

## Research papers

# Uncertainty-observed virtual battery model for an electric vehicle parking lot enabling charger-sharing modelling

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## ABSTRACT

With the increase in the penetration of electric vehicles (EVs), there is a substantial need for a proper solution to meet the EVs' charging demand. Due to the high investment cost of charging stations, the efficient operation of EV chargers is crucial. In this regard, in this paper, charger-sharing charging has been proposed to charge multiple EVs with a single EV charger. However, the existing models cannot model uncertain EV parking lots (EVPLs) with charger-sharing charging. In addition, most presented methods for uncertainty modelling of EVPLs are hard to implement in planning and large-scale system-level studies due to their complicated process and high computational burden. Therefore, in this paper, a virtual battery model has been proposed to model an EVPL enabling the charger-sharing charging modelling considering the uncertainty of arrival and departure. Our proposed approach models the EVPL as a battery with time-variant parameters obtained from EVs' arrival and departure patterns. The proposed virtual battery model has been validated by comparing its performance on day-ahead (DA) and real-time (RT) power market participation of a 24-bus distribution system owning 12 EVPLs with the scenario-based method. The results show that its performance is similar to scenario-based uncertainty modelling while its computational burden is around 2.24% of the scenario-based model. In addition, the results indicate how by employing our proposed charging-sharing charging, EVPLs can dramatically increase their profit as a result of increasing the number of hosted EVs. In this context, a sufficiently high charging tariff motivates the EVPL owner to accommodate a substantial number of EVs. With only 200 EV chargers, the EVPL can host approximately 3200 EVs, given the characteristics of EVs and EV chargers outlined in the case study section. In contrast, the exclusive charger approach allows only 200 EVs to enter the parking facility and undergo charging.

## 1. Introduction

### 1.1. Motivation

The increasing penetration of EVs in recent years makes it a principal factor in the operational and planning decisions on energy and energy-related sectors [1]. In this regard, studying the optimal operation and planning of EV charging facilities and infrastructures such as EV Parking Lots (EVPLs) and charging stations is of undeniable importance [2]. This way, the performance of the EV charging stations could be studied from different points of view; such as EV owners, parking lot or charging station owners, power system operators, potential investors, and policymakers [3]. Studies that try to model EVs from the point of view of EV owner deal with a single EV charging modelling as EV owners own single EV [4]. However, for studying EV-related problems from other agents' points of view, it is crucial to have a proper model for the aggregation of EVs such as EVPLs. In this regard,

there are several papers that tried to provide a model that simulates the operation of EV aggregation. In [5], a storage model is presented to model the charging of an EVPL. In this study, the historical data of an existing parking lot is utilized to obtain the parameters of the equivalent storage model.

### 1.2. Research literature

In modelling electric vehicles, there are uncertain parameters such as arrival and departure time. However, some papers in the field neglected uncertainty, mainly for the sake of simplicity, to allow for the deployment of straightforward and low-computational burden approaches. For instance, Authors in [6] proposed an approach for optimal scheduling of an EV aggregator for determining charge and discharge strategy. The presented model for the EV aggregator is similar to [5] without considering uncertainty. Ref. [7] proposes an approach

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**Nomenclature****Indices**

|             |   |
|-------------|---|
| $cl$        | EV class  |
| $t$         | Time  |
| $w_{EV}$    | Scenario for arrival and departure of EVs in the scenario-based approach. |
| $w_{RC EV}$ | Scenario for arrival and departure of EVs in realization.                 |
| $w_{RC rt}$ | Scenario for RT electricity price in realization.                         |
| $w_{rt}$    | Scenario for RT electricity price in the scenario-based approach.         |

**Parameters**

|                           |   |
|---------------------------|---|
| $\Delta t$                | Duration of time period.                                    |
| $\eta_{ch}$               | Charging efficiency.  |
| $\eta_{dc}$               | Discharging efficiency.                                     |
| $\lambda_t^{DA}$          | DA electricity price at time t [€/kW].                      |
| $\lambda_{w,t}^{pur,rt}$  | RT purchasing price at scenario w time t [€/kW].            |
| $\lambda_{w,t}^{sell,rt}$ | RT selling price at scenario w time t [€/kW].               |
| $C_{ins}^{PL,ch}$         | Total charging capacity of the installed EV chargers [kWh]. |
| $Cap_{cl}^{Ev}$           | Battery capacity of the EV class cl [kWh].                  |
| $Sh_{cl}$                 | Share of EVs class cl from all available EVs.               |
| $SOC_{cl}^{arr}$          | Arriving SOC of the EVs class cl.                           |
| $SOC_{cl}^{dep}$          | Departing SOC of the EVs class cl.                          |

**Variables**

|                        |  |
|------------------------|--|
| $E_t^{arr}$            | Added energy to EVPL due to EVs arrival at time t [kWh].         |
| $E_t^{dep}$            | Depleted energy from EVPL due to EVs departure at time t [kWh].  |
| $E_t^{PL}$             | EVPL stored energy in time t [kWh].                              |
| $E_{w,t}^{PL}$         | EVPL stored energy in real time for scenario w [kWh].            |
| $N^{EV,ent}$           | Number of total arrived EVs in a day.                            |
| $N_t^{Ev,arr}$         | Number of arrived EVs at time t.                                 |
| $N_t^{Ev,dep}$         | Number of departed EVs at time t.                                |
| $P_{cl}^{ch,max}$      | Maximum charging power of the EVs class cl [kW].                 |
| $P_{cl}^{ch,min}$      | Minimum charging power of the EVs class cl [kW].                 |
| $P_{cl}^{dc,max}$      | Maximum discharging power of the EVs class cl [kW].              |
| $P_{cl}^{dc,min}$      | Minimum discharging power of the EVs class cl [kW].              |
| $P_i/Q_i/V_i/\theta_i$ | Active/reactive power injection/voltage amplitude/angle of bus i |
| $P_t^{DA,sc}$          | EVPL scheduled power in DA stage at time t [kW].                 |

|                     |  |
|---------------------|--|
| $P_t^{PL,ch}$       | Charging power of EVPL at time t [kW].                     |
| $P_t^{PL,dc}$       | Discharging power of EVPL at time t [kW].                  |
| $P_{w,t}^{sell,rt}$ | Purchasing power of EVPL in real time for scenario w [kW]. |
| $P_{w,t}^{sell,rt}$ | Selling power of EVPL in real time for scenario w [kW].    |

of the arrival and departure time of EVs has been neglected because of the deterministic formulation of the problem. However, neglecting uncertainty will result in unreliable analysis and inaccurate outputs.

A part of the literature attempted to consider EV uncertainties in EV-related studies. Some studies addressed the uncertainty by generating a value for uncertain parameters of each EV that is a part of the EV aggregation based on the Probability Distribution Function (PDF) of the uncertain parameters. In [9], an approach is proposed to assist the distribution system operator with the optimal operation of the system in the presence of an EVPL and demand response programme. In this paper, the EV modelling is based on a single EV. However, EV uncertainties have been taken into account by generating random values for the uncertain parameter of each EV based on truncated normal distribution. The other approach deployed in a part of literature to consider EV uncertainty is to generate a single scenario by random value generation based on parameters' pdf or random value selection among historical data. Ref. [10] studies EV aggregation's role in increasing the sustainability of the power system. It is given that number of EVs arriving at the charging station is fixed. This way, random values are generated for each EV's arrival and departure based on the PDF resulting from historical data. In [11], the impact of a PV-equipped EVPL operation on the distribution network is investigated. This paper modelled EVPLs based on the expected loading profile generated by randomly selected EVs' parking duration.

Some other papers in the literature generate multiple scenarios for uncertain EV parameters and proceed with scenario-based approaches to utilize the generated scenarios. Ref. [12], similar to [5], proposed a storage-equivalent model for EV parking addressing EV uncertainties via a scenario-based stochastic formulation. Authors in [13] presented transactive energy management for EVPLs equipped. Multiple scenarios are generated for the arrival and departure times based on the normal distribution, and energy management is performed according to the generated scenarios. The uncertainty of EVs in [14] that intends to integrate EVPLs in the distribution system, is handled via introducing 24 groups of EVs classified based on arrival and departure time. The arrival and departure times are generated from the truncated normal distribution function. In [15], authors proposed an approach for the optimal bidding strategy of EV aggregators. EV aggregator charging is modelled based on its expected strategy in the demand response programme. To consider EV uncertainties, the problem is studied in a few scenarios generated based on the type of EV. Similarly, in [16], energy management of an EVPL is conducted under a load reduction-based demand response programme. To take into account the uncertainty, eight scenarios are generated for the reference power profile of the parking lot based on the driving cycles of EVs. Authors in [17], for conducting optimal planning of EVPLs generate several scenarios for EV uncertain parameters using Monte Carlo simulation. For decreasing the simulation time Kantorovich distance technique is used to reduce the number of scenarios. Ref. [18] proposed a stochastic approach for optimal charge and discharge scheduling of a parking lot. This way, similar to [13], different scenarios are generated based on the truncated normal distribution. However, the total number of arrived EVs is given a fixed parameter. Ref. [19] utilized a scenario-based approach for formulating the uncertain smart EV charging to evaluate the profitability of compressed air energy storage. Authors in Ref. [20] investigated the flexibility potential of EV parking lots in both V2G and

for optimal participation of an EVPL-owned microgrid in the energy and reserve market without considering EV uncertainties. EVPL modelling in this work is based on a single EV model. Ref. [8] proposed an approach for load scheduling of EVs to reduce their charging cost by optimally providing ancillary service for the grid. The uncertainty

G2V modes as a component of microgrids. In this regard, a stochastic programming methodology is used to address the EV uncertainty via multiple scenarios.

