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Line maintenance within transmission expansion planning: a multistage framework

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Abstract: Maintenance in transmission networks is an economical way to reduce upgrading network costs without decreasing its reliability. Hence, new studies regarding transmission expansion planning (TEP) must take into account the effects of maintenance in order to obtain realistic and economic expansion investment plans. This work presents a novel framework for multistage TEP, considering line maintenance, i.e. the expansion cost of the transmission system, network losses, costs of old-line replacement and maintenance, cost of newly constructed line maintenance, and cost of replaced line maintenance, are simultaneously optimised. The advantage of this approach is the fact that the lifetimes of the lines that are replaced, retained, and added to the transmission system are changing during the expansion horizon. These lifetimes have an impact on the maintenance expenses. Annual maintenance costs are also affected by the inflation rate. Hence, both the lifetime and inflation rate roles are integrated into the proposed model. The robustness and effectiveness of the model are tested on the IEEE 24-bus test system, using a particle swarm optimisation algorithm. The results show that the proposed formulation finds more economic investment plans for TEP when compared with those found using static formulations considering the maintenance available in specialised literature.

Nomenclature

Indices

i, q	indices for corridors
$t, t1$	indices for year
n, m	indices for bus

Sets

$\Omega^{\mathrm{b}}, \Omega^{\mathrm{c}}$	set of all buses, and set of all corridors
$\Omega^{\mathrm{ec}}, \Omega^{\mathrm{s}}$	set of old corridors including lines, and set of old corridors including substations

Parameters

C^{L}	per unit cost of power losses ($/MWh)
$C_{\mathrm{i}}^{\mathrm{C}}$	construction cost of a line circuit in corridor i ($)
$C_{\mathrm{i}}^{\mathrm{R}}, C_{\mathrm{i}}^{\mathrm{S}}$	replacement cost of a line circuit, and construction cost of a substation 138/230 kV in corridor *i* ($)
$\underline{C_i^{\mathrm{M}}}, \underline{C_i^{\mathrm{r}}}$	fixed maintenance, and repair cost of a line circuit in corridor *i* ($)
k^{L}	losses coefficient
$m_{\mathrm{i}}, M_{\mathrm{i}}$	feature constant, and its maximum value for a line circuit in corridor *i*
$n_{\mathrm{i}}^{\mathrm{rl}}, n_{\mathrm{i}}^{\mathrm{l0}}$	regular, and initial life of a line circuit in corridor *i* (year) ($n_{\mathrm{i}}^{\mathrm{l0}} = 0$ for new and replaced line circuits)
nr_{i}	number of replaced circuits in corridor *i*
$\underline{n_i}$	initial number of circuits in corridor *i*
$\underline{n_i^{\mathrm{s}}}$	initial number of substations in corridor *i*
$\overline{n_i}$	maximum number of circuits in corridor *i*
$\overline{n_i^{\mathrm{s}}}$	maximum number of substations in corridor *i*
$\overline{\overline{P}}_i$	maximum active power of a line circuit in corridor *i* (MW)
r_i', γ_i'	resistance (Ω/km) and susceptance (Ω^{-1}/km) of each circuit per kilometre of corridor *i*
T, I, β	planning horizon (year), inflation rate, and growth rate
Vl_i, ℓ_i	voltage level (kV), and length (km) of corridor *i*

Variables

n_i, n^{s}_i	number of new circuits, and substations in corridor i
$\Delta\theta_{i,t}$	difference between voltage phase angle of start and end buses of corridor *i* in year *t* (radian)
$\Delta\theta_{i,t,q}$	$\Delta\theta_{\mathrm{i,\ t}}$after outage of a line circuit in corridor *q*

Functions

$A_i\,(t)$	effective age (lifetime) of a line in corridor *i* after performing maintenance in year t until end of line's life (year)
$B_i\,(t)$	effective age (lifetime) of a line in corridor *i* before performing maintenance in year *t* until end of line's life (year)
C_i^{M}	maintenance cost of a line circuit in corridor *i* ($)
$C^{\mathrm{L}}{}_{i,t}$	losses cost of corridor *i* in year *t* ($)
$\mathrm{CM}_i^{\mathrm{old}}$	maintenance cost of an old line circuit within corridor *i* ($)
$\mathrm{CM}_{i,t}^{\mathrm{new}}$	maintenance cost of a new line circuit in corridor *i* from the beginning of year *t* (when it is installed in the network) to the end of the planning horizon ($)
$\mathrm{CM}_{i,t}^{\mathrm{old}}$	maintenance cost of an old line circuit within corridor *i* in year *t* ($)
$D_{n,t}$	total demand on bus *n* at the end of year *t* (MW)
$G_{n,t}$	total generation on bus *n* at the end of year *t* (MW)
K_i	maintenance cost coefficient of a line circuit in corridor *i* ($)
$K_{i,t}{}^{\mathrm{new}}$	K_i for new circuits in year *t*
$K_{i,t}{}^{\mathrm{old}}$	K_{i} for old circuits in year *t*
$m_{i,t}{}^{\mathrm{new}}$	m_i for new circuits in year *t*
$m_{i,t}{}^{\mathrm{old}}$	m_i for old circuits in year *t*
$n_{i,t}, n^{\mathrm{s}}{}_{i,t}$	n_i, and $n^{\mathrm{s}}{}_i$ in year *t*
$nr_{i,t}$	nr_i in year *t*
$nl_{i,t}^{\mathrm{old}}$	lifetime of an old line circuit in corridor *i* at the beginning of year *t* (year)

$nl_{i,t}^{\text{new}}$	lifetime of a new line circuit in corridor *i* from the end of year *t* (1 year after its installation) until the end of the planning horizon (year)
$P_{i,t}$	active power of corridor *i* in year *t* (MW)
$P_{i,t,q}$	$P_{\text{i,t}}$ after outage of a line circuit in corridor *q* (MW)
$P_{nm,t}$	active power transmitted from buses *n* to m in year *t* (MW)
$P_{nm,t,q}$	$P_{nm,t}$ after outage of a line circuit in corridor *q* (MW)
$P^{\text{L}}{}_{i,t}$	active power losses of corridor *i* in year *t* (MW)
$\overline{P_{i,t}}$	maximum active power of corridor *i* in year *t* (MW)
$\overline{P_{i,t,q}}$	$\overline{P_{i,t}}$ after outage of a line circuit in corridor *q* (MW)
$r_{i,t}$	resistance of corridor *i* in year *t* (Ω)
$v_{i,t}$	decision variable that is 1 when old circuits of corridor *i* are replaced by new ones in year *t* ($n_{\text{i}}^{10}=0$)
$w_{i,t}$	binary variable that is 1 when replacement does not happen for corridor *i* in year *t* ($n_{\text{i}}^{10}\neq 0$)
$\gamma_{i,t}$	susceptance of corridor *i* in year *t* (Ω^{-1})
$\gamma_{i,t,q}$	$\gamma_{i,t}$ after outage of a line circuit in corridor *q* (Ω^{-1})

1 Introduction

Studies regarding transmission expansion planning (TEP) are commonly classified as static or multistage (dynamic). Static TEP finds the optimal place and number of new circuits that should be added to the transmission network in a specific planning horizon; here, the line construction times are not specified by the planner, and the total expansion investment is carried out at the beginning of the planning horizon. On the other hand, the multistage TEP divides the static approach into several time intervals, and the optimal expansion design is outlined over the whole planning period [1, 2].

