



Aalborg Universitet

AALBORG UNIVERSITY  
DENMARK

## Modal Parameter Identification from Responses of General Unknown Random Inputs

Ibrahim, S. R.; Asmussen, J. C.; Brincker, Rune

*Publication date:*  
1995

*Document Version*  
Early version, also known as pre-print

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Ibrahim, S. R., Asmussen, J. C., & Brincker, R. (1995). *Modal Parameter Identification from Responses of General Unknown Random Inputs*. Dept. of Building Technology and Structural Engineering, Aalborg University. Fracture and Dynamics Vol. R9544 No. 72

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- ? Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- ? You may not further distribute the material or use it for any profit-making activity or commercial gain
- ? You may freely distribute the URL identifying the publication in the public portal ?

### Take down policy

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

**FRACTURE & DYNAMICS**  
**PAPER NO. 72**

To be presented at the 14th International Modal Analysis Conference,  
Dearborn, Michigan, USA, February 12-15, 1996

---

**S. R. IBRAHIM, R. BRINCKER & J. C. ASMUSSEN**  
**MODAL PARAMETER IDENTIFICATION FROM RESPONSES OF GE-**  
**NERAL UNKNOWN RANDOM INPUTS**  
**DECEMBER 1995** **ISSN 1395-7953 R9544**

---

The FRACTURE AND DYNAMICS papers are issued for early dissemination of research results from the Structural Fracture and Dynamics Group at the Department of Building Technology and Structural Engineering, University of Aalborg. These papers are generally submitted to scientific meetings, conferences or journals and should therefore not be widely distributed. Whenever possible reference should be given to the final publications (proceedings, journals, etc.) and not to the Fracture and Dynamics papers.

## FRACTURE AND DYNAMICS PAPERS

PAPER NO. 43: M. Krawczuk: *A New Finite Element for Static and Dynamic Analysis of Cracked Composite Beams*. ISSN 0902-7513 R9305.

PAPER NO. 44: A. Rytter: *Vibrational Based Inspection of Civil Engineering Structures*. Ph.D.-Thesis. ISSN 0902-7513 R9314.

PAPER NO. 45: P. H. Kirkegaard & A. Rytter: *An Experimental Study of the Modal Parameters of a Damaged Steel Mast*. ISSN 0902-7513 R9320.

PAPER NO. 46: P. H. Kirkegaard & A. Rytter: *An Experimental Study of a Steel Lattice Mast under Natural Excitation*. ISSN 0902-7513 R9326.

PAPER NO. 47: P. H. Kirkegaard & A. Rytter: *Use of Neural Networks for Damage Assessment in a Steel Mast*. ISSN 0902-7513 R9340.

PAPER NO. 48: R. Brincker, M. Demosthenous & G. C. Manos: *Estimation of the Coefficient of Restitution of Rocking Systems by the Random Decrement Technique*. ISSN 0902-7513 R9341.

PAPER NO. 49: L. Gansted: *Fatigue of Steel: Constant-Amplitude Load on CCT-Specimens*. ISSN 0902-7513 R9344.

PAPER NO. 50: P. H. Kirkegaard & A. Rytter: *Vibration Based Damage Assessment of a Cantilever using a Neural Network*. ISSN 0902-7513 R9345.

PAPER NO. 51: J. P. Ulfkjær, O. Hededal, I. B. Kroon & R. Brincker: *Simple Application of Fictitious Crack Model in Reinforced Concrete Beams*. ISSN 0902-7513 R9349.

PAPER NO. 52: J. P. Ulfkjær, O. Hededal, I. B. Kroon & R. Brincker: *Simple Application of Fictitious Crack Model in Reinforced Concrete Beams. Analysis and Experiments*. ISSN 0902-7513 R9350.

PAPER NO. 53: P. H. Kirkegaard & A. Rytter: *Vibration Based Damage Assessment of Civil Engineering Structures using Neural Networks*. ISSN 0902-7513 R9408.

PAPER NO. 54: L. Gansted, R. Brincker & L. Pilegaard Hansen: *The Fracture Mechanical Markov Chain Fatigue Model Compared with Empirical Data*. ISSN 0902-7513 R9431.

PAPER NO. 55: P. H. Kirkegaard, S. R. K. Nielsen & H. I. Hansen: *Identification of Non-Linear Structures using Recurrent Neural Networks*. ISSN 0902-7513 R9432.

PAPER NO. 56: R. Brincker, P. H. Kirkegaard, P. Andersen & M. E. Martinez: *Damage Detection in an Offshore Structure*. ISSN 0902-7513 R9434.

PAPER NO. 57: P. H. Kirkegaard, S. R. K. Nielsen & H. I. Hansen: *Structural Identification by Extended Kalman Filtering and a Recurrent Neural Network*. ISSN 0902-7513 R9433.

PAPER NO. 58: P. Andersen, R. Brincker, P. H. Kirkegaard: *On the Uncertainty of Identification of Civil Engineering Structures using ARMA Models*. ISSN 0902-7513 R9437.

PAPER NO. 59: P. H. Kirkegaard & A. Rytter: *A Comparative Study of Three Vibration Based Damage Assessment Techniques*. ISSN 0902-7513 R9435.

