Aalborg University

DENMARK

Laboratory Study of Dispersion of Buoyant Surface Plumes

Petersen, Ole ; Larsen, Torben

[Link to publication from Aalborg University](#)

Laboratory Study of Dispersion of Buoyant Surface Plumes

by

Ole Petersen and Torben Larsen
Dept. of Civil Engineering
University of Aalborg
Sohngaardsholmsvej 57, DK-9000 Aalborg

Abstract

A laboratory study on surface dispersion of buoyant plumes in open channel turbulence is made, where the buoyancy is due to both salinity and heat. The measured parameters are the downstream derivative of a plume width and height, which are integral-characteristics of the distributions of density-differences. Other methods as infra-red sensing are used for visualizing purposes. The results are used to calibrate an integral model of the dispersion. Conclusions are that the dispersion of a bouyant surface plume can be treated as the superposition of a bouyancy induced stretching and turbulent diffusion, reduced in the vertical direction by the density gradient. The result implies a simplification of the description of near-field dilution.

1. Introduction

Dispersion and dilution of discharges of buoyant effluents is a problem of practical and theoretical interest, as for example in studies of sea outfalls for disposal of sewage. In the present work we use a laboratory model to study how the dispersion of buoyant surface plumes in channel shear flows depends on ambient turbulence and density difference.

The significance of buoyancy to dispersion of surface plumes was first pointed out by Larsen and Sørensen, 1968, who realized that the dilution was the result of a lateral stretching due to density differences and vertical diffusion by ambient turbulence, which is reduced by the presence of a stable density gradient. Their theoretical considerations were later confirmed from experiments on heated water plumes by Weill and Fischer, 1979. The reduction in vertical dispersion of a heated surface layer in open channel flow, where the turbulence is generated by shear near the bottom, has been studied by Schiller and Sayre, 1975.

In the present experiments the plumes are formed by heated water discharged into a fresh or saline stream providing a wide range of density differences. The primary data are the downstream change of integral parameters as plume height or plume width, derived from measured distributions of density differences. These data enable us to obtain a consistent calibration of an integral model which combines the effects of turbulence and buoyancy.
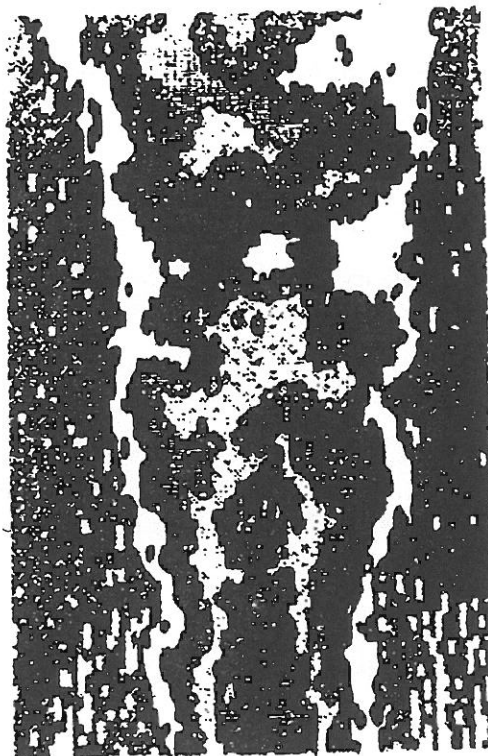

Fig. 1. Thermal image of water surface plume. $1.1°C$ between contours. $U_o = 10\ cm/s$, rough bed. $\Delta T_o = 20°C$.

The outline of the paper is as follows: First is a brief presentation of the theoretical background for plume dispersion, followed by a description of the laboratory model. Next is presented the results of the experiments which are used to calibrate the model and finally are given some conclusions on plume dispersion.

2. Theory

Buoyancy induced fronts

A lens of light water of weight $\rho + \Delta\rho$, confined to a volume of height h and width b, will rapidly be stretched into a thin layer, due to buoyancy induced pressure gradients. Taking the pressure distribution as hydrostatic, the celerity of the front, V, can be found using Bernoulli's equation as

$$V = \sqrt{2g \cdot \frac{\Delta\rho}{\rho} \cdot h} \qquad (1)$$

The disturbance propagates in a direction perpendicular to the front. For a continuous discharge, which is conveyed downstream with velocity U_o, and assuming that $V << U_o$, geometrical considerations yields

Fig. 2. Sketch of definitions.