The multistage expansion planning in power systems is a widely studied topic, as dynamic planning models were introduced in the early 1980s [3–6]. For instance, [3] solves a pseudodynamic model for expansion planning of power distribution systems, aiming minimisation of distribution feeders and substation expansion costs, as well as energy and demand losses. In this model, first, static expansion planning problem is solved. Then, successive concatenated single year expansion plans (the optimal sizes and locations of feeders and substations) are found from first year to last year of the planning horizon. Moreover, in [4], a dynamic mixed-integer programming model for distribution system expansion planning was presented. The objective was to minimise investment, power loss costs, reconductoring cost for feeders, and substation expansion costs. The proposed model provides considerable cost savings for the electrical utility, but piece-wise linear approximation of the non-linear terms does not result in an accurate solution.

In order to propose a more accurate formulation for the distribution expansion planning problem, in [5], the voltage drop constraints were included in the model presented in [3]. The authors demonstrated how the inclusion of these constraints significantly affected the quality of solution found; concluding that considering voltage drops in the multistage distribution expansion planning increases complexity of the problem. For this, in [6], a genetic algorithm (GA) was applied to solve the dynamic distribution expansion planning problem instead of mathematical methods used in [3–5]. The model proposed in [6] was a more efficient formulation compared to those in [3–5], as it considered constraints related to network radiality, voltage drops, and reliability. Although the proposed models in [3–6] represent a dynamic formulation for expansion planning of distribution systems, important issues like maintenance and lifetime of lines were disregarded.

Recently, various papers have been presented in the field of multistage TEP [2, 7–20]. In [7], a method called network topology was proposed in order to reduce the number of variables in the long-term composite multistage generation and TEP, thereby minimising the investment cost of generation, lines, and fuel cost. In [8], an efficient GA was presented, tackling the multistage TEP. The GA presented has a set of specialised genetic operators and an efficient initial population generation that finds high-quality suboptimal topologies for large-size and high-complexity systems. In [9], a new formulation was presented for the multistage TEP. The objective function includes the expansion and generation costs and one reliability criterion (power not supplied). The authors concluded that the proposed model can increase the quality of service in modern power systems. Moreover, the multistage TEP considering the transmission system security and congestion in a competitive electricity market was solved in [10], showing that the approach enables more realistic assessments of generation and transmission investment decisions.

Later, in [11], a multiyear dynamic TEP model was presented, one that considered incentive payments to independent power producers. In [13], a multistage hybrid generation and TEP was proposed, considering non-uniform geographical fuel supply costs. Furthermore, an objective function, including the investment cost of new lines and reliability worth as a loss of load cost for the TEP problem, was presented in [14]. In [15], a model was introduced for multistage TEP in restructured power systems, which involved formulating the multistage TEP as a minimisation of line investment costs, congestion costs, and load curtailment costs due to single line outages. Also, multistage transmission and generation expansion planning considering generation reliability was formulated in [17].

Moreover, in [18], a novel mixed-integer linear programming model was presented for a multi-stage TEP considering high-voltage alternating and direct current lines. In [19], a dynamic TEP

Table 1 *Main concepts of proposed models*

Concepts	Model
pseudodynamic expansion planning for distribution systems	[3, 5]
dynamic expansion planning for distribution systems	[4, 6]
dynamic expansion planning for transmission systems (TEP)	[8]
comparing static TEP with dynamic TEP	[12]
dynamic TEP considering N–1 contingency	[20]
dynamic TEP considering generation reliability	[9]
dynamic TEP considering electricity market	[10]
dynamic TEP considering generation uncertainty	[16]
dynamic TEP considering independent power producers (IPPs)	[11]
dynamic TEP considering network reliability	[14]
dynamic TEP considering load curtailment	[15]
dynamic TEP considering generation expansion	[7]
dynamic TEP considering generation expansion and fuel costs	[13]
dynamic TEP considering generation expansion and reliability	[17]
dynamic TEP considering high-voltage DC (HVDC) lines	[18]
dynamic TEP considering CVSR	[19]

problem considering continuously variable series reactor (CVSR) was solved. It was shown that CVSRs can delay construction of new transmission lines by controlling the lines impedance. Finally, in [20], $N-1$ contingency constraint was formulated more optimally in the multi-stage TEP using exact linearisation of non-linear terms. In order to illustrate differences of proposed models, Table 1 lists main concept of each model.

It is important to remark that several approaches have previously tackled the dynamic TEP [1]. However, in these models, multi-stage TEP has been solved disregarding line maintenance. To the best of the authors' knowledge, the multistage TEP problem has not been optimised simultaneously with maintenance. Maintenance activities can improve network reliability and reduce the total expansion cost of the transmission system. The lifetime of the transmission lines is extended if the maintenance increases [21]. Hereafter, despite the multistage TEP model limitations concerning the system size and the system modelling complexity, considering maintenance is possible when the effect of line lifetimes is considered within the problem formulation.

Line lifetimes have an important role in TEP because decisions about the construction of new lines depend on the initial and regular lifetime of transmission lines. Although the effects of line maintenance on TEP were formulated in [21, 22] they were not accurately studied, due to the fact that the model was presented as static, while maintenance effects must be accounted every year. In simple terms, in the static model presented in [21, 22], maintenance has been considered only for one year (the last year of planning horizon), while transmission lines needed to be maintained when they installed in the network until the end of the planning horizon. In the multistage framework, the maintenance of old lines is considered on every year of the planning horizon, while the maintenance of new ones is accounted since the installation year until the last year of the expansion horizon. In addition, in [21, 22], only the maintenance of old transmission lines was considered; new line maintenance was disregarded, so the wear-and-tear that these lines suffer was ignored during the expansion horizon.

