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Paper's Title:

Ostrowski Type Inequalities for Lebesgue Integral: a Survey of Recent Results

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Abstract:

The main aim of this survey is to present recent results concerning Ostrowski type inequalities for the Lebesgue integral of various classes of complex and real-valued functions. The survey is intended for use by both researchers in various fields of Classical and Modern Analysis and Mathematical Inequalities and their Applications, domains which have grown exponentially in the last decade, as well as by postgraduate students and scientists applying inequalities in their specific areas.

12: Paper Source PDF document

Paper's Title:

Bounds on the Jensen Gap, and Implications for Mean-Concentrated Distributions

Author(s):

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Abstract:

This paper gives upper and lower bounds on the gap in Jensen's inequality, i.e., the difference between the expected value of a function of a random variable and the value of the function at the expected value of the random variable. The bounds depend only on growth properties of the function and specific moments of the random variable. The bounds are particularly useful for distributions that are concentrated around the mean, a commonly occurring scenario such as the average of i.i.d. samples and in statistical mechanics.

11: Paper Source PDF document

Paper's Title:

A Multivalued Version of the Radon-Nikodym Theorem, via the Single-valued Gould Integral

Author(s):

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Abstract:

In this paper we consider a Gould type integral of real functions with respect to a compact and convex valued not necessarily additive measure. In particular we will introduce the concept of integrable multimeasure and, thanks to this notion, we will establish an exact Radon-Nikodym theorem relative to a fuzzy multisubmeasure which is new also in the finite dimensional case. Some results concerning the Gould integral are also obtained.

10: Paper Source PDF document

Paper's Title:

Solving Two Point Boundary Value Problems by Modified Sumudu Transform Homotopy Perturbation Method

Author(s):

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Abstract:

This paper considers a combined form of the Sumudu transform with the modified homotopy perturbation method (MHPM) to find approximate and analytical solutions for nonlinear two point boundary value problems. This method is called the modified Sumudu transform homotopy perturbation method (MSTHPM). The suggested technique avoids the round-off errors and finds the solution without any restrictive assumptions or discretization. We will introduce an appropriate initial approximation and furthermore, the residual error will be canceled in some points of the interval (RECP). Only a first order approximation of MSTHPM will be required, as compared to STHPM, which needs more iterations for the same cases of study. After comparing figures between approximate, MSTHPM, STHPM and numerical solutions, it is found through the solutions we have obtained that they are highly accurate, indicating that the MSTHPM is very effective, simple and can be used to solve other types of nonlinear boundary value problems (BVPs).

9: Paper Source PDF document

Paper's Title:

Strong Convergence Theorem for a Common Fixed Point of an Infinite Family of J-nonexpansive Maps with Applications

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Abstract:

Let E be a uniformly convex and uniformly smooth real Banach space with dual space E^*. Let $\{T_i\}^{\infty}{}_{i=1}$ be a family of J-nonexpansive maps, where, for each i,~T_i maps E to 2^{E^*}. A new class of maps, *J-nonexpansive maps* from E to E^*, an analogue of nonexpansive self maps of E, is introduced. Assuming that the set of common *J-fixed points* of $\{T_i\}^{\infty}{}_{i=1}$ is nonempty, an iterative scheme is constructed and proved to converge strongly to a point x^* in $\cap^{\infty}{}_{n=1}F_JT_i$. This result is then applied, in the case that E is a real Hilbert space to obtain a strong convergence theorem for approximation of a common fixed point for an infinite family of nonexpansive maps, assuming existences. The theorem obtained is compared with some important results in the literature. Finally, the technique of proof is also of independent interest.

9: Paper Source PDF document

Paper's Title:

Countable Ordinal Spaces and Compact Countable Subsets of a Metric Space

Author(s):

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Abstract:

We show in detail that every compact countable subset of a metric space is homeomorphic to a countable ordinal number, which extends a result given by Mazurkiewicz and Sierpinski for finite-dimensional Euclidean spaces. In order to achieve this goal, we use Transfinite Induction to construct a specific homeomorphism. In addition, we prove that for all metric space, the cardinality of the set of all the equivalence classes, up to homeomorphisms, of compact countable subsets of this metric space is less than or equal to aleph-one. We also show that for all cardinal number smaller than or equal to aleph-one, there exists a metric space with cardinality equals the aforementioned cardinal number.

7: Paper Source PDF document

Paper's Title:

Ostrowski Type Fractional Integral Inequalities for Generalized (s,m,φ)-preinvex Functions

Author(s):

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Abstract:

In the present paper, the notion of generalized *(s,m,φ)*-preinvex function is introduced and some new integral inequalities for the left hand side of Gauss-Jacobi type quadrature formula involving generalized *(s,m,φ)*-preinvex functions along with beta function are given. Moreover, some generalizations of Ostrowski type inequalities for generalized *(s,m,φ)*-preinvex functions via Riemann-Liouville fractional integrals are established.

7: Paper Source PDF document

Paper's Title:

Inequalities for Discrete F-Divergence Measures: A Survey of Recent Results

Author(s):

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School of Computer Science & Applied Mathematics,
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URL: http://rgmia.org/dragomir

Abstract:

In this paper we survey some recent results obtained by the author in providing various bounds for the celebrated *f*-divergence measure for various classes of functions *f*. Several techniques including inequalities of Jensen and Slater types for convex functions are employed. Bounds in terms of Kullback-Leibler Distance, Hellinger Discrimination and Varation distance are provided. Approximations of the *f*-divergence measure by the use of the celebrated Ostrowski and Trapezoid inequalities are obtained. More accurate approximation formulae that make use of Taylor's expansion with integral remainder are also surveyed. A comprehensive list of recent papers by several authors related this important concept in information theory is also included as an appendix to the main text.

6: Paper Source PDF document

Paper's Title:

On operators for which $T^2 \geq -T^{*2}$

Author(s):

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Abstract:

In this paper we introduce the new class of operators for which $T^2 \geq -T^{*2}$ acting on a complex Hilbert space *H*. We give some basic properties of these operators. we study the relation between the class and some other well known classes of operators acting on *H*.

6: Paper Source PDF document

Paper's Title:

The boundedness of Bessel-Riesz operators on generalized Morrey spaces

Author(s):

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Abstract:

In this paper, we prove the boundedness of Bessel-Riesz operators on generalized Morrey spaces. The proof uses the usual dyadic decomposition, a Hedberg-type inequality for the operators, and the boundedness of Hardy-Littlewood maximal operator. Our results reveal that the norm of the operators is dominated by the norm of the kernels.

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Paper's Title:

On The Rayleigh-Love Rod Accreting In Both Length And Cross-Sectional Area: Forced And Damped Vibrations

Author(s):

M.L.G. Lekalakala[1], M. Shatalov[2], I. Fedotov[3], S.V. Joubert[4]

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[2,3,4]Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria, South Africa.

Abstract:

In this paper an elastic cylindrical rod that is subjected to forced and damped vibrations is considered. The rod is assumed to be isotropic. The applied external force of excitation is assumed to be harmonic, and the damping force is that of Kelvin-Voigt. The longitudinally vibrating rod is fixed at the left end and free at the other end. The rod is assumed to be accreting in length and cross-sectional area as it vibrates. The problem arising and the dynamics of the vibrating rod are described and investigated within the Rayleigh-Love theories of the rod. A partial differential equation describing the longitudinal displacement of the rod is formulated. The formulated partial differential equation, together with the corresponding boundary conditions as per the configuration of the rod, is solved numerically using the Galerkin-Kantorovich method. The frequency of vibration of the harmonic exciting force is kept constant in this investigation.

It is shown that in this periodically forced viscoelastic damped vibration, all the modes of vibration are subjected to the resonance behaviour within a proper time interval, depending on the length of the accreting rod.

5: Paper Source PDF document

Paper's Title:

Properties of *q*-gamma and *q*-beta functions derived from the *q*-Gauss-Pólya inequalities

Author(s):

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Abstract:

We consider log-convexity and other properties of several functions related to q-gamma and q-beta functions. These properties are consequences of the general inequality, so-called q-analogue of the Gauss-Pólya inequality. Various inequalities involving these special functions are also given.

5: Paper Source PDF document

Paper's Title:

Inequalities for the Area Balance of Functions of Bounded Variation

Author(s):

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Private Bag 3, Johannesburg 2050,
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Abstract:

We introduce the *area balance* function associated to a Lebesgue integrable function f:[a,b] →**C** by

$$AB_f(a,b,\cdot):[a,b]\to\mathbb{C},\ AB_f(a,b,x):=\frac{1}{2}\left[\int_x^b f(t)\,dt-\int_a^x f(t)\,dt\right].$$

Several sharp bounds for functions of bounded variation are provided. Applications for Lipschitzian and convex functions are also given.

5: Paper Source PDF document

Paper's Title:

Hyponormal and *K*-Quasi-Hyponormal Operators On Semi-Hilbertian Spaces

Author(s):

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Abstract:

Let H be a Hilbert space and let A be a positive bounded operator on H. The semi-inner product $< u|v>_A := <Au|v>, u,v \in H$ induces a semi-norm $\|.\|_A$ on H. This makes H into a semi-Hilbertian space. In this paper we introduce the notions of hyponormalities and k-quasi-hyponormalities for operators on semi Hilbertian space $(H,\|.\|_A)$, based on the works that studied normal, isometry, unitary and partial isometries operators in these spaces. Also, we generalize some results which are already known for hyponormal and quasi-hyponormal operators. An operator $T \in B_A(H)$ is said to be (A, k)-quasi-hyponormal if

$$(T^\sharp)^k\left(T^\sharp T - TT^\sharp\right)T^k \geq_A 0 \text{ or equivalently } A(T^\sharp)^k\left(T^\sharp T - TT^\sharp\right)T^k \geq 0.$$

5: Paper Source PDF document

Paper's Title:

Credibility Based Fuzzy Entropy Measure

Author(s):

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Abstract:

Fuzzy entropy is the entropy of a fuzzy variable, loosely representing the information of uncertainty. This paper, first examines both previous membership and credibility based entropy measures in fuzzy environment, and then suggests an extended credibility based measure which satisfies mostly in Du Luca and Termini axioms. Furthermore, using credibility and the proposed measure, the relative entropy is defined to measure uncertainty between fuzzy numbers. Finally we provide some properties of this Credibility based fuzzy entropy measure and to clarify, give some examples.

5: Paper Source PDF document

Paper's Title:

Stability of an Almost Surjective epsilon-Isometry in The Dual of Real Banach Spaces

Author(s):

Minanur Rohman, Ratno Bagus Edy Wibowo, Marjono

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Abstract:

In this paper, we study the stability of epsilon-isometry in the dual of real Banach spaces. We prove that the almost surjective epsilon-isometry mapping is stable in dual of each spaces. The proof uses Gâteaux differentiability space (GDS), weak-star exposed points, norm-attaining operator, and some studies about epsilon-isometry that have been done before.

5: Paper Source PDF document

Paper's Title:

Robust Error Analysis of Solutions to Nonlinear Volterra Integral Equation in L^p Spaces

Author(s):

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Abstract:

In this paper, we propose a novel strategy for proving an important inequality for a contraction integral equations. The obtained inequality allows us to express our iterative algorithm using a "for loop" rather than a "while loop". The main tool used in this paper is the fixed point theorem in the Lebesgue space. Also, a numerical example shows the efficiency and the accuracy of the proposed scheme.

5: Paper Source PDF document

Paper's Title:

Hermite-Hadamard Type Inequalities for *k*-Riemann Liouville Fractional Integrals Via Two Kinds of Convexity

Author(s):

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Abstract:

In this article, a fundamental integral identity including the first order derivative of a given function via *k*-Riemann-Liouville fractional integral is established. This is used to obtain further Hermite-Hadamard type inequalities involving left-sided and right-sided *k*-Riemann-

Liouville fractional integrals for *m*-convex and *(s,m)*-convex functions respectively.

