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Notes on the Implications of Ignoring Bayes' Rule in Search and Rescue Practice in the UK

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Abstract

Thomas Bayes (1702-1761) has had a big influence on the science of inference since he discovered the mathematically correct way of adjusting probabilities to account for new evidence. Nonetheless, it is still the case that in practice it is not always clear where and when to apply the rule he derived, or the consequences of not doing so. In this note, the effects of not doing so when searching an area of ground for a missing person (misper), where the chances of finding them depends both on whether they are there and how well the ground is searched, is investigated. This investigation suggests that within the range of probabilities that generally apply to search operations in rural settings in the UK, the widespread failure to apply Bayes' rule may incline search managers to widen search areas more than is warranted by the evidence and may thereby reduce overall search effectiveness (ceteris paribus).

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Discussion

If, upon tossing a coin, we were to get four 'heads' in a row we would nonetheless know that, unless the coin is biased in some way, the next toss is no more likely to result in a 'head' than a 'tail,' since with coin tosses past and future probabilities are unconnected and there is no useful information to exploit in what happened last time. If, on the other hand, the future is conditional on the past then we need to apply Bayes' rule to make use of the prior information. With three marbles in a bag; red, blue, and yellow for example, we know that the likelihood of our picking the yellow one having previously picked the red one is 16.5% since there was a 1 in 3 (33%) chance of picking red, followed by a 1 in 2 (50%) chance of subsequently picking the yellow out instead of the blue.

At an intuitive level we know that this rule must apply to the case of finding someone in a field since this depends both on how well the area is searched (the Probability of Detection, POD) and the likelihood of the misper having walked into the field at some point in the past (the Probability of their being in the Area, POA). In SAR practice in the UK, however, what we do is to simply multiply POA and POD together (Cooper, 2022, Charnes and Cooper, 1958), to determine the probability of finding a misper (Probability of Success, POS). This calculation is standard practice whether it is done manually, or using search management programmes such as 'SarMan,' and in this note we are simply comparing the results of doing it this way compared to using the alternative Bayesian calculation in case there are any practical implications arising from this. That it could have implications rather depends on what is done with the POS information, but typically search managers, in considering which segment of a search area to task a search party to, will look for the highest POA in deciding where to send them. If for example a POA of 20% is attached to a segment in the West and 10% to one in the East, then the party will be sent West regardless of whether Bayes' rule is applied or not. The difference between the traditional approach and the Bayesian alternative comes when a search party subsequently returns to 'control' (usually a control vehicle from which the search manager is organising the search) having not found the misper. If, for example, a party is believed to have covered the segment with a POD of 50% the search manager has a difficult decision to make since this means that in effect only half the segment was searched and half of 20% is 10%. This means that the same probability now applies to sending the party East and sending them back to the West since in effect there is a remaining POA of 10%, equal to the previous POA x (1-POD). If, instead, we apply Bayes' rule to the estimation of these new POAs, using equations (A) and (B) below, we may get a different result and quite possibly a different plan, which is better in theory and might be better in practice (see discussion below).

A. $POA_{new} = POA_{previous} \times ((1\text{-}POD)/(1\text{-}POA_{previous} \times POD))$ for the area searched and
B. $POA_{new} = POA_{previous} \times (1/(1\text{-}POA_{previous} \times POD))$ for the areas that haven't (which may be areas nearby that are being considered for searching but are yet to be searched, and a catch-all, Rest of World, category that may never be searched).

The probability of the misper being in the segment that has been searched will be lowered by the amount given by (A), while for everywhere else the probabilities will go up in line with (B). Repeatedly allocating returning search parties according to the best new POAs will enable a search manager to maximise the chances of finding the misper given the resources available (Colwell, 1994), both by shifting resources between segments in a given search area and by comparing new POAs to those that have yet to be constructed for segments outside the current search area.

