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Abstract. In engineering, the design of a product relies heavily on a design specification; a co-creation of customer and engineer which captures the requirements. Subjectivity is intrinsic to this process. Whilst engineers typically have a high appreciation of the technical aspects of design, the detailed knowledge of environmental and socioeconomic (ESE) implications are often held elsewhere. As such, efficient and effective design is critically dependent on the processes underpinning knowledge transfer. However, the information interfaces between engineering and the requirements of our swiftly changing civilisation remain indirect and suboptimal, and the unintended consequences of design choices are becoming increasingly serious.

Transdisciplinary engineering bridges knowledge boundaries interfacing with engineering (e.g. social science). This paper explores whether topology (a branch of pure mathematics) presents an opportunity to analyse the complex interdependency of transdisciplinary engineering information. Topology and geometry describe the structure of objects such as connectedness or the number of holes and have recently provided a suite of powerful and robust tools for analysing high-dimensional data sets. However, the real-world implementation of the term topology is still evolving. Interviews with engineering organisations, revealed that topology is almost exclusively interpreted as 'Topology Optimisation' in the context of advanced design and manufacturing. To date, mathematical processes for critically and systematically examining the topology of systems have not been transferred through to the engineering industry. This paper compares how topology is interpreted by the engineering industry, compared to academic literature, and reflects on the opportunities of applying the mathematical theory of topological analysis to transdisciplinary engineering data.

Keywords. Design, Socioeconomic, Topological data analysis, Topology, Transdisciplinary engineering.

Introduction

The UK is committed to sustainable energy generation and material use through initiatives such as the Circular Economy Package policy statement [1] and the Net Zero

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Strategy [2]. In 2024, the means by which these overarching environmental goals will be met remains unclear. Whilst industrial strategies exist, these are regularly changed to align with the current social and economic priorities. For example, the strategy referenced in the Circular Economy Package [1, 3], was superseded by a Plan for Growth in 2021 [4]. In this proposal, whilst there is a focus on skills and innovation [4] in the context of economic, social and environmental (here abbreviated as ESE, and in other disciplines known as environmental, societal and governance, ESG) implications, there is little definition of the means by which the technical ambitions will be met. The specifics of how engineering – the fundamental discipline underpinning the creation of products, infrastructure and processes – will achieve these technical requirements is omitted.

Concurrent design has been the paradigm of optimum-efficiency in engineering for the last 30 years [5]. As opposed to the waterfall approach, where a project flows between tasks, concurrent engineering allows relevant design requirements to be considered at the same time [6]. It encompasses aspects including (but not limited to) supply chain, technical requirements, and manufacturing, through to recycling and disposal (see Figure 1). However, the design process relies heavily on a product design specification (PDS) – a co-creation of customer and engineer – to capture product requirements. Subjectivity is inherent in this process, and the specification is limited by the perspective of engineer and customer, and therefore the implementation of concurrent design has often fallen short of concept.

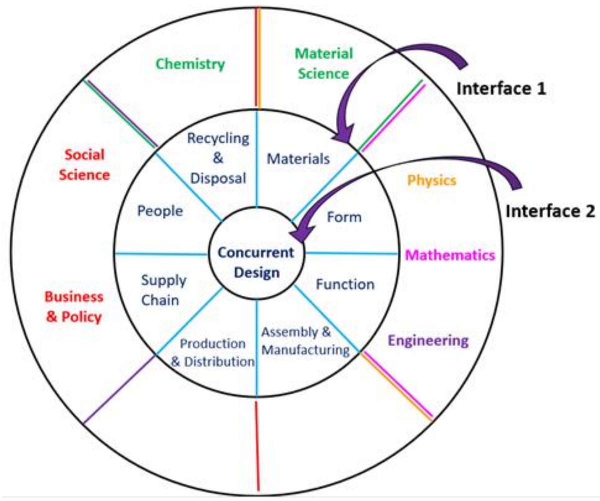


Figure 1. Facets of concurrent design and interface with academic disciplines

Design engineers typically have a high appreciation of technical aspects of design and an overview of the remaining considerations, for example commercial aspects [7]. Detailed knowledge of non-technical aspects is often held elsewhere, making effective concurrent design critically dependent on communication for knowledge transfer. Bryson and Rusten (2011) distinguish between engineering design which is "concerned with technical operations and the structural integrity" and industrial design that "focusses on enhancing the relationship between the thing and the consumer" [7]. The purpose of this paper is to highlight the need for a broader definition of engineering design that includes a concern and appreciation of ESE and explore a method by which this could be achieved. The aim of this research is to investigate how topological analysis is understood in the disciplines of engineering and explore whether the translation of a more accurate mathematical definition into design engineering could offer a vehicle to transdisciplinary considerations. To satisfy this aim, this paper has several objectives:

- To present design engineering as a process which is widely adopted in engineering education and industry, and to juxtapose this against socioeconomic examples of the consequences of design decisions.
- To highlight the need for a broader definition of engineering design that includes an appreciation of ESE and how this translates to design requirements or constraints.
- To demonstrate differences in the understanding of ‘topology’ between industrial engineering and mathematics.
- To highlight that concepts from graph theory and topology offer powerful tools to model and analyse the complex systems inherent in design engineering.

1. Background

1.1. Transdisciplinary Information in Design Engineering

Any one decision made during the engineering design process can have multi-faceted, interdependent and prolonged onward consequences. In the design of a part, engineers should have a reasonable appreciation of how the choice of materials, geometry and tolerances impacts manufacturability. However, it cannot be assumed that all engineers have a comprehensive appreciation of the design constraints and opportunities associated with every manufacturing technique. For example, with reference to additive manufacturing (AM) which has seen rapid development in recent years, design is referred to as a “black-art” where an informed perspective requires “in situ training, hands-on experience and a trial-and-error approach encompassing the entire design and manufacturing framework” [8].

Taking an arbitrary product, designed using CAD, with a specified mechanical function and preferred material(s), the final geometry (the form) will be fixed during the detailed design phase using knowledge of manufacturability and assembly, and mechanical validation. Often, there is some iteration between the detailed design and manufacturing stage, which may involve prototyping. The geometry may also be influenced by additional requirements (specified in the PDS), such as human factors, quality control, cost and weight (for a comprehensive overview of a PDS see [9]). However, the creation of the PDS is inherently subjective; similarly, the implementation of design requirements needs comprehensive knowledge of the requirements. For example, whether human factors, encompassing accessibility and gender dynamics, may hinge on the knowledge within the design team itself.

Whilst comprehensive and holistic knowledge within an engineering team, and a concurrent design process, is the ideal situation; there are boundaries to knowledge. Knowledge is acquired through education, personal development and experience. New knowledge can be gained, but this will impact the time and cost associated with the design process. In addition, knowledge is constantly changing, the nuances of which are generally best understood by specialists. In Figure 1, this is demonstrated by interfacing the facets of concurrent design with academic disciplines. Yet, it is not simply ineffective knowledge transfer that leads to a deficit in the implementation of concurrent design. Engineering is recognised for its challenges with psychological inertia [10]; in essence, engineers tend to operate within the framework of their conditioning, and approach problem-solving in a manner that aligns with their past successful experiences.

There are computational tools, which can inform the engineer of the environmental impacts of their design decisions. For example, Ansys Granta MI Enterprise (Canonsburg, PA, USA) [11] can provide materials and associated environmental footprint assessments. Comprehensive, up-to-date, materials information, integrated into CAD and/or Product Lifecycle Management, can enable informed design choices in the context of engineering properties, cost, legislation and environmental impacts [12]. However, it is important to note that such software only offers a snapshot of the implications linked to material choices. Some remain neglected, for example, the indoor air pollution impacts of 3D printing [13].

1.2. Socioeconomic and Environmental Implications of Design Engineering

Whilst human factor considerations may fall within the knowledge base of a design engineer, socioeconomic considerations often do not. Through two examples (one economic and one social), we illustrate some of the ramifications that limited awareness regarding ESE consequences during the product design process can have.

