flodin model are commonly used to describe growth model of a population. In this paper, we study the effect of trimmed data on parameter estimation results of those models. We use chicken weight data cited from literature. Parameter values of the models from the complete data and the trimmed data are compared. Then, the sensitivity index of all parameters is evaluated. We found that that sensitivity order of the models from the highest sensitivity was the Morgan-Mercer-Flodin, Weibull, Richards, logistic and Gompertz growth model. For practical applications, Gompertz model and Richards are recommended in order to modeling growth of a population.**

Keywords— growth model; parameter estimation; chicken weight; trimmed data.

I. INTRODUCTION

Mathematical growth models have been widely applied to explain body weight and age relationship in veterinary sciences. From the mathematical growth model, one can evaluate some important and practical parameters, e.g. the mature weight, the maturing rate and the growth rate of an animal. The parameters are beneficial tool to give estimations of the daily feed needs or to evaluate the effect of environmental condition on the weight growth of an animal. In addition, the mathematical growth models could be applied to forecast the optimum slaughter age. Therefore, mathematical growth models could be considered as an optimization instrument for the animal production [1, 2, 3].

The mathematical growth model could be classified into two groups, namely empirical growth models and the empirical growth model and dynamical growth models (the growth model derived from ordinary differential equations). The empirical growth models include Weibull growth model and MMF (Morgan-Mercer-Flodin) growth model. The Weibull and the MMF growth model have been applied to describe chicken growth dynamic [4]. The dynamical growth model include logistic growth model, Gompertz growth model, and Richards growth model. These dynamical growth models have been used the growth kinetics of many animals, including chicken [5], mammal [6], fish [7], reptile [8] and amphibian [9].

Topal and Bolukbasi reported that the MMF, Weibull and Gompertz the MMF, Weibull and Gompertz growth model can be useful for describing chicken growth performance, since these models were the best fitted models [4]. Aggrey found that the Richards and Gompertz growth model have the best fitted model in explaining rooster and hen growth dynamics [5]. Zadeh and Golshani also reported that the Richards growth model provided the best fit to the growth curve of Iranian Gulian sheep [10].

A mathematical growth model could be said as a good model if the model give accurately predicted result and it is robust with trimmed data. In this context, we compare robustness of some mathematical growth model due to trimmed data effect. We use sensitivity index to measure robustness performance of the models. We use chicken weight data cited from literature.

This paper is organized as follows. Section 2 briefly presents some mathematical growth models. Section 3 presents effect of trimmed data on robustness performance of the selected models. Finally, conclusions are presented in Section 4.

II. SOME MATHEMATICAL GROWTH MODELS

In this section, we briefly present some mathematical growth models including empirical growth models and dynamical growth models. Let $y(t)$ represents chicken body weight at time t. The Weibull and MMF growth model are given by

$$y(t) = K - (K - A)\exp(-Bt^D), \quad (1)$$

and

$$y(t) = \frac{AB + Ct^D}{B + t^D}, \quad (2)$$

respectively. Here, K is chicken mature weight, while A, B, C, D are empirical parameters [4].

Logistic growth model is derived from the following differential equation

$$\frac{dy}{dt} = ry\left(1 - \frac{y}{K}\right), y(0) = y_0 > 0. \quad (3)$$

Here r is per capita growth rate. The logistic growth model is analytical solution of Eq. (3), which is given by [5, 11]

$$y(t) = \frac{K}{1 + exp\left(-r(t - t_{inf})\right)} \quad (4)$$

where $t_{inf} = \frac{1}{r}\ln\left(\frac{K}{y_0}\right)$. Here t_{inf} is the inflection time, where at chicken growth is maximum at the inflection time.

The Gompertz growth model is derived from the following Gompertz differential equation

$$\frac{dy}{dt} = ry\ln\left(\frac{K}{y}\right), y(0) = y_0 > 0. \quad (5)$$

The exact solution of Eq. (5) represents the Gompertz growth model. The Gompertz growth model is given by

$$y(t) = \frac{K}{exp\left(\exp\left(-r(t-t_{inf})\right)\right)} \quad (6)$$

where $t_{inf} = \frac{1}{r}\ln\left(\ln\left(\frac{K}{y_0}\right)\right)$.

The Richards growth model is derived from the Richards differential equation

$$\frac{dy}{dt} = ry\left(1-\left(\frac{y}{K}\right)^{\beta}\right), y(0) = y_0 > 0. \quad (7)$$

Here β is the shape parameter in the Richards differential equation. For β=1, then the Richards differential equation could be simplified into logistic differential equation. Hence, Richards differential equation could be considered as an extension of the logistic differential equation. The exact solution of the Richards differential equation in Eq. (7) is given by

$$y(t) = \frac{K}{\left[1+\beta\exp\left(-r^{*}(t-t_{inf})\right)\right]^{1/\beta}} \quad (8)$$

where $r^{*} = r\beta,\ t_{inf} = \frac{1}{r\beta}\ln\left(\frac{\left(\frac{K}{y_0}\right)^{\beta}-1}{\beta}\right)$.

III. EFFECT OF TRIMMED DATA ON THE ROBUSTNESS PERFORMANCE

In this section, we study effect of trimmed data on robustness performance of the growth models presented in the previous section. We used rooster weight data cited from literature [5, 11]. The rooster weight data (y) at the day (t) is presented in the Table 1. At the first step, we estimate parameters in the growth model before trimmed data. We estimate the paramaters such that the mean square error (MSE) which is given by

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \quad (9)$$

is minimum. Here, y_i and $\hat{y}_i$ are rooster weight data and predicted rooster weight at the i-th day, while n is number of observation data.