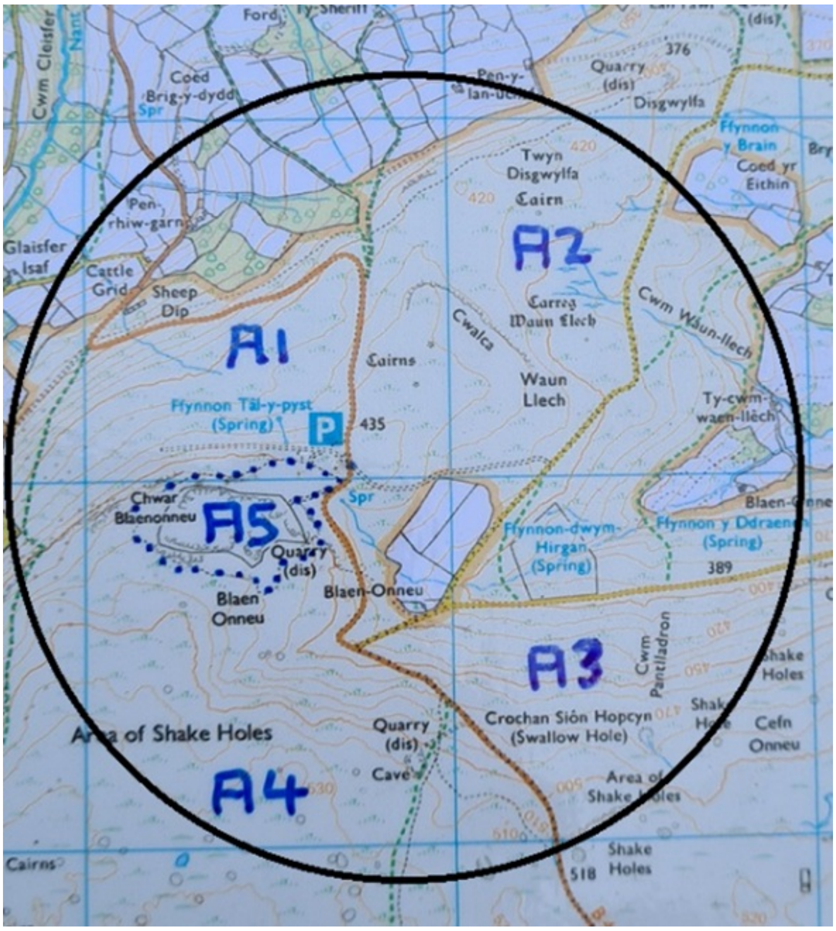
In addition to the issue of what maths to apply, there are a couple of additional practical challenges to directing a search through adjusted POAs in the manner described above. One is that the aforementioned construction of original POAs are a bit of a guess in the first place: Following Mattson (1976), search managers have to combine a lot of different types of information together. There is some widely available statistics to help them with this in the UK, (such as can be found in Perkins et al, 2011), but they will also be expected to combine this with a lot of unquantifiable information about the misper's habits provided by police, friends, and family, as well as all manner of local geographic knowledge from search team members, (since in the UK Mountain Rescue and Lowland Search teams are made up of local volunteers). A second problem is that there is disagreement on how exactly any POA adjustment should be done, and although one might assume that in a paper about Bayes' rule the author would recommend following Syrotuck (1975) and apply Bayes' rule every time, to do so would entail overlooking some significant practical difficulties. The problem is that to correctly maintain the relativities between each new POA, (given that it is the relativities that will determine which areas to prioritise), every POA segment need to be adjusted each time a search party returns to control and although modern technology means this is not the burden it once was, it is still the case that search manager may struggle to keep their intuitive feel for numbers that are continually moving about. In an attempt to help, some have suggested ensuring that all the POAs always add up to one, but such normalisation is not without its problems (Cooper et al, 2003) as although this stops scales moving along with the numbers themselves, it can still leave search managers scratching their heads over what the numbers are saying, regardless of the fact that they sum to the same number each time (Cooper, 2000).

Little wonder then that rigorous POA adjusting is seldom done in practice and the simple alternative of keeping PODs high (70% or more) has become the norm (on the assumption that this will ensure that each searched segment falls below the POA of any newly created segment), with searches therefore tending to evolve by expansion rather than by re-working. Consequently, in what follows the focus is on the first round results and the issue of whether to repeat or expand the search at that point, rather than cover multiple re-iterations, which, in the author's experience of over 20 years in Mountain Rescue in the UK, seldom occur, except perhaps where a crime is indicated.