5: Paper Source PDF document

Paper's Title:

Lower and Upper Bounds for the Point-Wise Directional Derivative of the Fenchel Duality Map

Author(s):

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Abstract:

In this paper, we introduce the point-wise directional derivative of the Fenchel duality map and we study its properties. The best lower and upper bounds of this point-wise directional derivative are also given. We explain how our functional results contain those related to the positive bounded linear operators.

5: Paper Source PDF document

Paper's Title:

A Generalization of Viete's Infinite Product and New Mean Iterations

Author(s):

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Abstract:

In this paper, we generalize Viéte's infinite product formula by use of Chebyshev polynomials. Furthermore, the infinite product formula for the lemniscate sine is also generalized. Finally, we obtain new mean iterations by use of these infinite product formulas.

5: Paper Source PDF document

Paper's Title:

Stability of the D-Bar Reconstruction Method for Complex Conductivities

Author(s):

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Abstract:

In 2000, Francini solved the inverse conductivity problem for twice-differentiable conductivities and permittivities. This solution was considered to be the first approach using D-bar methods with complex conductivities. In 2012, based on Francini's work, Hamilton introduced a reconstruction method of the conductivity distribution with complex values. The method consists of six steps. A voltage potential is applied on the boundary. Solving a D-Bar equation gives the complex conductivity. In this paper, the stability of the D-Bar equation is studied via two approximations, t^{exp} and t^{B}, for the scattering transform. The study is based on rewriting the reconstruction method in terms of continuous operators. The conductivity is considered to be non smooth.

5: Paper Source PDF document

Paper's Title:

On Singular Numbers of Hankel Matrices of Markov Functions

Author(s):

Vasily A. Prokhorov

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Abstract:

Let $E \subset (01,1)$ be a compact set and let μ be a positive Borel measure with support supp $\mu=E$. Let

$$D_n = \left(\frac{1}{2\pi} \int_E x^{i+j} d\mu(x) \right)_{i,j=0}^{n}.$$

In the case when $E=[a,b]\subset(-1,1)$ and μ satisfies the condition $d\mu/dx>0$ a.e. on E, we investigate asymptotic behavior of singular numbers $\sigma_{kn,n}$ of the Hankel matrix D_n, where $k_n/n\rightarrow\theta\in[0,1]$ as $n\rightarrow\infty$. Moreover, we obtain asymptotics of the Kolmogorov, Gelfand and linear k-widths, $k=k_n$, of the unit ball $A_{n,2}$ *of* $P_n\cap L_2(\Gamma)$ in the space $L_2(\mu,E)$, where $\Gamma=\{z:|z|=1\}$ and P_n is the class of all polynomials of the degree at most n.

4: Paper Source PDF document

Paper's Title:

Some New Generalizations of Jensen's Inequality with Related Results and Applications

Author(s):

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Abstract:

In this paper, some new generalizations of Jensen's inequality are presented. In particular, upper and lower bounds for the Jensen gap are given and compared analytically and numerically to previously published bounds for both the discrete and continuous Jensen's inequality cases. The new bounds compare favorably to previously proposed bounds. A new method based on a series of locally linear interpolations is given and is the basis for most of the bounds given in this paper. The wide applicability of this method will be demonstrated. As by-

products of this method, we shall obtain some new Hermite-Hadamard inequalities for functions which are 3-convex or 3-concave. The new method works to obtain bounds for the Jensen gap for non-convex functions as well, provided one or two derivatives of the nonlinear function are continuous. The mean residual life function of applied probability and reliability theory plays a prominent role in construction of bounds for the Jensen gap. We also present an exact integral representation for the Jensen gap in the continuous case. We briefly discuss some inequalities for other types of convexity, such as convexity in the geometric mean, and briefly discuss applications to reliability theory.

4: Paper Source PDF document

Paper's Title:

On the Constant in a Transference Inequality for the Vector-valued Fourier Transform

Author(s):

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Abstract:

The standard proof of the equivalence of Fourier type on R^d and on the torus T^d is usually stated in terms of an implicit constant, defined as the minimum of a sum of powers of sinc functions. In this note we compute this minimum explicitly.

4: Paper Source PDF document

Paper's Title:

Bartle Integration in Lie Algebras

Author(s):

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Abstract:

Using Bartle's bilinear vector integral we define stochastic integrals of bounded operator valued functions with respect to Stieltjes measures associated with the generators of the Heisenberg and Finite Difference Lie algebras. Our definition also covers the Square of White Noise and sl/2 Lie algebras.

4: Paper Source PDF document

Paper's Title:

Presentation a mathematical model for bone metastases control by using tamoxifen

Author(s):

Maryam Nikbakht, Alireza Fakharzadeh Jahromi and Aghileh Heydari

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Abstract:

Bone is a common site for metastases (secondary tumor) because of breast and prostate cancer. According to our evaluations the mathematical aspect of the effect of drug in bone metastases has not been studied yet. Hence, this paper suggested a new mathematical model for bone metastases control by using tamoxifen. The proposed model is a system of nonlinear partial differential equations. In this paper our purpose is to present a control model for bone metastases. At end by some numerical simulations, the proposed model is examined by using physician.

4: Paper Source PDF document

Paper's Title:

Some interesting properties of finite continuous Cesàro operators

Author(s):

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Abstract:

A complex scalar λ is called an extended eigenvalue of a bounded linear operator T on a complex Banach space if there is a nonzero operator X such that $TX = \lambda XT$, the operator X is called extended eigenoperator of T corresponding to the extended eigenvalue λ.

In this paper we prove some properties of extended eigenvalue and extended eigenoperator for C_1 on $L^p([0,1])$, where C_1 is the Cesàro operator defined on the complex Banach spaces $L^p([0 , 1])$ for $1<p<\infty$ by the expression

$$(C_1 f)(x) = \frac{1}{x}\int_0^x f(t)dt.$$

4: Paper Source PDF document

Paper's Title:

Partial Semigroup Algebras Associated to Partial Action

Author(s):

Bahman Tabatabaie Shourijeh and Sahar Moayeri Rahni

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Abstract:

For a given inverse semigroup S, we introduce the notion of algebraic crossed product by using a given partial action of S, and we will prove that under some condition it is associative. Also we will introduce the concept of partial semigroup algebra $K_{Par}(S)$, and we show that the suitable quotient of $K_{Par}(S)$ is a kind of crossed product.

4: Paper Source PDF document

Paper's Title:

Strong Convergence Theorems for a Common Zero of an Infinite Family of Gamma-Inverse Strongly Monotone Maps with Applications

Author(s):

Charles Ejike Chidume, Ogonnaya Michael Romanus, and Ukamaka Victoria Nnyaba

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Abstract:

Let E be a uniformly convex and uniformly smooth real Banach space with dual space E^* and let $A_k:E\to E^*$, $k=1, 2, 3, \ldots$ be a family of inverse strongly monotone maps such that $\cap^{\infty}_{k=1} A_k^{-1}(0)\neq\emptyset$.
A new iterative algorithm is constructed and proved to converge strongly to a common zero of the family.
As a consequence of this result, a strong convergence theorem for approximating a common J-fixed point for an infinite family of gamma-strictly J-pseudocontractive maps is proved. These results are new and improve recent results obtained for these classes of nonlinear maps.
Furthermore, the technique of proof is of independent interest.

4: Paper Source PDF document

Paper's Title:

Convergence and Stability Results for New Three Step Iteration Process in Modular Spaces

Author(s):

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Abstract:

The aim of this paper is to introduce a new iteration process (5) for ρ-contraction mappings in Modular spaces. We obtain some analytical proof for convergence and stability of our iteration process (5). We show that our iteration process (5) gives faster convergence results than the leading AK iteration process (4) for contraction mappings. Moreover, a numerical example (using the Matlab Software) is presented to compare the rate of convergence for existing iteration processes with our new iteration process (5).

4: Paper Source PDF document

Paper's Title:

Global Analysis on Riemannian Manifolds

Author(s):

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Abstract:

In this paper, an exposition of the central concept of global analysis on a Riemannan manifold is given. We extend the theory of smooth vector fields from open subsets of Euclidean space to Riemannan manifolds. Specifically, we prove that a Riemannian manifold admits a unique solution for a system of ordinary differential equations generated by the flow of smooth tangent vectors. The idea of partial differential equations on Riemannian manifold is highlighted on the unit sphere.

3: Paper Source PDF document

Paper's Title:

Generalized Weighted Trapezoid and Grüss Type Inequalities on Time Scales

Author(s):

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Abstract:

In this work, we obtain some new generalized weighted trapezoid and Grüss type inequalities on time scales for parameter functions. Our results give a broader generalization of the results due to Pachpatte in [14]. In addition, the continuous and discrete cases are also considered from which, other results are obtained.

3: Paper Source PDF document

Paper's Title:

New Implicit Kirk-Type Schemes for General Class of Quasi-Contractive Operators in Generalized Convex Metric Spaces

Author(s):

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Abstract:

In this paper, we introduce some new implicit Kirk-type iterative schemes in generalized convex metric spaces in order to approximate fixed points for general class of quasi-contractive type operators. The strong convergence, T-stability, equivalency, data dependence and convergence rate of these results were explored. The iterative schemes are faster and better, in term of speed of convergence, than their corresponding results in the literature. These results also improve and generalize several existing iterative schemes in the literature and they provide analogues of the corresponding results of other spaces, namely: normed spaces, CAT(0) spaces and so on.

3: Paper Source PDF document

Paper's Title:

Iterative Approximation of Zeros of Accretive Type Maps, with Applications

Author(s):

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Abstract:

Let E be a reflexive real Banach space with uniformly Gâteaux differentiable norm. Let $J:E\rightarrow E^*$ be the normalized duality map on E and let $A:E^*\rightarrow E$ be a map such that AJ is an accretive and uniformly continuous map. Suppose that $(AJ)^{-1}(0)$ in nonempty. Then, an iterative sequence is constructed and proved to converge strongly to some u^* in $(AJ)^{-1}(0)$. Application of our theorem in the case that E is a real Hilbert space yields a sequence which converges strongly to a zero of A. Finally, non-trivial examples of maps A for which AJ is accretive are presented..

3: Paper Source PDF document

Paper's Title:

The Dynamics of an Ebola Epidemic Model with Quarantine of Infectives

Author(s):

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Abstract:

The recurrent outbreaks of ebola in Africa present global health challenges. Ebola is a severe, very fatal disease with case fatality rates of up to 90%. In this paper, a theoretical deterministic model for ebola epidemic with quarantine of infectives is proposed and analyzed. The model exhibits two equilibria; the disease free and endemic equilibrium points. The basic reproduction number, R_0, which is the main threshold, is obtained and the stability of the equilibrium points established. Using parameter values drawn from the 2014 West Africa ebola outbreak, a numerical simulation of the model is carried out. It is found that the dynamics of the model are completely determined by R_0 and that a quarantine success rate of at least 70% is sufficient to contain the disease outbreak.

3: Paper Source PDF document

Paper's Title:

On Closed Range C^*-modular Operators

Author(s):

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Abstract:

In this paper, for the class of the modular operators on Hilbert C^*-modules, we give the conditions to closedness of their ranges. Also, the equivalence conditions for the closedness of the range of the modular projections on Hilbert C^*-modules are discussed. Moreover, the mixed reverse order law for the Moore-Penrose invertible modular operators are given.