We used Lavenberg-Marquardt algorithm fo find the optimal parameters for the optimization problem given in Eq. (9). Estimation results of the Weibull, MMF, logistic, Gompertz and the Richards growth model for the rooster weight and the mean squared error of the models are presented in the Table 2. From the Table 2, we found that the Weibull was the best models, while the logistic growth model was the worst model. We also obtained that accuracy of the Weibull model and the Richards model did not considerably differ. We also found that mean squared error of the Richards model and the Gompertz model did not significantly differ. This was apparently caused by the shape parameter β in the Richards model was almost zero.

Table 1. Means of the rooster weight data (y)

t (days)	y (grams)	t (days)	y (grams)
0	37	42	519.72
3	41.74	45	577.27
6	59.19	48	633.59
9	79.94	51	667.18
12	102.96	54	717.17
15	132.13	57	786.35
18	170.18	71	1069.28
21	206.56	85	1326.49
24	250.71	99	1589.71
27	285.27	113	1859.26
30	324.92	127	2015.44
33	372.83	141	2142.31
36	417.41	155	2220.54
39	469.13	170	2262.63

Table 2. Estimated parameters value for the whole data

Growth Model	Parameters	Estimated value	MSE
Weibull	K	2426.1709	347.743
	A	58.2211	
	B	0.000197	
	D	1.8699	
MMF	A	67.7095	793.779
	B	14411.3917	
	C	2996.0317	
	D	2.1030	
Logistic	K	2279.9041	1887.461
	r	0.0403	
	t_{inf}	74.6775	
Gompertz	K	2539.6505	384.666
	r	0.0220	
	t_{inf}	63.4975	
Richards	K	2512.9724	376.277
	r^{*}	0.0230	
	t_{inf}	64.3072	
	β	0.0541	

In order to study the effect of trimmed data, we also estimated parameters of the models for trimmed data at the end of the original data (the data from t = 0 until 127 days). We estimated parameters in the models for the trimmed data. We presented estimation results for the trimmed data in the Table 3. From the Table 3, we found that the Weibull model and the MMG model were the best models, while the logistic growth model was the worst model. It indicates that the empirical models are more suit when they are applied in a short data. We also obtained that accuracy of the Gompertz model and the Richards model did not considerably differ.

Table 3. Estimated parameters value for the trimmed data

Growth Model	Parameters	Estimated value	MSE
Weibull	K	2992.8983	111.903
	A	41.9675	
	B	0.000334	
	D	1.6772	
MMF	A	43.5080	139.359
	B	5355.9663	
	C	4540.7213	
	D	1.7266	
Logistic	K	2132.0511	1708.691
	r	0.0433	
	t_{inf}	70.3077	
Gompertz	K	2694.6160	230.084
	r	0.0206	
	t_{inf}	66.8981	
Richards	K	2694.3571	230.053
	r*	0.0206	
	t_{inf}	66.8987	
	β	0.0002	

In order to measure effect of trimmed data on robustness performance of the models, we defined a sensitivity index of all parameters in the model. For any parameter α, we defined the sensitivity index as

$$SI_\alpha = \left|\frac{\alpha-\alpha_{trim}}{\alpha}\right|, \alpha \neq 0. \quad (10)$$

Here α_{trim} is the parameter value after trimmed data process. Sensitivity index of all parameters was presented in the Table 4.

Table 4. Sensitivity index of all parameters

Growth Model	Parameters	Sensitivity index	Average value
Weibull	K	0.2336	0.3278
	A	0.2792	
	B	0.6954	
	D	0.1031	
MMF	A	0.3574	0.4201
	B	0.6284	
	C	0.5156	
	D	0.1790	
Logistic	K	0.0649	0.0659
	r	0.0744	
	t_{inf}	0.0585	
Gompertz	K	0.0610	0.0594
	r	0.0636	
	t_{inf}	0.0536	
Richards	K	0.0722	0.3034
	r*	0.1049	
	t_{inf}	0.0403	
	β	0.9961	

From the Table 2 and Table 3, we found that the mean squared error of the Weibull model and the MMF model drastically increased due to adding a few data. From the Table 4, we found that average value of the sensitivity index varied from 5.94% until 42.01%. In addition, we found that the shape parameter β in the Richards model was very sensitive, while the remaining parameters in the Richards model were robust. Furthermore, we found that the Gompertz growth model was a robust model with respect to trimmed data. We also obtained that sensitivity index of the empirical model were more sensitive than the dynamical model studied in this paper. Hence we found that the empirical growth model were more sensitive than the dynamical growth models. For practical applications, Gompertz model and Richards are recommended in order to describing a population growth.

IV. CONCLUSIONS

We have studied effect of trimmed data on parameter estimation results of some empirical models (Weibull and Morgan-Mercer-Flodin) and some dynamical models (logistic, Gompertz and Richards growth model). We found that the empirical models were more sensitive than the dynamical models. We also found that the dynamical models were more robust with respect to trimmed data. For practical applications, Gompertz model and Richards are recommended in order to modeling growth of a population.