In addition to the mentioned approaches, parameter prediction based on historical data is the other method for considering EV uncertainties. In this regard, [21] that tries to deploy the potential of EVPLs for increasing the distribution system reliability, models an existing EVPL as available storage capacity. This way, sequential Monte Carlo simulations are utilized to determine the equivalent model based on the historical data of the parking lot. Ref. [22] presented a data-driven approach to predict EV owners' behaviour in an EVPL. Then, EVPL is modelled as an aggregation of single EVs. Authors in [23] deployed an energy storage model for EV aggregators in order to present a robust optimization approach for their participation in energy markets. EV aggregator modelling in this work is based on forecasted demand for different types of EVs under the aggregator contract. Ref. [24] presented an approach for defining the optimal operational strategy for an EVPL equipped with local photovoltaic generation. To consider the charging pattern and EV availability uncertainty, historical data from a real-case parking lot has been utilized to forecast the charging characteristic. EVs' arrival and departure uncertainty in [25] that study the optimal operation of an EVPL in a microgrid is addressed via deploying a Markov chain model for predicting the EV availability using historical data.

### 1.3. Research gap

EVPL (or EV aggregation) uncertainty modelling based on prediction using historical data (Refs. [5,21,23,24], and [25]) is just applicable to the studies related to existing parking lots. Such approaches cannot be employed for planning studies. Because in such approaches the core of the modelling is forecasting the uncertain parameters using the historical data of the EV charging stations, while for planning studies, historical data is not available for the Parking lots that have not yet been constructed.

In addition, there are challenges in utilizing single or multiple-scenario generation (Refs. [9] to [18]) for uncertainty modelling. Firstly, the limited number of generated scenarios cannot totally reflect the characteristics of the PDF. Moreover, it imposes complexity on the deployed approach due to stochastic aspects that add to the formulation and modelling. Furthermore, it will produce a computational burden when several uncertain parameters exist, and the number of generated scenarios is high. Besides the mentioned disadvantages of the scenario generation approach, specifically, there is a critical difficulty in deploying this approach for considering EV uncertainties in the studies related to EV aggregation charging. In this type of EV uncertainty modelling, for all EVs, based on the selected PDF, random values are generated for arrival and departure times. According to the generated values for each EV, the number of arrived EVs at the parking lot in each hour will be defined. Therefore, in such approaches, the number of arrived/departed EVs (in a day) is a parameter and predefined. This way, the scenarios are generated with regard to the total arrived/departed EVs, and then the generated scenarios are utilized in the deployed stochastic method. Consequently, scenario generation cannot work for studies where the number of EVs is unknown or is a variable of the problem, especially for some sorts of planning problems.

Furthermore, the literature review indicates that almost all papers that studies EVPLs in the power system assumed that EVPL charging in the charging stations or EVPLs is exclusive-charger based. In other words, for every single EV in the parking lot, there is one charger that is connected to an EV all the time that the EV is parked in the parking space. In such studies, it is assumed that all parking spaces in the parking lot are equipped with a separate EV charger. However, knowing that Each EV is parked in the parking for several hours while the charging process takes a few hours, there is no need to equip each parking space with a separate charger. In this regard, our paper

proposes a charger-sharing approach that empowers EVPL operators to deploy one charger for multiple EV spaces. This way, more efficient operational and planning decisions could be made for EVPL owners that assist them in increasing their operational profit (for operation purposes) or decreasing investment costs (for planning purposes). It is noteworthy that the nature of our proposed virtual battery model allows us to study the EVPL with the charger-sharing approach, while the other EVPL models that have been presented yet (due to their limitations) cannot cope with the charger-sharing approach.

### 1.4. Contributions

Although several studies have been conducted to model the charging of aggregated EVs, a straightforward compact model with low computational burden that is able to reflect EVs arrival and departure uncertainties and model charger-sharing charging is missing. The proposed virtual battery model for modelling the EVPL (consisting of unidirectional and bidirectional EV chargers) scheduling enables the study of aggregated EV charging in interaction with other energy system components from the point of view of the charging station owner, power system operator, and all agents that intend to study the system for the operation or planning purpose. Moreover, using the virtual battery model, EV aggregation charging could be studied like a battery. This way, using the CDF of the truncated normal distribution, the number of arriving, departing, and parked EVs in each hour is calculated based on the maximum number of hosted EVs in the day, and the characteristics of the proposed virtual model are determined according to the obtained values. Therefore, the proposed modelling can be utilized for large-scale operational or planning studies that deal with aggregated EV charging when arrival and departure uncertainty is taken into consideration without imposing additional computational burden on the simulation. Furthermore, the efficient performance of the proposed virtual battery model has been validated by comparing its performance on DA and RT markets with the scenario-based method. The comparison between our paper and related papers in the literature in terms of uncertainty model, EV model, planning applicability, accuracy, computational burden, and charger-sharing modelling is presented in Table 1. It is noteworthy that the High, Mid, and Low terms used for describing the accuracy and computational burden are related to the overall EV aggregation and uncertainty model in the papers. The terms are determined in comparison with our approach, and most of the mentioned papers have resealable accuracy and computational burden.

The other contribution of this paper lies in proposing an approach that empowers the EV charging place operator to utilize the charging stations in the most efficient way. This way, the EVPL can host the most possible number of EVs. From the planning point of view, for charging a specified number of EVs, there is no need to equip each parking lot with one EV charger. Instead, multiple EVs can be charged by one EV charger as depicted in Fig. 2. Note that EV chargers in (a) exclusive charger and (b) charger-sharing are the same in terms of type and charging capacity. The idea is raised from the fact that each EV may be parked for several hours while the charging process may last a few hours. Therefore, there is no need to equip each parking space with a charging station. Instead, using the proposed charger-sharing approach, one charging station could be deployed for charging multiple EVs parked in different parking spaces. It is worth mentioning that among the EVs that share one charger, one EV is charged at each time. Therefore, the proposed model could also assist in planning decisions on the optimal number of EV charging stations for a parking lot. It should be noted that our proposed virtual battery model enables us to study and model EVPLs' charger-sharing operation, while the other EV aggregation models cannot model the charger-sharing charging. In this regard, in the charger-sharing charging mode, the maximum number of hosted EVs by EVPLs during the whole day is a decision variable of the problem that should be optimally determined in the optimization problem involving all of the EVPL and EVs constraints. The graphical abstract of our paper is depicted in Fig. 1. To summarize, the contributions of this paper are listed below.



**Table 1**  
EV modelling in the literature.

| Ref       | Uncertainty model     | EV model        | AfP | ac   | CB   | CS |
|-----------|-----------------------|-----------------|-----|------|------|----|
| Ref. [4]  | Parameter prediction  | Single EV-based | ✗   | Mid  | Low  | ✗  |
| Ref. [5]  | Parameter prediction  | Storage model   | ✗   | Mid  | Low  | ✗  |
| Ref. [6]  | ✗                     | Storage model   | ✓   | Low  | Low  | ✗  |
| Ref. [7]  | ✗                     | Single EV-based | ✓   | Low  | Low  | ✗  |
| Ref. [8]  | ✗                     | Single EV-based | ✓   | Low  | Low  | ✗  |
| Ref. [9]  | Random generation     | Single EV-based | ✓   | High | High | ✗  |
| Ref. [10] | Random generation     | Storage model   | ✓   | High | High | ✗  |
| Ref. [11] | Random generation     | Load profile    | ✓   | High | High | ✗  |
| Ref. [12] | Scenario generation   | Storage model   | ✓   | High | High | ✗  |
| Ref. [13] | Scenario generation   | Storage model   | ✓   | High | High | ✗  |
| Ref. [14] | Scenario generation   | Storage model   | ✓   | High | High | ✗  |
| Ref. [15] | Scenario generation   | Storage model   | ✓   | High | High | ✗  |
| Ref. [16] | Scenario generation   | Load profile    | ✓   | High | High | ✗  |
| Ref. [17] | Scenario generation   | Storage model   | ✓   | High | High | ✗  |
| Ref. [18] | Scenario generation   | Storage model   | ✓   | High | High | ✗  |
| Ref. [19] | Scenario generation   | Storage model   | ✓   | High | High | ✗  |
| Ref. [20] | Scenario generation   | Storage model   | ✓   | High | High | ✗  |
| Ref. [21] | Parameter prediction  | Storage model   | ✗   | Mid  | Low  | ✗  |
| Ref. [22] | Parameter prediction  | Single EV       | ✗   | Mid  | Low  | ✗  |
| Ref. [23] | Parameter prediction  | Load profile    | ✗   | Mid  | Low  | ✗  |
| Ref. [24] | Parameter prediction  | Storage model   | ✗   | Mid  | Low  | ✗  |
| Ref. [25] | Parameter prediction  | Storage model   | ✗   | Mid  | Low  | ✗  |
| Our paper | PDF-based calculation | Virtual battery | ✓   | High | Low  | ✓  |

AfP: Applicable for Planning, ac: accuracy, CB: computational burden, CS: Charger-sharing.

