

# Optimal Generic Energy Storage System Offering in Day-Ahead Electricity Markets

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**Abstract**—This paper models the offers and bids of a generic storage system in an electricity market through stochastic mixed integer linear programming. The objective function aims at maximizing the profit from buying or selling energy for a general storage system. Some parameters such as storage system efficiency, losses of the energy stored and marginal costs are parameterized to evaluate the offers and bids. Market prices are forecasted for 24 hours using AR, MA and ARIMA time series models. The problem is tested for a case study, analyzing the behaviour of the offers and bids. Also, the results obtained are studied and relevant conclusions are presented.

**Index Terms**—Storage system, day-ahead market, sale offer, purchase bid, price forecasting, stochastic mixed integer linear programming.

## NOMENCLATURE

### Indexes

$t$  Index referring to a period [hour].  
 $w$  Index referring to a scenario.

### Parameters

$B^{MAX}$  Upper limit of the purchase bid [MW].  
 $B^{MIN}$  Lower limit of the purchase bid [MW].  
 $c^{STG}$  Marginal cost of the energy stored [€/MWh].  
 $e^{MAX}$  Upper limit of the energy stored [MWh].  
 $e^{MIN}$  Lower limit of the energy stored [MWh].  
 $\eta$  Efficiency of the sale offers and the purchase bids.  
 $\gamma$  Loss efficiency of the energy stored.  
 $\lambda_{t,w}$  Day-ahead market price in period  $t$  and scenario  $w$  [€/MWh].  
 $O^{MAX}$  Upper limit of the sale offer [MW].  
 $O^{MIN}$  Lower limit of the sale offer [MW].  
 $\rho_w$  Probability of each scenario.

### Continuous Variables

$b_t$  Power offer/bid in the day-ahead market associated to the storage system in period  $t$  [MW].  
 $bid_t$  Power purchase bid in the day-ahead market associated to the storage system in period  $t$  [MW].  
 $e_t$  Energy stored in period  $t$  [MWh].  
 $e_t^{LOSS}$  Energy loss of the energy stored in period  $t$  [MWh].

$of_t$  Power sale offer in the day-ahead market associated to the storage system in period  $t$  [MW].  
 $PF$  Profit from sales and purchases of energy in the day-ahead market [€].

## I. INTRODUCTION

In European electricity markets, generators send their energy offers the day before the day-ahead market closes. Conventional energy and hydro power were the main kind of generation in the years previous to 2000. After the Kyoto Protocol, the installation of renewable energy was increased, including wind power, mini-hydro, photovoltaic, biomass, etc.

Renewable energy has changed the behaviour of electricity markets, where there are two main kinds of uncertainty: i) internal uncertainty that comes from the electricity market (prices) and ii) external uncertainty that comes from variable generation like wind and photovoltaic power.

Storage systems have increased their participation in electricity markets [1] due to the high penetration of renewable energy such as wind power [2]. Mature storage systems like reversible hydro power sell energy in the day-ahead and reserve markets [3]–[8].

Other storage technologies are used in reserve markets for energy compensation [9]. However, storage systems are usually employed to regulate the excess or lack of energy in electrical grids and will be needed for a future sustainable network [10]. Hence, storage systems are useful to smooth the demand curve, increasing energy consumption (buying energy) during valley hours and injecting energy (selling energy) during peak hours.

Although storage systems do not always produce energy [11], they can be profitable in the day-ahead market [12]. The viability of balancing wind generation with energy storage is analyzed in [13].

Another issue is the association between renewable technologies and storage systems [14] since storage systems can mitigate part of the uncertainty coming from renewable energy [15].

This paper proposes a model for offering/bidding energy by means of a generic energy storage system in the day-ahead market. Moreover, the optimal offer or bid of the storage

system is analyzed with several parameters like the efficiency of the storage system, losses of the energy stored and marginal costs of the energy stored.

The main contributions of this paper are:

- Modeling a generic energy storage system using offer/purchase bids in the day-ahead market through stochastic mixed integer linear programming.
- Providing information on the behaviour of a generic energy storage system using several values of the main storage parameters.
- Analyzing the behaviour of charging/discharging of the storage system to be profitable in electricity markets.