## FRACTURE AND DYNAMICS PAPERS

PAPER NO. 60: P. H. Kirkegaard, J. C. Asmussen, P. Andersen & R. Brincker: *An Experimental Study of an Offshore Platform*. ISSN 0902-7513 R9441.

PAPER NO. 61: R. Brincker, P. Andersen, P. H. Kirkegaard, J. P. Ulfkjær: *Damage Detection in Laboratory Concrete Beams*. ISSN 0902-7513 R9458.

PAPER NO. 62: R. Brincker, J. Simonsen, W. Hansen: *Some Aspects of Formation of Cracks in FRC with Main Reinforcement*. ISSN 0902-7513 R9506.

PAPER NO. 63: R. Brincker, J. P. Ulfkjær, P. Adamsen, L. Langvad, R. Toft: *Analytical Model for Hook Anchor Pull-out*. ISSN 0902-7513 R9511.

PAPER NO. 64: P. S. Skjærbæk, S. R. K. Nielsen, A. Ş. Çakmak: *Assessment of Damage in Seismically Excited RC-Structures from a Single Measured Response*. ISSN 1395-7953 R9528.

PAPER NO. 65: J. C. Asmussen, S. R. Ibrahim, R. Brincker: *Random Decrement and Regression Analysis of Traffic Responses of Bridges*. ISSN 1395-7953 R9529.

PAPER NO. 66: R. Brincker, P. Andersen, M. E. Martinez, F. Tallavó: *Modal Analysis of an Offshore Platform using Two Different ARMA Approaches*. ISSN 1395-7953 R9531.

PAPER NO. 67: J. C. Asmussen, R. Brincker: *Estimation of Frequency Response Functions by Random Decrement*. ISSN 1395-7953 R9532.

PAPER NO. 68: P. H. Kirkegaard, P. Andersen, R. Brincker: *Identification of an Equivalent Linear Model for a Non-Linear Time-Variant RC-Structure*. ISSN 1395-7953 R9533.

PAPER NO. 69: P. H. Kirkegaard, P. Andersen, R. Brincker: *Identification of the Skirt Piled Gullfaks C Gravity Platform using ARMAV Models*. ISSN 1395-7953 R9534.

PAPER NO. 70: P. H. Kirkegaard, P. Andersen, R. Brincker: *Identification of Civil Engineering Structures using Multivariate ARMAV and RARMAV Models*. ISSN 1395-7953 R9535.

PAPER NO. 71: P. Andersen, R. Brincker, P. H. Kirkegaard: *Theory of Covariance Equivalent ARMAV Models of Civil Engineering Structures*. ISSN 1395-7953 R9536.

PAPER NO. 72: S. R. Ibrahim, R. Brincker, J. C. Asmussen: *Modal Parameter Identification from Responses of General Unknown Random Inputs*. ISSN 1395-7953 R9544.

PAPER NO. 73: S. R. K. Nielsen, P. H. Kirkegaard: *Active Vibration Control of a Monopile Offshore Structure. Part One - Pilot Project*. ISSN 1395-7953 R9609.

**Department of Building Technology and Structural Engineering  
Aalborg University, Sohngaardsholmsvej 57, DK 9000 Aalborg  
Telephone: +45 98 15 85 22    Telefax: +45 98 14 82 43**

**FRACTURE & DYNAMICS**  
**PAPER NO. 72**

To be presented at the 14th International Modal Analysis Conference,  
Dearborn, Michigan, USA, February 12-15, 1996

---

**S. R. IBRAHIM, R. BRINCKER & J. C. ASMUSSEN**  
**MODAL PARAMETER IDENTIFICATION FROM RESPONSES OF GE-**  
**NERAL UNKNOWN RANDOM INPUTS**  
**DECEMBER 1995** **ISSN 1395-7953 R9544**

---

# MODAL PARAMETER IDENTIFICATION FROM RESPONSES OF GENERAL UNKNOWN RANDOM INPUTS

S. R. Ibrahim<sup>1</sup>, R. Brincker<sup>2</sup> and J. C. Asmussen<sup>2</sup>

<sup>1</sup>Department of Mechanical Engineering  
Old Dominion University  
Norfolk, Virginia, USA

<sup>2</sup>Department of Building Technology and  
Structural Engineering  
Aalborg University, Sohngaardsholmsvej  
57, 9000 Aalborg, Denmark

**ABSTRACT.** Modal parameter identification from ambient responses due to a general unknown random inputs is investigated. Existing identification techniques which are based on the assumptions of white noise and or stationary random inputs are utilized even though the inputs conditions are not satisfied. This is accomplished via adding, in cascade, a force conversion system to the structure's system under consideration. The input to the force conversion system is white noise and the output of which is the actual force(s) applied to the structure. The white noise input(s) and the structure's responses are then used to identify the combined system. Identification results are then sorted as either structural parameters or input force(s) characteristics.

