



## Interference Management And Game Theoretic Analysis of Cognitive Radio

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*Publication date:*  
2010

*Document Version*  
Early version, also known as pre-print

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Di Taranto, R. (2010). *Interference Management And Game Theoretic Analysis of Cognitive Radio*. Department of Electronic Systems, Aalborg University.

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# **Interference Management And Game Theoretic Analysis of Cognitive Radio**

Ph.D. Thesis

by

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# Abstract

Cognitive Radio systems are intended to dynamically access the spectrum that is underutilized by its owner at certain time, geographical location or frequency. Dynamic spectrum access presents a great opportunity to increase the available bandwidth, but it has also posed new challenges to the research community. This Ph.D. thesis deals with interference management in Cognitive Radio systems: interference management is a *conditio sine qua non* for cognitive radio systems, as they can re-use the primary resources underused or not utilized by the respective owners, provided that primary transmissions are not harmed. The first contribution of this Ph.D. thesis is on the challenge in deploying spectrum principles that significantly improves spectrum utilization efficiency without losing the benefits associated with static spectrum allocation, i.e., without provoking harmful interference toward the licensed primary users. We have identified conditions and proposed solutions/techniques for optimal usage of radio spectrum, by allowing coexistence on the same spectrum resources between primary and cognitive users. The second part of this Ph.D. thesis is dedicated to model the interactions and system dynamics between independent primary and cognitive user and to derive rules of local action at the independent cognitive users that result in stable and efficient system operation. We have modeled our scenario via a non-cooperative power control game so that the corresponding Nash equilibriums are taken as stable operation points for secondary users. The last contribution of this Ph.D. thesis has been dedicated to propose a technique for different treatment of primary interference at the cognitive receivers: under given conditions, primary interference at cognitive users should be decoded and not treated as noise. These different approaches to interference management at Cognitive Radio systems contribute to the increasing set of techniques that will make Cognitive Radio possible to deploy.



# Dansk Resumé

Cognitive Radio systemer er beregnede til dynamisk at tilgå spektrum der ikke udnyttes fuldt ud i tid, sted eller frekvens. Dynamisk spektrum-adgang giver en stor mulighed for at øge den tilgængelige båndbredde, men det har også givet nye udfordringer for forskere. Denne afhandling er fokuseret på interferens management i Cognitive Radio systemer: interferens management er nødvendig forudsætning for Cognitive Radio systemer da de kan genbruge de primære ressourcer som er underudnyttede eller ikke anvendt af de respektive ejere på betingelse af at primære transmissioner ikke skades. Det første bidrag i denne Ph.D afhandling omhandler udfordringen i at indføre spektrum-principper, der forbedrer spektrumanvendelsen betydeligt uden at miste fordelene ved statisk spektrum tildeling. Dvs. uden at fremprovokere skadelig interferens for licenserede primære brugere. Vi har identificeret vilkår og foreslået løsninger samt teknikker til at opnå optimal udnyttelse af radiospektrum, ved at tillade at primære og cognitive brugere sameksisterer i de samme spektrumressourcer. Den anden del af denne Ph.D. afhandling er dedikeret til at modellere interaktioner og dynamikker mellem selvstændige primære og cognitive brugere og til at udlede regler for lokale handlinger til selvstændige cognitive brugere, som resulterer i et stabilt og effektivt system. Vi har modelleret vores scenarie som et ikke-kooperativt spil til styring, hvor de tilsvarende Nash equilibria benyttes som stabile arbejds punkter for de cognitive brugere. Det sidste bidrag i denne Ph.D. afhandling har været dedikeret til at foreslå en ny teknik til behandling af den primære brugers interferens i cognitive modtagere: under bestemte vilkår, bør den primære brugers interferens blive dekodet af cognitive brugere og ikke blive behandlet som støj. Disse forskellige tilgange til interferens management i Cognitive Radio systemer bidrager til det stigende antal af teknikker, der gør det muligt at realisere Cognitive Radio.



# Preface

This is a Ph.D. thesis, focusing on interference management for cognitive radio systems. The thesis summarized the results I have obtained during my 3 years at the Antennas, Propagation and Radio Networking (APNet), Department of Electronic Systems, Aalborg University. The thesis was prepared under the supervision of Associate Professor Petar Popovski, Associate Professor Hiroyuki Yomo and Professor Gert Frølund Pedersen.

This work has been financed by Danish Research Council for Technology and Production Sciences, (project no. 274-05-0490).

The thesis consists of 10 papers covering the topic, and a complete list of all publication is also included.





# Acknowledgments

I would like to thank my supervisors, Associate Professor Petar Popovski, Associate Professor Hiroyuki Yomo and Professor Gert Frølund Pedersen for their continuous support and guidance through my Ph.D. Studies.

I also express my gratitude to Professor Kentaro Nishimori, the first person I have worked with in Aalborg University, within a joint project with NTT Corporation, Japan.

Furthermore, I would like to express my gratitude to Professor Osvaldo Simeone for inviting me to New Jersey Institute of Technology (NJIT), USA.

Many thanks also to Doctor Zoran Utkovski from Ulm University for his precious collaboration during his stay here in Aalborg and to Doctor Jesper Ødum Nielsen for having introduced, helped and explained me topics relevant to the measurements in cognitive radio scenarios.

I would also like to thank the sponsor of this Ph.D. Project, the Danish Research Council for Technology and Production Sciences and more in general, I would like to express my profound gratitude to Denmark, Aalborg University and Danes for giving me the opportunity to continue my studies within telecommunications.

I am thankful to the members of Antenna, Propagation and Networking (APNet) Section for their assistance and comments. And I appreciate the efforts of the administrative staffs at the Department of Electronic Systems: Charlotte Skindbjerg Pedersen, Kirsten Nielsen and Dorthe Sparre.

I would also thank all my friends I have met here in Aalborg and with whom I have spent an unforgettable time: A special hug to the members of the Aalborg Universitets Kaffeklub!

It is one of my greatest pleasures to express my infinite gratitude to the Chief Watchmen, Rene Nielsen, for being the only special person allowing me to play piano during these years. Tak skal du ha'!

This Ph.D. work is dedicated to my family and my village, Deliceto.

« Essere un pianista e un musicista non è una professione. È una filosofia, uno stile di vita che non può basarsi né sulle buone intenzioni né sul talento naturale. Bisogna avere prima di tutto uno spirito di sacrificio inimmaginabile. »

**Arturo Benedetti Michelangeli**



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# 1. Introduction

## 1.1 Wireless Communication and Traditional Spectrum Regulation

Wireless technology is rapidly proliferating into all aspects of computing and communication: The distances involved in wireless communication range from a few meters to thousands of kilometers (e.g., for radio communications). The number of wireless devices (especially mobile phones and computers today) is expected to increase to ~100 billion by the year 2025 [1]. However, the anticipated exponential growth of wireless devices and applications relies on the ability of the wireless research community to design radio technologies that continue to achieve high spectral efficiency even with increasing deployment density. In other words, reliable and efficient spectrum access is vital for the expected growth and innovation of wireless technologies.

The usage of radio spectrum and the regulations of radio emissions are coordinated by national regulatory bodies. As part of radio regulation, the radio spectrum is divided into frequency bands. Today the radio spectrum is divided in licensed and unlicensed frequency bands [2]. Unlicensed frequency bands are parts of the radio spectrum in which any type of radio service is permitted. Any type of radio system that meets a predefined set of regulatory requirements can be used over these bands. Those requirements regulate, among other, radio parameters such as limit of the radiated power, out of band emissions, and antenna characteristics. Examples of unlicensed frequency bands are *Industrial, Scientific and Medical* (ISM) such as the 2.4 GHz band and *Unlicensed Information Infrastructure* (U-NII) bands (in the United States), such as 5 GHz band. With unlicensed frequency bands, radio systems coordinate the usage of radio resources autonomously based on distributed rules (and possibly following netiquettes guidelines) while operating and there is no guarantee on how efficiently the frequency resources are used. With licensed frequency bands, operators have the exclusive right to use the radio resources of the assigned bands for providing radio services. Typical licensed radio services are for example radio-navigation and radio-location, mobile communication or TV-broadcasting. The operator does not have to share radio resources with other operators.

Static allocation of licensed frequency bands is favorable and advantageous from many points of view. An undeniable benefit is the simplicity: For a given spectrum allocation, there is no ambiguity about who can use the spectrum and enforcing policy is relatively easy. Moreover, radios have historically been fixed functionality devices designed for a specific frequency band. Consequently, using a specific frequency band requires a significant investment in infrastructure that can only be used in that band. Such an investment only makes sense if there is some guarantee of continued access to that band. Giving exclusive licenses encourages investments in infrastructure, which ultimately benefits society in the form of new services. Finally, equipments transmitting on a dedicated frequency are simpler (interference management is not an issue, in general) and their deployment is less problematic (a single operator can manage deployment, without needing to be concerned about arbitrary competing users).



## 1.2 Changes in Spectrum Regulation: Dynamic Spectrum Usage

While static allocation has many advantages and has generally served well in the past, it can lead to very inefficient usage of spectrum. Many recent studies have shown that many allocated frequency bands are significantly underutilized: According to the Federal Communication Commission [3]-[4], temporal and geographical variations in the utilizations of the assigned spectrum range from 15% to 85%. The today's dramatic increase in the access to the limited spectrum is therefore straining the effectiveness of the traditional spectrum policy. As a consequence, the limited available spectrum and the inefficiency in the spectrum usage necessitate a new communication paradigm to exploit wireless spectrum efficiently: Aiming at enhancing the allocation and use of spectrum so as to foster innovation, competition and the efficient use of the spectrum, one solution is to migrate from the current *static* spectrum access to *dynamic spectrum sharing* (DSS). Dynamic spectrum sharing is a collection of techniques for better utilizing radio spectrum as a function of time, space and frequency. In this context *cognitive radio* (also referred to as secondary system or secondary user) has been proposed as a novel means to achieve such flexible spectrum management, which drastically increases the efficiency of spectrum utilization. Cognitive radio networks [2] [3] are allowed to access the spectrum that is underutilized by its owner (called primary system) at certain time, geographical location or frequency. A cognitive radio senses the spectral environment over a wide frequency band and exploits this information to opportunistically provide wireless links that can best meet the demand of the cognitive user, while not harming the rights of the incumbent primary users. A cognitive radio first senses the spectrum environment in order to learn the frequency spectrum unoccupied/underutilized by primary users. Once such spectrum is found, the cognitive radio adapts its transmission power, frequency band, modulation, etc., so that it minimizes the interference to primary users [5]-[7]. Even after starting the transmission, the cognitive radio should be able to detect or predict the appearance of a primary user so that it makes the spectrum available for the primary user. The cognitive radio technology enables the user to: 1. Determine which portion of the spectrum is available and detect the presence of licensed users when a user operates in a licensed band (*spectrum sensing*), 2. Select the best available channel (*spectrum management*), 3. Coordinate access to this channel with other users (*spectrum sharing*), 4. Vacate the channel when a licensed user is detected (spectrum mobility).

When considering the coexistence between primary and cognitive users there are two distinct ways of sharing radio spectrum: The so-called Underlay and Overlay models [5]. In the Underlay model, the secondary user(s) use the same spectrum at the same time as the primary licensee. The peculiarity of this approach is that secondary users have the intent to cause as little interference as possible to the primary users. Ultra Wideband (UWB) technologies are particularly suited for this type of spectrum sharing because signals are spread over large portions of spectrum and the signal strength is around the RF noise level (this allows a UWB signal to operate on occupied spectrum with a very low power output, and not cause any interference). This model relies on measuring the ambient noise and the interference caused in

the operating range and maintaining it under a predefined threshold [8]. In the Overlay model, spectrum is shared explicitly in one of the three following ways:

1. Opportunistic (where spectrum is used whenever the licensee does not use it),
2. Cooperative (where frequencies are allocated centrally based on real-time negotiation with the licensee),
3. Mixed (where sharing is cooperative when possible and opportunistically otherwise).

More specifically, in the Opportunistic model, secondary users use the so-called “spectrum holes”, i.e., regions of time-space-frequency that are not used by the licensee, while the Cooperative model encompasses the cases where the primary user knowingly lets the secondary use the spectrum in some space/time/frequency (this is also referred to as *property rights model* or spectrum leasing [9]).

### **1.3 Driving factors for Cognitive Radio development**

Cognitive Radio systems, with their ability to access the spectrum that is underutilized by its owner at certain time, geographical location or frequency, and dynamic spectrum sharing have presented a great opportunity to increase the available bandwidth, and have posed many challenging problems for researchers. The main challenge in deploying spectrum principles is that of significantly improving spectrum utilization efficiency without losing the benefits associated with static spectrum allocation. Undoubtedly, the first issue is to develop wireless devices and networks that can opportunistically operate in different frequency bands. Other challenges are in the spectrum policy domain. There is pressing need for policies for dynamic spectrum access that lead to efficient spectrum use, are practical, protect the rights of license holders and maintain acceptable service quality. Moreover, there are also significant economic issues to take into consideration. First of all, the interests of primary users, who have made significant investment in infrastructure, must be protected. Moreover, it must be economically attractive, and on the long run convenient, to manufacturers and service providers to develop and deploy equipment for opportunistic spectrum access by secondary users. To summarize, given the many technology, spectrum policy and economic challenges to face, the development and growth of cognitive radio technologies is a very complex problem [2]. During the last years and still today, the research community has studied the tight coupling between wireless technology, spectrum policy and economics for cognitive radio networks (see [10] and references therein).

Another issue to be taken into account when developing cognitive radios is that, despite its intrinsic agile characteristics, they will have to follow some Regulation when implemented in practice: Radio requirements and spectrum aspects are regulated by governmental authorities and protection of primary users is a must. The Federal Communications Commission (FCC) is an independent United States government agency charged with regulating interstate and international communications by radio, television, wire, satellite and cable. Within the European Community spectrum policy involves global decisions made by the World (Administrative)

Radio Conference of the International Telecommunication Union (ITU) and regional decisions by the Conférence Européenne des Administrations des Postes et des Télécommunications (CEPT) encompassing all European countries. Similar regulatory boards exist worldwide. There are many regulatory issues yet to be fully addressed in the context of cognitive radio. As an example, let us consider the definition of harmful interference from a secondary user toward primary users. In general, interference is harmful if it is large enough to cause a significant disruption in service. However, defining a significant disruption in service (for example in terms of minimum guaranteed SINR or loss of coverage area in the primary system) is often a non-technical and largely subjective question [10].

Besides Regulation, another key point for the future of cognitive radio technologies is the standardization process: It will push the evolution towards next generation cognitive networks functionalities and services, and will ensure the convergence and interoperability in a future wireless network, while bringing undeniable benefits from business point of view, by enabling it to be mass produced and accepted. Realizing the importance of coordinated work around cognitive radio standardization, several IEEE standards groups are working on issues related to spectrum sharing; for instance, IEEE 802.22 [11] is aiming to create Wireless Regional Area Networks (WRANs) based on cognitive radio through the secondary usage of TV bands. IEEE SCC41 [12] (preceded by the IEEE 1900 task force) efforts are also developing standards for technologies, architectures and facilitators to realize dynamic spectrum access networks. Furthermore, IEEE 802.16h and IEEE 802.11y are working on interference management and efficient resource allocation in shared (e.g., unlicensed) bands. Another ongoing standardization process is in connection with the further evolution of the 3GPP Long Term Evolution (LTE), which is shifting toward the LTE-Advanced [12]. Even if a lot of standardization work has been conducted in recent years and more is in progress, the cognitive standardization process is certainly only in its early steps and represents one of the biggest challenges for the future of the cognitive radio technologies.

## **1.4 Thesis motivation and contribution**

When considering the introduction of cognitive radio systems in order to enable coexistence between primary and cognitive users a number of challenges can be identified as discussed above. However, it is clear that the central issue in Cognitive Radio systems is the treatment of interference: It is manifest the need to determine how interference will affect the regulation, standardization and design of future cognitive radio systems. In this thesis we research a number of approaches for the interference management in the context of cognitive radio system. Specifically, three different aspects are considered here:

- a)* Interference avoidance
- b)* Interference control through game theoretic model
- c)* Interference cancellation through decoding

## **A. Interference Avoidance**

### ***A.1 Background***

Interference is a fundamental nature of wireless communication systems, in which multiple transmissions often take place simultaneously over a common communication medium. In the context of coexistence between primary and cognitive users, interference management is one of the key issues to consider: Cognitive radios are allowed to access the spectrum that is owned by a legacy system as long as it is unused/underutilized at certain time and geographic location, or if the interference level toward the primary user is below a required level. Additionally, in order to achieve successful data transmission, the cognitive radio also needs to minimize the interference from primary user. Such a dynamic spectrum management plays an important role for cognitive radios to increase the opportunity of successful data transmission under the interference from primary users. This has driven research toward interference avoidance issues which is a significant concern when many users are sharing the spectrum. Studies are being done on geographical separation that would be required and detectors that are needed to ‘listen’ before the radios can ‘talk’. New algorithms are being developed for interference avoidance and their impacts are being investigated. The novel functionalities of the cognitive radio networks such as spectrum sensing, spectrum management, spectrum mobility and spectrum sharing have received tremendous amount of global attention and the impact of these functions on upper layer protocols are being looked into. Cross layer designing of these networks is also a very exciting challenge which is under investigation.

### ***A.2 Contribution***

Under the interference management framework, the first challenge we have faced in [13]-[14]-[15] was to develop dynamic spectrum management techniques for spectrum sharing, signal/power adaptation, radio resource allocation and interference management when the cognitive users are equipped with adaptive array antennas and use beam forming to maximize the gain toward the intended receiver and/or to minimize the interference from/to the primary users. Specifically, we have studied cognitive mesh networks that dynamically utilize the frequency band allocated to a primary system. When the cognitive mesh networks are located within the communication area of the primary system, there can be two different types of intersystem interference to handle: the interference from the primary system toward the cognitive mesh network and the interference from cognitive mesh network to primary system. We have investigated how to reduce/avoid these two types of interference by introducing adaptive array antennas at the cognitive receivers and transmitters.

In [16]-[17] we consider coexistence between a primary transmitter and a cognitive user under the *commons model*, where the primary operates with a certain gratuitous margin, which allows accommodating transmissions in the secondary system without degrading the target performance of the primary: We analyze the problem of selecting the maximal possible power in the

secondary system while not violating the outage probability in the primary system. We show that in the Rayleigh faded primary link, the power assignment in the secondary system is more favorable when the secondary transmitter is equipped with lower number of antennas. Conversely, in the case of unfaded primary link, we show that the increase of the number of antennas for the secondary systems results in the increase of the cumulative value for the power used by all the antennas at the secondary transmitter.

## **B. Interference control through game theoretic model**

### **B.1 Background**

Another challenge of this research project is to understand the system dynamics of cognitive radio and derive the rules of local action that can result in stable and efficient system operation (i.e., a situation in which no one has anything to gain by changing its strategy unilaterally).

The terminals in cognitive radio system adapt their local actions (transmission power, frequency timing, sensing methods, modulation parameters, etc.) according to the surrounding environment so that the individual links as well as the overall system perform in an efficient and stable manner. In order to achieve the system stability, it is vital to understand how such local interactions are affecting the overall system behavior at a large scale. In the cognitive radio system, depending on the employed local actions, there can be a positive or negative macro-level system behavior. The positive behavior results in the efficient and stable use of radio spectrum while the negative behavior produces completely unused radio spectrum. Game theory [18] [19] [20] has been utilized to analyze such a system dynamics with many self-organizing entities that cooperate and compete with each other. Generally speaking, game theory models strategic interactions among agents using formalized incentive structures. It not only provides game models for efficient self-enforcing distributed design, but also derives well defined equilibrium criteria to study the optimality of game outcomes for various games scenarios (static or dynamic, complete or incomplete information, non-cooperative or cooperative). These game models and equilibrium criteria have been extensively studied in the scenarios of dynamic spectrum sharing to achieve efficient and fair solutions for different network architectures (centralized/distributed), spectrum allocation behaviors (cooperative/non-cooperative) or spectrum access techniques (overlay/underlay).

The use of game theory to analyze the performance of cognitive radio networks is not straightforward. We mention three particularly challenging areas: a) Assumption of rationality: Game theory is founded on the hypothesis that players act rationally, in the sense that each player has an objective function that it tries to optimize given imposed constraints on its choices of actions by conditions in the game. Although nodes in a cognitive network can be programmed to act in a rational manner, the steady state outcome of rational behavior need not be socially desirable. Indeed, a major contribution of game theory is that it formally shows that individually

rational, objective-maximizing behavior does not necessarily lead to socially optimal states. Moreover, the assumption of perfect rationality, on some practical occasions, does not accurately reflect empirically observed behavior (e.g., wide-spread existence of peer-to-peer file sharing networks in the absence of any punishment/reward schemes). b) Realistic scenarios require complex models: The dynamic nature of cognitive networks leads to imperfection or noise in actions observed by a node. Such imperfections need to be modeled with reasonably complex games of imperfect information and/or games of imperfect monitoring. In addition, modeling of wireless channel and interactions between protocols at the different layers involves complex and, at times, non-linear mathematical analysis. c) Choice of utility functions: It is difficult to assess how a node will value different levels of performance and what trade-offs it is willing to make. The problem is exacerbated by a lack of analytical models.

Game theory has been applied to all the layers of the protocol stack [21]. Distributed power control [22] and selection of an appropriate signaling waveform [23] are physical adaptations that have been studied at the physical layer of cognitive radio nodes. The medium access control problem, with many users contending for access to a shared communication medium, lends itself naturally to a game theoretic formulation. In these medium access control games, selfish users seek to maximize their utility by obtaining an unfair share of access to the channel [24]. Functionalities of the network layer include the establishment and updating of routes and the forwarding of packets along these routes. Issues such as the presence of selfish nodes and a network, convergence of different routing techniques as the network changes, and the effects of different node behavior on routing have been analyzed using game theory [21]. At the transport layer, game theoretic models have been developed to analyze the robustness of congestion control algorithms to the presence of selfish nodes in the network. However, the bulk of the research has been focused on wired networks [21].

## **B.2 Contribution**

In [25] we study power control game for spectrum sharing among two secondary users with maximum interference constraint at a measurement point: Under this framework, unlicensed users can operate on the same frequencies as licensed signals, provided they can quantify and bound their additional interference. We model this scenario as 2-player non cooperative power control game and we identify the conditions for existence of unique/multiple Nash equilibria.

In [26] we study the coexistence between primary and cognitive users under the framework of the so-called *property-rights model*: Primary users are aware of the existence of primary terminals and they can grant the secondary users the possibility to use the primary band, provided that such a secondary operation does not provoke excessive interference at the primary users. The rationale is that primary users lease part of their resources and charge the secondaries in price per bit, such that a primary user is motivated to maximize the secondary aggregate throughput. In turn, the secondary users can start their own transmissions using the primary band. This scenario is modeled via a non-cooperative power control game so that the corresponding

Nash equilibriums are taken as stable operation points for secondary users. We use Stackelberg games as an appropriate analytical framework to study the described scenario of spectrum leasing.

## **C. Interference Cancellation through Decoding**

### ***C.1 Background***

If not exploiting a spectrum hole, a cognitive user always needs to operate under interference from primary system. Such interference is commonly treated as noise [27], but information-theoretic approaches provide more sophisticated treatment of interference. As first outlined in [28], in opportunistic interference cancellation the cognitive receiver opportunistically decodes the primary user's message, which then subtracts it off its received signal. This intuitively cleans up the channel for the cognitive pair's own transmission. In [29] authors increase the cognition even further and assume the cognitive radio has the primary codebooks as well as the message to be transmitted by the primary sender. This allows for a form of asymmetric cooperation between the primary and cognitive transmitters.

### ***C.2 Contribution***

We investigate the problem of spectrally efficient operation of cognitive user under interference from primary user. A secondary user observes a multiple access channel of two users, the secondary and the primary transmitter. Under the practical assumption that the secondary system knows only the primary codebooks (but not the messages) the secondary receiver applies Opportunistic Interference Cancellation (OIC) and decodes the primary signal when such an opportunity is created by the rate selected in the primary transmitter (PTX) and the power received from the PTX. We investigate the power allocation in the secondary system when opportunistic interference cancellation is used over multiple channels and we show that a significant gain can be achieved with the proposed technique. Also, we investigate how the secondary transmitter should select its rate in order to meet its target outage probability under different assumption about the channel state information (CSI) available at the secondary transmitter.

## 2. Aims

This thesis focuses on interference management within the coexistence between cognitive radio and primary licensed systems: In this context, interference management is of paramount importance and interference treatment is a central issue. The main goals of this thesis can be summarized as follows:

- ✓ To identify conditions and to propose solutions/techniques for optimal usage of radio spectrum, by allowing coexistence on the same spectrum resources between primary and cognitive users.
- ✓ To derive rules of local action at the independent cognitive users that result in stable and efficient secondary system operation (e.g., Nash equilibriums and Pareto optimal points), while at the same time guaranteeing protection to the primary users.
- ✓ To propose a technique for different treatment of primary interference at the cognitive receivers: under given conditions, interference should be decoded and not treated as noise.

The thesis is divided into three parts. The first part investigates how a cognitive system should operate in order to use efficiently portions of unused/underutilized primary spectrum. The second part is dedicated to model the interaction between independent primary and cognitive users using game theory, and to identify stable and efficient outcomes of the resulting games. The third part investigates the advantages obtained by applying *opportunistic interference cancellation* at a cognitive receiver.



### 3. Summary of Papers

#### 3.1 Paper 1

##### **Spatial opportunity for cognitive radio systems with heterogeneous path loss conditions**

*Nishimori K.; Di Taranto R.; Yomo H.; Popovski P.; Takatori Y; Prasad R.; Kubota S.;*

Vehicular Technology Conference, 2007. VTC2007

Page: 2631 - 2635

##### *Motivation*

Frequency utilization can be dramatically increased if a given system is allowed to use not only its own frequency bands but also the frequencies allocated to another system, but under-utilized. Following this principle, if there is a certain frequency band that is unutilized by a conventional system at a particular time or geographical location, that band can be temporary exploited by a cognitive system.

##### *Paper*

In this paper, the co-existence between a primary system with a long-range communication and a secondary system with a short range transmission is considered. The spectrum reusability within the primary service area depends on the amount of interference which the primary and secondary systems cause each other. We investigate the *spatial opportunity* when a short-range secondary system coexists with a large range primary system that employs directional antennas. Since the primary and secondary systems have different features and usage scenarios (e.g., different antenna heights) different path loss models are needed when considering the interference from the primary toward the cognitive system. This paper derives the spatial opportunity for cognitive radio systems when heterogeneous propagation path loss conditions are considered between the communication link within the primary system and the interference from the primary to secondary system.

##### *Main results*

The reusable area for the secondary systems is evaluated when the several scenarios are considered in actual systems with various antenna heights of the primary and secondary systems. We show that the different coefficients on path loss due to the different antenna heights between the primary and secondary systems largely affect the spatial opportunity for secondary users. The accurate modeling of propagation environment is a key to understand the actual benefits brought by cognitive radios.

### 3.2 Paper 2

#### **Simple Antenna Pattern Switching and Interference-induced Multi-hop Transmissions for Cognitive Radio Networks**

*Di Taranto R.; Nishimori K.; Popovski P.; Yomo H.; Takatori Y.; Prasad R.; Kubota S.;*

2nd IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks, 2007. DySPAN 2007. Page:543-546

##### *Motivation*

Cognitive radios dynamically utilize the frequency band allocated to a primary system, which is not being utilized at a particular time and geographic location. A key challenge to realize such an efficient coexistence between the primary and cognitive radio systems is the interference management. When a cognitive radio system is located within the communication area of a primary system, there can be two different types of inter-system interference to handle: the interference from the primary system to the cognitive radio and the interference from the cognitive radio to the primary system.

##### *Paper*

This paper investigates how to reduce the interference from the primary system toward the cognitive radio. To this aim, cognitive receivers are equipped with adaptive array antennas. As a side effect of the array adaption, the antenna pattern created according to the interference condition from the primary system can cause serious degradation of transmission quality between the cognitive radio nodes. In order to avoid this detrimental effect we first introduce a simple antenna pattern switching at the cognitive receiver, and second we propose a combination of the above-mentioned antenna switching with interference induced multi-hop transmissions.

##### *Main results*

Numerical results have shown that the proposed combination of antenna switching and interference-induced multi-hop transmissions can significantly improve the achievable rate in the cognitive radio networks: We can obtain an improvement by the proposed method in average transmission rate larger than 9 bit/sec/Hz.

### 3.3 Paper 3

#### **Cognitive Mesh Network under Interference from Primary System**

*Di Taranto R.; Yomo H.; Popovski P.; Nishimori K.; Prasad R.;*

10<sup>th</sup> International Symposium on Wireless Personal Multimedia Communication, WPMC 2008,

##### *Motivation*

Cognitive mesh networks dynamically utilize the frequency band allocated to a primary system. A key challenge to realize such an efficient coexistence between the primary and cognitive mesh network is the interference management. When the cognitive mesh networks are located within the communication area of the primary system, there can be two different types of intersystem interference to handle: the interference from the primary system toward the cognitive mesh network and the interference from cognitive mesh network to primary system.

##### *Paper*

In this paper, we investigate how to reduce/avoid these two types of interference by introducing adaptive array antennas at the cognitive receivers and transmitters. Specifically, effectiveness of antenna pattern switching proposed in Paper 1 is investigated when it is applied in a mesh network environment. Also, 3-element array antenna is introduced at the cognitive radio nodes with a simple procedure to heuristically select the pattern to reduce interference.

##### *Main results*

We have compared the performances of 2-element and 3-element linear array when they are applied to a cognitive mesh network. We have shown that the antenna switching and 3-element array with a simple heuristic algorithm can significantly improve the networking performance as compared with the 2-element array antenna.

### 3.4 Paper 4

#### **Cognitive Mesh Network under Interference from Primary User**

*Di Taranto R.; Yomo H.; Popovski P.; Nishimori K.; Prasad R.;*

Wireless Personal Communications, Volume 45, Number 3 / May, 2008,

Page 385-401

#### *Motivation*

This paper presents in a more comprehensive and exhaustive study of paper 3.

#### *Paper*

This paper is a generalization of paper 3. Same problems are faced; more general approach and solutions to the problems are proposed.

#### *Main results*

We study the impact of the cognitive transmissions on the outage performance of the primary system. We show that the 3-element array causes higher outage probability at the primary system, than antenna switching and 2-element array: This happens because the performance of the 3-element array depends on the channel state information (CSI) most largely among the three schemes. Moreover, we show that the tradeoff between the performance of primary and cognitive networks can be controlled by a threshold parameter on interference caused by cognitive transmitter to primary receivers. We also study network link metrics, i.e., percentage of unavailable links and percentage of mesh full connectivity.

### 3.5 Paper 5

#### **Outage Margin and Power Constraints in Cognitive Radio with Multiple Antennas**

*Popovski P.; Utkovski Z.; Di Taranto R.;*

IEEE 10th Workshop on Signal Processing Advances in Wireless Communications, 2009.

SPAWC '09.

Page 111-115

#### *Motivation*

In the *commons model*, the primary should operate with a certain gratuitous margin, which allows accommodating transmissions in the secondary system without degrading the target performance of the primary. The margin can take several forms: time, frequency, interference. The cognitive system performs spectrum sensing and identifies its transmission opportunity.

#### *Paper*

In this paper we consider scenarios that deal with interference margin: secondary transmissions are allowed as long as the outage probability in the primary system is kept at or below an acceptable value. We analyze the problem of selecting the maximal possible power in the secondary system while not violating the outage probability in the primary system.

#### *Main results*

In this paper we investigate two different configurations at the cognitive transmitter: When it is equipped with one or two antennas. We determine the maximal allowable power at the cognitive transmitter. The conclusion is that the sum of the powers for the two antennas is different from the power allowed for a single antenna system (for a fixed outage probability in the primary system). The relation between these powers depends on the propagation conditions in the primary system. When the primary system has a strong line-of-sight the usage of two antennas is much more advantageous than the usage of a single antenna. Conversely, with a weak line-of-sight component in the primary link, the gain of having two antennas is decreased as the total power used by the two antennas should be less than the power that is allowed for a single-antenna secondary transmitter.

### 3.6 Paper 6

#### **The Effect of Interference Statistics on the Power Levels in Multi-Antenna Cognitive Radios**

*Utkovski Z.; Popovski P.; Di Taranto R.;*

Submitted to International Journal on Communication Networks and Distributed Systems, special issue on Cognitive Wireless Networks, January 2010

##### *Motivation*

This paper extends and completes the results and conclusions in paper 5: The analysis is generalized to the case where the cognitive transmitter is equipped with an arbitrary number of antennas.

##### *Paper*

Paper 5 showed that the maximal allowable power in the secondary system depends on the propagation conditions in the primary system. In this paper, we analyze two extreme cases of propagation in the primary link: a) Rayleigh fading and b) unfaded link with constant SNR.

##### *Main results*

We have shown that in the Rayleigh faded primary link, the power assignment in the secondary system is more favorable for lower number of antennas. Conversely, in the case of unfaded primary link, the increase of the number of antennas for the secondary systems results in the increase of the cumulative value for the power used by all the antennas at the secondary transmitter. Hence, when the direct component in the primary link is substantial, the usage of multiple antennas brings benefits also in terms of maximum transmitting power.

### 3.7 Paper 7

#### **Two Players Non-Cooperative Iterative Power Control for Spectrum Sharing**

*Di Taranto R.; Yomo P.; Popovski P.;*

PIMRC 2008. IEEE 19th International Symposium on Personal, Indoor and Mobile Radio Communications, 2008.

Page: 1 – 5

#### *Motivation*

Recently the FCC has introduced another possible form of coexistence between primary and cognitive users: Unlicensed users can operate on the same frequencies as licensed signals, provided they can quantify and bound the additional interference.

#### *Paper*

In this paper, we consider the coexistence between primary and cognitive users within the framework described above. Cognitive user, who observes the channel availability dynamically, considers a channel to be available for transmission if its transmission on that channel would result in increase of the interference from cognitive system toward the primary system below a fixed threshold. We model this scenario as a 2-player non cooperative power control game.

#### *Main results*

We study the possible outcomes of the 2-player game and identify the conditions for existence of unique/multiple Nash equilibria. We evaluate the performances of these Nash equilibria and compare them with the social optimal solution by carrying out computer simulations. We show that the proposed solution can achieve a satisfactory performance in terms of the total transmitting power at the two secondary users.

### 3.8 Paper 8

#### **Efficient Spectrum Leasing via Randomized Silencing of Secondary Users**

*Di Taranto R.; Popovski P.; Simeone O.; Yomo H.;*

submitted to IEEE Transactions on Wireless Communications, December 2009

##### *Motivation*

One of the possible forms of coexistence between primary and cognitive users is under the so-called *property-rights model*: Primary users are aware of the existence of primary terminals and they can grant the secondary users the possibility to use the primary band, provided that such a secondary operation does not provoke excessive interference at the primary users.

##### *Paper*

In this paper the rationale is that primary users lease part of their resources and charge the secondaries in price per bit, such that a primary user is motivated to maximize the secondary aggregate throughput. In turn, the secondary users can start their own transmissions using the primary band. This scenario is modeled via a non-cooperative power control game so that the corresponding Nash equilibriums are taken as stable operation points for secondary users. We use Stackelberg games as an appropriate analytical framework to study the described scenario of spectrum leasing.

##### *Main results*

For the most general setting of channel gains, we investigate the conditions for NE for a subset of power allocations. When the scenario is symmetric in the sense that all the secondary users have the same channel gains in the direct/interfering links, we prove that only two optimal power allocations exist. Finally for the case of general channel gains with strong interference, we show that there is a unique Nash equilibrium in the game.



### 3.9 Paper 9

#### **Opportunistic Interference Cancellation in Cognitive Radio Systems**

*Popovski P.; Yomo H.; Nishimori K.; Di Taranto R.; Prasad R.;*

2nd IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks, 2007. DySPAN 2007.

Page: 472 – 475

##### *Motivation*

A cognitive user operating under interference from primary system generally treats the primary signal as additional noise. However information-theoretic approaches provide more sophisticated treatment of interference: under some conditions interference can be decoded and cancelled.

##### *Paper*

The departing point in this paper is that it is reasonable to assume that secondary users can decode the primary signal, as it is a legacy system. We advocate that secondary users should apply *Opportunistic Interference Cancellation* (OIC) and decode the primary signal when such an opportunity is created by the rate selected in the primary system and the power received at the secondary system.

##### *Main results*

We investigate the power allocation in the secondary system when opportunistic *interference cancellation* is used over multiple channels. We show that the optimal power allocation can be achieved with *intercepted water-filling* instead of the conventional water filling. The results show a significant gain for the rate achieved with OIC.

### 3.10 Paper 10

#### **Outage Performance in Cognitive Radio Systems with Opportunistic Interference Cancellation**

*Di Taranto R.; Popovski P.;*

submitted to IEEE Transactions on Wireless Communication, January 2010

##### *Motivation*

Generally, a cognitive receiver needs to operate under interference of a primary system. Such interference is commonly treated as noise, but information-theoretic approaches provide more sophisticated treatment of interference.

##### *Paper*

In this paper, a secondary receiver observes a multiple access channel of two users, the secondary and the primary transmitter, respectively. The secondary receiver applies opportunistic interference cancellation and decodes the primary signal when such an opportunity is created by the rate selected at the primary transmitter and the power received from the primary transmitter. The goal of this paper is to investigate how the secondary transmitter should select its rate in order to meet its target outage probability under different assumption about the channel state information (CSI) available at the secondary transmitter.

##### *Main Results*

We have evaluated the maximum secondary rate that has a predetermined outage probability at the secondary receiver. We have studied opportunistic interference cancellation for three different cases of CSI available at the secondary transmitter, and for each of them we have identified the region of achievable primary and secondary rates. Simulation results have shown that the best secondary performance is always obtained when the secondary transmitter knows the instantaneous channel gain toward its intended receiver.

## 4. Discussions

In this thesis we have researched three different approaches for the interference management in the context of coexistence between primary and cognitive systems: a) interference avoidance, b) interference control through game theoretic model, and c) interference cancellation through decoding.

### a) Interference avoidance

The first contribution of this thesis is on the challenge in deploying spectrum principles that significantly improves spectrum utilization efficiency without losing the benefits associated with static spectrum allocation, i.e., without provoking harmful interference toward the licensed primary users. In paper 1 we have discussed the possibility for a short-range cognitive radio to be located within the service area of the primary system. There are locations in the service area of the primary system where the cognitive radio can reuse the frequency of the primary system without disturbing it or being disturbed by the primary system. We say that in those locations there is *spatial opportunity* for communication in the secondary system. The key point in paper 1 is the study of the impact of heterogeneous path loss conditions on spatial opportunity for cognitive radio systems: primary and secondary systems have different features and usage scenarios; therefore we use different path loss models for links in primary and secondary systems. Our results clarified that the different coefficients on the path loss due to the different antenna heights between the primary and secondary systems largely affects the spatial opportunity for secondary users. In paper 2 we propose a simple antenna switching and relay technique for cognitive radios using adaptive array antennas to effectively reduce the interference from primary system. While the adaptive array can create null toward the direction of primary system, the created antenna pattern can result in non-optimized pattern between cognitive nodes. In order to solve such a problem in paper 2 we introduce a simple antenna pattern switching. With this technique, each node is equipped with three antennas and each node tries to select the antenna configuration resulting in the best antenna pattern for each link. Furthermore we propose a combination between the above simple antenna pattern switching and the interference-induced multi-hop transmission where the multi-hop transmission is utilized when a link in the network suffers from the low transmission quality due to the created antenna pattern. Numerical results in paper 2 show that the proposed techniques can significantly improve the achievable data rate in cognitive radio networks. Papers 3 and 4 extend these results in a more comprehensive scenario where a cognitive mesh network operates under the interference from a primary licensed system. A 3-element linear array is introduced with the relative heuristic procedure to select the pattern. The focus of these studies is not only on the available bandwidth on each link, but also networking connectivity metrics are evaluated.

Paper 5 considers coexistence between primary and cognitive systems under the framework of the Underlay model: in the *commons model*, the primary operates with a certain gratuitous margin, which allows accommodating transmission in the secondary systems without degrading

the target performance in the primary. In this context, we deal with situations where cognitive radios can transmit only if they keep the outage probability at the primary system at or below an acceptable value. In papers 5 we analyze the problem of selecting the maximal possible power in the secondary system while not violating the outage probability in the primary system. We consider the cases where the secondary transmitter is equipped with one or two antennas (with equal power allocated to each antenna) and derive the maximal allowed power per antenna. The conclusion is that the sum of the powers of the two antennas is different from the power allowed for a single antenna system. Interestingly, the relation between these powers depends on the propagation conditions in the primary system. When the primary system has a strong Line-Of-Sight, the usage of two antennas is much more advantageous than the use of a single antenna. Paper 6 extends these results to the case where the secondary transmitter is equipped with an arbitrary number of transmitting antennas: same conclusions hold.

## **b) Interference control through game theoretic model**

Game theory can be roughly divided into two broad areas: non-cooperative (or strategic) games and cooperative (or coalitional) games. Non-cooperative game theory deals largely with how intelligent individuals interact with one another in an effort to achieve their own goals. On the contrary, in cooperative game groups of players enforce cooperative behaviors, and thus the game becomes competition between coalitions of players. Game theory not only provides models for efficient self-enforcing distributed design, but also derives well defined equilibrium criteria (e.g., Nash equilibrium) to study the optimality of game outcomes for various game models. In paper 7 we study power control at two cognitive users when they coexist on the same frequencies with a primary system under the constraint that unlicensed users can quantify and bound their additional interference. We assume that there is one primary node, called measurement point, which upon violation of the interference threshold iteratively backs off cognitive users, until the threshold is no more violated. We model this scenario as 2-player non-cooperative power control game, where the two players are selfish and rational and want to maximize their own utility. We identify the possible outcomes of this game and the conditions for the existence of unique/multiple Nash equilibria. We analyze the property of the Nash equilibria in our game, and compare their performances with the social optimal solution. We also show that our proposed solution can achieve a satisfactory performance in terms of the total transmitting rate at the two secondary users. In paper 8 we focus on the *property-rights model*: Primary users are aware of the existence of secondary terminals and they can grant the secondary users the possibility to use the primary band, provided that such a secondary operation does not provoke excessive interference at a given primary receiver (PRX). A primary (licensed) user leases part of its resources to independent secondary (unlicensed) terminals in exchange for a tariff in dollars per bit, under the constraint that secondary transmissions do not cause excessive interference at the primary receiver (PRX). The PRX selects a power allocation (PA) for the secondary user that maximizes the secondary rate (and thus its revenue) and enforces it by the

following mechanism: Upon violation of a predefined interference level, PRX keeps silencing randomly selected secondary users, until the aggregate secondary interference is below the required threshold. This mechanism ensures that secondary users may be not willing to deviate from the allocated PA. Specifically this scenario gives rise to a Stackelberg game, in which the primary determines the PA and a Nash equilibrium constraint is imposed on the PA to ensure the secondary users do not have incentives to deviate, given their knowledge of the silencing mechanism run at the PRX. In principle, the primary should find the set of PAs that are NE and among them choose the one that maximizes the aggregate secondary utility, and thereby the revenue of the primary. For the most general setting of channel gains, we investigate the conditions for NE for a subset of PAs. When the scenario is symmetric in the sense that all secondary users have the same channel gain in the direct/interfering links, we prove that only two optimal power allocations exist. Finally, for the case of general channel gains with strong interference, we show that there is a unique NE of the game.

### **c) Interference cancellation through decoding**

Because of its spectrum-sharing nature, a cognitive radio network inevitably operates in interference-intensive environments. Interference management plays a key role when considering the coexistence between primary and cognitive systems. Historically interference has always been treated as a noise: this is the same approach used since 1920. Recently by using information theory a more sophisticated treatment of interference has been proposed in wireless communications. In paper 9 we have investigated the problem of spectrally efficient operation of a cognitive radio operating under interference from a primary system. The key point in paper 9 is that the cognitive receiver should not always treat the primary interference as noise, but should try to decode it and subtract from the received signal when possible. Specifically, the cognitive receiver should apply *opportunistic interference cancellation* (OIC) and decode the primary signal when such opportunity is created by the rate selected at the primary transmitter and the power received at the cognitive receiver. In paper 9 we derive the achievable data rate in the cognitive system when OIC is applied. When the primary signal is decodable at the cognitive receiver, we devise a method applied by the cognitive transmitter to achieve the maximal possible rate in the secondary system. This method has a practical significance, since it enables rate adaptation without requiring any action from the primary system. Moreover, we have investigated the power allocation at the cognitive transmitter when OIC is applied over multiple channels: optimal power allocation can be achieved with intercepted water-filling (a method proposed in paper 9) instead of the conventional water-filling. Overall, results have shown a significant gain for the rate achieved by OIC. We have used the same approach in paper 10: the cognitive receiver, operating under interference from the primary system, applies opportunistic interference cancellation. The novelty of paper 10 lies on investigating how the secondary transmitter should select its rate in order to meet its target outage probability under different assumptions about the available CSI. We study three different cases and for each of them

identify the region of achievable primary and secondary rates. By numerical evaluations we show that the best secondary performance is always obtained when the cognitive transmitter knows the instantaneous channel gain toward its intended receiver (rather than the instantaneous channel gain toward the primary receiver).