This paper is organized as follows: Section II explains the problem, Section III describes the mathematical problem, equations and variables, Section IV shows the main parameters, price scenarios and the simulation process. In Section V the main results are presented and Section VI shows the general conclusions.

## II. PROBLEM DESCRIPTION

The problem proposed evaluates the behaviour of a storage system for offering energy in the day-ahead electricity market. The model has to select the best hours to buy energy and the best hours to sell it, maximizing the profits of this trade in the electricity market.

Fig. 1 shows a diagram including the necessary information to evaluate the energy offering of a generic storage system.

The market equilibrium point of the supply curve and demand curve shows the marginal price. Hence, market prices (marginal prices) are decisive when making the decision to buy or sell energy. Different prices in 24 hours enable the purchase bid/sale offer to be profitable.

The uncertainty of market prices complicates the decision, so, as a consequence, the problem formulation is presented in the next section. The problem is formulated using stochastic programming, where it is cast as a stochastic mixed integer linear problem in order to maximize the profit of the storage system. Thereby, the main decision of the model is to choose the specific amount of energy per hour to be purchased/sold the next day.

## III. MATHEMATICAL FORMULATION

Stochastic mixed integer linear programming is used to solve the problem, where the objective aims at obtaining the maximum profit from buying/selling energy in the day-ahead market.

The objective function and several constraints are presented next.

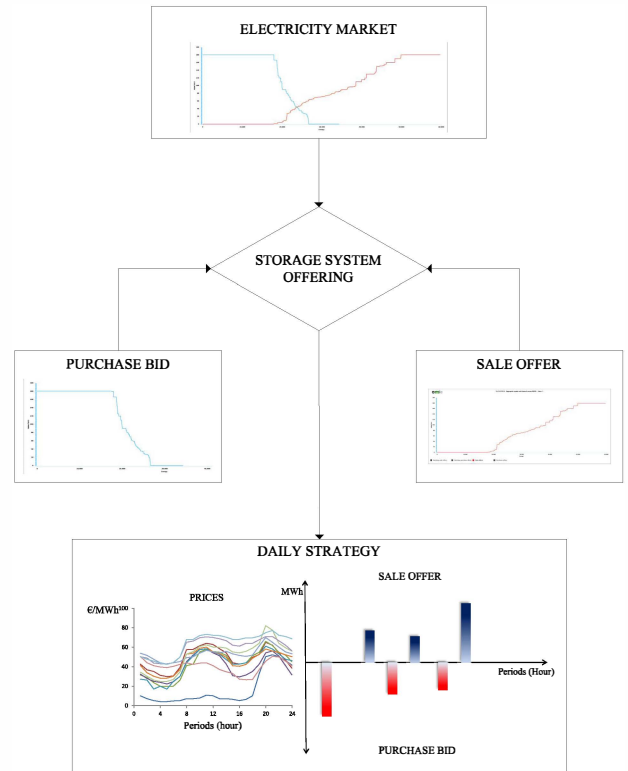


Fig. 1. Diagram of the problem.

### A. Objective Function

The objective function maximizes the profits from buying and selling energy for a generic energy storage system in the electricity market.

$$\max PF \quad (1)$$

where

$$PF = \sum_w \rho_w \left[ \sum_t \left( \lambda_{t,w} \cdot b_t - c^{STG} \cdot e_t \right) \right]. \quad (2)$$

Equation (2) is the total profit from buying or selling energy. Purchase bids or sale offers are represented by variable  $b_t$ , which does not depend on scenarios. The model decides the best quantity of energy that has to be bought and, after that, this energy may be sold. The costs and losses of the storage system are taken into account, too. The marginal cost is considered as a production cost, including maintenance and operating expenses.

Depending on the values of the marginal cost and storage loss, the storage system may not be active because of the economic losses incurred.

### B. Constraints

The main constraints are divided into three blocks: offer, energy level and losses.

1) *Offer Constraints*: The limits of the sale offers are (3) and (4), and the limits of the purchase bids are fixed in (5) and (6), where  $B^{MAX}$  is 0 and  $B^{MIN}$  is a negative value, so  $bid_t$  is a negative value too. Therefore, the sale offers are positive values and the purchase bids are negative values. Finally, (7) decides if the storage system buys or sells energy. Furthermore, the energy can be purchased and sold at the same time. Therefore, simultaneous charge and discharge can be profitable.

$$of_t \leq O^{MAX}; \quad (3)$$

$$of_t \geq O^{MIN}; \quad (4)$$

$$bid_t \leq B^{MAX}; \quad (5)$$

$$bid_t \geq B^{MIN}; \quad (6)$$

$$b_t = of_t + bid_t. \quad (7)$$

2) *Energy Level Constraints*: The energy level is measured through a continuous variable,  $e_t$ . The limits are fixed in (8) and (9).