3: Paper Source PDF document

Paper's Title:

A Comparison Between Two Different Stochastic Epidemic Models with Respect to the Entropy

Author(s):

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Abstract:

In this paper at first a brief history of mathematical models is presented with the aim to clarify the reliability of stochastic models over deterministic models. Next, the necessary background about random variables and stochastic processes, especially Markov chains and the entropy are introduced. After that, entropy of SIR stochastic models is computed and it is proven that an epidemic will disappear after a long time. Entropy of a stochastic mathematical model determines the average uncertainty about the outcome of that random experiment. At the end, we introduce a chain binomial epidemic model and compute its entropy, which is then compared with the DTMC SIR epidemic model to show which one is nearer to reality.

3: Paper Source PDF document

Paper's Title:

Cubic Alternating Harmonic Number Sums

Author(s):

Anthony Sofo

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Abstract:

We develop new closed form representations of sums of cubic alternating harmonic numbers and reciprocal binomial coefficients. We also identify a new integral representation for the $\zeta(4)$ constant.

2: Paper Source PDF document

Paper's Title:

Some New Inequalities of Hermite-Hadamard and Fejér Type for Certain Functions with Higher Convexity

Author(s):

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Abstract:

In this paper, we present some new inequalities of Hermite-Hadamard or Fejér type for certain functions satisfying some higher convexity conditions on one or more derivatives.
An open problem is given also.
Some applications to the logarithmic mean are given.

2: Paper Source PDF document

Paper's Title:

MSplit Equality for Monotone Inclusion Problem and Fixed Point Problem in Real Banach Spaces

Author(s):

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Abstract:

In this paper a new iterative algorithm for approximating a common solution of split equality monotone inclusion problem and split equality fixed point problem is introduced. Using our algorithm, we state and prove a strong convergence theorem for approximating an element in the intersection of the set of solutions of a split equality monotone inclusion problem and the set of solutions of a split equality fixed point problem for right Bregman strongly nonexpansive mappings in the setting of p-uniformly convex Banach spaces which are also uniformly smooth. We also give some applications.

2: Paper Source PDF document

Paper's Title:

Some Inequalities of the Hermite-Hadamard Type for k-Fractional Conformable Integrals

Author(s):

C.-J. Huang, G. Rahman, K. S. Nisar, A. Ghaffar and F. Qi

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Abstract:

In the paper, the authors deal with generalized k-fractional conformable integrals, establish some inequalities of the Hermite-Hadamard type for generalized k-fractional conformable integrals for convex functions, and generalize known inequalities of the Hermite-Hadamard type for conformable fractional integrals.

2: Paper Source PDF document

Paper's Title:

An Analytical Solution of Perturbed Fisher's Equation Using Homotopy Perturbation Method (HPM), Regular Perturbation Method (RPM) and Adomian Decomposition Method (ADM)

Author(s):

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Abstract:

In this paper, Homotopy Perturbation Method (HPM), Regular Pertubation Method (RPM) and Adomian decomposition Method (ADM) are applied to Fisher equation. Then, the solution yielding the given initial conditions is gained. Finally, the solutions obtained by each method are compared

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2: Paper Source PDF document

Paper's Title:

Some Convergence Results for Jungck-Am Iterative Process In Hyperbolic Spaces

Author(s):

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Abstract:

In this paper, we introduce a new three steps iterative process called Jungck-AM iterative process and show that the proposed iterative process can be used to approximate fixed points of Jungck-contractive type mappings and Jungck-Suzuki type mappings. In addition, we establish some strong and Δ-convergence results for the approximation of fixed points of Jungck-Suzuki type mappings in the frame work of uniformly convex hyperbolic space. Furthermore, we show that the newly proposed iterative process has a better rate of convergence compare to the Jungck-Noor, Jungck-SP, Jungck-CR and some existing iterative processes in the literature. Finally, stability, data dependency results for Jungck-AM iterative process is established and we present an analytical proof and numerical examples to validate our claim.

2: Paper Source PDF document

Paper's Title:

Action of Differential Operators On Chirpsconstruct On L^∞

Author(s):

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Abstract:

We will study in this work the action of differential operators on L∞ chirps and we will give a new definition of logarithmic chirp. Finally we will study the action of singular integral operators on chirps by wavelet characterization and Kernel method.

2: Paper Source PDF document

Paper's Title:

An Existence of the Solution to Neutral Stochastic Functional Differential Equations Under the Holder Condition

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Abstract:

In this paper, we show the existence and uniqueness of solution of the neutral stochastic functional differential equations under weakened H\"{o}lder condition, a weakened linear growth condition, and a contractive condition. Furthermore, in order to obtain the existence of a solution to the equation we used the Picard sequence.

1: Paper Source PDF document

Paper's Title:

Some Identities for Ramanujan - Göllnitz - Gordon Continued fraction

Author(s):

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Abstract:

In this paper, we obtain certain *P--Q* eta--function identities, using which we establish identities providing modular relations between Ramanujan-Göllnitz-Gordon continued fraction *H(q)* and *H(q^n)* for *n= 2, 3, 4, 5, 7, 8, 9, 11, 13, 15, 17, 19, 23, 25, 29* and *55*.

1: Paper Source PDF document

Paper's Title:

C*-valued metric projection and Moore-Penrose inverse on Hilbert C*-modules

Author(s):

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Abstract:

Let *t* be a regular operator between Hilbert C^*-modules and $t^\dagger$ be its Moore-Penrose inverse. We give some characterizations for $t^\dagger$ based on C^*-valued metric projection. Moore-Penrose inverse of bounded operators and elements of a C^*-algebra is studied as a special case.

1: Paper Source PDF document

Paper's Title:

A Subordination Theorem for Analytic Functions

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Abstract:

It is shown that if f is analytic in $D=\{z:|z|<1\}$, with $f(0)=f'(0)-1=0$, then for $\alpha > 0,\ \gamma > 0,\ f'(z)\left(\frac{f(z)}{z}\right)^{\alpha-1} \prec \left(\frac{1+z}{1-z}\right)^{\beta(\gamma)}$ implies $\left(\frac{f(z)}{z}\right)^{\alpha} \prec \left(\frac{1+z}{1-z}\right)^{\gamma}$, where $\beta(\gamma) = \gamma + \frac{2}{\pi}\arctan\left(\frac{\gamma}{\alpha}\right)$, and that β(γ) is the largest number such that this implication holds.

1: Paper Source PDF document

Paper's Title:

A Note on Divergent Fourier Series and λ-Permutations

Author(s):

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Abstract:

We present a continuous function on [-π,π] whose Fourier series diverges and it cannot be rearranged to converge by a λ-permutation.

1: Paper Source PDF document

Paper's Title:

Bilinear Regular Operators on Quasi-Normed Functional Spaces

Author(s):

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Abstract:

Positive and regular bilinear operators on quasi-normed functional spaces are introduced and theorems characterizing compactness of these operators are proved. Relations between bilinear operators and their adjoints in normed functional spaces are also studied.

1: Paper Source PDF document

Paper's Title:

New Refinements of Hölder's Inequality

Author(s):

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Abstract:

In this paper, we define two mappings, investigate their properties, obtain some new refinements of Hölder's inequality.

1: Paper Source PDF document

Paper's Title:

Extreme Curvature of Polynomials and Level Sets

Author(s):

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Abstract:

Let f be a real polynomial of degree n. Determining the maximum number of zeros of kappa, the curvature of f, is an easy problem: since the zeros of kappa are the zeros of f'', the curvature of f is 0 at most n-2 times. A much more intriguing problem is to determine the maximum number of relative extreme values for the function kappa. Since kappa'=0 at each extreme point of kappa, we are interested in the maximum number of zeros of kappa'. In 2004, the first author and R. Gordon showed that if all the zeros of f'' are real, then f has at most n-1 points of extreme curvature. We use level curves and auxiliary functions to study the zeros of the derivatives of these functions. We provide a partial solution to this problem, showing that f has at most n-1 points of extreme curvature, given certain geometrical conditions. The conjecture that f has at most n-1 points of extreme curvature remains open.

1: Paper Source PDF document

Paper's Title:

Some properties of k-quasi class Q* operators

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Abstract:

In this paper, we give some results of *k*-quasi class Q^* operators. We proved that if *T* is an invertible operator and *N* be an operator such that *N* commutes with T^*T, then *N* is *k*-quasi class Q^* if and only if TNT^{-1} is of *k*-quasi class Q^*. With example we proved that exist an operator *k*-quasi class Q^* which is quasi nilpotent but it is not quasi hyponormal.

1: Paper Source PDF document

Paper's Title:

Applications of the Structure Theorem of (w_1,w_2)-Tempered Ultradistributions

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Abstract:

Using a previously obtained structure theorem for (w_1, w_2)-tempered ultradistributions, we prove that these ultradistributions can be represented as initial values of solutions of the heat equation.

1: Paper Source PDF document

Paper's Title:

Euler Series Solutions for Linear Integral Equations

Author(s):

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Abstract:

In this work, we seek the approximate solution of linear integral equations by truncation Euler series approximation. After substituting the Euler expansions for the given functions of the equation and the unknown one, the equation reduces to a linear system, the solution of this latter gives the Euler coefficients and thereafter the solution of the equation. The convergence and the error analysis of this method are discussed. Finally, we compare our numerical results by others.

1: Paper Source PDF document

Paper's Title:

Coefficient Estimates for Certain Subclasses of Bi-univalent Sakaguchi Type Functions by using Faber Polynomial

Author(s):

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Abstract:

In this work, considering a general subclass of bi-univalent Sakaguchi type functions, we determine estimates for the general Taylor-Maclaurin coefficients of the functions in these classes. For this purpose, we use the Faber polynomial expansions. In certain cases, our estimates improve some of those existing coefficient bounds.

1: Paper Source PDF document

Paper's Title:

Invariant Subspaces Close to Almost Invariant Subspaces for Bounded Linear Operators

Author(s):

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Abstract:

In this paper, we consider some features of almost invariant subspace notion. At first, we restate the notion of almost invariant subspace and obtain some results. Then we try to achieve an invariant subspace completely close to an almost invariant subspace. Also, we introduce the notion of "almost equivalent subspaces" to simply the subject related to almost invariant subspaces and apply it.

1: Paper Source PDF document

Paper's Title:

Local Boundedness of Weak Solutions for Singular Parabolic Systems of *p*-Laplacian Type

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Abstract:

We study the local boundedness of weak solutions for evolutional p-Laplacian systems in the singular case. The initial data is belonging to Lebesgue space $L^{\infty}(0,T;W^{(1,p)}(\Omega,\mathbf{R}^n))$. We use intrinsic scaling method to treat the boundedness of weak solutions. The main result is to make the local boundedness of weak solution for the systems well-worked in the intrinsic scaling.

1: Paper Source PDF document

Paper's Title:

Euler-Maclaurin Formulas for Functions of Bounded Variation

Author(s):

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Abstract:

The first-order Euler-Maclaurin formula relates the sum of the values of a smooth function on an interval of integers with its integral on the same interval on $\boldsymbol{R}$. We formulate here the analogue for functions that are just of bounded variation.

1: Paper Source PDF document

Paper's Title:

The Functional Equation With an Exponential Polynomial Solution and its Stability

Author(s):

Young Whan Lee[1], Gwang Hui Kim[2*], and Jeong Il Kim[3]

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Abstract:

In this paper, we prove that the unique continuous solution of the functional equation

is

where $p_n(x)$ is a polynomial with degree n and

We also obtain the superstability and stability of the functional equation with the following forms, respectively:

and

1: Paper Source PDF document

Paper's Title:

Convergence Speed of Some Random Implicit-Kirk-type Iterations for Contractive-type Random Operators

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Abstract:

The main aim of this paper is to introduce a stochastic version of multistep type iterative scheme called a modified random implicit-Kirk multistep iterative scheme and prove strong convergence and stability results for a class of generalized contractive-type random operators. The rate of convergence of the random iterative schemes are also examined through an example. The results show that our new random implicit kirk multistep scheme perform better than other implicit iterative schemes in terms of convergence and thus have good potentials for further applications in equilibrium problems in computer science, physics and economics.

