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Publication date:
1995

Document Version
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

Citation for published version (APA):
Kirkegaard, P. H., Andersen, P., & Brincker, R. (1995). *Identification of Civil Engineering Structures using Multivariate ARMAV and RARMAV Models*. Dept. of Building Technology and Structural Engineering, Aalborg University. Fracture and Dynamics Vol. R9535 No. 70

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**FRACTURE & DYNAMICS
PAPER NO. 70**

Aalborg Universitetsbibliotek

530005198363



To be presented at the International Conference on Identification in Engineering Systems
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ISSN 1395-7953 R9535

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IDENTIFICATION OF CIVIL ENGINEERING STRUCTURES USING MULTIVARIATE ARMAV AND RARMAV MODELS

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ABSTRACT

This paper presents how to make system identification of civil engineering structures using multivariate auto-regressive moving-average vector (ARMAV) models. Further, the ARMAV technique is extended to a recursive technique (RARMAV). The ARMAV model is used to identify measured stationary data from an offshore structure while the recursive RARMAV identification technique is used on simulated data generated by the non-linear finite element program SARCOF modelling a 6-storey 2-bay concrete structure subjected to amplitude modulated Gaussian white noise filtered through a Kanai-Tajimi filter. The results show the usefulness of the approaches for identification of civil engineering structures excited by natural excitation.

1. INTRODUCTION

Since the end of the sixties the interest in the system identification based on time domain models has increased, and now literature on system identification is very much dominated by time domain methods. In Ljung [1] and Söderström et al. [2] the basic features of system identification based on time and frequency domain approaches are highlighted. For many years the identification techniques based on scalar auto-regressive moving average (ARMA) models in the time domain have attracted limited interest concerning structural engineering applications. A factor contributing to this situation is that ARMA models have been developed primarily by control engineers and applied mathematicians. Further, ARMA models have been primarily developed concerning systems for where limited a priori knowledge is available, whereas the identification of structural systems relies heavily on the understanding of physical concepts. Comparison of the structural time domain identification using ARMA representation with frequency domain techniques has shown that the ARMA time domain modelling approaches can be superior to Fourier approaches for the identification of structural systems since e.g. leakage and resolution bias problems are avoided. These findings make identification techniques utilizing ARMA algorithms interesting for modal parameter estimation. Especially, with respect to damage detection where modal parameters are used as damage indicators. Therefore, in recent years the application of scalar ARMA models as well as vector ARMA models (ARMAV) to the description of structural systems subjected to stationary ambient excitation has become more common, see e.g. Gersch et al. [3], Pandit et al. [4], Hac et al. Kozin et al. [5], Hamamontou et al. [6], Li et al. [7], Hoen [8], Prevosto et al. [9], Kirkegaard et al. [10] and Andersen et al. [11]. All these references are related to identification of linear time-invariant structures. However, in order to deal with time-varying systems a recursive implementation (RARMAV) of a multivariate ARMAV model can be used, see e.g. Ljung [1] and Söderström et al. [2]. The aim of this paper is to present how multivariate ARMAV and RARMAV models can be used for identification of time-invariant as well as time-variant civil engineering structures by calibrating the models directly to the measured time series. The ARMAV and RARMAV identification techniques are evaluated in two examples with experimental data from an offshore structure and simulated data from a 6-storey RC-structures subjected to an earthquake, respectively.

2. THEORY

This section describes the relationship between an Auto-Regressive Moving-Average Vector model (ARMAV) and the governing differential equation for a linear n -degree of freedom elastic system.