Thus, it is very useful to have a general, time-dependent model that explicitly considers the wear-and-tear of new and old transmission lines within the planning period. Also, the model should include the maintenance of all (new and old) transmission lines from the beginning to the end of the expansion horizon. In the present paper, a new framework is introduced to dynamically evaluate maintenance effects on the multistage TEP. Here, the expansion cost of the transmission system, network losses, cost of old-line replacement and maintenance, cost of newly constructed line maintenance, and cost of replaced-line maintenance are optimised simultaneously, considering line lifetimes and annual inflation rates. The proposed model is tested on the IEEE reliability test system, using a particle swarm optimisation (PSO) algorithm, which has shown great performance in power system optimisation [23, 24].

The main aims and contributions of the present paper are as follows:

(i) To formulate the dynamic effect of the lifetimes of both old and new transmission lines on maintenance.
(ii) To model the maintenance effects of new and replaced transmission lines on TEP.

2 Problem formulation

The proposed multistage TEP is formulated using a DC model and is represented as a non-linear programming problem, using the set of (1)–(22). This formulation minimises the objective function shown in (1), where the first and second terms represent the total construction costs of new lines and substations during the whole planning horizon, respectively, i.e. the expansion cost of the transmission system. The third term expresses the total active losses cost of the transmission system at the end of the planning time, while the fourth term represents the investment related to old circuits that must be replaced. The term $(1+\beta)^{t-1}/(1+I)^{t-1}$ annualises the cost for the present year based on line construction times (t), growth rate (β), and inflation rate (I). The fifth term describes the maintenance costs of the old line circuits that are replaced by new ones, from the time of replacement to the end of the expansion horizon. The sixth term represents the maintenance cost of old-line circuits (see (9)). Finally, the last term of the objective function expresses the maintenance cost of new line circuits from the time of their installation in the network until the end of the planning horizon. (see (1)) s.t.

$$C_{i,t}^{\mathrm{L}} = 8760k^{\mathrm{L}}C^{\mathrm{L}}P_{i,t}^{\mathrm{L}}. \tag{2}$$

$$P_{i,t}^{\mathrm{L}} = r_{i,t}(P_{i,t})^2/|Vl_i|^2. \tag{3}$$

$$P_{i,t} = \gamma_{i,t}\Delta\theta_{i,t}. \tag{4}$$

$$r_{i,t} = \ell_i r'_i \bigg/ \left(\underline{n_i} + \sum_{t_1=1}^{t} n_{i,t1}\right). \tag{5}$$

$$\gamma_{i,t} = \ell_i \left(\underline{n_i} + \sum_{t_1=1}^{t} n_{i,t1}\right)\gamma'_i. \tag{6}$$

$$v_{i,t} = \begin{cases} 0 & \text{for } n_i^{l0} + t \le n_i^{rl} \\ 1 & \text{ow} \end{cases}. \tag{7}$$

$$n_i^{l0} = \begin{cases} n_i^{l0} & \text{for } n_i^{l0} + t \le n_i^{rl} \\ 0 & \text{ow} \end{cases}. \tag{8}$$

$$w_{i,t} = \begin{cases} 1 & \text{for } n_i^{l0} + t \le n_i^{rl} \text{ and } n_i^{l0} \ne 0 \\ 0 & \text{ow} \end{cases}. \tag{9}$$

$$\mathrm{CM}_{i,t}^{\mathrm{new}} = K_{i,t}^{\mathrm{new}}\underline{C_i^{\mathrm{M}}}. \tag{10}$$

$$\mathrm{CM}_{i,t}^{\mathrm{old}} = K_{i,t}^{\mathrm{old}}\underline{C_i^{\mathrm{M}}}. \tag{11}$$

$$G_{n,t} = D_{n,t} + \sum_{n,m\in\Omega^b}^{n\ne m} P_{nm,t} \tag{12}$$

$$G_{n,t} = D_{n,t} + \sum_{n,m\in\Omega^b}^{n\ne m} P_{nm,t,q} \tag{13}$$

$$\begin{aligned} \min \mathrm{Obj} = {} & \sum_{t=1}^{T}\sum_{i\in\Omega^{\mathrm{c}}} C_i^{\mathrm{C}} n_{i,t}\frac{(1+\beta)^{t-1}}{(1+I)^{t-1}} + \sum_{t=1}^{T}\sum_{i\in\Omega^{\mathrm{s}}} C_i^{\mathrm{S}} n_{i,t}^{\mathrm{s}}\frac{(1+\beta)^{t-1}}{(1+I)^{t-1}} \\ & + \sum_{t=1}^{T}\sum_{i\in\Omega^{\mathrm{c}}} C_{i,t}^{\mathrm{L}}\frac{(1+\beta)^{t-1}}{(1+I)^{t-1}} + \sum_{t=1}^{T}\sum_{i\in\Omega^{\mathrm{ec}}} v_{i,t}\underline{n_i}C_i^{\mathrm{R}}\frac{(1+\beta)^{t-1}}{(1+I)^{t-1}} \\ & + \sum_{t=1}^{T}\sum_{i\in\Omega^{\mathrm{ec}}}\sum_{t_1=t}^{T} v_{i,t}\underline{n_i}\mathrm{CM}_{i,t}^{\mathrm{new}}\frac{(1+\beta)^{t_1-1}}{(1+I)^{t_1-1}} + \sum_{t=1}^{T}\sum_{i\in\Omega^{\mathrm{ec}}} w_{i,t}\underline{n_i}\mathrm{CM}_{i,t}^{\mathrm{old}} \\ & \times \frac{(1+\beta)^{t-1}}{(1+I)^{t-1}} + \sum_{t=1}^{T}\sum_{i\in\Omega^{\mathrm{c}}}\sum_{t_1=t}^{T} n_{i,t}\mathrm{CM}_{i,t}^{\mathrm{new}}\frac{(1+\beta)^{t_1-1}}{(1+I)^{t_1-1}}, \end{aligned} \tag{1}$$

$$D_{n,t} = D_n^0(1 + LGF)^t \tag{14}$$

$$\left|P_{i,t}\right| \le \overline{P_{i,t}} \tag{15}$$

$$\left|P_i^{tq}\right| \le \overline{P_i^{tq}}\,. \tag{16}$$

$$0 \le \sum_{t=1}^{T} n_{i,t} \le \overline{n_i} - \underline{n_i}\,. \tag{17}$$

$$0 \le \sum_{t=1}^{T} n_{i,t}^{\mathrm{s}} \le \overline{n_i^{\mathrm{s}}} - \underline{n_i^{\mathrm{s}}}\,. \tag{18}$$