1: Paper Source PDF document

Paper's Title:

A Note on Taylor Expansions Without the Differentiability Assumption

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Abstract:

We introduce new Taylor expansions when the function is not differentiable.

1: Paper Source PDF document

Paper's Title:

The Influence of Fluid Pressure in Macromechanical Cochlear Model

Author(s):

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Abstract:

An increase of pressure in the structure of cochlea may cause a hearing loss. In this paper, we established the relationship between the fluid pressure and the amplitude of displacement of Basilar Membrane to clarify the mechanisms of hearing loss caused by increasing of this pressure. So, a mathematical cochlear model was formulated using finite difference method in order to explain and demonstrate this malfunction in passive model. Numerical simulations may be considered as helpful tools which may extend and complete the understanding of a cochlea dysfunction.

1: Paper Source PDF document

Paper's Title:

Dynamical Analysis of HIV/AIDS Epidemic Model with Two Latent Stages, Vertical Transmission and Treatment

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Abstract:

We discuss about dynamical analysis of HIV/AIDS epidemic model with two latent stages, vertical transmission and treatment. In this model, the spreading of HIV occurs through both horizontal and vertical transmission. There is also treatment for individual who has been HIV infected. The latent stage is divided into slow and fast latent stage based on the immune condition which varies for each individual. Dynamical analysis result shows that the model has two equilibrium points: the disease-free equilibrium point and the endemic equilibrium point. The existence and global stability of equilibrium points depend on the basic reproduction number R_0. When $R_0 < 1$, only the disease-free equilibrium point exists. If $R_0 > 1$, there are two equilibrium points, which are the disease-free equilibrium point and the endemic equilibrium point. Based on the result of stability analysis, the disease-free equilibrium point is globally asymptotically stable if $R_0 < 1$, while if $R_0 > 1$ and $p=q$, the endemic equilibrium point will be globally asymptotically stable. In the end, we show some numerical simulations to support the analytical result.

1: Paper Source PDF document

Paper's Title:

On a subset of Bazilevic functions

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Abstract:

Let *S* denote the class of analytic and univalent functions in $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. For $\alpha \geq 0$, the subclass $B_1\alpha$ of *S* of Bazilevic functions has been extensively studied. In this paper we determine various properties of a subclass of $B_1\alpha$, for $\alpha \geq 0$ which extends early results of a class of starlike functions studied by Ram Singh.

1: Paper Source PDF document

Paper's Title:

On an extension of Edwards's double integral with applications

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Abstract:

The aim of this note is to provide an extension of the well known and useful Edwards's double integral. As an application, new class of twelve double integrals involving hypergeometric function have been evaluated in terms of gamma function. The results are established with the help of classical summation theorems for the series ${}_3F_2$ due to Watson, Dixon and Whipple. Several new and interesting integrals have also been obtained from our main findings.

1: Paper Source PDF document

Paper's Title:

Some New Mappings Related to Weighted Mean Inequalities

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Abstract:

In this paper, we define four mappings related to weighted mean inequalities, investigate their properties, and obtain some new refinements of weighted mean inequalities.

1: Paper Source PDF document

Paper's Title:

A new approach to the study of fixed point for simulation functions with application in G-metric spaces

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Abstract:

The purpose of this work is to generalize the fixed point results of Kumar et al. [11] by introducing the concept of (α,β)-Z-contraction mapping, Suzuki generalized (α,β)-Z-contraction mapping, (α,β)-admissible mapping and triangular (α,β)-admissible mapping in the frame work of G-metric spaces. Fixed point theorems for these class of mappings are established in the frame work of a complete G-metric spaces and we establish a generalization of the fixed point result of Kumar et al. [11] and a host of others in the literature. Finally, we apply our fixed point result to solve an integral equation.

1: Paper Source PDF document

Paper's Title:

Composite Variational-Like Inequalities Given By Weakly Relaxed ζ-Semi-Pseudomonotone Multi-Valued Mapping

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Abstract:

In this article, we introduce a composite variational-like inequalities with weakly relaxed ζ-pseudomonotone multi-valued maping in reflexive Banach spaces. We obtain existence of solutions to the composite variational-like inequalities with weakly relaxed ζ-pseudomon -otone multi-valued maps in reflexive Banach spaces by using KKM theorem. We have also checked the solvability of the composite variational-like inequalities with weakly relaxed ζ-semi-pseudomonotone multi-valued maps in arbitrary Banach spaces using Kakutani-Fan-Glicksberg fixed point theorem.

1: Paper Source PDF document

Paper's Title:

The Jacobson Density Theorem for Non-Commutative Ordered Banach Algebras

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Abstract:

The Jacobson density theorem for general non-commutative Banach algebras states as follows: Let π be a continuous, irreducible representation of a non-commutative Banach algebra *A* on a Banach space *X*. If $x_1,x_2,...,x_n$ are linearly independent in *X* and if $y_1,y_2,...,y_n$ are in *X*, then there exists an a∈ A such that $\pi(a)x_i=y_i$ for $i=1,2,...,n$. By considering ordered Banach algebras *A* and ordered Banach spaces *X*, we shall establish an order-theoretic version of the Jacobson density theorem.

1: Paper Source PDF document

Paper's Title:

Several Applications of a Local Non-convex Young-type Inequality

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Abstract:

A local version of the Young inequality for positive numbers is used in order to deduce some inequalities about determinants and norms for real quadratic matrices and norms of positive operators on complex Hilbert spaces.

1: Paper Source PDF document

Paper's Title:

On Ruled Surfaces According to Quasi-Frame in Euclidean 3-Space

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Abstract:

This paper aims to study the skew ruled surfaces by using the quasi-frame of Smarandache curves in the Euclidean 3-space. Also, we reveal the relationship between Serret-Frenet and quasi-frames and give a parametric representation of a directional ruled surface using the quasi-frame. Besides, some comparative examples are given and plotted which support our method and main results.

1: Paper Source PDF document

Paper's Title:

On The Degree of Approximation of Periodic Functions from Lipschitz and Those from Generalized Lipschitz Classes

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Abstract:

In this paper we have introduced some new trigonometric polynomials. Using these polynomials, we have proved some theorems which determine the degree of approximation of periodic functions by a product of two special means of their Fourier series and the conjugate Fourier series. Many results proved previously by others are special case of ours.

The Australian Journal of Mathematical Analysis and Applications

AJMAA

Volume 13, Issue 1, Article 9, pp. 1-10, 2016

THE BOUNDEDNESS OF BESSEL-RIESZ OPERATORS ON GENERALIZED MORREY SPACES

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Received 23 May, 2016; accepted 22 June, 2016; published 4 July 2016, 2016.

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ABSTRACT. In this paper, we prove the boundedness of Bessel-Riesz operators on generalized Morrey spaces. The proof uses the usual dyadic decomposition, a Hedberg-type inequality for the operators, and the boundedness of Hardy-Littlewood maximal operator. Our results reveal that the norm of the operators is dominated by the norm of the kernels.

Key words and phrases: Bessel-Riesz operators, Hardy-Littlewood maximal operator, generalized Morrey spaces, Boundedness, Kernels.

2000 *Mathematics Subject Classification.* Primary 42B20; Secondary 26A33, 42B25, 26D10, 47G10.

ISSN (electronic): 1449-5910

The first and second authors are supported by ITB Research & Innovation Program 2015.

1. INTRODUCTION

We begin with the definition of Bessel-Riesz operators. For $\gamma \geq 0$ and $0 < \alpha < n$, we define

$$I_{\alpha,\gamma} f(x) := \int_{\mathbb{R}^n} K_{\alpha,\gamma}(x-y) f(y)\, dy,$$

for every $f \in L^p_{loc}(\mathbb{R}^n)$, $p \geq 1$, where $K_{\alpha,,\gamma}(x) := \frac{|x|^{\alpha-n}}{(1+|x|)^{\gamma}}$. Here, $I_{\alpha,\gamma}$ is called *Bessel-Riesz operator* and $K_{\alpha,\gamma}$ is called *Bessel-Riesz kernel*. The name of the kernel resembles the product of Bessel kernel and Riesz kernel [13]. While the Riesz kernel captures the local behaviour, the Bessel kernel take cares the global behaviour of the function. The Bessel-Riesz kernel is used in studying the behaviour of the solution of a Schrödinger type equation [8].

For $\gamma = 0$, we have $I_{\alpha,0} = I_\alpha$, known as *fractional integral operators* or *Riesz potentials*[12]. Around 1930, Hardy and Littlewood [5, 6] and Sobolev [11] have proved the boundedness of I_α on Lebesgue spaces via the inequality

$$\|I_\alpha f\|_{L^q} \leq C\, \|f\|_{L^p},$$

for every $f \in L^p(\mathbb{R}^n)$, where $1 < p < \frac{n}{\alpha}$, and $\frac{1}{q} = \frac{1}{p} - \frac{\alpha}{n}$. Here C denotes a constant which may depend on α, p, q, and n, but not on f.

For $1 \leq p \leq q$, the *(classical) Morrey space* $L^{p,q}(\mathbb{R}^n)$ is defined by

$$L^{p,q}(\mathbb{R}^n) := \left\{ f \in L^p_{loc}(\mathbb{R}^n) : \|f\|_{L^{p,q}} < \infty \right\},$$

where $\|f\|_{L^{p,q}} := \sup_{r>0, a\in\mathbb{R}^n} r^{n(1/q-1/p)} \left(\int_{|x-a|<r} |f(x)|^p dx \right)^{1/p}$. For these spaces, we have an inclusion property which is presented by the following theorem.

Theorem 1.1. *For $1 \leq p \leq q$, we have $L^q(\mathbb{R}^n) = L^{q,q}(\mathbb{R}^n) \subseteq L^{p,q}(\mathbb{R}^n) \subseteq L^{1,q}(\mathbb{R}^n)$.*

On Morrey spaces, Spanne [10] has shown that I_α is bounded form $L^{p_1,q_1}(\mathbb{R}^n)$ to $L^{p_2,q_2}(\mathbb{R}^n)$ for $1 < p_1 < q_1 < \frac{n}{\alpha}$, $\frac{1}{p_2} = \frac{1}{p_1} - \frac{\alpha}{n}$, and $\frac{1}{q_2} = \frac{1}{q_1} - \frac{\alpha}{n}$. Furthermore, Adams [1] and Chiarenza and Frasca [2] reproved it and obtained a stronger result which is presented below.

Theorem 1.2. *[Adams, Chiarenza-Frasca] If $0 < \alpha < n$, then we have*

$$\|I_\alpha f\|_{L^{p_2,q_2}} \leq C\, \|f\|_{L^{p_1,q_1}},$$

for every $f \in L^{p_1,q_1}(\mathbb{R}^n)$ where $1 < p_1 < q_1 < \frac{n}{\alpha}$, $\frac{1}{p_2} = \frac{1}{p_1}\left(1 - \frac{\alpha q_1}{n}\right)$, and $\frac{1}{q_2} = \frac{1}{q_1} - \frac{\alpha}{n}$.

For $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ and $1 \leq p < \infty$, we define the *generalized Morrey space*

$$L^{p,\phi}(\mathbb{R}^n) := \left\{ f \in L^p_{loc}(\mathbb{R}^n) : \|f\|_{L^{p,\phi}} < \infty \right\},$$

where $\|f\|_{L^{p,\phi}} := \sup_{r>0, a\in\mathbb{R}^n} \frac{1}{\phi(r)} \left(\frac{1}{r^n} \int_{|x-a|<r} |f(x)|^p\, dx \right)^{1/p}$. Here we assume that ϕ is almost decreasing and $t^{n/p}\phi(t)$ is almost decreasing, so that ϕ satisfies the *doubling condition*, that is, there exists a constant C such that $\frac{1}{C} \leq \frac{\phi(r)}{\phi(v)} \leq C$ whenever $\frac{1}{2} \leq \frac{r}{v} \leq 2$.

