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Yang, Zhenyu; Izadi-Zamanabadi, Roozbeh

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Design of Reconfigurable Nonlinear Control Using On-Line Piecewise Affine System Approximation

Zhenyu Yang * **R. Izadi-Zamanabadi** **

* *Department of Electronic Systems/Esbjerg Institute of Technology, Aalborg University, Niels Bohrs Vej 8, 6700 Esbjerg, Denmark (e-mail: yang@cs.aaue.dk)*
** *Department of Electronic Systems, Aalborg University, Fredrik Bajers Vej 7C, 9220 Aalborg Ø, Denmark (e-mail: riz@es.aau.dk)*

Abstract:
This paper proposes a novel reconfigurable control design approach for a class of nonlinear systems using a piecewise affine system approximation. The piecewise affine model is obtained in an on-line manner through linearizing the considered nonlinear system along system's real-time trajectory. The H_∞ control synthesis is employed for the local reconfigurable control design. A supervisory framework is used to determine the updating of the affine system models, the reconfiguration module and the possibly nonlinear state estimation as well. The proposed method is illustrated in a ship propulsion system and the results showed the power and flexibility of the proposed method for nonlinear reconfigurable control design. The payoff of these benefits is the computation load and complexity of the reconfigured system.

1. INTRODUCTION

The objective of control reconfiguration is to recover the faulty system's operation or functionality to the same or degraded but acceptable level as that of the nominal system (Blanke et al. [2003], Patton [1997]). The design of reconfigurable control for nonlinear systems is always a challenging topic (Kanev and Verhaegen [2000], Yang and Stoustrup [2000]). In recent decades there has been increasing interest in the use of so-called *Multiple Model Approaches (MMAs)* to handle complicated nonlinear systems (Chang and Davison [1999], Morse [1997], Murray-Smith and Johansen [1997]). Basing on the divide-and-conquer strategy, a typical MMA partitions the entire operating range of the considered system into a set of local operating regimes, where each of them is associated with a locally valid model or controller, so that the analysis and design of controls for these local regimes could be easier (Murray-Smith and Johansen [1997]). The switches of different models and/or controllers need to be supervised by a proper logic diagram (Chang and Davison [1999], Morse [1996]). For many cases, the development of this logic program needs to be coordinated with the design of local controls so as to guarantee the global system's stability and performance (Morse [1997]).

As one of early research work using multiple linear models for control reconfiguration design, Huang and Stengel [1990] proposed a Multiple Model Adaptive Control (MMAC) method. The main idea of this method is that all potential fault scenarios need to be defined as a set of fault hypotheses. Each fault hypothesis corresponds to one candidate model and one corresponding control mode. During the system operation, the control mode corresponding to the model with the highest likehood will be selected for controlling the current system operation. However, the MMAC method can be very complicated for a large number of faults. Furthermore, this method can only handle anticipated fault scenarios. Nevertheless, the acquisition and coordination of these local models, especially when some linear models are not *priori*, are still in a quite *ad hoc* situation. From the systematical point of view, Sontag [1981] proposed a *Piecewise Linear (PL) approach* for handling nonlinear control system design using a set of linear maps. However, how to apply this theoretical approach into practice is not clear yet. Morse [1996] introduced a supervisory framework for coordinating switches of a set of linear controllers into a feedback loop, so that the output of the controlled system tracks a set-point. Chang and Davison [1999] extended Morse's framework into the MIMO consideration. However, both approaches are under assumption that the system, which need to be controlled, is modeled as a linear system, and these linear controllers and the model known beforehand.

This paper proposes a reconfigurable control design approach for a class of nonlinear systems using an on-line piecewise affine system approximation. The piecewise affine system model has been extensively studied and used in hybrid system research (Bemporad et al. [2000], Collins and van Schuppen [2004]). Comparing with these existing work, the difference of the proposed method is that the piecewise affine model are not known beforehand. it can only be obtained in a on-line manner, i.e., through linearization of the considered nonlinear system along the system's real-time trajectory. A local-region reconfigurable control can be designed using some linear reconfigurable control design methods based on a corresponding affine system model which is obtained around some specific trajectory point. Then, a supervisory framework is needed so

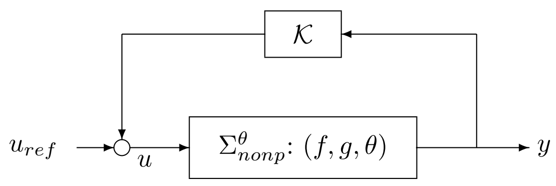

Fig. 1. considered Reconfigurable Control Configuration

as to determine the updating of the affine system model, the reconfiguration module and possibly nonlinear state estimation as well. Comparing with our previous work Yang et al. [2000], the contribution here is that the robust control mixer method proposed in (Yang and Stoustrup [2000], Yang et al. [2007]) is employed for the local reconfigurable control design, so that the reconfiguration module is extended to be a dynamic-type, and the constant bias in the piecewise affine system model can be systematically handled by H_{∞} control design by regarding them as system disturbances. The proposed method is more powerful and flexible than the previous work. However, the payoff of these benefits is the complexity of the reconfigured system.