$$\overline{P_{i,t}} = \left(\underline{n_i} + \sum_{t_1=1}^{t} n_{i,t1}\right)\overline{\overline{P}}_i\,. \tag{19}$$

$$P_{i,t,q} = \begin{cases} \gamma_{i,t,q}\Delta\theta_{i,t,q} & \text{if } \gamma_{i,t,q} \ge 0 \\ 0 & \text{ow} \end{cases}. \tag{20}$$

$$\gamma_{i,t,q} = \begin{cases} \ell_i\left(\underline{n_i} + \sum_{t_1=1}^{t} n_{i,t1} - 1\right)\gamma'_i & \text{for } i = q \\ \gamma_{i,t} & \text{ow} \end{cases}. \tag{21}$$

$$\overline{P_{i,t,q}} = \begin{cases} \left(\underline{n_i} + \sum_{t_1=1}^{t} n_{i,t1} - 1\right)\overline{\overline{P}}_i & \text{for } i = q \\ \overline{P_{i,t}} & \text{ow} \end{cases}. \tag{22}$$

$$1 \le t \le T\,. \tag{23}$$

The solution will represent an expansion plan that satisfies the set (2)–(23), with minimum construction, replacement, losses, and maintenance costs. Hereinafter, (2) states the annual losses cost of each corridor, while (3) describes the active power losses of each corridor for a specified year. Equation (4) shows the DC power flow through the transmission lines at each year of the planning horizon. Moreover, (7) avoids the replacement of old lines in each corridor for each year of the planning horizon if the line's age is less than or equal to its regular lifetime. Constraint (8) ensures that a line circuit is not replaced twice in the planning horizon. Equations (10) and (11) establish the maintenance costs of the new and old line circuits in terms of the maintenance cost coefficients and their fixed maintenance value. The coefficients depend on the initial operation periods of the line circuits and the planning year. These time-dependent factors show the effect of a line's life on the maintenance costs (see Section 2.1).

Furthermore, (12) and (13) represent the DC power flow balance for each bus under normal conditions and under a line circuit outage ($N-1$ safe criterion), respectively. In (12) and (13), the total demand on each bus is calculated using (14). Equations (15) and (16) model the power flow limits for each corridor under normal and $N-1$ conditions, respectively. In addition, (17) and (18) show the right-of-way constraint and the maximum number of substations, respectively. Constraint (19) represents the maximum power flow under normal conditions, while (20) and (22) model the power flow and maximum power flow for $N-1$ conditions, respectively. Finally, (21) represents the susceptance under $N-1$ conditions, and (23) divides the planning horizon (T) into several stages (years) t.

2.1 Effect of a line's lifetime on the maintenance cost coefficient

Predefined or specific maintenance expenditures are required to provide a regular lifetime for transmission lines under normal operational conditions. If the line is operated for a longer time, the maintenance budget must increase. This fact can be described mathematically in static TEP, using the age reduction factor of old transmission lines in corridor i (b_i), as follows [21, 25]:

$$b_i = A_i(T)/B_i(T) = 1 - \left(\left(\mathrm{CM}_i^{\mathrm{old}} - \underline{C_i^{\mathrm{M}}}\right)/\underline{C_i^{\mathrm{r}}}\right)^{1/m_i}, \tag{24}$$

where $A_i(T) = n_i^{rl} – n_i^{l0} – T$ and $B_i(T) = n_i^{rl} – n_i^{l0}$. It should be noted that fixed maintenance and repair costs are allocated to a line before and after a failure, respectively. In simple terms, a line needs to be maintained before failure, but after failure it has to be repaired. This relation can be defined as (25) for multistage TEP, using the dynamic age reduction factor:

$$b_{i,t} = A_i(t)/B_i(t) = 1 - \left(\left(\mathrm{CM}_{i,t}^{\mathrm{old}} - \underline{C_i^{\mathrm{M}}}\right)/\underline{C_i^{\mathrm{r}}}\right)^{1/m_{i,t}^{\mathrm{old}}}. \tag{25}$$

The following algebraic equations are obtained by replacing $A_i(t) = n_i^{rl} - nl_{i,t}^{\mathrm{old}} - t$ and $B_i(t) = n_i^{rl} - nl_{i,t}^{\mathrm{old}}$ in (25):

$$\left(n_i^{rl} - nl_{i,t}^{\mathrm{old}} - t\right)/\left(n_i^{rl} - nl_{i,t}^{\mathrm{old}}\right) = 1 - \left(\left(\mathrm{CM}_{i,t}^{\mathrm{old}} - \underline{C_i^{\mathrm{M}}}\right)/\underline{C_i^{\mathrm{r}}}\right)^{1/m_{i,t}^{\mathrm{old}}}. \tag{26}$$

$$\begin{aligned} t/\left(n_i^{rl} - nl_{i,t}^{\mathrm{old}}\right) &= \left(\left(\mathrm{CM}_{i,t}^{\mathrm{old}} - \underline{C_i^{\mathrm{M}}}\right)/\underline{C_i^{\mathrm{r}}}\right)^{1/m_{i,t}^{\mathrm{old}}} \\ &= \left(\underline{C_i^{\mathrm{M}}}/\underline{C_i^{\mathrm{r}}}\right)^{1/m_{i,t}^{\mathrm{old}}}\left(\left(\mathrm{CM}_{i,t}^{\mathrm{old}}/\underline{C_i^{\mathrm{M}}}\right) - 1\right)^{1/m_{i,t}^{\mathrm{old}}}. \end{aligned} \tag{27}$$

Solving for $\mathrm{CM}_{i,t}^{\mathrm{old}}/\underline{C_i^{\mathrm{M}}}$ yields

$$\mathrm{CM}_{i,t}^{\mathrm{old}}/\underline{C_i^{\mathrm{M}}} = 1 + \left(t/\left(n_i^{rl} - nl_{i,t}^{\mathrm{old}}\right)\right)^{m_{i,t}^{\mathrm{old}}}\left(\underline{C_i^{\mathrm{r}}}/\underline{C_i^{\mathrm{M}}}\right). \tag{28}$$

$$K_{i,t}^{\mathrm{old}} = 1 + \left(t/\left(n_i^{rl} - nl_{i,t}^{\mathrm{old}}\right)\right)^{m_{i,t}^{\mathrm{old}}}\left(\underline{C_i^{\mathrm{r}}}/\underline{C_i^{\mathrm{M}}}\right). \tag{29}$$