#### 4.1 Implications of the Thesis

The contributions presented in this thesis are expected to have impact in the field of interference management in the context of coexistence between legacy and cognitive radio systems. Nevertheless some of proposed approaches may have wider applicability in communication engineering. The implications can be classified into three groups:

***Feasibility of Cognitive Radio in the future:*** Cognitive radio is an exciting technology that promises to increase the efficiency of our current spectrum usage. In this thesis, we have answered some of the fundamental questions that must be addressed before the FCC (or similar) will consider allowing cognitive devices into more frequency bands. There are still more questions to be answered before cognitive radio can become a reality, however the studies and results presented in this thesis can be used as reference to decide/influence the future directions in which cognitive radio could be developed.

***Game theory for Wireless Communication:*** Game theory is a field of applied mathematics that describes and analyzes interactive decision situations. Still today there is a growing interest in adopting game-theoretic methods to model interaction between independent agents in cognitive radio. The application of game theory in this thesis on one hand shows the difficulties in using mathematical model that can encompass all the complicated interactions between primary and cognitive users. On the other hand, it shows that very interesting insights can be obtained by applying game theory. The main intuition we have gained is the introduction of a new punishment mechanism to enforce policy in a power control game, but certainly an appropriate use of the numerous other branches of game theory will strongly impact the future cognitive radio networks.

***Standard and Regulations:*** The studies carried out in this thesis contribute to a better understanding of the dynamics characterizing the interactions between primary and cognitive users. As a consequence the results and conclusions from this thesis can possibly be used to influence the definitions of both standards and regulations for cognitive radios. Some of the issues addressed in this thesis will be keys for regulatory bodies to alter/decide their policy and rule making. Also, results in this thesis can be used to some extent to affect the standardization process in coming years. For example, the proposed *opportunistic interference cancellation* technique with its disruptive idea of decoding interference (rather than treating it always as noise) could be a revolutionary novelty for standards and regulations.

## 4.2 Overall perspective

Interference is the central issue for the expected future success of Cognitive Radio systems: It can substantially limit the reliability and the throughput of any wireless system. Additionally, interference management is a *conditio sine qua non* for cognitive radio systems, as they can re-use the primary resources underused or not utilized by the respective owners, provided that primary transmissions are not harmed. In this thesis I have investigated three different philosophies for interference management in the context of cognitive radio system. I expect these different approaches to provide answers to different needs that will certainly arise in the near future with the introduction of cognitive radio systems. Different technologies, different geographical/physical conditions, economic constraints, regulations etc. will pose different problems to be possibly solved with the interference management at cognitive radios. The interference avoidance approach could be used in the digital TV (DTV) frequency bands, especially in rural areas (generally characterized by low density in the deployment of primary receivers). In this case, there will be very likely a significant amount of white spaces and therefore interference avoidance approach could be reasonable to implement at cognitive radios: This fairly simple approach to the interference management makes sense when there is a certain amount of unused spectrum resources. The insights gained by modeling interference control through game theoretic models could be used for situations where cognitive radios are deployed in urban areas and are in the proximity of a primary base station system willing to lease part of its spectrum (e.g., because they are underutilized or not used in certain periods of time). Clearly this scenario requires some synchronization between primary and secondary users, but this cost is sustainable in very populated urban areas where spectrum scarcity is a central problem. The last approach to interference treatment, where cognitive radios intelligently exploit the knowledge and the structure of primary interference to achieve improved reliability and throughput, could be appropriate for advanced cognitive radio systems supposed to operate with relatively low transmitting power even in highly interfered geographical, most probably urban, areas: No cognitive transmission would have an acceptable quality of service unless part of the primary interference is decoded and cancelled. With this approach to interference management, cognitive radio system will have possibility to operate in very adverse conditions without provoking harmful interference toward very sensitive primary system (e.g., in airport areas).

## 5. Conclusions

Today spectrum is regulated such that bands are allocated exclusively to a particular service, often with only a single system licensed to use that band in any given location. Over time, this has led to significant over-allocation and under-utilization of spectrum, slowing down wireless deployments. To realize efficient spectrum usage, we must migrate from current static spectrum access to dynamic spectrum access. One promising solution in this direction is the cognitive radio. Cognitive radio initiates a revolution regarding the spectrum allocation considerations by putting forward a new concept called opportunistic spectrum usage, which involves the soft usage of current licensed and unlicensed spectrum. This concept proposes that licensed bands can be utilized by secondary users at times when they are not being utilized by their owners, leading to the most efficient exploitation of the entire spectrum. In this opportunistic way of spectrum usage, it has to be guaranteed by the unlicensed systems that their operation does not affect the primary users. This Ph.D. thesis deals with some of the many challenges which research community has to face with the introduction of cognitive radio principles. The first contribution of this Ph.D. thesis is on the challenge in deploying spectrum principles that significantly improves spectrum utilization efficiency without losing the benefits associated with static spectrum allocation, i.e., without provoking harmful interference toward the licensed primary users. We have identified conditions and proposed solutions/techniques for optimal usage of radio spectrum, by allowing coexistence on the same spectrum resources between primary and cognitive users. The second part of this Ph.D. thesis is dedicated to model the interactions and system dynamics between independent primary and cognitive user and to derive rules of local action at the independent cognitive users that result in stable and efficient system operation. We have modeled our scenario via a non-cooperative power control game so that the corresponding Nash equilibriums are taken as stable operation points for secondary users. The last contribution of this Ph.D. thesis has been dedicated to propose a technique for different treatment of primary interference at the cognitive receivers: under given conditions, primary interference at cognitive users should be decoded and not treated as noise.



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## 7. List of Publications



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### 7.1 Patents and Patents Application

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## **8. Publications included in the thesis**

### **8.1 Paper 1**

**Spatial opportunity for cognitive radio systems with heterogeneous path loss conditions**

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Vehicular Technology Conference, 2007. VTC2007

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# Spatial opportunity for cognitive radio systems with heterogeneous path loss conditions

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**Abstract**—In this paper, the possibility for a short-range cognitive radio (secondary communication system) to be located within the service area of the primary system is discussed. Although the secondary system interferes with the primary system, there can be certain locations in the service area of the primary system where the cognitive radio can reuse the frequency of the primary system without disturbing it or being disturbed by the primary system. We say that in those locations there is a *spatial opportunity* for communication in the secondary system. The primary and secondary systems have different features and usage scenarios, which result in different deployment conditions. This gives rise to differences between the path loss for a link in the primary system and the path loss for the interference from the primary to secondary systems. This paper investigates the impact of the heterogeneous path loss conditions on the spatial opportunity for the secondary system. Our approach can be applied as a general way to investigate the coexistence among multiple systems. We clarify that the change of the path loss coefficient due to the different antenna heights of the primary and secondary systems can largely affect the spatial opportunity for frequency reuse by the cognitive radios.

**Index Terms**—Cognitive radio, reusable area, heterogenous propagation path loss conditions, directional antenna, antenna height

## I. INTRODUCTION

Due to the large popularity of mobile phones and wireless LAN systems, the high data transmission with limited frequency band becomes key issue for wireless communication systems. We can in fact dramatically improve the frequency utilization if we allow a system to dynamically utilize not only the given frequency band but also the frequency which is allowed to the other systems, but under-utilized. In the research environment this concept is referred to as *cognitive behavior* [1]. Following this principle, if there is a certain frequency band that is unutilized by a conventional system at a particular time and geographic location, that band can be temporary exploited by a cognitive system. This kind of spectrum that is not utilized by its owner at specific time and location is called *spectrum hole*. Cognitive radios try to find the spectrum holes automatically, and utilize these frequency bands while avoiding interference with other systems. Thus, by using cognitive systems, we can significantly enhance the frequency utilization per area.

As a target of cognitive radio systems, the co-existence between the primary system with a long-range communication and secondary system with a short range transmission is considered [2]. The spectrum reusability within the primary service area depends on the amount of interference which the primary and secondary systems cause to each other. We investigated the *spatial opportunity* when a short-range secondary system should coexist with a large-range primary system that employs directional antenna [3]. We derived the spatial availability as the function of antenna and propagation's parameters and clarified that the spatial opportunities of cognitive systems could be significantly enhanced when assuming the directional antenna with narrow beamwidth for the primary systems [3].

Since the primary and secondary systems have different features and usage scenarios, deployment conditions of the primary and secondary systems are largely different with each other. For example, the base stations are located on top of the buildings when the long-range transmission for the primary systems is considered. On the other hand, transceivers of secondary systems are placed at various locations (indoor and outdoor environments with high and low antenna heights). Thus, the difference between the deployment conditions on the primary and secondary systems gives rise to difference between the path loss for a link within the primary system and the path loss for the interference from the primary to secondary system.

This paper derives the spatial opportunity for cognitive radio systems when the heterogeneous propagation path loss conditions are considered between the communication link within the primary system and the interference from the primary to secondary system. To achieve this, we introduce a novel propagation loss model [4]. This model can be utilized even if antenna heights of both the transmitter and receiver are varied. In other words, the heterogenous path loss conditions can be determined according to our target scenarios [4]. The reusable area for the secondary systems is evaluated when the several scenarios are considered in actual systems with various antenna heights of the primary and secondary systems.

The rest of this paper is organized as follows. In Section 2 we show our target system and motivation. Section 3 derives



the spatial opportunity for cognitive radio systems when we consider heterogeneous propagation loss conditions, where two different path loss coefficients are used for the link between primary users and the interference for primary to secondary transceivers. In Section 4, the spatial opportunity for the secondary systems is clarified when various antenna heights for the primary and secondary systems are assumed.

## II. TARGET AND MOTIVATION

Figure 1 shows an example of our target scenario. The characteristics of the systems with the large and small areas largely differ from each other. Our strategy exploits the features of both systems in order to reach an efficient coexistence [2][3].

As can be seen in the figure, we assume a fixed wireless system with a point to point communication between the base station (BS) and the subscriber station (SS). Fixed Wi-MAX can be considered as an actual example for such primary system. The BS antenna is positioned at high location. The SS antenna is placed at high or low location. Both the BS and SS use directional antennas to enlarge the service area: the communicable range at the direction except the main beam becomes much smaller than that of main beam, and this gives the chance to deploy the cognitive devices in the side lobe directions [3].

On the other hand, we consider a secondary system with much smaller communication range than the primary system, which attempts to reuse the frequency band given to the primary system. Since the communication is locally restricted, such secondary system has high potential to flexibly find available space within service area of the primary system. There are several examples for such short range system. As a typical system, the indoor W-LAN can be considered. However, our target is not restricted in indoor scenario. Even in outdoor scenarios, there are chances to deploy for the short-range systems such a W-LAN. In indoor scenario, we previously clarified that the secondary system can use the frequency band inside the area of the primary system with high probability [3]. Thus, in this paper, the outdoor scenario for the secondary systems is considered. We assume omnidirectional antenna in the secondary system, which is generally used in such short-range systems. The secondary system can reuse the spectrum inside the primary service area only if it can tolerate the interference from primary system and if it does not cause unacceptable interference to the primary receiver. By exploiting such *spatial opportunity*, the frequency utilization can be largely enhanced.

The propagation path loss largely affects the spatial opportunity for cognitive radio systems [5]. Kitao's model [6] is used in our previous work, because this model can be applied for 0.4 to 8 GHz band and this model is effective when considering co-existence among various systems for future wireless communication. A general expression for the path loss  $L(d)$  ( $d$ : Distance between transmitter and receiver) can be written as follows:

$$L(d) = \alpha \cdot \log_{10}(d) - \beta \text{ [dB]} \quad (1)$$

where,  $\alpha$  and  $\beta$  become  $42.7$  and  $32.7 \log_{10}(H_{BS}) - 20.2 \log_{10}(f_c) - 55.4$ , respectively, in Kitao's model.  $H_{BS}$

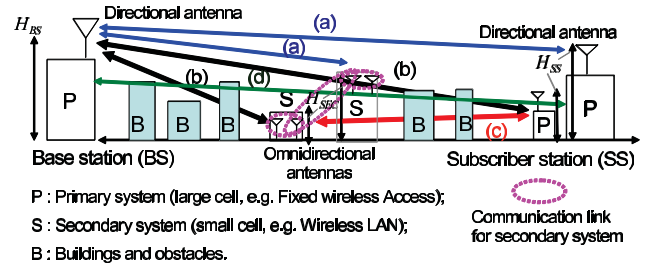


Fig. 1. Example of our target in cognitive radio system

and  $f_c$  denote the antenna height of BS and carrier frequency, respectively.

Looking at our target scenario shown in Fig.1, we can notice that the propagation path losses are obviously different among BS to/from SS, BS to/from secondary systems and SS to/from secondary systems, because the antenna heights for the primary and secondary systems are different. We can consider four kinds of path loss conditions from (a) to (d) in Fig. 1, when secondary systems or SS are placed at various locations.

- (a) Higher Tx and Rx than surrounding buildings
- (b) Higher Tx and lower Rx than surrounding buildings
- (c) Lower Tx and Rx than surrounding buildings
- (d) Others

Kitao's model can deal with condition (b), because the antenna heights of Tx and Rx are restricted over 30 m and below 2m, respectively. Since there are various antenna heights when the coexistence between the primary and secondary systems is considered, the appropriate propagation path loss is necessary to accurately deal with not only condition (b) but also conditions (a), (c), and (d). In order to consider such a situation, we introduce the path loss model in [4]. In this model, we can consider not only the path loss in (b) but also the path loss in (a), (c) and (d). In Sect. III-B, we explain this model in detail.

## III. REUSABLE AREA FOR COGNITIVE SYSTEMS WITH HETEROGENEOUS PATH LOSS CONDITIONS

### A. Definition of reusable area

In order to quantify the opportunities for the secondary users, we first define the *reusable area* for the secondary system. To this end, we need to consider all the possible cases of interference between the primary and secondary systems. When the primary and secondary systems coexist, the following types of interference can arise: Figure 2 shows the relationship between the communicable and interference areas. With the term communicable area we mean the service range in which the BS and SS can communicate using directional antennas. The interference area has instead a wider definition and will be in detail described in the following. We have to consider in fact four kinds of interference in cognitive radio systems:

- $I_1$  Interference from Primary BS to Secondary system
- $I_2$  Interference from Primary SS to Secondary system
- $I_3$  Interference from Secondary system to Primary BS
- $I_4$  Interference from Secondary system to Primary SS

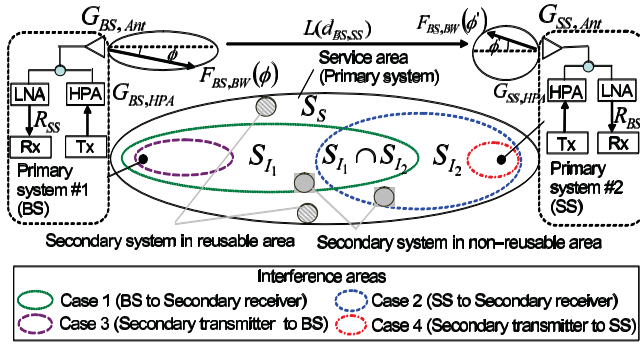


Fig. 2. Relation between communicable area and interference area.

In this paper, we assume that each receiver in the secondary system should operate above the minimum guaranteed received power. With such an assumption, if the target Carrier to Interference Ratio (CIR) is not satisfied at a certain spatial position due to the interference  $I_1$  or  $I_2$ , we consider that the spectrum is not reusable by the secondary devices. We refer to the sets of those locations as *interference areas*, respectively  $S_{I_1}$  and  $S_{I_2}$ . On the other hand, the interference areas due to the cases  $I_3$  and  $I_4$  also exist. However, since the secondary system has typically much lower transmission power and antenna gain than the primary user, it is reasonable to assume that the interference area due to  $I_3$  ( $I_4$ ) is included in the area due to  $I_1$  ( $I_2$ ). This relationship is depicted in Figure 2. Here, the *service area* of the primary system is the area where the primary receiver can achieve its target CIR when there is no interference. We finally define the *reusable area* as the service area outside the interference area, thus, outside the area  $S_{I_1} \cup S_{I_2}$  in Figure 2.

In order to analyze communicable and interference areas in Fig. 2, the definitions of the antenna gains and beam patterns for both the transmitter and receiver are required. In fact, the gain  $G_{Ant}$  can be approximately expressed as a function of the 3dB antenna beamwidth in horizontal and vertical planes ( $\theta_{BW}^h$  and  $\theta_{BW}^v$  [7]). Since the antenna gain  $G_{BS, Ant}$  on the BS is generally higher than the one  $G_{SS, Ant}$  of the SS, in this paper we set the beamwidth of the BS and SS in the vertical plane to 10 and 30 degrees, respectively. The antenna response  $F_{BW}(\phi) = \cos^n(\phi)$  ( $n$  is a positive value [8]) is a function of  $\theta_{BW}^h$  too [9]. To determine the sidelobe level, we impose that the integration of the  $G_{Ant}F_{BW}(\phi)$  from 0 to  $2\pi$  should be constant regardless of the beamwidth, in order to keep this model closer to the real antenna pattern.

We determine the transmit power  $G_{HPA}$  and the minimum received power  $R_{min}$ . These parameters are generally determined by the actual system, and become fixed values in the simulations. In this paper  $G_{HPA}$  and  $R_{min}$  for the primary system are set respectively to 33dBm and -80dBm with a 20MHz bandwidth, as specified by one of Fixed Wi-Max standard [2]. For the secondary system,  $G_{HPA}$  and  $R_{min}$  are set respectively to 2dBm and -65 dBm with a 20MHz bandwidth. These values refer to IEEE802.11a systems [10].

To obtain the communication area, we calculate the distance between the transmitter and receiver, in which the target CIR is

TABLE I  
PATH LOSS MODEL WITH HETEROGENEOUS PATH LOSS CONDITIONS

$L = [51 - 8 \log_{10}(H_{Tx}H_{Rx})] \log_{10}(d) + 8.4 \log_{10}(H_{Tx}H_{Rx}) + 20 \log_{10}(f_c/2.2) + 14$	
$L$	: Path loss (dB)
$d$	: Distance between transmitter and receiver (200-3000 m)
$H_{Tx}$	: Antenna height at the transmitter (3.5-30 m)
$H_{Rx}$	: Antenna height at the Receiver (1.5-30 m)
$f_c$	: Frequency (2 GHz band)

satisfied. By using the relationship among the transmit power, antenna gain at Tx and Rx, and propagation path loss, we can obtain the following equation:

$$\log_{10}(d) = \frac{1}{\alpha} \cdot [G_{HPA} + G_{Tx, Ant} + F_{Tx, BW}(\phi) + \beta + G_{Rx, Ant} + F_{Rx, BW}(\phi') - R_{min} + CIR] \quad (2)$$

where, if we do not need to consider the interference (or can neglect the term of CIR),  $d$  becomes the communicable distance of the primary system along direction  $\phi$ . To simplify the definition of communicable and interference areas,  $\phi'$  is assumed to be zero in the above equation. In other words, since  $F_{Rx, BW}(\phi')$  becomes 0dB, we assume that the receiver of the primary system can receive the signal with the maximum beam direction regardless of the direction on the transmitter. The distance  $d$  is a function of the response angle of  $F_{Tx, BW}(\phi)$ . The communicable area  $S_C$  can be finally obtained by integrating  $d$  in Eqn. (2) over all the directions.

We set the actual values of parameters for the secondary system in a similar way so that the interference distances for  $I_1$  and  $I_2$  and then these area  $S_{I_1}$  and  $S_{I_2}$  are obtained. Finally, the re-usable area is derived as follows:

$$Pr_1 = \left(1 - \frac{S_{I_1}}{S_C}\right) \times 100 \quad [\%] \quad (3)$$

$$Pr_2 = \left(1 - \frac{S_{I_2}}{S_C}\right) \times 100 \quad [\%] \quad (4)$$

$$Pr_{Total} = \left(1 - \frac{S_{I_1} \cup S_{I_2}}{S_C}\right) \times 100 \quad [\%] \quad (5)$$

where  $Pr_1$  and  $Pr_2$  represent the percentage reusable area for interference  $I_1$  and  $I_2$  from BS and SS to the secondary system, respectively.  $Pr_{Total}$  denotes the percentage reusable area for total interference area.

### B. Path loss model with heterogeneous path loss conditions

Table I presents the path loss model which Ichitsubo et al., proposed [4]. To obtain this equation, a lot of measurements were carried out with 2GHz band in urban areas of Japan. Moreover, the path losses obtained by the measurement result and the equation in Table I agree well with each other, when the transmissions of Tx32m-Rx32m, Tx32m-Rx15m, and Tx32m-Rx2m are considered. As can be seen in Table I, since  $\alpha$  and  $\beta$  become a function of the antenna heights of transmitter and receiver, this model can be useful for all cases from (a) to (d) in Fig. 1. Thus, we can accurately evaluate the reusable area when various kinds of location for the secondary system and SS are considered.

### C. Derivation of reusable area with heterogeneous propagation path loss conditions

In this subsection, we derive the reusable area using Eqns. (2), (3) and (4). By using Eqn. (2), the communicable distance  $d_{BS, SS}$ , and interference distances for  $I_1$  and  $I_2$   $d_{BS, SEC}$ ,  $d_{SS, SEC}$  can be expressed as follows:

$$\begin{aligned} d_{BS, SS}(\phi) &= 10^{P_C/\alpha_C} 10^{F_{BS, BW}(\phi)/\alpha_C} \\ &= K_C l_C(\phi) \end{aligned} \quad (6)$$

$$\begin{aligned} d_{BS, SEC}(\phi) &= 10^{P_{I_1}/\alpha_{I_1}} 10^{F_{BS, BW}(\phi)/\alpha_{I_1}} \\ &= K_{I_1} l_{I_1}(\phi) \end{aligned} \quad (7)$$

$$\begin{aligned} d_{SS, SEC}(\phi) &= 10^{P_{I_2}/\alpha_{I_2}} 10^{F_{SS, BW}(\phi)/\alpha_{I_2}} \\ &= K_{I_2} l_{I_2}(\phi) \end{aligned} \quad (8)$$

$$\begin{aligned} P_C &= G_{HPA} + G_{BS, Ant} + \beta_C + G_{SS, Ant} \\ &\quad - R_{min, PRI} \end{aligned} \quad (9)$$

$$\begin{aligned} P_{I_1} &= G_{HPA} + G_{BS, Ant} + \beta_{I_1} + G_{SEC, Ant} \\ &\quad - R_{min, SEC} + CIR_{SEC} \end{aligned} \quad (10)$$

$$\begin{aligned} P_{I_2} &= G_{HPA} + G_{SS, Ant} + \beta_{I_2} + G_{SEC, Ant} \\ &\quad - R_{min, SEC} + CIR_{SEC} \end{aligned} \quad (11)$$

where  $C$ ,  $I_1$  and  $I_2$  represent the terms regarding  $S_C$ ,  $S_{I_1}$  and  $S_{I_2}$ , respectively.  $P$  is prepared to separate the term of only  $F(\phi)$  from Eqn.(2). The communicable and interference areas  $S_C$ ,  $S_{I_1}$  and  $S_{I_2}$  are expressed by integrating Eqns. (6), (7) and (8), respectively. Since  $K_C$ ,  $K_{I_1}$ , and  $K_{I_2}$  are constant values, the re-usable area  $Pr_1$  and  $Pr_2$  can be derived by using Eqns. (6), (7) and (8):

$$\begin{aligned} Pr_1 &= \left[ 1 - \left( \frac{K_{I_1}}{K_C} \right)^2 \frac{\int_0^{2\pi} l_{I_1}^2(\delta) d\delta}{\int_0^{2\pi} l_C^2(\sigma) d\sigma} \right] \times 100 \quad [\%] \\ &= \left[ 1 - X_1 \frac{Y_{I_1}}{Y_C} \right] \times 100 \quad [\%] \end{aligned} \quad (12)$$

$$\begin{aligned} Pr_2 &= \left[ 1 - \left( \frac{K_{I_2}}{K_C} \right)^2 \frac{\int_0^{2\pi} l_{I_2}^2(\delta) d\delta}{\int_0^{2\pi} l_C^2(\sigma) d\sigma} \right] \times 100 \quad [\%] \\ &= \left[ 1 - X_2 \frac{Y_{I_2}}{Y_C} \right] \times 100 \quad [\%] \end{aligned} \quad (13)$$

In  $Pr_1$  and  $Pr_2$ , we first focus on constant terms  $X_1$  and  $X_2$  with  $\phi$ . These terms can be expressed by Eqns. (9), (10) and (11):

$$X_1 = 10^{2Z_1/\alpha_C \alpha_{I_1}} \quad (14)$$

$$X_2 = 10^{2Z_2/\alpha_C \alpha_{I_2}} \quad (15)$$

$$\begin{aligned} Z_1 &= (\alpha_C - \alpha_{I_1})(G_{HPA} + G_{BS, Ant}) - \alpha_{I_1} G_{SS, Ant} \\ &\quad + \alpha_C G_{SEC, Ant} + \alpha_C \beta_{I_1} - \alpha_{I_1} \beta_C \\ &\quad + \alpha_{I_1} R_{min, PRI} - \alpha_C (R_{min, SEC} + CIR_{SEC}) \end{aligned} \quad (16)$$

$$\begin{aligned} Z_2 &= (\alpha_C - \alpha_{I_2})(G_{HPA} + G_{SS, Ant}) - \alpha_{I_2} G_{BS, Ant} \\ &\quad + \alpha_C G_{SEC, Ant} + \alpha_C \beta_{I_2} - \alpha_{I_2} \beta_C \\ &\quad + \alpha_{I_2} R_{min, PRI} - \alpha_C (R_{min, SEC} + CIR_{SEC}) \end{aligned} \quad (17)$$

From the above equations, we can find several interesting insights into the relationship between the reusable area and the antenna and propagation's parameters. When  $Z_1$  and  $Z_2$  increase/decrease, we can confirm that  $Pr_1$  and  $Pr_2$

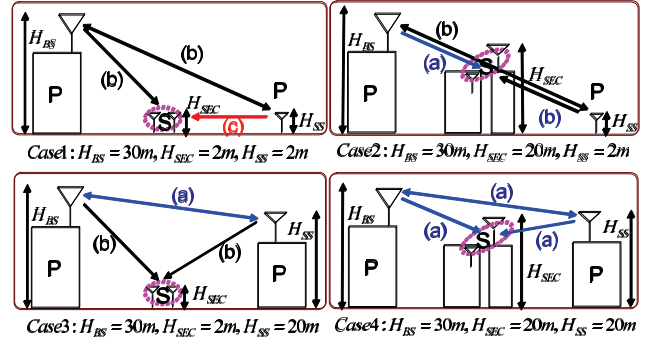


Fig. 3. Conditions with heterogeneous path loss in numerical calculation

decrease/increase. First, we notice that the antenna gain on the BS or SS doesn't affect the percentage re-usable area in  $Pr_1$  or  $Pr_2$ , respectively, when  $\alpha_C$  equals  $\alpha_{I_1}$  or  $\alpha_{I_2}$ . In other words, we can say that the effect due to the antenna gain on the BS or SS is small for  $Pr_1$  or  $Pr_2$ , respectively. If the path loss coefficient ( $\alpha_{I_1}/\alpha_{I_2}$ ) from the BS/SS to secondary system or the antenna gain on SS/BS increases,  $Pr_1/Pr_2$  increases. An opposite behavior can be seen when the path loss coefficient from the BS to SS ( $\alpha_C$ ) or the antenna gain on the secondary system increases.

We next transform the term of  $Y$ . Since the antenna response  $F(\phi)$  can be expressed as the function of  $\cos^n$ ,  $Y$  can be approximated as followings:

$$\begin{aligned} Y &\simeq \int_0^{\phi_{SL}} [\cos(\phi)]^a d\phi + 2 \int_{\phi_{SL}}^{\pi} (SL_{BW})^{40/\alpha} \\ &= \frac{\Gamma\left(\frac{a+3}{2}\right)}{(a+1)\Gamma\left(\frac{a+2}{2}\right)} + (SL_{BW})^{40/\alpha} [\pi - \phi_{SL}] \end{aligned} \quad (18)$$

where  $SL_{BW}$  and  $\phi_{SL}$  represent the sidelobe level and the angle at which  $F_{BW}(\phi_{SL})$  equals to  $SL_{BW}$ , respectively  $\Gamma(\cdot)$  is Gamma function and  $a = 40n/\alpha$ . Consequently,  $Y_1/Y_C$  is determined by the difference between  $\alpha_C$  and  $\alpha_{I_1}$ . By the numerical calculations, we confirm that the range of  $Y_1/Y_C$  is from 0.8 to 1.2 when the antenna heights of Tx and Rx are changed from 2 to 30 m.

## IV. NUMERICAL RESULTS

We evaluate the reusable area by using four cases shown in Fig. 3, where different path loss conditions (a), (b) and (c) are considered. Although we consider the fixed wireless system as the primary system as mentioned in Sect.2, a mobile terminal can be also assumed as the SS for the primary system in Case 1 and 2 with the model introduced in Sect.3. On the other hand, we suppose that the primary system is fixed wireless access/relay in Case 3 and 4. The antenna height of SS can be assumed to be high in these cases. In actual scenarios, transceivers of secondary systems are placed at various heights. For example, secondary system on Case 1 or 3 is regarded as the hot-spot scenario in outdoor environment. Case 2 or 4 assumes the usage of W-LAN between buildings (e.g.: relay/mesh network). Thus, the high antenna location of the secondary system in Case 2 or 4 is assumed.

Figure 4 presents the reusable area versus the BW of BS and SS when Case 1 to 4 are considered in Fig. 3. The target CIR is set to 20 dB in this calculation. Figure 4 first indicates that the directional antenna with narrower beamwidth increases the percentage reusable area in all cases even if we assume the heterogeneous propagation path loss conditions. As can be seen in Fig. 4, the absolute value of percentage reusable areas are largely changed among Case 1 to 4. In Case 3, the differences between communicable and interference areas ( $S_C$  and  $S_I$ ) obviously becomes large. This means that  $S_I$  becomes much smaller than  $S_C$ . Thus, the reusable area of Case 3 becomes highest in all cases.

On the other hand, since the antenna height of the SS is low (2m) on Case 1 and 2, the communicable area  $S_C$  in Case 1 or 2 decreases compared with Case 3 or 4. When we compare secondary systems between Case 1 and 2, the interference area  $S_I$  in Case 2 is greater than that in Case 1, because the antenna height of the secondary system in Case 2 is much higher than that in Case 1. As a result, the reusable area in Case 2 becomes much smaller than that in Case 1.

The result also indicates that the reusable area in Case 4 become almost same with that in Case 1. Although both  $S_C$  and  $S_I$  in case 4 become large compared with those in Case 1, the ratio of  $S_C$  and  $S_I$  is not different between Case 1 and 4. In other word, when the antenna heights between the secondary system and SS are almost same, the reusable area becomes almost constant regardless of the antenna heights of the secondary system and SS.

To understand how the difference of the antenna height on the secondary systems affects on the reusable area more clearly, we compare the percentage reusable areas versus the antenna height on the secondary system in Fig. 5. We evaluate two cases that the antenna heights of BS and SS are set to be 30, 2 m (Case 1 and 2) and 30, 20 m (Case 3 and 4).

The results show that the percentage reusable area greatly decreases as the antenna height on the secondary systems becomes higher. Particularly, when the antenna height of SS becomes lower than the secondary system, the percentage reusable area is dramatically reduced with the broader beamwidth for the primary system. This means that the ratio of  $S_C$  and  $S_I$  is largely changed according to the difference of antenna heights between the secondary system and SS and the beamwidth for the primary system.

## V. CONCLUSION

This paper derived the reusable area for cognitive radio systems with heterogeneous propagation path loss conditions for the transmission with the primary system and the interference from primary to secondary systems. We introduced a model which can be applied for evaluations of the spatial opportunity on the secondary system when it coexists with the primary system under various combinations of antenna heights. We clarified that the different coefficients on path loss due to the different antenna heights between the primary and secondary systems largely affect the spatial opportunity for the secondary users. Such an accurate modeling of propagation environment is a key to understand the actual benefits brought by cognitive radios.

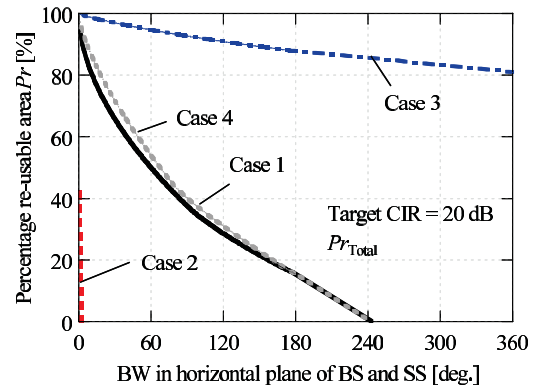


Fig. 4. The percentage reusable area with the heterogeneous path loss conditions

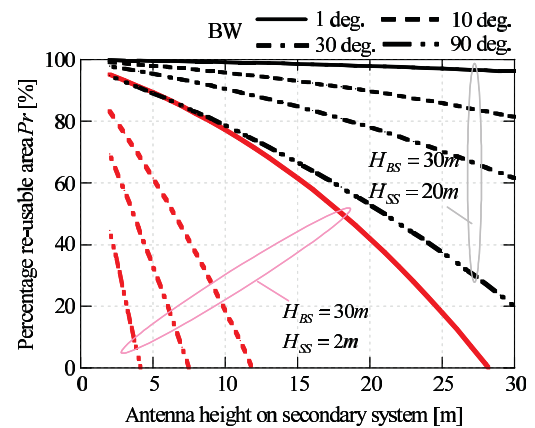


Fig. 5. The percentage reusable area versus the antenna height of the secondary system

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## 8.2 Paper 2

### **Simple Antenna Pattern Switching and Interference-induced Multi-hop Transmissions for Cognitive Radio Networks**

*Di Taranto R.; Nishimori K.; Popovski P.; Yomo H.; Takatori Y.; Prasad R.; Kubota S.;*

2nd IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks, 2007. DySPAN 2007. Page:543-546





# Simple Antenna Pattern Switching and Interference-induced Multi-hop Transmissions for Cognitive Radio Networks

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**Abstract**—This paper proposes a simple antenna switching and a relay technique for cognitive radios using adaptive array antenna to effectively reduce the interference from the primary system. While the adaptive array antenna can create null towards the direction of primary system, the created antenna pattern can result in non-optimized pattern between cognitive nodes. In order to solve such a problem, this paper first introduces a simple antenna pattern switching. With this technique, each cognitive radio node is equipped with three antennas and each node tries to select the antenna configuration resulting in the best antenna pattern for each link. Furthermore, we propose a combination between the above simple antenna switching and the interference-induced multi-hop transmission where the multi-hop transmission is utilized when a link in the network suffers from the low transmission quality due to the created antenna pattern. Our numerical results show that the proposed techniques can significantly improve the achievable data rate in cognitive networks.

**Index Terms**—Cognitive radio, interference cancellation, adaptive array, multi-hop communication, antenna pattern switching

## I. INTRODUCTION

Cognitive radios dynamically utilize the frequency band allocated to a primary system, which is not being utilized at a particular time and geographic location. [1]–[3]. A key challenge to realize such an efficient co-existence between the primary and cognitive radio systems is the interference management. When a cognitive radio system is located within the communication area of a primary system, there can be two different types of inter-system interference to handle: the interference

from the primary system to the cognitive radio and vice versa.

This paper investigates how to reduce the first type of interference while assuming that the second type of interference can be avoided by regulating the transmission of cognitive radio (e.g the possible area and the maximum transmission power). In order to reduce the interference from the primary system we introduce adaptive array antenna at the cognitive receivers [6]. As a side effect of the array adaptation, the antenna pattern created according to the interference condition from the primary system can cause serious degradation of transmission quality between the cognitive radio nodes. For example, if the antenna pattern created by a cognitive receiver to form the null toward the direction of the primary interference does not have enough gain toward the cognitive transmitter, there can be loss for the received signal power between the cognitive nodes. In order to solve this problem, this paper introduces two techniques. First, we introduce simple antenna pattern switching. With this method, there are three antennas in each node of cognitive radio and the node tries to select the antenna configuration resulting in the best antenna pattern for each link. Second, we propose a combination of antenna switching with interference-induced multi-hop transmission. Such a multi-hop transmission can alleviate the degradation of signal quality when the secondary transmitter is located in the same direction as the null created to cancel the primary interference. We show by computer simulation that the proposed method can significantly improve the achievable data rate in the cognitive radio networks.



## II. THE TARGET SCENARIO

We consider the co-existence between a primary system with a long-range communication and a secondary system<sup>1</sup> with a short-range transmission. The primary system has a *licence* to utilize a certain spectrum band, such as the cellular and TV broadcast networks. Secondary network tries to use the primary band while avoiding the interference from/to the primary systems. The spectrum reusability within the primary service area depends on the amount of interference which the primary and the secondary system cause to each other [4].

Our target scenario is depicted in Fig. 1. The broadcast transmissions, e. g. TV systems, are considered as the primary systems. The secondary network consists of several nodes which have array antennas, used to cancel the interference from the primary systems. We assume that the secondary system can generate an interference cancellation pattern by using the channel state information between the primary and secondary systems. The secondary nodes try to transmit signals to each other while creating null pattern toward the interference from the primary system. In order to simplify the problem, we assume that the transmissions of cognitive radios are regulated so that they don't cause interference to primary receivers.

To realize the interference cancellation, an adaptive array is adopted. As shown in Fig. 1, the adaptive array is realized by a two-element linear array antenna, in order to achieve hardware simplification for the secondary devices. However, when the cognitive receiver creates a null toward the direction of the primary interference, it can happen that the cognitive receiver does not have enough gain toward the cognitive transmitter because of the null pattern. In this case there is loss in the received signal power at the cognitive receivers. Since this situation arises by the linear antenna symmetry, we cannot eliminate such a grating null. In order to solve this problem, we propose new techniques using the antenna switching and relay transmissions.

## III. SIMPLE ANTENNA PATTERN SWITCHING AND INTERFERENCE-INDUCED MULTI-HOP TRANSMISSION

Figure 2 shows the proposed hardware configuration of antenna pattern switching. The proposed configuration has two antenna arrangements (x-axis and y-axis) at a cognitive node. These two antenna arrays create different grating nulls at different angle while they make null toward the same direction of the interference from

<sup>1</sup>In this paper, we use the terms of cognitive radio and secondary system interchangeably.

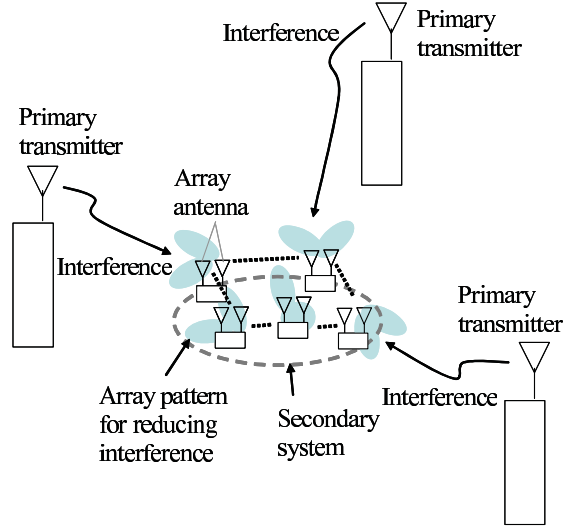


Fig. 1. The target scenario to investigate the coexistence between the primary and the secondary systems.

primary system. The reason why the different grating null is generated is described later. In order to simplify the hardware configuration, we switch the antenna 2 and 3 by using a simple SPDT (Single-Pole Dual-Throw) switch according to the transmission quality between secondary nodes.

Figure 3 shows an example of operation of the proposed antenna switching. The secondary receiver cancels the interference from the primary transmitter (PT). However, a grating null arises at the different angle when using the linear array antenna. This grating null itself cannot be controlled by the receiver. When this grating null is in the direction of secondary transmitter, the signal quality of the secondary system is largely degraded. In the proposed scheme, we utilize the difference of the grating nulls between array antenna on X-axis and Y-axis at the receiver site. Figure 3 shows that the antenna response is higher towards the direction of ST if the array antenna on Y-axis is selected.

The proposed antenna switching can reduce the impact of the grating null on the signal quality. However, the degradation in the secondary transmission cannot be avoided when the angle of arrivals of primary interference matches with the direction of ST.

We select the path of nodes and the combination of pattern which maximizes the achievable transmission rate in the secondary system.

## IV. EFFECTIVENESS OF THE PROPOSED METHOD

### A. Simulation Setup

Figure 4 shows the deployment of the primary and secondary systems in our evaluation. Table I shows the simulation parameters.

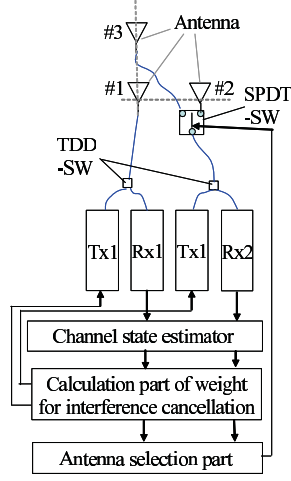


Fig. 2. Configuration of proposed interference cancellation with simple antenna switching

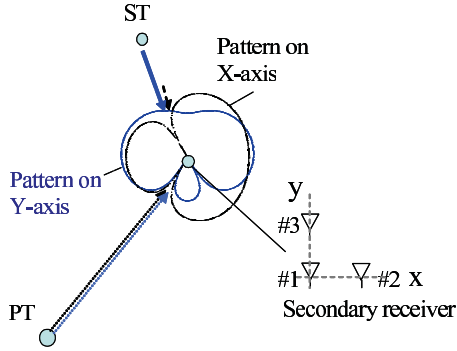


Fig. 3. Operation of proposed antenna switching at the receiver (PT: Primary transmitter, ST: Secondary transmitter)

As it can be seen in Figure 4, we assume a simple scenario, with a single primary transmitter and a secondary system that consists of three nodes. We assume that the three antenna elements at each cognitive radio node are arranged as shown in Figure 4. In order to evaluate the influence of the angle of arrival (AOA) on the interference from the primary system, we change the rotating angle  $\theta_R$  of the secondary system. However, we always keep the same antenna arrangement regardless of the rotating angle  $\theta_R$ . The element spacing is 0.5 wavelengths. The Signal-to-Interference Ratio (SIR) is the ratio between the desired signal in the secondary system and the interference from the primary system in the initial conditions i. e. prior to the array adaptation. The SIR is set to be 0 dB as the severe condition of the interference from the primary system. As the conventional method, we consider the two-element array antenna used at the secondary nodes without multi-hop operation. The array is located on x-axis, regardless of the rotating angle. For both conventional and proposed methods, the array weight is given based on Zero forcing

TABLE I  
SIMULATION PARAMETERS

Parameter	Value
Frequency	5GHz
Transmit power of the primary system	33 dBm
Transmit power of the secondary system	19 dBm
Antenna gain of the secondary system	10dBi
Antenna gain of the secondary system	2dBi
Modulation schemes	QPSK
Angular spread	0–360 deg.
Path loss coefficient	3.5
SNR at between secondary nodes	20 dB

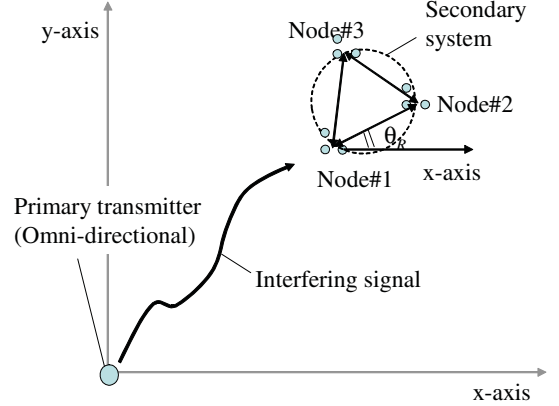


Fig. 4. Antenna deployment between primary and secondary systems used in the simulation

algorithm.

The achievable transmission rate in the secondary network is evaluated in order to compare the conventional scheme and the proposed method. In the case of the conventional scheme, the total achievable transmission rate  $R_{conv}$  in the secondary system is defined as follows:

$$R_1 = \log_2(1 + \text{SINR}_{12}) \quad (1)$$

$$R_2 = \log_2(1 + \text{SINR}_{31}) \quad (2)$$

$$R_3 = \log_2(1 + \text{SINR}_{23}) \quad (3)$$

$$R_{conv.} = \sum_{k=1}^3 R_k \quad (4)$$

where  $\text{SINR}_{ij}$  is Signal to Interference and Noise Ratio for the link from node  $i$  to  $j$ . On the other hand, for the proposed method, two antenna patterns are first generated at each node. Then, the pattern combination between two nodes to give the maximum received SINR is selected for each link. From these SINRs,  $R_1$ ,  $R_2$ , and  $R_3$  are calculated. Then we choose the link which gives the minimum rate, and check if the multi-hop transmission can give higher rate or not. If so, the rate with multi-hop is utilized to calculate the achievable rate  $R_{selection}$ . The calculation is summarized as the

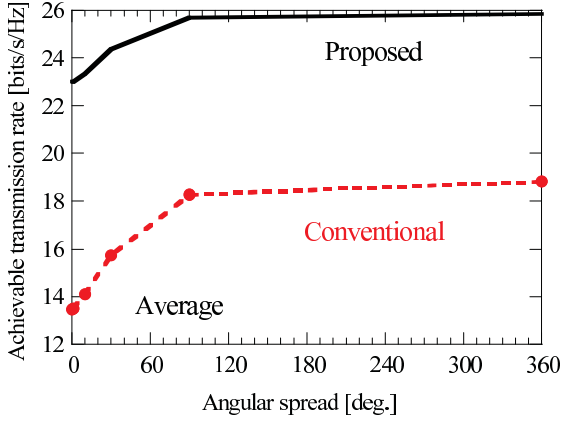


Fig. 5. Achievable transmission rate vs. angular spread by using the proposed scheme

following equations:

$$R_{min} = \min(R_1, R_2, R_3) \quad (5)$$

$$R_{multi-hop} = \frac{R_i R_j}{R_i + R_j} \quad (6)$$

$$R_{selection} = \max(R_{min}, R_{multi-hop}) + R_i + R_j \quad (7)$$

where  $R_{multi-hop}$  is the achievable transmission rate when applying multi-hop scheme to the link with the worst rate.  $R_i$  and  $R_j$  are the rate in the links which are not the worst link.