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A Comparison of Continuous Genetic Algorithm and Particle Swarm Optimization in Parameter Estimation of Gompertz Growth Model

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Abstract. Genetic algorithm and Particle Swarm Optimization are heuristic optimization methods inspired by genetic principles and swarm behavior phenomena, respectively. Those two methods are initiated by random generation of initial populations (initial solutions), fitness evaluation of every solution, solution updating until a termination condition are met. It is well known that those two methods are not always converge to an optimal solution. Those methods sometimes converge to suboptimal solutions, solution near the optimal solution. In this paper, continuous genetic algorithm and particle swarm optimization were implemented to estimate parameters in the Gompertz growth model from rooster weight data cited from literature. Although the best results of the two models were not significantly differs, we found that the particle swarm optimization method was more robust than the continuous genetic algorithm. Hence, the particle swarm optimization method is more recommended than the continuous genetic algorithm.

Keywords: Gompertz growth model, rooster weight dynamic, parameter estimation, particle swarm optimization.

INTRODUCTION

Mathematical models are useful tool to describe many real problems. A mathematical model is usually began by identification of a real problem. Then one could construct a suitable mathematical model and determining mathematical solution of the model. Finally, one should interpret mathematical solution of the model into real problem points of view. A mathematical model might occur in either a deterministic model or a probabilistic (stochastic) model. Mathematical model validation could be performed whenever relevant data from real phenomena are available. If the predicted results from a mathematical model fit the real data, then the model is said a good model. When the predicted results from the model differ significantly the real data, then the model should be improved and modified.

Most mathematical models contain one or more parameters. The parameters should be estimated in order to accurately perform model simulation. Parameter estimation of a mathematical model could be considered as an optimization problem. Deterministic optimization methods such as conjugate gradient method, Nelder-Mead method or Newton method could be applied to estimate parameters in a mathematical model whenever analytical solution of the model could be presented in closed form [1]. Unfortunately, deterministic optimization methods such as Nelder-Mead or Newton method fail to converge into global minimum of a function if the function has many local minima [2]. Moreover, some mathematical models occur in non-linear ordinary differential equation systems, so exact solution (closed form solution) of the model could not be determined. In this case, heuristic method such as particle swarm optimization and genetic algorithm method could be implemented to estimate parameter values from the models.

Particle swarm optimization and genetic algorithm are optimization methods based on a population-based stochastic search process [3, 4]. Particle swarm optimization methods and modified particle swarm optimization have been widely applied in many areas, including performance improvement of Artificial Neural Network [5, 6], scheduling problems [7, 8], flowshop scheduling problem [9], traveling salesman problem [10], vehicle routing problem [11, 12] and clustering technique [13]. Genetic algorithm has been in parameter estimation in poultry growth model [14, 15] and parameter estimation for dynamical system model [1, 16].

Some authors compared performance of particle swarm optimization and genetic algorithm in some research are. Yang et al. compared the methods in a Hidden Markov Model training [17]. Wang et al. have been compared performance of genetic algorithm and particle swarm optimization in relativistic backward wave oscillator [18]. Islam et al. have compared performance of some nature inspired algorithms including genetic algorithm and particle swarm optimization in function optimization of some benchmark functions [19]. In this paper, we compared performance of continuous genetic algorithm and particle swarm optimization in parameter estimation of Gompertz growth model.

The remainder of this paper is organized as follows. Section 2 briefly presents particle swarm optimization and genetic algorithm procedure. Comparison of particle swarm optimization and genetic algorithm in parameter estimation of Gompertz growth model will be presented in Section 3. Finally, conclusions are presented in Section 4.

CONTINUOUS GENETIC ALGORITHM AND PARTICLE SWARM OPTIMIZATION

Genetic algorithm is inspired from principles of genetic and natural selection in a life organism. Therefore, many terms such as gene chromosome, individual, parent, selection, mating, crossover, offspring in genetic algorithm are adopted from biology. From mathematical point of view a gene represents a variable, while a chromosome or an individual represents a solution. Genetic algorithm has at least the following elements, namely populations of chromosomes, selection according to fitness, crossover to produce new offspring, and random mutation of new offspring [20].

We can transform an optimization problem into a minimization problem. Here is genetic algorithm procedure for finding either optimal or suboptimal solution of a minimization problem [2, 16, 20]:

(1) Define the objective function (the cost function) and decision variables related to the optimization problem.
(2) Determine parameter values in genetic algorithm, namely number of generations/number of iteration, crossover probability, and mutation probability/mutation rate.
(3) Generate initial solutions (initial population) from the search space/solution space.
(4) Evaluate cost function of each solution (individual). In a minimization problem, all individuals (chromosomes) are ranked from the lowest cost to the highest cost.
(5) Select part of solutions (individuals) for the next generation as parent individuals. Only the best solutions are kept for the next generation, while the remaining chromosomes are should be substituted by better individuals. Fraction of all population that survives for the next generation is determined by selection rate parameter. The typical value of selection rate parameter is 50%.
(6) Carry out mating process from parent individuals.
(7) Do crossover process to generate offspring individual.
(8) Perform mutation process to part of solutions to generate solutions.
(9) Test termination condition. If the termination condition did not satisfy yet, then go to the fourth step.

The particle swarm optimization algorithm was invented by Eberhart and Kennedy in 1995. The algorithm has similarities with evolutionary computation methods such as genetic algorithm. The particle swarm optimization algorithm is initialized with a population of random solutions and searches optimal solution updating generations. But particle swarm optimization algorithm does not have crossover and mutation operators. Potential particles (solutions) in the particle swarm optimization algorithm move through the solution space by following the current optimum particles [4].