The rest of this paper is organized as: Section 2 formulates the reconfigurable control design problem; Section 3 presents some theoretical results regarding the piecewise affine approximation; Section 4 briefs the robust reconfigurable control method; Section 5 states the model updating algorithm; Section 6 illustrates the application of the proposed approach in a ship propulsion control system; Finally Section 7 concludes the paper.

2. PROBLEM FORMULATION

Consider a class of nonlinear control system, denoted as Σ^{θ}_{nonp}, described by a general form:

$$\Sigma^{\theta}_{nonp}: \begin{cases} \dot{x}(t) = f(x(t), u(t), \theta(t)), \ x(t_0) = x_0 \\ y(t) = g(x(t), u(t), \theta(t)) \end{cases} \quad (1)$$

where $x(t) \in X \subseteq R^n$ is the state vector, x_0 is the initial state, $u(t) \in U \subseteq R^m$ is the input (controllable) vector, $y(t) \in R^r$ is the output vector and $\theta(t) \in R^p$ is the *fault parameter vector*, meaning that each entry $\theta_i(t), \ i = 1, \cdots, p$ in this vector represents one specific fault scenario in the considered system. If the system is fault-free, there is $\theta(t) = 0$. Vector field $f : X \times U \times R^p \mapsto X$ and $g : X \times U \times R^p \mapsto R^r$ both are continuous and differentiable functions w.r.t. x and u, respectively.

In the following, we denote the nominal system as Σ^{θ}_{nonp} with output $y_n(t)$ and dynamical fields f_n and g_n, respectively. Denote the faulty system as $\Sigma^{\theta_f}_{nonp}$ when some fault occurs in the considered system. Suppose $\theta(t) \neq 0$ for $t \geq t_f > 0$ represents the situation that a fault occurs at instant t_f. Furthermore, we assume that the fault vector $\theta(t)$ can be provided by some on-line fault detection and diagnosis mechanism in this work.

Suppose that we are not allowed to change the existing system structure and parameters except adding some extra configuration for control reconfiguration. This situation is quite common when we want to improve some existing industrial systems to be fault tolerant ones. Normally the industry is reluctant to completely change their systems which development and deployment could be very cost and time consuming. Nevertheless, to enhance the system's capability or reliability by adding extra components/subsystems into the existing systems sounds more simple and acceptable to industry. Thereby we assume that an extra feedback control module denoted as $\mathcal{K}$ in Fig.1 is allowed to be added into the already-existing control system, denoted as Σ^{θ}_{nonp}, for control reconfiguration purpose. $\mathcal{K}$ is referred to as *Reconfiguration Modules (RM)* in this paper. Denote the feedback control system as Σ^{θ}_{non}. Then, a reconfigurable control design problem can be proposed as a suboptimal problem, i.e.,

To synthesize compensator $\mathcal{K}$ such that the reconfigured closed-loop system, denoted as:

$$\Sigma^{cr}_{non}: \begin{cases} \dot{x}_{cr}(t) = f_{cr}(x_{cr}(t), u_{ref}(t), \theta_f, \mathcal{K}), \\ y_{cr}(t) = g_{cr}(x_{cr}(t), u_{ref}(t), \theta_f, \mathcal{K}), \end{cases} \quad (2)$$

satisfies $\forall (x, u_{ref}) \in X \times U$, there is

$$J(x, u_{ref}, \theta_f, \mathcal{K}) < \alpha_{x, u_{ref}}, \quad (3)$$

where

$$J \hat{=} \beta_{x, u_{ref}} \| g_n(x, u_{ref}, 0) - g_{cr}(x, u_{ref}, \theta_f) \|_2 \quad (4)$$

where $\alpha_{x, u_{ref}}$ is some given constant used to evaluate the reconfiguration quality, and $\beta_{x, u_{ref}}$ is some weighting function depending on operating point (x, u_{ref}).

In the following, the RM $\mathcal{K}$ will be synthesized using the model matching strategy (Huang and Stengel [1990], Yang et al. [2007]) based on an on-line piecewise affine system approximation. The piecewise affine model is obtained through linearization of the considered nonlinear system along the system's real-time trajectory. A supervisory control is developed for the updating of the affine system model and module $\mathcal{K}$.