In 1994, Nakai [9] obtained the boundedness of I_α from $L^{p_1,\phi}(\mathbb{R}^n)$ to $L^{p_2,\psi}(\mathbb{R}^n)$ where $1 < p_1 < \frac{n}{\alpha}$, $\frac{1}{p_2} = \frac{1}{p_1} - \frac{\alpha}{n}$ and $\int_r^\infty v^{\alpha-1}\phi(v)dv \leq Cr^\alpha\phi(r) \leq C\psi(r)$ for every $r > 0$. Nakai's result may be viewed as an extension of Spanne's. Later on, in 2009, Gunawan and Eridani [3] extended Adams-Chiarenza-Frasca's result.

Theorem 1.3. *[Gunawan-Eridani] If $\int_r^\infty \frac{\phi(v)}{v} dv \leq C\phi(r)$, and $\phi(r) \leq Cr^\beta$ for every $r > 0$, $-\frac{n}{p_1} \leq \beta < -\alpha$, $1 < p_1 < \frac{n}{\alpha}$, $0 < \alpha < n$, then we have*

$$\|I_\alpha f\|_{L^{p_2,\psi}} \leq C\, \|f\|_{L^{p_1,\phi}}$$

for every $f \in L^{p_1,\phi}(\mathbb{R}^n)$ where $p_2 = \frac{\beta p_1}{\alpha+\beta}$ and $\psi(r) = \phi(r)^{p_1/p_2}$, $r > 0$.

The proof of the boundedness of I_α on Lebesgue spaces, Morrey spaces, or generalized Morrey spaces, usually involves *Hardy-Littlewood maximal operator*, which is defined by

$$Mf(x) := \sup_{B \ni x} \frac{1}{|B|} \int_B |f(y)|\, dy,$$

for every $f \in L^p_{loc}(\mathbb{R}^n)$ where $|B|$ denotes the Lebesgue measure of the ball $B = B(a, r)$ (centered at $a \in \mathbb{R}^n$ with radius $r > 0$). It is well known that the operator M is bounded on $L^p(\mathbb{R}^n)$ for $1 < p \le \infty$ [12, 13] and also on Morrey spaces $L^{p,q}$ for $1 < p \le q \le \infty$ [2].

Next, we know that $I_{\alpha,\gamma}$ is guaranteed to be bounded on generalized Morrey spaces because $K_{\alpha,\gamma}(x) \le K_\alpha(x)$ for every $x \in \mathbb{R}^n$. The boundedness of $I_{\alpha,\gamma}$ on Lebesgue spaces can also be proved by using Young's inequality, as shown in [7].

Theorem 1.4. [7] *For* $\gamma > 0$ *and* $0 < \alpha < n$, *we have* $K_{\alpha,\gamma} \in L^t(\mathbb{R}^n)$ *whenever* $\frac{n}{n+\gamma-\alpha} < t < \frac{n}{n-\alpha}$. *Accordingly, we have*

$$\|I_{\alpha,\gamma} f\|_{L^q} \le \|K_{\alpha,\gamma}\|_{L^t} \|f\|_{L^p}$$

for every $f \in L^p(\mathbb{R}^n)$ *where* $1 \le p < t'$, $\frac{n}{n+\gamma-\alpha} < t < \frac{n}{n-\alpha}$, $\frac{1}{q} + 1 = \frac{1}{p} + \frac{1}{t}$.

Using the boundedness of Hardy-Littlewood maximal operator, we also know that $I_{\alpha,\gamma}$ is bounded on Morrey spaces.

Theorem 1.5. [7] *For* $\gamma > 0$ *and* $0 < \alpha < n$, *we have*

$$\|I_{\alpha,\gamma} f\|_{L^{p_2,q_2}} \le C\, \|K_{\alpha,\gamma}\|_{L^t} \|f\|_{L^{p_1,q_1}}$$

for every $f \in L^{p_1,q_1}(\mathbb{R}^n)$ *where* $1 < p_1 < q_1 < t'$, $\frac{n}{n+\gamma-\alpha} < t < \frac{n}{n-\alpha}$, $\frac{1}{p_2} + 1 = \frac{1}{p_1} + \frac{1}{t}$, *and* $\frac{1}{q_2} + 1 = \frac{1}{q_1} + \frac{1}{t}$.

In 1999, Kurata *et al.* [8] proved the boundedness of $W \cdot I_{\alpha,\gamma}$ on generalized Morrey spaces where W is a multiplication operator. A similar result to Kurata's can be found in [3]. In the next section, we shall reprove the boundedness of $I_{\alpha,\gamma}$ on generalized Morrey spaces using a Hedberg-type inequality and the boundedness of Hardy-Littlewood maximal operator on these spaces.

Theorem 1.6. *(Nakai) For* $1 < p \le \infty$, *we have*

$$\|Mf\|_{L^{p,\phi}} \le C\, \|f\|_{L^{p,\phi}},$$

for every $f \in L^{p,\phi}(\mathbb{R}^n)$.

Our results show that the norm of Bessel-Riesz operators is dominated by the norm of their kernels on (generalized) Morrey spaces.

2. MAIN RESULTS

For $\gamma > 0$ and $0 < \alpha < n$, one may observe that the kernel $K_{\alpha,\gamma}$ belongs to Lebesgue spaces $L^t(\mathbb{R}^n)$ whenever $\frac{n}{n+\gamma-\alpha} < t < \frac{n}{n-\alpha}$, where

$$\sum_{k \in \mathbb{Z}} \frac{(2^k R)^{(\alpha-n)t+n}}{(1 + 2^k R)^{\gamma t}} \sim \|K_{\alpha,\gamma}\|^t_{L^t}$$

(see [7]). With this in mind, we obtain the boundedness of $I_{\alpha,\gamma}$ on generalized Morrey spaces as in the following theorem.

Theorem 2.1. *Let $\gamma > 0$ and $0 < \alpha < n$. If $\phi(r) \le Cr^{\beta}$ for every $r > 0$, $-\frac{\alpha t'}{p_1} \le \beta < -\alpha$, $1 < p_1 < t'$, and $\frac{n}{n+\gamma-\alpha} < t < \frac{n}{n-\alpha}$, then we have*

$$\|I_{\alpha,\gamma} f\|_{L^{p_2,\psi}} \le C\,\|K_{\alpha,\gamma}\|_{L^t}\|f\|_{L^{p_1,\phi}},$$

for every $f \in L^{p_1,\phi}(\mathbb{R}^n)$ where $p_2 = \frac{\beta p_1}{\alpha+\beta}$, and $\psi(r) = \phi(r)^{p_1/p_2}$.

Proof. Let $\gamma > 0$ and $0 < \alpha < n$. Suppose that $\phi(r) \le Cr^{\beta}$ for every $r > 0$, $-\frac{\alpha t'}{p_1} \le \beta < -\alpha$, $1 < p_1 < t'$, $\frac{n}{n+\gamma-\alpha} < t < \frac{n}{n-\alpha}$. Take $f \in L^{p_1,\phi}(\mathbb{R}^n)$ and write

$$I_{\alpha,\gamma} f\,(x) := I_1\,(x) + I_2\,(x)\,,$$

for every $x \in \mathbb{R}^n$ where $I_1\,(x) := \int_{|x-y|<R} \frac{|x-y|^{\alpha-n} f(y)}{(1+|x-y|)^{\gamma}} dy$ and $I_2\,(x) := \int_{|x-y|\ge R} \frac{|x-y|^{\alpha-n} f(y)}{(1+|x-y|)^{\gamma}} dy$, $R > 0$.

Using dyadic decomposition, we have the following estimate for I_1:

$$\begin{aligned} |I_1(x)| &\le \sum_{k=-\infty}^{-1} \int_{2^k R \le |x-y| < 2^{k+1}R} \frac{|x-y|^{\alpha-n}\,|f\,(y)|}{(1+|x-y|)^{\gamma}} dy \\ &\le C_1 \sum_{k=-\infty}^{-1} \frac{\left(2^k R\right)^{\alpha-n}}{(1+2^k R)^{\gamma}} \int_{2^k R \le |x-y| < 2^{k+1}R} |f\,(y)|\,dy \\ &\le C_2\, Mf\,(x) \sum_{k=-\infty}^{-1} \frac{\left(2^k R\right)^{\alpha-n+n/t} \left(2^k R\right)^{n/t'}}{(1+2^k R)^{\gamma}}. \end{aligned}$$

We then use Hölder's inequality to get

$$|I_1\,(x)| \le C_2 Mf\,(x) \left(\sum_{k=-\infty}^{-1} \frac{\left(2^k R\right)^{(\alpha-n)t+n}}{(1+2^k R)^{\gamma t}} \right)^{1/t} \left(\sum_{k=-\infty}^{-1} \left(2^k R\right)^{n} \right)^{1/t'}.$$

Because we have

$$\left(\sum_{k=-\infty}^{-1} \frac{\left(2^k R\right)^{(\alpha-n)t+n}}{(1+2^k R)^{\gamma t}} \right)^{1/t} \le \left(\sum_{k\in\mathbb{Z}} \frac{\left(2^k R\right)^{(\alpha-n)t+n}}{(1+2^k R)^{\gamma t}} \right)^{1/t} \sim \|K_{\alpha,\gamma}\|_{L^t}\,,$$

we obtain $|I_1(x)| \le C_3\,\|K_{\alpha,\gamma}\|_{L^t}\, Mf\,(x)\, R^{n/t'}$.

To estimate I_2, we use Hölder's inequality again:

$$\begin{aligned} |I_2\,(x)| &\le C_4 \sum_{k=0}^{\infty} \frac{\left(2^k R\right)^{\alpha-n}}{(1+2^k R)^{\gamma}} \int_{2^k R \le |x-y| < 2^{k+1}R} |f\,(y)|\,dy \\ &\le C_4 \sum_{k=0}^{\infty} \frac{\left(2^k R\right)^{\alpha-n}}{(1+2^k R)^{\gamma}} \left(2^k R\right)^{n/p_1'} \left(\int_{2^k R \le |x-y| < 2^{k+1}R} |f\,(y)|^{p_1}\,dy \right)^{1/p_1}. \end{aligned}$$

It follows that

$$\begin{aligned} |I_2\,(x)| &\le C_5\,\|f\|_{L^{p_1,\phi}} \sum_{k=0}^{\infty} \frac{\left(2^k R\right)^{\alpha-n+n/t}}{(1+2^k R)^{\gamma}} \phi\left(2^k R\right) \left(2^k R\right)^{n/t'} \\ &\le C_6\,\|f\|_{L^{p_1,\phi}} \sum_{k=0}^{\infty} \frac{\left(2^k R\right)^{\alpha-n+n/t}}{(1+2^k R)^{\gamma}} \left(2^k R\right)^{\beta+n/t'}. \end{aligned}$$

Another use of Hölder's inequality gives

$$|I_2(x)| \le C_6 \|f\|_{L^{p_1,\phi}} \left(\sum_{k=0}^{\infty} \frac{\left(2^k R\right)^{(\alpha-n)t+n}}{\left(1+2^k R\right)^{\gamma t}} \right)^{1/t} \left(\sum_{k=0}^{\infty} \left(2^k R\right)^{\beta t'+n} \right)^{1/t'}.$$