The particle swarm optimization algorithm starts by randomly choosing initial (particles) solutions within the search space. Then, fitness function of the current position of every particle should be evaluated. If the fitness value is better than the previous best value, then the local best position of a particle is updated. The global best is updated based on the best fitness value found by any of the neighbor.

The particle swarm optimization algorithm consists of the following steps, which are repeated until some termination condition is met [4, 21]:

(1) Evaluate the fitness of every particle (solution). For a maximization problem, the greater the objective function then the greater of the fitness will be. On the other hand, for a minimization problem, the smaller the objective function then the greater of the fitness will be.
(2) Update particle best (local best) position and global best position.

(3) Update velocity of every particle using the following equation

$$v_i(t+1) = wv_i(t) + c_1 r_1 (lbest(t) - x_i(t)) + c_2 r_2 (gbest(t) - x_i(t)), \quad (1)$$

where $v_i(t)$ and $x_i(t)$ are velocity of particle i and position of particle i at discrete time t, $lbest(t)$ and $gbest(t)$ are local best and global best position at time t, r_1 and r_2 are uniform distributed random number between zero and one.

(4) Update position of every particle using the following equation

$$x_i(t+1) = x_i(t) + v_i(t+1). \quad (2)$$

In Eq. (1), w is the inertia weight, while c_1 and c_2 are cognitive coefficient and social coefficient respectively. The value of the inertial coefficient is typically between 0.8 and 1.2, while the values of cognitive coefficient and social coefficient are usually close to 2. In order to prevent the particles from moving very far beyond the search space, velocity clamping technique could be applied to limit the maximum velocity of every particle. For a search space bounded by the range $[x_{min}, x_{max}]$, the velocity is limited within the range $[-v_{max}, v_{max}]$ where $v_{max} = m(x_{max} - x_{min})$ for some constant m, $0.1 \leq m \leq 1$. Some common stopping conditions in particle swarm optimization include a predetermined number of iterations, a number of iterations since the last update of global best solution or a preset target fitness value [21].

A COMPARISON OF CONTINUOUS GENETIC ALGORITHM AND PARTICLE SWARM OPTIMIZATION

In this section, we compared performance of continuous genetic algorithm and particle swarm optimization in parameter estimation of Gompertz growth model. Gompertz growth model is derived from the following Gompertz differential equation

$$\frac{dy}{dt} = ry \ln\left(\frac{K}{y}\right), y(0) = Y_0. \quad (3)$$

Here, $y(t)$ is population size at time t. The exact solution of the Gompertz differential equation in Eq. (3) could be represented as

$$y(t) = \frac{K}{exp(exp(-r(t - t_{inf})))}, \quad (4)$$

where $t_{inf} = \frac{1}{r} \ln\left(\ln\left(\frac{K}{Y_0}\right)\right)$. The Gompertz growth model has three parameters namely intrinsic growth (r), carrying capacity (K), and inflection time (t_{inf}) parameter. From biological point of view, the fastest growth of a population occur at the inflection time.

In this paper, the Gompertz growth model is applied to describe rooster growth where the data is cited from literature [15, 22]. The rooster growth data is shown in the Table 1.

Since $y(t)$ is the rooster weight at time t, then the carrying capacity parameter (K) could be interpreted as the rooster mature weight or the maximum weight that can be attained by the rooster. Parameters in the Gompertz model (K, r, t_{inf}) are estimated such that the mean absolute percentage error (MAPE)

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \left|\frac{y_i - \widehat{y_i}}{y_i}\right| \quad (5)$$

is maximum. Here n is number of observation data.

We applied continuous genetic algorithm and particle swarm optimization to estimate parameters in the Gompertz growth model. Here, optimal parameters in the Gompertz growth model was found from the following search space

$$\Omega = \left\{(K, r, t_{inf}) \in R^3 : K \in [2000, 5000], r \in [0, 0.1], t_{inf} \in [30, 100]\right\}. \quad (6)$$

We applied particle swarm optimization method described in the previous section with the inertia weight parameter $w = 1$, the cognitive coefficient parameter $c_1 = 2$ and the social coefficient parameter $c_2 = 2$ respectively. We

TABLE 1. Means of the rooster weight data (y).

t (days)	y (grams)	t (days)	y (grams)
0	37	42	519.72
3	41.74	45	577.27
6	59.19	48	633.59
9	79.94	51	667.18
12	102.96	54	717.17
15	132.13	57	786.35
18	170.18	71	1069.28
21	206.56	85	1326.49
24	250.71	99	1589.71
27	285.27	113	1859.26
30	324.92	127	2015.44
33	372.83	141	2142.31
36	417.41	155	2220.54
39	469.13	170	2262.63

TABLE 2. The best estimation results of the particle swarm optimization and the continuous genetic algorithm.

Methods	K	r	t_{inf}	MAPE
PSO	2468.54	0.023646	60.68	0.039334
GA (m = 0.05)	2445.65	0.023870	60.15	0.039465
GA (m = 0.1)	2435.97	0.023800	60.21	0.039225
GA (m = 0.2)	2437.81	0.023740	60.34	0.039117
GA (m = 0.3)	2468.65	0.023749	60.67	0.039326
GA (m = 0.4)	2444.90	0.023750	60.41	0.039173
GA (m = 0.5)	2468.70	0.023628	60.77	0.039185

m = mutation rate.

also applied continuous genetic algorithm for various mutation rate (m) namely $m = 0.05, 0.1, 0.2, 0.3, 0.4, 0.5$. For both algorithm, number of population is set to 100 individuals (particles). We applied both methods for 50 trials while for every trial the methods were terminated after 500 iterations. The best estimation results of the particle swarm optimization and the continuous genetic algorithm was presented in the Table 2.