3. PIECEWISE AFFINE APPROXIMATION

In the following the fault vector θ is treated as a system parameter instead of a system state. Firstly, consider the nonlinear system Σ^{θ}_{nonp} described by (1) within an open ball-type neighborhood, denoted as $\mathcal{B}(x_0, u_0)$, of one operating point $(x_0, u_0) \in X \times U$ with radius δ_0. according to the nonlinear system theory (Isidori [1995]), there is:

Definition 1: The θ-parameterized affine system, denoted as Σ^{θ}_{linp}, is called the *Local Affine Approximation (LAA)* of the nonlinear system Σ^{θ}_{nonp} within $\mathcal{B}(x_0, u_0)$, if the *θ-parameterized* vector functions $f(x, u, \theta)$ and $g(x, u, \theta)$ both are $\mathcal{C}^1$ differentiable w.r.t. x and u at point (x_0, u_0), respectively, where

$$\Sigma^{\theta}_{linp}: \begin{cases} \dot{x}_{lin} = A_0(\theta) x_{lin} + B_0(\theta) u + \phi_0(\theta), \\ y_{lin} = C_0(\theta) x_{lin} + D_0(\theta) u + \psi_0(\theta), \end{cases} \quad (5)$$

with $x_{lin}(t_0) = x_0$ and $(x_{lin}, u) \in \mathcal{B}(x_0, u_0))$, and

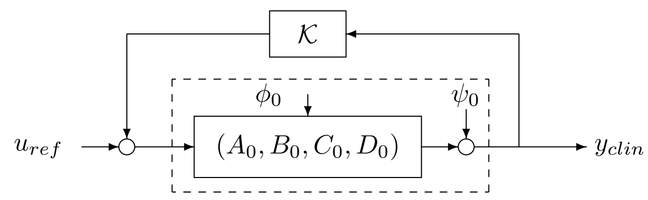

Fig. 2. Affine Approximation with reconfiguration module

$$
\begin{aligned}
A_0(\theta) &= \frac{\partial f(x,u,\theta)}{\partial x}|_{(x_0,u_0)};\\
B_0(\theta) &= \frac{\partial f(x,u,\theta)}{\partial u}|_{(x_0,u_0)};\\
C_0(\theta) &= \frac{\partial g(x,u,\theta)}{\partial x}|_{(x_0,u_0)};\\
D_0(\theta) &= \frac{\partial g(x,u,\theta)}{\partial u}|_{(x_0,u_0)};\\
\phi_0(\theta) &= f(x_0,u_0,\theta) - A_0(\theta)x_0 - B_0(\theta)u_0;\\
\psi_0(\theta) &= g(x_0,u_0,\theta) - C_0(\theta)x_0 - D_0(\theta)u_0.
\end{aligned} \tag{6}
$$

Lemma 2: The LAA has the following properties:

- (i) For a fixed (x_0, u_0), the LAA is unique;
- (ii) If f and g belong to $\mathcal{C}^\infty$ and all corresponding derivatives are bounded w.r.t. x and u within $\mathcal{B}(x_0, u_0)$, then, there exist two positive reals related to θ and δ_0, denoted as $\alpha_x(\theta, \delta_0)$ and $\alpha_y(\theta, \delta_0)$, respectively, satisfying $\forall t \in (t_0, t_1), \forall u \in U$, there is
$$
\begin{aligned}
\|\dot{x}(t) - \dot{x}_{lin}(t)\|_2 &< \alpha_x(\theta, \delta_0),\\
\|y(t) - y_{lin}(t)\|_2 &< \alpha_y(\theta, \delta_0),
\end{aligned} \tag{7}
$$
where the interval (t_0, t_1), $t_1 > t_0$ denotes the interval when the system operates within $\mathcal{B}(x_0, u_0)$, and $t_1 - t_0$ is often referred to as *dwell time* (Morse [1996]).

The control reconfiguration strategy showed in Fig.1 is also employed to the corresponding LAA Σ^θ_{linp}. This configuration leads to a linear closed-loop control system, which is denoted as Σ^θ_{lin}. Then there is

Theorem 3: Suppose that the linear system Σ^θ_{linp} is the LAA of the nonlinear system Σ^θ_{nonp} within $\mathcal{B}(x_0, u_0)$ and the following conditions are fulfilled:

- $\mathcal{K}$ is a LTI system, and
- $I_{r\times r} + D_0(\theta)\mathcal{K}$ is invertible.

Then the linear closed-loop control system Σ^θ_{lin} is also the LAA of nonlinear control system Σ^θ_{non} within $\mathcal{B}(x_0, u_{ref0})$ with $u_{ref0} = u_0 + \mathcal{K}g(x_0, u_0, \theta)$.