### B. Simulation results

Figure 5 shows the average achievable rate of the proposed and conventional schemes against the angular spread. As it can be seen in Figure 5, for small values of angular spread, the probability that secondary transmission is degraded is very high. We can obtain an improvement by the proposed method in the average transmission rate larger than 9 bit/sec/Hz. With the small angular spread, the signal is received at the receiver from a limited direction. Therefore, if the secondary transmitter is located in the direction of the grating null, the signal quality is largely degraded. On the other hand, with the larger angular spread, the signal is received more uniformly from different angles, which reduces the degradation due to the grating null. Moreover, we can see from Figure 5 that the achievable transmission rate is significantly improved by the proposed scheme regardless of the value of angular spread.

## V. DISCUSSION AND CONCLUSION

In this paper we have investigated how we can improve the achievable transmission rate when the sec-

ondary system employs the interference cancellation with adaptive array antenna to reduce the primary interference. We have introduced antenna pattern switching which adaptively chooses the antenna pattern generating the favorable grating null according to the interference condition from the primary system. Moreover, the usage of relay technique was introduced to enhance the transmission quality in the secondary system. Our numerical results have shown that our proposed method can significantly improve the achievable rate in the cognitive radio networks.

In this paper, we assumed that the transmission of cognitive radios is regulated so that it does not cause the harmful interference to the primary receivers. In our future work, we will investigate how the antenna pattern and relay transmission should be controlled to minimize such interference. We will also extend our proposed scheme to handle multiple primary systems coexisting with cognitive radios. Another interesting study is to study the gain brought by our proposed method in the network consisting of more users in a network.

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### **8.3 Paper 3**

#### **Cognitive Mesh Network under Interference from Primary System**

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10<sup>th</sup> International Symposium on Wireless Personal Multimedia Communication, WPMC 2008,



# Cognitive Mesh Network under Interference from Primary System

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**Abstract**—In a commonly accepted usage scenario, a cognitive radio appears as a secondary user of certain spectrum which is licensed to another, primary system. A prominent example of cognitive system is a mesh network operating under the interference from primary system. For such a scenario, we propose techniques for efficient secondary usage of spectrum, which rely on the adaptive array antenna in order to reduce the interference between the primary and the cognitive system. In order to keep the hardware complexity as low as possible, the number of antennas at each cognitive node should be small. However, with the simplest 2-element linear adaptive array, the created antenna pattern can result in non-optimized pattern between cognitive nodes in mesh network. In order to solve such a problem, in a previous work we have proposed an interference cancellation/avoidance using an antenna switching when the cognitive nodes are equipped with 2-element linear array antenna. In this paper, we investigate the effectiveness of the antenna pattern switching when it is applied in a cognitive mesh environment. Moreover, we introduce a simple procedure to heuristically select the pattern when the cognitive nodes are equipped with 3-element linear array. Our numerical results show that the proposed techniques can significantly increase the available bandwidth and networking connectivity with small complexity when a cognitive mesh network is located inside the communication area of the primary system.

**Index Terms**—Cognitive radio, dynamic spectrum sharing, interference management, adaptive array, antenna switching, networking connectivity

## I. INTRODUCTION

Cognitive mesh networks dynamically utilize the frequency band allocated to a primary system [1], [2]. A key challenge to realize such an efficient coexistence between the primary and cognitive mesh network is the interference management [3], [4]. When the cognitive mesh networks are located within the communication area of the primary system, there can be two different types of inter-system interference to handle: the interference from the primary system to cognitive mesh network and the interference from cognitive mesh network to primary system.

In this paper, we investigate how to reduce/avoid these two types of interference by introducing adaptive array antenna at the cognitive receivers and transmitters. Adaptive array systems can sense the presence of interference sources and suppress them, while simultaneously enhancing the reception of the desired signal [5]. The interference suppression does not require prior knowledge of the interfering signals, but only capability to detect the direction of arrival for the interference and a corresponding adaptation of the array. In such a way, the cognitive nodes can reuse the primary frequency band even if they are located in a highly-interfered area. In order to keep the hardware complexity as low as possible, the number of antennas at each cognitive node should be small. However, the adaptation of the simplest linear array with 2-element antenna to cancel the interference has consequences for the communication in the cognitive mesh and causes additional problems.

The first problem occurs with the interference cancellation at the cognitive receivers. When the cognitive receiver creates a null

towards the direction of the primary interference, a side effect can be that the obtained antenna pattern produces an insufficient antenna gain towards the corresponding cognitive transmitter. This effect can occur under certain geometrical conditions, thereby deteriorating the communication performance in the cognitive mesh network. The second problem occurs when considering the avoidance of the interference that the cognitive mesh network induces to the primary system. Generally, there can be many receivers in the primary system, when we assume multi-user system, such as cellular networks. Hence it is essential for the cognitive system not to cause an excessive interference towards each of the primary receivers. In order to do this, many nulls would be needed for the cognitive transmitter but that implies a large number of antennas in the linear array and in this case the hardware complexity grows significantly. In particular, the increase in the number of transceivers as well as antennas implies higher hardware costs, when considering small terminal stations. Furthermore, as the number of elements in the linear array increases, the degree of freedom for possible beam pattern significantly increases, and the pattern-optimization problem becomes much more complicated.

In [6] we have proposed an interference cancellation/avoidance using an antenna switching with a simple hardware configuration, when the cognitive transmitters/receivers are equipped with 2-element array antenna.

In this paper, 1) we investigate the effectiveness of the antenna pattern switching when it is applied in a mesh network environment and 2) we introduce a simple procedure to heuristically select the pattern to reduce interference towards the primary system while maximizing the transmission rate in cognitive mesh link when the cognitive nodes are equipped with 3-element linear array antenna. We show that the 3-element linear array antenna with heuristic pattern selection slightly outperforms the antenna switching technique in terms of network connectivity. Moreover, our numerical results show that the antenna switching, with simpler hardware configuration, outperforms the 3-element array in terms of usable links in the mesh.

## II. TARGET SCENARIO

We consider the coexistence between the primary system with a long or middle-range communication and cognitive mesh network with a short range transmission. The primary system has a license to use a certain spectrum band, such as cellular networks. The cognitive mesh network tries to reuse the same frequency spectrum while decreasing/avoiding the interference from/to primary system. The spectrum reusability within the primary service area can be significantly improved by reducing the amount of interference which the primary and the cognitive mesh network can cause to each other. Our target scenario is depicted in Figure 1. The primary system broadcasts the signal over its service area. There is a single primary transmitter (base station) and multiple, uniformly-located primary



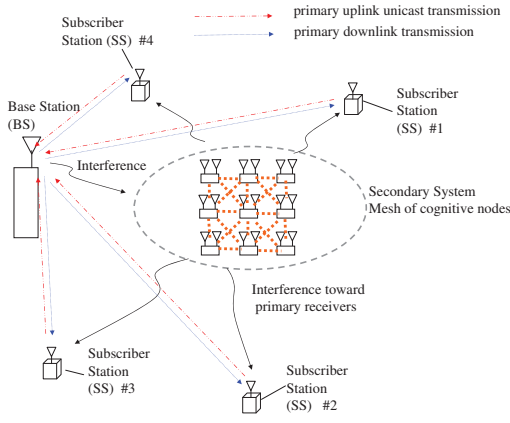


Fig. 1. The target scenario

receivers (subscriber stations) within the service area. The secondary system is a cognitive network, whose nodes are connected with mesh topology as shown in Figure 1. Each cognitive node tries to establish connections with all the other nodes in the mesh network. The departing point in this paper is the assumption that the cognitive system tries to detect the pilot signal transmitted *a)* by the primary transmitter e.g. when a downlink transmission is established in the primary system and *b)* by the primary receivers e.g. when they establish an uplink unicast transmission with the primary transmitter. By using such a pilot signal from primary system, cognitive system estimates the channel state information (CSI), as channel response, *a)* from the primary transmitter to each cognitive receiver and *b)* from all primary receivers to each cognitive transmitter. In this paper, we treat the case when the secondary system attempts to communicate during a downlink transmission in primary system. This case is more interesting than the uplink because generally larger channel resources in time and frequency domain are assigned in downlink channel than in uplink channel. Moreover, the interference towards the primary system is more critical during a downlink transmission. In our scenario there is one primary transmitter and multiple randomly-located primary receivers. Thus, during a primary uplink transmission cognitive system must avoid interference only toward the primary transmitter, while interference management toward all the primary receivers is needed during a downlink transmission in primary system. We assume that the cognitive system knows the noise power, transmitter power and antenna gain of the primary receivers. With such information cognitive transmitter can estimate the interference-to-noise-ratio in primary receivers. Cognitive nodes transmit and receive signals recurrently over the primary band. All the cognitive nodes are equipped with array antenna. When a cognitive node transmits signal, it can generate an interference avoidance pattern by using adaptive array. On the other hand, when a cognitive node is in reception mode, it can generate a pattern to cancel the interference from the primary system. We assume that 2-element or 3-element linear array antenna is adopted as hardware configuration in cognitive nodes.

### III. SIMPLE ANTENNA PATTERN SWITCHING AND 3-ELEMENTS HEURISTIC ALGORITHM

#### A. Simple Antenna Pattern Switching

Figure 2 shows the proposed hardware configuration of antenna pattern switching. It has two antenna arrangements, i.e. x-axis (antenna #1 and #2) and y-axis (antenna #1 and #3), so that these two linear antenna arrays create different grating nulls at different angle while making null towards the same direction of

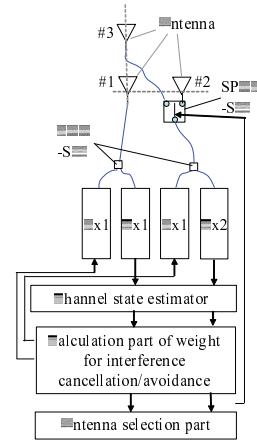


Fig. 2. Configuration of the proposed interference cancellation with simple antenna switching

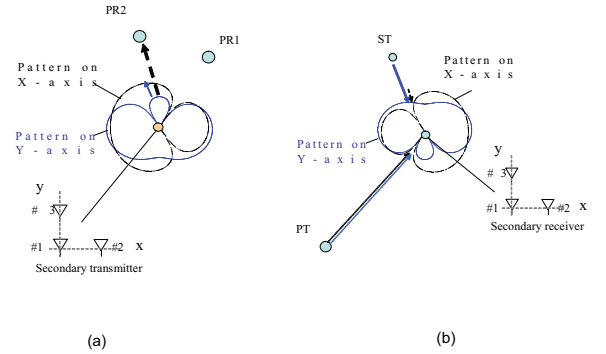


Fig. 3. Operation of the proposed antenna switching at the transmitter (3a) and receiver (3b)

the interference from/to primary system. In order to simplify the hardware configuration, we switch the antenna 2 and 3 by using a simple SPDT (Single - Pole Dual - Throw) switch according to the transmission quality between secondary nodes. The basic premise in this method is that, in each cognitive node, each of the two possible antenna arrangements creates a different grating null while creating the same 'desired' null. We exploit this characteristic of linear array for both interference avoidance and cancellation in the cognitive receivers and transmitters. Figure 3a illustrates an example of operation of the proposed antenna switching at the transmitter site. In Figure 3a, the primary receiver (PR) 1 is the closest among all primary receivers. Since only two element array is considered for each of the possible configurations, only one null can be controlled at cognitive transmitter. Thus, we employ the interference cancellation using only CSI of PR1. However, as it can be seen in Figure 3a, there is higher interference to PR 2 when using the antenna pattern on the x-axis, which may not satisfy the required interference condition for primary receiver. In such a case, the antenna switching selects the antenna arrangements on y-axis.

Next, Figure 3b illustrates an example of operation of the antenna switching at the receiver site. As it can be seen in Figure 3b, the cognitive receiver can cancel the interference from the primary transmitter (PT). A grating null arises at a different angle when using the linear array antenna. This grating null itself cannot be controlled

at the receiver. When this grating null direction almost agrees with the direction of the cognitive transmitter, the transmission quality of the cognitive system is degraded. Figure 3b shows that the antenna response is not degraded if the array antenna on y-axis is selected. Thus, with these two examples we can see that the antenna switching, exploiting the fact that different antenna arrangements have different grating nulls, can be applicable both for reception and transmission mode. With the proposed antenna pattern switching two patterns (corresponding to the antenna arrangements on x-axis and y-axis and obtained with a method based on Zero-Forcing algorithm) are always available at the cognitive receiver. When considering the cognitive transmitter two different patterns are generated for each of the two possible antenna arrangements: one that cancel the interference and one that maximizes the antenna gain towards the intended secondary receiver. But before starting transmission, it has to be checked if these four transmitting patterns cause harmful interference to any of the primary receivers. To this end, each cognitive transmitter estimates, for each pattern, the possible interference-to-noise-ratio (INR) at the primary receivers. We assume that the cognitive system knows the following parameters of the primary receivers: noise power, transmitter power and antenna gain. With this assumption, the cognitive nodes first estimates the path loss gain,  $GL$ , between subscriber station and cognitive nodes (during uplink primary transmission), as follows:

$$GL = \frac{INR_{sec-rx} \times P_{n-sec}}{P_{ss-tx} \times G_{ss-tx} \times G_{sec-rx}} \quad (1)$$

where,  $INR_{sec-rx}$  is the interference-to-noise-ratio at the cognitive node ( $I$  in the pilot signal from subscriber station towards cognitive node),  $P_{ss-tx}$  and  $G_{ss-tx}$  are the transmitting power and the antenna gain in the subscriber station,  $P_{n-sec}$  and  $G_{sec-rx}$  are the noise power and the antenna gain in the cognitive node. Then, exploiting this estimated path loss gain and the information about noise power and antenna gain in the primary receiver, the cognitive transmitter estimates the interference-to-noise-ratio,  $INR_{ss-rx}$ , in the subscriber station, as follows:

$$INR_{ss-rx} = \frac{P_{n-sec} \times P_{sec-tx} \times INR_{sec-rx}}{P_{n-ss} \times P_{ss-tx}} \quad (2)$$

where  $P_{sec-tx}$  is the transmitting power in the cognitive transmitter,  $P_{n-ss}$  and  $P_{ss-tx}$  are noise power and the transmitting power in the subscriber station. With this method, the cognitive transmitter estimates the INR level in dB in all the primary receivers (subscriber stations). If the maximum INR is less than a parameter called *INR threshold* we decide that such pattern is available at the cognitive transmitter. Among all the available patterns at cognitive transmitter and receiver, the set of pattern resulting in the best signal-to-interference-plus-noise-ratio (SINR) in the cognitive transmission is selected. Thus, with proposed antenna switching, the observed SINR at the cognitive receiver is the maximum SINR among the possible combinations of patterns at the cognitive transmitter and receiver, satisfying the condition on the INR threshold. On the contrary, cognitive transmission is not allowed if any transmission pattern at cognitive transmitter causes harmful interference toward primary receivers.

#### B. Heuristic Algorithm for 3-element linear array

When the cognitive nodes are equipped with the 3-element linear array antenna, many strategies can be implemented to create pattern at cognitive nodes, in order to achieve the best rate under the interference from/to primary system. The optimality also depends on how accurately each cognitive node can estimate CSI. Therefore, in order to fully exploit the potential of the 3-element linear array antenna, we need a complicated optimization process for creating

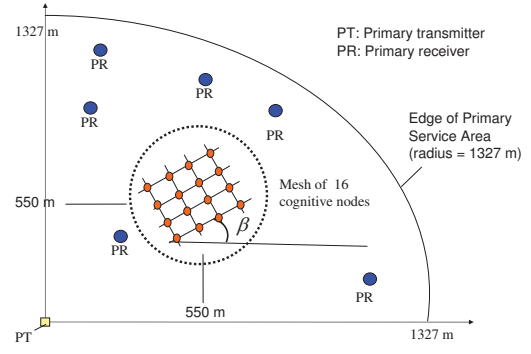


Fig. 4. Geometry relationship between the primary and cognitive systems in the simulation

the beam pattern. In order to avoid such a complexity, in this paper, we introduce a simple heuristic algorithm to create the pattern for 3-element linear array applied to cognitive mesh network.

With this method, the cognitive receiver always creates one pattern which has one null towards the primary transmitter and maximum gain in the direction of the corresponding cognitive transmitter. On the other hand, the cognitive transmitter first creates the following three patterns: 1) two nulls towards the two closest primary receivers, 2) one null toward the closest primary receiver and maximization of antenna gain in the direction of the corresponding primary receiver, 3) maximization of antenna gain in the direction of the corresponding cognitive receiver, regardless of primary receivers.

For any of the created pattern, each cognitive transmitter checks whether cognitive transmitter can make their transmission without causing harmful interference to all the primary receivers in the same way as the antenna-switching described in Sec. III-A. Then, among the available patterns at cognitive transmitter and receivers, the best non-interfering pattern in terms of the observed SINR at the cognitive receiver is selected for each case.

### IV. NUMERICAL EVALUATION OF THE PROPOSED SCHEME

#### A. Simulation Conditions

Fig. 4 shows the geometrical relationship between the primary and cognitive system for our simulation. We consider a single sector cell with 90 degrees as the primary service area. Primary service area radius is 1327 m. The primary receivers are uniformly located inside the primary service area. The mesh of cognitive nodes, whose center is (550m, 550m), is located inside the primary service area. 4X4 mesh is considered. Uniformly-rotated angle  $\beta$  is introduced in order to consider different positions of cognitive mesh network. Given  $n$  nodes in the mesh,  $n(n-1)$  possible links in the network are considered.

The path loss coefficient for propagation between the cognitive transmitter and receiver is set to 3.5 [7], while the path loss coefficient between primary and secondary nodes is obtained from the Ichitsubo model [8]. Rayleigh fading environment is assumed and the angular spread is set to 0 [degree]. We assume that cognitive antenna receives 8 reflected and scattered waves. The amplitude of each of these waves is assumed to be the same while random phase is generated for each of them. The SNR at the edge of primary service area is fixed to 20dB (i. e. when the interference from cognitive system is not considered). The element spacing of the 2- and 3-element array antenna is 0.5 wavelengths, and the array axis is randomly determined. The array weight which cancels/avoids interferences from/to primary system is given by using the method based on ZF algorithm. In our scenario the



estimation of CSI can not be perfect, and there can be a difference between ideal and estimated CSI. Let's for example consider the estimation of CSI between the primary transmitter and the antenna 1 on the  $k$ -th cognitive receiver. In this case, the estimated CSI is obtained as follows:

$$h'_{1k} = h_{1k} + \frac{1}{\gamma} \times \Delta h_{1k} \quad (3)$$

where  $h'_{1k}$  and  $h_{1k}$  represent estimated and ideal CSI, respectively,  $\Delta h_{1k}$  denotes the estimation error of CSI, and  $\gamma$  is the received SNR of the pilot signal transmitted by the primary transmitter. Similarly, the estimated CSI between each primary receiver and each antenna on the cognitive transmitters is obtained. In this case, as  $\gamma$ , we use the received SNR of the pilot signal transmitted by the primary receivers.

In our study we evaluate and compare link and network layer metrics of a cognitive mesh network whose nodes are equipped respectively with *a)* 2-element linear array antenna, *b)* 3-element linear array antenna with heuristic algorithm and *c)* simple antenna pattern switching.

We define the "Percentage of unavailable cognitive link" as follows:

$$P_d = \frac{k}{n \times (n-1)} \times 100 \quad (4)$$

where,  $k$  is the number of links in which cognitive mesh network *a)* is not allowed to start transmission (i.e. there is no available pattern at the cognitive transmitter due to the harmful interference towards primary system) or *b)* doesn't satisfy the target SINR at the cognitive receiver (we assume that a link in the mesh is connected if SINR at the cognitive receiver is greater than 10dB).  $n$  is the total number of nodes in the mesh.

We also define "Percentage of mesh full connectivity" as the percentage of the ratio between the number of *connected* mesh we observed in the simulation and the total number of trials in the simulation. A mesh is said to be *connected* if there exists a bi-directional connection between any two nodes in the mesh. The connection can be established using either a direct link (between two nodes in the mesh) or a multi-hop connection (getting packets from the source to the destination nodes in the mesh requires usage of intermediate forwarding nodes).

Moreover, in our study we investigate the mean number of hops needed to get packets from source to destination nodes in the cognitive mesh network, when the shortest path between the source and the destination is selected.

Besides the above performance metric for cognitive transmissions, we also define a measure for primary system. We evaluate the outage probability of the primary receiver, which is defined as the percentage of the ratio between the number of primary links which cannot satisfy the target SINR of the primary system and the number of primary links (a pair of primary transmitter and receivers) generated during our simulation. That is, the above probability shows how often the performance of the primary system is degraded below the required level. As described in Sec. III, the secondary transmitter checks whether it can start transmission or not by using the possible INR in primary receivers and INR threshold. However, cognitive nodes cannot know the received power of the desired primary's signal at each primary receiver. Therefore, the outage at primary receiver can still occur, e.g. when the primary receiver does not have sufficient level of received power from primary transmitter. Furthermore, the estimation of the INR at primary receivers is made based on the imperfect CSI. Therefore, the CSI estimation error can also degrade the outage probability. In our evaluation, we set the target SINR of the primary system to 20 [dB].

TABLE I  
OUTAGE PROBABILITY OF PRIMARY RECEIVER [%]

	2 element	antenna switching	3 element
INR Threshold = 5 dB	1.45	1.76	1.89
INR Threshold = 6 dB	1.71	1.95	2.78
INR Threshold = 7 dB	2.12	2.31	3.23

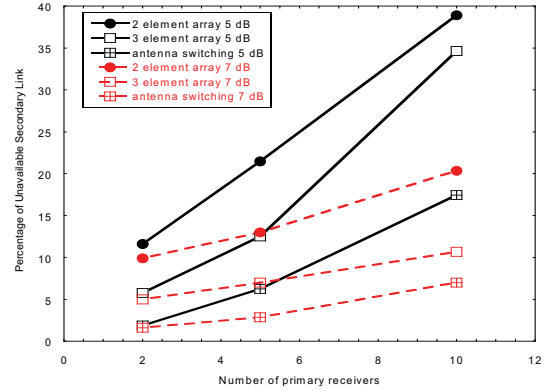


Fig. 5. Percentage of unavailable cognitive link vs. number of primary receivers (INR threshold = 5 and 7 dB)

### B. Simulation results

First, we analyze the impact of the cognitive transmissions on the outage performance of the primary system. Table I shows the outage probability of primary receiver for different schemes with different value of the INR threshold. From this table, we can see that the outage probability of the primary system is controlled to be the order of a few percent with each scheme. As expected, we can observe higher outage probability for higher INR threshold which gives more chances for the cognitive transmitter to start its transmission. We can also notice that 3-element linear array has higher outage probability than the other two schemes. The performance of 3-element linear array depends the CSI estimation most largely among 3 schemes as it requires CSI of 2 links when it creates 2 nulls toward the primary receivers. In our scenario, the estimation of CSI is not ideal, and the 3-element linear array is largely affected by such an imperfectness. This causes a bit higher outage probability for the 3-element linear array.

Figure 5 shows the percentage of unavailable cognitive link against the number of primary receivers with the INR threshold of 5 and 7 dB. This disconnectivity is function of number of primary receivers, and can be strongly reduced by using antenna pattern switching and 3-elements array antenna. Effectiveness of these two proposed techniques is clear, regardless of number of primary receivers. We can also see that the disconnectivity is improved (i.e., the percentage becomes lower) as we increase the interference threshold. This is because, with the larger value of interference threshold, the cognitive transmitter has more chances to initiate its transmission. However, this improvement is at the cost of the performance of the primary receiver, as shown in Table I. Therefore, the trade-off between the cognitive and primary performance should be controlled by changing the interference threshold. Moreover we can see that the antenna switching outperforms the 3-element array with heuristic scheme in terms of available links in the mesh. Generally the 3-element array causes higher interference towards primary receivers than the 2-element array. Therefore, when the INR threshold is fixed, the 2-element array has more opportunity to start transmission.

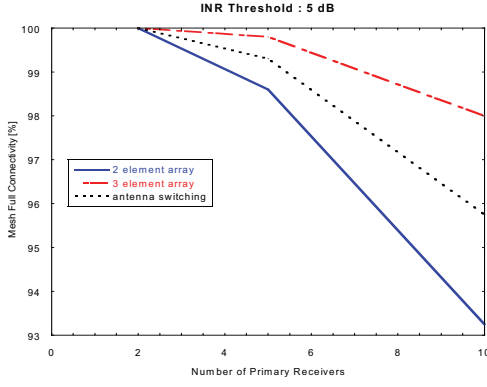


Fig. 6. Percentage of mesh full connectivity vs. number of primary receivers (INR threshold = 5 dB)

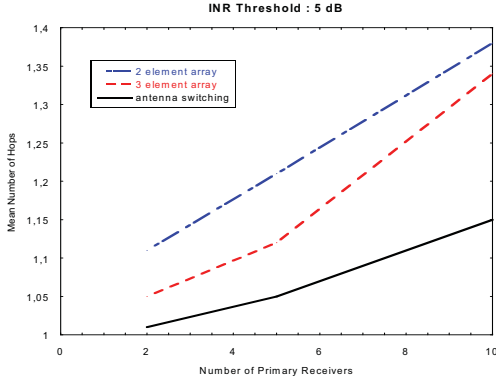


Fig. 7. Mean number of hops versus primary receivers (INR threshold = 5 dB)

Figure 6 shows the percentage of mesh full connectivity against the number of primary receivers with the INR threshold set to 5 dB. As seen from the figure, the more the primary receivers is, the less the percentage of full connectivity is. This happens because when the number of primary receivers increases, number of unavailable links in the mesh increases. We can notice that the 3-element array has higher mesh connectivity than antenna switching. This happens because in the range of disconnectivity for secondary links shown in Figure 5, the event when the mesh is not fully connected occurs very rarely, and mostly in specific situations where the 3-element array can exploit its potential to avoid interference toward two primary receivers. Let's for example consider a scenario with two primary receivers highly interfered from the primary mesh network. The 3-element array has a potential to avoid interference toward two primary receivers, while the antenna switching can avoid interference towards only one primary receiver. In this specific situation only the mesh equipped with the 3-element array is fully connected.

Finally, we investigate the average number of hops, with the shortest path, needed to get packets from source to destination in the cognitive mesh network. This result refers only to connected mesh (i.e. when there is possibility to establish a bidirectional connection between any two nodes in the mesh). As seen in Figure 7, the average number of hops increases when the number of primary receivers increases. With a lot of primary receivers, the percentage of unavailable links in the mesh is higher. Thus, even when the mesh is fully connected, the mean number of required hops to deliver a packet is increased. It is worth noting that the worse the performance is in terms of unavailable links (Figure 5), the higher the required number

of hops is for each of the three antenna arrangement (Figure 7).

## V. DISCUSSION AND CONCLUSION

In this paper, we have investigated how we can increase the available bandwidth and the networking connectivity when a cognitive mesh network is located inside the communication area of the primary system and employs the interference cancellation/avoidance with adaptive array antenna. We have compared the performances of the 2-element and 3-element linear array when they are applied to a cognitive mesh network. We have shown that the trade-off between the performance of primary and cognitive mesh networks can be controlled by a threshold parameter. We have also shown that the antenna switching and 3-element linear antenna with the simple heuristic algorithm can significantly improve the networking performance as compared with the 2-element linear array. The antenna switching can achieve competitive performance with simpler hardware complexity as compared with the 3-element linear array with heuristic algorithm.

The investigation with multiple types of primary networks is one of interesting future work. In that case, interference management from many primary system towards the cognitive mesh network and from cognitive mesh network towards many primary receivers is needed. It will be therefore necessary to find a more elaborate solution to this issue. Moreover, an interesting work could be to derive upper/lower bounds for primary and cognitive performance with optimized beam pattern for 3-element array.

## ACKNOWLEDGEMENT

This work is in part supported by Danish Research Council for Technology and Production Sciences, (project no. 274-05-0490).

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## **8.4 Paper 4**

### **Cognitive Mesh Network under Interference from Primary User**

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Wireless Personal Communications, Volume 45, Number 3 / May, 2008,  
Page 385-401



## Cognitive Mesh Network Under Interference from Primary User

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Published online: 27 February 2008  
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**Abstract** In a commonly accepted usage scenario, a cognitive radio appears as a secondary user of certain spectrum which is licensed to another, primary system. A prominent example of cognitive system is a mesh network operating under the interference from primary system. For such a scenario, we propose techniques for efficient secondary usage of spectrum, which rely on the adaptive array antenna in order to reduce the interference between the primary and the cognitive system. In order to keep the hardware complexity as small as possible, the number of antennas at each cognitive node should be small. However, with the simplest 2-element linear adaptive array, the created antenna pattern can result in non-optimized pattern between cognitive nodes in the mesh network. In order to solve such a problem, this paper introduces a simple antenna pattern switching where each cognitive node is equipped with three antennas, and tries to select the antenna configuration constituting 2-element linear array with the best antenna pattern for each link. The proposed configuration requires three antennas but only two transceiver chains, which can reduce the hardware complexity. We also introduce 3-element linear array and design a simple procedure to heuristically select the pattern. Our numerical results show that the proposed techniques can significantly increase the available bandwidth and networking connectivity with small complexity when a cognitive mesh network is located inside the communication area of the primary system.

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**Keywords** Cognitive radio · Dynamic spectrum sharing · Interference management · Adaptive array · Antenna switching · Networking Connectivity

## 1 Introduction

The large popularity of mobile phones and Wireless Local Area Network (WLAN) has challenged these systems with an increasing demand to offer the target service quality within a constrained frequency bandwidth. As the utilization of a given spectrum portion becomes saturated, additional spectrum allocation is needed to meet those demands. However, measurements studies [1] have shown that the legacy system owning certain frequency spectrum, called *primary system*, exhibits an inefficient spectrum usage due to the absence of communication traffic in certain periods/regions. In order to efficiently increase the spectrum utilization, one solution is to enable an unlicensed system (*cognitive system*) to access the spectrum which is unused by the primary system. Such an approach to the spectrum usage is termed “dynamic spectrum sharing” by the research community [2–6].

A prominent example of cognitive system is a mesh network under the interference from primary system. A key challenge to achieve an efficient co-existence between mesh network and primary system is the interference management. When the cognitive mesh networks are located within the communication area of the primary system, there can be two different types of inter-system interference to handle: the interference from the primary system to cognitive mesh network and the interference from cognitive mesh network to primary system.

In this paper, we investigate how to reduce/avoid these two types of interference by introducing adaptive array antenna at the cognitive receivers and transmitters. Adaptive array systems can sense the presence of interference sources and suppress them, while simultaneously enhancing the reception of the desired signal [7]. The interference suppression does not require prior knowledge of the interfering signals, but only capability to detect the direction of arrival for the interference and a corresponding adaptation of the array. In such a way, the cognitive nodes can reuse the primary frequency band even if they are located in a highly-interfered area. In order to keep the hardware complexity as small as possible, the number of antennas at each cognitive node should be small. However, the adaptation of the simplest linear array with 2-element antenna to cancel the interference has consequences for the communication in the cognitive mesh and causes additional problems.

The first problem occurs with the *interference cancellation* at the cognitive receivers. When the cognitive receiver creates a null towards the direction of the primary interference, a side effect can be that the obtained antenna pattern produces an insufficient antenna gain towards the corresponding cognitive transmitter. This effect can occur under certain geometrical conditions, thereby deteriorating the communication performance in the cognitive mesh network. The second problem occurs when considering the *avoidance of the interference* that the cognitive mesh network induces to the primary system. Generally, there can be many receivers in the primary system, when we assume multi-user system, such as cellular networks. Hence it is essential for the cognitive system not to cause an excessive interference towards each of the primary receivers. In order to do this, many nulls would be needed for the cognitive transmit pattern. Nevertheless, the number of nulls is limited when considering a small number of elements in the array antenna. One solution could be to simply increase the number of antennas in the linear array, but in this case the hardware complexity becomes much more significant. Particularly, to increase the number of transceivers as well as the antennas implies higher hardware costs, when considering small terminal stations. Furthermore, as the

number of elements in the linear array increases, the degree of freedom for possible beam pattern significantly increases, and the pattern-optimization problem becomes much more complicated.

In this paper we analyze and compare the performances of the 2-element and 3-element linear array antenna when they are applied to cognitive mesh network. Interference avoidance/cancellation patterns are created at the cognitive transmitters/receivers equipped with linear array antenna. In order to improve the network performance of cognitive system with smaller complexity (1) we propose an interference cancellation/avoidance using an antenna switching with a simple hardware configuration, and (2) we introduce a simple procedure to heuristically select the pattern to reduce interference towards the primary system while maximizing transmission rate in cognitive mesh link when the cognitive nodes are equipped with 3-element linear array antenna. In the former case, there are three antennas in each node of cognitive system and the node tries to select the antenna configuration constituting 2-element linear array with the best antenna pattern for each link. This scheme requires three antennas, but only two transceiver chains at each node, which can reduce the hardware complexity. The different antenna arrangements create different grating nulls at different angles while making a null towards the same direction of interference from/to the primary system. The key point in this proposal is to efficiently utilize the fact that these grating nulls are different and each node can select the one which reduces more the interference to the primary receivers. We show that the 3-element linear array antenna with heuristic pattern selection slightly outperforms the antenna switching technique in terms of network connectivity. Moreover, our numerical results show that the antenna switching, with simpler hardware configuration outperforms the 3-element array in terms of usable links in the mesh. A link in the mesh is said to be usable if (a) the cognitive transmitter is allowed to start transmission (i.e. it doesn't provoke harmful interference toward primary system) and (b) the target quality at the cognitive receiver is satisfied.

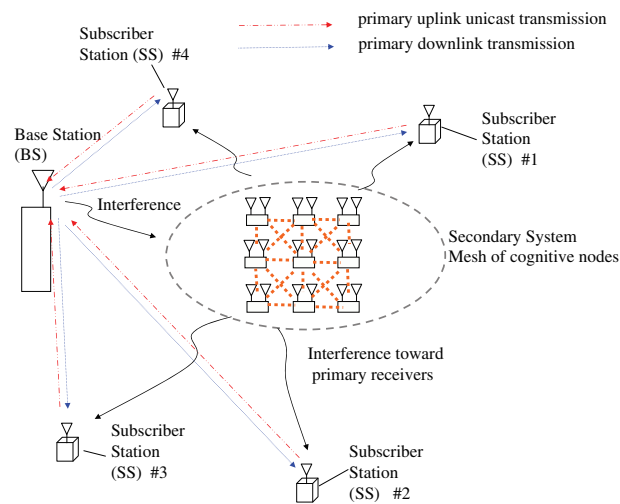
The rest of the paper is organized as follows. Section 2 describes our target scenario and the problem to realize the target system. Section 3 presents the proposed antenna pattern switching and a simple heuristic algorithm to decide the beam pattern with 3-element linear array. The simulation results and discussions are given in Sect. 4. Section 5 concludes the paper with possible future work.

## 2 Target Scenario and Problem Definition

### 2.1 Scenario

We consider the coexistence between the primary system with a long or middle-range communication and cognitive mesh network with a short range transmission. The primary system has a license to use a certain spectrum band, such as cellular networks. The cognitive mesh network tries to reuse the same frequency spectrum while decreasing/avoiding the interference from/to primary system. The spectrum reusability within the primary service area can be significantly improved by reducing the amount of interference which the primary and the cognitive mesh network can cause to each other. Our target scenario is depicted in Fig. 1. The primary system broadcasts the signal over its service area. There is a single primary transmitter (base station) and multiple, uniformly-located primary receivers (subscriber stations) within the service area. The secondary system is a cognitive network, whose nodes are connected with mesh topology as shown in Fig. 1. Each cognitive node can potentially have a connection with any other node in the mesh network.





**Fig. 1** The target scenario

The departing point in this paper is the assumption that the cognitive system tries to detect the pilot signal transmitted (a) by the primary transmitter e.g. when a downlink transmission is established in the primary system and (b) by the primary receivers e.g. when they establish an uplink unicast transmission with the primary transmitter. By using such a pilot signal from primary system, cognitive system estimates the channel state information (CSI), as channel response, (a) from the primary transmitter to each cognitive receiver and (b) from each primary receiver to each cognitive transmitter. In this paper, we treat the case when the secondary system attempts to communicate during a downlink transmission in primary system. This case is more interesting than the uplink because generally larger channel resources in time and frequency domain are assigned in downlink channel than in uplink channel. Moreover, the interference towards the primary system is more critical during a downlink transmission. In our scenario there is one primary transmitter and multiple randomly-located primary receivers. Thus, during a primary uplink transmission cognitive system must avoid interference only towards the primary transmitter, while interference management towards all the primary receivers is needed during a downlink transmission in primary system. Cognitive nodes transmit and receive signals recurrently over the primary band. All the cognitive nodes are equipped with array antenna. When a cognitive node transmits signal, it can generate an interference avoidance pattern by using the adaptive array. On the other hand, when a cognitive node is in reception mode, it can generate a pattern to cancel the interference from the primary system. We assume that 2-element and 3-element linear array antenna is adopted as hardware configuration in cognitive nodes. Before starting transmission, the secondary transmitters first check if they create harmful interference to any of the primary receivers. To this end, each cognitive transmitter estimates, for each pattern, the possible interference-to-noise-ratio (INR) in primary receivers, where  $I$  is the interfering signal power from cognitive transmitter to primary receiver and  $N$  is the noise power in primary receiver.

We assume that the cognitive system knows the following parameters of the primary receivers: noise power, transmitter power and antenna gain. With this assumption, the cognitive nodes first estimate the path loss gain,  $G_{I_i}$ , between subscriber station and cognitive nodes (during uplink primary transmission), as follows:

$$G_l = \frac{\text{SNR}_{\text{pilot}} \times N_{p,\text{Sec}}}{P_{\text{SS},Tx} \times G_{\text{SS},Tx} \times G_{\text{Sec},Rx}} \quad (1)$$

where,  $\text{SNR}_{\text{pilot}}$  is the signal-to-noise-ratio at the cognitive node ( $S$  is the average received power of the pilot signal from subscriber station toward cognitive node,  $N$  is the noise in the cognitive receiver),  $P_{\text{SS},Tx}$  and  $G_{\text{SS},Tx}$  are the transmitting power and the antenna gain in the subscriber station,  $N_{p,\text{Sec}}$  and  $G_{\text{Sec},Rx}$  are the noise power and the antenna gain in the cognitive node. Then, exploiting this estimated path loss gain and the information about noise power and antenna gain in the primary receiver, the cognitive transmitter estimates the interference-to-noise-ratio,  $\text{INR}_{\text{SS},Rx}$ , in the subscriber station, as follows:

$$\text{INR}_{\text{SS},Rx} = \frac{N_{p,\text{Sec}} \times P_{\text{Sec},Tx} \times \text{SNR}_{\text{pilot}}}{N_{p,\text{SS}} \times P_{\text{SS},Tx}} \quad (2)$$

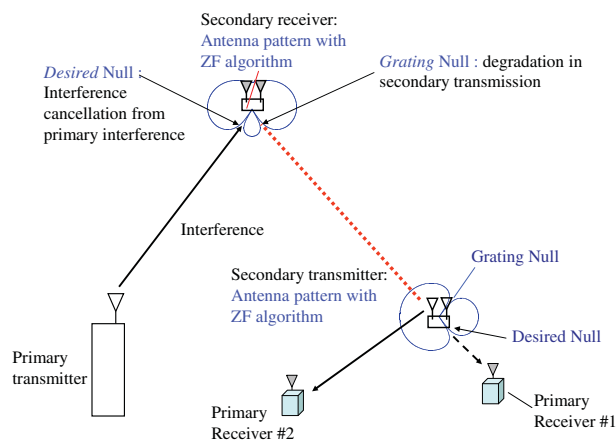
where  $P_{\text{Sec},Tx}$  is the transmitting power in the cognitive transmitter,  $N_{p,\text{SS}}$  is the noise power in the subscriber station. If the maximum INR among all the primary receivers is less than a parameter called *INR threshold* we decide that such pattern is available at the cognitive transmitter. This INR threshold controls the amount of opportunity for the secondary transmitters to start their transmission. Higher interference threshold implies higher probability for the secondary transmitter to be able to start its transmission.

## 2.2 Problems Using 2-element Linear Adaptive Array

The adaptive array antenna has a potential to alleviate the interference problem in primary/cognitive coexistence. However, the introduction of the array antenna at each cognitive node requires additional hardware complexity especially when we apply such cognitive techniques to small devices. In order to limit such a complexity, the number of antenna elements at each node should be as small as possible. However, if we use the simplest 2-element linear array for cognitive mesh network, we have the following two problems.

The first problem occurs when we consider the interference from the primary to cognitive system. In this case, although the 2-element linear array can reduce the interference from the primary system by creating a null towards the direction of primary interference, as a side effect, the created antenna pattern can cause serious degradation of transmission quality between the cognitive nodes. As seen in Fig. 2, it can happen that the antenna pattern created by the cognitive receiver does not have enough gain towards the cognitive transmitter. Thus, there can be a loss for the received signal between cognitive nodes. In this figure, this happens because the cognitive transmitter is in the direction of the 'grating' null created in the cognitive receiver applying the zero-forcing (ZF) algorithm. "Grating" null is the undesired null generated in the antenna pattern due to the antenna pattern symmetry. The "desired" null is the null we create in order to cancel the interference from primary transmitter.

The second problem occurs when considering the avoidance of the interference that the cognitive system can create to the primary system. Generally, multiple mobile terminals are connected to a single base station in the primary system. The cognitive system must avoid the interference towards all the mobile receivers in the primary system. The problem is that the number of nulls is limited when the cognitive nodes are equipped with few elements linear array antenna. In fact, as seen in Fig. 2, only one null can be controlled at the cognitive transmitter when it is equipped with 2-element array. In this figure, it happens that the antenna pattern created by the cognitive transmitter to avoid interference toward the primary receiver #1 (i.e. the most interfered), provokes new interference toward primary receiver #2. One strategy to solve this problem could be to increase the number of antennas in the linear array. In fact, in the case of  $n$ -element array,  $(n - 1)$  interferences can be ideally avoided, assuming



**Fig. 2** Problem using conventional adaptive array

that perfect CSI can be estimated for all the  $(n - 1)$  sources of interference. Such an ideal estimation is not always possible with practical conditions. Moreover when increasing the number of antennas, the hardware complexity becomes much higher. Furthermore the degree of freedom for possible beam pattern significantly increases, and the pattern-optimization problem becomes much more complicated.

In order to solve these problems related to adaptive array performance, in the next section we propose a simple antenna pattern switching technique (Sect. 3.1) and a simple procedure to heuristically select the antenna pattern in cognitive transmitters equipped with 3-elements linear array antenna (Sect. 3.2).

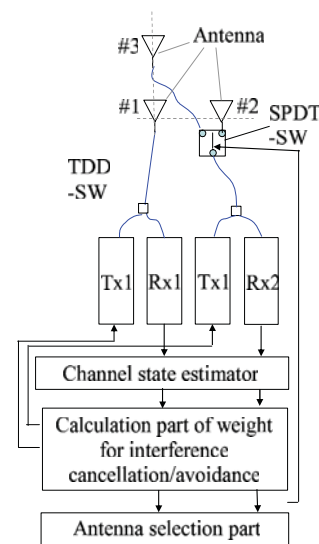
### 3 Simple Antenna Pattern Switching and 3-elements Heuristic Algorithm

#### 3.1 Simple Antenna Pattern Switching

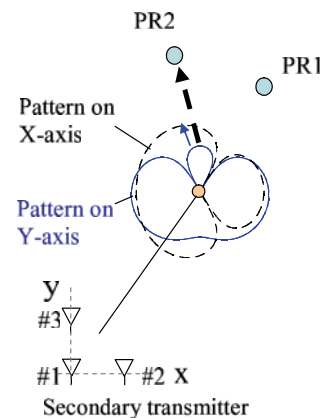
Figure 3 shows the proposed hardware configuration of antenna pattern switching. It has two antenna arrangements, i.e.  $x$ -axis (antenna #1 and #2) and  $y$ -axis (antenna #1 and #3), so that these two linear antenna arrays create different grating nulls at different angle while making null towards the same direction of the interference from/to primary system. In order to simplify the hardware configuration, we switch the antenna 2 and 3 by using a simple SPDT (Single—Pole Dual—Throw) switch according to the transmission quality between secondary nodes. The basic premise in this method is that, in each cognitive node, each of the two possible antenna arrangements creates a different grating null while creating the same ‘desired’ null. We exploit this characteristic of linear array for both interference avoidance and cancellation in the cognitive transmitters and receivers.

