Development of Applicabe Methods for Evaluating the Safety of Offsshore Structures

part 2

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PAPER NO. 11

J. D. SØRENSEN, P. THOFT-CHRISTENSEN & G. SIGURDSSON

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1. INTRODUCTION

The research project »Development of applicable methods for evaluating the safety of offshore structures» was initiated in August 1984. It is in 1985 supported by the Danish Ministry of Energy with D. kr. 380.000. The results obtained will be published in a series of papers. The first paper [1] was published in January 1985 and describes the computer program package UNZIP by which the reliability of 3-dimensional structures can be evaluated.

The present paper describes progress made during the first four months of 1985. A new program package called MEKBETA is described. By this program fundamental mechanisms can be identified automatically. Also significant mechanisms in plane and space frame and lattice structures are identified automatically. Finally, the reliability of the structural system is estimated on the basis of the identified significant mechanisms. The automatic generation of fundamental mechanisms is based on the method suggested by Watwood [4].

The computer program MEKBETA has been tested on three simple examples.

2. RELIABILITY ANALYSIS OF DUCTILE ELASTO-PLASTIC FRAME STRUCTURES USING A SET OF FAILURE MECHANISMS

In this paper only frame an lattice structures which are made of ductile elasto-plastic materials are considered. It is assumed that the members of these structures fail by plastic yielding.

Failure of a structural system can be defined as formation of a mechanism or as the event that a critical number of hinges has been formed. In [1] a method is described which can be used to evaluate the reliability of a structural system when failure is defined by one of these two definitions. The method uses a number of elastic analyses. However, this method is not very suitable for the first mentioned definition of failure, if the structure is highly redundant. The reason is that mechanisms are formed after formation of a large number of hinges and therefore, it can be difficult and expensive to identify the most significant failure modes. On the other hand, a method based on a plastic limit analysis is very suitable when failure is defined as formation of a mechanism.

From plastic limit analysis of ductile structures it is well-known that all possible mechanisms can be written as linear combinations of a set of independent mechanisms (called fundamental mechanisms ([2] and [3])). The number of fundamental mechanisms for a given structural system is $m = n - r$, where n is the number of possible locations of yield hinges and r is the degree of redundancy. Of course, all possible sets of fundamental mechanisms are not equally suitable as a basis for identification of significant failure modes.

Each fundamental mechanism fulfils the compatibility conditions, but does not necessary have positive external work.

Watwood [4] has suggested a method to generate automatically a set of fundamental mechanisms (see section 3). Alternatively, in simple structures a set of fundamental mechanisms can be identified by inspection. Among the m fundamental mechanisms it is assumed that $m_e > 0$ mechanisms are real mechanisms, E_M i.e. mechanisms where the external work is different from zero. The remaining $m_f = m - m_e$ mechanisms are called fictitious mechanisms, F_M. Whether a fundamental mechanism is real or fictitious depends on the external loads.

New mechanisms are formed by linear combination of fundamental mechanisms. Since the fundamental mechanisms fulfil the compatibility conditions the new mechanisms will also do so.

The safety margin for the i^{th} mechanism can be written

$$M_i = \sum_{j=1}^{n} a_{ij} R_j - \sum_{j=1}^{n_p} b_{ij} P_j \qquad (1)$$

where $\bar{R} = (R_1, R_2, \ldots, R_n)$ models the yield moment/force capacity and $\bar{P} = (P_1, P_2, \ldots P_{n_p})$ models the loads. $\bar{R}$ and $\bar{P}$ are assumed to be modelled as normally distributed stochastic variables. Further, it is assumed that the loads can be modelled as concentrated static forces/moments which are placed in the nodes of the structural model, $\bar{\bar{a}}$ and $\bar{\bar{b}}$ are matrices which contain coefficients of influence (for a fictitious mechanism $b_{ij} = 0, j = 1, 2, \ldots, n_p$). n_p is the number of not fully correlated load variables. $\bar{R}$ and $\bar{P}$ are assumed to be independent with expected values $\bar{\mu}_R \geqslant 0$ and $\bar{\mu}_P \geqslant 0$, and covariance matrices $\bar{\bar{C}}_R$ and $\bar{\bar{C}}_p$.

The reliability index β; for the i^{th} safety margin is

$$\beta_i = \frac{\sum_{j=1}^{n} a_{ij}\mu_{Rj} - \sum_{j=1}^{n_p} b_{ij}\mu_{Pj}}{\sum_{j=1}^{n}\sum_{k=1}^{n} a_{ij}a_{ik}C_{Rjk} + \sum_{j=1}^{n_p}\sum_{k=1}^{n_p} b_{ij}b_{ik}C_{Pjk}} \qquad (2)$$

Assume that the real fundamental mechanisms are numbered so that $\beta_1 \leqslant \beta_2 \leqslant \ldots \leqslant \beta_{m_e}$.

