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H. Uğur Köylüoğlu[1], Søren R. K. Nielsen[2] and Ahmet Ş. Çakmak[3]

ABSTRACT

A hysteretic mechanical formulation is derived to quantify local, modal and overall damage in reinforced concrete (RC) shear frames subject to seismic excitation. Each interstorey is represented by a Clough and Johnston (1966) hysteretic constitutive relation with degrading elastic fraction of the restoring force. The local maximum softening damage indicators are based on the variation of the local stiffness and strength deterioration. The modal damage indicators are calculated from the variation of the eigenfrequencies of the structure. A statistical analysis is performed where a sample 5 storey shear frame is subject to simulated earthquake excitations, which are modelled as a stationary Gaussian stochastic process with Kanai-Tajimi spectrum, multiplied by an envelope function. The relationship between local, modal and overall damage indices is investigated statistically.

1. INTRODUCTION

For RC structures modelled by non-linear mechanical theories, local damage can be quantified by the degradation of local stiffness and strength. Damage indicators are quantities characterizing the damage state of the structure after an earthquake excitation, and such can be used in decision-making during design, or in case of post-earthquake reliability and repair problems. The maximum softening damage indicators (MSDI) used here measure the maximum relative reduction of the vibrational frequencies for an equivalent linear system with slowly varying stiffness during a seismic event, hence, display the combined damaging effects of the maximum displacement ductility of the structure during extreme plastic deformations and the stiffness deterioration in the elastic regime. The authors have previously studied different merits of the MSDI concept, including applications to available real data, testing the Markov property, prediction of future performance and reliability of damaged structures in a series of papers, the latter being Köylüoğlu et al. (1995).

2. HYSTERETIC MODEL FOR MDOF SHEAR FRAMES

Consider an n storey RC shear frame. The relative displacement between the ith and $(i+1)$th storeys is designated x_i, and x_1 signifies the displacement of the first storey relative to ground surface excited by the horizontal acceleration $\ddot{u}_g$. With a shear force of the magnitude $Q_i m_i$ where m_i is the storey mass, the equations of motion in terms of the relative displacements are :

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[1] Koç University, TR-80860 İstinye, İstanbul, Turkey.
[2] Aalborg University, DK-9000, Aalborg, Denmark.
[3] Princeton University, Princeton, NJ 08544, USA.

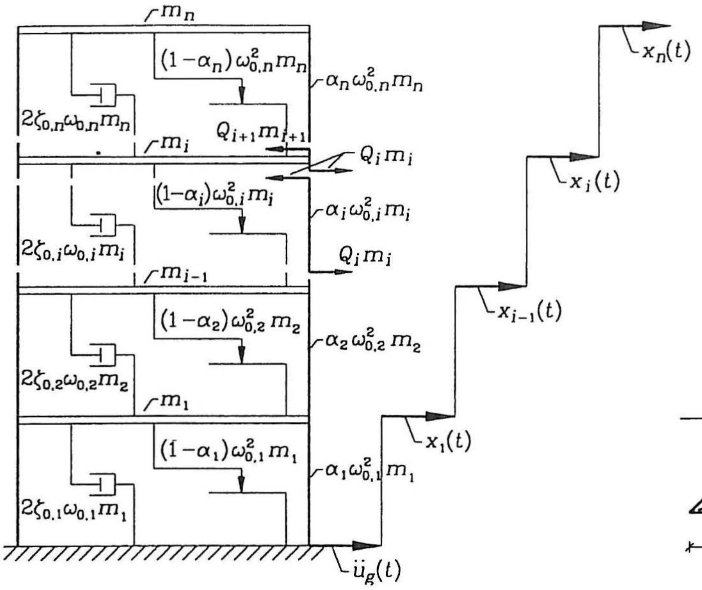

Figure 1. MDOF shear frame.