Form the Table 2, we found that best result (minimum of MAPE) of the continuous genetic algorithm and the particle swarm optimization method were not significantly differ. The mean average percentage error for the Gompertz growth model obtained from the methods were around 3.9 %. The results indicated that the Gompertz growth model could be applied to describe rooster growth dynamic. It also indicated that particle swarm optimization and continuous genetic algorithm were successfully implemented in parameter estimation of the Gompertz growth model.

Particle swarm optimization and continuous genetic algorithm are essentially probabilistic methods. Hence, the two methods will generally produce different optimal/sub optimal solution in every trial/calculation/experiment. Statistics of the MAPE of the both methods was presented in the Table 3.

From the Table 3, we found that the MAPE average of various mutation rate in the continuous genetic algorithm did not significantly differ. Although the best result (minimum of MAPE) of the continuous genetic algorithm and the particle swarm optimization method were not significantly differ, the MAPE average of the particle swarm optimization method was smaller than the continuous genetic algorithms one. We also found that the MAPE variance of the genetic algorithms were more higher than the particle swarm optimization variance. This results indicate that the particle swarm optimization method was more robust than the continuous genetic algorithm.

CONCLUSIONS

We have implemented particle swarm optimization and continuous genetic algorithm in parameter estimation of the Gompertz growth model. Although the best results of the two models were not significantly differs, we found that the

TABLE 3. Statistics of Mean Absolute Percentage Error.

Methods	Number of trials	Average of MAPE	Standard deviation	Minimum	Median	Maximum	p-value
PSO	50	0.045661 [a]	0.003743	0.039334	0.045548	0.058081	
GA (m = 0.05)	50	0.092665 [b]	0.051349	0.039465	0.077008	0.267170	
GA (m = 0.1)	50	0.093638 [b]	0.044163	0.039225	0.087674	0.202533	p-value <
GA (m = 0.2)	50	0.084986 [b]	0.037714	0.039117	0.080257	0.244075	0.0005
GA (m = 0.3)	50	0.098225 [b]	0.054169	0.039326	0.084729	0.270992	
GA (m = 0.4)	50	0.092345 [b]	0.066085	0.039173	0.070386	0.400521	
GA (m = 0.5)	50	0.091441 [b]	0.051345	0.039185	0.077055	0.325060	

m = mutation rate.
[a,b] different superscripts showed significant difference between group at the level 0.05.

particle swarm optimization method was more robust than the continuous genetic algorithm.

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On the fractional order of von Bertalanffy growth model

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ABSTRACT The von Bertalanffy growth model has been applied in describing fisheries growth and bullhead growth. The model was derived from the von Bertalanffy ordinary differential equation. On the other hand, fractional derivative was used in the study of viscoelastic medium (a medium with viscosity and elasticity properties), image signal processing, and population growth modeling. In this paper, we extent the classical von Bertalanffy differential used into the fractional order model. We also study equilibria and their stability of the fractional order model. In addition, we applied the fractional order model to describe the dynamic of poultry growth, where the poultry growth data was cited from the literature.

Keywords: Fractional order, von Bertalanffy growth model, poultry growth.

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1. INTRODUCTION

Mathematical growth models were widely used to describe body weight dynamic in veterinary sciences. By applying a suit mathematical growth model, one can evaluate some important and practical parameters, e.g. the mature weight, the maturing rate and the growth rate of an animal. The parameters are useful tool to give estimations of the daily feed needs or to evaluate the effect of environmental condition on the weight growth of an animal. Moreover, the mathematical growth models could be used to predict the optimum slaughter age. Hence, mathematical growth models could be considered as an optimization instrument for the animal production (Lopez et al., 2000; Vázquez et al., 2012; Teleken et al., 2017).

Growth of an animal could be defined as body weight increase or dimension increase of the animal against time. Mathematical growth curve can be applied to describe dynamics of body weight or dynamic of body dimension of an animal (Keskin et al., 2010; Hossein-Zadeh and Golshani, 2016). Many mathematical growth models have been used to describe dynamic of body weight of an animal. The models include logistic, von Bertalanffy, Gompertz and Richards growth model. The growth models have been applied to describe the growth kinetics of many animals, including chicken/broiler (Rogers et al., 1987; Aggrey, 2002; Roush and Branton, 2005; Topal and Bolukbasi, 2008), mammal (Franco et al., 2011; Hossein-Zadeh and Golshani, 2016), fish (Lester et al., 2004; Santos et al., 2013), reptile (Bardsley et al., 1995) and amphibian (Mansano et al., 2013).

Fractional order mathematical model has many advantages than the classical integer order model where there is memory effect in the case (Rihan, 2013). Fractional order model has been applied in many varied and widespread fields including biological systems (Arafa et al., 2012), physics (Debnath, 2003), chemistry (Yuste et al., 2004), medicine (Ferdy, 2012) and finance (Chen, 2008). In biological system modelling, El-Sayed et al. generalized logistic growth model into fractional-order logistic growth model. They also studied stability, existence, uniqueness and numerical solution of the fractional-order logistic equation (El-

Sayed et al., 2007). Motivated by the work of El-Sayed et al. (2007), we extent the classical von Bertalanffy growth model into fractional order von Bertalanffy growth model. We also study equilibria and equilibria stability of the fractional order model. In addition, we apply the fractional order model to describe the dynamic of poultry growth, where the poultry growth data was cited from the literature.