- Proposing a novel charger-sharing approach for EVPL charging which defines the maximum number of EVs that an EVPL with a specified number of charging stations can host while committing to charging the EVs based on their desired final SOC upon their departure.
- Developing a straightforward compact virtual battery model for aggregated EV charging considering the uncertainty of departure and arrival enabling charging station owner, system operator, and other upper-level agents to study and model the EV parking station and its charging process (in both charger-sharing and exclusive charger mode) in interaction with other system components with a very low computational burden.
- Designing a two-stage validation framework for evaluating the performance of the proposed virtual battery model in market participation of a 24-bus distribution system with high penetration of EVs

The rest of the paper is organized as follows. In Section 2, the proposed virtual battery model is presented. Section 3 explains the process for validating our proposed virtual model for considering uncertainty. Simulation results for the presented case study are discussed in Section 4. Finally, our conclusions are presented in 5.

## 2. Proposed virtual battery model

In our proposed approach, the EVPL is modelled as a time-variant storage system based on EVs' arrival and departure times as presented in Fig. 3. While a battery is characterized with its constant maximum charging and discharging power ( $P_t^{ch,max}$  and  $P_t^{dc,max}$ ) as well as constant maximum and minimum SOC ( $SOC_t^{max}$  and  $SOC_t^{min}$ ), the proposed virtual battery is characterized with time-variant parameters  $P_t^{ch,max}$ ,  $P_t^{dc,max}$ ,  $SOC_t^{max}$ , and  $SOC_t^{min}$ . For each hour, the mentioned parameters are determined based on the arriving, departing and parked EVs in that hour. In our model, the uncertainty of arrival and departure time is taken into account to define the parameters of the equivalent storage model. After determining the equivalent-storage model, energy management is done for optimal operation and planning of the parking lot. In this paper, we consider that the parking lot will host different classes of EVs with different battery capacities, charging and discharging power as well as initial and final SOCs.

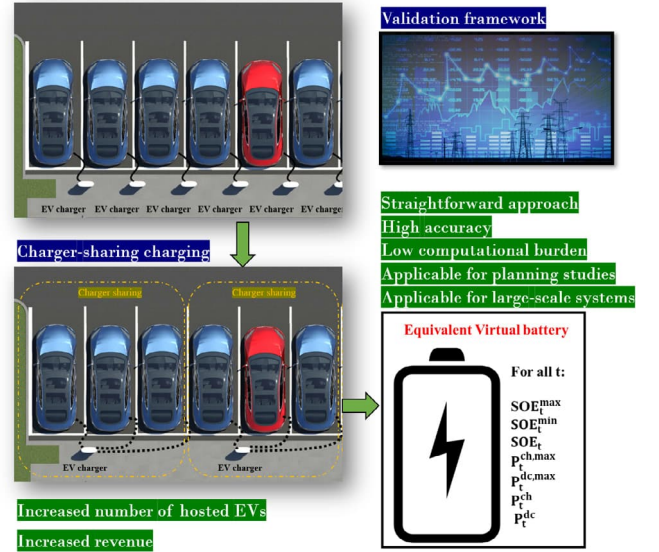
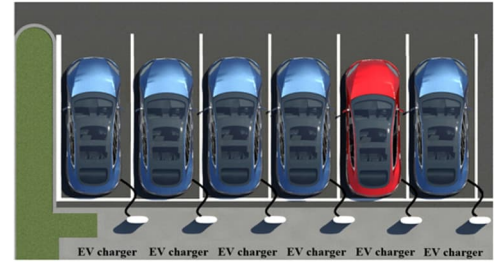


Fig. 1. The graphical abstract of our paper.

### (a) Exclusive charger charging approach



### (b) Charger sharing charging approach

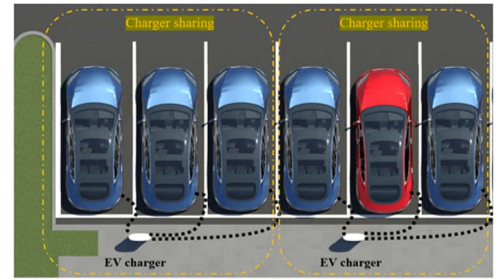


Fig. 2. (a) exclusive charger and (b) charger-sharing charging.

The stored energy in the parking lot in each hour is obtained based on the stored energy in the parking lot in the previous hour as well as stored energy in the arriving and departing EVs in that hour as represented in (1).  $\eta_{ch}$  and  $\eta_{dc}$  stand for the charging and discharging efficiency.  $\Delta t$  is the duration of a single time period.  $\Delta t$  is multiplied with the charging and discharging power to obtain the energy within the time period. Since in this paper, each single time period is equal to one hour,  $\Delta t$  is equal to 1. Therefore, each day consists of 24 time periods.

$$E_t^{PL} = E_{t-1}^{PL} - E_t^{dep} + E_t^{arr} + \Delta t \eta_{ch} P_t^{PL,ch} - (\Delta t / \eta_{dc}) P_t^{PL,dc} \quad (1)$$

To find the containing energy of the arriving and departing EVs, it is required to find the number of arriving and departing EVs in each

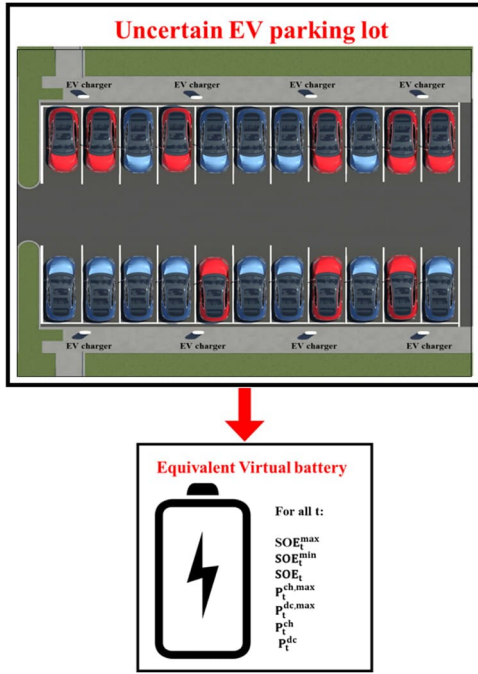


Fig. 3. Virtual battery model for an uncertain EVPL.

hour. According to the literature, the uncertainty of the arriving and departing time of the EVs is modelled with a truncated normal distribution function. It must be noted that the truncated normal distribution function has been widely used in the literature as a reliable PDF for modelling the uncertainty of arrival and departure such as [26,27], and [28]. This way, using the CDF of the truncated normal distribution, the number of arriving and departing EVs in each hour is calculated. If we consider that,  $N^{EV.ent}$  EVs could be parked in the whole day, the number of arriving and departing EVs in each hour is determined as represented in (2) and (3) where  $F_t^{TND}$  is cumulative distribution function (CDF) of truncated normal distribution. Eq. (4) presents how the CDF of truncated normal distribution with the upper bound  $b$  and lower bound  $a$  is calculated based on the CDF of the normal distribution with mean  $\mu$  and standard deviation  $\sigma$ . It should be noticed that in our proposed method, in the process of energy management, the maximum number of EVs that could be parked in the parking lot ( $N^{EV.ent}$ ) itself is determined according to the optimal operation of the system to utilize the charging stations most efficiently. The maximum number of EVs that could be parked should be less than the number of available EVs arriving in the parking as represented in (5).