## NOMENCLATURE

### Roman

c	damping coefficient
f	force
H	transfer matrix or function
i	index or $\sqrt{-1}$
j	index
k	stiffness
m	mass
n	white noise
s	Laplace variable
t	time
x	response

### Abbreviations

ARMA	Auto Regressive Moving Average
ARV	Vector Auto Regressive (model)
ITD	Ibrahim Time Domain
RDD	Random Decrement

### Subscripts

c	combined system
f	force system
s	structure

### Superscripts

T	transpose
---	-----------

## Greek and Symbols

$\zeta$	damping factor
$\phi$	mode shape
$\omega$	circular frequency
$\angle$	angle

## 1. INTRODUCTION

Modal parameter identification from ambient responses has gained considerable attention in recent years<sup>1,2</sup>. The literature reveals a multitude of cases of vibration testing of bridges, buildings, off shore structures, aircrafts, spacecrafts, ground vehicles, among others, utilizing responses due to wind, waves, traffic, road roughness, propulsion systems . . . etc. The advantages of such techniques are quite evident: The normal operation of the structure under test is not interrupted; no excitation cost; no measurements of inputs; continuous if not unlimited response records; suitable for structural integrity monitoring.

However, identification from ambient responses possesses two main disadvantages: first the input energy may be low to excite the modes of interest; secondly the input is assumed to be white noise or stationary random.

The general identification theories as applied to modal parameters estimation of vibrating structures can be classified into different main categories depending on the nature of the loading. Usually the loads are assumed to fall into one of the three following categories:

- Known and measurable force inputs time histories and locations,
- no force inputs, (utilizing structure's free response due to initial excitation), and
- white noise inputs.

However, there exist many structural applications, as pointed out earlier, where it is either impractical or uneconomical to use, or satisfy the conditions of, the above mentioned inputs. At the same time, these types of structures or applications offer the readily available and economical ambient or operational responses.

Such applications have traditionally been analyzed implementing identification techniques such as Frequency Response Functions, ARMA models and Random Decrement Techniques; among others. Such approaches, however, are based on the classical assumption of white noise inputs; a condition that is not usually satisfied.

In this paper, these techniques which require white noise inputs will be extended to apply to cases of general force inputs. This is accomplished by adding a pseudo second order system, in series with the second order system representing the structure, to which pseudo white noise inputs are applied. The responses of the combined system loaded by white noise are in reality the actual structure's response to the general force inputs.

Simulated and experimental results are presented in support of the proposed approach. The techniques implemented here are applied to a full scale structure in another publication<sup>3</sup>.

## 2. THEORY

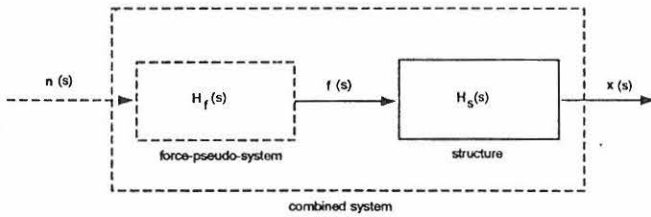


Figure 1. Block Diagram of Combined Structure's and Assumed Force's Systems

As shown in Figure 1, the structure whose transfer function is  $H_s(s)$  has an input of  $f(s)$  and an output of  $x(s)$ . The input  $f(s)$  is not white noise. A pseudo second order system of a transfer function  $H_f(s)$  is added in series to the system and is assumed to have the dynamic characteristics such that if the input to it is white noise, the output is the actual force to the structure  $f(s)$ . Now for the combined system in cascade the input is white noise and the output is the actual structure's response. Even though systems in cascade have been fully analyzed in dynamical systems theory<sup>4</sup>, the objective of the ensuing proof is to address the identification aspect of systems in cascade, particularly vibrating systems, to ensure that the modal parameters of the structural system are independently preserved and can be uniquely identified.

The following equations relate the input to output for both the structural system and the force pseudo system

$$x(s) = H_s(s)f(s) \quad (1)$$

$$f(s) = H_f(s)n(s) \quad (2)$$

Thus the transfer function of the combined system becomes

$$H_c(s) = H_f(s)H_s(s) \quad (3)$$

Now let

$$H_s(s) = \sum_{i=1}^{2n} \frac{a_i}{s - \lambda_i} \quad (4)$$

and

$$H_f(s) = \sum_{j=1}^{2m} \frac{b_j}{s - \alpha_j} \quad (5)$$

Then the combined system transfer function becomes

$$H_c(s) = \sum_i \frac{a_i}{s - \lambda_i} \sum_j \frac{b_j}{s - \alpha_j} \quad (6)$$

$$= \sum_i \sum_j \frac{a_i b_j}{(s - \lambda_i)(s - \alpha_j)} \quad (7)$$

utilizing partial functions, equation (7) becomes

$$H_c(s) = \sum_i \sum_j \left\{ \frac{a_i b_i (\lambda_i - \alpha_j)}{(s - \lambda_i)} + \frac{a_i b_i (\alpha_j - \lambda_i)}{s - \alpha_j} \right\} \\ = \sum_i \frac{a_i}{s - \lambda_i} (A \lambda_i - B) + \sum_j \frac{b_j (C \alpha_j - D)}{s - \alpha_j} \quad (8)$$

where

$$A = \sum_j b_j \quad (9)$$

$$B = \sum_j b_j \alpha_j \quad (10)$$

$$C = \sum_i a_i \quad (11)$$

$$D = \sum_i \alpha_i \lambda_i \quad (12)$$

Equation (8) verifies the stipulation that the modal parameters of the structural system and the force pseudo system are preserved and separable. The poles, in the denominator, are unaffected by combining the two systems. Thus frequencies and damping factors identification is expected to be correct. As well, the residues in the partial fractions are simply multiplied by a constant for each mode. Thus the mode shapes remain uniquely identifiable.