The balance of energy is evaluated in (10), where the energy bought or sold has an efficiency,  $\eta$ .

$$e_t \leq e^{MAX}; \quad (8)$$

$$e_t \geq e^{MIN}; \quad (9)$$

$$e_t = e_{t-1} - (of_t/\eta) - (bid_t \cdot \eta) - e_t^{LOSS}. \quad (10)$$

The energy in period  $t$  is the energy stored in period  $t-1$  minus the energy sold plus the energy bought, including the parameter for energy loss. The energy bought and stored ( $bid_t \cdot \eta$ ) has a negative sign in (10) because  $bid_t$  is a negative value.

3) *Loss Constraint*: The model also considers a term of stored energy losses in each period  $t$ .

These losses are equal to the energy stored in each period multiplied by the loss parameter,  $\gamma$ .

$$e_t^{LOSS} = e_t \cdot \gamma. \quad (11)$$

#### IV. CASE STUDY

##### A. Description

The model is tested for a storage system whose capacity is 10MW, also  $O^{MAX} = 10$  MW,  $e^{MAX} = 10$  MWh,  $B^{MAX} = 0$  MW,  $O^{MIN} = 0$  MW,  $e^{MIN} = 0$  MWh and  $B^{MIN} = -10$  MW.

The storage system buys or sells energy in the Spanish day-ahead market on 23 October, 2014. Market prices are forecasted as described in Section IV-B. The model is tested for several parameters. These parameters are the marginal cost ( $c^{STG}$ ), the efficiency of the sale offers and purchase bids ( $\eta$ ) and the losses of the energy stored ( $\gamma$ ). Parameter  $c^{STG}$  can have the values 0, 5, 10 and €15/MWh, parameter  $\eta$  ranges

from 0.8, 0.9 and 0.99, and parameter  $\gamma$  can have the values 0.01, 0.03, and 0.05.

Fig. 2 shows a diagram of the main input and output needed to simulate the model.

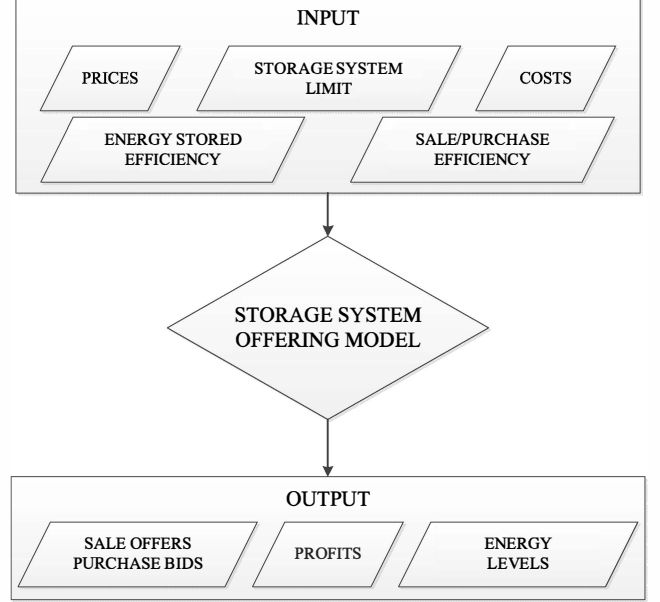


Fig. 2. Input and output of the simulations.

The main outputs are the purchase bids and the sale offers, where the energy scheduled profits the most from the generic storage system.

##### B. Market Price Scenarios

Price forecasts have been obtained using ECOTOOL [16], a toolbox implemented in MATLAB [17].