2.1 Continuous Time Model

In the continuous time domain an n -degree linear elastic viscous damped vibrating system is described to be a system of linear differential equations of second order with a constant coefficient given by a mass matrix M ($n \times n$), a damping matrix C ($n \times n$), a stiffness matrix K ($n \times n$), an input matrix S ($n \times r$) and a force vector $f(t)$ ($r \times 1$). Then the equations of motion for a linear multivariate system may in the time domain be expressed as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Sf(t) \quad (1)$$

where $x(t)$ is the displacement vector. The state space model corresponding to the dynamic equation (1) is

$$\dot{z}(t) = Ax(t) + Bf(t) \quad , \quad A = \begin{bmatrix} \mathbf{0} & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad B = \begin{bmatrix} \mathbf{0} \\ -M^{-1}S \end{bmatrix} \quad (2)$$

where $z(t)$ is the state vector. It is assumed that the system matrix A is asymptotically stable and can be eigenvector-eigenvalue decomposed as

$$A = U\mu U^{-1} \quad , \quad U = \begin{bmatrix} u_1 & \dots & u_{2n} \\ \mu_1 u_1 & \dots & \mu_{2n} u_{2n} \end{bmatrix} \quad \mu = \text{diag} [\mu_i] \quad , \quad i=1,2,\dots,2n \quad (3)$$

U is the matrix which columns are the scaled mode shapes u_i of the i th mode. μ is the continuous time diagonal eigenvalue matrix which contains the poles of the system from which the natural frequency ω_i and the damping ratio ζ_i of the i th mode can be obtained for under damped systems from a complex conjugate pair of eigenvalues as

$$\mu_i, \mu_i^* = -\omega_i \zeta_i \pm \omega_i \sqrt{1 - \zeta_i^2} \quad (4)$$

2.2 Discrete Time ARMAV Model

For multivariate time series, described by an m -dimensional vector $y(t)$, an ARMAV(p, q) model can be written with p AR-matrices and q MA-matrices

$$y(t) + \sum_{i=1}^p A_i y(t-i) = \sum_{j=1}^q B_j e(t-j) + e(t) \quad (5)$$

where the discrete-time system response is $y(t) = [y_1(t), y_2(t) \dots y_m(t)]^T$. A_i is an $m \times m$ matrix of auto-regressive coefficients and B_j is an $m \times m$ matrix, containing the moving-average coefficients. $e(t)$

is the model residual vector, an m -dimensional white noise vector function of time. Theoretically, an ARMAV model is equivalent to an ARV model with infinite order. The ARV is often preferred because of the linear procedure of the involved parameter estimation. The parameter estimation of the ARMAV model is a non-linear least squares procedure and requires some skill as well as large computation effort. A discrete state-space equation for equation (6) obtained by uniformly sampling the structural responses at time t is given by, e.g. Pandi et al. [4]

$$Z_t = FZ_{t-1} + W_t \quad (6)$$

with the state vector Z_t and the system matrix F , respectively are given by

$$Z_k = \{y(t)^T \ y(y-1)^T \ y(t-2)^T \ \dots \ y(t-p+1)^T\}^T, \quad F = \begin{bmatrix} -A_1 & -A_2 & \dots & -A_{p-1} & -A_p \\ I & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \dots & \cdot & \cdot \\ 0 & 0 & \dots & I & 0 \end{bmatrix} \quad (7)$$

W_t includes the MA terms of the ARMAV model. It is assumed that F can be decomposed as

$$F = L\lambda L^{-1}, \quad L = \begin{bmatrix} L_1\lambda_1^{p-1} & L_2\lambda_2^{p-1} & \dots & L_{pm}\lambda_{pm}^{p-1} \\ L_1\lambda_1^{p-2} & L_2\lambda_2^{p-2} & \dots & L_{pm}\lambda_{pm}^{p-2} \\ \cdot & \cdot & \dots & \cdot \\ L_1 & L_2 & \dots & L_{pm} \end{bmatrix}, \quad \lambda = \text{diag} [\lambda_i], \quad i=1,2,\dots,pm \quad (8)$$

The discrete state space model can now be used to identification of modal parameters and scaled mode shapes as follows, see Andersen et al. [11]. First, the discrete system matrix F is estimated by minimizing a quadratic error criterion $l(\epsilon)$ using a damped Gauss-Newton optimization algorithm and analytically gradients, see Appendix 1.

$$l(\epsilon) = \frac{1}{2} \epsilon^T \Lambda^{-1} \epsilon, \quad \epsilon(t, \theta) = y(t) - \hat{y}(t|t-1) \quad (9)$$