$$N_t^{EV.arr} = N^{EV.ent} (F_{t+0.5}^{TND,arr} - F_{t-0.5}^{TND,arr}); \forall t \quad (2)$$

$$N_t^{EV.dep} = N^{EV.ent} (F_{t+0.5}^{TND,dep} - F_{t-0.5}^{TND,dep}); \forall t \quad (3)$$

$$F^{TND}(x) = \frac{\Phi(x, \mu, \sigma) - \Phi(a, \mu, \sigma)}{\Phi(b, \mu, \sigma) - \Phi(a, \mu, \sigma)} \quad (4)$$

$$N^{EV.ent} \leq N^{EV.ava} \quad (5)$$

It is clear that the numbers that (2) and (3) generate are not necessarily integer numbers. However, because the purpose of calculating the number of arriving and departing EVs in each hour is to find the containing energy of the arriving and departing EVs as well as the maximum and minimum charging and discharging power of the parking lot, in addition to the fact that the number generation is intended to model the uncertainty, there is no crucial need to convert the generated numbers to integer numbers. Eqs. (6) and (7) represent the containing energy of the arrived EVs equivalent to their initial

SOC upon their arrival and the containing energy of the departed EVs equivalent to their final SOC upon their departure in each hour.  $Cap_{cl}$  refers to the battery capacity of EV class  $cl$ .

$$E_t^{arr} = N_t^{EV.arr} \left( \sum_{cl} Cap_{cl}^{EV} SOC_{cl}^{arr} Sh_{cl} \right); \forall t \quad (6)$$

$$E_t^{dep} = N_t^{EV.dep} \left( \sum_{cl} Cap_{cl}^{EV} SOC_{cl}^{dep} Sh_{cl} \right); \forall t \quad (7)$$

To obtain the EVPL's equivalent storage model, the hourly maximum and minimum bounds for charging and discharging power as well as stored energy in the EVPL should be defined. In this regard, firstly, the charging power of the parking lot for each hour should be less than the total charging capacity of the parking lot that is equal to the total maximum charging power of the charging stations in the parking lot as presented in (8). It is noteworthy that for the charger-sharing charging the number of EV chargers can be less than the number of parked EVs. Moreover, it should be noted that as the line that connects the EVPL to the grid is designed based on the capacity of the whole EV chargers existing in the parking lot, the line capacity is equal to the whole charging capacity of EV chargers. This way, as the charging power of EVPL in the charger-sharing approach is limited to the whole charging capacity of the EV chargers as represented in, the line capacity limitation will not be violated in the charger-sharing approach. In addition, the charging power should be less than the summation of the maximum charging power of the existing EVs in that hour as presented in (9). This way, the number of parked EVs in each hour is obtained according to the number of parked EVs in the previous hour and the number of EVs that arrive and depart the parking in that hour as represented in (12). Similarly, the discharging power of the EVPL is constrained as represented in (10) and (11).

$$0 \leq P_t^{PL,ch} \leq C_{ins}^{PL,ch}; \forall t \quad (8)$$

$$0 \leq P_t^{PL,ch} \leq N_t^{EV.par} \left( \sum_{cl} P_{cl}^{ch,max} Sh_{cl} \right); \forall t \quad (9)$$

$$0 \leq P_t^{PL,dc} \leq C_{ins}^{PL,dc}; \forall t \quad (10)$$

$$0 \leq P_t^{PL,dc} \leq N_t^{EV.par} \left( \sum_{cl} P_{cl}^{dc,max} Sh_{cl} \right); \forall t \quad (11)$$

$$N_t^{EV.par} = N_{t-1}^{EV.par} + N_t^{EV.arr} - N_t^{EV.dep}; \forall t \quad (12)$$

The stored energy in the parking lot in each hour must be less than the total allowed capacity of the parked EVs in that hour which is obtained by the maximum allowed SOC of the EVs. In addition, according to the minimum allowed SOC of the parked EVs in each hour, the lower bound of parking lot stored energy in that hour is calculated. This way, the stored energy in the Parking lot is constrained as represented in (13).

$$E_t^{PL,min} \leq E_t^{PL} \leq E_t^{PL,max}; \forall t \quad (13)$$

$$E_t^{PL,min} = N_t^{EV.par} \left( \sum_{cl} Cap_{cl}^{EV} SOC_{cl}^{min} Sh_{cl} \right); \forall t \quad (14)$$

$$E_t^{PL,max} = N_t^{EV.par} \left( \sum_{cl} Cap_{cl}^{EV} SOC_{cl}^{max} Sh_{cl} \right); \forall t \quad (15)$$

Moreover, the charging and discharging power cannot be nonzero simultaneously as presented in (16). To discard the non-linearity that (16) generates, the big M approach is used according to (17) and (18).

$$P_t^{PL,ch} P_t^{PL,dc} = 0; \forall t \quad (16)$$

$$0 \leq P_t^{PL,ch} \leq u_t M; \forall t \quad (17)$$

$$0 \leq P_t^{PL,dc} \leq (1 - u_t) M; \forall t \quad (18)$$

All in all, the EVPL considering the uncertainty of arrival and departure times is modelled as a virtual battery according to (1)–(18).

### 3. Validation approach

To evaluate our proposed virtual battery model, it is required to investigate its performance in a representative problem and compare it with other existing approaches. In this section, we have selected the optimal operation of a distribution system (DS) with high penetration of EVPLs in the DA market as a representative problem to assess the performance of our proposed approach and compare it with the scenario-based approach for EV uncertainty modelling that is widely used in this field of research. We selected the operational market participation problem for performance evaluation, as it validates the performance of our approach precisely. This way, to show the effectiveness of our virtual battery model in uncertainty modelling, the pricing of the DA and RT market has been selected in such a way that error in uncertainty evaluation imposes a cost to the EVPL. In other words, the selling price in RT is lower in comparison with the DA market, and the purchasing price in RT is higher than in DA. Therefore, if the parking lot could not consider EVs uncertainties and estimate its load profile correctly, it will be forced to trade in the RT market where both purchasing and selling in this market is more costly than in the DA market. This is the reason why we choose this kind of pricing mechanism for the evaluation of the success of our approach in uncertainty modelling. It should be noted that, because other EV uncertainty modelling methodologies cannot model the charger-sharing approach, in this section, we compare our proposed virtual battery modelling with the scenario-based modelling in the exclusive charger charging case. For the sake of simplicity, it is considered that all chargers in the parking lot are unidirectional chargers without the capability of EV discharge. In this case, the total number of the EVs that arrive in the whole day is equal to EV chargers. The market participation problems for each approach and their resulting cost calculation in realization scenarios are described in the below sections. The overall validation process flowchart is depicted in Fig. 4.

#### 3.1. Market participation with proposed virtual battery model

Via the proposed virtual battery model, the market participation of the DS is formulated as below where DS tries to minimize its cost in the DA market. It is noteworthy that our virtual battery model does not involve any stochastic process for RT status of EVs arrival and departure, and the DA stage decisions are made based on the characteristics of the virtual battery model that are extracted considering the PDF of the uncertain parameters.

$$OF = \min \sum_t P_t^{DA} \lambda_t^{DA}$$

s.t. (1)–(18) and DS constraints

#### 3.2. Market participation with scenario-based storage model

In the scenario-based approach for modelling EV uncertainty, based on the generated scenarios for the RT state of EVs from the truncated normal distribution function for all EVPLs, the optimal values regarding the DA market decisions of DS will be obtained via two-stage stochastic programming. This way, the parameters for the storage model of the EVPLs in each scenario are obtained by the EVs' arrival and departure in that scenario. In this regard, the number of arrived and departed EVs in each hour is generated for each scenario and based on that  $E_t^{arr}$ ,  $E_t^{dep}$ ,  $P_t^{PL, ch, max}$ ,  $E_t^{PL, min}$ , and  $E_t^{PL, max}$  are determined. Then, the optimal DA decisions of the DS will be obtained by solving (19), that is the objective function of the DS in the market participation problem, subject to (20)–(24) which are the constraints of the problem. The first term in the objective function indicates the DS cost in the DA stage, and the second term is the weighted cost of DS related to the RT stage considering the probability of each scenario. In this approach, the DA decisions are made according to the possible scenarios that may occur in RT and their expected costs. It should be noted that the RT price is

also an uncertain parameter. The RT price in each hour has a normal PDF with the mean value of the DA price in that hour and the standard deviation equal to  $0.3\lambda_t^{DA}$  as indicated in Fig. 5.