The prices have been forecasted using AR, MA and ARIMA models. In addition, ARIMA models from [18] are used to forecast prices on 23 October, 2014. The main ARIMA model is presented in (12).

$$\begin{aligned}
 & (1 - \phi_1 B^1 - \phi_2 B^2 - \phi_3 B^3 - \phi_4 B^4 - \phi_5 B^5) \\
 & \times (1 - \phi_{23} B^{23} - \phi_{24} B^{24} - \phi_{47} B^{47} - \phi_{48} B^{48} \\
 & - \phi_{72} B^{72} - \phi_{96} B^{96} - \phi_{120} B^{120} - \phi_{144} B^{144}) \\
 & \times (1 - \phi_{168} B^{168} - \phi_{336} B^{336} - \phi_{504} B^{504}) \log p_t \\
 & \times (1 - B)(1 - B^{24})(1 - B^{168}) \\
 & = c + (1 - \theta_1 B^1 - \theta_2 B^2)(1 - \theta_{24} B^{24}) \\
 & \times (1 - \theta_{168} B^{168} - \theta_{336} B^{336} - \theta_{504} B^{504}) \epsilon_t \quad (12)
 \end{aligned}$$

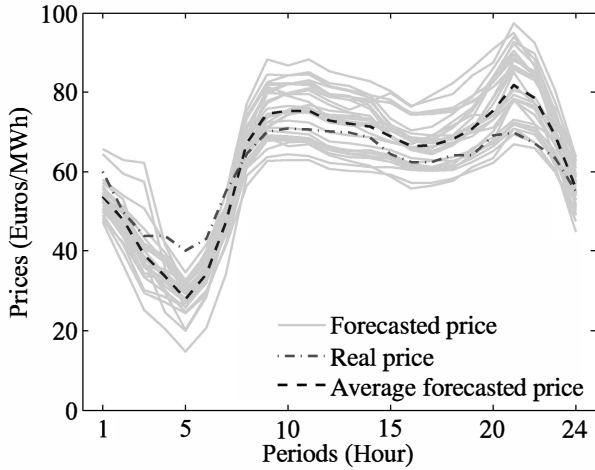


Fig. 3. Price scenarios, real prices and average forecasted prices on 23 October, 2014.

Next, the 30 forecasted prices (price scenarios), the real prices and the average forecasted prices are presented in Fig. 3.

The model is programmed in MATLAB [17] and GAMS [19] on a computer with 2 processors at 3.10 GHz and 256 GB of RAM.

## V. RESULTS

This section presents the main results divided into two parts. Firstly, the behaviour of the energy level, sale offers and purchase bids are shown. Secondly, the general behaviour of the storage system and the profits are presented.

### A. Effects on energy levels, sale offers and purchase bids of $c^{STG}$ , $\eta$ and $\gamma$ parameters

Normally, the purchase bid of a storage system occurs when the price is low, thus, in hour 5, as can be observed in Fig. 3, the storage system buys energy. The storage system sells energy near the higher price at hour 9. This behaviour is observed in Fig. 4, where the marginal cost  $c^{STG}$  is zero, the efficiency of buying and selling energy is 0.8 and the efficiency of the energy stored is 0.01.

However, the energy stored is sold at a price different from the high price owing to the losses of the energy stored, as shown in Fig. 5.

Finally, the impact of the marginal cost and the losses of the energy stored is reduced taking advantage of the increase in prices in different hours. This effect is observed in Fig. 6 and the time frame of the energy stored changes for different parameters.

Therefore, the normal behaviour of buying at low prices and selling at high prices shown in Fig. 4 occurs when the storage system has a low marginal cost and the losses of energy stored are also low.

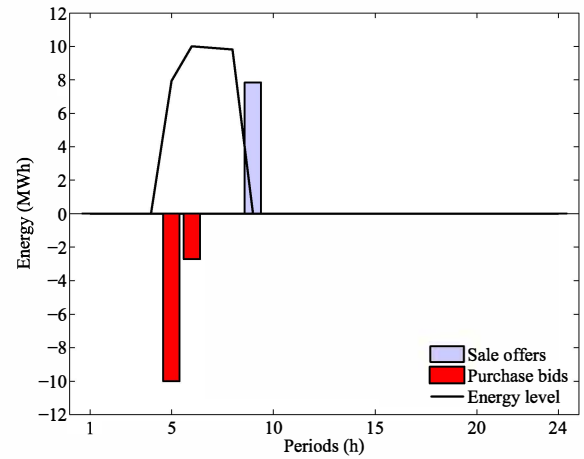


Fig. 4. Energy level, sale offers and purchase bids for  $c^{STG} = \text{€}0/\text{MWh}$ ,  $\eta = 0.8$  and  $\gamma = 0.01$ .