Proof: Firstly, we assume $\mathcal{K}$ is just a gain-matrix, denoted as K. The output equation of the closed-loop control nonlinear system is described as $y = g(x, u_{ref} - Ky, \theta)$. Define a r-dimension vector function F as:

$$
F(x, u_{ref}, y, \theta) = y - g(x, u_{ref} - Ky, \theta). \tag{8}
$$

Define $y_0 = g(x_0, u_0, \theta)$, and an $n+m+r$-dimension open neighbor set of (x_0, u_{ref0}, y_0) according to the set $\delta(x_0, u_0)$, denoted as $\delta(x_0, u_{ref0}, y_0)$. Then we can observe

(1) F is continuous within set $\delta(x_0, u_{ref0}, y_0)$;
(2) $\frac{\partial F}{\partial x}$, $\frac{\partial F}{\partial u_{ref}}$ and $\frac{\partial F}{\partial y}$ all exist and are continuous within the set $\delta(x_0, u_{ref0}, y_0)$;
(3) $F(x_0, u_{ref0}, y_0) = 0$;
(4) $\frac{\partial F}{\partial y}|_{(x_0, u_{ref0}, y_0)}$ is full rank, since
$$
\begin{aligned}
\frac{\partial F}{\partial y}|_{(x_0,u_{ref0},y_0)} &= (I_{r\times r} + \frac{\partial g}{\partial u}K)|_{(x_0,u_{ref0},y_0)}\\
&= I_{r\times r} + \frac{\partial g}{\partial u}|_{(x_0,u_0)}K = I_{r\times r} + D_0(\theta)K.
\end{aligned}
$$

Then, according to the implicit function theorem, it is noted that the equation $F(x, u_{ref}, y, \theta) = 0$ uniquely determines a r-dimensional vector function in terms of explicit y:

$$
y = \varphi(x, u_{ref}, \theta), \tag{9}
$$

which is defined within a neighbor open set of point (x_0, u_{ref0}), denoted as $\delta_y(x_0, u_{ref0})$, such that

- The $n + m + r$-dimension set $\{(x, u_{ref}, y)|\ y = \varphi(x, u_{ref}, \theta), (x, u_{ref}) \in \delta_y(x_0, u_{ref0})\} \subset \delta(x_0, u_{ref0}, y_0)$;
- $F(x, u_{ref}, \varphi(x, u_{ref}), \theta) \equiv 0, \forall (x, u_{ref}) \in \delta_y(x_0, u_{ref0})$;
- $y_0 = \varphi(x_0, u_{ref0}, \theta)$; and
- Function $\varphi(x, u_{ref}, \theta)$ has the continuous partial derivations within the set $\delta_y(x_0, u_{ref0})$, i.e.,
$$
\frac{\partial \varphi}{\partial x} = -(\frac{\partial F}{\partial y})^{-1}\frac{\partial F}{\partial x}, \qquad \frac{\partial \varphi}{\partial u_{ref}} = -(\frac{\partial F}{\partial y})^{-1}\frac{\partial F}{\partial u_{ref}}. \tag{10}
$$

According to Definition 1 a linear system to approximate the nonlinear relationship (9) within a open neighbor set of (x_0, u_{ref0}), denoted as $\delta'_y(x_0, u_{ref0})$, can be obtained as

$$
y_{lin} = C_{cl}(x_0, u_{ref0}, \theta)x + D_{cl}(x_0, u_{ref0}, \theta)u_{ref} + \psi_{cl}(\theta), \tag{11}
$$

where

$$
\begin{aligned}
C_{cl}(x_0, u_{ref0}, \theta) &= \frac{\partial \varphi(x, u_{ref}, \theta)}{\partial x}|_{(x_0,u_{ref0})};\\
D_{cl}(x_0, u_{ref0}, \theta) &= \frac{\partial \varphi(x, u_{ref}, \theta)}{\partial u_{ref}}|_{(x_0,u_{ref0})};\\
\psi_{cl}(\theta) &= (\varphi(x,u,\theta) - C_{cl}(x,u,\theta)x - D_{cl}(x,u,\theta)u)|_{x_0,u_{ref0}}.
\end{aligned}
$$

From the property (10), we can further have

$$
\begin{aligned}
C_{cl}(x_0, u_{ref0}, \theta) &= \frac{\partial \varphi(x, u_{ref}, \theta)}{\partial x}|_{(x_0,u_{ref0})}\\
&= (I_{r\times r} + D_0(\theta)K)^{-1}C_0(\theta),
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
D_{cl}(x_0, u_{ref0}, \theta) &= \frac{\partial \varphi(x, u_{ref}, \theta)}{\partial u_{ref}}|_{(x_0,u_{ref0})}\\
&= (I_{r\times r} + D_0(\theta)K)^{-1}D_0(\theta).
\end{aligned} \tag{13}
$$