Λ and ϵ are the weighting matrix and the prediction errors, respectively. After the discrete eigenvalues of F are estimated by solving the eigen-problem $\det(F - \lambda I) = 0$ which gives the pm discrete eigenvalues λ_i . The continuous eigenvalues can now be obtained by $\lambda = e^{\mu_i \Delta}$ which implies that the modal parameters can be estimated using (4). The scaled mode shapes are determined directly from the columns of the bottom $m \times pm$ submatrix of L . The number of discrete eigenvalues in general are larger or different from the number of continuous eigenvalues. Therefore, only a subset of the discrete eigenvalues will be structural eigenvalues. This means that the user has to separate the physical modes from the computational modes. The computational modes are related to the unknown excitation and the measurement noise processes. The separation can often be done by studying the stability of e.g. frequencies, damping ratios and mode shapes, respectively, for increasing AR model order. Often, it is also possible to separate the modes by selecting physical modes as the modes with corresponding damping ratios below a reasonable limit for the modal damping ratios. However, satisfactory results obtained using ARMAV models for system identification require that appropriate models are selected and validated. A throughout description of the problem of model selection and validation is given in e.g. Ljung [1].

2.3 Discrete Time Recursive RARMAV Model

In order to estimate a time-variant system using an ARMAV model the parameters of the model have to be estimated on-line by using a method for parameter estimation known as the Recursive Prediction Error Method (RPEM). Such an approach has two main advantages: 1) it requires much less memory in the computer since the calculations are done sequentially using only the latest segment of data, and 2) it can detect time varying characteristics at each time step. The RPEM algorithm for a multivariate ARMAV model can be formulated as, see e.g. Ljung [1]

$$\begin{aligned}\theta(t) &= \theta(t-1) + L(t)\epsilon(t, \theta(t-1)) \\ L(t) &= P(t-1)\psi(t)[\lambda(t)\Lambda_t + \psi(t)^T P(t-1)\psi(t)]^{-1} \\ P(t) &= [P(t-1) - L(t)\psi^T(t)P(t-1)]/\lambda(t)\end{aligned}\tag{10}$$

The parameter vector θ is given as

$$\theta = \text{col}(\{A_1, \dots, A_p, B_1, \dots, B_q\})\tag{11}$$

where $\text{col}(X)$ means stacking of all columns of a matrix X on top of each other. $\psi(t)$ is the gradient of the prediction $y(t|t-1)$ with respect to θ as described in appendix 1. Λ_t is a matrix that weighs together the relative importance of the components of ϵ . $\lambda(t)$ is a forgetting factor, a number somewhat less than 1. This means that one can assign less weight to older data that are no longer representative for the system. The choice of the forgetting factor is often very important. Theoretically, one must have $\lambda(t) = 1$ to get convergence. On the other hand, if $\lambda(t) < 1$ the algorithm becomes more sensitive and the parameter estimates can change quickly. For this reason it is often an advantage to allow the forgetting factor to vary with the time. A typical choice is to let $\lambda(t)$ tend exponentially to 1, see e.g. Söderström et al. [2].

In order to start the recursion initial values of the parameters need to be specified. It can be shown that for stable systems the effect of initial values diminishes very rapidly with time, thus they can be assumed zero. Further, one also have to specify initial values for the covariance matrix $P(t)$. If this matrix is initialized with small values the parameter estimates will not change too much from the initial estimates of the parameters. On the other hand, if $P(t)$ is initialized with large values, the parameter estimates will quickly jump away from the initial values of the parameter vector. The intermediate steps of the derivation of (10) are given in e.g. Ljung [1] where it also is shown that the RPEM represents a general family of recursive system identification methods. There are several other methods, such as e.g. the recursive pseudolinear regression, maximum likelihood estimation, and the recursive least square method, that all can be considered as the special forms of the RPEM method for particular forms of the ARMAV model.

3. EXAMPLE 1: Identification of an Offshore Structure using ARMAV Models

In this section the ARMAV system identification technique is used to identify the Gullfaks C offshore platform. The identification of the offshore structure and computational aspects are described in details in Kirkegaard et al. [10] and Andersen et al. [11], respectively.