$$\frac{db}{dx} = \frac{V}{U_o} \qquad (2)$$

and in the absence of mixing, conservation of mass requires that $h \cdot b = const$ thus

$$\frac{dh}{dx} = -\frac{h}{b}\frac{db}{dx} \qquad (3)$$

Turbulent diffusion

Taking for simplicity one dimensional diffusion, Fick's law postulates that the variance grows linear in time according to

$$\frac{db^2}{dt} = 2K \qquad (4)$$

where the plume width b, is defined as the standard deviation of the distribution of the diffusing substance and K is the Fickian diffusion coefficient. For the surface plume we obtain, inserting $dt = U_o \cdot dx$, and assuming a plane of symmetry at the water surface

$$\frac{db^2}{dx} = 2\frac{K_y}{U_o} \quad (5a) \qquad\qquad \frac{dh^2}{dx} = 2\frac{K_z}{U_o} \quad (5b)$$

Conservation of mass gives $h \cdot b \cdot \bar{c} = const$.

3. Experiments

Characterization of plumes

Assuming that a common set of local scales as a height, width and mean density difference applies to both diffusion and dispersion, is a simple consistent set of parameters derived

from the moments of the distribution of density differences. The density difference is used because this is the parameter of dynamic significance. We define a plume width and height as

$$b^2 = \frac{\int_\infty^\infty \int_0^d \Delta\rho y^2 \, dz \, dz}{\int_\infty^\infty \int_0^d \Delta\rho dz \, dy} \quad (6a) \qquad\qquad h^2 = \frac{\int_\infty^\infty \int_0^d \Delta\rho z^2 \, dz \, dy}{\int_\infty^\infty \int_0^d \Delta\rho dz \, dy} \quad (6b)$$

The excess mass per downstream distance, M, is the denominator in the two expressions. A mean density difference is defined assuming that the excess mass of the plume is contained in an area within two standard deviations from the centre of mass

$$\bar{\rho} = \frac{M}{2h4b} \tag{7}$$

As discussed in the next section, the instrumentation of the experiment is designed to allow a direct evaluation of these three parameters from a numerical integration of the measured density profiles.

Experimental facility

The experiments were made in a recirculating hydraulic flume, modified to use both saline and fresh water, as shown in Fig. 3. Salinity was introduced by adding common salt to the water. Artificial roughness of the bottom was supplied using triangular wooden list, 1 *cm* high placed with 5 *cm* spacing across the flume.

Fresh water was supplied by gravity from a 180 *l* insulated, constant head tank through a rotameter and a deaeration device. The water was discharged onto the surface through three circular nozzles ⌀70 mm, parallel to the main flow direction with the same velocity in the discharge and in the ambient water. In the experiments with fresh water in the flume, heated water was used to obtain the density difference, while cold water was used for the experiments with saline water; temperature here acting simply as a tracer.
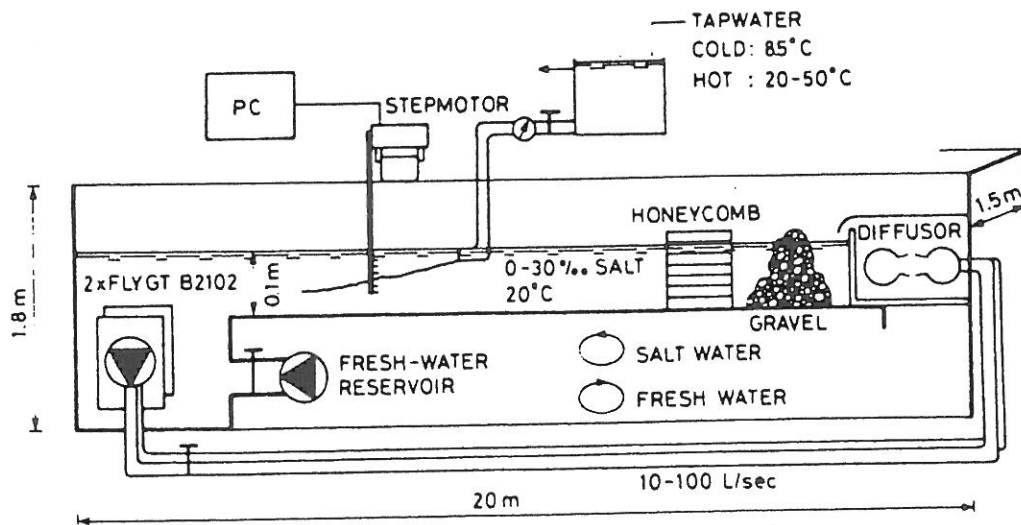

Fig. 3. Combined salt and fresh water flume.

Instrumentation

Temperature was measured using a column of Copper-Constantan, 0.5 *mm* thermocouples, connected to a PC with an A/D-converter. The transverse position of the thermocouples was also controlled from the PC by means of a step motor. Sensitivity of the temperature measurement was 0.01°C and 90% raise-time of the thermocouples was 0.04 *sec*.