During the COVID-19 pandemic, the exchange of raw materials and products was disrupted, producing massive shortages created by the immobilisation of global supply chains. Most prominently, the shortage of medical equipment emerged as a major constraint. Whilst there were some noteworthy successes, for example the rapid mobilisation of local initiatives to produce protective personal equipment (PPE). Such initiatives were largely enabled by AM, also known as 3D printing (3DP), which using Makerspaces and similar facilities, facilitated the rapid and localised manufacture of parts [14]. For more complex equipment, such as ventilators, with components that cannot be produced using AM, the nature of the global value chains (GVC), directly led to shortages with significant consequences for public health [15]. Whilst GVCs can minimise costs and maximise economic value, they also create risks including environmental impacts, and leave supply chains vulnerable to global scale risks including pandemics, climate change or events that disrupt shipping routes. For instance, the obstruction in the Suez Canal in March 2021 and the attacks on vessels in the Red Sea since October 2023, serve as notable examples.

Social impacts also result from engineering design decisions, for example, dangerous levels of indoor overheating due to the use of inappropriate building materials that absorb rather than reflect heat [16], leading to adverse mental and physical health conditions, particularly among vulnerable populations. As heat waves become more frequent and intense, indoor overheating is a growing issue in the UK, and the drive towards net zero increases the use of said building materials. Despite evidence spanning several decades on the role of certain forms of building insulation increasing overheating, guidance on design with consideration for overheating risks was only added in 2021 to Building Regulations [17], and only covers new build properties. As such, there is a significant risk that design choices made as part of net zero policies in the present day could provoke environmentally damaging maladaptation in the future via uptake of air conditioning. The chemical composition of building insulation, in combination with decreasing ventilation rates, has also been scrutinised in terms of indoor air quality.

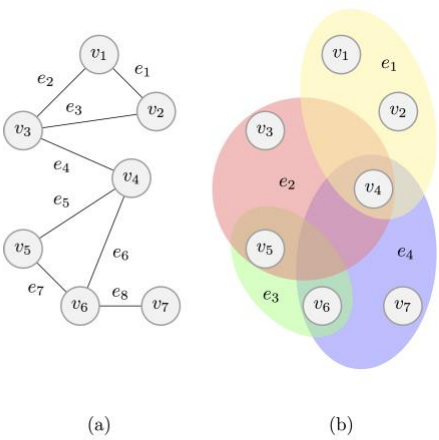
1.3. Research Hypothesis

This raises the question – where has the engineering design process failed to anticipate future ESE implications; could the situation have been avoided if there was more information available during the design process? If such data were available during the design process, how could it be accessed in such a way that was efficient to an engineer? The hypothesis of this research is that through promoting the use of concepts from the mathematical disciplines of graph theory and topology to represent and analyse relevant high-dimensional data sets, relationships between production design decisions and ESE implications may be revealed.

2. Mathematical Theory

2.1. Graphical Representation of Relationships between Objects

The relationship between objects, or elements within a system, can be represented by a graph. A graph consists of nodes and edges, where the nodes are the components of the system (e.g. design decisions and ESE implications), and the edges encode pairwise (binary) interactions between the nodes. Such representations of systems have been used widely to understand various types of complex systems such as gene expression, voting behaviour and characteristics of basketball players [18]. Real-world systems, such as socio-economic ones, often involve the interaction of groups of nodes (polyadic interaction) [19]. A hypergraph can be used to represent such polyadic interactions. In Figure 2, a graph representation of binary relationships and a hypergraph representation of polyadic interactions together with their respective incidence matrices are shown [20].



	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
v_1	1	1	0	0	0	0	0	0
v_2	1	0	1	0	0	0	0	0
v_3	0	1	1	1	0	0	0	0
v_4	0	0	0	1	1	1	0	0
v_5	0	0	0	0	1	0	1	0
v_6	0	0	0	0	0	1	1	1
v_7	0	0	0	0	0	0	0	1

(c)

	e_1	e_2	e_3	e_4
v_1	1	0	0	0
v_2	1	0	0	0
v_3	0	1	0	0
v_4	1	1	0	1
v_5	0	1	1	0
v_6	0	0	1	1
v_7	0	0	0	1

(d)

Figure 2. Example of an undirected graph (a) with its node-arc incidence matrix (c), together with an example of a hypergraph (b) with its incidence matrix (d).