## 3. THEORY VERIFICATION AND TESTING

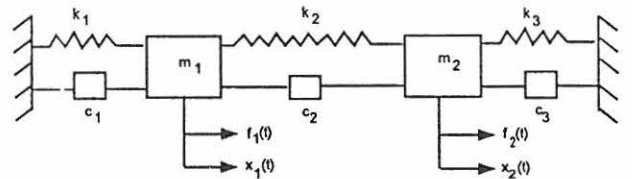


Figure 2. System for Simulation



To test the above theorems, a simulated test of a two degrees of freedom system is performed. The system is shown in Figure 2. Real experimental results are reported in another publication<sup>3</sup>. For the simulated experiment a white noise process is inputted to the force pseudo system which in this case assumed to be a single degree of freedom second order system the output of which is

$$F(w) = \frac{1}{w_f^2 - w^2 + 2i\zeta_f w_f w} \quad (13)$$

The load to the "structure" under test is simulated using a covariance equivalent ARMA model:

$$F(t_i) = AR_1 F(t_{i-1}) + AR_2 F(t_{i-2}) + N(t_i) + MA_1 N(t_{i-1}) \quad (14)$$

where  $N(t)$  is the white noise and  $AR_1$ ,  $AR_2$ ,  $MA_1$  are functions of  $w_f$ ,  $\zeta_f$  and sampling rate  $\Delta T$ .  $F(t_i)$  is applied equally to both degrees of freedom; thus  $F_1 = F_2 = F$ . Figure 3 shows an example of the spectrum, time history and normal probability plots for the white noise and the output of the pseudo force system which is the input to the structural system. The force pseudo system for these plots has  $w_f = 15.2578$  and  $\zeta_f = 0.005$ . These numbers are merely for illustration. For structural simulation  $w_f$  is taken as the average of the two natural frequencies of the system.

The structural system's parameters were chosen as:

$$k_1 = k_3 = 150, k_2 = 20 \quad m_1 = 1.0 \quad m_2 = 2.0 \quad (15)$$

and the damping matrix was selected as nonproportional of

the form

$$C = 0.02M + 0.001K + \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \quad (16)$$

Thus the system modal parameters were  $w_1 = 9.0947$  r/s  $w_2 = 13.1256$  r/s and  $w_f = 11.1101$  r/s (average of  $w_1$  and  $w_2$ ). For damping factors  $\zeta_1 = 0.0097$ ,  $\zeta_2 = 0.0102$  and  $\zeta_f = 0.005$ .

The theoretical mode shapes we calculated to be:

$$\begin{aligned} \phi^1 &= [1 \quad 4.3597]^T & \langle \phi^1 &= [0 \quad 2.7613]^T \\ \phi^2 &= [1 \quad 0.1148]^T & \langle \phi^2 &= [0 \quad 176.0168]^T \end{aligned} \quad (17)$$

Figure 4 shows spectra and time histories for the outputs.

The random decrement technique<sup>5,6</sup> "RDD" was used to convert systems random responses into free decay responses or correlation functions. Figure 5 shows the auto and cross RDD signatures using mass 1 as the triggering measurement and triggering was at every positive point. Figure 6 shows RDD signatures for triggering at local extremum of mass 1 response.

Modal parameter identification was performed using ARV<sup>7,8</sup> and ITD<sup>9</sup> methods. Table 1 shows identification results using signatures of Figure 5, and Table 2 is for those signatures of Figure 6.

From identification results, it can be seen that the system's characteristics as well as the force characteristics were identified.

Table 1. Modal Parameters estimated from RDD-Signatures (Every positive point trig. condition) by ARV and ITD Methods, and % Error from Theoretical Values

Parameter	Theory	ARV	% error	ITD	% error
$w_1$	9.0947	9.1085	0.1517	9.1075	0.1407
$w_2$	12.1256	13.1306	0.0881	13.1099	0.1106
$100.\zeta_1$	0.0097	0.0107	10.0810	0.0097	0.8711
$100.\zeta_2$	0.0102	0.0132	29.4118	0.0107	5.2745
$\Phi_1^1$	1.0000	1.0000	—	1.0000	—
$\Phi_2^1$	4.3597	4.3536	0.1899	4.3609	0.0292
$\langle \Phi_1^1$	0.0000	0.0000	—	0.0000	—
$\langle \Phi_2^1$	2.7613	7.3072	164.60	6.4866	134.9111
$\Phi_1^2$	1.0000	1.0000	—	1.0000	—
$\Phi_2^2$	0.1148	0.1170	1.9164	0.1199	4.4774
$\langle \Phi_1^2$	0.0000	0.0000	—	0.0000	—
$\langle \Phi_2^2$	176.0168	62.2515	7.8204	165.7310	5.8436
$w_f$	11.1101	11.0994	0.0963	11.1332	0.2079
$\zeta_f$	0.0050	0.0053	6.0000	0.0064	28.0000
$\Phi_{1f}$	—	1.0000	—	1.0000	—
$\Phi_{2f}$	—	1.1787	—	1.1347	—
$\langle \Phi_{1f}$	—	0.0000	—	0.0000	—
$\langle \Phi_{2f}$	—	176.3223	—	175.4883	—