As a tracer were used Rodamine-B. Concentration was measured using an Navitronic Q-200 in-situ fluorometer either directly or using a glass cuvette. Water was withdrawn to 250 *ml* samples using a column of 12 1 *mm* siphons. Salinity was measured by weighing a 25 *ml* pyknometer. An AGEMA Thermovision infra-red sensing system combined with digital image processing was used to measure surface temperatures in the plume. The instrumentation is described in details in Petersen et al., 1988.

Experimental procedure

In the experiment the plume is transversed by the column of thermocouples, measuring the mean density difference in 15-20 transverse positions, equally distributed across the plume. The instantaneous temperature is in each point converted to a density difference using an equation of state, relating density to temperature and salinity. From this a time average density difference is calculated. Dependency on temperature was found from a standard table, while dependency on salinity was calibrated by weighing. After the traverse the density profile is reduced to a plume width, height and a mean density difference. The mean and RMS temperature was also measured, allowing for calculation of the energy flux as a conserved quantity. The whole procedure is fully automated.

Experimental conditions

The range of conditions used was outlet excess temperature from $-12°$ to $35°C$, flow velocity $5-20$ *cm/sec*, Darcy bottom friction 0.006 to 0.017 for smooth and rough bed, water depth $10-25$ *cm* and density of ambient water $1000-1030$ kg/m^3.

4. Results

The undisturbed flow

Transverse dispersion was measured by recording the path of small wooden spheres. Positions were found using a videocamera and digital image analysis. Defining an effective dispersion coefficient by

$$K_y = \frac{U_o}{2} \frac{db^2}{dx} \tag{8}$$

K_y is found using (8) and measured transverse distributions. The results can be given as $K_y = a \cdot u_f \cdot d$ where $a = 0.12$. Fischer, 1979; Engelund, 1969.

The vertical dispersion coefficient, which is an integral property of the flow, is assumed to depend on the plume height. This is here described using a simple correlation of the form

$$K_{z_o} = \gamma u_f h \left(\frac{1}{\sqrt{3}} - \frac{h}{d} \right) \tag{9}$$

Using (9) and measured vertical distributions of tracer γ is found to 0.7.

5. Buoyant plumes

In Fig. 4 is shown shapes of average iso-density contours. Notice the distinct noses with relatively low mixing in the case with high buoyancy, Fig. 4a, a pattern characteristic of density fronts. Dye injections showed that the mixing in the central parts had character of vertical jets penetrating the plume causing a intermittent pattern of temperature fluctuations. With very high levels of buoyancy the plume is almost split into two parts. When turbulence is dominating the contours are more bellshaped, Fig. 4b. and the mixing irregular as it appears on the thermal image shown in Fig. 1.
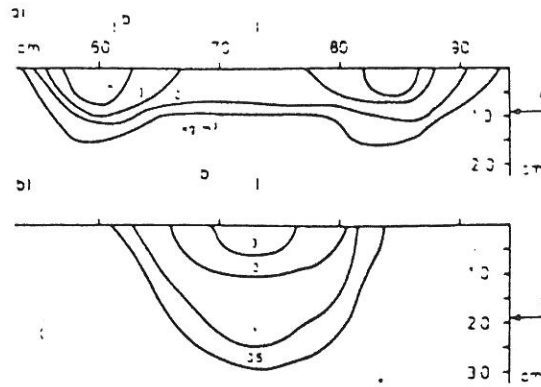

Fig. 4. Isodensity contours.
a) $U_o = 10$ cm/s rough bed
b) $U_o = 20$ cm/s rough bed

Plume dispersion

We assume that the total effect of buoyant and turbulent dispersion is linear in these two processes. Assuming changes in mean density difference to be linear, combining (2) and (5a) for the width and (3), (5b) for the height, yields

$$\frac{db}{dx} = \alpha \frac{V}{U_o} + \frac{K_y}{U_o b} \quad (10) \qquad \frac{dh}{dx} = -\alpha \frac{h}{b} \frac{V}{U_o} + \frac{K_z}{U_o h} \quad (11) \qquad h\, b\, \overline{\Delta \rho} = const. \quad (12)$$

where α has been introduced as an empirical factor.

It is not the aim here to resolve the complex mechanisms that results in the reduction of the vertical diffusion in the presences of vertical density gradients, so a simple form which relates the dispersion coefficient to a Richardson number is used. The reduction of the vertical dispersion coefficient is given as

$$\frac{K_z}{K_{z_o}} = \frac{1}{1 + \beta R_{io}} \qquad \text{where} \qquad R_{io} = \frac{g\, h\, \Delta \rho}{\rho u_f^2} \qquad (13)$$

is a plume Richardson number, β is another empirical constant and K_{z_o} is the dispersion coefficient, (9).