2.2. Topology: Its Use to Describe Objects and to Analyse Data Sets

2.2.1. Topological Spaces

Loosely speaking, in mathematical terms, a topological space is a space equipped with a structure, which is called a topology. This structure allows one to define topological characteristics of sets in the space, such as topological dimension, compactness and connectedness. These characteristics are examples of invariants of topological spaces, meaning that they remain unchanged under deformations (homeomorphisms). Another important invariant comes from algebraic topology, namely the concept of homotopy groups. The first and simplest homotopy group is the fundamental group, which records information about loops in a space. Intuitively, homotopy groups record information about the basic shape of an object, for instance its genus (intuitively its number of holes). For an introduction to topological spaces, we refer the reader to [21, 22].

A prominent example of a topological space is the three-dimensional space $\mathbb{R}^3$ equipped with the standard Euclidean structure, which relates to our common understanding of distance between points in space. An additive manufactured object can be viewed as a set in this topological space, and can be described by the above-mentioned topological characteristics. However, topological spaces are much more general. Indeed, topology deals with qualitative geometric information; it is relatively insensitive to the metric (measurement of distance), and as such is useful for situations where the understanding of the metric is coarse [23]. Within mathematics the concept of topology is an abstract notion, in which closeness is defined but cannot necessarily be measured numerically.

2.2.2. Topological Data Analysis

Topological data analysis (TDA) equips a data set with a metric, asks questions about the resulting topological space to build new insights into the original data set, and helps predict patterns and trends. The first fundamental step in the analysis is to unearth an appropriate metric on a given data set relevant to the underlying production design questions. Once a suitable metric is identified, different methods can be used to analyse the space, amongst them are the following.

1. Use the data set together with its metric to form a graph, namely, to connect points in the data set with an edge if their distance lies beneath a given threshold, and to analyse the homotropy of the resulting graph. This typically reveals how porous the given structure is, that is how correlated the data is. One can also use the metric to identify clustering and to obtain a representation as a hypergraph, which can be analysed analogously or combinatorically.
2. Use concepts from fractal geometry [24] such as box-counting dimension and coarse multi-fractal analysis, to measure the roughness of the data set and to identify hot and cold spots.
3. Use the data set together with its metric to build a subset of n-dimensional Euclidean space (i.e. a manifold) and to analyse the local and global curvature, search for peaks and troughs (mountains and valleys) and identify lower-dimensional sub-spaces/manifolds to understand how the data is clustering.

Identifying an appropriate metric and analysing the associated topological and geometric properties goes hand in hand: if the metric is too simple, it may not see the rich structures exhibited by the data set; and if it is too complex, the analysis might become too challenging to be able to identify important attributes. Therefore, a fine balance between discovery and analysis is required. An introduction to TDA is, for instance, given in [25].

3. Challenges

3.1. The Understanding of Topology in Engineering

To integrate a topological approach into engineering, it is important to understand to what extent the theory of topology has already been transferred into engineering. Through an academic lens; engineering as a discipline, and topology as a sub-discipline of mathematics, are active areas of research. The take-away message is that whilst there are examples of topological analysis in engineering (academia and industry), there is a disproportionate number of studies which focus purely on Topology Optimisation (TO).

3.2. The Understanding of Topology in Industry

3.2.1. Method

An interview approach was adopted to assess the technical and commercial awareness of topology in the engineering industry; to identify the current knowledge and understanding, and highlight novel application, as well as barriers to its progression. The majority of the results gained through these interviews are beyond the scope of the current research, and so included in an abridged format in this manuscript.

Semi-structured interviews were conducted with nine organisations. The unit of analysis was an industrial engineering organisation; for example, automotive, biomedical, design or manufacturing, represented by a suitably technically informed employee. The geographical homogeneity was not constrained to the UK. There were no additional inclusion or exclusion criteria. Participants gave informed consent to take part in the study; the data included in this manuscript is provided in an anoymised form. Participants were known to the investigators via: previous research collaboration/involvement or professional institute acquaintance. This study was conducted as part of a taught post-graduate degree program; ethical approval was granted by the Director of the Programme (Centre for Doctoral Training, Topological Design, School of Physics, University of Birmingham, UK).

3.2.2. Results

Eight face-to-face interviews were conducted, and one was conducted through a written response to a questionnaire. The commercial focus of the nine organisations was as follows: three engineering software, two automotive, two pharmacology/biotechnology, one manufacturing, and one research and development.