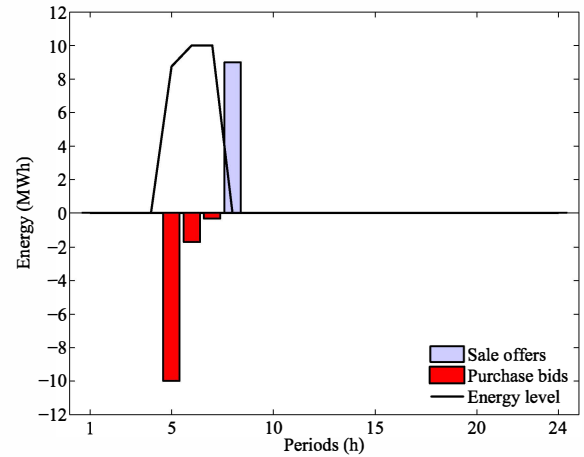


Fig. 5. Energy level, sale offers and purchase bids for  $c^{STG} = \text{€}5/\text{MWh}$ ,  $\eta = 0.9$  and  $\gamma = 0.03$ .

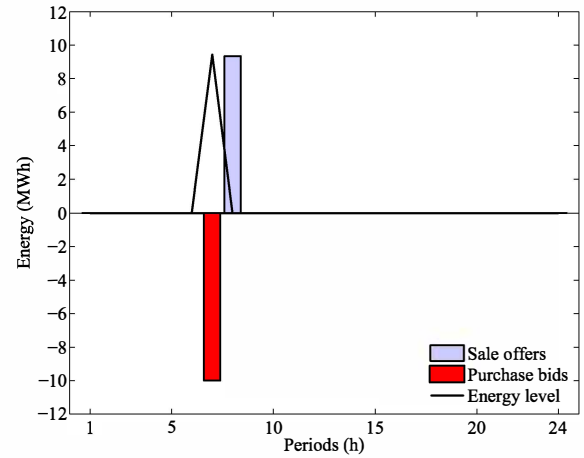


Fig. 6. Energy level, sale offers and purchase bids for  $c^{STG} = \text{€}15/\text{MWh}$ ,  $\eta = 0.99$  and  $\gamma = 0.05$ .

## B. General effects and profits

The energy and profits for several conditions of the storage system are studied in Table I and Table II.

The profits shown in Table I are higher for high values of  $\eta$  due to the advantages of buying/selling energy at different market prices.

When the storage system does not make any decision, the profit is zero because it would lose money otherwise. This effect starts when the marginal cost is equal to €5/MWh, where the worst case occurs with  $\eta = 0.8$ , and  $\gamma = 0.03$  and 0.05. However, for a marginal cost of €10/MWh and  $\eta = 0.9$ , the profit is also zero.

As a consequence of a higher efficiency,  $\eta = 0.99$ , the optimal offer can compensate the increase of the marginal cost.

On the other hand, the purchases are always higher than the sales as observed in Table II. The energy level varies between 9 MWh and 80 MWh to compensate for the effects of several parameters. Due to this, the storage system makes the most of the differences between market prices.

Due to the fact that the storage system has a cost, an efficiency and storage losses, it is impossible to charge and discharge at the same time and make a profit. Therefore, to take advantage of the differences in prices, the storage system has to charge depending on the parameters but near the lowest price and discharge when the difference among prices is the best to make a profit.