$$OF = \min \sum_t P_t^{DA} \lambda_t^{DA} +$$

$$\sum_{w_{EV}} \sum_{w_{rt}} \pi_{w_{EV}} \pi_{w_{rt}} (P_{w_{EV}, t}^{sell, rt} \lambda_{w_{EV}, t}^{sell, rt} + P_{w_{EV}, t}^{pur, rt} \lambda_{w_{EV}, t}^{pur, rt}) \quad (19)$$

s.t. DS constraints and

$$E_{w_{EV}, t}^{PL} = E_{w_{EV}, t-1}^{PL} - E_{w_{EV}, t}^{dep} + E_{w_{EV}, t}^{arr} + \Delta t \eta_{ch} P_{w_{EV}, t}^{PL, ch}, \forall t, w_{EV} \quad (20)$$

$$P_{w_{EV}, t}^{PL, ch} = P_t^{DA} + P_{w_{EV}, t}^{pur, rt} - P_{w_{EV}, t}^{sell, rt}, \forall t, w_{EV} \quad (21)$$

$$0 \leq P_{w_{EV}, t}^{PL, ch} \leq C_{ins}^{PL, ch}, \forall t, w_{EV} \quad (22)$$

$$0 \leq P_{w_{EV}, t}^{PL, ch} \leq P_{w_{EV}, t}^{PL, ch, max}, \forall t, w_{EV} \quad (23)$$

$$E_{w_{EV}, t}^{PL, min} \leq E_{w_{EV}, t}^{PL} \leq E_{w_{EV}, t}^{PL, max}, \forall t, w_{EV} \quad (24)$$

#### 3.3. Cost calculation in the realization scenarios

In this section, new realization scenarios are generated for EVs arrival and departure. Then, based on the determined DA power trade in the previous section and realization scenarios, the RT power trade are defined for each approach, and the total cost of the DS in the realization RT market is calculated according to the new scenarios generated to stand for the realization. In other words, realization scenarios are generated from the truncated normal PDF to represent the realization stage and  $E_t^{arr}$ ,  $E_t^{dep}$ ,  $P_t^{PL, ch, max}$ ,  $E_t^{PL, min}$ , and  $E_t^{PL, max}$  are determined. Then regarding the DA scheduled power of each approach, the traded power in RT is calculated, and the cost of the DS in the RT stage for realization scenarios resulting from each approach is obtained by solving (25) subject to (26)–(30). Then the total cost of the DS in the DA and RT stages are calculated and compared for the two approaches to give an overview of the performance of our proposed approach. Multiple scenarios are generated for the realization instead of single scenario to provide more reliable outputs.

$$OF = \min \sum_{w_{RCEV}} \sum_{w_{RCrt}} \pi_{w_{RCEV}} \pi_{w_{RCrt}}$$

$$(P_{w_{RCEV}, t}^{sell, rt} \lambda_{w_{RCEV}, t}^{sell, rt} + P_{w_{RCEV}, t}^{pur, rt} \lambda_{w_{RCEV}, t}^{pur, rt}) \quad (25)$$

s.t. to DS constraints and

$$E_{w_{RCEV}, t}^{PL} = E_{w_{RCEV}, t-1}^{PL} - E_{w_{RCEV}, t}^{dep} + E_{w_{RCEV}, t}^{arr} + \Delta t \eta_{ch} P_{w_{RCEV}, t}^{PL, ch}, \forall t, w_{RCEV} \quad (26)$$

$$P_{w_{RCEV}, t}^{PL, ch} = P_t^{DA, sc} + P_{w_{RCEV}, t}^{pur, rt} - P_{w_{RCEV}, t}^{sell, rt}, \forall t, w_{RCEV} \quad (27)$$

$$0 \leq P_{w_{RCEV}, t}^{PL, ch} \leq C_{ins}^{PL, ch}, \forall t, w_{RCEV} \quad (28)$$

$$0 \leq P_{w_{RCEV}, t}^{PL, ch} \leq P_{w_{RCEV}, t}^{PL, ch, max}, \forall t, w_{RCEV} \quad (29)$$

$$E_{w_{RCEV}, t}^{PL, min} \leq E_{w_{RCEV}, t}^{PL} \leq E_{w_{RCEV}, t}^{PL, max}, \forall t, w_{RCEV} \quad (30)$$

#### 3.4. Distribution system constraints

The constraints related to the power flow in the distribution system [29] is presented below.

$$P_{ij} = \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} (V_i - V_j) + \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2} (\theta_i - \theta_j) \quad (31)$$

$$P_i = \sum_{j=1, i \neq j}^{N_B} P_{i,j} \quad (32)$$

$$Q_{ij} = \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2} (V_i - V_j) - \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} (\theta_i - \theta_j) \quad (33)$$

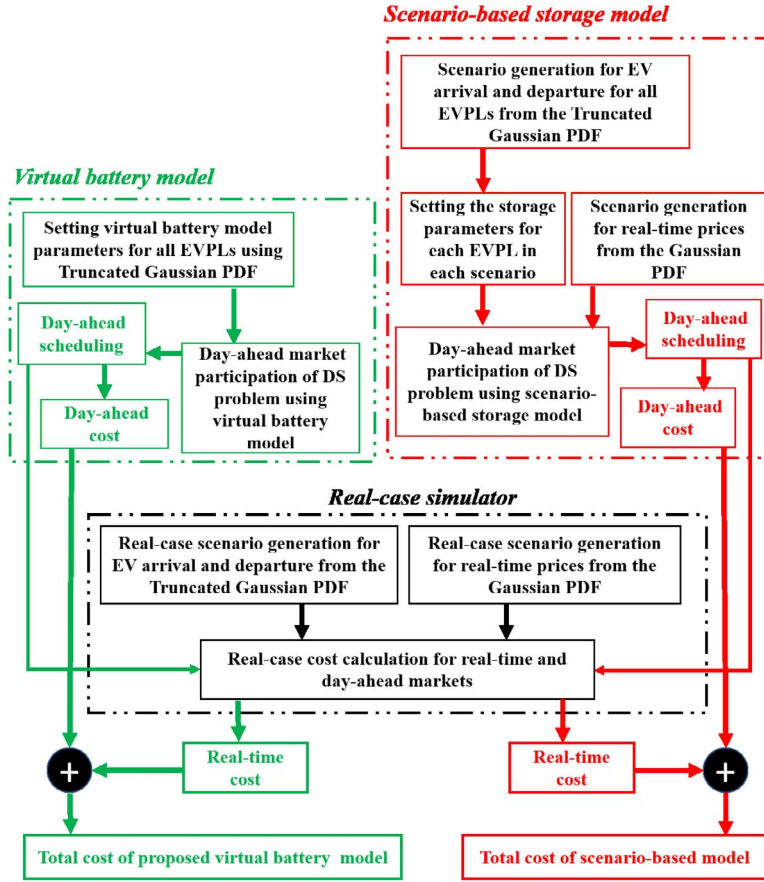


Fig. 4. Validation process flowchart.

$$Q_i = \sum_{j=1, i \neq j}^{N_B} Q_{i,j} \quad (34)$$

$$P_{ij}^2 + Q_{ij}^2 \leq S_{ij}^2 \quad (35)$$

for bus 0:

$$P_i^{RT} = P_i^{DN} - P_{L,i,t}^{Fix} \quad (36)$$

$$Q_i^{RT} = P_i^{DN} - Q_{L,i,t}^{Fix} \quad (37)$$

for EVPL buses:

$$P_i^{RT} = -P_{k,t}^{EVPL,RT} - P_{L,i,t}^{Fix} \quad (38)$$

$$Q_i^{RT} = -Q_{L,i,t}^{Fix} \quad (39)$$

for non-EVPL buses

$$P_i^{RT} = P_{L,i,t}^{Fix} \quad (40)$$

$$Q_i^{RT} = -Q_{L,i,t}^{Fix} \quad (41)$$

#### 4. Simulation results

As presented in the previous section, to validate the performance of our proposed virtual battery model and the profitability of the charger-sharing approach, we study the problem of optimal participation of an EVPL in DA and RT markets. This way, firstly, without considering the charging-sharing approach, our proposed model is evaluated by comparing its performance with the performance of the scenario generation approach. Then, the profitability of the charger-sharing approach in comparison with the exclusive charger is presented.

**Table 2**  
EV classes and characteristics.

| cl | $Cap_{cl}$ | $SOC_{cl}^{arr}$ | $SOC_{cl}^{dep}$ | $P_{cl}^{ch,max}$ | $P_{cl}^{dc,max}$ |
|----|------------|------------------|------------------|-------------------|-------------------|
| 1  | 15         | 0.33             | 0.85             | 7                 | 7                 |
| 2  | 20         | 0.33             | 0.85             | 10                | 10                |
| 3  | 20         | 0.16             | 0.85             | 10                | 10                |
| 4  | 15         | 0.4              | 0.85             | 7                 | 7                 |
| 5  | 20         | 0.1              | 0.85             | 10                | 10                |
| 6  | 15         | 0.45             | 0.85             | 7                 | 7                 |
| 7  | 10         | 0.5              | 0.85             | 5                 | 5                 |
| 8  | 10         | 0.2              | 0.85             | 5                 | 5                 |
| 9  | 15         | 0.33             | 0.85             | 7                 | 7                 |
| 10 | 20         | 0.2              | 0.85             | 10                | 10                |

##### 4.1. Case study

In this section, the two case studies used for the validation of the proposed approach and evaluation of the profitability of the charger-sharing approach are explained. Firstly, For the validation purpose, as depicted in 6, the distribution system of our case study is the modified version of the case study used in [30] with adding EVPL to the system. The DS owns 10 EVPLs buses. Secondly, for assessing the profitability of the proposed charger sharing approach, an EVPL equipped with 100 bidirectional and 100 unidirectional EV chargers is used. It has been assumed that ten classes of EV are available where the share of each EV class from all available EVs is equal to other classes. To protect the battery, it is assumed that  $SOC^{EV,min}$  and  $SOC^{EV,max}$  are 0.05 and 0.95, accordingly. Different EV classes information can be found in Table 2. Moreover, the DA price and generated scenarios for the RT price (purchasing and selling) are shown in Fig. 7.