3.1 Description of The Gullfaks C Gravity Platform

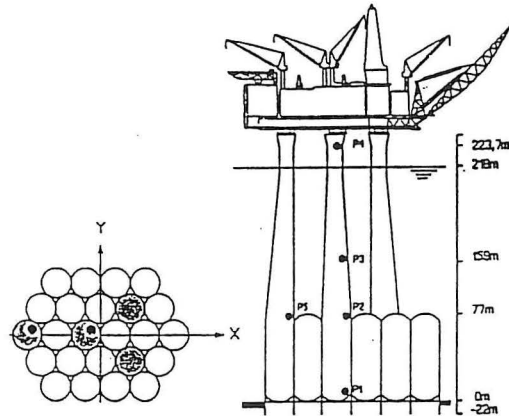


Figure 1: Elevation of the Gullfaks C Platform, Hoen [12].

The considered platform, shown in figure 1, was installed in May 1989 on 220 meters water depth in the North Sea, and was so far the largest and heaviest offshore gravity base concrete structure in the world. The dynamic motions were measured by means of 15 extremely sensitive linear and angular accelerometers. 13 of the accelerometers are located in the so-called utility shaft at different levels as indicated in figure 1. Two accelerometers recording accelerations in X and Y direction are placed at mudlevel (P1), at cell top level (P2), at the midpoint of the utility shaft (P3) and at the top of the utility shaft (P4), respectively. Further, two angular accelerometers are placed at location P3 and P4, respectively. The accelerations were sampled at 8 Hz during 20 minutes recording periods, giving time series of 9600 samples for each channel. In this paper 3 recording periods have been considered

A : 891226-0100, $H_s = 7.8$ m, $T_p = 11.7$ s.
B : 900101-0840, $H_s = 3.8$ m, $T_p = 20.5$ s.
C : 900108-0540, $H_s = 4.3$ m, $T_p = 9.6$ s

The description of recording period A shows that the data were sampled December 26, 1989, where the waves had a significant wave height $H_s = 7.8$ m and a wave peak period $T_p = 11.7$ s. By investigating the energy content of the time series no significant dynamically amplified response above 2 Hz was found. Therefore, prior to the system identification, the data were low-pass filtered beyond the new Nyquist frequency and resampled to 4 Hz. In the following only the two time series from the linear accelerometers at location P2, P3 and P4, respectively, have been used for the identification, i.e. 6 time series have been considered. The order of the ARMAV model was selected by incorporating the so-called FPE criteria, see e.g. Ljung [1] and it was found that only small improvements in the FPE criteria was obtained using a model with higher order than an ARMAV(6,5) model with 6 AR terms. This means that 36 eigenvalues will be estimated by the ARMAV(6,5). However, only a subset of these eigenvalues belongs to physical modes. Therefore, these were separated by studying the stability of e.g. frequencies, damping ratios and mode shapes, respectively, for increasing AR model order. Stabilization diagrams obtained by the ARMAV model show two physical modes just below 0.4 Hz.

Mode		A	B	C
1	f (Hz)	0.336	0.332	0.332
	ζ (%)	0.021	0.016	0.013
2	f (Hz)	0.371	0.373	0.371
	ζ (%)	0.014	0.014	0.015
3	f (Hz)	0.482	0.498	0.493
	ζ (%)	0.045	0.025	0.024
4	f (Hz)	0.556	0.580	0.580
	ζ (%)	0.091	0.088	0.049
5	f (Hz)	0.602	0.618	0.590
	ζ (%)	0.083	0.091	0.104
6	f (Hz)	1.269	1.272	1.260
	ζ (%)	0.024	0.014	0.016

Figure 2: Estimated frequencies and damping ratios for recording periods A,B and C, respectively.