Figure 4 illustrates an example of operation of the proposed antenna switching at the transmitter site. We assume that CSI (as channel response) from all primary receivers has been previously estimated. In Fig. 4, the primary receiver (PR) 1 is the closest among all primary receivers. Since only two element array is considered for each of the possible configurations, only one null can be controlled at cognitive transmitter. Thus, we employ the interference avoidance using only CSI of PR1. However, as it can be seen in Fig. 4, there

**Fig. 3** Configuration of the proposed interference cancellation with simple antenna switching



**Fig. 4** Operation of the proposed antenna switching at the transmitter

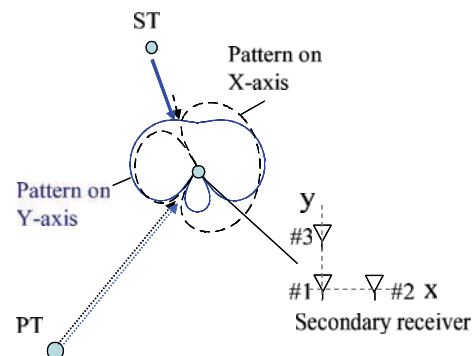


is higher interference to PR 2 when using the antenna pattern on the  $x$ -axis, which may not satisfy the required interference condition for primary receiver. In such a case, the antenna switching selects the antenna arrangements on  $y$ -axis.

Next, Fig. 5 illustrates an example of operation of the antenna switching at the receiver site. As it can be seen in Fig. 5, the cognitive receiver can cancel the interference from the primary transmitter (PT). A grating null arises at the different angle when using the linear array antenna. This grating null itself cannot be controlled at the receiver. When this grating null direction almost agrees with the direction of the cognitive transmitter, the transmission quality of the cognitive system is degraded. Figure 5 shows that the antenna response is not degraded if the linear array antenna on  $Y$ -axis is selected. Thus, with these two examples we can see that the antenna switching, exploiting the fact that different antenna arrangements have different grating nulls, can be applicable both for reception and transmission mode.

With the proposed antenna switching two patterns (corresponding to the antenna arrangements on  $x$ -axis and  $y$ -axis and obtained with a method based on ZF algorithm as described in Sect. 4.1) are always available at the cognitive receiver. When considering the

**Fig. 5** Operation of proposed antenna switching at the receiver



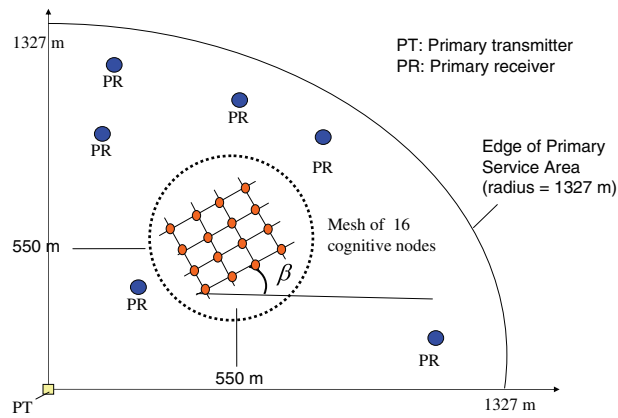
cognitive transmitter two different patterns are generated for each of the two possible antenna arrangements: one that avoids the interference toward the most interfered primary receiver and the other that maximizes the antenna gain towards the intended secondary receiver. Before starting transmission, the secondary transmitter first checks if these four transmitting patterns are available, i.e. not causing harmful interference to any of the primary receivers, with the method described in Sect. 2. Among the available patterns at cognitive transmitter and receiver, the set of pattern resulting in the best signal-to-interference-plus-noise-ratio (SINR) in the cognitive transmission is selected. Thus, with the proposed antenna switching, the observed SINR at the cognitive receiver is the maximum SINR among the possible combinations of patterns at the cognitive transmitter and receiver, satisfying the condition on the INR threshold. On the contrary, cognitive transmission is not allowed if any transmission pattern at cognitive transmitter causes harmful interference toward primary receivers.

### 3.2 Heuristic Algorithm for 3-element Linear Array

When the cognitive nodes are equipped with the 3-element linear array antenna, many strategies can be implemented to create pattern at cognitive nodes, in order to achieve the best rate under the interference from/to primary system. The optimality also depends on how accurately each cognitive node can estimate CSI. Therefore, in order to fully exploit the potential of the 3-element linear array antenna, we need a complicated optimization process for creating the beam pattern. In order to avoid such a complexity, in this paper, we introduce a simple heuristic algorithm to create the pattern for 3-element linear array applied to cognitive mesh network.

With this method, the cognitive receiver always creates one pattern which has one null towards the primary transmitter and maximum gain in the direction of the corresponding cognitive transmitter. On the other hand, the cognitive transmitter first creates the following three patterns: (1) two nulls towards the two most interfered primary receivers, (2) one null toward most interfered primary receiver and maximization of antenna gain in the direction of the corresponding cognitive receiver, (3) maximization of antenna gain in the direction of the corresponding cognitive receiver, regardless of primary receivers.

For any of the created pattern, each cognitive transmitter checks whether cognitive transmitter can make their transmission without causing harmful interference to all the primary receivers with the method described in Sect. 2. Then, among the available patterns at cognitive transmitter and receivers, the best available pattern in terms of the observed SINR at the cognitive receiver is selected for each case.



**Fig. 6** Geometry relationship between the primary and cognitive systems in the simulation

## 4 Numerical Evaluation of the Proposed Scheme

### 4.1 Simulation Conditions

Figure 6 shows the geometrical relationship between the primary and cognitive system for our simulation. We consider a single sector cell with  $90^\circ$  as the primary service area. Primary service area radius is 1,327 m. The primary receivers are uniformly located inside the primary service area. The mesh of cognitive nodes, whose center is (550m, 550m), is located inside the primary service area.  $4 \times 4$  mesh is considered. Uniformly-rotated angle  $\beta$  is introduced in order to consider different positions of cognitive mesh network. Given  $n$  nodes in the mesh,  $n(n - 1)$  possible links in the network are considered.

The propagation path loss coefficients from primary to cognitive system are obtained assuming Ichitsubo model [8]. This model can express the path loss when the antenna heights at transmitter and receiver are parameterized. In our scenario the antenna heights of the primary transmitter and receiver are 30 and 5 m, respectively. The propagation path loss coefficient between the cognitive transmitter and receivers (antenna height is fixed to 2 m for both) is 3.5 [10]. Rayleigh fading environment is assumed and the angular spread ranges from  $0^\circ$  to  $360^\circ$  [9]. We assume that cognitive antenna receives eight reflected and scattered waves. The amplitude of each of these waves is assumed to be the same while random phase is generated for each of them. The target SNR at the edge of primary service area is fixed to 20 dB (i.e. when the interference from cognitive system is not considered). The element spacing of the 2- and 3-element array antenna is 0.5 wavelengths, and the array axis is randomly determined. The array weight which cancels/avoids interferences from/to primary system is given by using the method based on ZF algorithm. When considering the 2-element linear array antenna, the weights are obtained as follows:

$$y_k = h'_{1k} \times w_{1k} + h'_{2k} \times w_{2k} = 0 \quad (3)$$

$$w_{1k} = \alpha \quad (4)$$

$$w_{2k} = \alpha \frac{h'_{1k}}{h'_{2k}} \quad (5)$$

where,  $h'_{1k}$  and  $h'_{2k}$  are the estimated CSI as channel responses from the primary transmitter to the antenna 1 and 2, respectively, on the  $k$ th node (or channel response from antenna 1 and 2 on the  $k$ th node to the most interfered primary receiver, in the case of interference avoidance).  $\alpha$  is a constant value which parameterizes the weights. In our scenario the estimation of CSI cannot be perfect, and there can be a difference between ideal and estimated CSI. Let's for example consider the estimation of CSI between the primary transmitter and the antenna 1 on the  $k$ th cognitive receiver. In this case, the estimated CSI is obtained as follows:

$$h'_{1k} = h_{1k} + \frac{1}{\gamma} \times \Delta h_{1k} \quad (6)$$

where  $h'_{1k}$  and  $h_{1k}$  represent estimated and ideal CSI, respectively,  $\Delta h_{1k}$  denotes the estimation error of CSI, and  $\gamma$  is the received SNR of the pilot signal transmitted by the primary transmitter. Similarly, the estimated CSI between each primary receiver and each antenna on cognitive transmitters is obtained. In this case, as  $\gamma$ , we use the received SNR of the pilot signal transmitted by the primary receivers.

In our study we evaluate and compare link and network layer metrics of a cognitive mesh network whose nodes are equipped respectively with (a) 2-element linear array antenna, (b) 3-element linear array antenna with heuristic algorithm and (c) simple antenna pattern switching.

We define the “Percentage of unavailable cognitive link” as follows:

$$P_d = \frac{k}{n \times (n - 1)} \times 100 \quad (7)$$

where,  $k$  is the number of links in which cognitive mesh network (a) is not allowed to start transmission (i.e. there is no available pattern at the cognitive transmitter due to the harmful interference towards primary system) or (b) doesn't satisfy the target SINR at the cognitive receiver (we assume that a link in the mesh is connected if SINR at the cognitive receiver is greater than 10dB).  $n$  is the total number of nodes in the mesh.

We also define “Percentage of mesh full connectivity” as follows:

$$P_f = \frac{C}{N} \times 100 \quad (8)$$

where,  $C$  is the number of *connected* mesh which we observed during the simulations and  $N$  is the total number of trials in the simulations. A mesh is said to be *connected* if there exists a bi-directional connection between any two nodes in the mesh. The connection can be established using either a direct link (between two nodes in the mesh) or a multi-hop connection. A multi-hop connection is established when getting packets from the source to the destination nodes in the mesh requires usage of intermediate forwarding nodes.

Moreover, in our study we investigate the mean number of hops needed to get packets from source to destination nodes in the cognitive mesh network, when the shortest path between the source and the destination is selected.

Besides the above performance metric for cognitive transmissions, we also define a measure for primary system. We evaluate the outage probability of the primary receiver, which is defined as:

$$P_o = \frac{L_d}{L_p} \times 100 \quad (9)$$

where  $L_p$  is the number of primary links (a pair of primary transmitter and receivers) generated during our simulation, and  $L_d$  is the number of primary links which cannot satisfy

**Table 1** Outage probability of primary receiver (%)

	Two element	Antenna switching	Three element
INR threshold = 5 dB	1.45	1.76	1.89
INR threshold = 6 dB	1.71	1.95	2.78
INR threshold = 7 dB	2.12	2.31	3.23

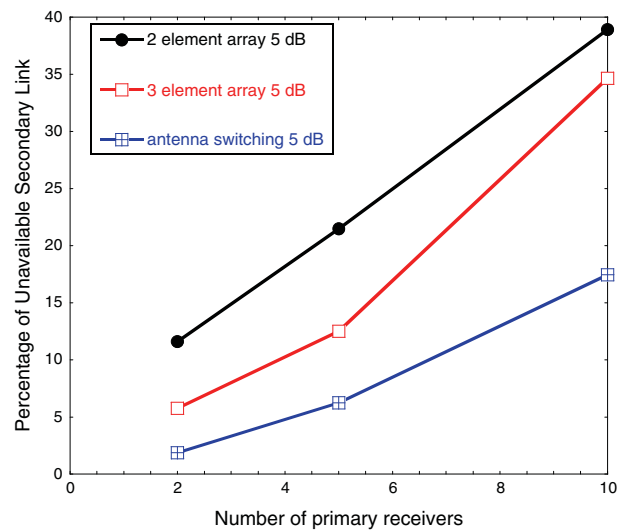
the target SINR of the primary system. That is, the above probability shows how often the performance of the primary system is degraded below the required level. As described in Sect. 3.1, the secondary transmitter checks whether it can start transmission or not by using the possible INR in primary receivers and INR threshold. However, cognitive nodes cannot know the received power of the desired primary's signal at each primary receiver. Therefore, the outage at primary receiver can still occur, e.g. when the primary receiver does not have sufficient level of received power from primary transmitter. Furthermore, the estimation of INR at primary receivers is made based on the imperfect CSI. Therefore, the CSI estimation error can additionally degrade the outage probability. In our evaluation, we set the target SINR of the primary system to 20 dB.

#### 4.2 Simulation Results

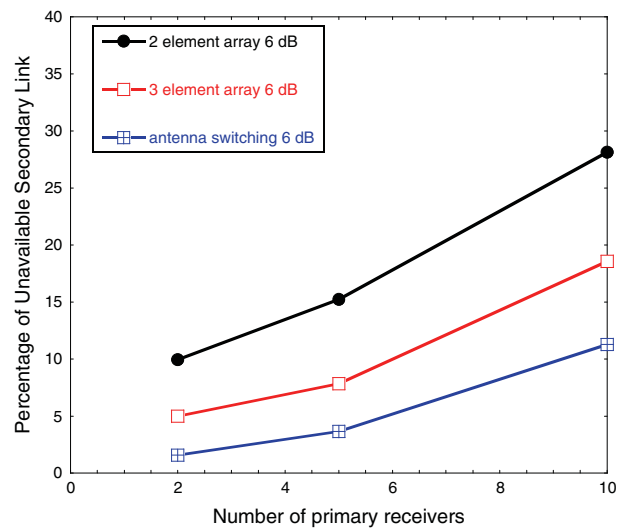
First, we analyze the impact of the cognitive transmissions on the outage performance of the primary system. Table 1 shows the outage probability of primary receiver for different schemes with different value of the INR threshold. From this table, we can see that the outage probability of the primary system is controlled to be the order of a few percent with each scheme. As expected, we can observe higher outage probability for higher INR threshold which gives more chances for the cognitive transmitters to start their transmissions. We can also notice that 3-element linear array has higher outage probability than the other two schemes. The performance of 3-element linear array depends the CSI estimation most largely among three schemes as it requires CSI of two links when it creates two nulls towards the primary receivers. In our scenario, the estimation of CSI is not ideal, and the 3-element linear array is largely affected by such an imperfectness. This causes a higher outage probability for 3-element linear array.

Figures 7–9 show the percentage of unavailable cognitive link against the number of primary receivers with the INR threshold of 5, 6, and 7 dB, respectively. This disconnection is function of number of primary receivers, and can be strongly reduced by using antenna pattern switching and 3-elements array antenna. Effectiveness of these two proposed techniques is clear, regardless of number of primary receivers. We can also see that the disconnection is improved (i.e. the percentage becomes lower) as we increase the interference threshold. This is because, with the larger value of the interference threshold, the cognitive transmitter has more chances to initiate its transmission. However, this improvement is at the cost of the performance of the primary receiver as shown in Table 1. Therefore, the trade-off between the cognitive and primary performance should be controlled by changing the interference threshold. Moreover we can see that the antenna switching outperforms the 3-element array with heuristic scheme in terms of available links in the mesh. Generally the 3-element array causes higher interference toward the primary receivers than the 2-element array. Therefore, when the INR threshold is fixed, the 2-element array has more opportunity to start transmission. It is worth noting that when the INR threshold is increased (Figs. 7–9), the difference of performance between antenna switching and 3-element array is decreased. This





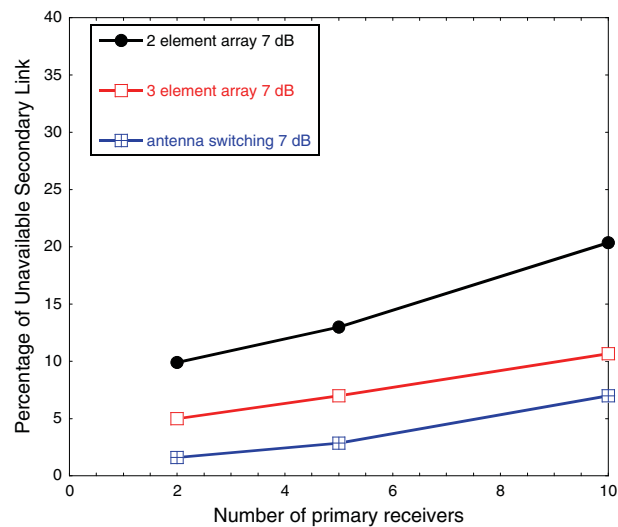
**Fig. 7** Percentage of unavailable cognitive link versus number of primary receivers (INR threshold = 5 dB)



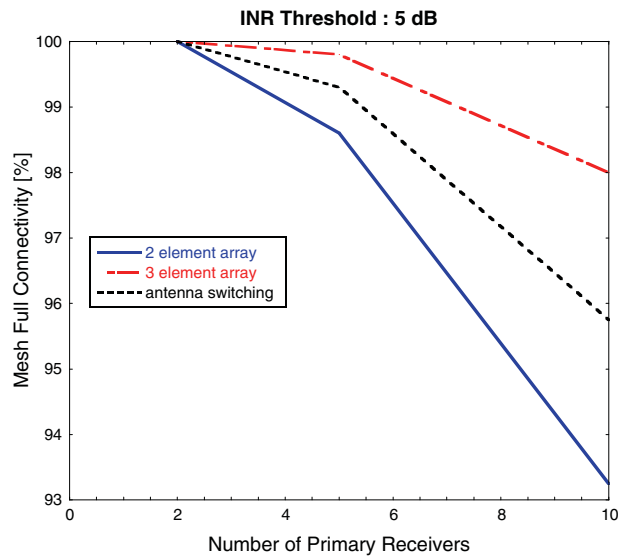
**Fig. 8** Percentage of unavailable cognitive link versus number of primary receivers (INR threshold = 6 dB)

happens because the 3-element array, causing higher interference toward primary receiver than the 2-element array, has more opportunity to start its transmission when the allowed interference toward the primary system is increased.

Figure 10 shows the percentage of mesh full connectivity against the number of primary receivers with the INR threshold set to 5 dB. As seen from the figure, when the number of primary receivers increases, the percentage of mesh full connectivity decreases. This happens because when the number of primary receivers increases, the number of unavailable links in the mesh increases, as previously shown in Fig. 7. We can notice that the 3-element array has higher mesh connectivity than antenna switching. This happens because in the range

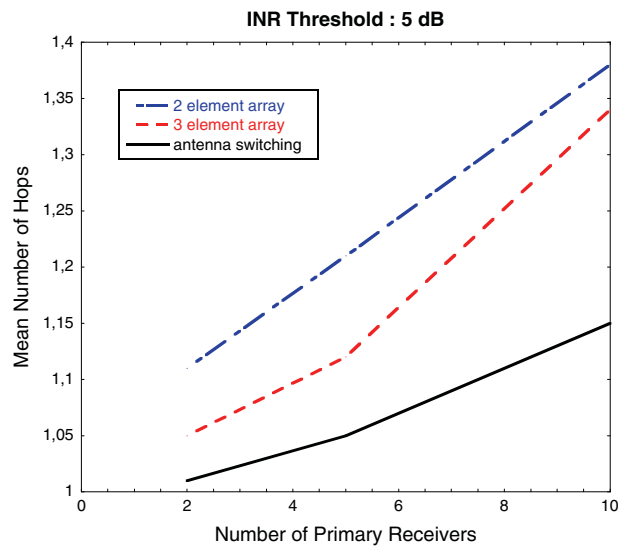


**Fig. 9** Percentage of unavailable cognitive link versus number of primary receivers (INR threshold = 7 dB)



**Fig. 10** Percentage of mesh full connectivity versus number of primary receivers (INR threshold = 5 dB)

of disconnectivity shown in Fig. 7 (INR threshold 5 dB), the event when the mesh is not fully connected occurs very rarely, and mostly in specific situations where the 3-element array can exploit its potential to avoid interference toward two primary receivers. An example of such a situation is a scenario with two primary receivers highly interfered from the cognitive mesh network. The 3-element array has the potential to avoid interference toward two primary receivers, while the antenna switching can avoid interference toward only one primary receiver. In this specific situation only the mesh equipped with the 3-element array is fully connected.



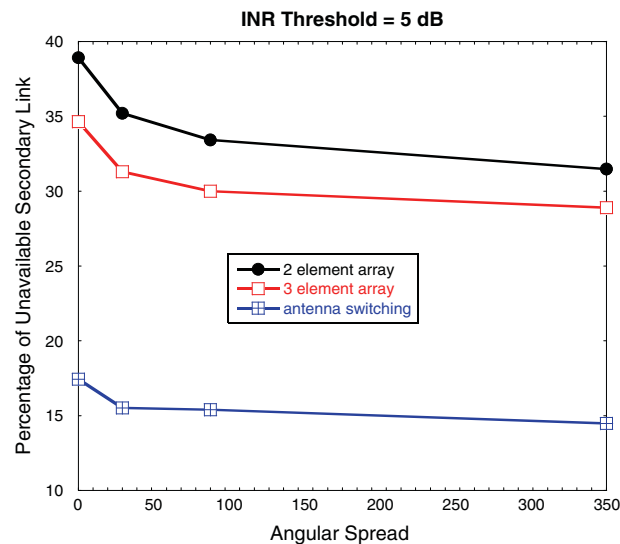
**Fig. 11** Mean number of hops versus primary receivers (INR threshold = 5 dB)

Moreover, we investigate the average number of hops, with the shortest path, needed to get packets from source to destination in the cognitive mesh network. This result refers only to connected mesh (i.e. when there is possibility to establish a bidirectional connection between any two nodes in the mesh). As seen in Fig. 11, the average number of hops increases when the number of primary receivers increases. With a lot of primary receivers, the percentage of disconnected links in the mesh is higher. Thus, even when the mesh is fully connected, the mean number of required hops to deliver a packet is increased. It is worth noting that the worse the performance in terms of unavailable links (Fig. 7), the higher the required number of hops for each of the three antenna arrangements (Fig. 11).

Finally, we evaluate the performance of the 2-element array, 3-element array with heuristic algorithm and antenna switching with different values of angular spread. Angular spread is an important parameter to determine the spread of multipath propagation. With the larger angular spread the signal is received from many different directions, therefore the concept of null toward a specific direction in the created pattern at the cognitive nodes disappears. Nevertheless, even with the larger angular spread, adaptive array antenna at cognitive nodes can minimize/avoid the interference from/to primary system, by taking into account the directions of arrival of all the incoming signals. Figure 12 confirms that the same tendency as in Figs. 7–9 is obtained for any value of angular spread when percentage of unavailable secondary links is considered. INR threshold is set to 5 dB and angular spread ranges from  $0^\circ$  to  $360^\circ$ . Moreover, as it can be seen in Fig. 12, when the angular spread increases, the percentage of unavailable link decreases. This happens because with the larger angular spread, the signal is received more uniformly from different angles, which reduces the degradation due to grating null. The reason why the antenna switching outperforms the 3-element array with heuristic scheme is identical with the explanation in relation to Figs. 7–9.

## 5 Conclusion

In this paper, we have investigated how we can increase the available bandwidth and the networking connectivity when a cognitive mesh network is located inside the communication



**Fig. 12** Percentage of unavailable secondary link versus angular spread (INR threshold = 5 dB)

area of the primary system and employs the interference cancellation/avoidance with adaptive array antenna.

We have introduced and compared the performances of the 2-element and 3-element linear array when they are applied to a cognitive mesh network. In order to achieve high networking performance with reduced complexity, we have introduced antenna pattern switching which adaptively chooses the antenna arrangement at cognitive nodes. We have also introduced a simple heuristic algorithm to select the pattern to reduce interference between cognitive mesh network and primary system when cognitive nodes are equipped with 3-element linear array antenna.

We have shown that the trade-off between the performance of primary and cognitive mesh networks can be controlled by a threshold parameter on interference caused from cognitive transmitter to primary receivers. We have also shown that the proposed antenna switching and 3-element linear antenna with the simple heuristic algorithm can significantly improve the networking performance as compared with the 2-element linear array. The proposed antenna switching can achieve competitive performance with simpler hardware complexity as compared with the 3-element linear array with heuristic algorithm.

The investigation with multiple types of primary networks is one of interesting future work. In that case, interference management from many primary system towards the cognitive mesh network and from cognitive mesh network towards many primary systems is needed. It will be therefore necessary to find a more elaborate solution to this issue. Moreover, an interesting work could be to derive upper/lower bounds for primary and cognitive performance with optimized beam pattern for 3-element linear array.

**Acknowledgements** This work is in part supported by Danish Research Council for Technology and Production Sciences, (project no. 274-05-0490).

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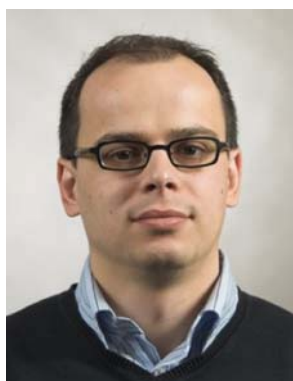
### Author Biographies



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**Ramjee Prasad** is a distinguished educator and researcher in the field of wireless information and multimedia communications. During February 1988–May 1999 he has been with the Telecommunications and Traffic-Control Systems Group of Delft University of Technology (DUT), The Netherlands, where he was actively involved in the area of wireless personal and multimedia communications (WPMC). He was head of the Transmission Research Section of International research Centre for Telecommunications Transmission and Radar (IRCTR) and also Founding Program Director of the Centre for Wireless Personal Communications (CWPC). As from June 1999 Ramjee Prasad joined as the Wireless Information Multimedia Communications chair and co-director of Centre for PersonKommunikation at Aalborg University, Denmark. From January 2004 he is Founding Director of the “Centre for TeleIn-frastruktur (CTIF)”. He has published over 500 technical papers, and authored and co-edited 15 books about Wireless Multimedia Communications (Artech House). His research interest lies in wireless networks,

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## 8.5 Paper 5

### **Outage Margin and Power Constraints in Cognitive Radio with Multiple Antennas**

*Popovski P.; Utkovski Z.; Di Taranto R.;*

IEEE 10th Workshop on Signal Processing Advances in Wireless Communications, 2009. SPAWC '09.

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# Outage Margin and Power Constraints in Cognitive Radio with Multiple Antennas

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**Abstract**—In the commons model for spectrum usage, the cognitive (secondary) users are allowed to use the spectrum as long as the target performance in the primary system is not violated. In this paper we consider primary system that has a target outage performance and the transmission power of the secondary transmitter (STX) should be appropriately not to violate the target outage probability for the primary terminals (PT). We have considered two types of STX, with single antenna (SISO STX) and two antennas (MISO STX) and analyzed the allowed secondary power. In general, the power level allowed for the SISO STX differs from the total power level allowed for the MISO STX. Our analysis shows that the relation between these power levels changes as the direct component (the K-factor) of the Ricean fading in the primary channel changes. For a large K-factor in the primary system, the total power allowed for a MISO STX is higher than the power allowed for a SISO STX system. The situation is reversed when the fading in the primary system has a low value of the K-factor and moves towards Rayleigh fading. This implies that, for example, when the direct component in the primary system is substantial, the usage of multiple antennas in the cognitive system has additional benefit, as it can use a higher power.

## I. INTRODUCTION

In the recent years, the rather conservative spectrum regulation has been challenged by the concepts of Cognitive Radio [1] and dynamic spectrum access [2]. The main idea behind these concepts is to allow *secondary spectrum usage*: a cognitive radio, also called secondary user, can transmit in a certain band allocated to a primary user (spectrum owner or licensee), provided that the communication in the primary system is not degraded more than a level that is deemed acceptable. This kind of cognitive radio operation is featured in the *commons spectrum usage model* [3], where the primary user is oblivious with respect to the operation of the secondary. Alternatively, there is the *property rights model* (or spectrum leasing) [4], where the primary user knowingly lets the secondary use the spectrum in some space/time.

In the commons model, the primary should operate with a certain *gratuitous margin*, which allows to accommodate transmissions in the secondary system without degrading the target performance of the primary. For example, if the primary transmitter knows the Signal-to-Noise Ratio (SNR) at the Primary Terminal (PT) and transmits at a rate just below the capacity achievable for that SNR, then the PT cannot stand more additional noise/interference and thus any secondary

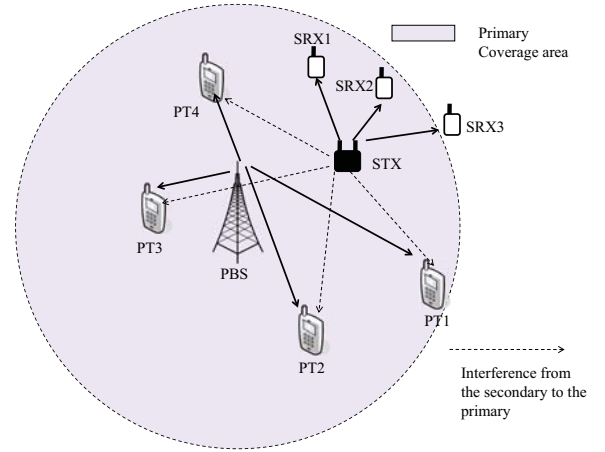


Fig. 1. Example scenario with Primary Base Station (PBS), Primary Terminals (PTs), Secondary Transmitter (STX), and Secondary Receivers (SRXs)

transmission is prevented. The margin can take several forms: (a) time - the primary communicates less than 100% of the time; (b) frequency - the primary is using only part of its allocated spectrum; (c) interference - the secondary can transmit by keeping the interference below some threshold. The secondary needs to perform *spectrum sensing* and identify its transmission opportunity, which in the cases (a) and (b) consists of detecting the spectrum hole [5], while in (c) it detects the interference induced to the primary receivers [6].

Here we consider scenarios that deal with the interference margin by keeping the outage probability in the primary system at or below an acceptable value. In Fig. 1 a primary Base Station (PBS) serves the primary terminals (PTs) in its coverage area. We assume that the coverage area is defined as the area in which the outage probability for the PTs is below a certain value  $\alpha_0$ . Referring to Fig. 1, the average power that PT1 receives from PBS is lower as compared to PT2 and therefore PT1 can stand less interference power from the secondary transmitters (STXs). If a PT is at the very edge of the coverage area, then no secondary transmission can take place, while the opportunity for secondary transmissions increases as the PT moves closer to the PBS. Besides that, this opportunity also increases as the channel gain between STX and the PT weakens. Clearly, the reception at the secondary

receivers (SRXs) is interfered by the PBS, but here this interference is treated as an increased noise level for SRX. For different treatment of this problem, refer to [7].

In this paper we analyze the problem of selecting the maximal possible power in the secondary system while not violating the outage probability in the primary system. We first investigate the case when the STX uses a single antenna and we derive the maximal allowable power. Next, we consider the case when the STX uses two antennas [8], with equal power allocated to each antenna and derive the maximal allowed power per antenna. The conclusion is that the sum of the powers for the two antennas is different from the power allowed for a single antenna system. Interestingly, the relation between these powers depends on the propagation conditions in the primary system. In general, when the primary system has a strong Line-Of-Sight (LOS) or direct component, the usage of two antennas is much more advantageous than the usage of a single antenna. Conversely, with a weak LOS component in the primary, the gain of having two antennas is decreased as the total power used by the two antennas should be less than the power that is allowed for a single-antenna STX.

## II. SYSTEM MODEL

The primary Base Station (PBS) uses fixed transmission rate  $R_p$  in the downlink. In the absence of interference, the signal received at the primary terminal (PT) is given by:

$$y_{pp} = h_{pp}x_p + z_p \quad (1)$$

where  $x_p$  is the signal sent by the PBS, normalized as  $E[|x_p|^2] = 1$ ,  $z_p$  is the Gaussian noise at the PT with variance  $E[|z_p|^2] = \sigma^2$ . The complex value  $h_{pp}$  is the channel coefficient between the PBS and the PT and the instantaneous SNR at the primary terminal is determined as:

$$\gamma_{pp} = \frac{|h_{pp}|^2}{\sigma^2} \quad (2)$$

We assume a block fading model [10], in which the instantaneous SNR is constant during the whole packet transmission. Considering normalized bandwidth, the achievable instantaneous rate is given by:

$$C(\gamma_{pp}) = \log_2(1 + \gamma_{pp}) \quad (3)$$

The minimal SNR that should be used to support rate  $R_p$  will be denoted by  $r_p = 2^{R_p} - 1$ . If the achievable instantaneous rate is lower than  $R_p$ , i. e. if  $r_p < \gamma_{pp}$ , then outage occurs. Let the maximal allowed outage probability be  $\alpha_0$ . If a given primary receiver has a probability of outage  $\alpha < \alpha_0$ , then this receiver has an *outage margin* and it can receive additional interference from the secondary transmission without violating the target operation regime of the primary system.

In our model we do not consider the effects of shadowing and the average SNR received by a given PT depends on the distance between the PBS and the PT, denoted by  $l$ . We will assume that the fading between PBS and PT has a Ricean distribution, such that the instantaneous SNR  $\gamma_{pp}$  at the PT has the following distribution [11]:

$$p_{\gamma_{pp}}(l) = \frac{1}{\bar{\gamma}_{ppD}(l)} e^{-\left(\frac{x}{\bar{\gamma}_{ppD}(l)} + K\right)} I_0\left(2\sqrt{\frac{Kx}{\bar{\gamma}_{ppD}(l)}}\right), \quad (4)$$

where  $\bar{\gamma}_{ppD}(l)$  is the mean of the diffuse component for a terminal at distance  $l$ , which is Rayleigh distributed.  $K$  is the Ricean factor, i. e. the ratio between the mean power of the LOS and the diffuse component.

Let  $P_{out}(l, R_p)$  denote the outage probability for a PT that is at the distance  $l$  from the PBS, for transmission rate  $R_p$ . The outage probability experienced by any primary terminal (PT) in the coverage area of the PBS should be  $P_{out}(l, R_p) \leq \alpha_0$  for any  $l$ . Clearly, the highest outage probability will be experienced by the terminals that are at the edge (distance  $L$ ) of the coverage area. For given maximal outage probability  $\alpha_0$  and given Ricean factor  $K$ , we can find the average diffuse component  $\bar{\gamma}_{ppD}(L)$  at the cell edge by setting:

$$P_{out}(L, R_p) = \alpha_0 \quad (5)$$

which means that the terminals at the edge will have zero outage margin and cannot experience any additional interference. On the other hand, if the primary terminal is at a distance  $l < L$ , then it can stand additional interference from a secondary transmitter (STX). In this paper we consider a single PT, that is used as a reference to determine the allowed power levels in the secondary system and the generalization to multiple PTs is straightforward, discussed in Section V.

The secondary transmitter (STX) has  $M_t$  transmit antennas. We assume that the signal transmitted from the  $i$ -th antenna is  $\sqrt{P_s}x_{s,i}$ , where  $E[|x_{s,i}|^2] = 1$  for each  $i$  and  $P_s$  is the average power that is equal for each antenna. Now the interfered signal at the primary receiver can be represented as:

$$y_{pp} = h_{pp}x_p + \sum_{i=1}^{M_t} h_{sp,i}\sqrt{P_s}x_{s,i} + z_p \quad (6)$$

where  $h_{sp,i}$  is the channel coefficient between the  $i$ -th antenna of the STX and the primary receiver PT. Each  $h_{sp,i}$  experiences independent Rayleigh fading [11]. This is reasonable, as the propagation model for the channel between the secondary/primary system is different from the propagation model within the primary system only. For example, the LOS component in the primary occurs due to the PBS being mounted at a designated location, while the STXs have a rather ad hoc position, with likely obstacles towards the PTs, having negligible LOS component. Furthermore, since the distances among the antennas are much smaller than the distance STX-PT, the average value  $E[|h_{sp,i}|^2] = g_{sp}$  does not change with  $i$ . We should note that in a context different than cognitive radio, an analysis of the outage probability in mobile radio systems with cochannel interferers in Ricean/Rayleigh fading environment was done in [9]. In this sense, the conclusions in [9] serve as a confirmation for our system model and verification for some of the results presented in this paper.

Following [9], [11], the instantaneous SNR  $\gamma_{sp,i}$  at the PT for the signal sent by the  $i$ -th antenna of STX is exponentially distributed with average value:

$$\bar{\gamma}_{sp,i} = \frac{P_s g_{sp}}{\sigma^2} \quad (7)$$

Note that STX does not know the instantaneous SNRs to PRX, it only knows the average value  $\bar{\gamma}_{sp,i}$ . Assuming that

STX transmits by using Gaussian codebooks, the instantaneous achievable rate in the primary system is:

$$R_p^{\max} = C \left( \frac{\gamma_{pp}}{1 + \sum_{i=1}^{M_t} \gamma_{sp,i}} \right) \quad (8)$$

The secondary receiver (SRX) has a single antenna. Each antenna of STX uses equal power  $P_s$  and  $h_{s,i}$  denotes the channel coefficient between the transmit antenna  $i$  and SRX. The channel between each transmitting antenna and the SRX experiences independent Rayleigh fading and the SNR of the signal transmitted from the  $i$ -th antenna and received at the  $j$ -th antenna is  $\gamma_{ss} = P_s \frac{|h_{s,i}|^2}{\sigma^2}$ . The noise variance at SRX also contains the interference that the secondary system experiences from the primary.

### III. PERMISSIBLE POWER LEVELS IN THE SECONDARY SYSTEM

The secondary system should choose the power  $P_s$  in a way that it does not violate the outage performance for the primary system. Therefore, we need to make an additional assumption: The secondary system knows  $\bar{\gamma}_{ppD}(l)$  in the primary system and also knows  $g_{sp}$ . The value of  $g_{sp}$  can be inferred by listening to the uplink transmissions of PT. On the other hand, the determination of  $\bar{\gamma}_{ppD}(l)$  requires either explicit signaling from the PT to the STX or that STX knows the location of the PT or another indirect way of knowing. Such an indirect way can be e. g. by having the STX overhear the transmissions of the PBS and based on the ACK/NACK sent by the PT, assess the outage probability at the PT in the absence of interference. This value of  $P_{\text{out}}$  has a one-to-one correspondence with  $\bar{\gamma}_{ppD}(l)$ , assuming that the Ricean factor  $K$  is known a priori.

#### A. Secondary SISO System

Here the STX uses a single antenna. An outage for the primary system occurs when:

$$R_p > C \left( \frac{\gamma_{pp}}{1 + \gamma_{sp}} \right) \Leftrightarrow r_p > \frac{\gamma_{pp}}{1 + \gamma_{sp}} \quad (9)$$

After transforming, the probability of outage is written:

$$\begin{aligned} \Pr \left( \gamma_{sp} \geq \frac{\gamma_{pp} - r_p}{r_p} \right) &= \int_0^\infty \Pr \left( \gamma_{sp} \geq \frac{x - r_p}{r_p} \right) p_{\gamma_{sp}}(x) dx \\ &= \int_0^{r_p} \frac{1}{\bar{\gamma}_{ppD}} e^{-\left(\frac{x}{\bar{\gamma}_{ppD}} + K\right)} I_0 \left( 2\sqrt{\frac{Kx}{\bar{\gamma}_{ppD}}} \right) dx + \\ &\quad \int_{r_p}^\infty \frac{1}{\bar{\gamma}_{ppD}} e^{-\left(\frac{x - r_p}{r_p \bar{\gamma}_{sp}} + \frac{x}{\bar{\gamma}_{ppD}} + K\right)} I_0 \left( 2\sqrt{\frac{Kx}{\bar{\gamma}_{ppD}}} \right) dx, \end{aligned} \quad (10)$$

where in the first integral we use  $\Pr \left( \gamma_{sp} \geq \frac{x - r_p}{r_p} \right) = 1$  for  $x \leq r_p$ . For a fixed probability of outage, the integrals can be evaluated numerically in order to find the mean  $\bar{\gamma}_{sp}$  of the permissible power of the secondary transmitter.

#### B. Secondary MISO system

Let STX use two transmit antennas (MISO STX). Here the outage analysis is different, since the sum of the instantaneous SNRs is not exponentially distributed [9]. Let us assume that the two antennas equally share the power, which results in equal contribution to the instantaneous SNR at the primary receiver,  $\bar{\gamma}_{sp,1} = \bar{\gamma}_{sp,2}$ . The latter is a consequence of the fact that the path loss and any large-scale fading towards the primary system is identical for both antennas. The assumption of equal transmit power at each antenna is easily justifiable if we assume that the secondary system operates in a transmit diversity regime. The PDF of the sum of the instantaneous SNRs  $\gamma_{sp,1} + \gamma_{sp,2}$  is given as

$$p_{\gamma_{sp,1} + \gamma_{sp,2}}(x) = \frac{x}{\bar{\gamma}_{sp,1}^2} e^{-\frac{x}{\bar{\gamma}_{sp,1}}} \quad (11)$$

and the CDF as

$$P_{\gamma_{sp,1} + \gamma_{sp,2}}(x) = 1 - \left( 1 + \frac{x}{\bar{\gamma}_{sp,1}} \right) e^{-\frac{x}{\bar{\gamma}_{sp,1}}} \quad (12)$$

If we were to design the secondary system only based on the outage calculation in the primary system, and without making any assumptions about the number of antennas in the secondary system, the permissible power level of the secondary system would be  $\bar{\gamma}_{sp}$ , as given by (10). However, in the case of MISO STX, we will show that due to the properties of the Erlang distribution, as well as of the Rice distribution of the signal in the primary system, the permissible power level of the secondary system will in general be different, i.e.  $\bar{\gamma}_{sp,1} + \bar{\gamma}_{sp,2} \neq \bar{\gamma}_{sp}$ . We represent the difference by introducing a factor  $c$  such that  $\bar{\gamma}_{sp,1} = \bar{\gamma}_{sp,2} = c\bar{\gamma}_{sp}$  where, in general,  $c \neq 0.5$ . The exact evaluation of the permissible power in the case of two antennas (and thus the factor  $c$ ) follows from the outage analysis. In the case of two antennas, the probability that the primary system is in outage is given by

$$\begin{aligned} &\Pr \left( \gamma_{sp,1} + \gamma_{sp,2} \geq \frac{\gamma_{pp} - r_p}{r_p} \right) \\ &= \int_0^{r_p} \frac{1}{\bar{\gamma}_{ppD}} e^{-\left(\frac{x}{\bar{\gamma}_{ppD}} + K\right)} I_0 \left( 2\sqrt{\frac{Kx}{\bar{\gamma}_{ppD}}} \right) dx + \\ &\quad \int_{r_p}^\infty \frac{1}{\bar{\gamma}_{ppD}} e^{-\left(\frac{x - r_p}{r_p c \bar{\gamma}_{sp}} + \frac{x}{\bar{\gamma}_{ppD}} + K\right)} I_0 \left( 2\sqrt{\frac{Kx}{\bar{\gamma}_{ppD}}} \right) dx + \\ &\quad \int_{r_p}^\infty \frac{1}{\bar{\gamma}_{ppD}} \left( \frac{x - r_p}{r_p c \bar{\gamma}_{sp}} \right) e^{-\left(\frac{x - r_p}{r_p c \bar{\gamma}_{sp}} + \frac{x}{\bar{\gamma}_{ppD}} + K\right)} I_0 \left( 2\sqrt{\frac{Kx}{\bar{\gamma}_{ppD}}} \right) dx. \end{aligned} \quad (13)$$

The integrals can be evaluated numerically. As the next section shows, the factor  $c$  depends on several factors, such as the Ricean factor  $K$ , the power levels of the primary and the secondary transmitter,  $\bar{\gamma}_{pp}$  and  $\bar{\gamma}_{sp}$ , and the information rate in the primary system  $R_p$ .

For a system with  $M_t$  transmit antennas and equal transmit powers, the CDF of the instantaneous SNR is given by

$$P_{\gamma_{sp,1} + \dots + \gamma_{sp,M_t}}(x) = 1 - \sum_{m=0}^{M_t-1} \frac{(x)^m}{(c\bar{\gamma}_{sp})^m m!} e^{-\left(\frac{x}{c\bar{\gamma}_{sp}}\right)}. \quad (14)$$

Therefore, the probability of outage is given by

$$\Pr \left( \gamma_{sp,1} + \gamma_{sp,2} + \dots + \gamma_{sp,M_t} \geq \frac{\gamma_{pp} - r_p}{r_p} \right) = \int_0^{r_p} \frac{1}{\bar{\gamma}_{ppD}} e^{-\left(\frac{x}{\bar{\gamma}_{ppD}} + K\right)} I_0 \left( 2\sqrt{\frac{Kx}{\bar{\gamma}_{ppD}}} \right) dx + \int_{r_p}^{\infty} P_{\gamma_{sp,1} + \dots + \gamma_{sp,M_t}} \left( \frac{x - r_p}{r_p} \right) \frac{1}{\bar{\gamma}_{ppD}} e^{-\left(\frac{x}{\bar{\gamma}_{ppD}} + K\right)} I_0 \left( 2\sqrt{\frac{Kx}{\bar{\gamma}_{ppD}}} \right) dx. \quad (15)$$

#### IV. PERFORMANCE EVALUATION

In this section we evaluate the effect of the number of antennas on the power levels that are allowed for usage in the secondary system. In all the evaluation scenarios we have assumed that the signal that the PT receives from the primary BS has a Ricean distribution, with a factor of  $K$ . The average SNR of the diffuse component from the primary signal measured at the PT is denoted by  $\bar{\gamma}_{ppD}$ . On the other hand, the signal that the PT receives from the secondary system is Rayleigh-distributed, with average SNR of  $\bar{\gamma}_{sp}$ .