Table 2. Modal Parameters Estimated from RDD-Signatures (Local extremum trig. condition) by ARV and ITD Methods, and % Error from Theoretical Values

Parameter	Theory	ARV	% error	ITD	% error
$\omega_1$	9.0947	9.0997	0.0550	9.1011	0.0706
$\omega_2$	13.1256	13.1307	0.0389	13.10448	0.1585
$\zeta_1$	0.0097	0.0083	14.9869	0.0082	15.8557
$\zeta_2$	0.0102	0.0140	37.3725	0.0136	33.7445
$\Phi_1^1$	1.0000	1.0000	—	1.0000	—
$\Phi_2^1$	4.3597	4.3118	0.3418	4.3518	0.1929
$\langle \Phi_1^1 \rangle$	0.0000	0.0000	—	0.0000	—
$\langle \Phi_2^1 \rangle$	2.7613	5.6025	102.8930	5.5074	99.4495
$\Phi_1^2$	1.0000	1.0000	—	1.0000	—
$\Phi_2^2$	0.1148	0.1183	3.0488	0.1229	7.0557
$\langle \Phi_1^2 \rangle$	0.0000	0.0000	—	0.0000	—
$\langle \Phi_2^2 \rangle$	176.0168	160.9737	8.5464	162.0385	7.9415
$\omega_f$	11.1101	11.1078	0.0207	11.1099	0.0018
$\zeta_f$	0.0050	0.0063	25.4600	0.6569	31.3800
$\Phi_{1f}$	—	1.0000	—	1.0000	—
$\Phi_{2f}$	—	1.1.694	—	1.1659	—
$\langle \Phi_{1f} \rangle$	—	0.0000	—	0.0000	—
$\langle \Phi_{2f} \rangle$	—	175.5213	—	175.1389	—

#### 4. CONCLUSIONS

Ambient random responses treated with random decrement and time domain identification techniques are effective and economical in modal identification of structures. This approach, among others, is classically based on the assumption of stationary random input. Non-stationary random inputs results in the identification of extraneous modal parameters which belong to the forcing system rather than the structure being tested. However, it is shown that forcing function dynamic characteristics have no effect on the accuracy of structural parameter identification. Techniques need to be developed to assist in sorting out structural dynamic properties from those of the inputs.

#### 5. ACKNOWLEDGEMENTS

This work is partially supported by The Danish Research Council. First author was a Summer Visiting Professor at Aalborg University, Denmark during parts of this work.

#### 6. REFERENCES

- Ventura, C. E.; Felber, A. J. and Prion, H. G. L., "Seismic Evaluation of a Long Span Bridge by Modal Testing," Proceedings of the 12th International Modal Analysis Conference; 1994; pp. 1309–1315.
- Ventura, C. E.; Schuster, N. D. and Felber, A. J., "Ambient Vibration Testing of A 32 Story Reinforced Con-

crete Building During Construction," Proceedings of the 13th International Modal Analysis Conference; 1995; pp. 1164–1170.

- Asmussen, J. C.; Ibrahim, S. R. and R. Brincker, "Random Decrement and Regression Analysis of Bridges Traffic Responses," Proceedings of the 14th International Modal Analysis Conference, 1996.
- Cannon, R. H., Jr.; Dynamics of Physical Systems — McGraw Hill Book Co., 1967.
- Brincker, R. and Krenk, S., "Estimation of Correlation Functions by Random Decrement Technique," Proceedings of the 9th International Modal Analysis Conference, 1991.
- Ibrahim, S. R.; Wentz, K. R. and Lee, J., "Damping Identification Using a Multi-triggering Random Decrement Technique," Journal of Mechanical Systems and Signal Proceeding, Vol. 1 (4); Oct. 1987; pp. 389–397.
- Pandit, S. M., Modal and Spectrum Analysis: Data Dependent Systems in State Space. John Wiley & Sons Publishers, 1991.
- Anderson, P., Brincker, R. and Kirkegaard, P. H., "On the Uncertainty of Identification of Civil Engineering Structures Using ARMA Models," Proceedings of the 13th Modal Analysis Conference, 1995, pp. 1115–1121.
- Ibrahim, S. R., et al., "Large Modal Survey Testing Using the Ibrahim Time Domain Identification Technique," AIAA Journal of Spacecrafts and Rockets; Vol. 19; No. 5, Sept.-Oct. 1982, pp. 459–465.