Mode	A: Mag. (Phase)	B: Mag. (Phase)	C: Mag. (Phase)
1-P4-Y	1.000 (0.0)	1.000 (0.0)	1.000 (0.0)
1-P4-X	0.147 (173.3)	0.085 (128.5)	0.027 (163.7)
1-P3-Y	0.375 (-0.2)	0.378 (-0.2)	0.372 (-0.2)
1-P3-X	0.046 (172.1)	0.029 (103.8)	0.003 (53.7)
1-P2-Y	0.099 (180.8)	0.097 (-181.1)	0.096 (180.1)
1-P2-X	0.009 (167.3)	0.006 (111.7)	0.001 (46.9)
2-P4-Y	0.074 (-40.1)	0.037 (-17.7)	0.095 (4.0)
2-P4-X	1.000 (0.0)	1.000 (0.0)	1.000 (0.0)
2-P3-Y	0.021 (-6.3)	0.004 (-60.4)	0.022 (6.9)
2-P3-X	0.398 (-0.279)	0.397 (-0.3)	0.398 (-0.3)
2-P2-Y	0.007 (126.6)	0.002 (-221.8)	0.008 (182.1)
2-P2-X	0.089 (-1.7)	0.090 (-1.3)	0.089 (-1.3)
3-P4-Y	1.000 (0.0)	0.916 (-14.2)	0.980 (-7.5)
3-P4-X	0.344 (-181.6)	0.147 (36.5)	0.115 (-188.7)
3-P3-Y	0.927 (-4.5)	1.000 (0.0)	1.000 (0.0)
3-P3-X	0.143 (-189.1)	0.050 (16.1)	0.065 (-198.5)
3-P2-Y	0.476 (-183.4)	0.639 (177.8)	0.586 (-174.1)
3-P2-X	0.064 (-194.5)	0.013 (180.4)	0.034 (-189.2)

Figure 3: Magnitude and phase (Deg.) of mode shapes (ARMAV)

Physical modes are also indicated at approximately 0.5 Hz, 0.6 Hz, 1.0 Hz, 1.26 Hz, 1.4 Hz and 1.6 Hz, respectively. The stabilization diagrams also give an important indication about the order of the models to be used for the parameter estimation process.

The ARMAV(6,5) model is validated by visual inspection of the spectrum for the residuals which suggests that the residuals are close to a white-noise sequence, since the peaks are distributed in all frequencies. Further, from a test of the autocorrelation function of the residuals it was found that the autocorrelation function remains, for the most part, within the 99% confidence level limits, and therefore validates the model. The estimated natural frequencies and damping ratios for the first six modes, presented in figure 2, show only a slightly deviation between the different sea states. The figures also show that the results for modes 4 and 5, respectively, are not as stable as the three other modes. The results for modes 1, 2 and 6, respectively, are very well identified for all the three recording periods. In Kirkegaard et al. [10] it is found that modes 1 and 6 are bending modes close to the Y-direction while mode 2 is a bending mode close to the X-direction. Modes 4 and 5 which are more unstable seem to be bending modes in Y-direction and X-direction, respectively. Mode 3 is the first torsional mode about the vertical axis which is found by investigating the time series from the angular accelerometer about the vertical axis. The magnitude and phase of the mode shapes for the first three modes are given in figure 3. A closer investigation of how complex mode shapes could be interpreted as damped mode shapes could be done as proposed in Hoen [12]. It is shown that the damped mode shapes at an arbitrary position of a structure may be described by the product of an exponentially decaying function and an ellipse. In Hoen [12] the Gullfaks C structure has also been identified using a Markov Block Hankel matrix factorization method. The results from this paper and Hoen [12] correspond very well.

4. EXAMPLE: Identification of an Equivalent Linear Model for a Non-Linear RC-Structure

In this example the system RARMAV identification technique described in section 2 will be investigated in a simulation study. The investigations will be based on time series simulated by a non-linear finite element program SARCOF, Mørk [12], which has been verified to be able to predict accurately the response of deteriorating RC-structures with well-defined structural parameters (bending stiffnesses of cracked and uncracked of all beam-elements must be specified). The program estimates the fundamental eigenfrequency of the equivalent linear structure at each time step. The computer program SARCOF is a non-linear finite element program which is able to handle severe inherent material non-linearities and it is able to handle the following items such as 1) Unsymmetric cross-sections with different yield capacities at positive and negative bending 2) Interaction of bending moments and axial forces 3) Stiffness and strength degradation during plastic deformation 4) Pinching effect of moment-curvature relation due to shear loading and 5) Finite extensions of plastic zones at the end of the beams. The program is based on a full non-linear description of the internal degrees of freedom, which controls the hysteresis. In order to save computer time, the external degrees of freedom, i.e. the global displacements are described by truncated expansion in the eigenmodes of the undamaged structure, see Mørk [12].