TABLE I  
PROFITS AND COSTS FOR ALL VALUES OF THE PARAMETERS

$c^{STG}$ (€/MWh)	$\eta$	$\gamma$	PF (€)	Costs (€)	
0	0.8	0.01	212.1	0.0	
		0.03	174.6	0.0	
		0.05	138.6	0.0	
	0.9	0.01	334.3	0.0	
		0.03	296.8	0.0	
		0.05	260.1	0.0	
	0.99	0.01	546.2	0.0	
		0.03	458.3	0.0	
		0.05	383.3	0.0	
	5	0.8	0.01	24.0	188.1
			0.03	0.0	0.0
			0.05	0.0	0.0
0.9		0.01	140.2	194.1	
		0.03	104.9	143.7	
		0.05	81.4	142.9	
0.99		0.01	237.4	199.0	
		0.03	202.2	196.6	
		0.05	178.4	103.0	
10		0.8	0.01	0.0	0.0
			0.03	0.0	0.0
			0.05	0.0	0.0
	0.9	0.01	14.4	177.3	
		0.03	0.0	0.0	
		0.05	0.0	0.0	
	0.99	0.01	110.8	198.0	
		0.03	94.1	196.1	
		0.05	77.7	194.3	
	15	0.8	0.01	0.0	0.0
			0.03	0.0	0.0
			0.05	0.0	0.0
0.9		0.01	0.0	0.0	
		0.03	0.0	0.0	
		0.05	0.0	0.0	
0.99		0.01	31.9	153.0	
		0.03	21.9	144.2	
		0.05	12.5	141.4	

TABLE II  
SALE OFFERS, PURCHASE BIDS, ENERGY LEVELS AND LOSSES OF ENERGY STORED FOR ALL VALUES OF THE PARAMETERS

$c^{STG}$ (€/MWh)	$\eta$	$\gamma$	Sales (MW)	Purchases (MW)	Energy levels (MWh)	Losses (MWh)	
0	0.8	0.01	7.8	-12.7	37.6	0.4	
		0.03	7.5	-13.2	36.9	1.1	
		0.05	7.3	-13.6	36.2	1.8	
	0.9	0.01	8.9	-11.4	38.8	0.4	
		0.03	8.7	-12.1	38.4	1.2	
		0.05	8.6	-12.7	38.1	1.9	
	0.99	0.01	19.8	-21.0	80.0	0.8	
		0.03	19.8	-22.3	70.0	2.1	
		0.05	19.8	-23.2	60.0	3.0	
	5	0.8	0.01	7.8	-12.7	37.6	0.4
			0.03	0.0	0.0	0.0	0.0
			0.05	0.0	0.0	0.0	0.0
0.9		0.01	8.9	-11.4	38.8	0.4	
		0.03	9.0	-12.1	28.7	0.9	
		0.05	9.0	-12.7	28.6	1.4	
0.99		0.01	9.9	-10.5	39.8	0.4	
		0.03	9.6	-11.0	39.3	1.2	
		0.05	9.9	-11.1	20.6	1.0	
10		0.8	0.01	0.0	0.0	0.0	0.0
			0.03	0.0	0.0	0.0	0.0
			0.05	0.0	0.0	0.0	0.0
	0.9	0.01	7.9	-10.0	17.7	0.2	
		0.03	0.0	0.0	0.0	0.0	
		0.05	0.0	0.0	0.0	0.0	
	0.99	0.01	9.9	-10.3	19.8	0.2	
		0.03	9.9	-10.7	19.6	0.6	
		0.05	9.9	-11.1	19.4	1.0	
	15	0.8	0.01	0.0	0.0	0.0	0.0
			0.03	0.0	0.0	0.0	0.0
			0.05	0.0	0.0	0.0	0.0
0.9		0.01	0.0	0.0	0.0	0.0	
		0.03	0.0	0.0	0.0	0.0	
		0.05	0.0	0.0	0.0	0.0	
0.99		0.01	9.9	-10.2	10.2	0.1	
		0.03	9.5	-10.0	9.6	0.3	
		0.05	9.3	-10.0	9.4	0.5	

## VI. CONCLUSIONS

This section presents the main conclusions as follows:

- Purchase bids at low prices and sale offers at high prices take place when the losses of the energy stored are very small.
- Energy losses, marginal costs and efficiencies are compensated with the increase of prices, as can be observed in Fig. 4, Fig. 5 and Fig. 6.
- A higher purchase/sale efficiency,  $\eta$ , increases the energy offered, but the purchase bid can be lower or higher for several values of  $\eta$ .
- The efficiency of buying and selling energy,  $\eta$ , is the more relevant parameter. Even with a high marginal cost,  $c^{STG}$ , the profit can be higher with respect to other values of the parameters if parameter  $\eta$  has a high value.
- The storage system does not charge and discharge at the same time due to cost, efficiency and losses parameters, the charge and discharge being not profitable.

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