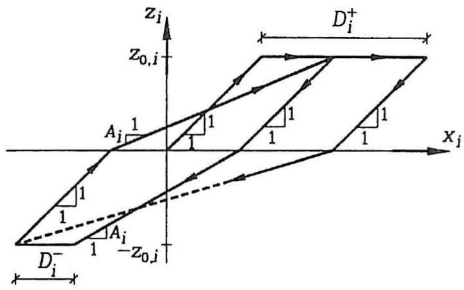

Figure 2. Clough-Johnston hysteretic model.

$$\left.\begin{array}{llll} \ddot{x}_1 = \mu_2 Q_2 - Q_1 - \ddot{u}_g & , \quad t > 0 & & \\ \ddot{x}_i = \mu_{i+1} Q_{i+1} - (\mu_i + 1) Q_i + Q_{i-1} & , \quad t > 0 & , & i = 2, 3, \cdots, n-1 \\ \ddot{x}_n = -(\mu_n + 1) Q_n + Q_{n-1} & , \quad t > 0 & & \end{array}\right\} \quad (1)$$

$$\mu_i = \frac{m_i}{m_{i-1}} \quad (2)$$

$$Q_i = 2\zeta_{0,i}\omega_{0,i}\dot{x}_i + \omega_{0,i}^2(\alpha_i x_i + (1-\alpha_i) z_i) \quad (3)$$

$$\dot{z}_i = k(\dot{x}_i, z_i, D_i; z_{0,i})\dot{x}_i \quad (4)$$

$$\dot{D}_i = g(\dot{x}_i, z_i; z_{0,i})\dot{x}_i \quad (5)$$

$$\alpha_i = \left(\frac{2z_{0,i}}{2z_{0,i} + D_i}\right)^{n_{0,i}} \quad (6)$$

$$\begin{array}{l} k(\dot{x}_i, z_i, D_i; z_{0,i}) = H(z_i)\{A_i H(\dot{x}_i)(1 - H(z_i - z_{0,i})) + H(-\dot{x}_i)\} + \\ H(-z_i)\{A_i H(-\dot{x}_i)(1 - H(-z_i - z_{0,i})) + H(\dot{x}_i)\} \end{array} \quad (7)$$

$$H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{8}$$

$$g(\dot{x}_i, z_i; z_{0,i}) = H(\dot{x}_i)H(z_i - z_{0,i}) - H(-\dot{x}_i)H(-z_i - z_{0,i}) \tag{9}$$

$$A_i = \frac{z_{0,i}}{z_{0,i} + D_i} \tag{10}$$

$\alpha_i(D_i)$ = elastic fraction of the restoring force which is a function of damage. The Clough-Johnston model deals with the stiffness degradation by changing the slope A_i of the elastic branches as the accumulated plastic deformations D_i^+ and D_i^- at positive and negative yielding increase, see Fig.2. $D_i = D_i^+ + D_i^-$ = total accumulated plastic deformations. A novelty is the modelling of $\alpha_i(D_i)$ as a non-increasing function of the damage parameter D_i. Since, $\alpha_i(D_i)$ measures the fraction of the restoring force from linear elastic behaviour, this fraction must decrease from 1 as more and more parts of the structure become plastic.

3. MODAL, LOCAL AND OVERALL MSDI

The modal MSDI for the jth mode, $\delta_{M,j}$, is defined as

$$\delta_{M,j} = \max\left\{1 - \frac{T_{0,j}}{T_j(t)}\right\} \quad , \quad 0 \leq \delta_{M,j} \leq 1 \tag{11}$$

$T_{0,j}$ = jth and $T_j(t)$ = jth period of the linear and equivalent linear structure. Locally, a hysteretic loop-averaged softening value $S_i(t)$ is defined using the average slope $\overline{m}_i$.

$$\overline{m}_i = \frac{2z_{0,i}}{2z_{0,i} + D_i(t)} \tag{12}$$

$$S_i(t) = 1 - \sqrt{\overline{m}_i(1 - \alpha_i) + \alpha_i} \tag{13}$$

$S_i(t)$ is non-decreasing during a seismic event and fully correlated to $D_i(t)$. The ith local MSDI $S_{M,i}$ is defined as the maximum of $S_i(t)$. $S_i(t) = 0$ denotes no local damage in the columns and $S_i(t) = 1$ means total collapse of columns under the ith storey. A scalar numerical overall damage indicator is defined with weights of modal participation factors for modal $\hat{\delta}_M$ and with equal weights for local MSDI $\hat{S}_M$.