Fig. 2 considers the outage probability experienced in the primary system as a function of the ratio between the average diffuse primary component and the average interference power received from the secondary, which is denoted by  $\frac{\bar{\gamma}_{ppD}}{\bar{\gamma}_{sp}}$ . For each point on the x-axis, we keep  $\bar{\gamma}_{sp}$  fixed and we change only  $\bar{\gamma}_{ppD}$ . When MISO STX is used, then  $\bar{\gamma}_{sp}$  denotes the sum of the SNRs contributed by both antennas and it is set to be, in its average value, equal to the SNR caused by a SISO STX. The transmission rate in the primary system  $R_p$  is fixed to  $R_p = 3$  bits/channel use and we measure the outage probability in the primary system. The Ricean factor  $K$  is taken as a parameter and for each value of  $K$ , we plot two curves, for SISO and MISO STX, respectively. A rather straightforward fact is that, for given  $\bar{\gamma}_{ppD}$ , the outage probability decreases as  $K$  increases, because larger  $K$  implies larger primary SNR. An interesting observation arises when we compare the outage performance in the primary system experienced for 1/2-antennas in the secondary system and how it changes when  $K$  increases. When  $K = 0$  i. e. the primary system also experiences Rayleigh fading, it can be noted that two antennas in the secondary system cause more performance loss at the primary terminal. This changes when  $K$  increases - e. g. for  $K = 8$  the outage caused by the MISO STX is much lower than the outage caused by a SISO STX, with identical total transmit power. This change in the relation might be surprising and slightly counterintuitive, but it results from the properties of the Rice distribution as well as the Bessel function which appear in the integrals in (10) and (13).

Fig. 3 gives a different perspective on the issue. The primary rate is fixed and for each curve, the outage probability is fixed. Let us show the curve interpretation by example. For the curve  $P_{out} = 0.07$  and the value  $K = 6$ , the power fraction per antenna is 0.56. If, on the other hand, the power limit for the secondary had been calculated for a SISO STX, then this power fraction would have been 0.5, i. e. half power to each antenna. Hence, the Ricean factor of  $K = 6$  allows

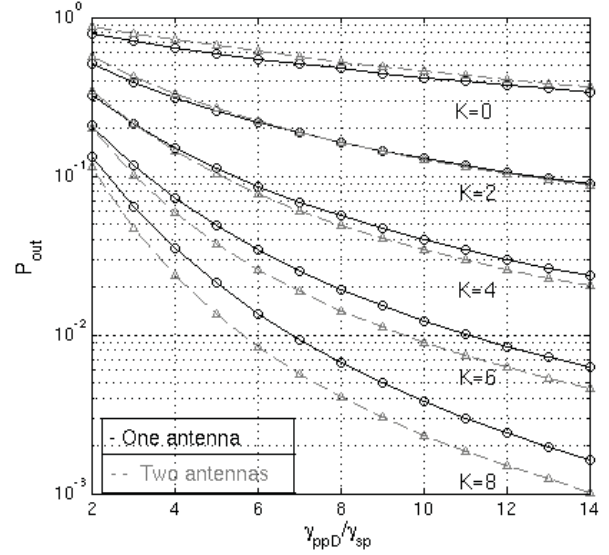


Fig. 2. Outage probability in the primary system as a function of the ratio between the average diffuse component of the primary signal and the average SNR for the interfering signal from the secondary at the primary terminal. The transmission rate is fixed to  $R_p = 3$  and the Ricean factor  $K$  is a parameter.

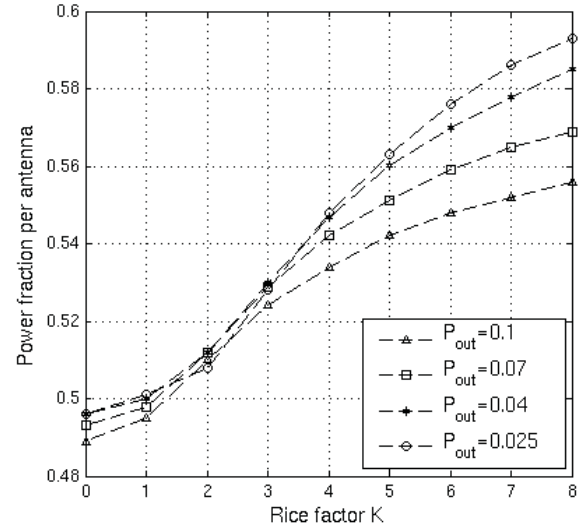


Fig. 3. Fraction of the power that can be used by each of the antennas for 2-antenna secondary transmitter relative to the power used by a 1-antenna secondary transmitter, for a fixed outage probability in the primary system.

a larger total average power in the secondary system when two antennas are used. Thus, if the average SNR from the interfering power when the STX uses a single antenna is  $\bar{\gamma}_{sp}$ , the total average SNR received from the two antennas can be  $1.12\bar{\gamma}_{sp}$  and still the same outage probability will be experienced in the primary system. When  $K$  is large and the target outage probability is small, the average power in the case with two antennas can be larger for more than 20%.

Finally, Fig. 4 shows how these power constraints affect the performance in the secondary system. The SISO curve shows the bit error rate (BER) when the secondary system uses single antenna, BPSK modulation and a power constraint imposed by the outage probability in the primary system when



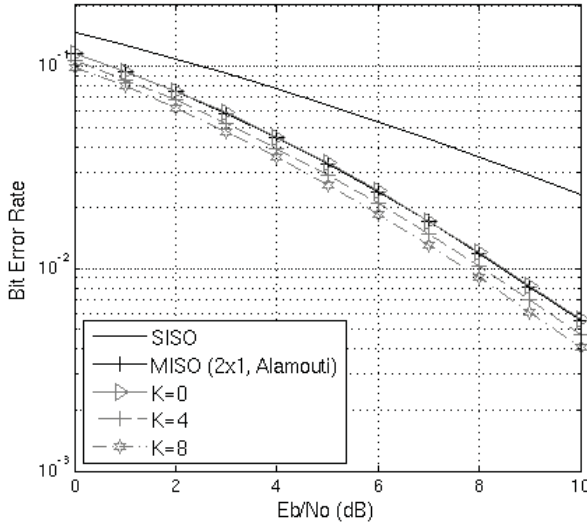


Fig. 4. Bit Error Rate (BER) performance for  $2 \times 1$  secondary system that uses the Alamouti scheme and determines the average power according to the outage probability in the primary system.

$K = 0$ . Note that, to obtain these curves, the average SNR in the primary system is kept constant, such that when  $K$  is increased, the diffuse component of the primary signal is accordingly decreased. The curve MISO shows the BER for the  $2 \times 1$  MISO system in which each antenna uses half of the power determined for the MISO system. For the other three curves ( $K = 0$ ,  $K = 4$  and  $K = 8$ ), the power per antenna is determined by the actual outage probability experienced when a 2-antenna STX interferes to the primary. In line with the previous results, when  $K$  increases, more power per antenna is allowed for the STX, which improves the BER performance.

## V. DISCUSSION

We have considered only a single PT and a single STX. When there are multiple PTs, the analysis presented here can readily be used - the power constraint is calculated with respect to each PT and the strictest power constraint is taken into account. Furthermore, in the analysis above we have assumed that both antennas used equal power. If the STX knows the channel state information towards the SRX, then, in general, the optimal spatial multiplexing is attained by having unequal power across the antennas. Hence, for this case the outage analysis of the primary system should include the statistics of the secondary channel, via the allocated powers.

When multiple STXs are active simultaneously, the analysis is quite different. For example, if there are two SISO STXs, it is different from the case of a single 2-antenna STX, because the average powers received from each STX are in general, not equal. If the STXs cooperate, then the problem is conceptually similar, only that we need to account for the statistics of the average SNR for each user and determine how to allocate the power to the individual users. If the STX do not cooperate, then the analysis should be put in an appropriate game-theoretic framework, where the secondary users will tune their power in order to meet the outage probability at the primary, while optimizing their own achieved rates or BERs.

## VI. CONCLUSION AND FUTURE WORK

We have considered communication scenarios in which the secondary (cognitive) user is allowed to transmit along with the transmissions in the primary system, not violating the target outage performance in the primary system. The secondary transmitter (STX) can guarantee the outage probability for a primary terminal (PT) by appropriate selection of the transmit power. The propagation channel in the primary system experiences Ricean fading with a certain  $K$ -factor, while the interference power from each secondary transmitter to the PT is Rayleigh-distributed. We have considered two types of STX, with single antenna (SISO STX) and two antennas (MISO STX), respectively, and determined the allowed power. We have shown that the power allowed in the SISO STX is different from the total power that is used in a MISO STX. Furthermore, the relation between the power for the SISO STX and the total power for the MISO STX changes as the direct component changes for the propagation channel in the primary system. For a large  $K$ -factor in the primary system, the total power allowed for a MISO STX is higher than the power allowed for a SISO STX system. The situation is reversed for a Rayleigh-like fading in the primary system (low  $K$ ). We have also quantified how this power difference affects the BER performance in the secondary system. An immediate topic for future work is to generalize the analysis to  $N$  transmitting antennas and find the asymptotic difference in the allowed power when  $N$  goes to infinity. A more practical extension of the work is to consider multiple secondary users and determine their allowed power levels that satisfy the target outage performance for the primary terminals.

## VII. ACKNOWLEDGMENT

Zoran Utkovski acknowledges the support of the COST2100 action during the stay at the Aalborg University.

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## 8.6 Paper 6

### **The Effect of Interference Statistics on the Power Levels in Multi-Antenna Cognitive Radios**

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Submitted to International Journal on Communication Networks and Distributed Systems, special issue on Cognitive Wireless Networks, January 2010





# The Effect of Interference Statistics on the Power Levels In Multi-Antenna Cognitive Radios

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## Abstract

In the commons model for spectrum usage, the cognitive (secondary) users are allowed to use the spectrum as long as the target performance in the primary system is not violated. In this paper we consider secondary transmitters (STXs) with single antenna (SISO STX) and multiple antennas (MISO STX). The analysis shows that the power level allowed for the SISO STX differs from the total power level allowed for the MISO STX. The relation between these power levels changes as the direct component (the K-factor) of the Ricean fading in the primary channel changes. For a large K-factor, the total power allowed for a MISO STX is higher than the power allowed for a SISO STX system. The situation is reversed when the K-factor is low. Hence, when the direct component in the primary system is substantial, the usage of multiple antennas in the cognitive system has additional benefit, as it can use a higher power.

## Index Terms

Cognitive radio, dynamic spectrum access, interference statistics, multiple-antenna systems

## I. INTRODUCTION

The spectrum regulation has recently been put under a revision by introducing the concepts of cognitive radio [1] and dynamic spectrum access [2]. The main idea behind these concepts is

to allow *secondary spectrum usage*: a cognitive radio, also called secondary user, can transmit in a certain band allocated to a primary user (spectrum owner or licensee), provided that the communication in the primary system is not degraded below a predefined acceptable level. This is called *commons spectrum usage model* [4], where the primary user is oblivious with respect to the operation of the secondary and it operates with a certain *gratuitous margin*, in order to be able to accommodate secondary transmissions without experiencing performance degradation.

We consider scenarios in which the activity of the secondary systems should keep outage probability in the primary system below an acceptable value. In Fig. 1 a primary Base Station (PBS) serves the primary terminals (PTs) in its coverage area. The *coverage area* is defined as the area in which the outage probability for the PTs is below a certain value  $\alpha_0$ . On Fig. 1, the average power that PT1 receives from PBS is lower as compared to PT2 and therefore PT1 can stand less interference power from the secondary transmitters (STXs). If PT is at the edge of the coverage area, then no secondary transmission can take place, while the opportunity for secondary transmissions increases as the PT moves closer to the PBS or the channel gain between STX and the PT weakens. Clearly, the reception at the secondary receivers (SRXs) is interfered by the PBS, but here this interference is treated as an increased noise level for SRX.

In this paper we analyze the problem of selecting the maximal possible power in the secondary system while not violating the outage probability in the primary system. Our previous results [3] have indicated that the maximal allowable power in the secondary system depends on the propagation conditions in the primary system. We analyze two extreme cases of propagation in the primary: (a) Rayleigh fading and (b) unfaded link with constant SNR. In the Rayleigh-faded primary link, the power assignment in the secondary system is more favorable for lower number of antennas. Conversely, in the case of unfaded primary link, the increase of the number of antennas for the secondary transmitter is resulting in the increase of the cumulative value for the power used by all antennas of STX. This effect implies that, for example, in Line-Of-Sight primary systems, the usage of multiple antennas is not only justified due to the increased capacity, but also due to the statistics of the interference that it creates for the primary receivers.

## II. SYSTEM MODEL

The primary Base Station (PBS) uses fixed transmission rate  $R_p$  in the downlink. In the absence of interference, the signal received at the primary terminal (PT) is given by:

$$y_{pp} = h_{pp}x_p + z_p \quad (1)$$

where  $x_p$  is the signal sent by the PBS, normalized as  $E[|x_p|^2] = 1$ ,  $z_p$  is the Gaussian noise at the PT with variance  $E[|z_p|^2] = \sigma^2$ . The complex value  $h_{pp}$  is the channel coefficient between the PBS and the PT and the instantaneous SNR at the primary terminal is determined as  $\gamma_{pp} = \frac{|h_{pp}|^2}{\sigma^2}$ .

We assume a block fading model [11], i.e a constant instantaneous SNR during the whole packet transmission. Considering normalized bandwidth, the achievable instantaneous rate is:

$$C(\gamma_{pp}) = \log_2(1 + \gamma_{pp}) \quad (2)$$

The minimal SNR that should be used to support rate  $R_p$  will be denoted by  $r_p = 2^{R_p} - 1$ . If the achievable instantaneous rate is lower than  $R_p$ , i. e. if  $r_p < \gamma_{pp}$ , then outage occurs. Let the maximal allowed outage probability be  $\alpha_0$ . If a given primary receiver has a probability of outage  $\alpha < \alpha_0$ , then it has an *outage margin* and can receive additional interference from the secondary transmission without violating the target operation regime of the primary system.

In our model we do not consider the effects of shadowing and the average SNR received by a given PT depends on the distance between the PBS and the PT, denoted by  $l$ . We will assume that the fading between PBS and PT has a Ricean distribution with Ricean factor  $K$ , such that the instantaneous SNR  $\gamma_{pp}$  at the PT has the following distribution [12]:

$$p_{\gamma_{pp}}(l, x) = \frac{1}{\bar{\gamma}_{ppD}(l)} e^{-\left(\frac{x}{\bar{\gamma}_{ppD}(l)} + K\right)} I_0\left(2\sqrt{\frac{Kx}{\bar{\gamma}_{ppD}(l)}}\right), \quad (3)$$

where  $\bar{\gamma}_{ppD}(l)$  is the mean of the diffuse component for a terminal at distance  $l$ , which is Rayleigh distributed and  $K$  is the ratio between the mean power of the LOS and the diffuse component.  $I_0(\cdot)$  is the modified Bessel function of order zero.

Let  $P_{\text{out}}(l, R_p)$  denote the outage probability for a PT that is at the distance  $l$  from the PBS, for transmission rate  $R_p$ . The outage probability experienced by any primary terminal (PT) in

the coverage area of the PBS should be  $P_{\text{out}}(l, R_p) \leq \alpha_0$  for any  $l$ . Clearly, the highest outage probability will be experienced by the terminals that are at the edge (distance  $L$ ) of the coverage area. For given maximal outage probability  $\alpha_0$  and given Ricean factor  $K$ , we can find the average diffuse component  $\bar{\gamma}_{ppD}(L)$  at the cell edge by setting  $P_{\text{out}}(L, R_p) = \alpha_0$ , which means that the terminals at the edge will have zero outage margin and cannot experience any additional interference. On the other hand, if the primary terminal is at a distance  $l < L$ , then it can stand additional interference from a secondary transmitter (STX).

The secondary transmitter (STX) has  $M_t$  transmit antennas. We assume that the signal transmitted from the  $i$ -th antenna is  $\sqrt{P_s}x_{s,i}$ , where  $E[|x_{s,i}|^2] = 1$  for each  $i$  and  $P_s$  is the average power that is equal for each antenna. The interfered signal at the primary receiver is:

$$y_{pp} = h_{pp}x_p + \sum_{i=1}^{M_t} h_{sp,i} \sqrt{P_s}x_{s,i} + z_p \quad (4)$$

where  $h_{sp,i}$  is the channel coefficient between the  $i$ -th antenna of the STX and the primary receiver PT. Each  $h_{sp,i}$  experiences independent Rayleigh fading [12]. This is reasonable, as the propagation model for the channel between the secondary/primary system is different from the propagation model within the primary system. For example, the LOS component in the primary occurs due to the PBS being mounted at a designated location, while the STXs have a rather ad hoc position, with likely obstacles towards the PTs, having negligible LOS component. Furthermore, since the distances among the antennas are much smaller than the distance STX-PT, the average value  $E[|h_{sp,i}|^2] = g_{sp}$  does not change with  $i$ .

It should be noted that in a context different from cognitive radio, an outage analysis in systems with co-channel interferes in Ricean/Rayleigh fading environment was done in [10]. Although there are certain common aspects in the analysis performed there and the one in this work, the setup here is different. Additionally, besides the mathematical analysis, we focus on the representation of the results in the context of cognitive radio. In this sense, the results presented here are interesting and novel. Further, they reshape the understanding and the use of both gratuitous margin and outage analysis in the design of cognitive radio systems.

We denote by  $\gamma_{sp,i}$  the instantaneous  $SNR$  at the PT for the signal sent by the  $i$ -th antenna

of STX. Following [10], [12],  $\gamma_{sp,i}$  is exponentially distributed with average value  $\bar{\gamma}_{sp,i} = \frac{P_s g_{sp}}{\sigma^2}$ . Note that STX does not know the instantaneous SNRs to PRX, it only knows the average value  $\bar{\gamma}_{sp,i}$ . Assuming Gaussian codebooks at STX, the instantaneous achievable rate in the primary system is:

$$R_p^{\max} = C \left( \frac{\gamma_{pp}}{1 + \sum_{i=1}^{M_t} \gamma_{sp,i}} \right) \quad (5)$$

The secondary receiver (SRX) has a single antenna. Each antenna of STX uses equal power  $P_s$  and  $h_{s,i}$  denotes the channel coefficient between the transmit antenna  $i$  and SRX. Each channel experiences independent Rayleigh fading and the SNR of the signal transmitted from the  $i$ -th antenna and received at the  $j$ -th antenna is  $\gamma_{ss} = P_s \frac{|h_{s,i}|^2}{\sigma^2}$ . The noise variance at SRX also contains the interference that the secondary system experiences from the primary.

### III. PERMISSIBLE POWER LEVELS IN THE SECONDARY SYSTEM

The secondary system should choose the power  $P_s$  in a way that it does not violate the outage performance for the primary system. Therefore, we need to make an additional assumption: The secondary system *knows*  $\bar{\gamma}_{ppD}(l)$  in the primary system and also knows  $g_{sp}$ . The value of  $g_{sp}$  can be inferred by listening to the uplink transmissions of PT. On the other hand, the determination of  $\bar{\gamma}_{ppD}(l)$  requires either explicit signaling from the PT to the STX or that STX knows the location of the PT or another indirect way of knowing. Such an indirect way can be e. g. by having the STX overhear the transmissions of the PBS and based on the ACK/NACK sent by the PT, assess the outage probability at the PT in the absence of interference. This value of  $P_{\text{out}}$  has a one-to-one correspondence with  $\bar{\gamma}_{ppD}(l)$ , if the Ricean factor  $K$  is known a priori.

#### A. STX with Single Antenna

In the following we assume that the STX uses a single antenna. All the calculations assume that PT is at distance  $l$  from the PBS. In order to simplify the notation, in the further text we will omit the dependency of the variables on  $l$ , however we will have on mind that the average received powers at PT from both PBS and STX depend on  $l$ .

An outage for the primary system occurs when:

$$R_p > C \left( \frac{\gamma_{pp}}{1 + \gamma_{sp}} \right) \Leftrightarrow r_p > \frac{\gamma_{pp}}{1 + \gamma_{sp}} \quad (6)$$

After transforming, the probability of outage is written:

$$\begin{aligned} P_{out}(R_p) &= \Pr \left( \gamma_{sp} \geq \frac{\gamma_{pp} - r_p}{r_p} \right) \\ &= \int_0^\infty \Pr \left( \gamma_{sp} \geq \frac{x - r_p}{r_p} \right) p_{\gamma_{pp}}(x) dx \\ &= \int_0^{r_p} \frac{1}{\bar{\gamma}_{ppD}} e^{-\left( \frac{x}{\bar{\gamma}_{ppD}} + K \right)} I_0 \left( 2\sqrt{\frac{Kx}{\bar{\gamma}_{ppD}}} \right) dx + \int_{r_p}^\infty \frac{1}{\bar{\gamma}_{ppD}} e^{-\left( \frac{x - r_p}{r_p \bar{\gamma}_{sp}} + \frac{x}{\bar{\gamma}_{ppD}} + K \right)} I_0 \left( 2\sqrt{\frac{Kx}{\bar{\gamma}_{ppD}}} \right) dx, \end{aligned} \quad (7)$$

where in the first integral we use  $\Pr \left( \gamma_{sp} \geq \frac{x - r_p}{r_p} \right) = 1$  for  $x \leq r_p$ . For a fixed probability of outage,  $\bar{\gamma}_{sp}$  can be calculated iteratively by numerical evaluation of the integrals.

Here we will address two extreme cases which describe the asymptotic system behavior. In the first case we consider Rayleigh distribution in the primary system, i.e. a Rician factor  $K = 0$ . In the second case we consider a constant power in the primary system, which corresponds to  $K \rightarrow \infty$ , i.e. line-of-sight environment and no diffuse component.

1)  $K=0$ : The case  $K = 0$  corresponds to a Raleigh distribution in the primary system. The outage probability is given as:

$$\begin{aligned} P_{out}(R_p) &= \Pr \left( \gamma_{sp} \geq \frac{\gamma_{pp} - r_p}{r_p} \right) \\ &= \int_0^\infty \left( 1 - e^{-\frac{r_p(1+x)}{\bar{\gamma}_{pp}}} \right) \frac{1}{\bar{\gamma}_{sp}} e^{-\frac{x}{\bar{\gamma}_{sp}}} dx = 1 - \frac{e^{-\frac{r_p}{\bar{\gamma}_{pp}}}}{\bar{\gamma}_{ss} \left( \frac{r_p}{\bar{\gamma}_{pp}} + \frac{1}{\bar{\gamma}_{ss}} \right)}. \end{aligned} \quad (8)$$

2)  $K \rightarrow \infty$ : The case  $K \rightarrow \infty$  corresponds to an unfaded link in the primary system, i.e a pure LOS signal. The outage probability can be then calculated as:

$$P_{out}(R_p) = \Pr \left( \gamma_{sp} \geq \frac{\gamma_{pp} - r_p}{r_p} \right) = e^{-\frac{\bar{\gamma}_{pp} - r_p}{r_p \bar{\gamma}_{sp}}} \quad (9)$$

### B. STX with Multiple Antennas

Let STX use  $M_t$  transmit antennas (MISO STX). Here the outage analysis is different, since the sum of the instantaneous SNRs is not exponentially distributed [10]. Let us assume that the  $M_t$  antennas equally share the power, which results in equal contribution to the instantaneous SNR at the primary receiver,  $\bar{\gamma}_{sp,1} = \bar{\gamma}_{sp,2} = \dots = \bar{\gamma}_{sp,M_t}$ . The latter is a consequence of the fact that the path loss and any large-scale fading towards the primary system is identical for all antennas. The assumption of equal transmit power at each antenna is easily justifiable if we assume that the secondary system operates in a transmit diversity regime. We denote the sum of the instantaneous SNRs as  $S_{sp} = \sum_{i=1}^{M_t} \gamma_{sp,i}$ . The PDF of  $S_{sp}$  is given as:

$$p_{S_{sp}}(x) = \frac{x^{M_t-1}}{\bar{\gamma}_{sp,1}^{M_t} (M_t - 1)!} e^{-\frac{x}{\bar{\gamma}_{sp,1}}} \quad (10)$$

and the CDF as

$$P_{S_{sp}}(x) = 1 - \sum_{m=0}^{M_t-1} \frac{x^m}{(\bar{\gamma}_{sp,1})^m m!} e^{-\left(\frac{x}{\bar{\gamma}_{sp,1}}\right)}. \quad (11)$$

If we were to design the secondary system only based on the outage calculation in the primary system, and without making any assumptions about the number of antennas in the secondary system, the permissible power level of the secondary system would be  $\bar{\gamma}_{sp}$ , as given by (7). However, in the case of MISO STX, we will show that due to the properties of the Erlang distribution, as well as of the Rice distribution of the signal in the primary system, the permissible power level of the secondary system will in general be different, i.e.  $\bar{\gamma}_{sp,1} + \dots + \bar{\gamma}_{sp,M_t} \neq \bar{\gamma}_{sp}$ . This conclusion differs from the usual analysis of the power margin which allows for accommodation of secondary users in the system. With other words, when designing the system based on the outage analysis in the primary system, we should consider the number of antennas of the STX, as well as the signal statistics in both primary and secondary system.

We represent the difference by introducing a factor  $c$  such that  $\bar{\gamma}_{sp,1} = \dots = \bar{\gamma}_{sp,M_t} = c \bar{\gamma}_{sp}$ . As argued, in general,  $c \neq \frac{1}{M_t}$ . The exact evaluation of the permissible power in the case of  $M_t$  antennas (and thus the factor  $c$ ) follows from the outage analysis. For the outage probability in



the primary system we have:

$$\begin{aligned}
P_{out}(R_p) &= \Pr \left( S_{sp} \geq \frac{\gamma_{pp} - r_p}{r_p} \right) \\
&= \int_0^{r_p} \frac{1}{\bar{\gamma}_{ppD}} e^{-\left( \frac{x}{\bar{\gamma}_{ppD}} + K \right)} I_0 \left( 2\sqrt{\frac{Kx}{\bar{\gamma}_{ppD}}} \right) dx \\
&\quad + \int_{r_p}^{\infty} \left[ 1 - P_{S_{sp}} \left( \frac{x - r_p}{r_p} \right) \right] \frac{1}{\bar{\gamma}_{ppD}} e^{-\left( \frac{x}{\bar{\gamma}_{ppD}} + K \right)} I_0 \left( 2\sqrt{\frac{Kx}{\bar{\gamma}_{ppD}}} \right) dx. \tag{12}
\end{aligned}$$

The integrals can be evaluated numerically. As we can see, the factor  $c$  depends on several factors, such as the Ricean factor  $K$ , the power levels of the primary and the secondary transmitter,  $\bar{\gamma}_{pp}$ ,  $\bar{\gamma}_{sp}$ , and the rate in the primary system  $R_p$ . Again, in order to demonstrate the asymptotic behavior and compare with the SISO case, we address the cases when  $K = 0$  and  $K \rightarrow \infty$ .

1)  $K=0$ : For Rayleigh distribution in the primary system, from (10) and (11) for the outage probability we have:

$$\begin{aligned}
P_{out}(R_p) &= \Pr (\gamma_{pp} \leq r_p(1 + S_{sp})) \\
&= \int_0^{\infty} \left( 1 - e^{-\frac{r_p(1+x)}{\bar{\gamma}_{ppD}}} \right) \frac{x^{M_t-1}}{(c\bar{\gamma}_{sp})^{M_t} (M_t-1)!} e^{-\frac{x}{c\bar{\gamma}_{sp}}} dx \\
&= 1 - \frac{e^{-\frac{r_p}{\bar{\gamma}_{ppD}}}}{(c\bar{\gamma}_{sp})^{M_t} \left( \frac{r_p}{\bar{\gamma}_{ppD}} + \frac{1}{c\bar{\gamma}_{sp}} \right)^{M_t} (M_t-1)!} \int_0^{\infty} y^{M_t-1} e^{-y} dy, \tag{13}
\end{aligned}$$

where we use the substitution  $y = x \left( \frac{r_p}{\bar{\gamma}_{ppD}} + \frac{1}{c\bar{\gamma}_{sp}} \right)$ .

Since  $\int_0^{\infty} y^{M_t-1} e^{-y} dy = \Gamma(M_t) = (M_t-1)!$ , where  $\Gamma(\cdot)$  is the Gamma function, we have:

$$P_{out}(R_p) = \Pr (\gamma_{pp} \leq r_p(1 + S_{sp})) = 1 - \frac{e^{-\frac{r_p}{\bar{\gamma}_{ppD}}}}{(c\bar{\gamma}_{sp})^{M_t} \left( \frac{r_p}{\bar{\gamma}_{ppD}} + \frac{1}{c\bar{\gamma}_{sp}} \right)^{M_t}} \tag{14}$$

2)  $K \rightarrow \infty$ : For unfaded link in the primary system, the outage probability is:

$$\begin{aligned}
 P_{out}(R_p) &= \Pr(\gamma_{pp} \leq r_p(1 + S_{sp})) \\
 &= \Pr\left(S_{sp} \geq \frac{\gamma_{pp} - r_p}{r_p}\right) = \sum_{i=0}^{M_t-1} \frac{\bar{\gamma}_{pp}^m}{(c \bar{\gamma}_{sp})^m (m)!} e^{-\frac{\bar{\gamma}_{pp}}{c \bar{\gamma}_{sp}}} \quad (15)
 \end{aligned}$$

#### IV. PERFORMANCE EVALUATION

In this section we evaluate the effect of the number of antennas on the power levels that are allowed in the secondary system. We assume that the signal that the PT receives from the primary BS has a Ricean distribution, with a factor  $K$ . The average SNR of the diffuse component from the primary signal measured at the PT is denoted by  $\bar{\gamma}_{ppD}$  and the total average SNR of the primary signal measured at the PT is  $\bar{\gamma}_{pp}$ . The signal that the PT receives from the secondary system is Rayleigh-distributed, with average SNR of  $\bar{\gamma}_{sp}$ .

Fig. 2 and Fig. 3 consider the total transmit power allowed in the secondary transmission as a function of the number of transmit antennas in the secondary system,  $M_t$ . The average SNR of the signal in the primary system at the cell border is fixed at  $\bar{\gamma}_{pp}(L) = 24$  dB. The transmission rate in the primary system  $R_p$  is fixed to  $R_p = 3$  bits/channel use and for each curve, the outage probability is fixed as well. As discussed, we consider two extreme cases. In the first case we consider a Rayleigh distribution in the primary system, i.e a Ricean factor  $K = 0$ . In the second case we consider a pure line-of-sight signal in the primary system i.e. a Ricean factor  $K \rightarrow \infty$ . Let us show the curve interpretation by example. For the curve  $P_{out} = 0.025$ ,  $M_t = 8$  and  $K \rightarrow \infty$ , the fraction of the allowed power at each antenna is around 0.256, which corresponds to a total allowed average transmit power of 2.05 taken relative to the case when a single antenna is used. Hence, for a large Ricean factor, a substantially larger total average power is allowed in the secondary system when multiple transmit antennas are used, and still the same outage probability will be experienced in the primary system. When  $K$  is large and the target outage probability is small, the total average SNR can be up to  $2.5\bar{\gamma}_{sp}$ , as shown in Fig. 2. The situation is reversed for a Rayleigh fading in the primary system ( $K = 0$ ). In that case the total transmit power in the multiple antenna STX is actually smaller than the power

in the single antenna case, although the effect of using multiple antennas is minor, as shown in Fig. 3.

## V. CONCLUSION AND FUTURE WORK

We have considered communication scenarios in which the secondary (cognitive) user is allowed to transmit along with the transmissions in the primary system, not violating the target outage performance in the primary system. The secondary transmitter (STX) can guarantee the outage probability for a primary terminal (PT) by appropriate selection of the transmit power. The propagation channel in the primary system experiences Ricean fading with a certain  $K$ -factor, while the interference power from each secondary transmitter to the PT is Rayleigh-distributed. We have considered two types of STX, with single antenna (SISO STX) and  $M_t$  antennas (MISO STX), respectively, and determined the allowed power. We have shown that the power allowed in the SISO STX is different from the total power that is used in a MISO STX. Furthermore, the relation between the power for the SISO STX and the total power for the MISO STX changes as the direct component changes for the propagation channel in the primary system. Asymptotically, when the Ricean factor  $K \rightarrow \infty$ , the total power allowed for a MISO STX is significantly higher than the power allowed for a SISO STX system. The situation is reversed for a Rayleigh fading in the primary system ( $K = 0$ ). An immediate topic for future work is to consider multiple secondary users and determine their allowed power levels that satisfy the target outage performance for the primary terminals.

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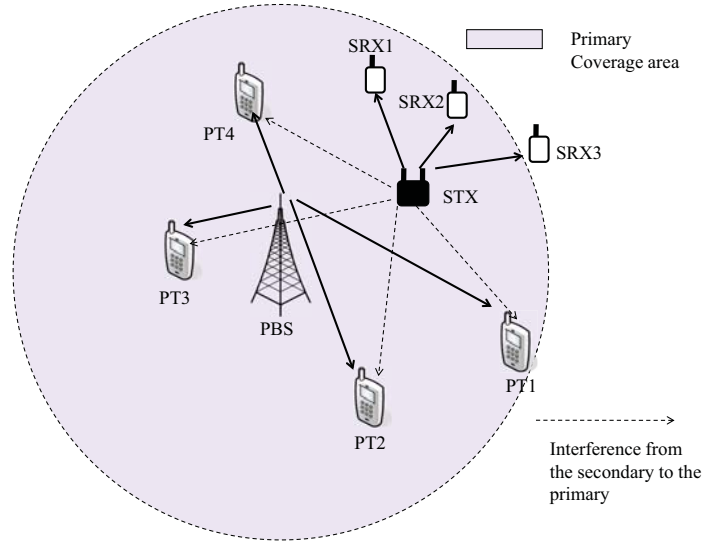


Fig. 1. Example scenario with Primary Base Station (PBS), Primary Terminals (PTs), Secondary Transmitter (STX), and Secondary Receivers (SRXs)

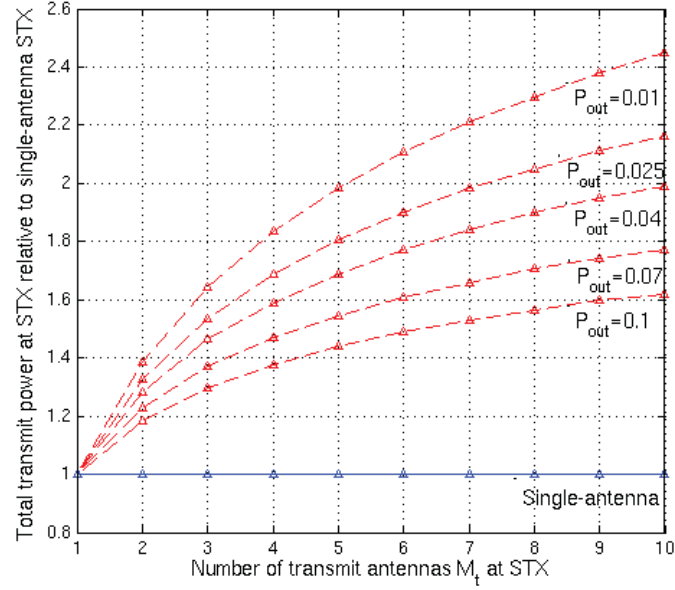


Fig. 2. The total power that can be used by a multiple-antenna secondary transmitter relative to the power used by a 1-antenna secondary transmitter, for a fixed outage probability in the primary system and Ricean factor  $K \rightarrow \infty$ .

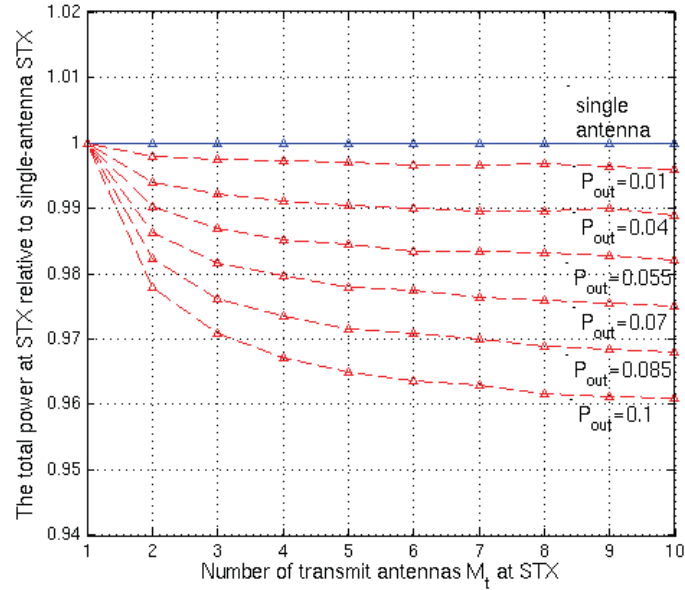


Fig. 3. The total power that can be used by a multiple-antenna secondary transmitter relative to the power used by a 1-antenna secondary transmitter, for a fixed outage probability in the primary system and Ricean factor  $K = 0$ .

## **8.7 Paper 7**

### **Two Players Non-Cooperative Iterative Power Control for Spectrum Sharing**

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PIMRC 2008. IEEE 19th International Symposium on Personal, Indoor and Mobile Radio Communications, 2008.

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# Two Players Non-Cooperative Iterative Power Control for Spectrum Sharing

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**Abstract**—To cope with limited spectrum resources, inefficient spectrum usage and overcrowded wireless communication, the Federal Communications Commission's (FCC) has introduced a new way to manage RF resources: the interference temperature model. With this model the key point is to let unlicensed users use licensed frequencies, provided they can quantify and bound their interference toward the primary license holders. In this paper we study power control for spectrum sharing among two secondary users with interference temperature limit (ITL) at a measurement point. We model this scenario as 2-player non-cooperative game. The 2 players are selfish and rational and strive to maximize their own utility. The measurement point, upon violation of ITL, iteratively backs off a randomly selected secondary user. We identify the possible outcomes of this game and the conditions for the existence of unique/multiple Nash equilibria. We analyze the property of the Nash equilibria in our games, and compare their performances with the social optimal solution. Our simulation results demonstrate that the proposed solution can achieve a satisfactory performance in terms of the total transmitting rate of the two secondary users.

**Index Terms**—Cognitive radio, Dynamic spectrum sharing, Interference management, Game theory, Power Control, Nash Equilibrium

## I. INTRODUCTION

The large popularity of mobile phones and Wireless Local Area Networks (WLAN) has challenged these systems with an increasing demand to offer the target service quality within a constrained frequency bandwidth. As the utilization of a given spectrum portion becomes saturated, additional spectrum allocation is needed to meet those demands. However, measurement studies [1] have shown that the legacy system owning certain frequency spectrum, called *primary system*, exhibits inefficient spectrum usage due to the absence of communication traffic in certain periods/regions. In order to efficiently increase the spectrum allocation, one solution is to enable an unlicensed system (*cognitive system*) to access the spectrum which is underutilized in space/time by the primary system, provided that the unlicensed devices do not obstruct the privileges of the licensed users [2]- [3]. The *interference temperature model* has been introduced to realize such sharing of licensed spectrum: in this model unlicensed users can operate on the same frequencies as licensed signals, provided they can quantify and bound the additional interference [4]- [6].

In this paper we focus our study on such an interference temperature model. We consider the coexistence between primary and cognitive users. Cognitive user, who observes the

channel availability dynamically, considers a channel to be available for transmission if its transmission on that channel would result in increase of the interference from cognitive system toward primary system below a parameter called *interference temperature limit*, ITL. This parameter is set to protect the primary users from any harmful interference. With secondary spectrum access, transmitting power control becomes of paramount importance to achieve satisfactory spectral efficiency. On one hand, each cognitive transmitter sharing the primary spectrum must select a transmission power such that the total interference doesn't violate the interference temperature limit, ITL. On the other hand, cognitive transmitter tries to use a power as high as possible, in order to increase the SINR at its intended receiver. In our simplified scenario there are *a)* one primary node, called measurement point (MP), which upon violation of ITL iteratively backs off cognitive users, until the ITL is no more violated, and *b)* two secondary users (two pairs of transmitter and receiver) trying to access the primary band. The problem treated in this work is closely related to [7], where authors assume that MP knows who is the secondary user generating the highest interference at the MP. In this work we assume that the measurement point knows the identities of the players in the game, but it has not an ability to figure out which secondary user(s) is responsible for the harmful interference (i.e. the measurement point can measure the total interference from all the players, but not the interference from each player). This assumption is realistic because no decoding of the interference signal is needed at the measurement point. Moreover it significantly changes the scenario in [7] and we have to introduce a completely different strategy: upon violation of ITL, the measurement point randomly selects one player to back off. In [7] the player responsible of harmful interference is always backed off. In this work the fundamental concept of personal responsibility disappears: the user to be backed off is randomly selected, regardless of his contribution to the harmful interference. As a consequence, in this work it is possible to identify specific situations where secondary players can have a rational motivation to violate the ITL.

We model our scenario as a 2-player non-cooperative power control game [8]. The 2 players are selfish and rational and strive to maximize their own utility. We study the possible outcomes of this game and identify the conditions for the existence of unique/multiple Nash equilibria. We evaluate the performances of these Nash equilibria and compare them with the social optimal solution by carrying out computer



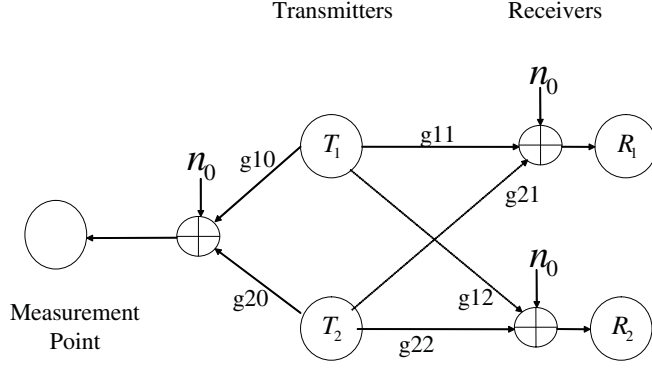


Fig. 1. System model for spectrum sharing array

simulations.

## II. TARGET SCENARIO

We consider the coexistence between primary and cognitive systems (Figure 1). In our model, there is one primary node, called measurement point, which has the ability to monitor and quantify the real-time interference toward the primary system and is responsible for the protection of all primary users. Besides, we consider two secondary users (one secondary user corresponds to a pair of transmitter and receiver) which are allowed to access the primary spectrum, provided their interference toward the primary system is below a parameter called Interference Temperature Limit, ITL. The ITL is a measurement of the power and bandwidth occupied by the interference. There are several equivalent definitions of interference temperature [9], but we use the following formulation in our paper:

$$T(f_c, B_c) = \frac{P_i(f_c, B_c)}{kB_c}, \quad (1)$$

where  $T(f_c, B_c)$  is the interference temperature for the channel  $c$ , with central frequency  $f_c$  and bandwidth  $B_c$ .  $P_i(f_c, B_c)$  is the average interference power in Watts centered at frequency  $f_c$  and covering the bandwidth  $B_c$  (in Hertz).  $k$  is the Boltzmann's constant ( $1.38 \times 10^{-23}$  Joules per Kelvin). This value is the maximum amount of tolerable interference for a given frequency band in a particular location. Any unlicensed transmitter utilizing this channel must guarantee that the total interference (i.e. its interference plus the existing interference) does not exceed the predefined threshold at any primary receiver within the transmitter interference area. We assume that interference toward primary users is not harmful as long as the interference temperature limit at the measurement point is not violated. Location and numbers of measurement points, as well as proper values of ITL are not object of study in this paper.

The protection of primary users is attained by limited interaction between the measurement point and the secondary

users. The secondary users' operation is transparent to the measurement point as long as the cognitive interference is lower than the ITL. On the contrary, if the ITL is violated, the measurement point starts an iterative backing off process: until the ITL is no more violated it randomly selects one cognitive user to back off. For example let's consider two cognitive users whose operations generate harmful interference. In this case, the measurement point first backs off a randomly selected secondary user. Then, if ITL is no more violated, the second cognitive transmitter can continue its transmission. Otherwise, no cognitive transmission is allowed. Generally, since we consider only two cognitive users in this work, at the end of the backing off process, either only one cognitive user is transmitting (if its interference at the measurement point is lower than ITL) or no cognitive transmission is allowed. In both cases protection of primary users is guaranteed with certainty.

## III. NON-COOPERATIVE POWER CONTROL GAME

In this section the power control issue described in the previous section is modeled as non-cooperative game in which each secondary user selfishly chooses its transmitting power in order to maximize its own utility. We identify the necessary and sufficient conditions for the existence of unique/multiple Nash equilibria, and analyze their properties and performances. Most of the game models are similar to [7], which we describe below with some modifications made for our scenario.

### A. Strategic Form Game

We assume at a certain time and geographic location a primary system offers a portion of its spectrum with bandwidth  $W$  to be shared among 2 spread spectrum cognitive users. We consider a game in strategic form. Let  $G = [N, \{P_i\}, \{U_i\}]$  denote the non-cooperative power control game. Here the two players in the game correspond to the secondary users in  $N$ . Each player  $i \in N$  selects a transmitting power (i.e. its strategy)  $p_i \in P_i = [p_i^{min}, p_i^{max}]$  with the objective of maximizing its utility. We assume that the strategy space  $P_i$  is continuous. The joint strategy space  $P = P_1 \times P_2$  is the Cartesian product of the individual strategy sets for the 2 players. In our game model we assume that each secondary user can choose a sufficiently high power level so that it causes harmful interference at the measurement point (i.e. ITL can be violated even if there is only one cognitive user transmitting at a given time). As it can be seen in Figure 1,  $g_{11}$  and  $g_{22}$  are the channel gain for users 1 and 2, while  $g_{12}$  and  $g_{21}$  are the intra-system gains between transmitter 1 (2) and receiver 2 (1).  $g_{10}$  and  $g_{20}$  represent respectively the channel gain between transmitter 1 (2) and measurement point.