Similarly, the state equation of the closed-loop nonlinear system can be expressed as $\dot{x} = f(x, u_{ref} - Ky, \theta)$. Take the linearization of the above nonlinear equation within an open neighbor set of (x_0, u_{ref0}), denoted as $\delta_x(x_0, u_{ref0})$, then the nonlinear closed-loop system can be approximated by a linear system, $\Sigma^{\theta cl}_{lin}$ described as

$$
\begin{cases}
\dot{x}_{lin} = A_{cl}(\theta)x_{lin} + B_{cl}(\theta)u_{ref} + \phi_{cl}(\theta)\\
y_{lin} = C_{cl}(\theta)x_{lin} + D_{cl}(\theta)u_{ref} + \psi_{cl}(\theta)
\end{cases} \tag{14}
$$

where

$$
\begin{aligned}
A_{cl} &= A_0(\theta) - B_0(\theta)K(I_{r\times r} + D_0(\theta)K)^{-1}C_0(\theta);\\
B_{cl} &= B_0(\theta) - B_0(\theta)K(I_{r\times r} + D_0(\theta)K)^{-1}D_0(\theta);\\
C_{cl} &= (I_{r\times r} + D_0(\theta)K)^{-1}C_0(\theta);\\
D_{cl} &= (I_{r\times r} + D_0(\theta)K)^{-1}D_0(\theta).
\end{aligned} \tag{15}
$$

Now we consider the closed-loop linear system as shown in Fig.2. A state space formulation of this control system, denoted as $\Sigma^{\theta cl'}_{lin}$, can be obtained

$$\begin{cases} \dot{x}_{lin} = A'_{cl}(\theta)x_{lin} + B'_{cl}(\theta)u_{ref} + \phi'_{cl}(\theta) \\ y_{lin} = C'_{cl}(\theta)x_{lin} + D'_{cl}(\theta)u_{ref} + \psi'_{cl}(\theta) \end{cases} \quad (16)$$

where it can be observed that A'_{cl}, B'_{cl}, C'_{cl}, D'_{cl} and $\phi'_{cl}(\theta)$, $\psi'_{cl}(\theta)$ have the same formats as (15). Thereby a nonlinear control-loop system, as shown in Fig.1, can be approximated by a closed-loop linear system, as shown in Fig.2, within a common neighbor open set of point (x_0, u_{ref0}).

In case that K is a LTI system, the consistency between (14) and (16) can still be kept. □

Corollary 4: If g belongs to $\mathcal{C}^\infty$ and all its corresponding derivatives are bounded w.r.t. x within $\mathcal{B}(x_0, u_0)$, then, there exist a positive real related to θ and δ_0, denoted as $\beta_{cy}(\theta, \delta_0)$, satisfying

$$\|y_{cn}(t) - y_{clin}(t)\|_2 < \beta_{cy}(\theta, \delta_0), \ \forall t \in (t_0, t_1), \forall u_{ref} \in U \quad (17)$$

where $y_{cn}(t)$ and $y_{clin}(t))$ represent the outputs of the closed-loop nonlinear and linear systems, respectively.

Corollary 5: The nonlinear control system Σ^θ_{non} within its whole operation range $X \times U$ can be linearly approximated by a piecewise affine linear system, denoted as $\{\Sigma^{\theta^i}_{lin}\}^N_{i=1}$, if there exists a set of ordered points $(x_i, u_{refi}) \in X \times U$ and a set of corresponding neighborhoods $\mathcal{B}(x_i, u_{refi})$ $i = 1, \cdots, N$, where N can be a finite integer or $+\infty$, satisfying

$$\begin{aligned} &\bigcup_{i=1}^{N} \mathcal{B}(x_i, u_{refi}) \supseteq X \times U, \quad \text{and} \\ &\mathcal{B}(x_i, u_{refi}) \bigcap \mathcal{B}(x_{i+1}, u_{refi+1}) \neq \phi. \end{aligned} \quad (18)$$

and functions f and g belong to $\mathcal{C}^1$ in x and u at any point (x_i, u_i), and all matrices $I_{r\times r} + D^i_0(\theta)\mathcal{K}$ are invertible $i = 1, \cdots, N$, where

$$\Sigma^{\theta^i}_{lin} : \begin{cases} \dot{x}^i_{clin} = A^i_c(\theta)x^i_{clin} + B^i_c(\theta)u_{ref} + \phi^i_c(\theta), \\ y^i_{clin} = C^i_c(\theta)x^i_{clin} + D^i_c(\theta)u_{ref} + \psi^i_c(\theta), \end{cases} \quad (19)$$

with $x^i_{clin}(t^i_0) = x_i$, $(x_{clin}, u_{ref}) \in \mathcal{B}(x_i, u_{refi})$ and

$$\begin{aligned} &A^i_c(\theta)\hat{=}(A^i_0(\theta) - B^i_0(\theta)K(I_{r\times r} + D^i_0(\theta)K)^{-1}C^i_0(\theta)); \\ &B^i_c(\theta)\hat{=}(B^i_0(\theta) - B^i_0(\theta)K(I_{r\times r} + D^i_0(\theta)K)^{-1}D^i_0(\theta)); \\ &C^i_c(\theta)\hat{=}(I_{r\times r} + D^i_0(\theta)K)^{-1}C^i_0(\theta); \\ &D^i_c(\theta)\hat{=}(I_{r\times r} + D^i_0(\theta)K)^{-1}D^i_0(\theta); \\ &\phi^i_c(\theta)\hat{=}\phi^i_0(\theta) - B^i_0(\theta)K(I_{r\times r} + D^i_0(\theta)K)^{-1}\psi^i_0(\theta); \\ &\psi^i_c(\theta)\hat{=}(I_{r\times r} + D^i_0(\theta)K)^{-1}\psi^i_0(\theta). \end{aligned} \quad (20)$$

The local affine approximation $(A^i_0, B^i_0, C^i_0, D^i_0, \phi^i_0, \psi^i_0)$ can be obtained by using (6) at point (x_i, u_i), where u_i satisfies $u_i = u_{refi} - g(x_i, u_i, \theta)$ for $i = 1, \cdots, N$.