Equation (29) shows the relation between the initial operation period and the maintenance cost coefficient for the old line circuits, where $nl_{i,t}^{\mathrm{old}} = n_i^{l0} + t - 1$ and $m_{i,t}^{\mathrm{old}}$ is calculated by (30). $m_{i,t}^{\mathrm{old}}$ shows relationship between maintenance cost and lifetime of a transmission line. Larger and smaller $m_{i,t}^{\mathrm{old}}$ correspond to younger and older lines, respectively.

$$m_{i,t}^{\mathrm{old}} = M_i - (M_i - 1)\left(nl_{i,t}^{\mathrm{old}}/n_i^{rl}\right)^{1/2}. \tag{30}$$

To define the relationship between the initial operation period and the maintenance cost coefficient for the new line circuits that were added to the network or that replaced the old lines during the planning horizon, (29) and (30) have to be written as (31) and (32):

$$K_{i,t}^{\mathrm{new}} = 1 + \left(t/\left(n_i^{rl} - nl_{i,t}^{\mathrm{new}}\right)\right)^{m_{i,t}^{\mathrm{new}}}\left(\underline{C_i^{\mathrm{r}}}/\underline{C_i^{\mathrm{M}}}\right). \tag{31}$$

$$m_{i,t}^{\mathrm{new}} = M_i - (M_i - 1)\left(nl_{i,t}^{\mathrm{new}}/n_i^{rl}\right)^{1/2}. \tag{32}$$

In the above relations, $nl_{i,t}^{\mathrm{new}} = t_1 - t$ ($t_1 = t, t+1, \ldots, T$). Equations (29) and (31) formulate the maintenance effects on multistage TEP as dynamic.

3 Solution method

In order to find a good-quality solution for the model proposed in Section 2, the discrete PSO algorithm presented in [26] was developed. Here, let N be the number of parameters (search space dimension) in the position vector ($\boldsymbol{X}_d$) and velocity vector ($\boldsymbol{V}_d$) for a particle d:

$$X = \begin{bmatrix} X_1 & X_2 & \ldots & X_j & \ldots & \boldsymbol{X}_d \end{bmatrix}^{\mathrm{T}}. \tag{33}$$

$$V = \begin{bmatrix} V_1 & V_2 & \ldots & V_j & \ldots & \boldsymbol{V}_d \end{bmatrix}^{\mathrm{T}}. \tag{34}$$

Moreover, (35) represents $\boldsymbol{X}_d$ in terms of NL_d and NS_d, which are the new line circuits and new substations proposed by particle d, respectively. These vectors are shown in (36) and (37), where $n_{i,t,d}$ and $n^{\mathrm{s}}{}_{i,t,d}$ [shown in (38) and (39), respectively] indicate the number of new circuits and substations of corridor i in year t that are proposed by particle d.

$$\boldsymbol{X}_d = [\mathrm{NL}_d, \mathrm{NS}_d]\,. \tag{35}$$

$$\begin{aligned}\mathrm{NL}_d = [&\overbrace{n_{1,1,d}, \ldots, n_{i,1,d}, \ldots, n_{|\Omega^c|,1,d}}^{t=1}, \overbrace{n_{1,2,d}, \ldots, n_{i,2,d}, \ldots, n_{|\Omega^c|,2,d}}^{t=2} \\ &, \ldots, n_{1,t,d}, \ldots, n_{i,t,d}, \ldots, \overbrace{n_{1,T,d}, \ldots, n_{|\Omega^c|,T,d}}^{t=T}]\,.\end{aligned} \tag{36}$$

$$\begin{aligned}\mathrm{NS}_d = [&\overbrace{n^s_{1,1,d}, \ldots, n^s_{i,1,d}, \ldots, n^s_{|\Omega^s|,1,d}}^{t=1}, \overbrace{n^s_{1,2,d}, \ldots, n^s_{i,2,d}, \ldots, n^s_{|\Omega^s|,2,d}}^{t=2} \\ &, \ldots, n^s_{1,t,d}, \ldots, n^s_{i,t,d}, \ldots, \overbrace{n^s_{1,T,d}, \ldots, n^s_{|\Omega^s|,T,d}}^{t=T}]\,.\end{aligned} \tag{37}$$

$$\sum_{t=1}^{T} n_{i,t,d} = \left\{0, 1, 2, \ldots, \overline{n_i} - \underline{n_i}\right\} \quad \forall i \in \Omega^{\mathrm{c}}\,. \tag{38}$$

$$\sum_{t=1}^{T} n^{\mathrm{s}}_{i,t,d} = \left\{0, 1, 2, \ldots, \overline{n^{\mathrm{s}}_i} - \underline{n^{\mathrm{s}}_i}\right\} \quad \forall i \in \Omega^{\mathrm{s}}\,. \tag{39}$$

Later, (5) and (6) are calculated. Then, (4) is determined, satisfying (12) (DC load flow in normal state); later, (3) is computed. Subsequently, (19), (21), and (22) are calculated. If (15) is met, (20) is calculated, considering constraint (13) (DC load flow when a line circuit fails). Moreover, (2), (7)–(11), and, consequently, the objective function (1) are determined if (16) is satisfied. Hereinafter, (1) is mapped into the fitness function using (40).

$$F = A/\mathrm{Obj}, \tag{40}$$

where $A = 10^{14}$ is a system-dependent constant, while constant A is used to prevent the fitness from obtaining values that are too small. The value of the fitness for all particles of initial population (X) is stored as shown in (41):

$$F = \left[F_1, F_2, \ldots, F_j, \ldots, F_d\right]. \tag{41}$$

A particle with the maximum fitness value (Fgbest) is stored as Xgp. The position (X_j) and velocity (V_j) of the jth particle are updated, according to (42) and (43) [27]:

$$V'_j = \mathrm{fix}\left(V_j + c_2 r_2 \left(\mathrm{Xgp} - X_j\right)\right), \tag{42}$$

$$X'_j = X_j + V'_j \quad \forall j = 1, 2, \ldots, d\,. \tag{43}$$

where r_2 is a random number between 0 and 1, and c_2 is a fixed amount.