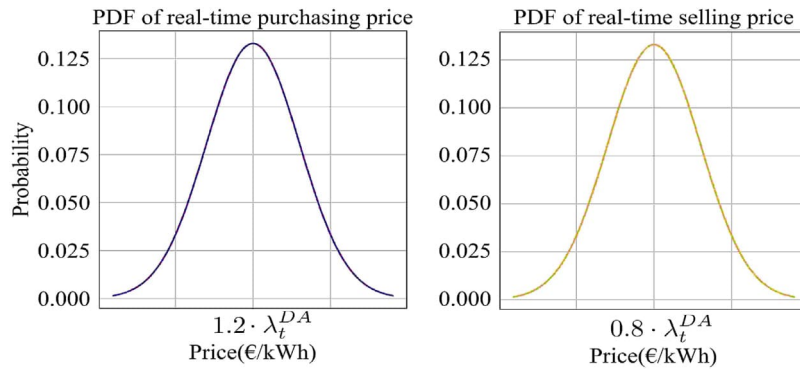


Fig. 5. PDF of RT purchasing and selling price for hour t.

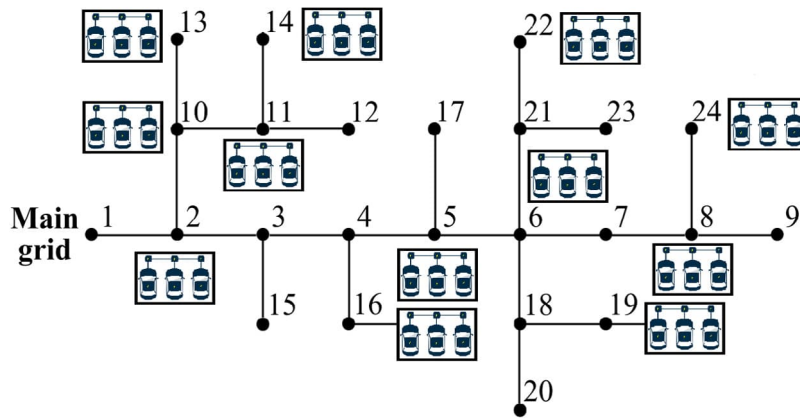


Fig. 6. Case study.

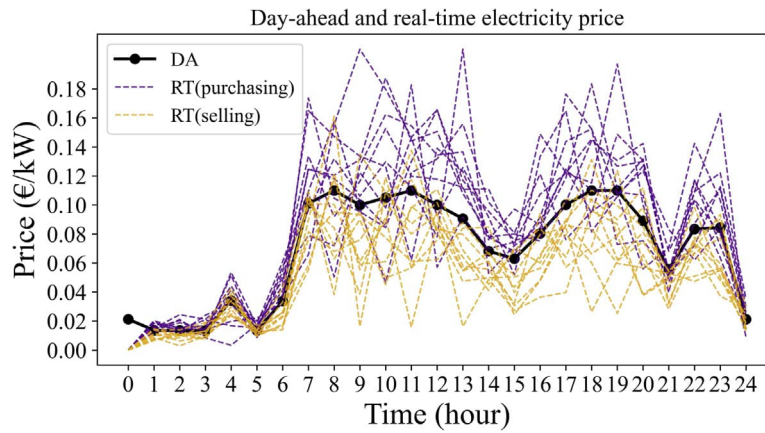


Fig. 7. DA and RT electricity price.

#### 4.2. Validation results

As explained in the previous section, to assess the performance of our proposed virtual battery model in considering the arrival and departure uncertainty, the operation cost of the EVPL-owned DS are compared with the scenario-based approach. The results show that the performance of our virtual battery model is similar to the scenario-based approach with 21 scenarios while its computational burden is way less than the scenario-based approach. Fig. 8(a) depicts that the DA power purchased from the upstream grid is very close for two approaches. However, they are not equal as shown in Fig. 8(b) where the difference in the DA power purchased from the upstream grid for the two approaches is depicted. This different day-ahead decisions results

in different real-time trades for different realization scenarios as shown in Fig. 9. However, overall the market strategy of two approaches are similar that results in similar total DS cost. Table 3 presents that the daily operational cost of the distribution system for providing power in the DA and RT markets using the scenario-based approach is 1692.06 €. While the DS operational cost with the proposed virtual batter model is 1689.78 €. The more important point is that the simulation time of our proposed approach is 4.92 s while the simulation time of the scenario-based approach is 219.53 s. This shows that the computational burden of the proposed virtual battery model is 2.242% of the computational burden of the scenario-based approach. This low computational burden of our proposed approach is of way more importance in the large scale problems where in addition to the uncertainty from numerous EVPLs,



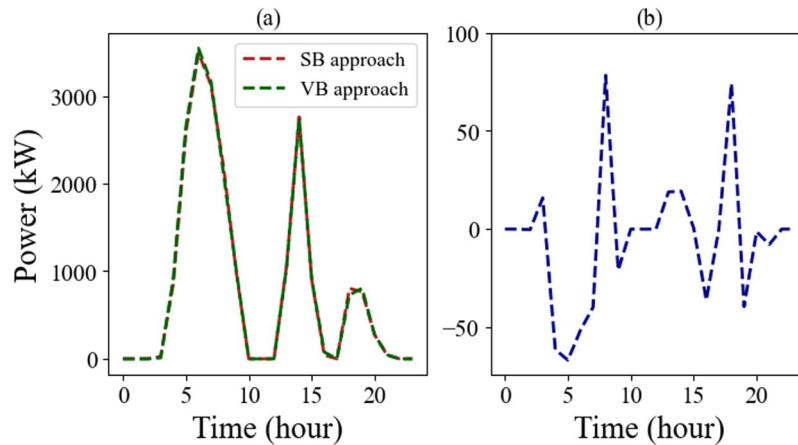


Fig. 8. (a) DA purchased power of DS for the proposed virtual battery and scenario-based approaches (b) difference of the DA purchased power for two approaches.

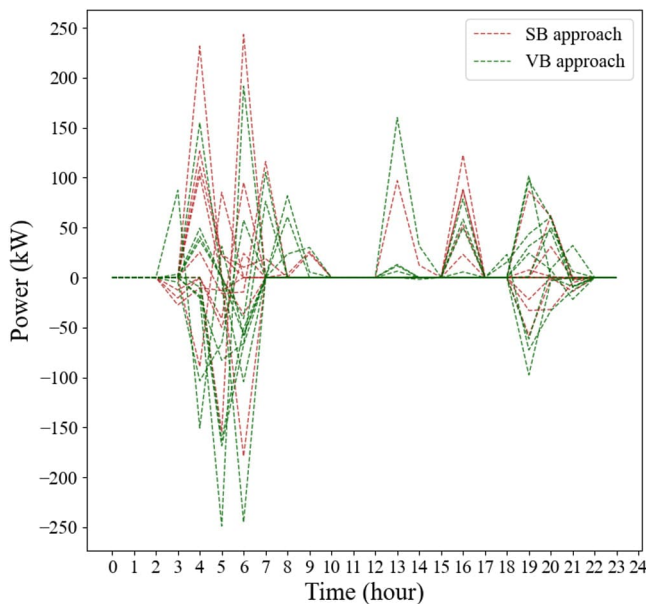


Fig. 9. RT traded power of EVPL in (a) scenario-based approach and (b) the proposed virtual battery model.

Table 3  
Operational cost and simulation time of the proposed virtual battery model and the scenario-based approach.

| Approach                       | Total cost | Simulation time |
|--------------------------------|------------|-----------------|
| Stochastic approach            | 1692.06    | 219.53          |
| Proposed virtual battery model | 1689.78    | 4.92            |

several other uncertainty resources, such as renewable generation exist as well. This way, for large-scale problems, while using the scenario-based approach, if the number of scenarios decreases the performance of the scenario-based approach worsens. However, the proposed virtual battery model will have similar performance for such large-scale problems without imposing any additional computational burden. This way, for large-scale problems, the proposed virtual battery model has a way greater performance than the scenario-based approach. Therefore, our proposed model is efficient model for deploying in the planning problem and operational studies from the system level point of view where the penetration of EVPLs is high. owing to its simple and compact formulation.