User  $i$ 's evaluation of the spectrum is characterized by a utility function  $U_i(\gamma_i)$ , where  $\gamma_i$  is the received SINR at user  $i$ 's receiver. For each user  $i$ , the received SINR is given by

$$\gamma_i(p_1, p_2) = \frac{p_i g_{ii}}{\sum_{j \neq i} p_j g_{ji} + N}, \quad (2)$$

where  $p_i$  is user  $i$ 's transmission power,  $N$  is the background noise that is assumed to be the same for all users. To satisfy

the ITL, the total received power at the measurement point must satisfy

$$p_1 g_{10} + p_2 g_{20} \leq T. \quad (3)$$

In this paper, we define the *logarithmic* function  $u_i$  as

$$u_i(\gamma_i) = \ln(\gamma_i). \quad (4)$$

This choice introduces some *fairness* in the game, as it gives higher utility to the system in disadvantage.

In our model, the set  $B$  of backed off users cannot start transmissions thus gaining utility 0. We denote as  $\mathbf{p}^b$  the power vector allocation after the backing off process. Three cases can be distinguished in our game, when considering the utility  $U_i$  accrued by user  $i$

$$U_i(\mathbf{p}) = \begin{cases} u_i(\mathbf{p}) & \text{if } \sum_{i=1}^2 p_i g_{i0} \leq T \\ u_i(\mathbf{p}^b) & \text{if } \sum_{i=1}^2 p_i g_{i0} > T \text{ and } i \text{ not in } B \\ 0 & \text{if } \sum_{i=1}^2 p_i g_{i0} > T \text{ and } i \in B. \end{cases} \quad (5)$$

In this paper we focus on a non-cooperative power control game: each player selects its transmitting power without coordination with the other. With this assumption, neither cooperation between the secondary users nor any central point is needed.

### B. Existence of Nash Equilibrium

The solution that is most widely used for game theoretic problems is the *Nash equilibrium*. In the rest of our paper, we will focus on finding or characterizing such point(s) in the strategy space.

**Definition 1:** A power profile  $\mathbf{p}^* = (p_1, p_2)$  is said to be a Nash equilibrium (NE) of  $G$  if for every  $i = 1, 2$

$$U_i(p_i, p_{-i}) \geq U_i(p'_i, p_{-i}) \quad (6)$$

for any  $p'_i \in P_i$ . By definition  $p_{-i}$  is the power profile of user  $i$ 's opponents (i.e. in our game with 2 players,  $p_{-1} = p_2$  and  $p_{-2} = p_1$ ).

At a Nash Equilibrium, given the power levels of the other players, no user can improve its utility level by making individual changes in its power. In other words, at a Nash equilibrium the power level chosen by a user constitutes a *best response* to the power actually chosen by the other players.

**Definition 2:**  $B_i$  is a best response by player  $i$  to  $p_{-i}$  if

$$B_i(p_{-i}) = \arg \max_{p_i \in P_i} U_i(p_i, p_{-i}). \quad (7)$$

**Theorem 1:** The power profile  $\mathbf{p} = (\frac{T}{g_{10}}, \frac{T}{g_{20}})$  is a Nash equilibrium in our game.

*Proof.* In this case each player generates an interference at the measurement point equal to the interference temperature limit (ITL), therefore the total interference from the two players is twice the ITL. Consequently, one player is randomly selected and backed off by the measurement point (the other player's interference doesn't violate the ITL). There are two possible outcomes in this lottery for player  $i$ : it is either backed off

(probability 0.5) or it can transmit alone (probability 0.5). Therefore the expected utility gained by the two players can be expressed as follows:

$$U_i(\frac{T}{g_{10}}, \frac{T}{g_{20}}) = \begin{cases} \ln(\frac{p_i g_{ii}}{N}) & \text{probability 0.5} \\ 0 & \text{probability 0.5.} \end{cases} \quad (8)$$

At this point none of the users has incentive to unilaterally deviate from this strategy. Let's assume user  $i$  increases its transmitting power to a value  $p'_i > \frac{T}{g_{i0}}$  (user  $i$  cannot increase its utility by decreasing its transmitting power, because the utility function  $U_i(p_i, p_{-i})$  is strictly increasing function). In this condition, user  $i$  is backed off with certainty by the measurement point, because it generates an interference at the measurement point higher than the interference threshold. Consequently, it will get utility 0.  $\square$

As discussed, upon violation of interference threshold the power profile  $\mathbf{p} = (\frac{T}{g_{10}}, \frac{T}{g_{20}})$  is the unique Nash equilibrium in our game. Nevertheless, there are specific situations where additional multiple Nash equilibria can exist in our game without violation of ITL. Let's for example consider a joint power strategy  $(\hat{p}_1, \hat{p}_2)$  such that the summation of the interferences of the two players at the measurement point is equal to the interference temperature limit. As previously stated, upon violation of ITL, the maximum power a rational player  $i$  can select is  $p_i = \frac{T}{g_{i0}}$ , if it doesn't want to be backed off with certainty by the measurement point. Given these conditions, if no player can increase its payoff even by unilaterally selecting its maximum transmitting power (i.e.  $p_i = \frac{T}{g_{i0}}$ ), then the joint power profile  $(\hat{p}_1, \hat{p}_2)$  is a Nash equilibrium without violation of ITL. Next theorem formalizes the necessary and sufficient conditions for the existence of such equilibria in our game.

**Theorem 2:** The power profile  $\mathbf{p} = (p_1, p_2)$  is a Nash equilibrium in our game if it satisfies the following conditions:

$$\begin{cases} (\frac{p_1 g_{11}}{p_2 g_{21} + N})^2 \geq \frac{T g_{11}}{g_{10} N}; \\ (\frac{p_2 g_{22}}{p_1 g_{12} + N})^2 \geq \frac{T g_{22}}{g_{20} N}; \\ p_1 g_{10} + p_2 g_{20} = T; \\ p_1 \geq 0; \\ p_2 \geq 0; \end{cases} \quad (9)$$

*Proof.* Let's consider a joint power strategy  $\mathbf{p} = (p_1, p_2)$  such that  $p_1 g_{10} + p_2 g_{20} = T$ . User 1 and 2 gain the following payoffs:

$$U_1(\mathbf{p}) = \ln(\frac{p_1 g_{11}}{p_2 g_{21} + N}) \quad (10)$$

$$U_2(\mathbf{p}) = \ln(\frac{p_2 g_{22}}{p_1 g_{12} + N}). \quad (11)$$

In order to determine whether the joint strategy  $\mathbf{p} = (p_1, p_2)$  is a Nash equilibrium we must prove that no player can improve its payoff unilaterally increasing its power. If rational player  $i$  increases its power, it must select a value  $p_i^* = \frac{T}{g_{i0}}$ . In fact, upon violation of ITL, user  $i$  will be backed off with probability 0.5 for any  $p_i < p_i^* \leq \frac{T}{g_{i0}}$ . Then the expected utility accrued by user  $i$  is the same as in equation (8). First two inequalities in system (9) summarizes this conditions for players 1 and 2. The third equation impose that the summation of interferences from the two players at the measurement point is equal to the ITL.  $\square$

The concept of Pareto-optimality is normally used to characterize the performance of Nash equilibrium.

*Definition 3:* A power allocation scheme is Pareto optimal if there is no other allocation in which some other utility is better off and no utility is worse off.

*Theorem 3:* The tight power profile  $\mathbf{p} = (p_1, p_2)$  satisfying system (9) is Pareto optimal.

*Proof.* We first prove (a) that tight power allocation is necessary condition for Pareto optimality. Then (b) we show that it gives also sufficient conditions, provided that system (9) is satisfied. (a) Suppose the power constraint is not tight (i.e.  $p_1 g_{10} + p_2 g_{20} < T$ ). Then each user can increase its power by a factor  $\alpha = \frac{T}{p_1 g_{10} + p_2 g_{20}}$ , which increases the SINR of each user. (b) Suppose ITL is violated. Then, each player maximizes its utility selecting a power  $p_i = \frac{T}{g_{i0}}$ . If system (9) is verified, by assumption each player has the highest utility when the tight power profile  $(p_1, p_2)$  is selected, regardless of the transmitting power of the other player (when ITL is violated only one player can transmit at a given time, see eqn.(8)). Therefore no player can improve the utility gained in  $(p_1, p_2)$  and this gives the condition for the Pareto optimality of this profile.  $\square$

We further analyze the conditions for the existence of multiple NEs given by system (9). This system is solvable in  $p_1$  and  $p_2$  if:

$$p_{1inf} = \frac{\sqrt{A}(\frac{Tg_{21}+Ng_{20}}{g_{20}})}{g_{11} + (\frac{g_{10}g_{21}}{g_{20}})\sqrt{A}} \leq p_1 \leq \frac{Tg_{22} - Ng_{20}\sqrt{B}}{g_{10}g_{22} + g_{12}g_{20}\sqrt{B}} = p_{1sup}; \quad (12)$$

$$p_{2inf} = \frac{\sqrt{B}(\frac{Tg_{12}+Ng_{10}}{g_{10}})}{g_{22} + (\frac{g_{20}g_{12}}{g_{10}})\sqrt{B}} \leq p_2 \leq \frac{Tg_{11} - Ng_{10}\sqrt{A}}{g_{20}g_{11} + g_{21}g_{10}\sqrt{A}} = p_{2sup}; \quad (13)$$

where

$$A = \frac{Tg_{11}}{g_{10}N}; B = \frac{Tg_{22}}{g_{20}N}; \quad (14)$$

System (9) has real solutions in  $p_1$  and  $p_2$  if

$$p_{1sup} - p_{1inf} > 0; \quad p_{2sup} - p_{2inf} > 0; \quad (15)$$

We define the following parameters  $\alpha, \beta$  and  $\gamma$  in order to determine the conditions to satisfy in the two previous inequalities:

$$\frac{T}{N} = \gamma; \quad (16)$$

$$\frac{g_{11}}{g_{10}} = \frac{g_{22}}{g_{20}} = \alpha; \quad (17)$$

We assume that the ratio between a) the gain in each secondary link and b) the gain of the link between each secondary transmitter and the measurement point is the same. This assumption is reasonable, because the distance between a secondary transmitter and its intended receiver is likely to be smaller than the distance between secondary transmitter and measurement point. We further assume that:

$$\frac{g_{11}}{g_{12}} = \frac{g_{22}}{g_{21}} = \beta; \quad (18)$$

TABLE I  
DISTANCES IN METERS

	d10=d20	d12=d21
$\alpha = \beta = 1$	10	10
$\alpha = \beta = 10$	21.5	21.5
$\alpha = \beta = 100$	46.5	46.5
$\alpha = \beta = 200$	58.4	58.4

With all the previous assumptions, it can be demonstrated that system (9) has solution in  $p_1$  and  $p_2$  if a value of  $\alpha$  satisfying the following condition exists:

$$\beta \frac{1 - \sqrt{1 - \frac{8}{\beta}}}{2\sqrt{\gamma}} < \sqrt{\alpha} < \beta \frac{1 + \sqrt{1 - \frac{8}{\beta}}}{2\sqrt{\gamma}}; \quad (19)$$

Equation (19) gives us some interesting insights. When considering noise limited systems ( $\text{SIR} \gg \text{SINR}$ , i.e. small values of  $\gamma$ ) there is more opportunity to find a value of  $\alpha$  which satisfies the conditions for the existence of NE, compared with the interference limited systems ( $\text{SNR} \gg \text{SINR}$ , i.e. big values of  $\gamma$ ). In a noise limited system, users are almost indifferent to the other users' operations. On the contrary, in interference limited systems, the intra system interference plays a determinant role. The gain in terms of utility attained by one player when the other player is backed off is considerable. In other words, interference limited systems have to satisfy an additional condition on intra system interference and this determines more strict conditions for the existence of Nash equilibriums.

#### IV. NUMERICAL RESULTS

In this section we evaluate the performance of the multiple Nash equilibria obtained as solutions of system (9). The distance between cognitive transmitter and its intended receiver is set to 10 m. The parameters  $\alpha$  and  $\beta$  (ranging from 1 to 200 in our simulations) determine the distances between the other entities in our scenario (Table I). The AWGN noise  $\sigma_i$  is  $1.0 \times 10^{-10}$ . Path gains are obtained by simple path model  $g_{ij} = k(\frac{d_0}{d_{ij}})^\alpha$  where  $k = 1.0 \times 10^{-6}$ ,  $d_0 = 10\text{m}$ ,  $\alpha = 3$  and  $d_{ij}$  is the distance between transmitter  $i$  and receiver  $j$ .

Considering only the cases in which system (9) is solvable, we compare the performance of one of the multiple Nash equilibria  $(p_1, p_2)$  of our game with the social optimal solution. In our simulations, due to the perfect symmetry in intra-channel interferences ( $g_{11} = g_{22}$ ,  $g_{12} = g_{21}$ ), the social optimal solution is obtained when each secondary user chooses a transmit power generating an equal distribution of the received power threshold at the measurement point, i.e.  $(p_{1opt}, p_{2opt}) = (\frac{T}{2g_{10}}, \frac{T}{2g_{20}})$  [7]. It is also important to notice that, provided that system (9) is solvable, the social optimal solution is always one of the multiple NEs of our game (*Theorem 1*). This is a consequence of the perfect symmetry assumed in our scenario.

Figure 2 shows the system utility versus  $\gamma$ . As it can be seen, the system utility for both social optimal and our NE increases with  $\gamma$ . This happens because with the higher threshold, both cognitive users can transmit with higher power, and this results

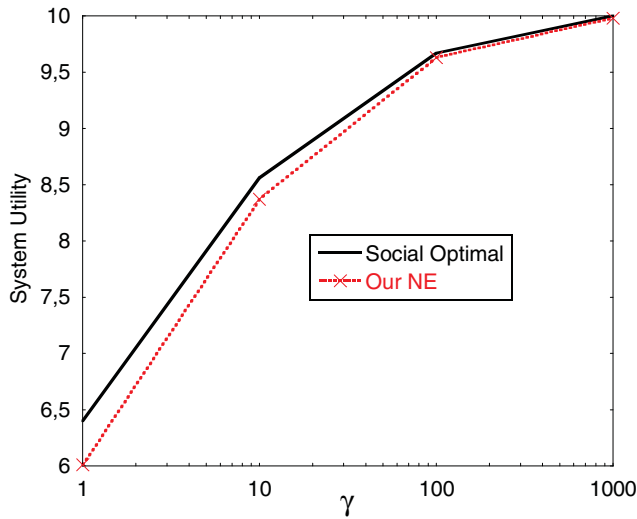


Fig. 2. System utility: Social Optimal and Nash Equilibrium of our game

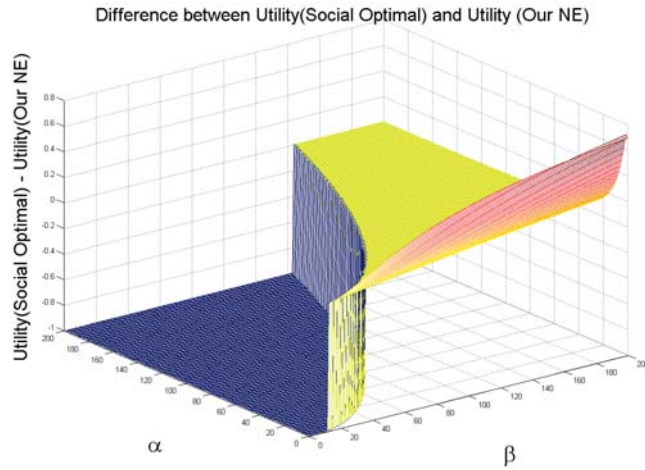


Fig. 3. System utility: Social Optimal - Our Nash Equilibrium ( $\gamma = 100$ )

in higher SINR at their intended receiver. Moreover for small values of  $\gamma$  the performance of our NE is slightly worse than the social optimal. With the low  $\gamma$ , as equation (19) confirms, intervals in  $\alpha$  which satisfy the conditions for the existence of Nash equilibria are wider. Therefore cognitive players have more opportunity to select a joint power strategy which is sensibly worse than the social optimal solution.

We also study the values of  $\alpha$  and  $\beta$  which satisfy equation (19), for fixed value of  $\gamma$ . The surface in Figure 3 shows the difference between the system utility gained *a)* with the social optimal solution and *b)* with one of the multiple NEs in our game. This surface assumes value -1 (blue color) for the values of  $\alpha$  and  $\beta$  which don't admit multiple NEs in our game.

As it can be seen in Figure 3, for a fixed value of  $\alpha$ , higher value of  $\beta$  gives *a)* more opportunity for the existence of multiple NEs and *b)* slightly worse performance of the NE in our game. This happens because with the higher  $\beta$  the

intrasystem interference is lower, therefore the gain in terms of utility attained by one player when the other player is backed off becomes more and more negligible. Consequently there is *a)* more opportunity to find a NE in which both users transmit simultaneously and *b)* higher possibility to select a joint power strategy worse than the social optimal solution. These considerations are confirmed by equation (19). We can also notice that higher values of  $\alpha$  require higher values of  $\beta$  for the existence of multiple Nash equilibria in our game. With higher  $\alpha$ , the channel gain toward the measurement point is lower, therefore cognitive users can transmit with higher power. The point is that, with the higher  $\beta$  the intrasystem interference is lower, therefore the two players don't harm each other considerably. Therefore the utility gain attained by one player when the other player is backed off is not sufficient.

## V. CONCLUSIONS

In this paper, we have analyzed spectrum sharing among two spread spectrum users with a constraint on the total interference temperature at a particular measurement point. Assuming limited coordination between measurement point and secondary users, we have modeled a non-cooperative power control game. With the assumption that secondary users are randomly backed off upon violation of ITL, we have identified all the possible outcomes of our game and the conditions for the existence of multiple Nash equilibria. Moreover, we have shown that our proposed solution achieves a satisfactory performance for different values of ITL.

In our future work, we will generalize our analysis to the case of multiple secondary users. In that case we need to consider many intra-system interferences: it will be therefore much more complicated to determine the conditions for the existence of Nash equilibria.

## ACKNOWLEDGEMENT

This work is in part supported by Danish Research Council for Technology and Production Sciences, (project no. 274-05-0490).

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## 8.8 Paper 8

### **Efficient Spectrum Leasing via Randomized Silencing of Secondary Users**

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submitted to IEEE Transactions on Wireless Communications, December 2009





# Efficient Spectrum Leasing via Centralized Control of a Random Secondary User Back Off

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## Abstract

In this paper, a primary (licensed) user leases part of its resources to independent secondary (unlicensed) terminals in exchange for a tariff in dollars per bit, under the constraint that secondary transmissions do not cause excessive interference at the primary receiver (PRX). The PRX selects a power allocation (PA) for the secondary user that maximizes the secondary rate (and thus its revenue) and enforces it by the following mechanism: Upon violation of a predefined interference level, PRX keeps backing off randomly selected secondary users, until the aggregate secondary interference is below the required threshold. This scenario gives rise to a Stackelberg game, in which the primary determines the PA and a Nash equilibrium (NE) constraint is imposed on the PA to ensure that secondary users do not have incentives to deviate. In principle, the primary should find the set of all PAs that are NE and among them choose the one that maximizes the aggregate secondary utility, and thereby the revenue of the primary. For the most general setting of channel gains, we investigate the conditions for NE for a subset of PAs. When the scenario is symmetric in the sense that all secondary users have the same channel gains in the direct/interfering links, we prove that only two optimal power allocations exist. Finally, for the case of general channel gains with strong interference, we show that there is a unique NE of the game.

## Index Terms

Cognitive radio, Dynamic spectrum sharing, Game theory, Power Control, Nash Equilibrium, Stackelberg game, Backing off process



## I. INTRODUCTION

Cognitive radio [1] and dynamic spectrum access [2] have recently challenged spectrum regulation, which had been rather conservative for many years. The main idea is to allow coexistence on the same spectral resource among primary (licensed) and secondary (unlicensed) users. In this paper, we focus on the *property-rights model* [2]: primary users are aware of the existence of secondary terminals and they can grant the secondary users the possibility to use the primary band, provided that such a secondary operation does not provoke excessive interference to the primary receivers (PRXs). The rationale is that primary users lease portion of their resources and charge the secondaries in price per bit, such that a primary user is motivated to maximize the aggregate secondary throughput. In turn, the secondary users can start their own transmissions using the primary band.

In our scenario, depicted on Figure 1, there is one PRX which receives signal from a primary base station and sets the maximum amount of tolerated interference,  $Q$ , from the  $N$  secondary users toward the PRX. The PRX is assumed to know all the channel gains in the system and allocates secondary powers aiming at maximizing secondary utility, and thus its revenue, while guaranteeing that the interference constraint is met. A secondary user is selfish and it would deviate from the prescribed power allocation (PA) if it can increase its utility. In order to discourage the secondaries from deviating and enforce a desired PA, the primary uses a back-off mechanism, such that a secondary user that is backed off needs to cease transmission. There are many options on how to select users to be backed off upon violation of  $Q$ , but we advocate randomized back-off as an effective strategy. The PRX knows the identities of the secondary users, since it admits them in the primary spectrum, but it does not have the ability to assess the interference created by each individual secondary user. We assume that PRX can measure the total interference from all the secondary users, but not the interference contribution from each individual secondary user. Thus, upon violation of the interference constraint  $Q$ , PRX cannot pinpoint which secondaries are responsible for the violation. Therefore PRX starts a backing off process meant to enforce desired behaviors to the secondary users: It randomly selects one user to back-off and shuts down its transmitter, regardless of the actual interference provoked by each user<sup>1</sup>. This assumption is realistic because no decoding of the interference signal is needed at

<sup>1</sup>The architecture and method to realize such signaling between primary and secondary are out of the scope of the paper.

the primary receiver PRX.

We assume that the secondary users follow the power allocation (PA) selected by the PRX only if they have no incentives to deviate. This fact is modeled via a non cooperative power control game so that the corresponding NEs [4] [5] (and the references therein) are taken as stable operating points for the secondary users. *Stackelberg games* [9] can be used as the appropriate analytical framework to study the scenario of spectrum leasing considered in this paper. In such a hierarchical game model, one agent (the competitive secondary network) acts subject to the strategy chosen by the other agent (the PRX leasing the primary spectrum). The latter in turn seeks maximization of the sum-secondary utility, under a maximum interference constraint to the primary system, and under an equilibrium (NE) constraint on the secondary activity. PRX's strategy that yields the optimal solution and the corresponding power response of the secondary network are jointly referred to as *Stackelberg equilibrium* [10]-[19].

#### A. Related Work

Previous works have investigated the application of Stackelberg games to modeling resource allocation problems in the context of cognitive radio, due to its effectiveness in modeling dynamic strategic decisions [10]. In [11] the authors propose a game theoretical approach that allows master-slave cognitive radio pairs to update their transmission powers and frequencies simultaneously. A Stackelberg game model is presented for frequency bands where a licensed user has priority over opportunistic cognitive radios. The main focus in [11] is the investigation of virtual price update algorithm used the licensee to admit secondary users. In [12] the authors propose a cooperative cognitive radio framework, where primary users, aware of the existence of secondary users, may select some of them to be cooperative relay, and in turn lease portion of the channel access time to them for their own data transmission. Reference [13] studies the problem of pricing uplink power in wide-band cognitive radio networks under the objective of revenue maximization for the service provider and while ensuring incentive compatibility for the users. In [14] a Stackelberg game between three entities (spectrum owner, primary users and secondary users) is presented under the opportunistic spectrum access model, where the secondary users share the channel with primary users in time.

Reference [15] considers how to allocate power among competing users sharing a frequency selective channel: The concept of Stackelberg equilibrium is introduced in order to characterize

the strategic behavior of a user by considering the response of its competing users. In [16] a “waterfilling” game in fading multiple access channels is studied where the base station is introduced as a player interested in maximizing a weighted sum of the individual rates. Reference [17] proposes Stackelberg game theoretic framework for distributive resource allocation over multiuser cooperative communication networks to improve the system performance and stimulate cooperation. In [18] the authors propose a distributed buyer/seller game to stimulate cooperation and improve the system performance in a multiuser cooperative network. The work [19] is primarily interested in the optimal design of an access point in a decentralized network.

Our work is not concerned with any of the aspects above. The central concept introduced here is the mechanism used by the primary in order to enforce certain efficient PAs in a spectrum leasing framework. To the best of our knowledge, this scenario has not been studied hitherto.

### *B. Paper Organization*

In Section II we define our target scenario and system model. Specifically, we discuss options for selecting users that are backed off and we single out random selection as an effective option in our work. In Section III, in a game with arbitrary channel gains, we identify conditions for a wideband state (where all users transmit at the same time without provoking excessive interference at the primary system) to be a NE. In section IV we focus on the special case of a symmetric network and we prove that, depending on the intra-secondary interference and on the channel gain in the direct link, only two optimal power allocations exist: a wideband state defined above, and an orthogonal state (e.g., TDMA) in which only one user transmits at a time. In Section V, in the case of asymmetric network, but strong interference, we show that the orthogonal state is the unique NE of the game. Section VI concludes the paper.

## II. TARGET SCENARIO AND SYSTEM MODEL

We consider coexistence between primary and secondary (cognitive) users, as illustrated in Figure 1. In our model, there is one primary receiver, PRX, that has a possibility to admit secondary users in the primary spectrum, in exchange for a certain revenue (paid in price per bit), provided that the interference at the PRX is maintained below a threshold  $Q$ . The value of  $Q$  is chosen by the PRX and is not object of study in this paper. We assume that the PRX can ideally measure and quantify the real-time interference toward the primary system. Moreover,

we consider an arbitrary number  $N$  of secondary users (pair of transmitter and receiver) that are willing to access the primary spectrum and are ready to pay for it. Secondary users follow the power allocation (PA) selected by the PRX only if they have no incentives to deviate. Upon violation of  $Q$ , a secondary user is backed off. In the next subsection, we discuss three different options for selecting the users to back off upon violation of  $Q$ .

#### A. Backoff Mechanisms to Enforce Policy

Since secondary users are selfish, there is no guarantee that they will adhere to the assigned PA unless it is a NE of the game modeling their preferences, discussed below. Here we list three different strategies which the PRX could possibly use and we advocate the choice of a strategy based on iterative random backing off.

*Strategy 1:* Assume that the PRX has the ability to decode the signals and thus quantify the interference from each secondary user. In this condition, the PRX can enforce any PA, as each user is individually responsible for its interference at the PRX. For example, in [20], the authors assume that the primary system has the ability to trace the interference of individual secondary users and prove the existence of a unique NE. This scenario is not of interest here, as we assume that the PRX cannot decode the secondary signals.

*Strategy 2:* Assume again that the PRX does not decode signals from secondary users. However, given that both PRX and all secondary users know all the channel gains, all are aware of, for given PA, which secondary users are disadvantaged and thus have incentives to deviate. If some users deviate from the allocated PA and cause violation of  $Q$ , then the PRX can decide to silence the subset of penalized secondary users, because they have incentive to deviate. Such a strategy at the PRX can easily give rise to unfair behavior: For example, user  $A$ , knowing that a given power allocation penalizes user  $B$ , will most likely deviate by increasing its power, because he is aware that in any case  $B$  will be backed off as a “usual suspect”.

*Strategy 3:* Assume that the PRX does not have the ability to decode the interference signals. In this case, upon violation of the  $Q$ , PRX starts an iterative backing off process: Until  $Q$  is no longer violated, it randomly selects one cognitive user to be backed off, i.e., to shut off its transmitter. The key point here is that secondary users are gambling whenever they increase their transmitting power: There is no certainty upon user deviation. For example, let us consider two cognitive users whose interference exceeds the interference limit  $Q$ . In this case, the PRX

first backs off a randomly selected secondary users. Then, if  $Q$  is no longer violated, the second cognitive transmitter can continue its transmissions. Otherwise, no secondary transmission is allowed. Generally, for an arbitrary number of users  $N$ , at the end of the backing off process, the protection of primary users is guaranteed with certainty.

We assume that secondary users have the freedom to deviate from the first PA, but they are obliged to obey a backoff command, because generally the back off is done by user and thus the *culprit* can always be identified. Note that when certain type of PA is imposed by the PRX backing off is part of the allocation, i.e., the primary knows in advance that backoffs should occur: in these cases some backoff may include multiple users if necessary. For example, when PRX allocates the PA in correspondence of which each secondary generates the maximum tolerated interference  $Q$  at the PRX, it knows in advance that, in a  $N$ -player game,  $N - 1$  users have to be backed off: In this case the PRX randomly selects all the  $N - 1$  users to back off at once, instead of backing off one of them at a time. On the other hand, if a user deviates from the PA, then there may be backoffs that were not initially planned by the PRX. One might ask: how can the primary be sure that a given secondary will obey the backoff command? As a slight digression on this issue, one can consider ordinary framed ALOHA: it is designed under the assumption that all users will follow the command to enlarge the backoff window upon collision. If not, then additional inference algorithms should be applied in order to assess whether the terminals are obeying the commands. The complete mechanism for verifying that the users are obedient to the backoff command is outside the scope for this paper. For the purpose of this paper, we can assume that punishment can be in the form of a reputation mechanism. If a disobedient secondary user is found, then this user gains a bad reputation in a centralized reputation system and no primary user will grant access to that secondary user for extended (practically infinite) period of time. In short, we assume that there is a system that strongly discourages deviation upon a backoff command, but detailed specification and analysis is outside the scope for this work.

### III. ARBITRARY CHANNEL GAINS

Here we formalize the game to model preferences and competitive behavior of the secondaries under a backoff process. Later we discuss the optimal power allocation used by PRX.

Let  $\mathcal{G} = [\mathcal{N}, \{\mathcal{P}_i\}, \{R_i\}]$  denote the secondary non-cooperative power control game where the

$N$  players in the game correspond to the  $N$  secondary users. Each player  $i \in \mathcal{N} = \{1, 2, \dots, N\}$  selects a transmitting power, i.e., strategy,  $p_i \in \mathcal{P}_i = \{p_i \geq 0\}$  such as to maximize his utility  $R_i$ . The joint strategy space  $\mathcal{P}_i = \mathcal{P}_1 \times \mathcal{P}_2 \times \dots \times \mathcal{P}_N$  is the Cartesian product of the individual strategy sets for the  $N$  players. As shown in Figure 2,  $\gamma_i$  denotes the channel gain for secondary user  $i$ ,  $\epsilon_{ij}$  is the intra-secondary system gain between transmitter  $i$  and receiver  $j$ , and  $\alpha_i$  represents the channel gain between transmitter  $i$  and the PRX.

User  $i$ 's evaluation of the spectrum is characterized by a utility function  $R_i(\mathbf{p})$ , with  $\mathbf{p} = [p_1, \dots, p_N]$ . This depends on received SINR  $\gamma_i$  at user  $i$ 's receiver, given by

$$\gamma_i(\mathbf{p}) = \frac{p_i \gamma_i}{\sum_{j \neq i} p_j \epsilon_{ji} + N_0} \quad (1)$$

where  $p_i$  is user  $i$ 's transmission power.  $N_0$  is the noise power, assumed to be the same for all users. The total interfering power at the PRX must satisfy

$$P_I = p_1 \alpha_1 + p_2 \alpha_2 + \dots + p_N \alpha_N \leq Q, \quad (2)$$

where  $Q$  is the the maximum amount of tolerable interference at the PRX. The PRX backs off randomly selected secondary users, one at a time, until condition (2) is satisfied. The final set  $\mathcal{S} \subseteq \mathcal{N}$  of non-backed-off users represents the set of users that are allowed to transmit satisfying the interference constraint. We define the *capacity* function  $r_i$  as

$$r_i(\mathbf{p}, \mathcal{S}) = \begin{cases} \log_2 \left( 1 + \frac{p_i \gamma_i}{\sum_{j \in \mathcal{S}, j \neq i} p_j \epsilon_{ji} + N_0} \right) & \text{if } i \in \mathcal{S} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

The utility function is defined as the average rate with respect to the back-off process as

$$R_i(\mathbf{p}) = E_{\mathcal{S}}[r_i(\mathbf{p}, \mathcal{S})]. \quad (4)$$

The distribution of the set of transmitting users  $\mathcal{S}$  depends on  $\mathbf{p}$ ,  $Q$  and the channel gains  $\alpha_i$ , since PRX backs off one random user until (2) is satisfied. The  $i$ th user utility is written as:

$$R_i(\mathbf{p}) = \sum_{\mathcal{S} \subseteq \mathcal{N}: i \in \mathcal{S}} \Phi(\mathbf{p}, \mathcal{S}) r_i(\mathbf{p}, \mathcal{S}) \quad (5)$$

$$\text{where} \quad \Phi(\mathbf{p}, \mathcal{S}) = \Pr[\text{set } \mathcal{S} \text{ selected at the end of the back-off process} | \mathbf{p}]. \quad (6)$$

For instance, suppose that  $N = 2$ ,  $\alpha_i = \epsilon_{ij} = \gamma_i = 1$ , and  $p_i \leq \frac{Q}{2}$ . The utility is  $R_i(\mathbf{p}) = \log_2 \left( 1 + \frac{p_i}{p_j + N_0} \right)$  ( $i = 1, 2$  and  $j \neq i$ ), because no backing-off process at the PRX is needed, and

the two players can always transmit at the same time, i.e.,  $\Phi(\mathbf{p}, \{1, 2\}) = 1$  and  $r_i(\mathbf{p}, \{1, 2\}) = \log_2(1 + \frac{p_i}{p_j + N_0})$ . On the contrary, if  $\frac{Q}{2} < p_i \leq Q$  ( $i = 1, 2$ ), the interference at the PRX exceeds  $Q$  and the PRX randomly selects one user to back-off so that there is always one user transmitting at any time. The utility can be therefore written as  $R_i(\mathbf{p}) = \frac{1}{2} \log_2(1 + \frac{p_i}{N_0})$  because  $\Phi(\mathbf{p}, \{1, 2\}) = 0$ ,  $\Phi(\mathbf{p}, \{i\}) = \frac{1}{2}$  and  $r_i(\mathbf{p}, \{i\}) = \log_2(1 + \frac{p_i}{N_0})$ .

Note that the utility (5) is not necessarily equal to the achievable rate, which would entail that the secondary operate over multiple blocks and thus attain a rate averaged over time. Equation (5) may as well be taken as a reasonable metric to be used by the secondary users to guide their choices. The PRX attempts to maximize the sum-utility of the secondaries  $\sum_{i=1}^N R_i(\mathbf{p})$ , under the condition that no secondary will deviate its allocation from  $\mathbf{p}$ . In order to address the latter point, we utilize the concept of Nash Equilibrium (NE).

*Definition 1:* A power profile  $\mathbf{p}^* = (p_1, p_2, \dots, p_N)$  is said to be a NE of the strategic game  $\mathcal{G}$  if for every  $i = 1, 2, \dots, N$

$$R_i(p_i, p_{-i}) \geq R_i(p'_i, p_{-i}) \quad (7)$$

for any  $p'_i \neq p_i \in \mathcal{P}_i$ , where  $p_{-i}$  denotes the transmission powers of all the users except user  $i$ . At a NE, given the powers of the other users, no user can increase its utility by deviating through unilateral changes in its power. We define  $\mathcal{NE}$  such that  $\mathbf{p} \in \mathcal{NE}$  if and only if it is NE.

Defining as  $\mathcal{NE}$  the set of all NEs, the problem to be solved at the PRX is then:

$$\begin{aligned} & \max_{\mathbf{p}} \sum_{i=1}^N R_i(\mathbf{p}) \\ & \text{s.t. } \mathbf{p} \in \mathcal{NE} \\ & (p_1 + \dots + p_N)\alpha \leq Q \end{aligned} \quad (8)$$

In correspondence of (8), the PRX allocates the secondary powers  $\mathbf{p} = (p_1, \dots, p_N)$  so as to maximize the secondary system utility under the maximum interference constraint to the primary system, and under the constraint that  $\mathbf{p}$  is one of the NEs of the game. This is an optimization problem with equilibrium constraints also referred to as Stackelberg game [10]-[19].

#### A. Solving the General Problem

In general, solving (8) requires to explore all NEs. However, finding the set of all PAs that are NE is, in general, a tedious task. In this section, instead, for the general model at hand, we



focus on two specific solutions: The wideband allocation  $\mathbf{p}_{WB} = (\frac{Q}{N\alpha}, \frac{Q}{N\alpha}, \dots, \frac{Q}{N\alpha})$ , where each user provokes identical interference  $\frac{Q}{N}$  and there is no back off, and the orthogonal allocation  $\mathbf{p}_{OR} = (\frac{Q}{\alpha}, \frac{Q}{\alpha}, \dots, \frac{Q}{\alpha})$  that has  $N - 1$  planned backoffs, as only one user can be left to transmit.

$\mathbf{p}_{WB}$  is relevant when channel gains in the direct links are sufficiently high. For example, when the intended receiver is much closer to the transmitter compared to the other receivers that are interfered by that transmitter, likely no secondary will have incentives to deviate and no backoff will be caused. This can be advantageous both for the primary (PRX need not to back off users) and for secondaries that are interested in non-bursty communications (i. e. a secondary prefers to transmit for long time at low rates rather than at high rates in short time).

The next proposition identifies the conditions upon which the wideband transmission  $\mathbf{p}_{WB}$  and the orthogonal transmission  $\mathbf{p}_{OR}$  are NEs in our game. It should be noted that the policy  $\mathbf{p}_{OR}$  provides the same utility, when (5) is interpreted as achievable rate, as one in which the PRX selects randomly one on the  $N$  users for transmission.

*Proposition 1:* In the game  $\mathcal{G}$  defined above *i)*  $\mathbf{p}_{OR}$  is always a NE, and *ii)* there exists a minimum direct link gain  $\gamma^*$  such that if  $\gamma_i > \gamma^*$ , (where  $\gamma^*$  is generally a function of the interference gains  $\epsilon_{ij}$ ) for  $i = 1, \dots, N$ , then  $\mathbf{p}_{WB}$  is NE in our game.

*Proof:* See Appendix A.

The main conclusion here is that,  $\mathbf{p}_{OR}$  is always a NE and thereby considered as an allocation strategy by the PRX. When the channel gain in the direct link of each secondary user is sufficiently large with respect to the interference (see previous Proposition),  $\mathbf{p}_{WB}$  becomes a NE and thus appears as an alternative to the primary. When  $\mathbf{p}_{WB}$  is NE, no user  $i$  ( $i = 1, \dots, N$ ) can increase its utility by unilaterally increasing its transmitting power to  $\frac{Q}{\alpha_i}$ . But it can be seen from (5) that the utility of user  $i$ , with this deviation, will be equal to the one obtained with  $\mathbf{p}_{OR}$ . Therefore the aggregate secondary utility in correspondence of  $\mathbf{p}_{WB}$ , when is NE, is higher than in correspondence of  $\mathbf{p}_{OR}$ . Since the primary is paid in price per bit, it will always allocate  $\mathbf{p}_{WB}$  when it is NE. To summarize, whenever  $\mathbf{p}_{WB}$  is a NE, the PRX will tend to select this PA over  $\mathbf{p}_{OR}$ . Otherwise, it can always select  $\mathbf{p}_{OR}$ , this being a NE of the considered game, albeit not guaranteed to maximize the aggregate secondary utility.

To illustrate Proposition 1, on Figure 3 we plot  $\gamma^*$ , which is the minimal value of  $\gamma$  for which  $\mathbf{p}_{WB}$  is a NE, assuming symmetry in channel conditions  $\gamma_i = \gamma$ ,  $\alpha_i = \alpha = 0.1$ ,  $\epsilon_{ij} = \epsilon = 0, 5, 10, 20$  (for  $i, j = 1, \dots, N$  and  $i \neq j$ ) and  $Q = 1$ . The x-axis represents the number



of secondaries. It can be seen that in the absence of intra-system interference,  $\epsilon = 0$ ,  $\mathbf{p}_{WB}$  is always a NE in a game with  $N = 2$  and 3 secondaries, regardless of  $\gamma$ . In fact, due to (5) the user's utilities are the result of a tradeoff between transmission probability and SINR. In the absence of intra-system interference, there is no gain in terms of lower intra-system interference for the deviating user when one or more of the other secondaries are backed off. When  $N = 2$  or  $N = 3$ , the probability to be backed off for the deviating secondary is considerable and always dominates the advantages coming by transmitting with higher power. When  $N > 3$ , the situation is different and a minimum value of  $\gamma^* > 0$  is needed to sustain  $\mathbf{p}_{WB}$ . Note that  $\gamma^*$  increases with  $N$ . On the one hand, the probability to be backed off decreases as the number of secondaries increases, thus giving higher motivation for deviation. On the other hand, when  $\gamma$  is large users prefer to transmit for the longest time possible without violation of  $Q$ . These two conflicting needs explain why  $\gamma^*$  increases with the number of secondaries  $N$ .

Figure 3 shows also the influence of the intra-system interference, corresponding to different values of  $\epsilon$ . For fixed  $N$ , the value of  $\gamma^*$  increases with  $\epsilon$ . This is because upon increasing  $\epsilon$ , the secondaries harm each other more and more and therefore a given user may sensibly increase its utility upon deviation. Similarly as before, it is always possible to find a minimum value  $\gamma^*$  where all the secondaries prefer not to violate the  $Q$ . This happens because the tradeoff between the lower intra-system interference and the backing off probability upon deviation is not advantageous for the deviating secondary when  $\gamma$  is very high.

### B. Simulation results for Rayleigh fading channels

Here we evaluate the secondary aggregate utility in correspondence of the PA selected by the PRX. As a reference, we evaluate the PA that maximizes the aggregate secondary utility, while respecting the interference constraint (2), but is not necessarily a NE. That PA is obtained as a solution to the optimization problem modeling the case where secondary users are not selfish and can be written as:

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{i=1}^N R_i(\mathbf{p}) \\ \text{s.t.} \quad & p_1 \alpha_1 + \dots + p_N \alpha_N \leq Q \end{aligned} \tag{9}$$

We consider a Rayleigh fading scenario, where the channels  $\gamma_i$ ,  $\alpha_i$  and  $\epsilon_{ij}$  fade independently. We assume that all the  $\alpha_i$  ( $i = 1, \dots, N$ ) fades independently with average  $\bar{\alpha}$ , all the  $\gamma_i$  ( $i =$

$1, \dots, N$ ) fade independently with average  $\bar{\gamma}$ , and all the  $\epsilon_{ij}$  ( $i = 1, \dots, N, j = 1, \dots, N, i \neq j$ ) fade independently with average  $\bar{\epsilon}$ . Recall (Proposition 1) that  $\mathbf{p}_{OR}$  is always NE while it exists a condition on the direct link of each secondary user for  $\mathbf{p}_{WB}$  to be NE.

Figure 4 refers to scenario with three secondary users ( $N=3$ ) where  $\bar{\alpha} = 1$  dB,  $\bar{\epsilon} = 1$  dB and  $\bar{\gamma}$  ranges between 1 and 50 dB. We average the performances of four different power allocations over 10000 channel realizations. We plot the average secondary aggregate utility with the four following power allocation: *a*) optimal power allocation (obtained numerically for each particular fading instance), *b*)  $\mathbf{p}_{WB}$ , regardless if it is a NE, *c*)  $\mathbf{p}_{OR}$ , *d*)  $\mathbf{p}_{WB}$  if it is NE, and  $\mathbf{p}_{OR}$  otherwise. It can be noted that, for small values of  $\bar{\gamma}$ , the secondary aggregate utility is larger with  $\mathbf{p}_{OR}$  than with  $\mathbf{p}_{WB}$ . This happens because when the channel gain in the direct link is, on average, equal to channel gain of the interfering links between the secondary users, the intra-secondary interference strongly affects the SINR at the receiver. Therefore, each user benefits transmitting without intra-secondary system interference, as with  $\mathbf{p}_{OR}$ . The situation is reversed when  $\bar{\gamma}$  increases: in this case the direct link is on average sensibly larger than the interfering links and favors the non-risky  $\mathbf{p}_{WB}$ , since the interference is not very significant. Note that for  $\bar{\gamma} < 10$ ,  $\mathbf{p}_{WB}$  is not a NE and thus its aggregate secondary utility may be smaller than for  $\mathbf{p}_{OR}$ .

Figure 5 shows the probability *a*) that the secondary aggregate utility in correspondence of  $\mathbf{p}_{WB}$  is higher than in correspondence of  $\mathbf{p}_{OR}$ , and *b*) that  $\mathbf{p}_{WB}$  is NE and therefore allocated by the PRX. It is important to note that in order for  $\mathbf{p}_{WB}$  to be NE, the condition in *a*) is not sufficient; on the contrary we know with certainty that if  $\mathbf{p}_{WB}$  is NE, then the aggregate secondary utility in its correspondence is higher than with  $\mathbf{p}_{OR}$ . When  $\bar{\gamma}$  increases, the probability that  $\mathbf{p}_{WB}$  outperform  $\mathbf{p}_{OR}$  increases considerably towards 1. The same tendency can be observed about the probability for  $\mathbf{p}_{WB}$  to be NE, albeit this probability attains much lower values than 1. For fixed value of  $\bar{\gamma}$ , the probability for  $\mathbf{p}_{WB}$  to be a NE is higher for  $N = 3$  compared to  $N = 5$ . This happens because in our game setting, users are randomly selected for backing off upon violation of  $Q$ , and therefore the probability to be backed off decreases as the number of secondaries increases: a higher value of  $\bar{\gamma}$  is needed with  $N = 5$  for  $\mathbf{p}_{WB}$  to be a NE.