4. NUMERICAL INVESTIGATIONS

Consider a five storey RC shear frame. All storeys have the same mass, stiffness and damping characteristics. The parameters are : $\mu_i = 1$, $\omega_{0,i} = 7\pi$ sec^{-1}, $\zeta_{0,i} = 0.03$, $z_{0,i}$ = 24 mm and $n_{0,i} = 0.8$ for $i = 1, 2, \cdots, 5$. Then, the first two eigenfrequencies are 1.00 Hz and 2.90 Hz. The ground excitation, $\ddot{u}_g(t)$ is taken as $\ddot{u}_g(t) = E(t)V(t)$, with

$$E(t) = \begin{cases} c_1 t & , \quad t < t_0 \\ c_2 e^{-c_3(t-t_0)} & , \quad t > t_0 \end{cases} \tag{14}$$

$$S_{VV}(\omega) = \frac{\omega_g^4 + 4\zeta_g^2\omega_g^2\omega^2}{(\omega^2 - \omega_g^2)^2 + 4\zeta^2\omega_g^2\omega^2} S_0 \tag{15}$$

For all cases, $t_0 = 7$ sec, $\zeta_g = 0.3$, $S_0 = 1$. A match in ω_g to the jth frequency of the structure denotes an earthquake exciting the jth mode the most. Two different types of ground motion exciting different modes and with statistically equivalent energy contents named Type A and B are utilized. For Type A, $(c_1, c_2, c_3) = (0.005, 0.035, 0.2)$. For Type B, $(c_1, c_2, c_3) = (0.00292, 0.02044, 0.2)$. The simulation of the stationary Gaussian stochastic processes is performed using the procedure of Shinozuka et al. (1991). A statistical analysis based on Monte Carlo simulations is performed. 30 realizations are generated and the results are tabulated below in Tables 1, 2 and 3. The results are consistent with the mode shapes. Type A excitation would cause the most damage. The coefficient of variation in the Type A excitation is observed to be relatively small compared to Type B. This shows that the reliability models can estimate small and sharper confidence intervals for severe damage compared to light damage.

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Table 1. Mean and coefficient of variation of the local and overall MSDI

Earthquake type	$S_{M,1}$	$S_{M,2}$	$S_{M,3}$	$S_{M,4}$	$S_{M,5}$	$\hat{S}_M$
Type A ($w_g = 6.6$)	0.449, 0.165	0.361, 0.192	0.196, 0.246	0.035, 0.492	0.000, -	0.208, 0.189
Type B ($w_g = 19.2$)	0.167, 0.397	0.090, 0.634	0.056, 0.743	0.024, 0.767	0.000, -	0.068, 0.447

Table 2. Mean and coefficient of variation of modal and overall MSDI

Earthquake type	$\delta_{M,1}$	$\delta_{M,2}$	$\delta_{M,3}$	$\delta_{M,4}$	$\delta_{M,5}$	$\hat{\delta}_M$
Type A ($w_g = 6.6$)	0.354, 0.197	0.223, 0.234	0.240, 0.222	0.206, 0.181	0.083, 0.174	0.301, 0.203
Type B ($w_g = 19.2$)	0.109, 0.439	0.073, 0.389	0.067, 0.476	0.061, 0.543	0.040, 0.498	0.093, 0.434