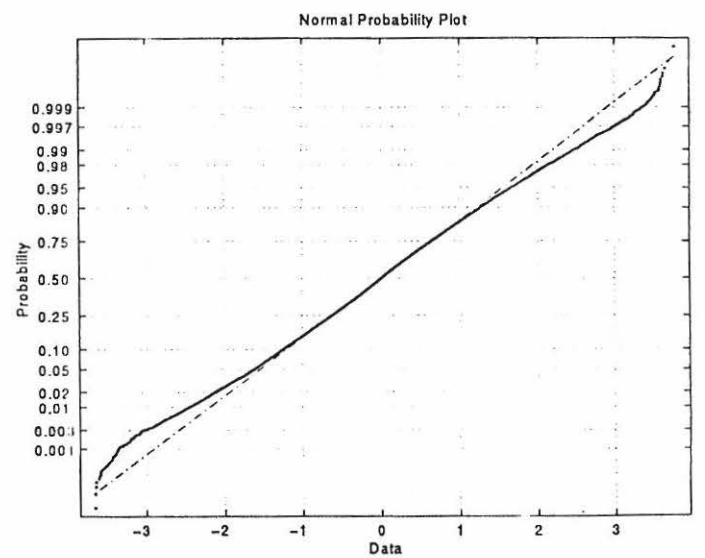
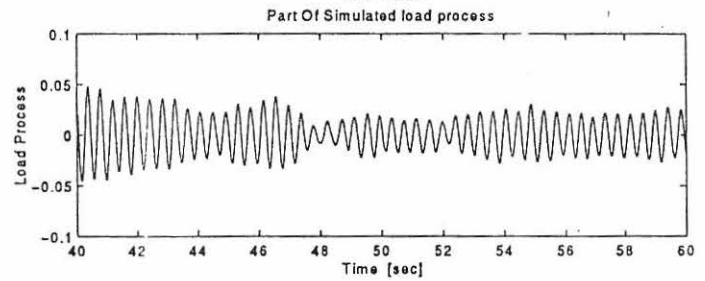
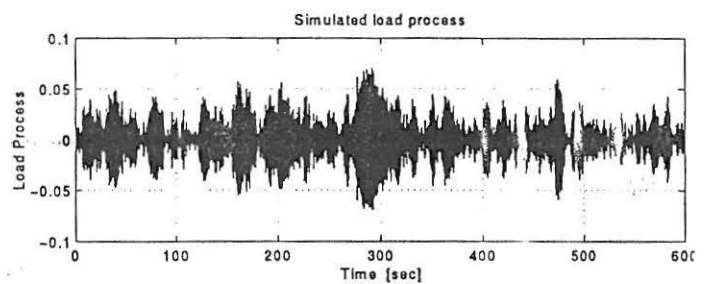
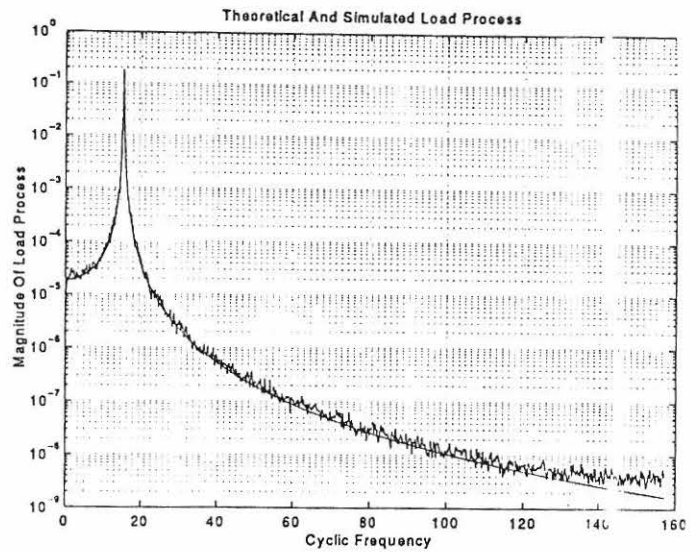
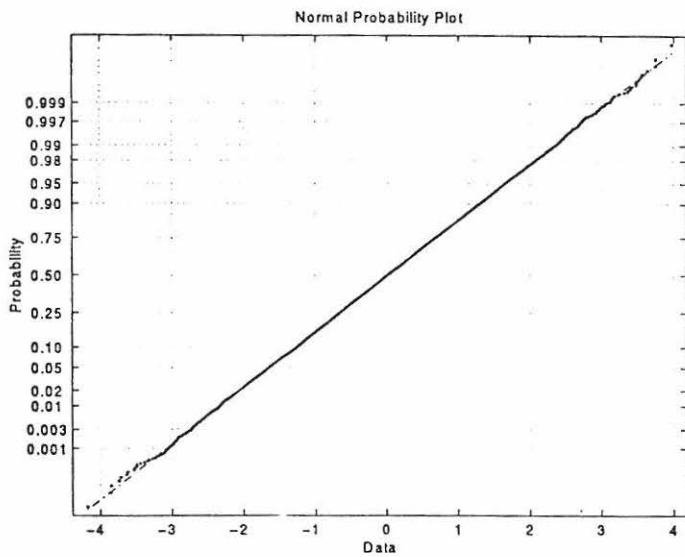
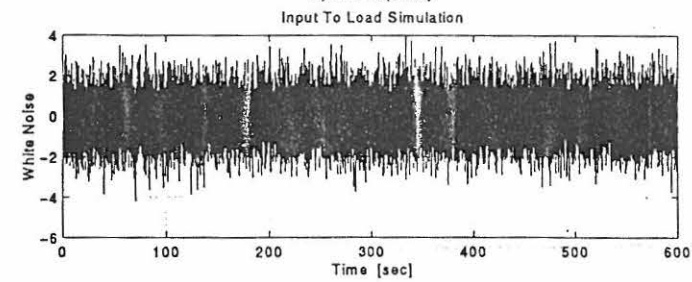
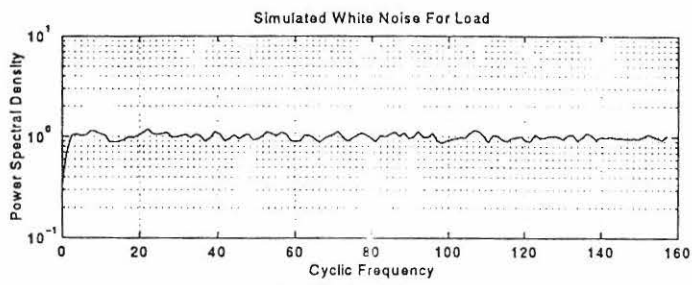