4.1 Test Structure

The computer model, which is a model of a laboratory test model, see figure 4 consists of two 6-storey, 2-bay frames working in parallel with storey weights, uniformly distributed, attached in between. The total height of the structure is 3.3 m and all storey heights are uniformly distributed. The columns and beams in the structure are 0.05 m wide, 0.06 m deep for the columns and beams, respectively. Furthermore, all columns and beams are symmetrically reinforced. The following values are used for the density $\rho = 2500 \text{ kg/m}^3$, the stiffness $E = 2.0 \cdot 10^{10} \text{ N/m}^2$ and the damping ratio $\zeta = 0.035$. The stiffness and strength deterioration are modelled using a Clough-Johnston hysteretic model. In this deteriorating model the limit value e_0 is taken as 26 and the decay parameter e_1 is taken as 12. The first and second eigenfrequencies of the structure are 2.98 Hz and 9.47 Hz, respectively.

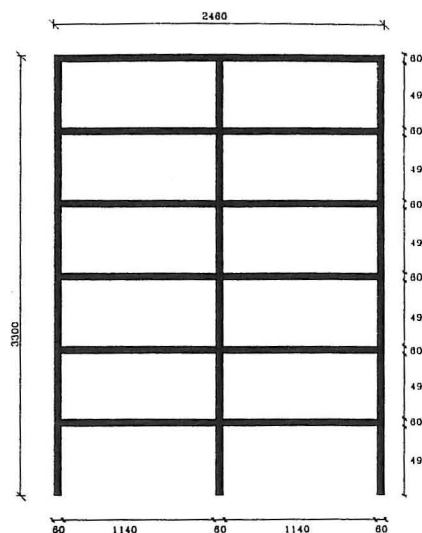


Figure 4: Computer model for the 6-storey 2-bay reinforced concrete frame in mm.

The excitation applied to the test structure was a simulated earthquake run for the basement motions that were patterned after the North-South component of the acceleration history measured at El Centro during the Imperial Valley Earthquake of 1940. The acceleration process at the ground surface is determined as the response process of an intensity modulated Gaussian white noise filtered through a Kanai-Tajimi filter, see Tajimi [13] implying that the negative part of the ground

surface acceleration is estimated. Here the white noise is calculated as by broken line process model of Ruiz and Penzien, see Clough et al. [14]. The deterministic modulation function used is given by Jennings et al [15]. The damping ratio in the Kanai-Tajimi filter is chosen as 0.3 and the circular frequency is chosen as $18.0.s^{-1}$. In the modulation function the decay parameter c is 0.2. The excitation has maximum acceleration after 3 sec. and duration of the strong motion is 20 sec. The integrated dynamic system is in SARCOF solved by a 4th Runge-Kutta Scheme. The time step is selected as 0.008 sec., where it has been proven that no drift occurs in the simulated signal. Time series with a length of 30 sec. were simulated.

During the simulations SARCOF also gives the instantaneous periods of the structure. These periods fluctuate due to changes in stiffness which are normally very high and the stiffness changes are very fluctuating during an earthquake. The reason for this fluctuating behaviour is simply due to the fact that the structure changes rapidly from being in the elastic to the plastic regime. It is therefore necessary to perform a smoothing of the measured eigenperiod which corresponds to time-averaging the structural degradation. A time-averaging method of the instantaneous period has been proposed by Rodriguez-Gomez [16] and is based on the principle of a moving averaging window in the following way. The smoothed value $\langle T(t_i) \rangle$ at the time t_i is evaluated as