### 4.3. Charger-sharing approach profitability

In this section, the profitability of the charger-sharing charging approach is shown by evaluating its role in the operation cost reduction in comparison with the existing exclusive charger charging approach. To this end, the optimal operation of an existing EVPL with a specified number of EV chargers is investigated for the exclusive charging and charger-sharing charging approach assuming that there are sufficient parking spaces to host more EVs.

#### 4.3.1. Exclusive-charger charging

The charging and discharging power of EVPL, the maximum capacity of EV chargers for charging and discharging, and the maximum charging and discharging power possibility from parked EVs in the parking lot are depicted in Fig. 10. The maximum charging and discharging power possibility result from the number of parked EVs in each hour extracted from the arrival and departure time of EVs. The charging and discharging process is managed to meet the charging demand at the lowest cost. This way, the EVPL do the charging in the low-price hours and discharging in the high-price hours taking into account the Evs' arrival and departure behaviour ensuring that all EVs depart the parking lot at their desired departure time with their desired final SOC. As expected, in the exclusive charger charging, the big share of the EV chargers' charging and discharging capacity is not deployed. This way, the total charging capacity of the whole parking lot is 2000 kW, while in hour 14 when the charging power of the parking lot is at the highest level, the charging power is around 1100 kW which is way less than the total charging capacity. In other hours the situation is worse. In this regard, it could be understood that a big part of the charging capacity is unused. The purple arrows depict the unused charging capacity of chargers in different hours. This is the motivation for deploying the charger-sharing approach that empowers EVPLs for maximum utilization of the charging and discharging capacity of existing chargers. Moreover, the maximum charging and discharging power of the parked EVs is represented via the dotted curves. It is defined based on the number of EVs that are parked in the parking lot each hour and their nominal charging and discharging power. Therefore in the hours when no EV is parked in the parking lot, it is equal to 0, and as the number of parked EVs increases it rises. After hour 12, when the number of departed EVs is getting more than the arrived EVs, the number of parked EVs decreases resulting in a decrease the maximum charging and discharging power of the whole parked EVs.

Fig. 11 shows how the EVPL state of energy varies in each hour based on the previous hour's state of energy, energy stored from charging, energy depleted from discharging, added energy of the arrived EVs and extracted energy from departed EVs. In this regard, EVs by arriving

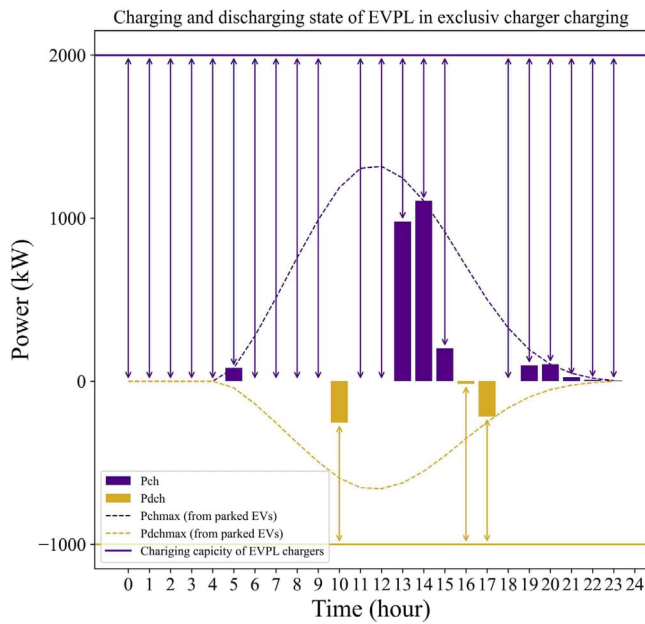


Fig. 10. Charging and discharging state of EVPL in exclusive-charger charging. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

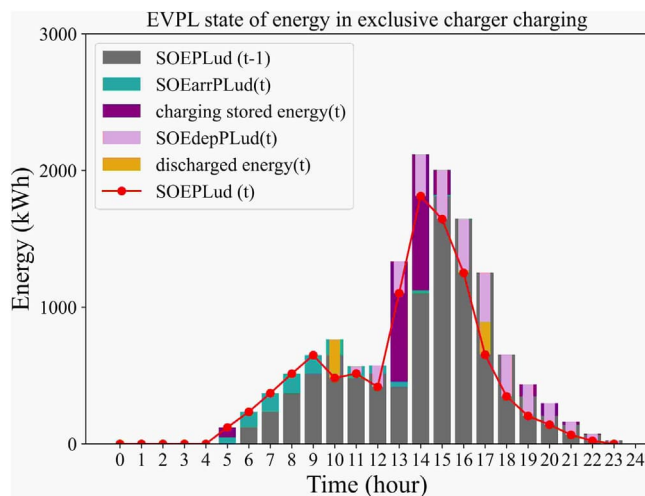


Fig. 11. EVPL state of energy in exclusive charger charging.

at the parking lot add their stored energy in their battery equivalent to their initial SOC to the containing energy of the parking lot and deplete the energy equivalent to their final SOC upon their departure from the containing energy of EVPL. Moreover, the equivalent energy of the EVs' charging/discharging is added/subtracted to/from the containing energy of EVPL. This way, the EVPL state of the energy varies in time while the energy demand of the EVs is met optimally ensuring that all EVs depart the parking lot at their desired departure time with their desired final SOC.

4.3.2. Charger-sharing charging

The performance of the charger-sharing approach in increasing the profit of EVPL is presented in this section. Firstly, Fig. 12 shows how this approach assists in better deploying the capacity of EV chargers in comparison with exclusive charger charging. In this regard, it is seen that the total capacity of the chargers is fully deployed in hours 13, 14, and 15. Comparing the results with the exclusive charger charging

Table 4

Number of accepted EVs for connecting to unidirectional and bidirectional EV chargers for different charging tariffs.

| Tariff | Accepted EVs for UDCH | Accepted EVs for BDCH |
|--------|-----------------------|-----------------------|
| 0.07   | 0                     | 160                   |
| 0.08   | 220                   | 181                   |
| 0.09   | 343                   | 221                   |
| 0.1    | 514                   | 514                   |
| 0.11   | 1143                  | 1143                  |
| 0.12   | 1601                  | 1601                  |
| 0.13   | 1601                  | 1601                  |

shows the profitability of the charger-sharing approach in utilizing the charging and discharging capacity of the chargers efficiently. This way, when it comes to using one charger for charging multiple EVs, EVPL can host more EVs (considering the parking space limitation) committing to charging the EVs based on their desired final SOC upon their departure. Therefore, it could gain more income from the efficient utilization of EV chargers. We considered seven cases with different charging tariffs assuming that in each case the tariff is acceptable by both the EVPL owner and EV owners. The results show that if there is no limitation on parking lot spaces, with the tariff of 0.09 €/kWh for charging EVs, the optimal number of hosted EVs would be 343 for the EVs that just want to be connected to UDEVCH and 221 for the EVs that are willing to be connected to BDEVCH. When the charging tariff is higher, it is profitable for EVPL to host more EVs (if EV chargers capacity allows it). Table 4 presents the optimal number of hosted EVs for different charging tariffs. It can be understood that when the charging tariff is high, hosting more EVs would be very important in comparison with the charging and discharging schedule, because just by hosting more EVs, EVPL would gain a considerable profit. This way, when the tariff increases the number of hosted EVs increases, but the limited capacity of EV chargers does not allow for accepting EVs more than a specific number. Because EVPL commits to charging the EVs based on their desired final SOC upon their departure, and with the limited number of chargers it can host a limited number of EVs. For instance, it is seen that when the tariff increases from 0.07 €/kW to 0.12 €/kW the number of accepted EVs increases, but when the tariff is 0.13 €/kW, the number of accepted EVs cannot increase due to the limitation of the EV chargers. Our charger-sharing charging approach's performance is evaluated by comparing the number of EV charges in EVPL and the number of accepted EVs. When the tariff is high enough to motivate the EVPL owner for hosting more EVs, EVPL can host around 3200 EVs with just 200 EV chargers (considering the characteristics of EVs and EV chargers explained in the case study section). However, when the exclusive charger approach is deployed, 200 EVs are accepted to enter the parking and be charged. This way, the charge-sharing approach has a substantial performance in utilizing the potential of installed EV chargers. In addition, this approach could be deployed in future EVPL planning studies to minimize the investment cost for EV chargers. In addition, Fig. 13 depicts the state of the energy of EVPL in each hour resulting from the optimal scheduling of the EVPL.