In the next step (iteration 2), if constraints (17) and (18) are met, the fitness function (40) is evaluated again for new particle X_j' after (5), (6), (4), (12), (3), (19), (21), (22), (15), (20), (13), (16), (2), (7)–(11), and (1) have been calculated. Then, the new fitness values are arranged as follows:

$$F' = \left[F'_1, F'_2, \ldots, F'_j, \ldots, F'_d\right]. \tag{44}$$

The global best fitness of (44) is defined as Fgbest′, and a particle with Fgbest′ is saved as Xgp′. According to (45), if the global best fitness of (44) is greater than Fgbest, its value and corresponding particle are assigned to Fgbest and Xgp, respectively; otherwise (ow), the old values (Fgbest and Xgp) are selected.

$$\begin{cases} \mathrm{Fgbest} = \mathit{Fgbest'} \;\&\; \mathrm{Xgbest} = \mathrm{Xgbest'} \text{ if } \mathrm{Fgbest'} \rangle \mathrm{Fgbest} \\ \mathrm{Fgbest} = \mathrm{Fgbest} \;\&\; \mathrm{Xgbest} = \mathrm{Xgbest} \;\; \mathrm{ow} \end{cases} . \tag{45}$$

If F'_j is bigger than F_j, then F'_j is called a *local best fitness*, and its value is replaced by Flbest$_j$; otherwise, F_j is replaced by Flbest$_j$. In this situation, the particle with greater fitness is called the *local particle* (Xlp). Equation (46) describes this process:

$$\begin{cases} \mathrm{Flbest}_j = F'_j \;\;\&\;\; \mathrm{Xlp}_j = X'_j & \text{if } F'_j \rangle F_j \\ \mathrm{Flbest}_j = F_j \;\;\&\;\; \mathrm{Xlp}_j = X_j & \mathrm{ow} \end{cases} . \tag{46}$$

Equation (47) updates the position of the jth particle (X'_j). Moreover, (48) represents the velocity of this particle in terms of a random number ($0 < r_1 < 1$), a fixed value (c_1), and the inertia weight (ω).

$$X''_j = X'_j + V''_j \quad \forall j = 1, 2, \ldots, d\,. \tag{47}$$

$$V''_j = \mathrm{fix}\left(\omega V'_j + c_1 r_1 \left(\mathrm{Xlp}_j - X'_j\right) + c_2 r_2 \left(\mathrm{Xgp} - X'_j\right)\right). \tag{48}$$

where ω is adjusted in a training process as follows:

$$\omega = 1/(1 + \ln z)\,. \tag{49}$$

In (49), z is the number of algorithm iterations. After updating the new particle, the process can begin all over again. Fig. 1 shows the flowchart of proposed method, in which the algorithm is started by generating initial population and is continued by satisfying all constraints and finally terminated via stop condition. The algorithm stop criteria are (i) when the fitness function achieves a target value or (ii) when the maximum number of iterations is reached.

4 Simulation results

To show the efficiency of the proposed model, the solution method was applied to the IEEE reliability test system (IEEE RTS) [28] in two scenarios. The first one employs the proposed dynamic framework; in the second one, a static framework (developed in the appendix) of the proposed model was used.

For both scenarios, the maximum number of circuits and substations ($\overline{n_i}$ and $\overline{n^{\mathrm{s}}_i}$) was set to 6; the regular lifetime (n_i^{rl}) for all corridors (all line circuits) was set to 30 years; inflation rate (I) and growth rate were set to 0.1 and 0.15, respectively [29].

The ages of all of the line circuits in each corridor (n_i^{l0}) at the beginning of the time horizon are listed in Table 2. Also, the considered planning horizon was 5 years ($T = 5$).

It is assumed that each corridor includes just one transmission line that each line consists of several circuits.

4.1 Scenario 1

The dynamic framework was tested on the IEEE RTS, and the results are presented in Tables 3–5. Table 3 shows the new line circuits that must be added to the network during the planning horizon. Table 4 lists the existing corridors that should be replaced by new transmission lines through the expansion horizon. In addition, a new 138/230 kV substation is constructed within corridor 10–11 in the third year of the planning horizon ($t = 3$) and within corridor 3–24 in the beginning of the planning horizon ($t = 1$).

Table 5 describes the related costs. Herein, CC represents the construction cost of new lines and substations, RC is the replacement cost of old lines, LC is the active losses cost, and MCR, MCN, and MCO indicate the maintenance cost of replaced lines, new lines, and old lines, respectively. Finally, TC represents the total cost of the transmission system.

4.2 Scenario 2

In order to compare the efficiency of the proposed approach, a static framework was developed, modifying the presented dynamic model (see Appendix). Hence, the same configuration proposed in Scenario 1 is adopted in this scenario, thereby establishing an appropriate and reasonable comparison between the dynamic and static formulations and showing important role of annual inflation rate in static and dynamic calculations.

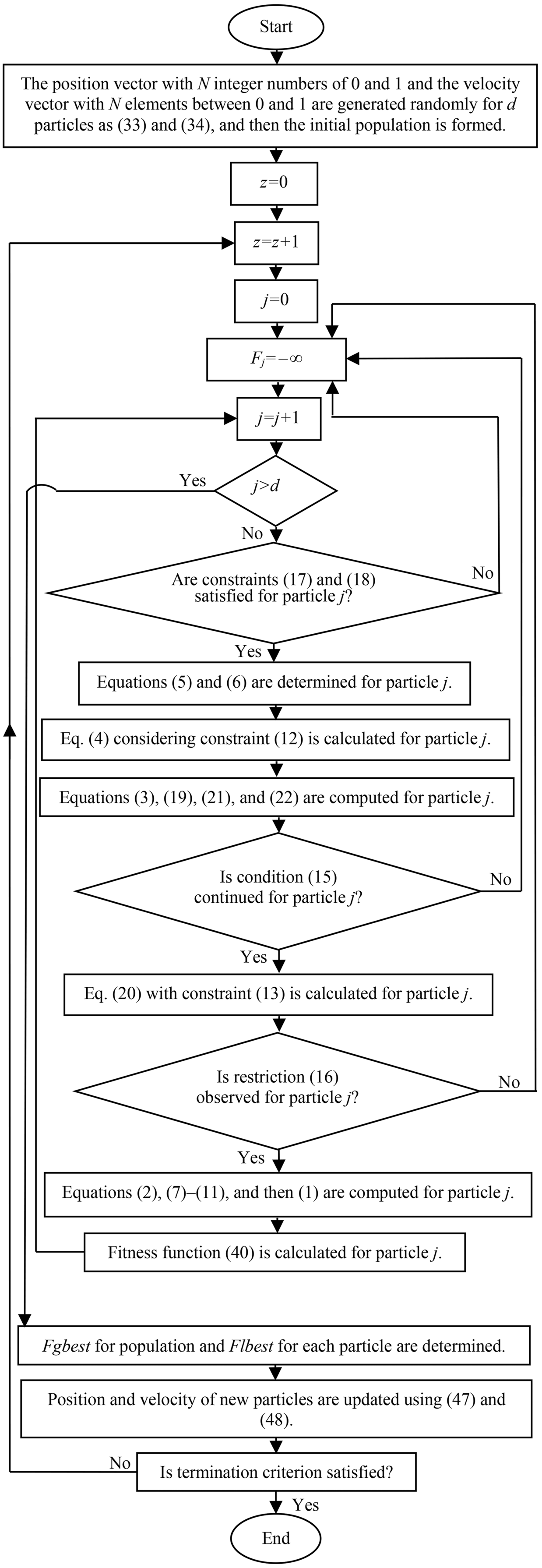

Fig. 1 *Flowchart of the proposed method*

In this manner, at the beginning of the planning horizon, all of the new line circuits are added and all of the replacements are made. In addition, the maintenance and network losses costs are calculated for both the new (replaced and added new line circuits) and old transmission lines from the start to the end of the planning horizon (5 years). Hence, the TC is calculated by (50)–(60). Likewise to Scenario 1, the static model was tested on the IEEE RTS; the results are provided in Tables 6–8. A new 138/230 kV