Calibration

The two empirical constants are estimated from the experimental results, employing an inverse technique. Using the measured values from one plume-section as initial conditions, the model is advanced forward and the outcome compared with measured values in the downstream section. Repeating this for all experiments one obtains a squared error sum as

$$E = \sum_{i=1}^{n} \overline{\Delta \rho} \left[\left(\frac{h_\bullet - h}{h} \right)^2 + \left(\frac{b_\bullet - b}{b} \right)^2 \right] \qquad (14)$$

where index $\bullet$ refers to calculated values. This particular form expresses the deviation in dilution between experiment and model. The value of the two constants which minimizes this error is estimated to $\alpha = 1.2$ and $\beta = 3.5$.
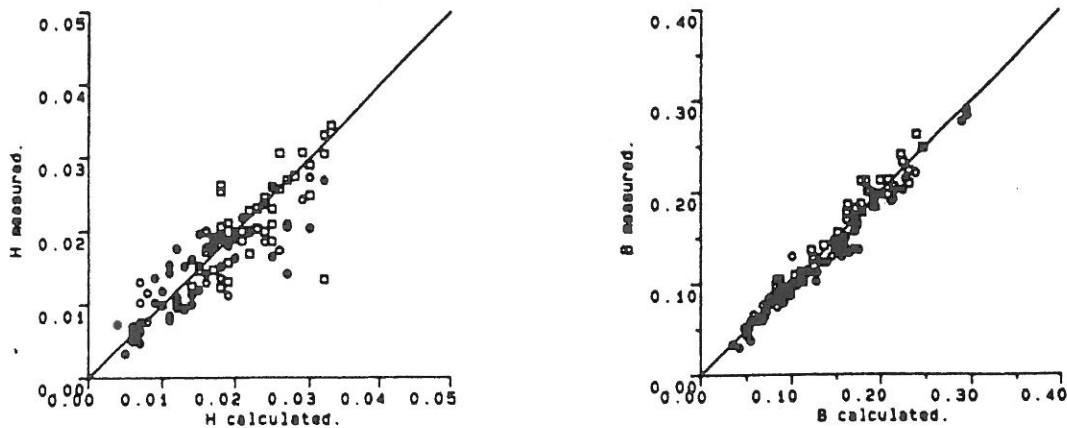
Fig. 5. Comparison between measured and predicted parameters
□ salt ○ fresh smooth bed ■● rough bed

Fig. 5 compares predicted and measured values of h and b, the full line being perfect agreement. Although there are some scatter in predictions of the height, there is good agreement between measured and estimated data.

6. Discussion and conclusions

Although direct comparison with other measurements are difficult due to differences in the choice of scales, there are at least a qualitative agreement on the value of α, Weill and Fischer, 1978. Benjamin, 1968. The reduction of turbulence in prescence of density gradients are well-established, Turner 1973, although quantitative predictions are still relatively uncertain.

The conclusions that emerges from this laboratory study are that the dispersion of a buoyant surface plume in channel shear flow can be treated as the superposition of a buoyancy induced stretching and reduced turbulent diffusion. The use of scales derived from moments of the density profile is valuable as a single set of measurable scales applies to both buoyant and turbulent dispersion.

Literature

Larsen, I. and Sørensen, T. 1968. *Buoyancy spread of waste water in coastal regions.* Proceedings of the Coastal Engineering Conf., p 1367, London.

Taylor, G.I. 1920. *Diffusion by continuous movements.* Proc. Roy. Soc. A, pp 196-212. London.

Weill, J. and Fischer, H.B. 1974. *Effect of stream Turbulence on Heated Water Plumes.* J. Hyd. Div. ASCE vol. 100 No. HY7, pp 951-970.

Schiller, E.J. and Sayre, W.W. 1975. *Vertical Temperature Profiles in Open Channel Flow.* J. Hyd. Div. ASCE vol 101, No. HY6, pp 749-761.

Fischer, H.B. et al. 1979. *Mixing in Inland and Coastal Waters.* Academic Press.

Benjamin, T.B. 1968. *Gravity currents and related phenomenae.* J. Fluid. Mech. vol 31, part 2, pp 209-248.

Petersen, Ole and Larsen, Torben 1988. *Infrared Sensing of Buoyant Surface Plumes.* Proc. of. Third Int. Symp. on Refined Flow Modelling and Turbulence Measurements, pp 701-707. IAHR. Tokyo.

Engelund, F.A. 1969. *Dispersion of floating particles in uniform channel flow.* J. Hyd. Div. ASCE vol. 95 No. HY4.

Turner, J.S. 1973. *Buoyancy effects in fluids* Cambridge University Press.