Table 1. Summary of participant interview responses

Industrial Organisation	Aware of the Definition of Topology	Application-based context of Topology
1	No	Topology optimisation (commercial software)
2	No	Topology optimisation (commercial software)
3	No	Topology optimisation (commercial software)
4	Yes	Topology optimisation (commercial software, in-house software), TDA
5	No	Topology optimisation (commercial software)
6	Yes	Topology optimisation (commercial software, in-house software), pure mathematics, solid modelling
7	No	Topology optimisation (commercial software)
8	Yes	Protein-protein interactions, TDA
9	Yes	Topological superconductors

Four participants of the nine organisations could define topology as outlined in Section 2 or had an awareness that was sufficiently close to this definition. Two organisations were familiar with and used topological data analysis. Examples of interview responses and outcomes are given in Tables 1 and 2.

Table 2. Participants' definitions of topology and/or geometry.

Industrial Organisation	Example Response/Outcome
1	"topology is synonymous with shape."
2	"topology is the geometry of the part, the functional geometry"
3	Topology was explained exclusively from the perspective of topology optimisation
4	"a geometric set of features" that is inclusive of material.
5	Understands topology exclusively from the perspective of topology optimisation
6	Defined topology well
7	Make no distinction between topology and shape/geometry
8	Defines topology, highlighting connectivity
9	Defines topology as being concerned with surfaces on a micro and macro scale

4. Discussion

4.1. New Perspectives

Whilst inter- or multi-disciplinary research, aimed at designing a system across disciplines, is common in engineering, the ambitious endeavor of creating a process to facilitate design across multiple disciplines requires a more innovative approach. Transdisciplinary research is an emerging concept, which has the characteristics of transcendence and integration, stakeholder involvement and problem solving to unify and produce societally useful knowledge [26]. To date, this approach is rooted in the amalgamation of various academic and stakeholder knowledge bases, therefore it is underpinned by communication and being receptive to new ideas. Furthermore, experience-based qualitative information can be difficult to comprehensively integrate back into the design process. It is a cross-disciplinary communication challenge, requiring bottom-up knowledge propagation which can result in fragmented and uncomprehensive application. The authors propose an alternative approach, to enable transdisciplinary transfer and amalgamation of knowledge, through the mathematical mapping of complex networks (graphs and hypergraphs) and analysis using TDA. It is important to note that putting an emphasis on calculability as opposed to mixed methods, risks over-rationalising irrational systems which can lead to inefficiency, unpredictability, incalculability and loss of control [27].

4.2. Study Reflections

The real-world, non-fiscal, impacts of design choices are creating increasingly more serious problems in our society (e.g. climate crisis), and the importance of environmental and socioeconomic (ESE) factors – known as the triple bottom line – is now widely acknowledged. This study is based on the assumption that knowledge of the ESE implications of design decisions is typically incomplete. Yet, if awareness exists, then there can be a tension between the fiscal value of the solution and the ESE implications, and whether they represent a measurable value. However, a new and rapidly-growing consumer segment is emerging that is highly ESE aware. To meet this, product design needs to become more aligned to the interests of this consumer group and this includes designs that resonate with ESE values.

This study highlights that the academic discipline of topology applied to design engineering offers powerful tools which are currently underutilised. It also indicates that to-date the interpretation of topology in the engineering industry is varied, and in some instances incorrect; where the concept of topology is related more to structural properties and materials used.

The limitations of this study are related to validity and reliability of the interview methodology, by which the quality of a case study research can be judged [28]. This study outlines a small proportion of the outcomes from this interview study, and therefore the quality will be assessed in line with future full assessments. It is important not to generalise the results and conclusions beyond the parameters outlined herein.

5. Conclusions

Through a generalist summary of production design considerations, and more specific examples of socioeconomic consequences of part, material or manufacturing choices, this study outlines that the unforeseen implications of design can have serious ramifications. It is proposed that topology offers an alternative method, to map complex qualitative ESE systems. This study has highlighted the limited awareness of this approach within the engineering industry, and offers a first step towards increasing this awareness, aiming towards the future acceleration of the commercialisation of topological design.

Acknowledgements

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