In this section, we have compared only two specific PAs, i.e.,  $\mathbf{p}_{WB}$  and  $\mathbf{p}_{OR}$ : In a game with arbitrary channel gain it is very difficult to identify all the possible NEs. Nevertheless, when considering specific cases, e.g., symmetric networks (Section IV) and strongly interfered scenarios (Section V), we are able to approach the problem of optimal PA in a more comprehensive

and exhaustive way.

#### IV. SYMMETRIC CHANNEL GAINS

In this section, we restrict our attention to a symmetric scenario where *a)* all the secondary users have the same channel gain toward the intended receiver  $\gamma_i = \gamma$  for  $i = 1, \dots, N$ , *b)* all the users have the same channel gain toward the PRX  $\alpha_i = \alpha$ , for  $i = 1, \dots, N$  and *c)* the intra-secondary interference between transmitter  $i$  and received  $j$  is the same for all the users  $\epsilon_{ij} = \epsilon$  for all  $i, j = 1, \dots, N$  with  $i \neq j$ . In Section III-A, we have compared two specific PAs in terms of overall secondary utility, while here we attack problem (8) more thoroughly. We identify the optimal power vector  $\mathbf{p}$  allocated by the PRX to the secondary users in two different cases. We first find  $\mathbf{p}$  that maximizes the aggregate secondary utility without necessarily being a NE. Next, we study the maximization problem as defined in (8).

##### A. Without NE constraint

Here we study problem (9) restricted to a symmetric network.

*Proposition 2:* There exists a  $\epsilon^* \geq 0$  such that the solution to (9) is *a) wideband transmission*  $\mathbf{p}_{WB} = (\frac{Q}{N\alpha}, \frac{Q}{N\alpha}, \dots, \frac{Q}{N\alpha})$  when  $\epsilon < \epsilon^*$ ; *b)  $\underline{\mathbf{p}}$* , in which all non-zero elements are equal to  $\frac{Q}{\alpha}$ .

*Proof:* See Appendix B.

The threshold  $\epsilon^*$  generally changes with the number  $N$  of secondaries and is function of  $Q$  and noise level at the receiver. It is worth noticing, that in a scenario with arbitrary channel gain, the solution to (8) can in principle be any power allocation satisfying the interference constraint (2) at the PRX. Note that  $\mathbf{p}_{OR}$  is a special case of  $\underline{\mathbf{p}}$ , in which no component is zero.

##### B. With NE constraint

Here we deal with problem (8). Given the result of the previous section,  $\mathbf{p}_{WB}$  and the orthogonal transmissions  $\underline{\mathbf{p}}$  are the only possible solutions. However, in (8) they have to satisfy an additional constraint: they must be a NE of the game  $\mathcal{G}$ . Note that only  $\mathbf{p}_{OR}$  among all possible vectors  $\underline{\mathbf{p}}$  is a NE, simply because it is evident that each user that has been allocated a power of zero has incentive to deviate.

The next proposition identifies the conditions under which *a)* they are NEs and *b)* they maximize the aggregate system throughput in (8).

*Proposition 3:* If  $\gamma > \gamma^*$ , with  $\gamma^*$  defined in Proposition 2, and if  $\epsilon < \epsilon^*$ , with  $\epsilon^*$  defined in Proposition 1,  $\mathbf{p}_{WB}$  solves problem (8); otherwise the solution is  $\mathbf{p}_{OR}$ .

*Proof:* Proposition 1 states that, in a game with arbitrary channel gains, a)  $\mathbf{p}_{OR}$  is always NE, and b)  $\mathbf{p}_{WB}$  is NE if  $\gamma_i > \gamma$ ,  $i = 1, \dots, N$ . Clearly, this results holds also for a game with symmetric channel gains. Proposition 2 guarantees that, in a game with symmetric channel gains, depending on  $\epsilon$ , there exist only two possible PAs which maximize the secondary aggregate utility:  $\mathbf{p}_{WB}$  or  $\mathbf{p}_{OR}$ . Proposition 3 combines the conclusions from Proposition 1 and 2 to identify the conditions upon which  $\mathbf{p}_{WB}$  maximizes the secondary aggregate utility and is NE (as it is requested in (8)). When both conditions are satisfied,  $\mathbf{p}_{WB}$  solves (8); otherwise the solution is  $\mathbf{p}_{OR}$ .  $\square$

Figure 6 illustrates Proposition 3 through a numerical example for a game with  $N = 5$ . Depending on the values of  $\gamma$  and  $\epsilon$  two regions can be identified in Figure 6: The region with vertical lines identifies situations where  $\mathbf{p}_{WB}$  is the solution of the Stackelberg game. The region with horizontal lines identifies the conditions under which  $\mathbf{p}_{OR}$  is the optimal solution. For  $\epsilon < \epsilon^*$ , the minimum value of  $\gamma$  to support  $\mathbf{p}_{WB}$  increases with  $\epsilon$ . This happens because with the larger intra-system secondary system interference  $\epsilon$ , higher  $\gamma$  is needed to support  $\mathbf{p}_{WB}$ . The vertical dashed line in correspondence of  $\epsilon = \epsilon^*$  specifies other two important regions in correspondence of which either  $\mathbf{p}_{WB}$  ( $\epsilon < \epsilon^*$ ) or  $\mathbf{p}_{OR}$  ( $\epsilon > \epsilon^*$ ) solve problem (9), without NEs constraint as per Proposition 3.

## V. OPTIMAL POWER ALLOCATION WITH STRONG INTERFERENCE

In the previous section we have made a restrictive assumption of perfect symmetry in the secondary system. Here, we still assume that  $\gamma$  and  $\alpha$  are the same for all the secondaries, but we relax the conditions for the interference, such that  $\epsilon_{ij}$  are not necessarily equal. In our game, the PRX backs off randomly selected secondary users until (2) is satisfied. We can therefore find the distribution of the set of transmitting users  $\mathcal{S}$ , and the probability  $\Phi(\mathbf{p}, \mathcal{S})$  that set  $\mathcal{S}$  is selected at the end of a given realization of the backing off process (see (5)-(6)). Therefore, our system is akin, interpreting (5) as achievable rates, to a TDMA or FDMA system, where a given set  $\mathcal{S}$  of users transmits for a fraction of the whole time/frequency slot. The fraction of time/frequency slot allocated to set  $\mathcal{S}$  equals  $\Phi(\mathbf{p}, \mathcal{S})$  in (6).

In our scenario, under the given power constraints, TDMA and FDMA schemes are equivalent, and this allows us to reuse some interesting results presented in [22] where the authors consider

the scenario of multiple multicarrier communication systems contending in a common frequency band in flat channels and prove some conditions for the optimality of flat frequency sharing and flat FDMA (frequency division multiple access). A channel is said to be *flat* in a given frequency band  $W = f_1 - f_2$  if the channel gains are constant for any frequency within that frequency band. For detailed description of results in [22] see Appendix C.

Normalizing  $\epsilon_{ij}$ ,  $\alpha$  and  $N_0$  by the direct channel gain  $\gamma$  (see Appendix C for details), we prove the following proposition in a strong intra-secondary system interference scenario. This case is interesting because it encompasses scenarios where secondary users are located in a small geographical area, and/or with low propagation attenuation and path loss (e.g., low frequencies).

*Proposition 4:* In a  $N$ -player game, if  $\epsilon_{ij} \geq \frac{1}{2}$ ,  $\forall i \neq j$ ,  $i, j = 1, 2, \dots, N$ ,  $\mathbf{p}_{OR}$  is the unique NE and therefore solves problem (8), which is an optimization problem with NE constraints.

*Proof:* See Appendix D

## VI. CONCLUSIONS

In this paper, we have designed robust spectrum leasing solutions among  $N$  secondary users with a constraint on the total interference at a particular primary receiver (where robustness is with respect to the selfish behavior of the secondary users). We have used a secondary network power control game to model the secondary behavior. First, we have introduced a back-off mechanism at the primary system in order to enforce a given power allocation on the secondary users that guarantees that the primary link target quality of service is not affected. Then, with the random back-off mechanism in place, we have identified some special power allocations that are NEs. In a scenario with arbitrary channel gains, conditions are identified upon which a wideband state, where all users transmit at the same time and provoke the same interference at the primary, is NE with respect to the secondary user preferences. In a symmetric network, we have proved that depending on the intra-secondary interference and on the channel gain in the direct link only two optimal power allocation exist: the wideband state and an orthogonal state in which only one user transmits at a time. We have also studied a specific asymmetric network with strong intra-secondary interference and proved that there is only one type of NE.

Our study opens a large number of interesting items which deserve future investigations. The first venue to be investigated would be an efficient algorithm for finding the set of all power allocations, such that each allocation is a Nash Equilibrium. An interesting direction is to extend

the results to OFDM scenarios (or any other system with multiple frequency channels). Another relevant extension is towards repeated games and reputation-based mechanisms.

#### ACKNOWLEDGEMENT

This work is in part supported by Danish Research Council for Technology and Production Sciences, (project no. 274-05-0490).

#### APPENDIX

##### A. Proof of Proposition 1 and Related Results

Here we provide the proof of Proposition 1. We start with a few useful Lemmas.

*Lemma 1:* When deviating from  $\mathbf{p}_{WB} = \left[ \frac{Q}{N\alpha_1}, \frac{Q}{N\alpha_2}, \dots, \frac{Q}{N\alpha_N} \right]$  player  $i$  must select one of the actions  $p^{(k)} = \frac{kQ}{N\alpha_i}$ , where  $k = 2, 3, \dots, N$ , in order to possibly increase its utility.

*Proof:* Let us assume that user 1 increases  $p_1$  to improve its utility (same considerations for the other secondaries). The rate  $r_1([p_1, \dots, p_N], \mathcal{S})$  is an increasing function of  $p_1$ . Instead, there are intervals of  $p_1$  resulting in the same  $\Phi([p_1, \dots, p_N], \mathcal{S})$  in (6). These intervals are defined as follows for  $j = 1, \dots, N - 1$ :  $I_j = \{ \frac{jQ}{N\alpha_1} < p_1 \leq \frac{(j+1)Q}{N\alpha_1} \}$ .

For any value of  $p_1 \in I_j$ , at the end of the backoff process, the user 1 is either backed off or it belongs to the subset of transmitting users  $\mathcal{S}$ . A rational user 1 will select the largest transmitting power within each interval  $I_j$  and this proves the Lemma.  $\square$

*Lemma 2:* In the game  $\mathcal{G}$  there are only  $N$  symmetric PAs that are candidates for NE:

$$\mathbf{p}_i = \left[ \frac{Q}{(N-i+1)\alpha_1}, \frac{Q}{(N-i+1)\alpha_2}, \dots, \frac{Q}{(N-i+1)\alpha_N} \right] \quad (10)$$

where  $i = 1, \dots, N$ . Note that  $\mathbf{p}_1 = \mathbf{p}_{WB}$  and  $\mathbf{p}_N = \mathbf{p}_{OR}$ .

*Proof:* Fix a symmetric allocation  $\mathbf{p} = [p_1, \dots, p_N]$  such that all the secondaries generate the same interference  $I = \alpha_i p_i$  at the PRX. If  $m^*$  is the largest integer for which  $m^* I \leq Q$ , then

$$\Phi(\mathbf{p}, \mathcal{S}) = \begin{cases} 0 & \text{if } |\mathcal{S}| \neq m^* \\ \frac{1}{\binom{N}{m^*}} & \text{if } |\mathcal{S}| = m^* \end{cases} \quad (11)$$

All the symmetric  $\mathbf{p}$  in which  $m^* I < Q$  cannot be NE, because each user can increase its transmitting power keeping  $\Phi(\mathbf{p}, \mathcal{S})$  constant and thus increasing its utility. Conversely, if  $\mathbf{p}$  is such that  $m^* I = Q$  no user can increase its power without decreasing his transmission probability

$\Phi(\mathbf{p}, \mathcal{S})$ . The  $N$  PAs given in (10) are the only ones that satisfy  $m^*I = Q$ .  $\square$

*Lemma 3:* In a game  $g$ , with  $N - k + 2$  players, if  $p_1 = \frac{kQ}{N\alpha_1}$  and  $p_j = \frac{Q}{N\alpha_j}$  for  $j = 2, \dots, N - k + 1$ , user 1 is backed off with probability  $P_b = \sum_{\mathcal{S}: i \notin \mathcal{S}} \Phi(\mathbf{p}, \mathcal{S}) = \frac{1}{N-k+2}$ .

*Proof:* User 1 (user  $j$ ) provokes an interference  $I_1 = \frac{kQ}{N\alpha_1}\alpha_1$  ( $I_j = \frac{kQ}{N\alpha_j}\alpha_j$ ) at the PRX. Therefore the total interference  $P_I$  at the PRX is:  $I = \frac{kQ}{N} + \frac{(N-k+1)Q}{N} = \frac{N+1}{N}Q$ . It is then sufficient to back-off one randomly selected secondary user, in order to avoid the violation of  $Q$  at the PRX. Therefore the probability to be backed off for all the users is  $P_b = \frac{1}{N-k+2}$ .

*Lemma 4:* In a game  $g$ , with  $N - k + 1 + m$  players, if  $p_1 = \frac{kQ}{N\alpha_1}$  and  $p_j = \frac{Q}{N\alpha_j}$  for  $j = 2, \dots, N - k + m$ , user 1 is backed off with probability  $P_b = \frac{m}{N-k+1+m}$ .

*Proof:* From Lemma 3, by induction:  $P_b\left(N - k + m + 2, \frac{kQ}{N\alpha_1}\right) = \frac{m+1}{N-k+m+2}$ , where  $P_b(N, kp_1)$  is defined as the probability to be backed off for user 1 in a game with  $N$  players, when it is transmitting with power  $kp_1$ , and  $p_{-1} = p_1$ .  $\square$

*Lemma 5:* In a  $N$ -player game  $\mathcal{G}$  the backing off probabilities in correspondence of the  $N - 1$  smart actions  $p_i = \frac{kQ}{N\alpha_i}$  (Lemma 1) are  $P_b(i) = \frac{k-1}{N}$ , for any  $k = 2, \dots, N$ .

*Proof:* Without loss of generality, let us focus on user 1. Define  $P_b(N, \frac{kQ}{N\alpha_1})$  the probability for player 1 to be backed off in a game with  $N$  players, with  $p_1 = \frac{kQ}{N\alpha_1}$  and  $p_{-1} = \frac{Q}{N\alpha_j}$ ,  $j = 2, \dots, N$  as done above. In this condition user 1 is either backed off as first, with probability  $\frac{1}{N}$ , or not, with probability  $\frac{N-1}{N}$ . If not, it means that another of the  $N - 1$  players has been backed off and therefore a new game with  $N - 1$  players can be identified, where player 1 still transmits with power  $p_1 = \frac{kQ}{N\alpha_1}$ . Using Lemma 3 and 4 we can write:

$$\begin{aligned} P_b\left(N, \frac{kQ}{N\alpha_1}\right) &= \frac{1}{N} + \left(1 - \frac{1}{N}\right)P_b\left((N-1), \frac{kQ}{N\alpha_1}\right) = \\ &= \frac{1}{N} + \left(1 - \frac{1}{N}\right)\left[\frac{1}{N-1} + \left(1 - \frac{1}{N-1}\right)P_b\left(N-2, \frac{kQ}{N\alpha_1}\right)\right] = \\ &= \frac{1}{N} + \frac{1}{N} + \dots + \frac{N-k+2}{N} \frac{1}{N-k+2} = \frac{k-1}{N}. \end{aligned} \quad (12)$$

*Proof of Proposition 1:*

i) With  $\mathbf{p}_{OR} = \left[\frac{Q}{\alpha_1}, \dots, \frac{Q}{\alpha_N}\right]$  each user generates an interference at the PRX equal to  $Q$ . On the one hand, no user can increase its utility by decreasing its power. On the other hand, if any user increases its power (thus provoking an interference at the PRX bigger than  $Q$ ), then it is



backed off with certainty and its utility is always zero. This shows that  $\mathbf{p}_{OR}$  is always a NE.

ii) Let us first focus on user 1. The utility of user 1 in correspondence of  $\mathbf{p}_{WB}$  can be written as follows,  $R_1(\mathbf{p}_{WB}) = \log_2\left(1 + \frac{x_1}{S}\right)$ , where  $x_1 = p_1\gamma_1$ ,  $S = N_0 + Y_2 + \dots + Y_N$  and  $Y_2 = p_2\epsilon_{21}$ ,  $Y_3 = p_3\epsilon_{31}$ , ...,  $Y_N = p_N\epsilon_{N1}$ . Following Lemma 1, when deviating from  $\mathbf{p}_{WB}$ , player 1 must select one of the following  $N - 1$  actions  $p^{(k)} = \frac{kQ}{N\alpha_1}$ , where  $k = 2, 3, \dots, N$ , in order to possibly increase its utility. The utility of user 1 in correspondence of  $p^{(k)}$  ( $k = 2, \dots, N$ ) is:

$$R_1(p^{(k)}, \mathbf{p}_{WB-1}) = \sum_{n=2}^{N-k+2} \sum_{m=n+1}^{N-k+3} \sum_{j=n+k-1}^N \frac{k!}{N!} \log_2\left(1 + \frac{kx_1}{S - Y_n - Y_m - \dots - Y_j}\right).$$

By definition,  $\mathbf{p}_{WB}$  is a NE if  $R_1(\mathbf{p}_{WB}) > R_1(p^{(k)}, \mathbf{p}_{WB-1})$ ,  $k = 2, \dots, N$ .

If we assume, without loss of generality, that  $Y_2 \geq Y_3 \geq \dots \geq Y_N$ , it follows (Lemma 5)

$$R_1(p^{(k)}, \mathbf{p}_{WB-1}) \leq \frac{N-k+1}{N} \log_2\left(1 + \frac{kx_1}{S - \sum_{j=2}^k Y_j}\right) = V_1(k).$$

Moreover,  $R_1(p^{(k)}, \mathbf{p}_{WB-1}) \leq V_1(k) < \frac{N-k+1}{N} \log_2(1 + kx_1) = W_1(k)$ .

Let us define the continuous function of  $z$ ,  $U(z) = (N - z + 1) \log_2(1 + zx_1)$ . Sufficient conditions for  $\mathbf{p}_{WB}$  to be a NE are then that  $\frac{dU}{dz} < 0, z \geq 1$ . The previous condition results in:  $\log_2(1 + x_1z) > x_1 \frac{N-z+1}{1+x_1z}$ .  $\log_2(1 + x_1z)$  is an increasing function of  $z$ , while  $\frac{N-z+1}{1+x_1z}$  is a decreasing function of  $z$ . Therefore it is sufficient to prove the previous inequality for  $z = 1$ .

Let us define  $a = \log(1 + x_1)$ ,  $b = N \frac{x_1}{1+x_1}$ . Then,  $\lim_{x_1 \rightarrow \infty} a = \infty$  and  $\lim_{x_1 \rightarrow \infty} b = N$ . This proves that it is always possible to  $x_1$  and thus  $\gamma_1^*$  such that  $\frac{dU}{dx} < 0$  for any  $x \geq 1$ . The same procedure can be repeated for all the  $N$  players, and finally  $\gamma^* = \max(\gamma_1^*, \gamma_2^*, \dots, \gamma_N^*)$ , QED  $\square$

## B. Proof of Proposition 2

We start proving that the optimum value for (9) is smaller than the optimum value for the maximization problem solved in [21].

$$\max_{\mathbf{p}} \sum_{i=1}^N R_i(\mathbf{p}) = \max_{\mathbf{p}} \sum_{S \subseteq N} \Phi(\mathbf{p}, S) \sum_{i=1}^N r_i(\mathbf{p}, S) \leq \max_{\mathbf{p}'} \sum_{S \subseteq N} \Phi(\mathbf{p}', S) \max_{p: \sum_s p_i \leq \frac{Q}{\alpha}} \sum_{i=1}^N r_i(\mathbf{p}, S) \quad (13)$$

$$\leq \max_{\mathbf{p}'} \sum_{S \subseteq N} \Phi(\mathbf{p}', S) \max_{p: \sum_N p_i \leq \frac{Q}{\alpha}} \sum_{i=1}^N r_i(\mathbf{p}, N) \leq \max_{p: \sum_N p_i \leq \frac{Q}{\alpha}} \sum_{i=1}^N r_i(\mathbf{p}, N). \quad (14)$$

Problem (14) has been solved by [21]. The solution to (14) is either a)  $[\frac{Q}{N\alpha}, \frac{Q}{N\alpha}, \dots, \frac{Q}{N\alpha}] = \mathbf{p}_{WB}$  ( $\epsilon < \epsilon^*$ ), or b)  $p_i = \frac{Q}{\alpha}$  and  $p_j = 0$  for  $j \neq i$  ( $\epsilon > \epsilon^*$ ). But (9) evaluated for  $\mathbf{p}_{WB}$  or  $p_i = \frac{Q}{\alpha}$  and  $p_j = 0$  for  $j \neq i$  equals the optimum value of (14). Therefore they are solutions of (9) as well.



### C. Results by Zhao and Pottie from [22]

Here we define parameters  $\epsilon_{ij}$ ,  $\alpha$  and  $N_0$  which are normalized by the direct channel gain  $\gamma$ :

$$\epsilon_{ijn} = \frac{\epsilon_{ij}}{\gamma}, \quad \alpha_n = \frac{\alpha}{\gamma}, \quad N_{0n} = \frac{N_0}{\gamma}. \quad (15)$$

[22] considers multiple multicarrier communication systems contending in a common frequency band in flat channels. There are two basic co-existence strategies for common flat channels: *Flat frequency sharing* and *Flat FDMA*. Given a flat channel in the band  $(f_1, f_2)$ :

$$N_{0n}(f) = N_{0n}; \alpha_{21}(f) = \alpha_{21}; \alpha_{12}(f) = \alpha_{12}; \forall f \in (f_1, f_2), \quad (16)$$

a flat frequency sharing of two users is defined as any power allocation in the form of

$$P_1(f) = p_1, P_2(f) = p_2, \forall f \in (f_1, f_2). \quad (17)$$

On the other hand, a flat FDMA of two users is defined as any power allocation in the form of

$$\begin{cases} P_1(f)P_2(f) = 0 \\ P_1(f) + P_2(f) = p', \quad \forall f \in (f_1, f_2) \end{cases} \quad (18)$$

The authors introduce a basic transformation from flat frequency sharing to flat FDMA: *flat FDMA re-allocation*. By noting the bandwidth,  $W = f_2 - f_1$ , a flat FDMA re-allocation is defined to be the following scheme that transforms a flat frequency sharing to a flat FDMA: (1) User 1 re-allocates all of its power within a sub-band  $W'_1 = \frac{p_1}{p_1+p_2}W$  with a flat power spectral density (PSD)  $p'_1 = p_1 + p_2$ ; (2) User 2 re-allocates all of its power within another disjoint sub-band  $W'_2 = \frac{p_2}{p_1+p_2}W$  with the same flat PSD  $p'_2 = p_1 + p_2$ . Similarly, the authors define flat frequency sharing schemes, flat FDMA schemes, and flat FDMA re-allocation in  $n$ -user flat channel access. The following Lemma is proved in [22]:

**Lemma 6:** Consider an  $N$ -user flat interference channel:  $N_{0n}(f) = N_{0n}$ ,  $\alpha_{ij}(f) = \alpha_{ij}$ . Let the  $N$  users use frequency sharing:  $P_i(f) = p_i, \forall f \in (f_1, f_2), i = 1, 2, 3, \dots, N$ . If  $\alpha_{ij} \geq \frac{1}{2}, \forall i \neq j$ , then with a flat FDMA re-allocation scheme, the rate of each user at least remains equal.

### D. Proof Proposition 4

To prove Proposition 4, we need the following Lemma:

**Lemma 7:** In a  $N$ -player game, if  $\epsilon_{ij} \geq \frac{1}{2}, \forall i \neq j$

$$U_i(\mathbf{p}_{OR}) = U_i(\mathbf{p}_N) \geq U_i(\mathbf{p}_j), \quad i = 1, 2, \dots, N \quad j = 1, 2, \dots, N - 1. \quad (19)$$

*Proof:* a) We first prove that, if  $\epsilon_{ij} \geq \frac{1}{2}$ ,  $\forall i \neq j$ , then  $U_i(\mathbf{p}_{OR}) \geq U_i(\mathbf{p}_{WB})$ , where  $i = 1, 2, \dots, N$  (i). When  $\mathbf{p}_{WB}$  is allocated by the PRX each user  $i$  transmits with power  $p_i = \frac{Q}{N\alpha}$  and coexists on the same channel with all the other  $N - 1$  users. Let us describe the FDMA-reallocation of  $\mathbf{p}_{WB}$  for a generic user  $i$  in the game:

$$W'_i = \frac{\frac{Q}{N\alpha}}{\frac{Q}{N\alpha} + \frac{Q}{N\alpha} + \dots + \frac{Q}{N\alpha}} = \frac{1}{N}; \quad (20)$$

$$p'_i = \frac{Q}{N\alpha} + \frac{Q}{N\alpha} + \dots + \frac{Q}{N\alpha} = \frac{Q}{\alpha}. \quad (21)$$

Note that  $\mathbf{p}_{OR}$  corresponds to a FDMA reallocation of  $\mathbf{p}_{WB}$ : each user can transmit with power  $p'_i = \frac{Q}{\alpha}$  using  $W'_i = \frac{1}{N}$  of the total bandwidth. Consequently, we can directly use results from Lemma 6 and this proves (i). b) We now prove (19). We consider the utility gained by each secondary in each of the  $N$  symmetric PA candidates for NE, see Lemma 2. For those PAs, any user  $j$  transmits at the same time with other  $N - i$  with probability  $P_b = \frac{N-i+1}{N}$ . As it can be seen in (10) with  $\mathbf{p}_i$  each player generates interference secondaries  $I = \frac{Q}{N-i+1}$  at the PRX, therefore there can be only  $N - i + 1$  secondary users transmitting at the same time. This also means that secondary user  $j$  transmits with probability  $P_{tx} = \frac{N-i+1}{N}$ .

Let us focus on the utility of user 1. In correspondence of any symmetric joint strategy profile  $\mathbf{p}_i$ , by putting  $P_{tx} = \frac{N-i+1}{N}$  the expected utility of user 1 is:

$$U_1(\mathbf{p}_i) = P_{tx}U_1(\mathbf{p}_{WB}(N - i + 1)) + (1 - P_{tx}) \times 0, \quad (22)$$

where  $U_1(\mathbf{p}_{WB}(N - i + 1))$  is the utility of user 1 in a game with  $N - i + 1$  players, when  $\mathbf{p}_{WB}$  is allocated. This happens because from user 1's point of view we can distinguish two sub-games in correspondence of  $\mathbf{p}_i$ : A sub-game A with  $N - j + 1$  players where user 1 always transmits along other other  $N - j$  players, and a sub-game B where user 1 is always backed off.

Now, from (i) which is valid also for a game with  $N - j + 1$  players we know that

$$U_1(\mathbf{p}_i) = P_{tx}U_1(\mathbf{p}_{WB}(N - i + 1)) \leq P_{tx}U_1(\mathbf{p}_{OR}(N - i + 1)). \quad (23)$$

But,  $P_{tx}U_1(\mathbf{p}_{OR}(N - j + 1))$  is the utility of user 1 in correspondence of  $\mathbf{p}_{OR}(N)$ ; in fact,

$$P_{tx} \frac{1}{N - j + 1} = \frac{N - j + 1}{N} \frac{1}{N - j + 1} = \frac{1}{N}, \quad (24)$$

which means that user 1 is allocated  $\frac{1}{N}$  of the total bandwidth (as with  $\mathbf{p}_{OR}(N)$ ) during which it can transmit with power  $\frac{Q}{\alpha}$ . This proves the Lemma.  $\square$

*Proof Proposition 4:* In a  $N$ -player game there are only  $N$  symmetric PAs that are candidates for NE. Following Lemma 7 we know that the unique NE among them is  $\mathbf{p}_{OR}$ : each user can increase its utility by unilaterally deviating from any symmetric PA toward  $\mathbf{p}_{OR}$ . This proves that none of the symmetric PAs is NE, except  $\mathbf{p}_{OR}$ . Here we prove that there cannot exist any other PA, even asymmetric, that can be a NE under a strong interference.

By definition, transmitting powers in an asymmetric PA can be ordered. Without loss of generality, let us assume that user 1 is always the secondary user transmitting with lower power. We will prove that user 1 will always deviate from any asymmetric PA: depending on its transmitting power, we can always identify one of the symmetric PAs from Lemma 2 by which user 1 gains at least equal utility. As a consequence, user 1 will deviate from any asymmetric joint power allocation, and this proves the Proposition.

We recall from Lemma 2 that in correspondence to each symmetric PA, user 1 transmits with power  $p_1 = \frac{Q}{N-i}$ , and transmitting probability  $P_{tx} = \frac{N-i}{N}$ ,  $i = 0, \dots, N-1$ . We prove that a) for  $0 < \bar{p}_1 < \frac{Q}{N\alpha}$ ,  $U_1(\bar{p}_1, \mathbf{p}_{-1}) < U_1\left(\frac{Q}{N\alpha}, \frac{Q}{N\alpha}, \dots, \frac{Q}{N\alpha}\right)$ , and b) for  $\frac{Q}{(N-i+1)\alpha} < \bar{p}_1 < \frac{Q}{(N-i)\alpha}$ ,  $U_1(\bar{p}_1, \mathbf{p}_{-1}) < U_1\left(\frac{Q}{(N-i)\alpha}, \frac{Q}{(N-i)\alpha}, \dots, \frac{Q}{(N-i)\alpha}\right)$ ,  $i = 1, \dots, N-1$ .

a) In correspondence of  $\mathbf{p}_{WB} = \left(\frac{Q}{N\alpha}, \frac{Q}{N\alpha}, \dots, \frac{Q}{N\alpha}\right)$ ,  $\Phi(\mathbf{p}_{WB}, (1, \dots, N)) = 1$ , (see (6)), therefore user 1 always transmits with power  $p_1 = \frac{Q}{N\alpha}$ . If  $0 < \bar{p}_1 < \frac{Q}{N\alpha}$  utility of user 1 is certainly lower than in correspondence of  $\mathbf{p}_{WB}$ : transmitting probability cannot be increased, and transmitting power is lower by assumption.

b) Whenever  $\frac{Q}{(N-i+1)\alpha} < \bar{p}_1 < \frac{Q}{(N-i)\alpha}$ , at least  $i$  users are backed off. This happens because, by assumption,  $p_1 > \frac{Q}{(N-i+1)\alpha}$  and  $p_1 < p_2 \leq p_3 \leq \dots \leq p_N$ : if all the secondaries had transmitted with power  $\frac{Q}{(N-i+1)\alpha}$ ,  $i-1$  backing off would have been needed, after which  $N-i$  users can remain to transmit without violation of  $Q$  at the PRX, and at the end of the backoff process there is no room for additional interference. This means that with  $(\bar{p}_1, \mathbf{p}_{-1})$  user 1 can transmit in the best case with probability  $P_{tx} = \frac{N-i}{N}$ . But in correspondence of the joint strategy profile  $\left(\frac{Q}{(N-i)\alpha}, \frac{Q}{(N-i)\alpha}, \dots, \frac{Q}{(N-i)\alpha}\right)$  user 1 transmits with probability  $P_{tx} = \frac{N-i}{N}$  and higher power. Since we have proven that the user deviates from the symmetric vector  $\left(\frac{Q}{(N-i)\alpha}, \frac{Q}{(N-i)\alpha}, \dots, \frac{Q}{(N-i)\alpha}\right)$ , then it follows that it will deviate from the vector  $(\bar{p}_1, \mathbf{p}_{-1})$ .  $\square$

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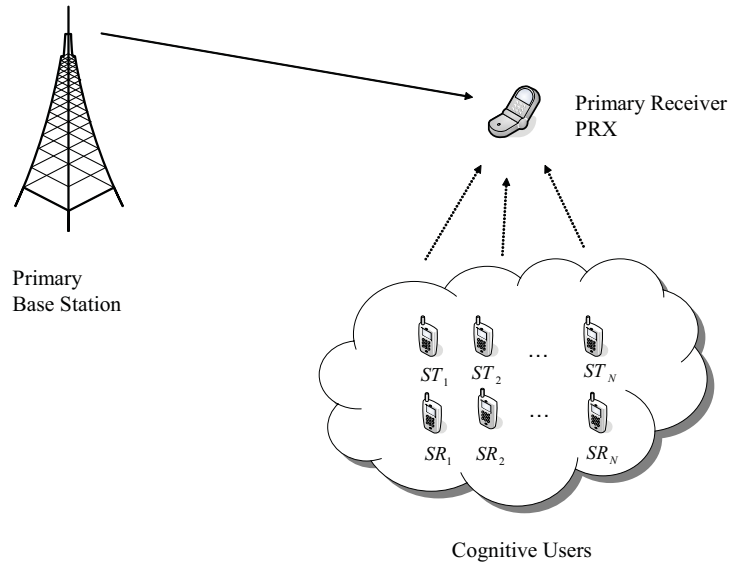


Fig. 1. System model for spectrum sharing among primary and cognitive users

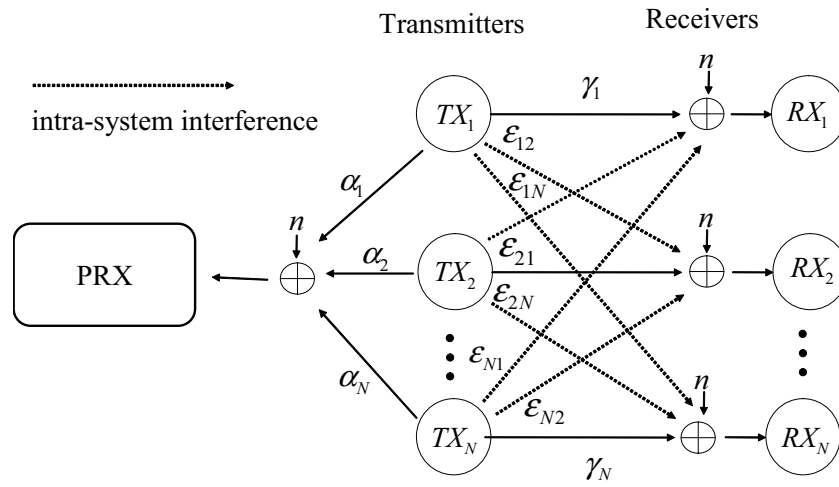


Fig. 2. System model for spectrum sharing

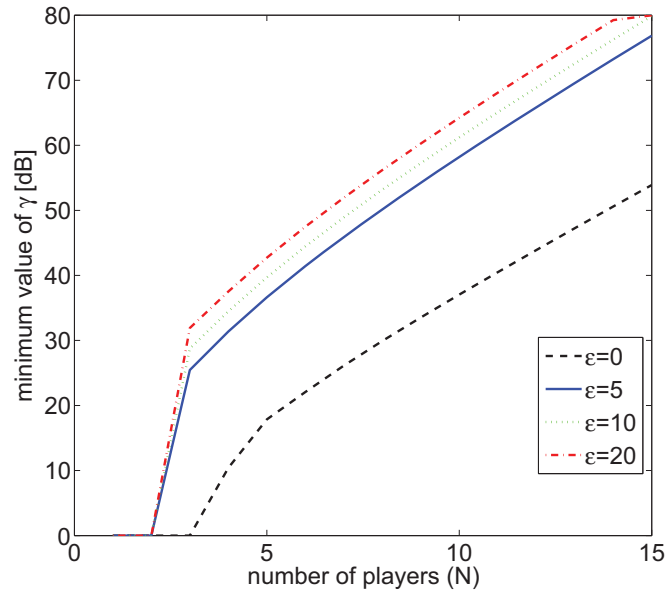


Fig. 3. Minimum  $\gamma$  for  $\mathbf{p}_{WB}$ , power allocation without violation of  $Q$ , to be NE ( $Q=1$ ,  $\alpha = 0.1$ ).

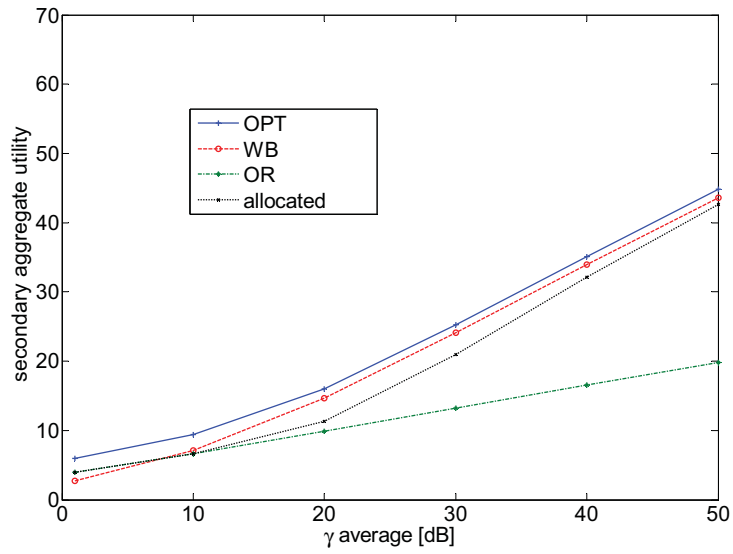


Fig. 4. Secondary aggregate utility versus average direct link  $\gamma$  average ( $N=3$ )

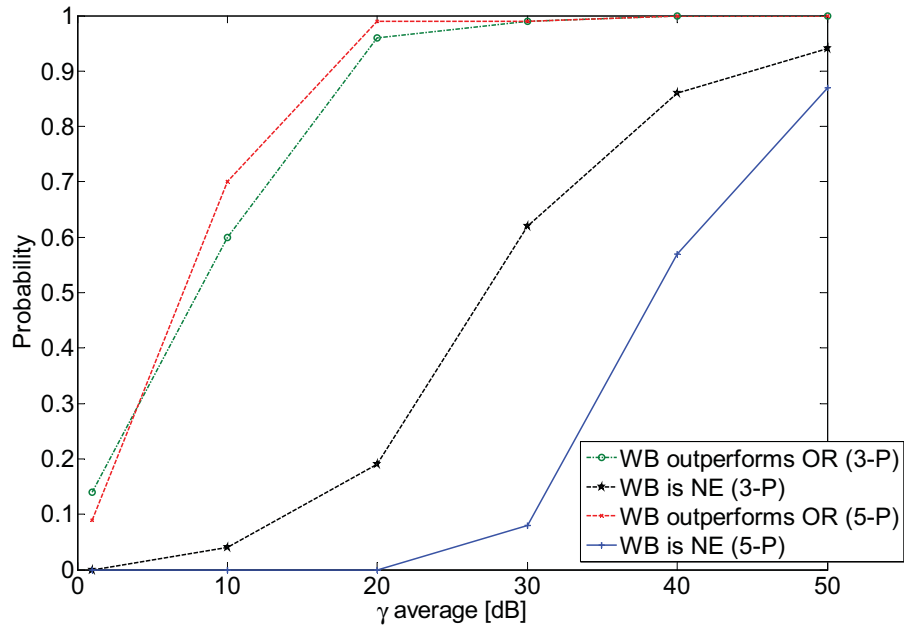


Fig. 5. Probability of allocation of wideband state  $\mathbf{p}_{WB}$  ( $N=3,5$ )

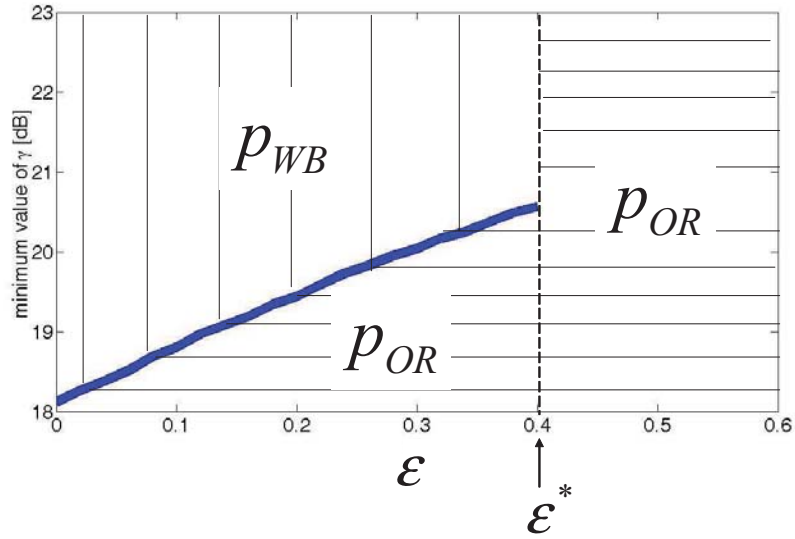


Fig. 6. Optimality region for the two power allocations  $\mathbf{p}_{WB}$  and  $\mathbf{p}_{OR}$  for problem (9).





## 8.9 Paper 9

### **Opportunistic Interference Cancellation in Cognitive Radio Systems**

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2nd IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks, 2007. DySPAN 2007.

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# Opportunistic Interference Cancellation in Cognitive Radio Systems

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**Abstract**—In this paper we investigate the problem of spectrally efficient operation of a cognitive radio (CR), also called Secondary System (SS), under an interference from the primary system (PS). A cognitive receiver (CRX) observes a multiple (MA) access channel of two users, the secondary and the primary transmitter, respectively. We advocate that the SS should apply *Opportunistic Interference Cancellation (OIC)* and decode the PS signal when such an opportunity is created by the rate selected in the PS and the power received from the PS. We derive the achievable data rate in the SS when OIC is applied. When the PS is decodable, we devise a method applied by the SS to achieve the maximal possible secondary rate. This method has a practical significance, since it enables rate adaptation without requiring any action from the PS. We investigate the power allocation in the SS when OIC is applied over multiple channels. We show that the optimal power allocation can be achieved with *intercepted water-filling* instead of the conventional water-filling. The results show a significant gain for the rate achieved by OIC.

## I. INTRODUCTION

A cognitive radio (CR) [1] is allowed to *reuse* the frequency spectrum which is assigned to a primary system (PS). A CR network (or secondary system (SS)) is allowed to use certain radio resource if it is not causing an adverse interference to the PS. Furthermore, the CR should achieve a spectrally efficient operation under the interference from the PS. One strategy [2] is to treat the PS signals as a noise and use only the radio resources where CR link can meet the target Signal-To-Interference-and-Noise-Ratio (SINR).

In this paper, our departing point is that it is reasonable that the SS can decode the PS signals, as the PS is a legacy system. On Fig. 1, the secondary receiver (SRX) receives both the signal from the primary Base Station (BS) and the secondary transmitter (STX). Hence, SRX observes a *multiple access (MA) channel* of two users: the desired STX and the undesired primary TX. However, the PS adapts its data rate for the primary terminals and the chosen transmission rate in the PS is *independent* of the SNR at which the PS signal is received by the SRX. Therefore, the SS should adapt its data rate by first considering whether the PS signal can be decoded. This is done by observing the received powers and the region of the achievable rates in the multiple access channel. We call this *opportunistic interference cancellation (OIC)*, as the decodability of the PS signal at the SRX depends on the opportunity created by the selection of the data rate in the PS and the SNR on the link between the primary BS and the SRX.

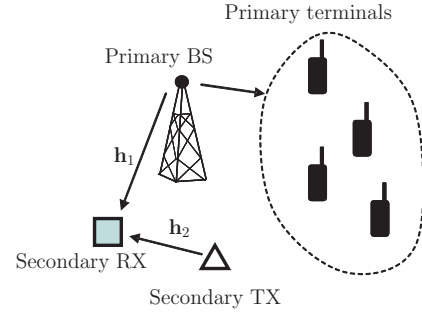


Fig. 1. The considered scenario where the primary transmitter is a Primary Base Station (BS), which adapts the transmission rates to the population of a Primary Terminals. The Secondary Transmitter (TX) knows the rates used in the primary system and accordingly adapts its transmission to the Secondary Receiver (RX).

We first derive the function by which the SS can adapt its rate by OIC over a single channel. When the PS signal is decodable, we introduce a method based on superposition coding by which any rate pair of the MA channel can be achieved without time sharing [3]. This has a practical significance, since the PS cannot be compelled to adapt the rate in a time-sharing manner. The derived rate adaptation function for the SS link is not a simple log-function of the power on SS the link. This has an impact on the power allocation in the SS when it is in case of multiple available channels, where the optimal power allocation can be achieved by *intercepted water-filling*.

## II. ASSUMPTIONS AND SYSTEM MODEL

The primary BS is using  $M$  channels and adapts the rate in each channel according to the scheduling policy and the channel state information (CSI) of the PS terminals, see Fig. 1. We assume that the rate adaptation in the PS is independent of the SS. The communication in the SS does not cause an adverse interference to the PS, since the SS to be a short-range radio system which uses a regulated low power.