Because $\beta t' + n < 0$ and $\sum_{k=0}^{\infty} \frac{\left(2^k R\right)^{(\alpha-n)t+n}}{\left(1+2^k R\right)^{\gamma t}} \lesssim \|K_{\alpha,\gamma}\|_{L^t}^t$, we obtain

$$|I_2(x)| \le C_7 \|K_{\alpha,\gamma}\|_{L^t} \|f\|_{L^{p_1,\phi}} R^{\beta} R^{n/t'}.$$

Summing the two estimates, we obtain

$$|I_{\alpha,\gamma} f(x)| \le C_8 \|K_{\alpha,\gamma}\|_{L^t} \left(Mf(x) R^{n/t'} + \|f\|_{L^{p_1,\phi}} R^{n/t'+\beta} \right),$$

for every $x \in \mathbb{R}^n$. Now, for each $x \in \mathbb{R}^n$, choose $R > 0$ such that $R^{\beta} = \frac{Mf(x)}{\|f\|_{L^{p_1,\phi}}}$. Hence we get a Hedberg-type inequality for $I_{\alpha,\gamma} f$, namely

$$|I_{\alpha,\gamma} f(x)| \le C_9 \|K_{\alpha,\gamma}\|_{L^t} \|f\|_{L^{p_1,\phi}}^{-\alpha/\beta} Mf(x)^{1+\alpha/\beta}.$$

Now put $p_2 := \frac{\beta p_1}{\alpha+\beta}$. For arbitrary $a \in \mathbb{R}^n$ and $r > 0$, we have

$$\left(\int_{|x-a|<r} |I_{\alpha,\gamma} f(x)|^{p_2} dx \right)^{1/p_2} \le C_9 \|K_{\alpha,\gamma}\|_{L^t} \|f\|_{L^{p_1,\phi}}^{1-p_1/p_2} \left(\int_{|x-a|<r} |Mf(x)|^{p_1} dx \right)^{1/p_2}.$$

We divide both sides by $\phi(r)^{p_1/p_2} r^{n/p_2}$ to get

$$\frac{\left(\int_{|x-a|<r} |I_{\alpha,\gamma} f(x)|^{p_2}\, dx \right)^{1/p_2}}{\psi(r) r^{n/p_2}} \le C_9 \|K_{\alpha,\gamma}\|_{L^t} \|f\|_{L^{p_1,\phi}}^{1-p_1/p_2} \frac{\left(\int_{|x-a|<r} |Mf(x)|^{p_1} dx \right)^{(1/p_2)}}{\phi(r)^{p_1/p_2} r^{n/p_2}},$$

where $\psi(r) := \phi(r)^{p_1/p_2}$. Taking the supremum over $a \in \mathbb{R}^n$ and $r > 0$, we obtain

$$\|I_{\alpha,\gamma} f\|_{L^{p_2,\psi}} \le C_{10} \|K_{\alpha,\gamma}\|_{L^t} \|f\|_{L^{p_1,\phi}}^{1-p_1/p_2} \|Mf\|_{L^{p_1,\phi}}^{p_1/p_2}.$$

By the boundedness of the maximal operator on generalized Morrey spaces (Nakai's Theorem), the desired result follows: $\|I_{\alpha,\gamma} f\|_{L^{p_2,\psi}} \le C \|K_{\alpha,\gamma}\|_{L^t} \|f\|_{L^{p_1,\phi}}$. ∎

We note that from the inclusion property of Morrey spaces, we have

$$\|K_{\alpha,\gamma}\|_{L^{s,t}} \le \|K_{\alpha,\gamma}\|_{L^{t,t}} = \|K_{\alpha,\gamma}\|_{L^t}$$

whenever $1 \le s \le t$, $\frac{n}{n+\gamma-\alpha} < t < \frac{n}{n-\alpha}$. Now we wish to obtain a more general result for the boundedness of $I_{\alpha,\gamma}$ by using the fact that the kernel $K_{\alpha,\gamma}$ belongs to Morrey spaces.

Theorem 2.2. *Let $\gamma > 0$ and $0 < \alpha < n$. If $\phi(r) \le C r^{\beta}$ for every $r > 0$, $-\frac{\alpha t'}{p_1} \le \beta < -\alpha$, $1 < p_1 < t'$, $\frac{n}{n+\gamma-\alpha} < t < \frac{n}{n-\alpha}$, then we have*

$$\|I_{\alpha,\gamma} f\|_{L^{p_2,\psi}} \le C \|K_{\alpha,\gamma}\|_{L^{s,t}} \|f\|_{L^{p_1,\phi}},$$

for every $f \in L^{p_1,\phi}(\mathbb{R}^n)$ where $1 \le s \le t$, $p_2 = \frac{\beta p_1}{\alpha+\beta}$, and $\psi(v) = \phi(v)^{p_1/p_2}$.

Proof. Let $\gamma > 0$ and $0 < \alpha < n$. Suppose that $\phi(r) \leq Cr^{\beta}$ for every $r > 0$, $-\frac{\alpha t'}{p_1} \leq \beta < -\alpha$, $1 < p_1 < t'$, $\frac{n}{n+\gamma-\alpha} < t < \frac{n}{n-\alpha}$. As in the proof of Theorem 2.1, we have $I_{\alpha,\gamma}f(x) := I_1(x) + I_2(x)$ for every $x \in \mathbb{R}^n$. Now, we estimate I_1 using dyadic decomposition as follow:

$$\begin{aligned}
|I_1(x)| &\leq \sum_{k=-\infty}^{-1} \int_{2^kR\leq|x-y|<2^{k+1}R} \frac{|x-y|^{\alpha-n}|f(y)|}{(1+|x-y|)^{\gamma}} dy \\
&\leq C_1 \sum_{k=-\infty}^{-1} \frac{\left(2^kR\right)^{\alpha-n}}{(1+2^kR)^{\gamma}} \int_{2^kR\leq|x-y|<2^{k+1}R} |f(y)|\, dy \\
&= C_2 Mf(x) \sum_{k=-\infty}^{-1} \frac{\left(2^kR\right)^{\alpha-n+n/s}\left(2^kR\right)^{n/s'}}{(1+2^kR)^{\gamma}},
\end{aligned}$$

where $1 \leq s \leq t$. By Hölder's inequality,

$$|I_1(x)| \leq C_2 Mf(x) \left(\sum_{k=-\infty}^{-1} \frac{\left(2^kR\right)^{(\alpha-n)s+n}}{(1+2^kR)^{\gamma s}} \right)^{1/s} \left(\sum_{k=-\infty}^{-1} \left(2^kR\right)^n \right)^{1/s'}.$$

We also have $\sum_{k=-\infty}^{-1} \frac{\left(2^kR\right)^{(\alpha-n)s+n}}{\left(1+2^kR\right)^{\gamma s}} \lesssim \int_{0<|x|<R} K_{\alpha,\gamma}^s(x)\, dx$, so that

$$|I_1(x)| \leq C_3 Mf(x) \left(\int_{0<|x|<R} K_{\alpha,\gamma}^s(x)\, dx \right)^{\frac{1}{s}} R^{n/s'} \leq C_3 \|K_{\alpha,\gamma}\|_{L^{s,t}} Mf(x) R^{n/t'}.$$

Next, we estimate I_2 by using Hölder's inequality. As in the proof of Theorem 2.1, we obtain

$$|I_2(x)| \leq C_4 \sum_{k=0}^{\infty} \frac{\left(2^kR\right)^{\alpha-n}}{(1+2^kR)^{\gamma}} \left(2^kR\right)^{n/p_1'} \left(\int_{2^kR\leq|x-y|<2^{k+1}R} |f(y)|^{p_1} dy \right)^{1/p_1}.$$

It thus follows that

$$\begin{aligned}
|I_2(x)| &\leq C_5 \|f\|_{L^{p_1,\phi}} \sum_{k=0}^{\infty} \frac{\left(2^kR\right)^{\alpha}\phi(2^kR)}{(1+2^kR)^{\gamma}} \frac{\left(\int_{2^kR\leq|x-y|<2^{k+1}R} dy\right)^{1/s}}{(2^kR)^{n/s}} \\
&\leq C_6 \|f\|_{L^{p_1,\phi}} \sum_{k=0}^{\infty} \phi(2^kR)\left(2^kR\right)^{n/t'} \frac{\left(\int_{2^kR\leq|x-y|<2^{k+1}R} K_{\alpha,\gamma}^s(x-y)dy\right)^{1/s}}{(2^kR)^{n/s-n/t}}
\end{aligned}$$

Because $\phi(r) \leq Cr^{\beta}$ and $\frac{\left(\int_{2^kR\leq|x-y|<2^{k+1}R} K_{\alpha,\gamma}^s(x-y)dy\right)^{1/s}}{\left(2^kR\right)^{n/s-n/t}} \lesssim \|K_{\alpha,\gamma}\|_{L^{s,t}}$ for every $k = 0, 1, 2, \ldots$, we get

$$\begin{aligned}
|I_2(x)| &\leq C_7 \|K_{\alpha,\gamma}\|_{L^{s,t}} \|f\|_{L^{p_1,\phi}} \sum_{k=0}^{\infty} \left(2^kR\right)^{\beta+n/t'} \\
&\leq C_8 \|K_{\alpha,\gamma}\|_{L^{s,t}} \|f\|_{L^{p_1,\phi}} R^{\beta} R^{n/t'}.
\end{aligned}$$

From the two estimates, we obtain

$$|I_{\alpha,\gamma}f(x)| \leq C_9 \|K_{\alpha,\gamma}\|_{L^{s,t}} \left(Mf(x)R^{n/t'} + \|f\|_{L^{p_1,\phi}} R^{n/t'+\beta} \right),$$

for every $x \in \mathbb{R}^n$. Now, for each $x \in \mathbb{R}^n$, choose $R > 0$ such that $R^{\beta} = \frac{Mf(x)}{\|f\|_{L^{p_1,\phi}}}$. Hence we get

$$|I_{\alpha,\gamma}f(x)| \leq C_9 \|K_{\alpha,\gamma}\|_{L^{s,t}} \|f\|_{L^{p_1,\phi}}^{-\alpha/\beta} Mf(x)^{1+\alpha/\beta}.$$

Put $p_2 := \frac{\beta p_1}{\alpha+\beta}$. For arbitrary $a \in \mathbb{R}^n$ and $r > 0$, we have

$$\left(\int_{|x-a|<r} |I_{\alpha,\gamma} f(x)|^{p_2}\, dx\right)^{1/p_2} \leq C_9 \|K_{\alpha,\gamma}\|_{L^{s,t}} \|f\|_{L^{p_1,\phi}}^{1-p_1/p_2} \left(\int_{|x-a|<r} |Mf(x)|^{p_1}\, dx\right)^{1/p_2}.$$

Divide both sides by $\phi(r)^{p_1/p_2}\, r^{n/p_2}$ and take the supremum over $a \in \mathbb{R}^n$ and $r > 0$ to get

$$\|I_{\alpha,\gamma} f\|_{L^{p_2,\psi}} \leq C_{10} \|K_{\alpha,\gamma}\|_{L^{s,t}} \|f\|_{L^{p_1,\phi}}^{1-p_1/p_2} \|Mf\|_{L^{p_1,\phi}}^{p_1/p_2},$$

where $\psi(r) := \phi(r)^{p_1/p_2}$. With the boundedness of the maximal operator on generalized Morrey spaces (Nakai's Theorem), we obtain

$$\|I_{\alpha,\gamma} f\|_{L^{p_2,\psi}} \leq C_{p_1,\phi} \|K_{\alpha,\gamma}\|_{L^{s,t}} \|f\|_{L^{p_1,\phi}},$$

as desired. ∎

Note that by Theorem 2.2 and the inclusion of Morrey spaces, we recover Theorem 2.1:

$$\|I_{\alpha,\gamma} f\|_{L^{p_2,\psi}} \leq C\ \|K_{\alpha,\gamma}\|_{L^{s,t}} \|f\|_{L^{p_1,\phi}} \leq C\ \|K_{\alpha,\gamma}\|_{L^{t}} \|f\|_{L^{p_1,\phi}}.$$

We still wish to obtain a better estimate. The following lemma presents that the Bessel-Riesz kernels belong to generalized Morrey space $L^{s,\sigma}(\mathbb{R}^n)$ for some $s \geq 1$ and a suitable function σ.