Before attempting to illustrate the effect of adopting Bayes' rule in such circumstances, it is worth mentioning that although calculating PODs is not the focus here, this can also be rather problematic. Often PODs are calculated by a search party simply walking towards, or tangentially, or even away from, an object such as a rucksack and indicating when they lose, or gain, sight of it. This 'rain dance' indicates how far apart (on average) searchers should walk when searching, and although the most diligent parties will adjust such 'sweep widths' as their estimates of how much ground they can effectively search is hit by changes in terrain, failing light, tiredness, and so on, it is inevitably a bit of a guess, although ways to improve it are frequently being suggested (Koester et al, 2014).

An illustration of the effect of adopting Bayes' rule

Although many of the assumptions regarding the relationship between searching and finding can be traced back to the need to find things in the relatively undifferentiated marine environment (Koopman, 1946), landscapes add a lot of complexity and consequently in this illustrative scenario a simple location with very few search segments (A1-A5 superimposed on the map below) is used. The point at which the missing person was last seen (PLS) was just to the east of the Parking sign shown on the map where she parked her car since the official car park was full of troops (who soon departed). Given that a car park is the ideal place to locate the control vehicle and to assemble search parties, this became the designated central initial planning point (IPP) for the search.



In the absence of additional information, such as a route favoured by the misper, it was decided to conduct line searches of the 5 segments A1-A5, which are broadly within the circle drawn on the map. Such circles are often included (either by hand or by programmes such as SarMan), to represent data collected on missing person behaviour over the years (Syrotuck 1976, Koester 2008). In this case the circle represent the distance from the PLS in which 50% of 'solo female walkers' are found according to Mountain Rescue England and Wales (Perkins et al 2011). With modern gps and map-enabled mobile phones it would be possible for searchers to actually skirt the edge of the circle, but in practice search managers will look for natural boundaries to make things easier and less distracting for their volunteers, with apps such as Sarman being able to add circles as well as the exact shapes of search segments and send these direct to the phones of search parties as required.

In theory a search manager would follow the rules of 'constrained optimisation' and maximise the overall POS given the resource constraints (Koopman 1980, Stone, 2007), but in addition to the practical problems mentioned previously with such maximisation they may also be unclear on what the constraints are; how many searchers will turn up, how quickly they will cover the ground, and whether this is quick enough to find the casualty alive (given that the likelihood of survival tends to go down with time, as Adams et al, (2007) show). This is not to imply zero progress on the searching side, with the advent of such things as 'watershed' maps that help to show the most likely routes mispers take when confronted by gullies and rivers (Doke, 2012), and of course the use of drones in the searching process.

In order to keep it simple in this illustration such details are omitted and the only complicating factors that stop us from simply dividing the area into four 1km squares (as denoted by the 1km grid lines on the map), is that one of the areas has a quarry which requires more effort to search because its relative topographical complexity narrows the required sweep widths to get a party into the desired 70% POD range. In effect, what this fifth area does is to allow the resources to be allocated unevenly to match the unevenness of the 'probability densities' on the ground, while keeping each search party group to a sensible number of 4 personnel, given that on the day 20 searchers turned up.

Results

Although the principle is the same, given that we have effectively eliminated the differences between segments in our illustration allows us to focus on the overall POA alone, which, with the overall original POA of 50% and POD of 70%, gives us a new POA figure of 15% using the traditional multiplication approach, and 23% using Bayes' rule. The 8% difference between the two is shown in bold alongside several others for a range of POAs and PODs in the table below:

Table showing extent to which Bayes' rule exceeds traditional approach in indicating need to re-do a previously searched area.

		POD					
POA		0.4	0.5	0.6	0.7	0.8	0.9
	0.3	2%	3%	3%	2%	2%	1%
	0.5	8%	8%	9%	**8%**	7%	4%
	0.7	16%	19%	20%	20%	18%	12%
	0.9	30%	37%	42%	46%	46%	38%

These results show that in considering where to search next, the likelihood of the misper being outside the search area is overstated by the traditional approach. A search manager may consequently find themselves a little more inclined (depending on what both the POAs and PODs are) to expand the search rather than re-do previously searched segments than is warranted by the data, unless we are looking at small POAs to begin with (as we may be in looking at segments), in which case the difference is small.

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Note: *As highlighted by the reviewers, there was a missing bracket in equation A. The original results remain unaffected by this error; in any case, this issue has now been corrected. (January 16, 2024)*

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