$$\langle T(t_i) \rangle = \frac{1}{T_a} \int_{t-\frac{T_a}{2}}^{t+\frac{T_a}{2}} T(t) dt \quad , \quad \delta(t) = 1 - \frac{T_0}{\langle T(t) \rangle} \quad (12)$$

where T_a is the length of the averaging window, which should be sufficiently large, so that the local peaks are removed. On the other hand, T_a should not be selected so large that intervals of increased plastic deformation are not displayed in $\langle T(t_i) \rangle$. The value $T_a = 2.4 T_0$ is recommended as a reasonable compromise, Rodriguez-Gomez [16], where T_0 is the 1st eigenperiod of the equivalent linear structure. Based on $\langle T(t_i) \rangle$, the instantaneous softening, $\delta(t)$, of a structure is defined in (12), see Çakmak et al.[17]. Obviously, the damage indicator $\delta(t)$ is non-decreasing with time and attains values in the range [0;1], where $\delta(t) = 0$ corresponds to an undamaged structure.

Figure 5 show the time variation of the softening estimated by the RARMAV system identification technique and the softening obtained from SARCOF, respectively for an earthquake at 1.6 g. The forgetting factor $\lambda(t)$ used in the RARMAV algorithm was chosen as $\lambda(t) = 0.99$. The results shown in figure 5 indicate that the multivariate RARMAV model perhaps can be recommended for earthquake-engineering applications where the softening has to be estimated.

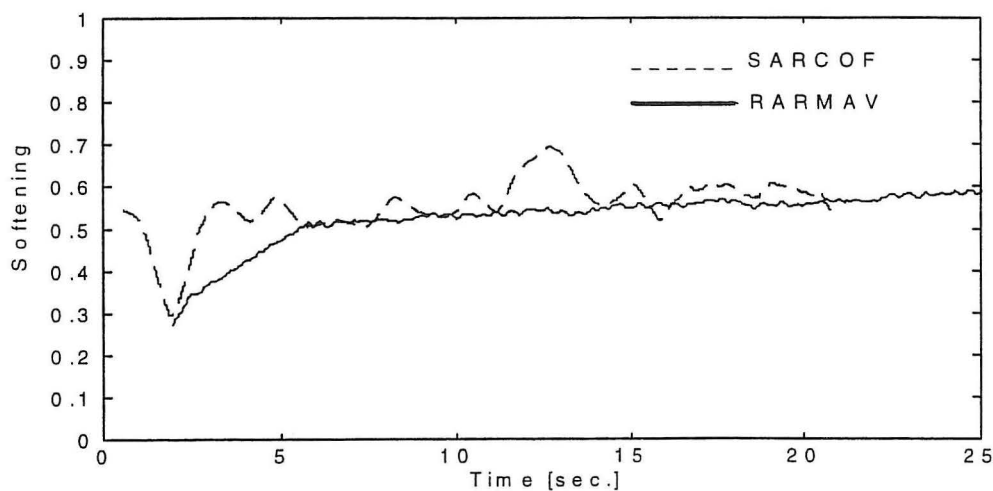


Figure 5: Softening estimated by an RARMAV model compared with simulated softening

5. ACKNOWLEDGEMENT

The authors wish to acknowledge Professor Ivar Iangen, Høgskolen i Stavanger, Norway for his kind release of information and data from the Gullfaks C platform.

6. CONCLUSIONS

This paper has considered use of the multivariate ARMAV and RARMAV models for system identification of an offshore structure under natural random excitation and an RC structure subjected to an earthquake, respectively. Based on the results presented in the paper there is a good reason to believe

- that the natural frequencies, damping ratios and mode shapes can be estimated very well for an offshore platform by using an ARMAV model.
- that an equivalent linear model for a time-variant RC-structure seems as it can be estimated using a recursive ARMAV model.

However, the possibility of using multivariate RARMAV techniques to identifications of time-variant civil engineering structures has to be investigated closer before a final conclusion can be made.

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APPENDIX 1.