Lemma 2.3. *Suppose that $\gamma > 0$ and $0 < \alpha < n$. If $\sigma : \mathbb{R}^+ \to \mathbb{R}^+$ satisfies*

$$\int_{0<r\leq R} r^{(\alpha-n)s+n-1} dr \leq C\sigma^s(R) R^n$$

for every $R > 0$, then $K_{\alpha,\gamma} \in L^{s,\sigma}(\mathbb{R}^n)$.

Proof. Suppose that the hypothesis holds. It is sufficient to evaluate the integral around 0. We observe that

$$\int_{|x|\leq R} K^s_{\alpha,\gamma}(x) dx \ = \ \int_{|x|\leq R} \frac{|x|^{(\alpha-n)s}}{(1+|x|)^{\gamma s}} dx \leq C \int_{0<r\leq R} r^{(\alpha-n)s+n-1} dr \leq C\sigma^s(R) R^n.$$

We divide both sides of the inequality by $\sigma^s(R)R^n$ and take s^{th}-root to obtain

$$\frac{\left(\int_{|x|\leq R} K^s_{\alpha,\gamma}(x) dx\right)^{1/s}}{\sigma(R) R^{n/s}} \leq C^{1/s}.$$

Now, taking the supremum over $R > 0$, we have

$$\sup_{R>0} \frac{\left(\int_{|x|\leq R} K^s_{\alpha,\gamma}(x) dx\right)^{1/s}}{\sigma(R) R^{n/s}} < \infty.$$

Hence $K_{\alpha,\gamma} \in L^{s,\sigma}(\mathbb{R}^n)$. ∎

By the hypothesis of Lemma 2.3 we also obtain $\frac{\left(\int_{2^kR<|x|\leq 2^{k+1}R} K^s_{\rho,\gamma}(x)dx\right)^{1/s}}{\sigma(2^kR)\left(2^kR\right)^{n/s}} \lesssim \|K_{\alpha,\gamma}\|_{L^{s,\sigma}}$ for every integer k and $R > 0$. Moreover, $\frac{\left(\sum_{k=-1}^{-\infty} K^s_{\alpha,\gamma}\left(2^kR\right)\left(2^kR\right)^n\right)^{1/s}}{\sigma(R)(R)^{n/s}} \lesssim \|K_{\alpha,\gamma}\|_{L^{s,\sigma}}$ holds for every $R > 0$. One may observe that $1 \leq s \leq \frac{n \ln R_1}{-\ln \sigma(R_1)}$ for every $R_1 > 1$. For $\sigma(R) = R^{-n/t}$, this inequality reduces to $1 \leq s \leq t$.

We shall now use the lemma to prove the following theorem.

Theorem 2.4. *Suppose that $\sigma : \mathbb{R}^+ \to \mathbb{R}^+$ satisfies the doubling condition and $\sigma(r) \le Cr^{-\alpha}$ for every $r > 0$, so that $K_{\alpha,\gamma} \in L^{s,\sigma}(\mathbb{R}^n)$ for $1 \le s < \frac{n}{n-\alpha}$, where $0 < \alpha < n$ and $\gamma > 0$. If $\phi(r) \le Cr^{\beta}$ for every $r > 0$, where $-\frac{n}{p_1} < \beta < -\alpha$, then we have*

$$\|I_{\alpha,\gamma} f\|_{L^{p_2,\psi}} \le C_{p_1,\phi} \|K_{\alpha,\gamma}\|_{L^{s,\sigma}} \|f\|_{L^{p_1,\phi}}$$

for every $f \in L^{p_1,\phi}(\mathbb{R}^n)$, where $1 < p_1 < \frac{n}{\alpha}$, $p_2 = \frac{\beta p_1}{\beta+n-\alpha}$ and $\psi(r) = \phi(r)^{p_1/p_2}$.

Proof. Let $\gamma > 0$ and $0 < \alpha < n$. Suppose that $\sigma : \mathbb{R}^+ \to \mathbb{R}^+$ satisfies the doubling condition and $\sigma(r) \le Cr^{-\alpha}$ for every $r > 0$, such that $K_{\alpha,\gamma} \in L^{s,\sigma}(\mathbb{R}^n)$ for $1 \le s < \frac{n}{n-\alpha}$. Suppose also that $\phi(r) \le Cr^{\beta}$ for every $r > 0$, where $-\frac{n}{p_1} < \beta < -\alpha$, $1 < p_1 < \frac{n}{\alpha}$. As in the proof of Theorem 2.1, we write $I_{\alpha,\gamma} f(x) := I_1(x) + I_2(x)$ for every $x \in \mathbb{R}^n$. As usual, we estimate I_1 by using dyadic decomposition:

$$\begin{aligned}
|I_1(x)| &\le \sum_{k=-\infty}^{-1} \int_{2^k R \le |x-y| < 2^{k+1}R} \frac{|x-y|^{\alpha-n} |f(y)|}{(1+|x-y|)^{\gamma}} dy \\
&\le C_1 \sum_{k=-\infty}^{-1} \frac{\left(2^k R\right)^{\alpha-n}}{(1+2^k R)^{\gamma}} \int_{2^k R \le |x-y| < 2^{k+1}R} |f(y)|\, dy \\
&= C_2 Mf(x) \sum_{k=-\infty}^{-1} \frac{\left(2^k R\right)^{\alpha-n+n/s} \left(2^k R\right)^{n/s'}}{(1+2^k R)^{\gamma}}
\end{aligned}$$

By using Hölder inequality, we obtain

$$|I_1(x)| \le C_2 Mf(x) \left(\sum_{k=-\infty}^{-1} \frac{\left(2^k R\right)^{(\alpha-n)s+n}}{(1+2^k R)^{\gamma s}} \right)^{1/s} \left(\sum_{k=-\infty}^{-1} \left(2^k R\right)^{n} \right)^{1/s'}.$$

But $\sum_{k=-\infty}^{-1} \frac{\left(2^k R\right)^{(\alpha-n)s+n}}{\left(1+2^k R\right)^{\gamma s}} \lesssim \int_{0<|x|<R} K_{\alpha,\gamma}^s(x)\, dx$, and so we get

$$\begin{aligned}
|I_1(x)| &\le C_2 Mf(x) \left(\int_{0<|x|<R} K_{\alpha,\gamma}^s(x)\, dx \right)^{\frac{1}{s}} R^{n/s'} \\
&\le C_2 \|K_{\alpha,\gamma}\|_{L^{s,\sigma}} Mf(x)\, \sigma(R)\, R^n \\
&\le C_3 \|K_{\alpha,\gamma}\|_{L^{s,\sigma}} Mf(x)\, R^{n-\alpha}.
\end{aligned}$$

Next, we estimate I_2 as follows:

$$\begin{aligned}
|I_2(x)| &\le C_4 \sum_{k=0}^{\infty} \frac{\left(2^k R\right)^{\alpha-n}}{(1+2^k R)^{\gamma}} \int_{2^k R \le |x-y| < 2^{k+1}R} |f(y)|\, dy \\
&\le C_4 \sum_{k=0}^{\infty} \frac{\left(2^k R\right)^{\alpha-n}}{(1+2^k R)^{\gamma}} \left(2^k R\right)^{n/p_1'} \left(\int_{2^k R \le |x-y| < 2^{k+1}R} |f(y)|^{p_1} dy \right)^{1/p_1} \\
&\le C_5 \|f\|_{L^{p_1,\phi}} \sum_{k=0}^{\infty} \frac{\left(2^k R\right)^{\alpha} \phi\left(2^k R\right) \left(2^k R\right)^{n}}{(1+2^k R)^{\gamma}} \frac{\left(\int_{2^k R \le |x-y| < 2^{k+1}R} dy \right)^{1/s}}{\left(2^k R\right)^{n/s}} \\
&\le C_6 \|f\|_{L^{p_1,\phi}} \sum_{k=0}^{\infty} \left(2^k R\right)^{n-\alpha+\beta} \frac{\left(\int_{2^k R \le |x-y| < 2^{k+1}R} K_{\alpha,\gamma}^s(x-y)\, dy \right)^{1/s}}{\sigma\left(2^k R\right) \left(2^k R\right)^{n/s}}
\end{aligned}$$

Because $\frac{\int_{2^kR\le|x-y|<2^{k+1}R} K^s_{\alpha,\gamma}(x-y)dy}{\sigma(2^kR)(2^kR)^n} \lesssim \|K_{\alpha,\gamma}\|^s_{L^{s,\sigma}}$ for every $k = 0, 1, 2, \ldots$, we obtain

$$\begin{aligned} |I_2(x)| &\le C_6 \|K_{\alpha,\gamma}\|_{L^{s,\sigma}} \|f\|_{L^{p_1,\phi}} \sum_{k=0}^{\infty} \left(2^kR\right)^{n-\alpha+\beta} \\ &\le C_7 \|K_{\alpha,\gamma}\|_{L^{s,\sigma}} \|f\|_{L^{p_1,\phi}} R^{n-\alpha+\beta}. \end{aligned}$$

It follows from the above estimates for I_1 and I_2 that

$$|I_{\alpha,\gamma}f(x)| \le C_8 \|K_{\alpha,\gamma}\|_{L^{s,\sigma}} \left(Mf(x) R^{n-\alpha} + \|f\|_{L^{p_1,\phi}} R^{n-\alpha+\beta}\right)$$

for every $x \in \mathbb{R}^n$. Now, for each $x \in \mathbb{R}^n$, choose $R > 0$ such that $R^\beta = \frac{Mf(x)}{\|f\|_{L^{p_1,\phi}}}$, whence

$$|I_{\alpha,\gamma}f(x)| \le C_9 \|K_{\alpha,\gamma}\|_{L^{s,\sigma}} \|f\|^{(\alpha-n)/\beta}_{L^{p_1,\phi}} Mf(x)^{1+(n-\alpha)/\beta}.$$

Put $p_2 := \frac{\beta p_1}{\beta+n-\alpha}$. For arbitrary $a \in \mathbb{R}^n$ and $r > 0$, we have

$$\left(\int_{|x-a|<r} |I_{\alpha,\gamma}f(x)|^{p_2} dx\right)^{1/p_2} \le C_9 \|K_{\alpha,\gamma}\|_{L^{s,\sigma}} \|f\|^{1-p_1/p_2}_{L^{p_1,\phi}} \left(\int_{|x-a|<r} |Mf(x)|^{p_1} dx\right)^{(1/p_2)}.$$

Divide the both sides by $\phi(r)^{p_1/p_2} r^{n/p_2}$ to get

$$\begin{aligned} \frac{\left(\int_{|x-a|<r} |I_{\alpha,\gamma}f(x)|^{p_2}dx\right)^{1/p_2}}{\psi(r) r^{n/p_2}} &= \frac{\left(\int_{|x-a|<r} |I_{\alpha,\gamma}f(x)|^{p_2}dx\right)^{1/p_2}}{\phi(r)^{p_1/p_2} r^{n/p_2}} \\ &\le C_9\|K_{\alpha,\gamma}\|_{L^{s,\sigma}}\|f\|^{1-p_1/p_2}_{L^{p_1,\phi}} \frac{\left(\int_{|x-a|<r} |Mf(x)|^{p_1}dx\right)^{(1/p_2)}}{\phi(r)^{p_1/p_2} r^{n/p_2}}, \end{aligned}$$

where $\psi(r) := \phi(r)^{p_1/p_2}$. Finally, take the supremum over $a \in \mathbb{R}^n$ and $r > 0$ to obtain

$$\|I_{\alpha,\gamma}f\|_{L^{p_2,\psi}} \le C_{10}\|K_{\alpha,\gamma}\|_{L^{s,\sigma}}\|f\|^{1-p_1/p_2}_{L^{p_1,\phi}}\|Mf\|^{p_1/p_2}_{L^{p_1,\phi}}.$$