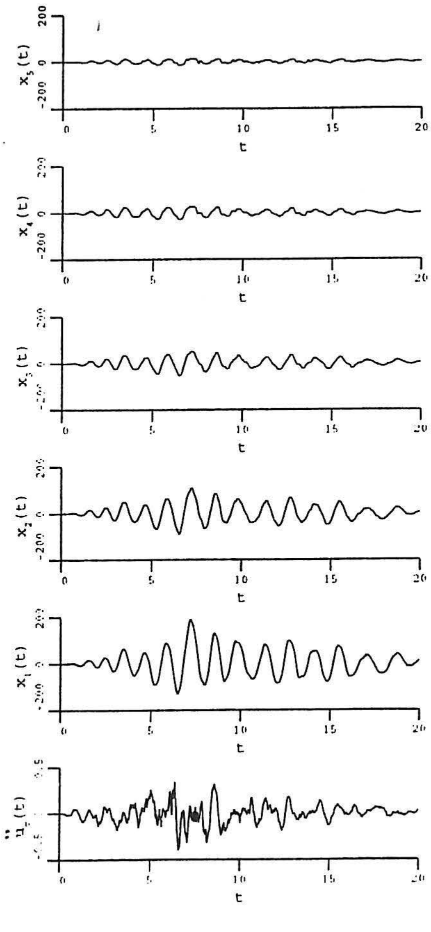

Figure 3. Type A excitation and $x_i(t)$ in mm.

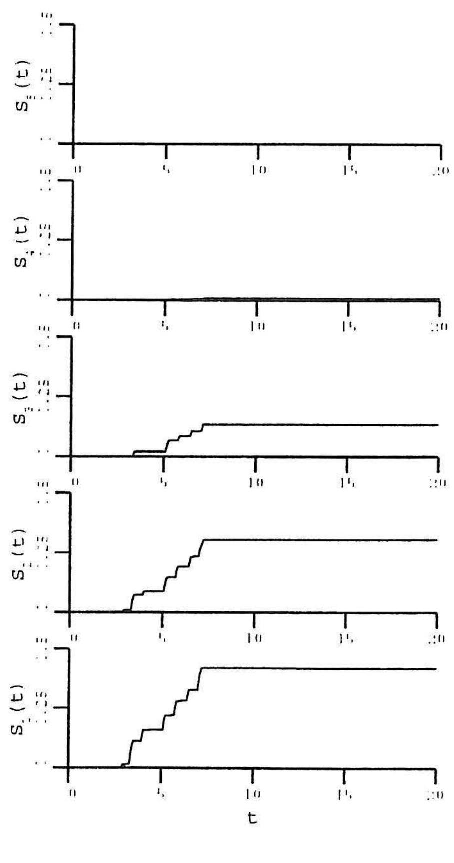

Figure 4. Corresponding $S_i(t)$.

Table 3. Correlation between the local, modal and overall MSDI for Type A excitation

	$s_{M,1}$	$s_{M,2}$	$s_{M,3}$	$s_{M,4}$	$s_{M,5}$	$\hat{s}_M$	$\delta_{M,1}$	$\delta_{M,2}$	$\delta_{M,3}$	$\delta_{M,4}$	$\delta_{M,5}$	$\hat{\delta}_M$
$s_{M,1}$	1.000	0.937	0.764	0.525	-	0.939	0.946	0.920	0.907	0.789	0.579	0.936
$s_{M,2}$		1.000	0.927	0.724	-	0.995	0.916	0.914	0.922	0.877	0.724	0.919
$s_{M,3}$			1.000	0.897	-	0.937	0.768	0.801	0.812	0.855	0.808	0.785
$s_{M,4}$				1.000	-	0.759	0.556	0.616	0.620	0.724	0.831	0.583
$s_{M,5}$					-	-	-	-	-	-	-	-
$\hat{s}_M$						1.000	0.916	0.918	0.919	0.878	0.743	0.919
$\delta_{M,1}$							1.000	0.993	0.990	0.920	0.755	0.999
$\delta_{M,2}$								1.000	0.995	0.947	0.811	0.997
$\delta_{M,3}$									1.000	0.963	0.826	0.995
$\delta_{M,4}$										1.000	0.935	0.936
$\delta_{M,5}$											1.000	0.783
$\hat{\delta}_M$												1.000

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Department of Building Technology and Structural Engineering
Aalborg University, Sohngaardsholmsvej 57, DK 9000 Aalborg
Telephone: +45 98 15 85 22 Telefax: +45 98 14 82 43