Figure 3. Spectra, Time Histories and Distribution of White Noise and Input to Structure

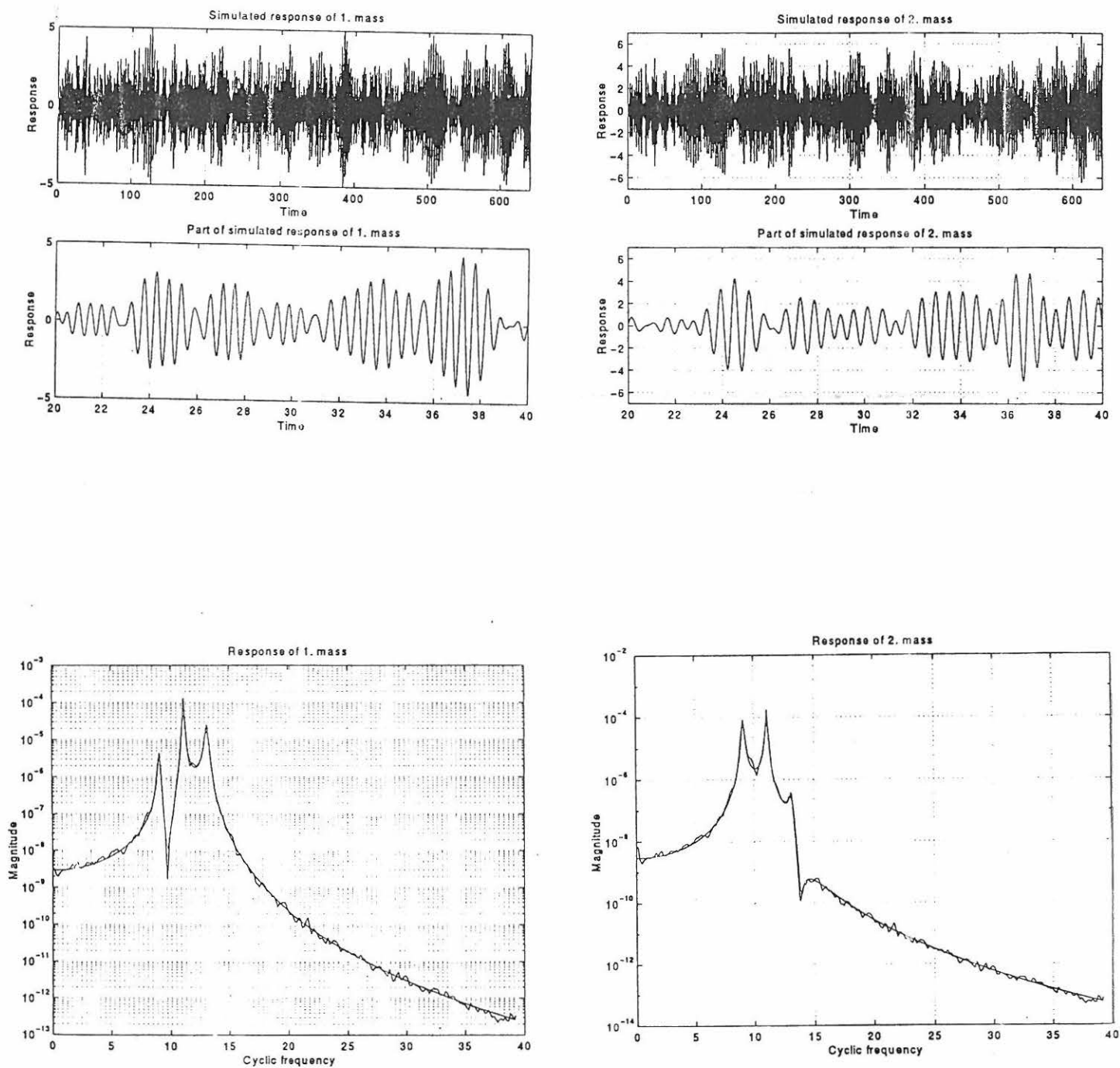


Figure 4. Time Histories and Spectra of Responses

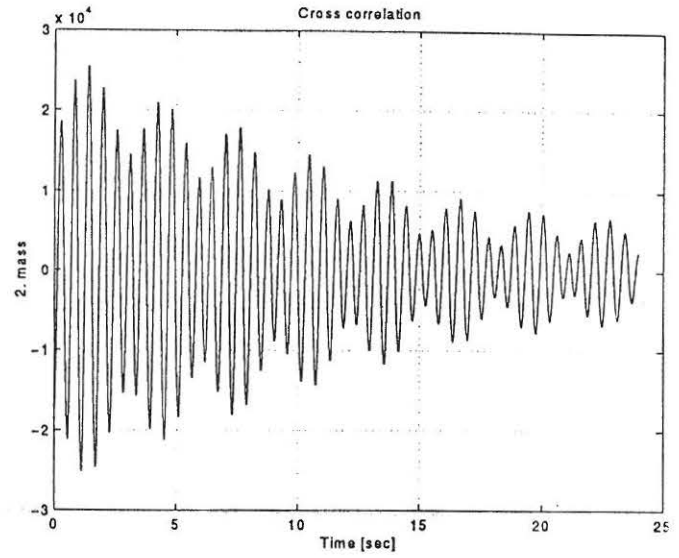
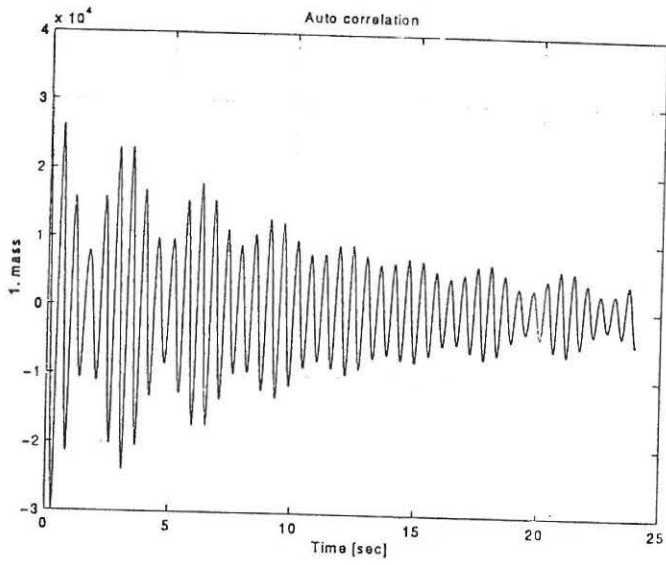