Based on one LAA, the compensator $\mathcal{K}$ can be designed using some standard linear reconfiguration design methods, such as the robust control mixer methods (Yang et al. [2000], Yang and Stoustrup [2000], Yang et al. [2007]).

4. LOCAL RECONFIGURATION DESIGN

Different with the control mixer method used in Yang et al. [2000], where the control mixers are limited to be gain matrices, the robust control mixer method extends the control mixer module to be more powerful dynamic-type. The standard H_∞ and μ control synthesis can be employed for this module development (Yang and Stoustrup [2000]).

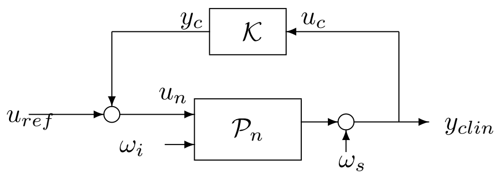

Fig. 3. The LTI reconfiguration system

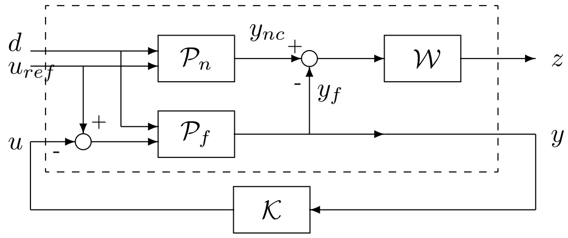

Fig. 4. The Augmented Control System Using $\mathcal{K}$

The feedback control configuration as shown in Fig.2 can be reformulated into the structure as shown in Fig.3, where $\mathcal{P}_n$ is the linear part of Σ^θ_{linp}. The constant bias parts of Σ^θ_{linp} are arranged as system process disturbance ω_i and measurement disturbance ω_s, respectively. The design problem of compensator $\mathcal{K}$ can be formulated as:

For a given real positive scalar constant γ, find out a real rational and proper compensating module $\mathcal{K}$, such that

$$\|\mathcal{W}(\mathcal{P}_n - \mathcal{P}_{fc}(\mathcal{K}))\|_\infty < \gamma, \quad (21)$$

under the condition that the reconfigured system $\mathcal{P}_{fc}(\mathcal{K})$ is internally stable, where $\mathcal{P}_n$ is the transfer function matrix of the nominal linear system, and $\mathcal{P}_{fc}(\mathcal{K})$ is the transfer function matrix of the reconfigured linear system by using module $\mathcal{K}$.

There are many numerical methods to deal with this γ-suboptimal problem after the original problem is formulated into a standard control design problem as shown in Fig.4, such as the two-Riccati-equation method and LMI based method. As pointed out by Yang et al. [2007], the γ can be regarded as a kind of quantitative evaluation of this control reconfiguration strategy. The infimum $\gamma^\star$ represents the best reconfiguration level that a LTI controller can achieve for the impaired System. This also gives some hints in the selection of threshold for updating LAA models. This design is linked to the objective (3) by the following statement.

Corollary 6(Yang et al. [2007]): Given a real scalar constant $\gamma > 0$, if there exists a real rational controller $\mathcal{K}$ which satisfies (21), then the tracking error between the nominal and reconfigured system is bounded by

$$\|y_n - y_{fc}(\mathcal{K}_i)\|_2 < \gamma\beta, \quad (22)$$

where β is the excitation level of the system, $\|[d^T \ r^T]^T\|_2 = \beta$.

5. ON-LINE UPDATING ALGORITHM

The adaptive algorithm proposed in Yang et al. [2000] can also be used here. The inspiration for that comes from the the following relationship:

$$\|y_n(t) - y_{cr}(t)\|_2 \leq (\underbrace{\|y_n(t) - y_{cnlin}(t)\|_2}_{cn-term} + \underbrace{\|\|y_{cnlin}(t) - y_{crlin}(t)\|_2}_{lcr-term} + \underbrace{\|y_{cr}(t) - y_{crlin}(t)\|_2}_{cr-term}). \quad (23)$$

This means that the reconfiguration design can be decomposed into two cooperative parts, i.e., the affine approximations of the nominal and the reconfigured nonlinear systems, corresponding to the *cn-term* and *cr-term* in equation (23); and local linear RM part, corresponding to the *lcr-term*.