The remaining of the paper is organized as follows. In Section 2, we present some fundamental concepts about fractional calculus and the stability of a fractional order differential equation system. In Section 3 we present fractional order von Bertalanffy growth model. Implementation of fractional order von Bertalanffy growth model will be presented in the Section 4. Finally, conclusions are presented in the last section.

2. SOME FUNDAMENTAL CONCEPTS

In this section, we present some fundamental concepts related to fractional calculus. Definition of Caputo fractional derivative is described in the Definition 2.1.

Definition 2.1. (Kilbas et al., 2006; Aguila-Camacho et al., 2014) The Caputo fractional derivative of order $\alpha \in R^+$ on the half axis R^+ is defined as follows

$${}_a^C D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(s)}{(t-s)^{\alpha-n+1}} ds, t > a \qquad (1)$$

where $n = \min\{k \in N: k > \alpha > 0\}$.

From the Definition 2.1, it is clear that the Caputo fractional derivative of any constant functions are zero. The Definition 2.2 presents description of the Caputo fractional non autonomous system.

Definition 2.2. (Li et al., 2010; Aguila-Camacho et al., 2014) Let $0 < \alpha < 1, x(t) \in R^d$. The Caputo fractional non autonomous system is defined as

$${}_a^C D_t^\alpha x(t) = f(t,x) \qquad (2)$$

with initial condition $x(a)$, where $f: [a,\infty) \times \Omega \to R^d, \Omega \subseteq R^d$. A constant vector $x^* \in R^d$ is said an equilibrium of Caputo fractional dynamic system in Eq. (2) if and only if $f(t,x^*) = 0$.

Li et al. (2010) proposed an extension of Lyapunov direct method for studying stability analysis of a nonlinear time-varying fractional order system. Li at al. used class-K function presented in the following definition.

Definition 2.3. (Li et al., 2010) A continuous function $g: [0,t) \to [0,\infty)$ is said to belong to class-K if it is a strictly increasing function and $g(0) = 0$.

Li et al. (2010) proposed the fractional order extension of Lyapunov direct method theorem presented in the Theorem 2.4.

Theorem 2.4. (Li et al., 2010) Let $x = 0$ be an equilibrium point of a fractional order non autonomous system presented in Eq. (2). Assume that there exists a Lyapunov function $V(t,x(t))$ and class-K functions g_i $(i = 1,2,3)$ satisfying

$$g_1(\|x\|) \le V(t,x(t)) \le g_2(\|x\|) \qquad (3)$$

$${}_a^C D_t^\alpha V(t,x(t)) \le g_3(\|x\|) \qquad (4)$$

where $0 < \alpha < 1$. Then the fractional order system in Eq. (2) is asymptotically stable.

In the following theorems, we present an important lemma in the Caputo fractional order dynamical system.

Lemma 2.5. (Aguila-Camacho et al., 2014) Let $x(t) \in R$ be a continuous and differentiable function. Then for any instant $t \ge a$ and for any $0 < \alpha < 1$

$$\frac{1}{2} {}_a^C D_t^\alpha x^2(t) \le x(t) {}_a^C D_t^\alpha x(t). \qquad (5)$$

From the Lemma 2.5, we find the following Lemma.

Lemma 2.6. Let $x(t) \in R$ be a continuous and differentiable function. Then for any instant $t \geq a, x^* \in R$ and for any $0 < \alpha < 1$

$\frac{1}{2}{}_a^C D_t^\alpha (x(t) - x^*)^2 \leq (x(t) - x^*){}_a^C D_t^\alpha x(t).$ (6)

Proof. By defining the auxiliary variable $y(t) = x(t) - x^*$ and by using the Lemma 2.5 we find that

$$\begin{aligned}\frac{1}{2}{}_a^C D_t^\alpha (x(t) - x^*)^2 &= \frac{1}{2}{}_a^C D_t^\alpha y^2(t)\\ &\leq y(t){}_a^C D_t^\alpha y(t)\\ &=. (x(t) - x^*){}_a^C D_t^\alpha (x(t) - x^*).\end{aligned}$$

Since Caputo fractional derivative is a linear operator and fractional derivative of any constant number x^* is zero, then we find that

$${}_a^C D_t^\alpha (x(t) - x^*) = {}_a^C D_t^\alpha x(t) - {}_a^C D_t^\alpha x^* = {}_a^C D_t^\alpha x(t).$$

Hence we find that

$$\frac{1}{2}{}_a^C D_t^\alpha (x(t) - x^*)^2 \leq (x(t) - x^*){}_a^C D_t^\alpha x(t). \blacksquare$$