Table 2 *Initial lifetimes of the line circuits (year)*

Corr.	n_i^{l0}	Corr.	n_i^{l0}	Corr.	n_i^{l0}	Corr.	n_i^{l0}
1–2	8	6–10	10	13–23	20	17–22	23
1–3	26	7–8	28	14–16	28	18–21	29
1–5	27	8–9	16	15–16	28	19–20	18
2–4	28	8–10	20	15–21	17	20–23	26
2–6	29	11–13	15	15–24	27	21–22	28
3–9	14	11–14	27	16–17	27	—	—
4–9	25	12–13	26	16–19	26	—	—
5–10	24	12–23	25	17–18	24	—	—

Table 3 *Proposed transmission expansion plan in Scenario 1*

Corr.	$t = 1$ year		$t = 2$ years		$t = 3$ years		$t = 4$ years		$t = 5$ years	
	$n_{i,1}$	V_i	$n_{i,2}$	V_i	$n_{i,3}$	V_i	$n_{i,4}$	V_i	$n_{i,5}$	V_i
1–2	3	138	0	–	0	–	0	–	0	–
2–5	0	–	0	–	0	–	0	–	1	138
2–8	3	138	0	–	0	–	0	–	0	–
2–9	0	–	0	–	0	–	0	–	1	138
2–10	0	–	0	–	0	–	0	–	1	138
6–7	1	138	0	–	0	–	0	–	0	–
6–8	0	–	0	–	0	–	1	138	0	–
6–10	2	138	0	–	0	–	0	–	0	–
7–8	0	–	0	–	2	138	0	–	0	–
9–10	1	138	0	–	0	–	0	–	0	–
9–11	1	138	0	–	0	–	0	–	0	–
11–17	0	–	0	–	1	230	0	–	0	–
11–18	1	230	0	–	0	–	0	–	0	–
11–23	4	230	0	–	0	–	0	–	0	–
12–13	0	–	1	230	0	–	0	–	0	–
13–24	1	230	0	–	0	–	0	–	0	–
14–15	0	–	1	230	0	–	0	–	0	–
14–18	0	–	0	–	1	230	0	–	0	–
14–22	1	230	0	–	0	–	0	–	0	–
16–22	0	–	1	230	0	–	0	–	0	–
19–23	1	230	0	–	1	230	0	–	0	—
21–23	1	230	1	230	0	–	0	–	0	–

Table 4 *Replaced lines in existing corridors for Scenario 1*

Corr.	$t = 1$ year		$t = 2$ years		$t = 3$ years		$t = 4$ years		$t = 5$ years	
	$nr_{i,1}$	V_i	$nr_{i,2}$	V_i	$nr_{i,3}$	V_i	$nr_{i,4}$	V_i	$nr_{i,5}$	V_i
1–3	0	–	0	–	0	–	0	–	1	138
1–5	0	–	0	–	0	–	1	138	0	–
2–4	0	–	0	–	1	138	0	–	0	–
2–6	0	–	1	138	0	–	0	–	0	–
7–8	0	–	0	–	1	138	0	–	0	–
11–14	0	–	0	–	0	–	1	138	0	–
12–13	0	–	0	–	0	–	0	–	1	230
14–16	0	–	0	–	1	230	0	–	0	–
15–16	0	–	0	–	1	230	0	–	0	–
15–24	0	–	0	–	0	–	1	230	0	–
16–17	0	–	0	–	0	–	1	230	0	–
16–19	0	–	0	–	0	–	0	–	1	230
18–21	0	–	2	230	0	–	0	–	0	–
20–23	0	–	0	–	0	–	0	–	2	230
21–22	0	–	0	–	1	230	0	–	0	–

substation has to be constructed within corridors 10–11 and 3–24. Fig. 2 compares the related costs of Scenario 1 with those of Scenario 2.

5 Discussion

It can be seen that both scenarios found similar investment plans for the 5-year planning horizon. In Fig. 2, a comparison of the investment plans is presented, showing that Scenario 1 reaches a lower TC (i.e. a reduction of US$27.58M is shown, when compared to Scenario 2). Therefore, the expansion plan proposed by the dynamic model is more economic than that of the static one.

Table 5 *Expansion and operation costs in Scenario 1 (Million $)*

Year	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	Total
CC	59.24	10.77	15.36	1.07	4.84	91.28
RC	0	2. 1	3.68	3.18	3.68	12.64
LC	21.2	22.8	24.57	29.67	34.87	133.11
MCR	0	0.69	0.85	0.45	0.28	2.27
MCN	7.68	1.13	1.06	0.11	0.27	10.25
MCO	4.19	7.02	8.39	10.13	9.02	38.75
TC	92.31	44.51	53.91	44.61	52.96	288.3

Table 6 *Proposed transmission expansion plan in Scenario 2*

Corr.	n_i	V_i	Corr.	n_i	V_i	Corr.	n_i	V_i
1–2	3	138	7–8	2	138	14–15	1	230
2–5	1	138	9–10	1	138	14–18	1	230
2–8	3	138	9–11	1	138	14–22	1	230
2–9	1	138	11–17	1	230	16–22	1	230
2–10	1	138	11–18	1	230	19–23	2	230
6–7	1	138	11–23	4	230	21–23	2	230
6–8	1	138	12–13	1	230	—	—	—
6–10	2	138	13–24	1	230	—	—	—

Table 7 *Lines replaced by new ones in existing corridors for Scenario 2*

Corr.	nr_i	V_i	Corr.	nr_i	V_i	Corr.	nr_i	V_i
1–3	1	138	11–14	1	138	16–17	1	230
1–5	1	138	12–13	1	230	16–19	1	230
2–4	1	138	14–16	1	230	18–21	2	230
2–6	1	138	15–16	1	230	20–23	2	230
7–8	1	138	15–24	1	230	21–22	1	230