The on-line updating algorithm is summarized in the following (Yang et al. [2000]):

- *Step 1*: Get a trajectory sample (x_i, u_{refi}) from the considered nonlinear system $\Sigma_{nonp}^{\theta_f}$, then obtain the LAA $\Sigma_{linp}^{\theta_f i}$ for $\Sigma_{nonp}^{\theta_f}$ at (x_i, u_{refi}). Meanwhile the LAA Σ_{linp}^{0i} corresponding to the fictitious nominal nonlinear system denoted as Σ_{nonp}^{0} also needs to be achieved at (x_i, u_{refi});
- *Step 2*: Design the RM $\mathcal{K}$ based on the LAA $\Sigma_{linp}^{\theta_f i}$ and Σ_{linp}^{0i}, where the second system serves as $\mathcal{P}_n$ as shown in Fig.4. Implement the developed $\mathcal{K}$ into the undergoing nonlinear system $\Sigma_{nonp}^{\theta_f}$, so that the reconfigured nonlinear system Σ_{non}^{cr} is obtained;
- *Step 3*: Obtain the LAA Σ_{lin}^{cri} of nonlinear Σ_{non}^{cr};
- *Step 4*: Keep the fictitious Σ_{linp}^{0i}, Σ_{lin}^{cri} and Σ_{nonp}^{0} running from the initial (x_i, u_{refi}) parallel (e.g., in software programs) to the real undergoing nonlinear system Σ_{non}^{cr} and monitor the inequality: $\|Err(t)\|_2 \leq Thres$, where $Err(t)$ represents combination of tracking error functions of linear models to corresponding nonlinear systems plus local CR design error, it could be the weighted right part of inequality (23).
- *Step 5*: When $\|Err(t)\|_2 \leq Thres$ is valid, keep the current nonlinear system and its RM $\mathcal{K}$ running. Otherwise, the fictitious linear systems as well as the compensating modules $\mathcal{K}$ need to be updated according to step (1)-(2).

6. RECONFIGURABLE CONTROL FOR A SHIP PROPULSION SYSTEM

6.1 Benchmark System

The ship propulsion system developed in Izadi-Zamanabadi and Blanke [1999] is used to test the proposed method.The Schematic diagram of the considered system is shown in Fig.27.

- The *Diesel Engine* generates a torque Q_{eng} controlled by its fuel index Y, and the transfer function of this part has the form:

$$Q_{eng}(s) = \frac{k_y e^{-\tau s}}{1 + \tau_c s} Y(s). \quad (24)$$

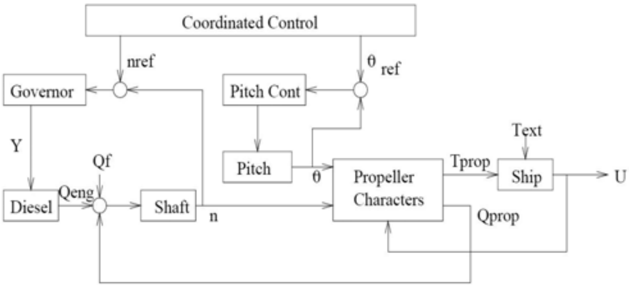

Fig. 5. Schematic diagram of the ship propulsion system

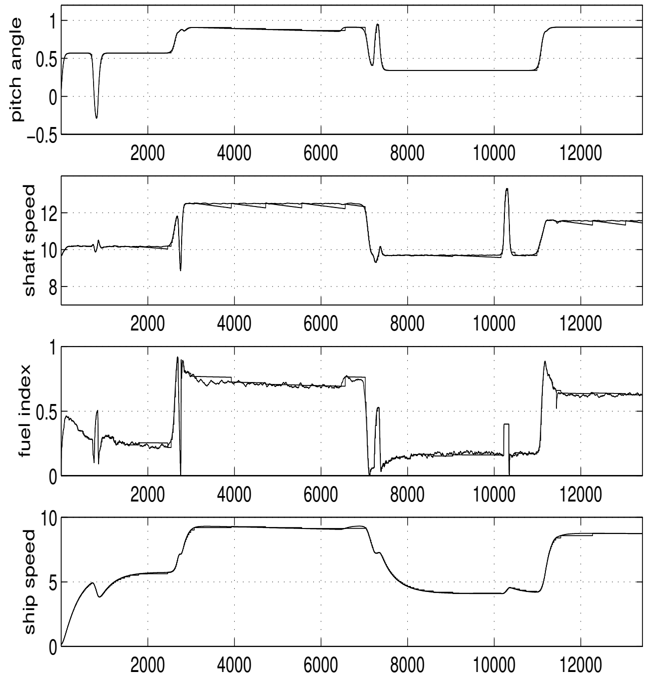

Fig. 6. Piecewise affine approximation to the nominal system

- The *Shaft* part generates speed n according to:

$$I_m \dot{n}(t) = Q_{eng}(t) - Q_{prop}(t) - Q_f(t). \quad (25)$$

- The thrust T_{prop} and torque Q_{prop} generated by the *Propeller* are determined by:

$$\begin{cases} T_{prop}(t) = & T_{|n|n}(\theta(t))|n(t)|n(t) + T_{|n|U}(\theta(t)) \\ & |n(t)|(1-w)U(t) \\ Q_{prop}(t) = & Q_0|n(t)|n(t) + Q_{|n|n}(\theta(t))|n(t)|n(t) \\ & +Q_{|n|U}(\theta(t))|n(t)|(1-w)U(t) \end{cases} \quad (26)$$