3. FRACTIONAL ORDER VON BERTALANFFY GROWTH MODEL

The von Bertalanffy growth model has been applied to analyze growth data in a wide range of studies. Let $w(t)$ represents the animal weight at time t. The von Bertalanffy growth model could be derived from the von Bertalanffy differential equation given by (Ohnishi et al., 2012)

$$\frac{dw}{dt} = aw^{2/3} - bw, w(0) = w_0. \quad (7)$$

Here a and b are anabolism and catabolism coefficient respectively. The model in Eq. (7) could be represented as

$$\frac{dw}{dt} = aw^{2/3}\left(1 - \left(\frac{w}{K}\right)^{1/3}\right), w(0) = w_0 \quad (8)$$

where $K = \left(\frac{a}{b}\right)^3$. The analytical solution of Eq. (8) is given by

$$w(t) = K\left(1 - \left(1 - \left(\frac{w_0}{K}\right)^{1/3}\right)exp(-rt)\right)^3, \quad (9)$$

where $r = \frac{1}{3}aK^{-1/3}$. We can simplify the von Bertalanffy growth model in Eq. (9) into

$$w(t) = K\left(1 - c\ exp(-rt)\right)^3 \quad (10)$$

where

$$c = 1 - \left(\frac{w_0}{K}\right)^{1/3}. \quad (11)$$

Here, the parameter r and K in Eq. (10) could be interpreted as the effective growth rate and the mature weight of the animal.

Motivated by the work of El-Sayed et al. (2007), we extent the von Bertalanffy growth model in Eq. (8) into the fractional order von Bertalanffy growth model which is given by

$$\frac{d^\alpha w}{dt^\alpha} = aw^{2/3}\left(1 - \left(\frac{w}{K}\right)^{1/3}\right), w(0) = w_0 \geq 0. \quad (12)$$

where $w \in \Omega = \{u \in R: u \geq 0\}$. Here α is fractional order where $0 < \alpha \leq 1$. For any positive initial value w_0 and $0 < \alpha < 1$, the exact solution of fractional order von Bertalanffy growth model in Eq. (11) can not be determined.

The fractional order von Bertalanffy growth model in Eq. (12) has two equilibria, namely $w = 0$ and $w = K$. Note that the equilibrium $w = 0$ is a singular point, since $f'(0)$ is not exist, where $f(w) = aw^{2/3}\left(1 - \left(\frac{w}{K}\right)^{1/3}\right)$. In the following theorem, we show that the equilibrium $w = 0$ is unstable.

Theorem 3.1. The equilibrium $w = 0$ is unstable.

Proof. We define Lyapunov function $U: \Omega \to R$ where $U(w) = w$. The function U is a positive definite function on the domain Ω. We find that

$$ {}_0^C D_t^\alpha w(t) = aw^{2/3}\left(1 - \left(\frac{w}{K}\right)^{1/3}\right). $$

We find that ${}_0^C D_t^\alpha w(t) > 0$ for every w sufficiently close to 0. Hence we find that the equilibrium $w = 0$ is not stable. ■

In the next theorem, we prove global stability of the equilibrium $w = K$.

Theorem 3.2. The equilibrium $w = K$ is asymptotically stable in $\Omega\backslash\{0\} = R^+$.

Proof. We define Lyapunov function $V: \Omega \to R$ where $V(w) = \frac{1}{2}(w-K)^2$. The function V is a positive definite function in the domain R^+. The Caputo derivative of V is

$$ {}_0^C D_t^\alpha V = {}_0^C D_t^\alpha \frac{1}{2}(w-K)^2 = \frac{1}{2}\,{}_0^C D_t^\alpha (w-K)^2. $$

Then by using Lemma 2.6 we find that

$$ {}_0^C D_t^\alpha V \leq (w-K)\,{}_0^C D_t^\alpha w(t) = aw^{\frac{2}{3}}(w-K)\left(1 - \left(\frac{w}{K}\right)^{\frac{1}{3}}\right). $$

It is clear that for any $0 < w < K$, $(w-K)\left(1 - \left(\frac{w}{K}\right)^{\frac{1}{3}}\right) < 0$. We also find that for any $w > K$, $(w-K)\left(1 - \left(\frac{w}{K}\right)^{\frac{1}{3}}\right) < 0$. Hence we find that for any $R^+\backslash\{K\}$, ${}_0^C D_t^\alpha V < 0$. Given the relation between positive definite functions and class-K functions (Slotine and Li, 1999; Aguila-Camacho et al., 2014), we can conclude that the equilibrium $w = K$ is globally asymptotically stable in R^+. ■

4. IMPLEMENTATION OF FRACTIONAL ORDER VON BERTALANFFY GROWTH MODEL

In this section, we apply the fractional order von Bertalanffy growth model in Eq. (12) for describing rooster growth. We estimate all parameters in the fractional order model, namely fractional order parameter (α), effective growth rate parameter (r), mature weight parameter (K) and day old chicken parameter (w_0) from some rooster weight data cited form the literature. Here, the effective growth rate parameter is related to the anabolism coefficient through the Eq. (11). The rooster weight data (w) at the day (t) is presented in the Table 1 (Aggrey, 2002; Windarto et al., 2014).

We apply particle swarm optimization to estimate the parameters in the fractional order von Bertalanffy growth model. The particle swarm optimization algorithm consists of the following steps, which are repeated until some stopping condition is met (Kuo et al., 2011):

(1) Evaluate the objective function of every particle (solution). For a maximization problem, the greater the objective function then the greater of the fitness will be. On the other hand, for a minimization problem, the smaller the objective function then the greater of the fitness will be.
(2) Update particle best (local best) position and global best position.
(3) Update velocity of every particle using the following equation

$$v_i(t+1) = zv_i(t) + c_1r_1(lbest - x_i(t)) + c_2r_2(gbest - x_i(t)), \quad (13)$$

Here $v_i(t)$ and $x_i(t)$ are velocity of particle i and position of particle i at discrete time t, $lbest(t)$ and $gbest(t)$ are local best and global best position at time t; $r1$ and $r2$ are uniform distributed random number between zero and one.