Table 8 *Costs from Scenario 2 (million $)*

CC	91.5	MCN	12.88
RC	12.74	MCO	18.13
LC	175.66	MCR	4.97
TC		315.88 M$	

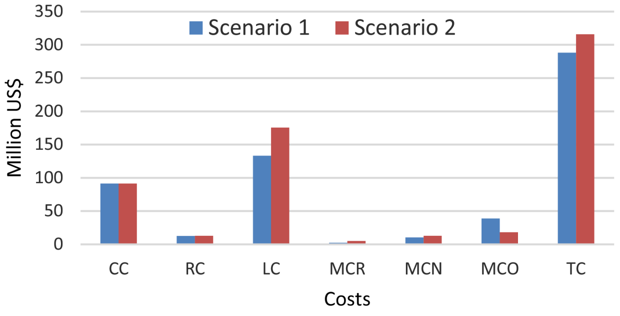

Fig. 2 *Comparison of costs for both scenarios*

Moreover, Scenario 1 saves US$0.22M and USM$0.1M in CC and RC, respectively, compared to Scenario 2. This difference is due to the fact that, in the static model, all of the new line circuits were constructed and replaced at the beginning of the planning horizon; the inflation rate does not affect these costs in Scenario 2. Hence, this shows that the construction and replacement costs in the static model are not precisely calculated.

Furthermore, the active losses cost, the MCR, and the MCN in Scenario 1 are less than those in Scenario 2 by US$42.55M, US $2.7M, and US$2.63M, respectively. This is due to the fact that in the dynamic model, some new and/or old lines are constructed or replaced during the expansion horizon, taking into account the power losses and the new line circuit maintenance from the moment in which they are installed in the network. On the other hand, in the static model, all of the new or replaced transmission lines are added to the network in the first year of the planning horizon.

Finally, the MCO in the dynamic model is US$20.62M more than the MCO in the static model. The maintenance cost coefficient of the old lines for Scenario 1 was considered to be dynamic, whereas this coefficient for Scenario 2 was considered static, i.e. the maintenance cost coefficient in the dynamic model is extended, with an increase in the line's lifetime, and the coefficient in the static evaluation is fixed and not changed during the planning period. Hence, the dynamic model showed itself to be more authentic, while the maintenance cost of the old lines in the static framework was inaccurately computed.

6 Conclusion

In this paper, a novel framework for dynamic multistage TEP, considering line maintenance, was presented. Moreover, the maintenance cost coefficient was formulated as a time-variant factor that changes with the increase in the lines' life during the planning horizon. In addition, the expansion cost of the transmission system, network losses, costs of old-line replacement and maintenance, cost of newly constructed line maintenance, and cost of replaced line maintenance were simultaneously optimised. The proposed dynamic framework was compared to a static approach in TEP. The results show that the maintenance cost of the new and replaced lines in the proposed model becomes less than that for a static approach, as the actual length of operation durations of the new lines are considered in the dynamic model (a fact that is disregarded in a static representation). Finally, it was shown that a dynamic approach for the TEP problem, considering line maintenance, finds economic and more realistic expansion plans compared to those found by static approaches. The proposed framework represents a novel approach for this problem.

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9 Appendix

In this section, a static framework for the TEP problem, considering line maintenance, is shown, based on [21]. Equation (50) represents the objective function, analogue to (1).

$$
\begin{aligned}
TC = {} & \sum_{i \in \Omega^c} C_i^C n_i + \sum_{i \in \Omega^s} C_i^S n_i^s + \sum_{t=1}^{T} \sum_{i \in \Omega^{ec}} v_{i,T} \underline{n_i} C_i^M \frac{(1+\beta)^{t-1}}{(1+I)^{t-1}} \\
& + \sum_{i \in \Omega^c} \sum_{t=1}^{T} 8760 k^L C^L P_{i,T}^L \frac{(1+\beta)^{t-1}}{(1+I)^{t-1}} + \sum_{i \in \Omega^{ec}} v_{i,T} \underline{n_i} C_i^R \\
& + \sum_{t=1}^{T} \sum_{i \in \Omega^{ec}} w_{i,T} \underline{n_i} C_i^M \frac{(1+\beta)^{t-1}}{(1+I)^{t-1}} + \sum_{t=1}^{T} \sum_{i \in \Omega^c} n_i C_i^M \frac{(1+\beta)^{t-1}}{(1+I)^{t-1}}
\end{aligned} \tag{50}
$$

Furthermore, (1) is subject to (17), (18), and (51)–(58).

$$n_i = \sum_{t=1}^{T} n_{i,t} n_i^s = \sum_{t=1}^{T} n_{i,t}^s \tag{51}$$

$$v_{i,T} = \begin{cases} 0 & \text{for } n_i^{l0} + T \le n_i^{rl} \\ 1 \text{ and } n_i^{l0} = 0 & ow \end{cases} \tag{52}$$

$$w_{i,T} = \begin{cases} 1 & \text{for } n_i^{l0} + T \le n_i^{rl} \text{ z } n_i^{l0} \ne 0 \\ 0 & ow \end{cases} \tag{53}$$

$$C_i^M = K_i \underline{C_i^M} \tag{54}$$

$$G_{n,T} = D_{n,T} + \sum_{n,m \in \Omega^b}^{n \ne m} P_{nm,T} \tag{55}$$

$$G_{n,T} = D_{n,T} + \sum_{n,m \in \Omega^b}^{n \ne m} P_{nm,T,q} \tag{56}$$

$$|P_{i,T}| \le \overline{P_{i,T}} \tag{57}$$

$$|P_{i,T,q}| \le \overline{P_{i,T,q}} \tag{58}$$

In the above equations, $P^{\mathrm{L}}_{\mathrm{i,T}}$, $P_{\mathrm{i,T}}$, and $P_{\mathrm{i,T,q}}$ are calculated for the complete planning horizon, i.e. replacing *t* with *T* in (3), (4), and (20), respectively. Moreover, the maintenance cost coefficient in (54) is determined by (59) and (60).

$$K_i = 1 + \left(T / \left(n_i^{rl} - n_i^{l0}\right)\right)^{m_i} \left(\underline{C_i^r} / \underline{C_i^M}\right) \tag{59}$$

$$m_i = M_i - (M_i - 1)\left(n_i^{l0} / n_i^{rl}\right)^{1/2} \tag{60}$$