The symbol  $y_m$  received at SRX at the  $m$ -th channel is:

$$y_m = h_{s,m} \sqrt{\mathcal{E}_m} x_{s,m} + h_{p,m} x_{p,m} + z_m \quad (1)$$

where  $h_{s,m}$  ( $h_{p,m}$ ) is the complex channel gain on the  $m$ -th channel from the STX(BS) to the SRX;  $\sqrt{\mathcal{E}_m} x_{s,m}$  is the signal transmitted by the STX, with a normalized expectation  $E[|x_{s,m}|^2] = 1$ , while  $\mathcal{E}_m$  is proportional to the energy used

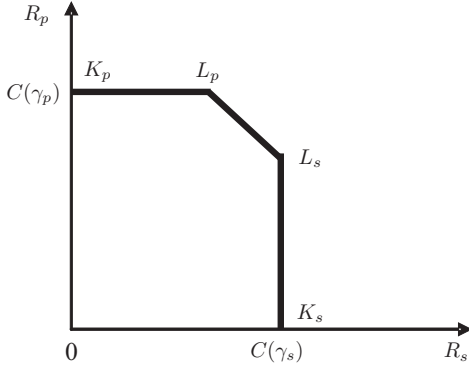


Fig. 2. The region of achievable rate pairs  $\mathcal{R} = (R_s, R_p)$  in a two-user multiple access channel.

in channel  $m$ .  $x_{p,m}$  is the normalized ( $E[|x_{p,m}|^2] = 1$ ) signal from the primary BS.  $z_m$  is the complex Gaussian noise with variance  $\sigma^2$ . Each channel has a normalized bandwidth  $W = 1$  [Hz], such that the time is measured in terms of number of symbols. The transmissions of the BS and the STX are assumed synchronized at the SRX, such that we consider the information-theoretic setting of the MA channel [3].

The PS serves the users in *scheduling epochs*. In each epoch, the primary BS decides accordingly the transmission rate  $R_{p,m}$  for the  $m$ -th channel. This information is broadcasted by the BS, such that the CR terminals learn about  $R_{p,m}$  for each  $m$ . Let  $\beta_{p,m}$  be the minimal SNR that enables successful decoding of a message sent at rate  $R_{p,m}$ . Then:

$$R_{p,m} = \log_2(1 + \beta_{p,m}) = C(\beta_{p,m}) \quad [\text{bps}] \quad (2)$$

Due to the bandwidth normalization, the spectral efficiency [bps/Hz] and the rate [bps] are equivalent. A scheduling epoch lasts for  $N$  symbols, where  $N$  is sufficiently large such that the primary BS can apply capacity-achieving transmissions. During each epoch, the channels  $h_{s,m}, h_{p,m}$  do not change. The *secondary SNR at the SRX for the channel  $m$*  is defined as  $\gamma_{s,m} = \frac{\mathcal{E}_m |h_{s,m}|^2}{\sigma^2} = \frac{\mathcal{E}_m}{\nu_m}$ , where  $\nu_m$  is the normalized noise energy at the  $m$ -th channel of the SRX. The *primary SNR at the SRX for the channel  $m$*  is  $\gamma_{p,m} = \frac{|h_{p,m}|^2}{\sigma^2}$ . The total average energy available for transmission on all channels is  $\sum_{m=1}^M \mathcal{E}_m = \mathcal{E}$ . In each scheduling epoch, the SS adapts the energy  $\mathcal{E}_m$  and the data rate  $R_{s,m}$  in each channel.

### III. OPPORTUNISTIC INTERFERENCE CANCELLATION

The concept of OIC will be introduced for  $M = 1$  channel (in this section we drop the subscript  $m$ ). The SRX can reliably decode both the primary/secondary signal if the rates  $R_p/R_s$  are within the capacity region of the MA channel (Fig. 2):

$$R_s \leq C(\gamma_s) \quad R_p \leq C(\gamma_p) \quad R_p + R_s \leq C(\gamma_s + \gamma_p) \quad (3)$$

The rate pairs  $\mathcal{R} = (R_s, R_p)$  at the points  $L_s$  and  $L_p$  are  $\mathcal{R}(L_s) = \left(C(\gamma_s), C\left(\frac{\gamma_p}{1+\gamma_s}\right)\right)$  and  $\mathcal{R}(L_p) = \left(C\left(\frac{\gamma_s}{1+\gamma_p}\right), C(\gamma_p)\right)$ , respectively. The rate pairs at the border are achieved by successive interference cancellation. In addition, the rates on the segment  $L_p L_s$  can be achieved by

*time-sharing* [3]: The two transmitters should use  $\mathcal{R}(L_s)$  for a fraction of time  $\theta$ , and  $\mathcal{R}(L_p)$  for the fraction of time  $1 - \theta$ . With  $\theta \in [0, 1]$ , any point on  $L_p L_s$  is achievable. However, in our scenario  $R_p$  is given *a priori* and time-sharing is not possible. Hence, the STX needs an alternative strategy to achieve the rate pairs  $\mathcal{R} \in L_p L_s$ . Let the PS have  $R_p$ , such that  $C\left(\frac{\gamma_p}{1+\gamma_s}\right) \leq R_p \leq C(\gamma_p)$ . We propose that STX uses *superposition coding* and transmits  $x_s = (1 - \alpha)x_s^{(1)} + \alpha x_s^{(2)}$ , where  $0 \leq \alpha \leq 1$  and  $E[|x_s^{(1)}|^2] = E[|x_s^{(2)}|^2] = 1$ , such that SRX receives  $y = h_s \left((1 - \alpha)x_s^{(1)} + \alpha x_s^{(2)}\right) + h_p x_p + z$ . The SRX decodes in three steps: **Step 1:**  $x_s^{(1)}$  is decoded from  $y$  by treating  $h_s \alpha x_s^{(2)} + h_p x_p$  as noise, after which  $y' = y - h_s (1 - \alpha)x_s^{(1)}$  is obtained; **Step 2:**  $x_p$  is decoded from  $y'$  by treating  $h_s \alpha x_s^{(2)}$  as a noise, after which  $y'' = y' - h_p x_p$  is obtained; **Step 3:**  $x_s^{(2)}$  is decoded from  $y''$ . From Step 2, by setting  $R_p = C\left(\frac{\gamma_p}{1+\alpha\gamma_s}\right)$ , we can determine the coefficient

$\alpha = \frac{\frac{\gamma_p}{\beta_p} - 1}{\frac{\gamma_p}{\gamma_s}}$ , where  $\beta_p = \frac{\gamma_p}{1+\alpha\gamma_s}$  from (2). The rate of  $x_s^{(1)}$  is  $R_s^{(1)} = C\left(\frac{(1-\alpha)\gamma_s}{1+\gamma_p+\alpha\gamma_s}\right)$  and the rate of  $x_s^{(2)}$  is  $R_s^{(2)} = C(\alpha\gamma_s)$ .

The total secondary rate is  $R_s = R_s^{(1)} + R_s^{(2)}$  and it can be easily verified that the rate pair  $(R_s, R_p)$  lies on the segment  $L_p L_s$ :  $R_s + R_p = C(\gamma_p + \gamma_s)$ .

In our scenario, the STX observes  $\gamma_p$  and  $R_p = C(\beta_p)$  as *a priori* given values and determines the maximal achievable rate  $R_s$  as a function of  $\gamma_s$ , with parameters  $\gamma_p$  and  $\beta_p$ :

$$R_s = F_{\gamma_p, \beta_p}(\gamma_s) \quad (4)$$

In absence of the PS signal,  $F_{\gamma_p=0, \beta_p}(\gamma_s) = C(\gamma_s)$ . The function  $F_{\gamma_p, \beta_p}(\gamma_s)$  reflects the policy of *opportunistic interference cancellation (OIC)*, where the CR makes the best possible use of the knowledge about the PS. A less optimal strategy would be to treat the PS signal as undecodable interference, even when  $\beta_p \leq \gamma_p$ . In order to determine  $F_{\gamma_p, \beta_p}(\gamma_s)$  we consider two regions for  $\gamma_p$ . When  $\gamma_p < \beta_p$ , SRX receiver cannot decode the PS signal, such that:

$$R_s = F_{\gamma_p, \beta_p}(\gamma_s)|_{\gamma_p < \beta_p} = C\left(\frac{\gamma_s}{1 + \gamma_p}\right) \quad (5)$$

In the second region  $\gamma_p \geq \beta_p$ , the SRX can decode the PS signal and  $R_s$  is chosen such that  $(R_p, R_s)$  belongs to the achievable rate region, determined for the given  $\gamma_p$  and  $\gamma_s$ . Depending on  $\gamma_s$ , there are two different cases:

- $\gamma_s \leq \frac{\gamma_p}{\beta_p} - 1$ : In this case the rate pair lies on the segment  $K_s L_s$  on Fig. 2:

$$R_s = F_{\gamma_p, \beta_p}(\gamma_s) = C(\gamma_s) \quad (6)$$

- $\gamma_s > \frac{\gamma_p}{\beta_p} - 1$ : In this case the rate pair lies on the segment  $K_s L_s$  on Fig. 2 and we use the proposed strategy with superposition coding, such that

$$R_s = F_{\gamma_p, \beta_p}(\gamma_s) = \log_2\left(\frac{1 + \gamma_p}{1 + \beta_p}\right) + C\left(\frac{\gamma_s}{1 + \gamma_p}\right) \quad (7)$$

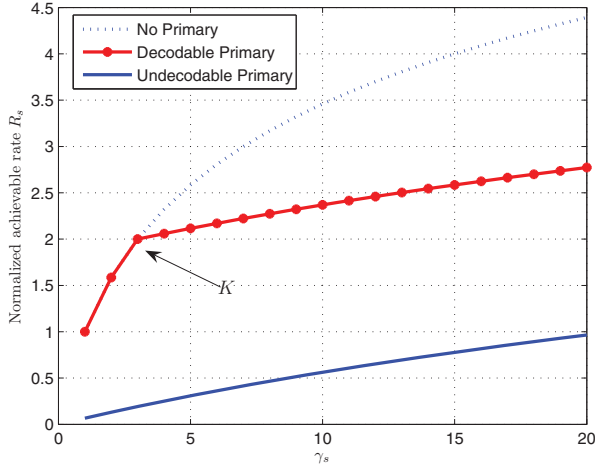


Fig. 3. Normalized achievable rate as a function of the secondary SNR  $\gamma_s$ . The abscissa is in the linear scale of  $\gamma_s$ . The case 'No primary' corresponds to  $\gamma_p = 0$ . In the other cases,  $\gamma_p = 20$ , the value  $\beta_p = 5$  when the primary is decodable, while it is  $\beta_p > 10$  when the primary is not decodable.

Fig. 3 exemplifies three different cases of the rate function  $F_{\gamma_p, \beta_p}(\gamma_s)$ . Note that, when  $\beta_p < \gamma_p$ , the function is non-differentiable at the point  $K$  with  $\gamma_s = \frac{\gamma_p}{\beta_p} - 1$ .

#### IV. OIC WITH MULTIPLE CHANNELS

In this section, we elaborate on how the energy should be allocated when the SS uses OIC over  $M > 1$  channels. We first consider the case  $M = 2$ . In absence of the primary interference ( $\gamma_p = 0$ ) the problem of rate/energy allocation in parallel Gaussian channels is [3]: Maximize  $C\left(\frac{\mathcal{E}_1}{\nu_1}\right) + C\left(\frac{\mathcal{E}_2}{\nu_2}\right)$  for  $\mathcal{E}_1 \geq 0, \mathcal{E}_2 \geq 0, \mathcal{E}_1 + \mathcal{E}_2 = \mathcal{E}$ . This optimization problem has the *water-filling solution*: If  $\mathcal{E} \leq \nu_2 - \nu_1$ , then  $\mathcal{E}_1 = \mathcal{E}$  and  $\mathcal{E}_2 = 0$ ; while if  $\mathcal{E} > \nu_2 - \nu_1$  then  $\mathcal{E}_1 = \frac{\mathcal{E} + \nu_2 - \nu_1}{2}$  and  $\mathcal{E}_2 = \frac{\mathcal{E} - \nu_2 + \nu_1}{2}$ . This conventional water-filling (CWF) can be interpreted as follows: While  $C\left(\frac{\mathcal{E}_1}{\nu_1}\right)$  is the faster-growing function, all the energy is poured in channel 1; when  $\mathcal{E}_1 = \nu_2 - \nu_1$ , then the rate in both channels starts to increase with identical pace, such that the energy  $\Delta\mathcal{E} = \mathcal{E} - (\nu_2 - \nu_1)$  should be equally distributed to both channels.

Let now  $\gamma_{p,1} = \gamma_{p,2} = \gamma_p > 0$ ,  $\nu_1 = \nu_2 = \nu$  and  $\beta_{p,1} = \beta_p > \gamma_p$ , but  $\beta_{p,2} < \gamma_p$ . The optimization problem is: Maximize  $\rho_s(\mathcal{E}_1, \mathcal{E}_2) = R_{s,1}(\mathcal{E}_1) + R_{s,2}(\mathcal{E}_2)$  for  $\mathcal{E}_1 \geq 0, \mathcal{E}_2 \geq 0, \mathcal{E}_1 + \mathcal{E}_2 = \mathcal{E}$ , where the achievable rates  $R_{s,1}(\mathcal{E}_1)$  and  $R_{s,1}(\mathcal{E}_2)$  are determined according to the function  $F_{\gamma_p, \beta_p}(\gamma_s)$ , discussed in the previous section. Here the Karush–Kuhn–Tucker conditions [4] cannot be directly applied, since the function  $\rho_s(\mathcal{E}_1, \mathcal{E}_2)$  is not a continuously differentiable function of  $(\mathcal{E}_1, \mathcal{E}_2)$ , as  $R_{s,1}(\mathcal{E}_1)$  is not a continuously differentiable function of  $\mathcal{E}_1$ . Still, due to the properties of the log-functions, the optimal solution can be described in the following way.

*Region  $\mathcal{E} < \mathcal{E}_{10}$ .* Here  $R_{s,1} = C\left(\frac{\mathcal{E}_1}{\nu}\right)$  and, as it grows faster than  $R_{s,2}$ , CWF implies  $\mathcal{E}_1 = \mathcal{E}$  and  $\mathcal{E}_2 = 0$ .

For the CWF, such an allocation would have continued until  $\mathcal{E} + \nu = \nu(1 + \gamma_p)$  i. e.  $\mathcal{E} = \nu\gamma_p$ . However, at  $\mathcal{E} = \mathcal{E}_{10} = \nu\left(\frac{\gamma_p}{\beta_p} - 1\right) < \nu\gamma_p$  the rate  $R_{s,1}$  starts to grow as a different function and we have to consider re-allocation. *Region  $\mathcal{E} = \mathcal{E}_{10} + \Delta\mathcal{E}$ ,* where  $\Delta\mathcal{E} > 0$  is sufficiently small. Let  $\mathcal{E}_1 = \mathcal{E}_{10} + \mathcal{E}_{11}$ , such that we can write:

$$\begin{aligned} R_{s,1} &= \log_2\left(\frac{1 + \gamma_p}{1 + \beta_p}\right) + \log_2\left(1 + \frac{\mathcal{E}_{10} + \mathcal{E}_{11}}{\nu(1 + \gamma_p)}\right) = \\ &= \log_2\left(\frac{\gamma_p}{\beta_p}\right) + \log_2\left(1 + \frac{\mathcal{E}_{11}}{\nu(1 + \gamma_p) + \mathcal{E}_{10}}\right) \end{aligned} \quad (8)$$

We can conclude that  $R_{s,2}$  grows with  $\mathcal{E}_2$  faster than  $R_{s,1}$  with  $\mathcal{E}_{11}$  for all points  $(\mathcal{E}_{11}, \mathcal{E}_2) = (0, \mathcal{E}_2)$  with  $0 \leq \mathcal{E}_2 < \mathcal{E}_{10}$ . Now the CWF imposes that  $\mathcal{E}_{11} = 0$  and  $\mathcal{E}_2 = \Delta\mathcal{E}$  when  $\Delta\mathcal{E} < \mathcal{E}_{10}$ .

*Region  $\mathcal{E} = 2\mathcal{E}_{10} + \Delta\mathcal{E}$ ,* where  $\Delta\mathcal{E} > 0$ . In this region, the energy of  $\mathcal{E}_{10} + \frac{\Delta\mathcal{E}}{2}$  is allocated to each channel.

The described solution is similar, yet not identical with the CWF solution and it can be interpreted as an *intercepted water-filling (IWF)*, see Figure 4. Note that in the absence of the upper “stone” block in channel 1, this figure would have represented a CWF. The region pinched between stone blocks of channel 1 and 2 can be thought of a leakage canal of zero volume, such that while  $\mathcal{E} < \mathcal{E}_{10}$  the lower basin of channel 1 is being filled only.

IWF produces the optimal solution when  $M > 2$  and the values of  $\nu_m, \gamma_{p,m}$  and  $\beta_{p,m}$  are arbitrary. We omit the rigorous proof here and provide only the main arguments. First, note that  $F_{\gamma_p, \beta_p}(\gamma_s)$  is always a concave function of  $\gamma_s$ . When  $\beta_p > \gamma_p$  the function is non-differentiable at one point, but is still concave, as it can be represented as a minimum of two concave functions [4]. In that case the IWF implements the steepest ascent algorithm, which leads to a globally optimal solution. The IWF implementation can be described by the following, rather visual, explanation. Based on  $\nu_m, \gamma_{p,m}, \beta_{p,m}$  we have to determine the height of the “stone” blocks for each

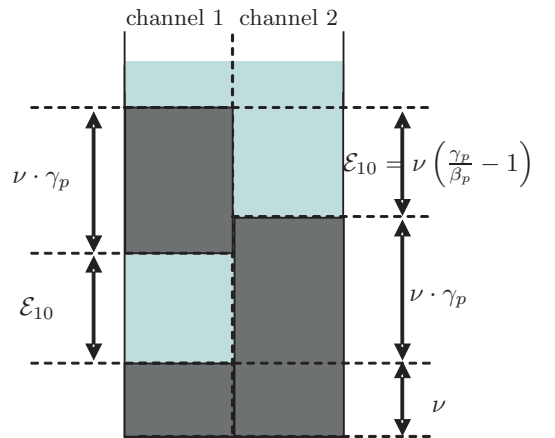


Fig. 4. Example of intercepted water-filling for two channels in which  $\nu_1 = \nu_2 = \nu$ ,  $\gamma_{p,1} = \gamma_{p,2} = \gamma_p$  and  $\beta_{p,1} = \beta_p > \gamma_p, \beta_{p,2} < \gamma_p$ .

TABLE I  
THE BLOCK HEIGHTS FOR INTERCEPTED WATER-FILLING (IWF)

Per-channel blocks for Intercepted Water-Filling	
•	If $\gamma_{p,m} < \beta_{p,m}$ , then the channel contains only one block of height $\nu_m(1 + \gamma_{p,m})$
•	If $\gamma_{p,m} \geq \beta_{p,m}$ , then the channel contains two blocks. The lower block starts from the bottom and has a height $\nu_m$ . The upper block starts at a height of $\nu_m + \nu_m \left( \frac{\gamma_{p,m}}{\beta_{p,m}} - 1 \right) = \nu_m \frac{\gamma_{p,m}}{\beta_{p,m}}$ . The height of the upper block is $\nu_m \gamma_{p,m}$ .

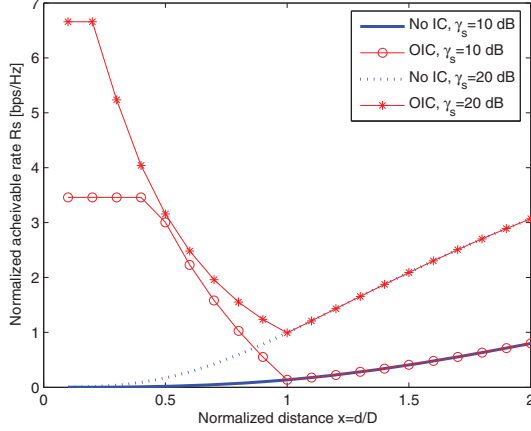


Fig. 5. Normalized achievable rate as a function of the normalized distance of the secondary RX from the primary BS. “No IC” is without Interference Cancellation. Here  $\beta_p = 20$  [dB], propagation coefficient is  $v = 3$ .

channel, as well as the position of the upper stone block. This is summarized in Table I. The upper block appears only in the channels in which the primary signal is decodable. With determined block levels/positions, the energy allocation can be done by water-filling and considering that the water is leaking through the side walls of the upper blocks in the channels. The total block height in a channel is equal to  $\nu_m(1 + \gamma_{p,m})$ . This implies that, when the energy is sufficiently high, such that the water-filling goes above the uppermost block, then the power allocation of IWF and CWF is identical.

## V. NUMERICAL RESULTS

We first illustrate a scenario with  $M = 1$ . The PS has a range of  $D$  meters and it adjusts its power so as to have a predefined SNR of  $\beta_p$  for a primary receiver at a distance  $D$  with LOS link to the BS. The SRX is at the distance  $d$  from the BS and a primary SNR of  $\gamma_p(x) = \frac{\beta_p}{x^v}$ , where  $x = \frac{d}{D}$  and  $v$  is the propagation coefficient. Fig. 5 depicts the normalized achievable rate as a function of the normalized distance  $x$ . Two values of  $\gamma_s$  are used, 10 and 20 dB, respectively and  $\gamma_s$  is a measure of the power applied in the SS. OIC leads to higher rate when  $x < 1$ , but is identical to the case without interference cancellation for  $x > 1$ , as the PS signal cannot be decoded when the SRX is at distances  $d > D$ . For OIC, the rate points in the region  $\frac{1}{(1+\gamma_s)^{\frac{1}{v}}} < x < 1$  are achieved by the described strategy of superposition coding.

Fig. 6 compares IWF and CWF for  $M = 10$  channels. For given  $\mathcal{E}$ , the normalized achievable rate is the sum of the rates

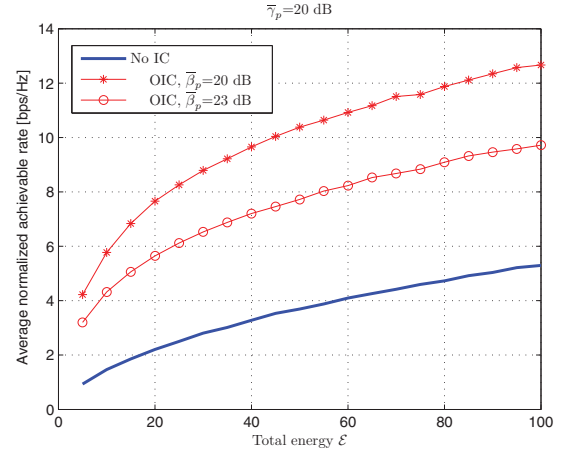


Fig. 6. Normalized average achievable rate in [bps/Hz] as a function of the average SNR  $\bar{\gamma}_s = \frac{\mathcal{E}}{\nu}$  on the secondary link. The number of channels is  $M = 10$ .

for all 10 channels and the value is obtained by averaging over  $10^4$  iterations. In each iteration,  $\nu_m = \frac{1}{\gamma_m}$  where  $\gamma_m$  is an exponentially distributed variable with average value 1, such that the average secondary SNR per channel is  $\frac{\mathcal{E}}{M}$ . In each iteration, the values  $\gamma_{p,m}$  is generated from exponential random variable with mean  $\bar{\gamma}_p = 20$  dB.  $\beta_{p,m}$  is generated from exponential random variable with mean value 20 dB and 23 dB, respectively, for each of the two OIC curves. We can see that IWF leads to significant rate improvements. When  $\beta_p > \bar{\gamma}_p$  the SRX has less opportunity to decode the primary, such that the improvement of IWF over CWF is decreased.

## VI. CONCLUSION

We have investigated the problem of spectrally efficient operation in a cognitive radio (CR)(or secondary system (SS)) under interference from a primary system (PS). We have shown that the SS should apply *Opportunistic Interference Cancellation (OIC)* and cancel the interference from the PS whenever such opportunity is created by (a) selection of the data rate in the PS and (b) the link quality between the primary TX and the secondary RX. We devise a method to obtain a maximal achievable rate in the SS whenever the primary signal is decodable. The derived rate adaptation function is then applied in case the SS uses multiple channels interfered by the PS. We show that the solution to the power/rate allocation problem is intercepted water-filling rather than the conventional water-filling. The numerical results confirm that the OIC can bring rate gains in the CR systems.

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## **8.10 Paper 10**

### **Outage Performance in Cognitive Radio Systems with Opportunistic Interference Cancellation**

*Di Taranto R.; Popovski P.;*

submitted to IEEE Transactions on Wireless Communication, January 2010





# Outage Performance in Cognitive Radio Systems with Opportunistic Interference Cancellation

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## Abstract

In this paper we investigate the problem of spectrally efficient operation of a cognitive radio (CR), also called secondary spectrum user, under an interference from the primary system. A secondary receiver (SRX) observes a multiple access (MA) channel of two users, the secondary and the primary transmitter, respectively. The SRX applies Opportunistic Interference Cancellation (OIC) and decodes the primary signal when such an opportunity is created by the rate selected in the primary transmitter (PTX) and the power received from the PTX. The goal of this paper is to investigate how the secondary transmitter (STX) should select its rate in order to meet its target outage probability under different assumptions about the CSI available at the STX. We study three different cases and for each of them identify the region of achievable primary and secondary rates. Our numerical results show that the best secondary performance is always obtained when the STX knows the instantaneous channel gain toward the intended receiver.

## Index Terms

Cognitive radio, Dynamic spectrum sharing, Opportunistic Interference Cancellation

## I. INTRODUCTION

The main idea behind the concept of Cognitive Radio (CR) [1] is to allow *secondary usage* of a spectrum licensed to another, primary spectrum user. If a CR device uses certain communication resource concurrently with the primary system, then it should use transmit power that will guarantee acceptable interference to the primary system. On the other hand, a cognitive (secondary) receiver needs to operate under interference of a primary system. Such interference is commonly treated as noise, but information-theoretic approaches provide more sophisticated treatment of interference. In [2], the primary system provides the secondary system with the primary messages in a non-causal manner. A more practical assumption is that the secondary system knows only the primary codebooks, but not the messages. Note that knowing the codebooks does not deteriorate the security in the primary system, as security is relying on encryption at the higher layers. With known codebooks, the secondary may be able to decode and cancel the primary interference [3]-[4].

Fig. 1 depicts the scenario with *Opportunistic Interference Cancellation (OIC)* in which the secondary receiver (SRX) receives the signal from the secondary transmitter (STX) along with the interference from the primary transmitter (PTX). The term “opportunistic” stands for the fact that the decodability of the primary signal depends on its rate as well as its power level at SRX. If STX knows the channel state information (CSI) for both the desired and the interfering SNR ( $\gamma_s$  and  $\gamma_p$ , respectively), then STX can select the highest possible secondary rate. On the other hand, if at least one of  $\gamma_s, \gamma_p$  is not known to the secondary transmitter, then for any selected secondary rate, an outage occurs if that rate is not decodable at SRX [5]. The goal of this paper is to investigate how the secondary should select its rate in order to meet the target outage probability under different assumptions about the CSI available to the secondary system. The results provide an interesting insight into the impact of the decodable interference: the knowledge of the interfering codebooks should motivate the CR to select higher (optimistic) transmission rates, even if the instantaneous CSI is not known at the transmitter.

## II. SYSTEM MODEL

We consider the CR network from Fig. 1. PTX communicates with a certain number of primary terminals (PT). The cognitive link consists of STX and SRX. STX is aware about the surrounding primary terminals and it decides the maximal power used for transmission which

guarantees that the primary receivers will not be disturbed. The actual methods for deciding the maximal transmitting power for STX are outside of the scope of this paper. Instead, our concern is, once this maximal power is known, how to improve the communication performance in the secondary system. We consider secondary communication under downlink interference from the primary system, since in that case a PRX is likely in a close proximity with respect to the STX, which decreases the allowed transmitting power of STX and thus improvement of the secondary spectral efficiency is of paramount importance.

On Fig. 1,  $\gamma_s$  and  $\gamma_p$  denote, respectively, the instantaneous SNR in the links STX-SRX and PTX-SRX. The PTX serves the users in *scheduling epochs*. In each epoch, the PTX uses a fixed transmission rate

$$R_p = \log_2(1 + \beta_p) = C(\beta_p), \quad (1)$$

where the bandwidth is normalized to 1 Hz and  $\beta_p$  is the minimal SNR required to decode  $R_p$ .  $R_p$  is known to STX and the primary interference is decodable at SRX if the instantaneous  $\gamma_p \geq \beta_p$ . The transmission rate used by STX is denoted by  $R_s$ . During a scheduling epoch, the SNRs are constant. In each new epoch, we assume that the channel on the links STX-SRX and PTX-SRX fade independently according to a Rayleigh distribution, and the averages of  $\gamma_s$  and  $\gamma_p$  are denoted by  $\bar{\gamma}_s$  and  $\bar{\gamma}_p$ , respectively, which are known to STX.

### III. OUTAGE ANALYSIS UNDER INCOMPLETE CSI

The transmissions of PTX and STX are assumed synchronized at the SRX, such that SRX observes a multiple access (MA) channel [8]. SRX can reliably decode both primary and secondary signal if the rate pair  $(R_s, R_p)$  is within the capacity region of the MA channel, see Fig. 2:

$$R_s \leq \log_2(1 + \gamma_s) \quad (2)$$

$$R_p \leq \log_2(1 + \gamma_p) \quad (3)$$

$$R_p + R_s \leq \log_2(1 + \gamma_s + \gamma_p) \quad (4)$$

When at least one of the SNRs  $\gamma_s, \gamma_p$  is not known at STX, then it is not guaranteed that  $R_s$  is selected such that the rate pair  $\mathcal{R} = (R_s, R_p)$  is in the capacity region of the MA channel. In the sequel, we consider three different cases of incomplete CSI.

A.  $\gamma_p$  is known and  $\gamma_s$  is not known

In this case, the multi access region in Fig. 2, varies along the abscissa depending on the instantaneous value of  $\gamma_s$ , which is unknown to the STX. Here we want to determine the outage probability at the SRX, when STX, knowing instantaneous  $\gamma_p$  and average  $\bar{\gamma}_s$ , selects rate  $R_s$ .

We distinguish two cases depending on the decodability of the primary signal at the SRX. The first case is if  $\gamma_p < \beta_p$ , which implies that the primary is not decodable at SRX and needs to be treated as a noise. Then the outage probability for a given secondary rate  $R_s$  is:

$$\Pr\left(\log_2\left(1 + \frac{\gamma_s}{1 + \gamma_p}\right) < R_s\right) = 1 - \exp\left[(-2^{R_s} - 1)(1 + \gamma_p)\right]. \quad (5)$$

Let us now consider the case  $\gamma_p \geq \beta_p$  and the primary signal is decodable. The maximal achievable secondary rate is:

$$R_{s,\max} = \min\left(\log_2(1 + \gamma_s), \log_2\left(\frac{1 + \gamma_s + \gamma_p}{1 + \beta_p}\right)\right), \quad (6)$$

and for known  $\gamma_p$ , it is a function of  $\gamma_s$ . This follows from (2) and (4) when we consider the case of fixed  $R_p$ . For a given secondary rate  $R_s$ , we determine the minimal  $\gamma_{s0}$ , such that (6) is satisfied by putting  $\gamma_s = \gamma_{s0}$ . Then outage occurs whenever the instantaneous  $\gamma_s < \gamma_{s0}$ , which is found as

$$\Pr(\gamma_s < \gamma_{s0}) = 1 - \exp\left(-\frac{\gamma_{s0}}{\bar{\gamma}_s}\right). \quad (7)$$

B.  $\gamma_s$  is known and  $\gamma_p$  is not known

If only  $\gamma_p$  is unknown, the MA capacity region changes due to the “vertical movement” of the capacity region on Fig. 2. For known  $\gamma_s$ , the maximal achievable secondary rate  $R_{s,\max}(\gamma_p)$  is a function of  $\gamma_p$ , plotted on Fig. 3. In absence of any interference we have

$$R_{s,\max}(\gamma_p = 0) = \log_2(1 + \gamma_s). \quad (8)$$

In the region where  $0 < \gamma_p < \beta_p$ , the primary signal cannot be decoded at SRX and it is treated as noise at the SRX, such that

$$R_{s,\max}(\gamma_p) = \log_2\left(1 + \frac{\gamma_s}{1 + \gamma_p}\right). \quad (9)$$

When  $\gamma_p$  grows beyond  $\beta_p$ , the primary becomes decodable at SRX. We first consider the interval  $\beta_p \leq \gamma_p \leq \beta_p(1 + \gamma_s)$ , where each  $\gamma_p$  is represented as  $\gamma_p = \beta_p(1 + \alpha\gamma_s)$  with  $0 \leq \alpha \leq 1$ .

It can be shown that, in this interval the maximal achievable rate  $R_s$  is found by considering the rate pair  $(R_s, R_p)$  that lies on the diagonal (slope -1) border of the capacity region on Fig. 2, such that:

$$R_{s,\max}(\gamma_p) = \log_2(1 + \gamma_s + \gamma_p) - R_p = \log_2\left(1 + \gamma_s \frac{1 + \alpha\beta_p}{1 + \beta_p}\right). \quad (10)$$

Finally, if  $\gamma_p > \beta_p(1 + \gamma_s)$ , then the primary signal can be decoded by treating the secondary signal as a noise, such that primary is completely canceled and the maximal secondary rate becomes independent of  $\gamma_p$ :

$$R_{s,\max}(\gamma_p) = \log_2(1 + \gamma_s) \quad (11)$$

We use Fig. 3 to determine the outage probability in this case. If  $R_s$  is less than the minimum of the function  $R_{s,\max}(\gamma_p)$ , i. e.  $R_s < \log_2\left(1 + \frac{\gamma_s}{1 + \beta_p}\right) = \mu$ , then the outage probability is zero, regardless of  $\gamma_p$ . Conversely, if  $R_s > \log_2(1 + \gamma_s)$ , then the outage probability is one, regardless of  $\gamma_p$ . If  $R_s$  is selected to be in the interval  $\mu \leq R_s \leq \log_2(1 + \gamma_s)$ , then it is seen from Fig. 3, that the actual secondary rate intersects  $R_{s,\max}(\gamma_p)$  in two points whose abscissas correspond to  $\gamma_A$  and  $\gamma_B$ : the outage probability is given by the integral of the probability density function (pdf) of  $\gamma_s$  (which is Rayleigh distributed) between  $\gamma_A$  and  $\gamma_B$ . Defining  $\beta_s = 2^{R_s} - 1$ ,  $\gamma_A$  and  $\gamma_B$  can be determined in closed form as follows:

$$\gamma_A = \frac{\gamma_s - \beta_s}{\beta_s}, \quad \gamma_B = \beta_p(1 + k\gamma_s), \quad (12)$$

where  $k = \frac{\beta_s(1 + \beta_p) - \gamma_s}{\gamma_s\beta_p}$ .

### C. Both $\gamma_s$ and $\gamma_p$ are not known

Here we want to determine the outage probability at the SRX, when STX, knowing only average  $\bar{\gamma}_p$  and  $\bar{\gamma}_s$ , selects rate  $R_s$ . For easier notation, we introduce  $\beta_s$  that corresponds to the selected rate as

$$R_s = \log_2(1 + \beta_s). \quad (13)$$

For given values of  $\beta_s$  and  $\beta_p$ , Fig. 4 depicts the region  $(\gamma_s, \gamma_p)$ , patterned with vertical lines, that renders the secondary signal undecodable at SRX. We explain the shape of this undecodability region by considering three intervals for  $\gamma_s$ . Note that, for each value of  $\gamma_s$ , we can plot the function  $R_{s,\max}(\gamma_p)$ , as on Fig. 3.

If  $\gamma_s < \beta_s$ , then, for each  $\gamma_s$ , the function  $R_{s,\max}(\gamma_p)$  lies below the line  $R_s = \log_2(1 + \beta_s)$ , such that the outage probability is one, regardless of  $\gamma_p$ . In the interval  $\beta_s \leq \gamma_s \leq \beta_s(1 + \beta_p)$ , it can be shown that for each fixed value of  $\gamma_s$ , the function  $R_{s,\max}(\gamma_p)$  has two intersecting points with the line  $R_s = \log_2(1 + \beta_s)$ , i. e.  $R_{s,\max}(\gamma_p) = R_s$  for  $\gamma_p = \gamma_A$  and  $\gamma_p = \gamma_B$ , as depicted on Fig. 3. Note that  $\gamma_A$  and  $\gamma_B$  are functions of  $\gamma_s$  and they approach each other as  $\gamma_s$  grows towards  $\beta_s(1 + \beta_p)$ . In this interval, for fixed  $\gamma_s$ , outage occurs if  $\gamma_A < \gamma_p < \gamma_B$ . Finally, for each  $\gamma_s \geq \beta_s(1 + \beta_p)$ , the function  $R_{s,\max}(\gamma_p)$  lies below the line  $R_s = \log_2(1 + \beta_s)$ , such that the outage probability is zero, regardless of  $\gamma_p$ .

The integral of the probability density function (pdf) of  $\gamma_s$  over the two-dimensional region patterned with vertical lines in Fig. 4 cannot be solved in closed form; therefore we evaluate the secondary outage probability numerically in the next section.

#### IV. NUMERICAL EVALUATION

In this section, we evaluate the maximum secondary rate  $R_s$  that has predetermined outage probability at SRX. We consider OIC for the three different cases of CSIT available at the STX, described in Sections III-A, III-B, III-C. As a reference, we consider the case in which averages  $\bar{\gamma}_p, \bar{\gamma}_s$  are known, but no OIC is applied. We assume that the channel on the links STX-SRX and PTX-SRX fade independently according to a Rayleigh distribution with averages  $\bar{\gamma}_s$  and  $\bar{\gamma}_p$ , which are parameters in our simulations.

Fig. 5 shows the maximum allowed secondary rate  $R_s$  (averaged over a large number of independent channel realizations in the links STX-SRX and PTX-SRX) that has outage probability equal to  $\xi = 0.1$ . We have set the parameter  $\bar{\gamma}_p = 20$  dB. As expected,  $R_s$  increases with  $\bar{\gamma}_s$  in all the cases, but the secondary rate is sensibly higher with OIC even in the case where the STX knows only the average  $\bar{\gamma}_s$  and  $\bar{\gamma}_s$ : the beneficial effects of OIC are not lost even if the STX does not know the instantaneous channel gains. When the STX known either the instantaneous  $\gamma_s$  or  $\gamma_p$  the maximum  $R_s$  is further improved: the SRX has an additional information and exploits it to maximize its average rate. It is also important to notice that knowing the instantaneous  $\bar{\gamma}_s$  always gives the highest secondary rate compared to knowledge of instantaneous  $\gamma_p$ .

Fig. 6 it shows the average value of the maximal allowable  $R_s$  for different values of  $\gamma_p$  when  $\xi = 0.1$  and  $\bar{\gamma}_s = 20$  dB. It is interesting to notice that, as expected, the maximum averaged  $R_s$  (for all the three cases with OIC) has a minimum in correspondence of  $\bar{\gamma}_p = 10$ . This happens

because the primary signal is not decodable at the SRX if  $\gamma_p < \beta_p$ :  $R_s$  decreases (on average) for increasing values of  $\overline{\gamma_p}$  below  $\beta_p$ . On the contrary, when  $\overline{\gamma_p}$  is on average larger than  $\beta_p$  the primary signal can be decoded and this explains why the secondary rate increases.

## V. CONCLUSIONS AND FUTURE WORK

In this paper we have investigated how the outage probability in the secondary system changes when various types of CSI information are at disposal at the secondary transmitter. We have studied three different cases, and for each of them we have identified the region of achievable primary and secondary rates. Moreover our simulation results have shown that the best secondary performance is always obtained when the STX knows the instantaneous channel gain toward the intended receiver. An interesting direction for further study is integration of the opportunistic interference cancellation with the process of spectrum sensing and power selection in the secondary system.

## ACKNOWLEDGEMENT

This work is in part supported by Danish Research Council for Technology and Production Sciences, (project no. 274-05-0490).

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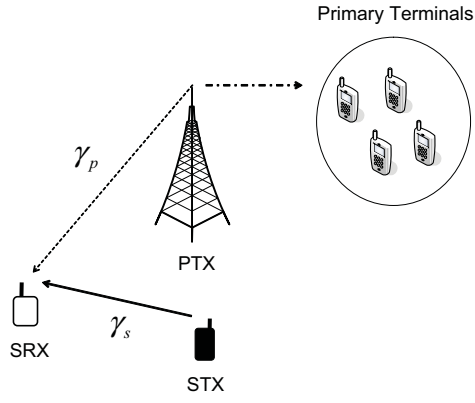


Fig. 1. The target scenario in which a primary transmitter (PTX) serves a group of Primary Terminals (PTs). A secondary receiver (SRX) is in the coverage area of the primary transmitter and thus experiences interference from PTX in addition to the desired signal from the secondary transmitter STX.

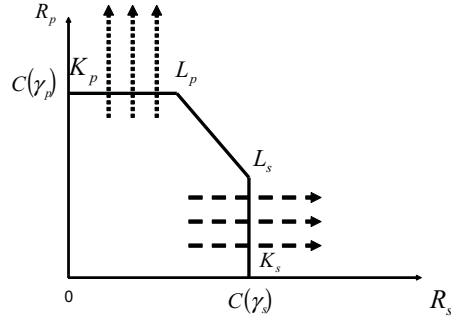


Fig. 2. The region of achievable rate pairs  $\mathcal{R} = (R_s, R_p)$ , in a two-user multi-access channel.

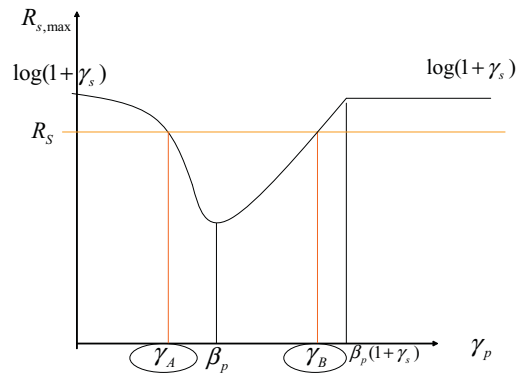


Fig. 3. Maximum achievable secondary rate in function of  $\gamma_p$ , for fixed  $\gamma_s$ .

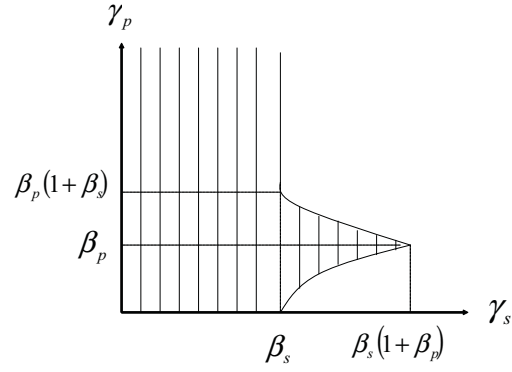


Fig. 4. Outage probability region: the region patterned with vertical lines renders the secondary signal undecodable at the SRX.

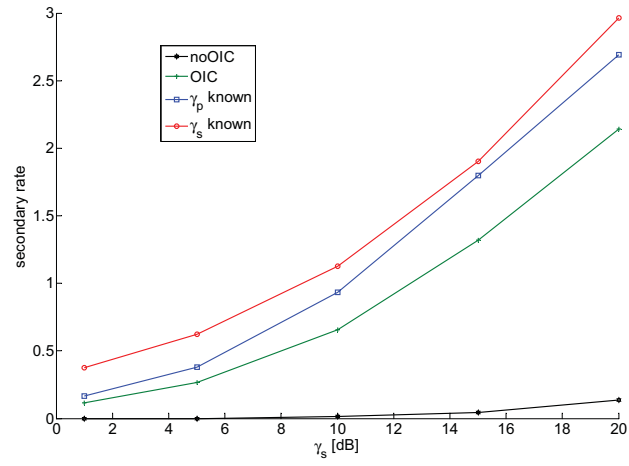


Fig. 5. Maximum (averaged) allowed secondary rate in function of  $\gamma_s$  ( $\overline{\gamma_p} = 20dB$ ,  $\xi = 0.1$ ).

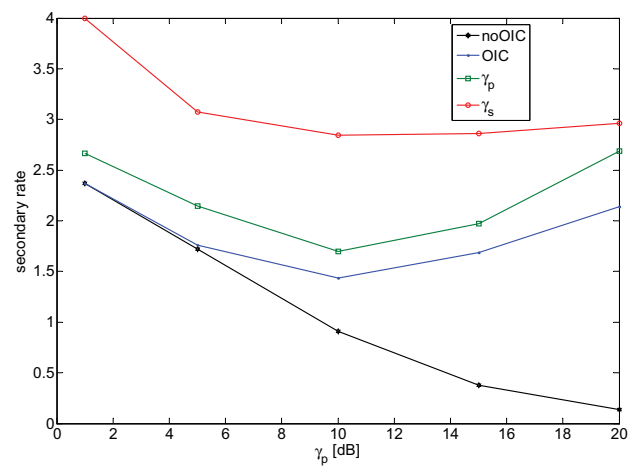


Fig. 6. Maximum (averaged) allowed secondary rate in function of  $\gamma_p$  ( $\overline{\gamma_s} = 20dB$ ,  $\xi = 0.1$ ).