(4) Update position of every particle using the following equation

$$x_i(t+1) = x_i(t) + v_i(t+1). \quad (14)$$

Table 1. Means of the rooster weight data (w)

t (days)	w (grams)	t (days)	y (grams)
0	37	42	519.72
3	41.74	45	577.27
6	59.19	48	633.59
9	79.94	51	667.18
12	102.96	54	717.17
15	132.13	57	786.35
18	170.18	71	1069.28
21	206.56	85	1326.49
24	250.71	99	1589.71
27	285.27	113	1859.26
30	324.92	127	2015.44
33	372.83	141	2142.31
36	417.41	155	2220.54
39	469.13	170	2262.63

In Eq. (1), z is the inertia weight, while c_1 and c_2 are cognitive coefficient and social coefficient respectively. The value of the inertial coefficient is typically between 0.8 and 1.2, while the values of cognitive coefficient and social coefficient are usually close to 2. In order to prevent the particles from moving very far beyond the search space, we applied velocity clamping technique to limit the maximum velocity of every particle. For a search space bounded by the range $[x_{min}; x_{max}]$, the velocity is limited within the range $[-v_{max}, v_{max}]$ where $v_{max} = m\,(x_{max} - x_{min})$ for some constant m, $0.1 < m < 1$. Some common stopping conditions in particle swarm optimization include a predetermined number of iterations, a number of iterations since the last update of global best solution or a preset target of the objective function value (Rini et al., 2011).

We use the particle swarm optimization method described with the inertia weight parameter $z = 1$, the cognitive coefficient parameter $c_1 = 2$ and the social coefficient parameter $c_2 = 2$ respectively. Number of particles is set to 50 particles. The particle swarm optimization algorithm is applied until 100 iterations. Parameters in the fractional order von Bertalanffy growth model (α, r, K, w_0) are estimated such that the mean square error (MSE) which is given by

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(w_i - \widehat{w_i})^2 \quad (15)$$

is minimum. Here, w_i and $\widehat{w_i}$ are rooster weight data and predicted rooster weight at the i-th day, while n is number of observation data. The estimation results of fractional order von Bertalanffy growth model are presented in the Table 2. The estimation results in Table 2 are the best results from 20 times application of particle swarm optimization method. The estimation results are not uniform, since particle swarm optimization uses pseudo-random numbers during implementation of the method.

Table 2. The estimation results of fractional order von Bertalanffy growth model

α	r (Effective growth rate)	K (Mature weight)	W_0 (day old chicken)	MSE
1	0.01489	2939.11	18.70	1200.53
1	0.01631	2683.90	11.34	1407.57
1	0.01633	2722.71	10.00	1084.73
1	0.01651	2734.73	10.00	956.22
1	0.01630	2728.94	10.75	994.43
1	0.01582	2769.14	14.92	921.35
1	0.01593	2798.74	14.64	1016.96
1	0.01636	2750.97	10.07	954.32
1	0.01663	2700.48	10.00	983.22
1	0.01416	2975.68	25.86	1206.97

From the Table 2, we find that the fractional order (α) equals one. In this case, the fractional order model could be simplified into the standard von Bertalanffy growth model. The analytical solution of the standard von Bertalanffy model was presented in Eq. (9). Since the exact solution of the model was known, we can use a deterministic optimization method e.g. Newton, Nelder-Mead, or Lavenberg-Marquardt method to estimate the parameters in the standard von Bertalanffy growth model. By applying Newton method, we find the optimal parameters are $r = 0.0158$, $K = 2787.48$, $w_0 = 14.54$, where the mean squared error MSE = 892.65. A comparison between observed and predicted rooster weight is shown in the Figure 1. From the figure, the predicted rooster weight of the model does not significantly differ from the observed data.

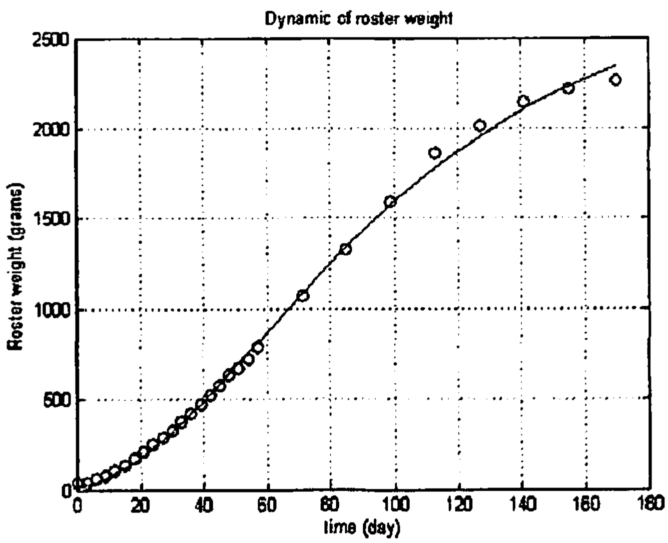

Figure 1. Comparison between observed and predicted rooster weight dynamic

5. CONCLUSION

We have generalized the standard (classical) von Bertalanffy growth model into a fractional order model. We also study stability property of the fractional order von Bertalanffy growth model. We further applied the fractional order model for describing dynamic of rooster weight. We found that for the rooster growth data, the fractional order model could be simplified into the standard classical von Bertalanffy growth model.

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