Because the maximal operator is bounded on generalized Morrey spaces (Nakai's Theorem), we conclude that $\|I_{\alpha,\gamma}f\|_{L^{p_2,\psi}} \le C_{p_1,\phi}\|K_{\alpha,\gamma}\|_{L^{s,\sigma}}\|f\|_{L^{p_1,\phi}}$. ∎

3. CONCLUDING REMARKS

The results presented in this paper, namely Theorems 2.1, 2.2, and 2.4, extend the results on the boundedness of Bessel-Riesz operators on Morrey spaces [7]. Similar to Gunawan-Eridani's result for I_α, Theorems 2.1, 2.2, and 2.4 ensures that $I_{\alpha,\gamma} : L^{p_1,\phi}(\mathbb{R}^n) \to L^{p_2,\phi^{p_1/p_2}}(\mathbb{R}^n)$. Notice that if we have $\sigma : \mathbb{R}^+ \to \mathbb{R}^+$ such that for $t \in (\frac{n}{n+\gamma-\alpha}, \frac{n}{n-\alpha})$, then $R^{-n/t} < \sigma(R)$ holds for every $R > 0$, then Theorem 2.4 gives a better estimate than Theorem 2.2. Now, if we define $\sigma(R) := \left(1 + R^{n/t_1}\right) R^{-n/t}$ for some $t_1 > t$, then $\|K_{\alpha,\gamma}\|_{L^{s,\sigma}} < \|K_{\alpha,\gamma}\|_{L^{s,t}}$. By Theorem 2.2 and the inclusion property of Morrey spaces, we obtain

$$\begin{aligned} \|I_{\alpha,\gamma}f\|_{L^{p_2,\psi}} &\le C\,\|K_{\alpha,\gamma}\|_{L^{s,\sigma}}\|f\|_{L^{p_1,\phi}} \\ &< C\,\|K_{\alpha,\gamma}\|_{L^{s,t}}\|f\|_{L^{p_1,\phi}} \\ &\le C\,\|K_{\alpha,\gamma}\|_{L^t}\|f\|_{L^{p_1,\phi}}. \end{aligned}$$

We can therefore say that Theorem 2.4 gives the best estimate among the three. Furthermore, we have also shown that, in each theorem, the norm of Bessel-Riesz operators on generalized Morrey spaces is dominated by that of Bessel-Riesz kernels.

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Australian Journal of Mathematical Analysis and Applications

Country	Australia - SIR Ranking of Australia
Subject Area and Category	Mathematics Analysis Applied Mathematics
Publisher	Australian Internet Publishing
Publication type	Journals
ISSN	14495910
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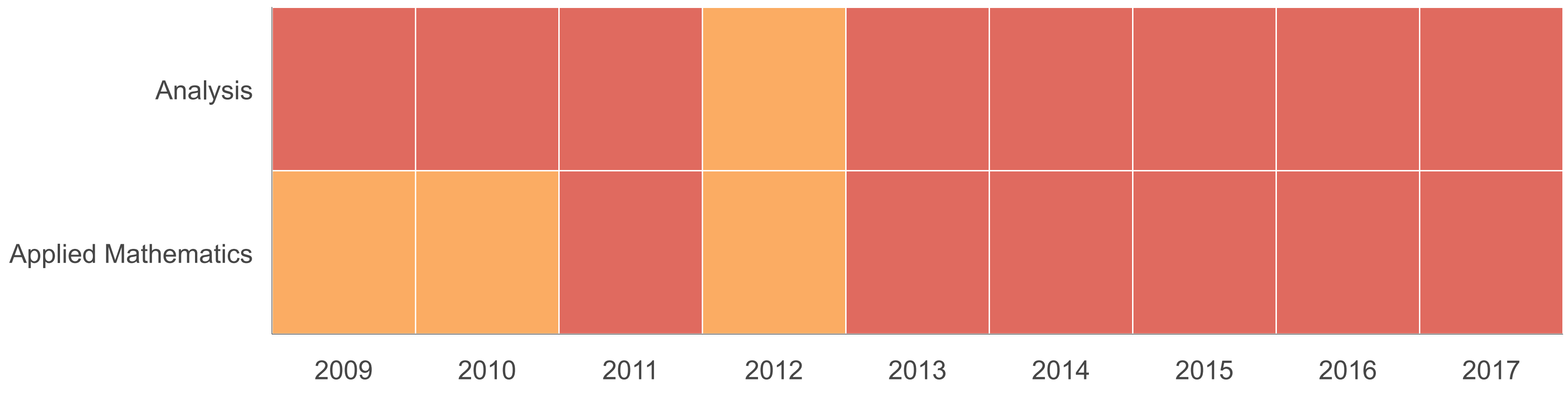

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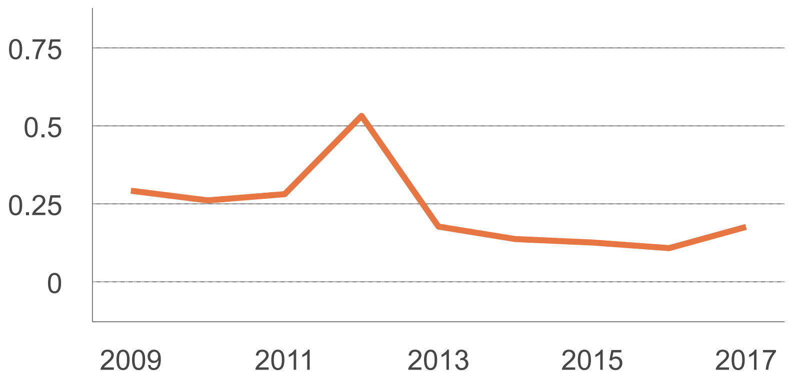

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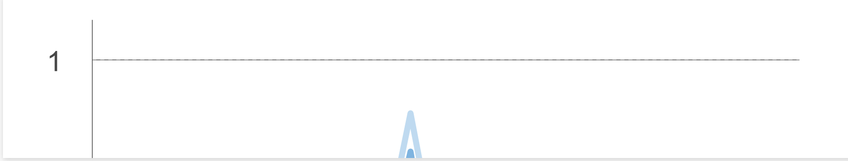

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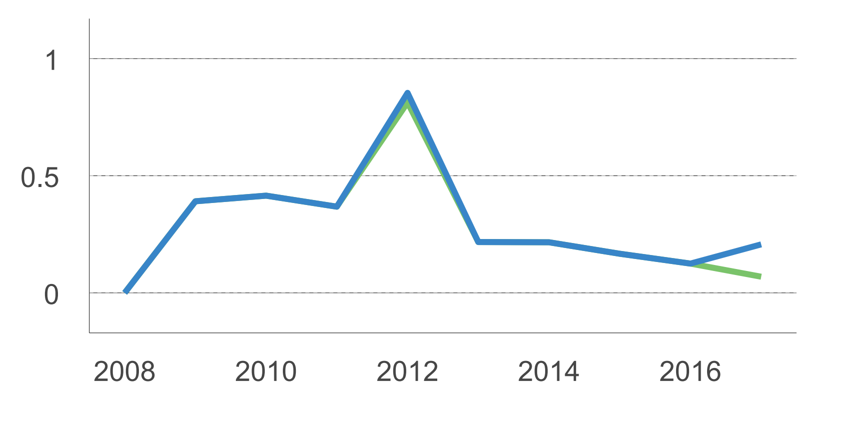

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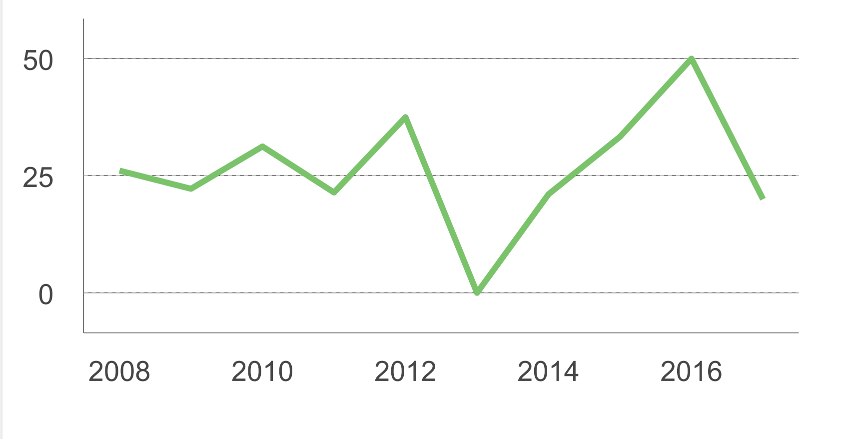

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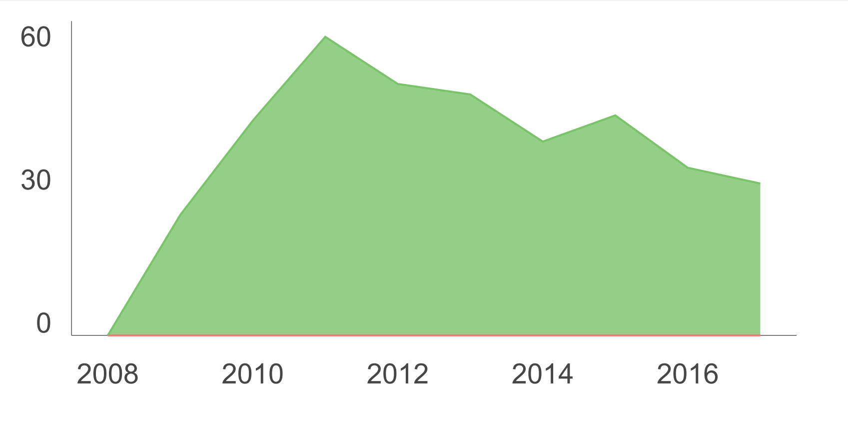

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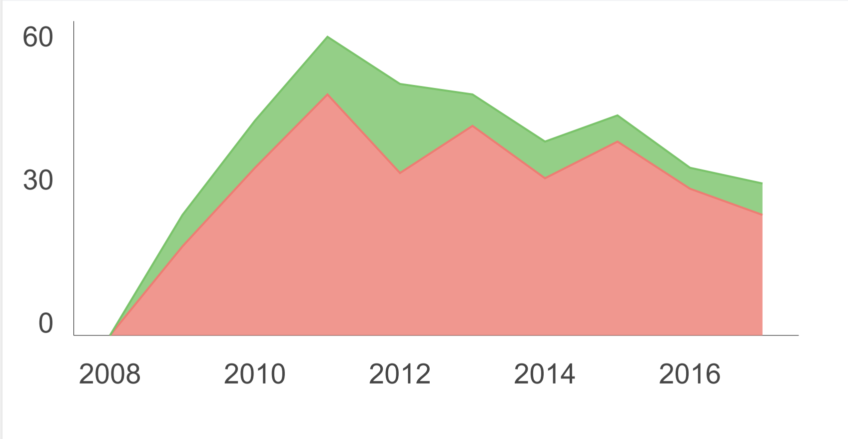

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EST MODUS IN REBUS

Horatio (Satire 1,1,106)