The one step-ahead predictor of the ARMAV model (6) is given by

$$\hat{y}(t|t-1) = - \sum_{i=1}^p A_i y(t-i) + \sum_{i=1}^q B_i e(t-i) \quad (14)$$

or equivalently by

$$\hat{y}(t|t-1) = \phi^T(t) \theta \quad (15)$$

$\phi^T(t)$ is the multivariate regression matrix of dimension $m \times (p+q)m^2$ defined as

$$\begin{aligned} \phi^T(t) &= \varphi^T(t) \otimes I_m \\ \varphi^T(t) &= \{-y^T(t-1), \dots, -y^T(t-p), e^T(t-1), \dots, e^T(t-q)\}^T \end{aligned} \quad (16)$$

where I_m is an $m \times m$ identity matrix, and \otimes is the Kronecker product. The $(p+q)m^2 \times 1$ parameter vector θ contains the stacked autoregressive matrices A_i and the moving average matrices B_i , in the following way

$$\theta = \text{col}(\{A_1, \dots, A_p, B_1, \dots, B_q\}) \quad (17)$$

where $col(X)$ means stacking of all columns of a matrix X on top of each other.

Because of the complexity of the ARMAV model it is vital that the gradient of the predictor is calculated in a recursive manner. Differentiating (21) with respect to the j th element of θ yields

$$\psi(t) = \frac{\partial \hat{y}(t|t-1)}{\partial \theta_j} = - \sum_{i=1}^p \frac{\partial A_i}{\partial \theta_j} y(t-i) + \sum_{i=1}^q \frac{\partial B_i}{\partial \theta_j} e(t-i) + \sum_{i=1}^q B_i \frac{\partial e(t-i)}{\partial \theta_j} \quad (18)$$

which can be formulated as a multivariate autoregressive process as

$$\frac{\partial \hat{y}(t|t-1)}{\partial \theta_j} = - \sum_{i=1}^q B_i \frac{\partial \hat{y}(t-ilt-i-1)}{\partial \theta_j} - \sum_{i=1}^p \frac{\partial A_i}{\partial \theta_j} y(t-i) + \sum_{i=1}^q \frac{\partial B_i}{\partial \theta_j} e(t-i) \quad (19)$$

In order to clarify this consider an ARMAV(1,1) with two channels, defined as

$$\begin{Bmatrix} y_1(t) \\ y_2(t) \end{Bmatrix} = - \begin{bmatrix} a_{111} & a_{112} \\ a_{121} & a_{122} \end{bmatrix} \begin{Bmatrix} y_1(t-1) \\ y_2(t-1) \end{Bmatrix} + \begin{Bmatrix} e_1(t) \\ e_2(t) \end{Bmatrix} + \begin{bmatrix} b_{111} & b_{112} \\ b_{121} & b_{122} \end{bmatrix} \begin{Bmatrix} e_1(t-1) \\ e_2(t-1) \end{Bmatrix} \quad (20)$$

The multivariate regression matrix is given by

$$\phi^T(t) = \begin{bmatrix} -y_1(t-1) & 0 & -y_2(t-1) & 0 & e_1(t-1) & 0 & e_2(t-1) & 0 \\ 0 & -y_1(t-1) & 0 & -y_2(t-1) & 0 & e_1(t-1) & 0 & e_2(t-1) \end{bmatrix} \quad (21)$$

and the parameter vector by

$$\theta = \{a_{111}, a_{121}, a_{112}, a_{122}, b_{111}, b_{121}, b_{112}, b_{122}\}^T \quad (22)$$

The gradient of the predictor with respect to, for example a_{111} , is then given by

$$\frac{\partial \hat{y}(t|t-1)}{\partial a_{111}} = - \sum_{i=1}^q B_i \frac{\partial \hat{y}(t-ilt-i-1)}{\partial a_{111}} - \begin{Bmatrix} y_1(t-1) \\ 0 \end{Bmatrix} \quad (23)$$

So principally for each element of θ there corresponds a multivariate autoregressive filter. If the model order is increased this does not mean that the number of autoregressive filters increases too, simply because the gradients of, e.g. the elements of A_2 , are the gradients of the elements of A_1 at the previous time step.

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