- The *Ship* dynamic is described as:

$$m\dot{U}(t) = X_{uu}U^2(t) + (1 - t_u)T_{prop}(t) \quad (27)$$

- The *Propeller-Pitch* control is a P controller (k_t), and the *Governor* is a PI controller (k_r, τ_i).

We refer to Izadi-Zamanabadi and Blanke [1999] for more details.

6.2 Piecewise Affine Approximation

First of all, the piecewise affine approximation is examined. One performance comparison of the piecewise affine system with the original nonlinear system under the nominal operation is shown in Fig.6. It can be clearly observed that the piecewise affine approximation works very well.

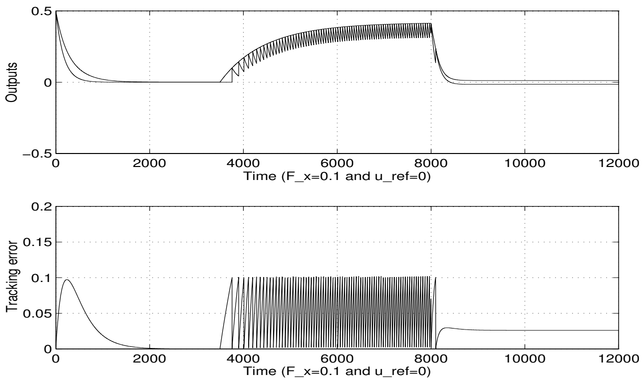

Fig. 7. Affine model updating in a high frequency

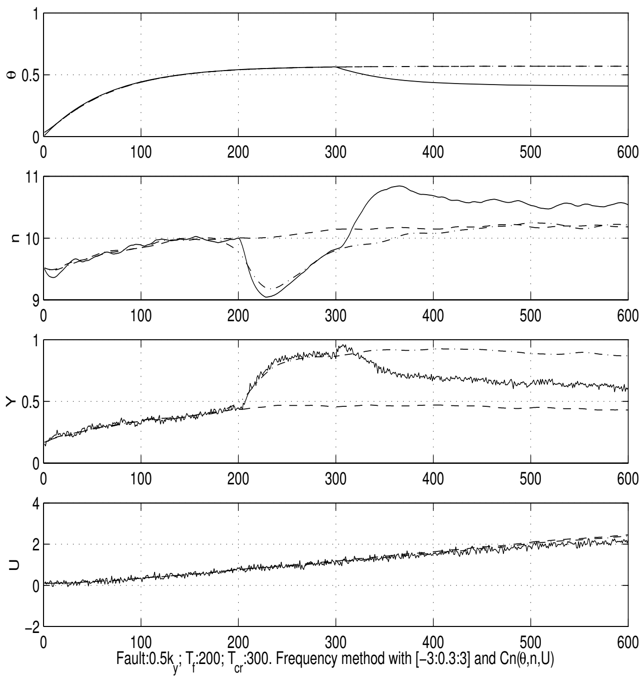

Fig. 8. Nonlinear System responses: Nominal([100,200]), faulty((200,300]) and reconfigured((300,600])

However, in some cases, the phenomenon of local affine model updating in a very high frequency can also be observed, especially during the transition period when the system transfers from one static operation into another one. A specific example is shown in Fig.7.

6.3 Reconfiguration Performance

A fault scenario - the engine loses half efficiency denoted as $k_y^f = 0.5k_y$ is considered for control reconfiguration design. As shown in Fig.8, this type of fault has no influence to the pitch loop. This fault causes lowdown of engine shaft speed (n), meanwhile the engine consume much more fuel (Y), even though the ship speed doesn't have obvious change. The reconfiguration makes the reconfigured system follow a new strategy, i.e., turn to a smaller pitch angle and then the engine consume less fuel comparing with the faulty situation.

7. CONCLUSION

The proposed reconfigurable control design approach for nonlinear systems is a kind of multiple model method. However, the difference compared with most research in this category lies in that the piecewise affine approximation are not known beforehand. It is obtained through online linearization of the considered nonlinear system along system's real-time trajectory. The robust control mixer method enhances the reconfiguration capability in terms of better performance and disturbance attenuation. However, the payoff of these benefits is the computation load and complexity of the reconfigured system.

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