4.4. Discussion

While comprehensive spatial-temporal models exist for considering the location of EV charging stations, in line with the papers in the literature that aims to model the EV parking lot for the power system and energy market studies such as [31], in our paper the spatial factor and location of the EV parking lot are incorporated within the characteristics of a Truncated normal distribution. This approach simplifies the modelling complexity and reduces computational burden, particularly for large-scale system studies where various power system-related components and associated uncertainties are involved. In this context, based on the historical data of the EV owners' behaviour in each region the patterns of arrival and departure will be shaped via

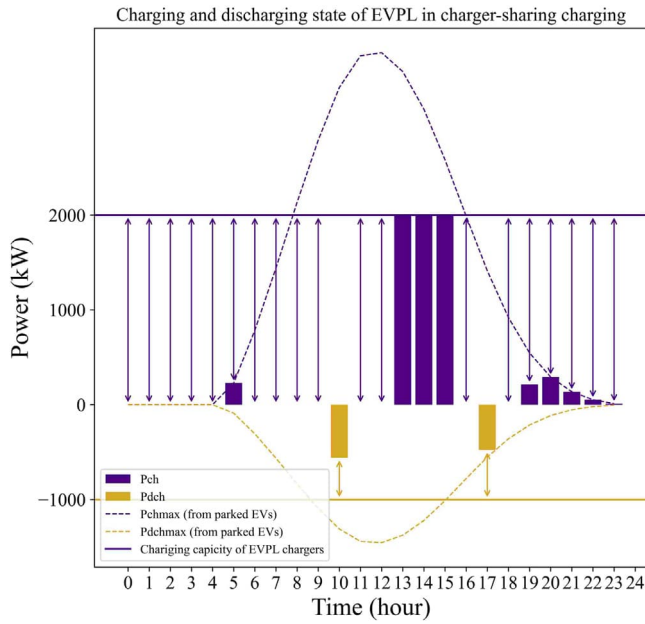


Fig. 12. Charging and discharging state of EVPL in charger-sharing charging.

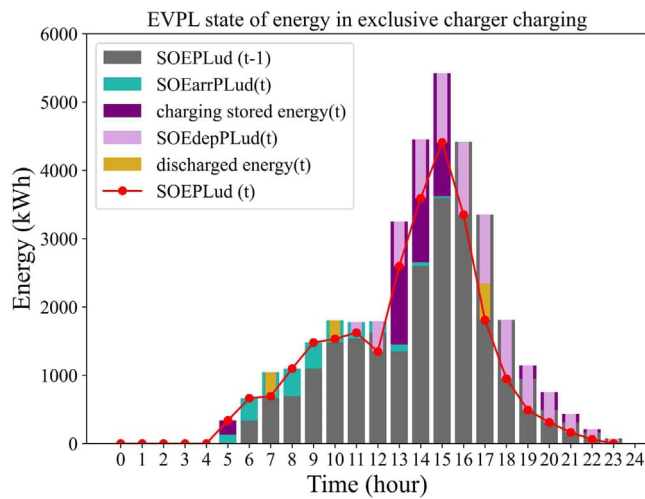


Fig. 13. EVPL state of energy in charger-sharing charging.

specific characteristics of the truncated normal distribution including mean, standard deviation, maximum value, and minimum value. In other words, it is assumed that EVs' arrival and departure behaviour for a parking lot located in a certain geographical area, for example, a district of a city, follow a certain pattern [31]. Consequently, the characteristics of the truncated normal distribution vary for EV parking lots located in different areas. Additionally, the maximum number of available EVs in the region, as represented in Eq. (5), serves as a determining factor for the maximum number of EVs that an EV parking lot can host during the whole day. This factor also varies for EV parking lots situated in different areas. In our case study, for the sake of simplicity and comparability, we have considered unique truncated normal distribution characteristics for different EV parking lots.

Another crucial aspect to note is that the proposed virtual model for the EV parking lot primarily serves power system operation and planning studies, offering a comprehensive overview of EV charging behaviour rather than exact real-time charging process management within the parking lot. Additionally, the key assumption here is that the arrival and departure patterns of EVs conform to the truncated normal

distribution. In this regard, if EVs' arrival and departure in the real-case scenario, follows the truncated normal distribution, considering the containing energy of the arriving and departing EVs and Eqs. (6) and (7), it is guaranteed that all of the EVs depart the parking lot in their desired time and with their desired final SOC based on the primary optimal power consumption solution (for example in the day-ahead stage). Otherwise, if the pattern does not exactly follow the truncated normal distribution, a minor adjustment in the primary charging process decisions guarantees the service quality of EV users.

For instance, in the validation section, we demonstrate that day-ahead decisions are based on the assumption of a truncated normal distribution for EV arrival patterns. However, in real-time operations, slight modifications are made to the charging schedule to cope with the actual EV arrival and departure patterns. The charger-sharing approach has no adverse impact on the users' service quality. This is because our model ensures compliance with the requirements of EV aggregation charging, considering both departure time and final SOC. The EV parking lot operator is in charge of prioritizing the charging schedule within the parked EVs in the parking based on their departure time. This aspect of the work is not within the scope of our paper, since our main purpose is to model the charging behaviour of the EV parking lot as an EV aggregation. However by considering the mentioned constraints the feasibility of the charging solution is guaranteed.

Moreover, Our proposed charger-sharing approach does not mandate charging only one EV at a time per charger. This suggestion aims to minimize potential technical barriers and reduce investment costs associated with charging-sharing assets. Therefore, our approach allows for the simultaneous charging of multiple EVs using a single charger. Furthermore, charging a single EV at a time per charger does not result in a queue or delayed departures beyond the preferred departure times. This is because the charging schedule of the whole EV parking lot satisfies the desired final SOC of the EVs upon their departure.

If multiple EVs are charging with one charger simultaneously, the charging power of each EV is a share of the total charging capacity of that charger. Therefore, the charging process takes more time for each EV. However, when a single EV is charged via a charger at each moment, it can utilize the whole charging capacity of the charger resulting in higher charging power, and less charging time. When the EV reaches its desired SOC, charging of the next EV assigned to the same charger is started. Therefore, there is no difference in terms of the final SOC of EVs, departure time, and the charging time of the group of EVs that are sharing one charger, for the two mentioned cases. As mentioned previously, the charger-sharing approach is presented for the EV parking lots where the EVs are parked for several hours, and it is intended to make the best use of the chargers for charging the aggregation of EVs. This way, instead of dedicating one charger for each parked car in the parking lot, multiple EVs can be charged via one charger during their stay in the parking lot. Therefore, it may not be applicable for charging stations that host EVs with short stays parked just for charging, as a means of alleviating the existing queue.

## 5. Conclusions

In this paper, a virtual battery model was proposed for modelling EVPLs considering the uncertainty of arrival and departure enabling modelling the charger-sharing charging to use an EV charger for charging multiple EVs. For validating our proposed model to assess its performance in reflecting the uncertainty of the arrival and departure of EVs, the cost of a distribution system owning multiple EVPLs in different buses in DA and RT markets was studied. In this regard, the performance of our proposed model was compared with the scenario-based approach. Validation results indicated that the performance of our virtual battery model was similar to the scenario-based approach in terms of cost. The daily operational cost of the distribution system for providing power in the DA and RT markets using the scenario-based approach is 1692.06 €, and the DS operation cost with the proposed



virtual batter model is 1689.78 €. However, its computational burden is way less than the scenario-based approach (2.24%). In addition, the performance of the EVPL with the charger-sharing charging approach was studied. The results indicated how the charger-sharing charging approach empowers the EVPL to host more EVs. According to the results, when the charging tariff increased, the EVPL could host more EVs, as hosting more EVs was profitable for EVPL. For EVPL equipped with 100 unidirectional and 100 bidirectional charging stations, when the charging tariff was high enough for EVPL to host as many EVs as possible, EVPL could host 3202 EVs. However, in the exclusive-charger charging approach, EVPL could host 200 EVs. This result showed how the charger-sharing charging approach facilitates the efficient deployment of EV chargers. Therefore, by employing charger-sharing charging, the investment cost for EVPL planning would also decrease dramatically paving the way for increasing penetration of EVs. In the future, the proposed virtual battery model will be used for modelling EV parking lots for large-scale distribution system management containing several uncertainty sources from RES. In addition, the charger-sharing approach will be further investigated to identify the possible technical difficulties.

### CRediT authorship contribution statement

**Mahoor Ebrahimi:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Miadreza Shafie-khah:** Writing – review & editing, Supervision, Investigation, Conceptualization. **Hannu Laaksonen:** Conceptualization, Supervision, Writing – review & editing.

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### Data availability

Data will be made available on request.

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