Figure 5. Random Drecement Signatures with Triggerring on Every Positive Point of Mass 1

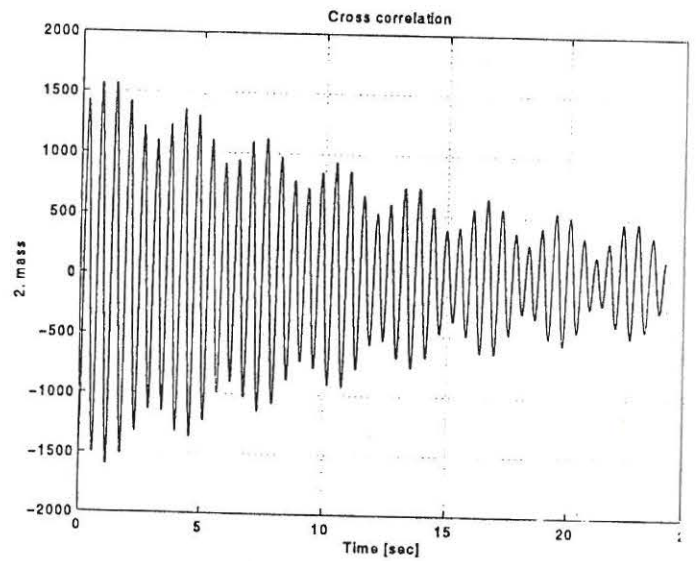
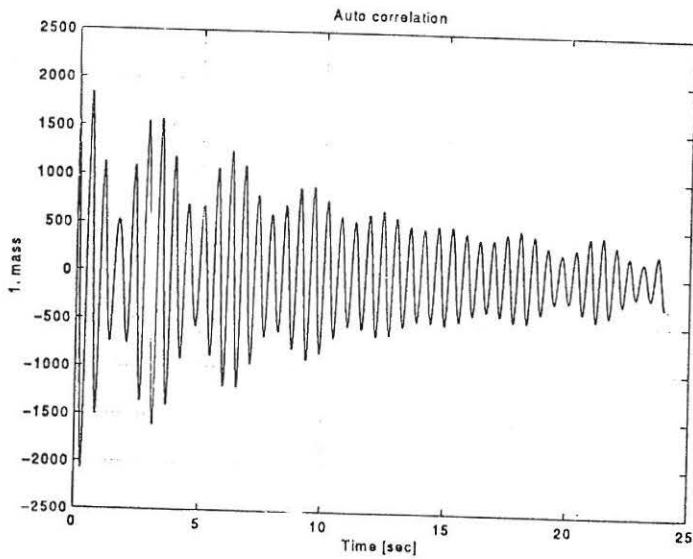


Figure 6. Random Drecement Signatures with Triggerring on Local Extremum of Mass 1