The chosen set of fundamental mechanisms can be described by the following equations:

For real mechanisms:

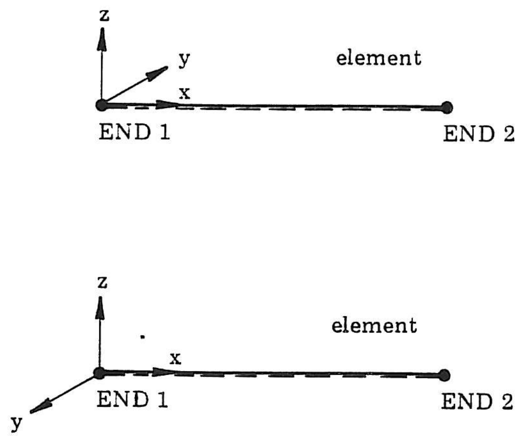
$$Z_i = \sum_{j=1}^{n} A_{ij} R_j - \sum_{j=1}^{n_p} B_{ij} P_j \qquad , i = 1, \ldots, m_e \qquad (3)$$

for fictitious mechanisms:

$$Z_i = \sum_{j=1}^{n} A_{ij} R_j \qquad , i = m_e + 1, \ldots, m \qquad (4)$$

where $a_{ij} = | A_{ij} |$ and $b_{ij} = B_{ij}$.

The sign of B_{ij} depends on the choice of global coordinate system. If a deformation has the same direction as the force/moment then $B_{ij}P_j > 0$.



END 1: local node 1 for the element

END 2: local node 2 for the element

— — — : lower side of element for bending about local y-axis

— · — : lower side of element for bending about local z-axis

Figure 1. Definition of lower sides of elements.

The sign of A_{ij} will be positive for compression when failure is axial and positive for rotation which gives tension in the lower side of the element when failure is in bending. In figure 1 the definition of lower sides in proportion to the local coordinate system is shown.

As an example consider the plane beam mechanism in figure 2. Here we get

$$Z = -R_1 + 2R_2 - R_3 + PL$$

i. e. $\bar{A} = (-1, 2, -1)$ and $\bar{B} = (-L)$.

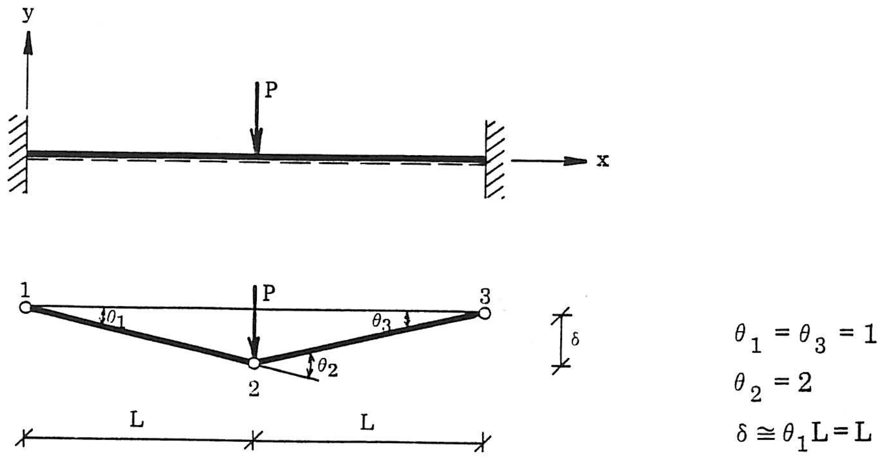


Figure 2. Plane beam mechanism.

As mentioned above new mechanisms are made by linear combination of the fundamental mechanisms. Combination of mechanisms i and j gives

$$Z = c\,Z_i + d\,Z_j$$

$$= \sum_{k=1}^{n} (c\,A_{ik} + d\,A_{jk})R_k - \sum_{k=1}^{n_p} (c\,B_{ik} + d\,B_{jk})P_k \tag{6}$$

where c and d are real numbers.

The corresponding safety margin is

$$M = \sum_{k=1}^{n} |c\,A_{ik} + d\,A_{jk}|R_k - \sum_{k=1}^{n_p} (c\,B_{ik} + d\,B_{jk})P_k \tag{7}$$

The reliability index β corresponding to M is determined from (2).

It is very expensive to identify all combinations of fundamental mechanisms and to calculate the failure probability of a series system with elements described by the safety margins of all posible mechanisms. Thus, it is very important to be able to identify the most significant mechanisms without first determining all mechanisms. The most significant mechanisms are the mechanisms that contribute the most to the failure probability of the structural system.

The method which is used in this paper for identification of the significant failure modes uses a failure tree formulation as the basis of the selection of mechanisms.

The real mechanisms which have β-indices in the interval $[\beta^F_{min}, \beta^F_{min} + \epsilon 1]$ are used as starting points for the failure tree. β^F_{min} is the β-index for the most significant of the real fundamental mechanisms and $\epsilon 1 \geq 0$ is a chosen constant. These selected mechanisms are in turn combined with the other fundamental mechanisms. Only mechanisms which have one or more yield hinges in common are combined. β-indices are calculated for the combined mechanisms.

The lowest β-index, β_{min}, among the combined mechanisms is determined, and the combined mechanisms with β-index in the interval $[\beta_{min}, \beta_{min} + \epsilon 2]$, where $\epsilon 2 \geq 0$ is a chosen constant, define the branches in the failure tree from the fundamental mechanisms which are used as starting points. These combined mechanisms now form the basis of new branches. Each branch symbolizes that the mechanism belonging to the node at the end of the branch is obtained by combination of a combined mechanism and a fundamental mechanism. It is expected that the reliability indices for the most significant mechanisms decrease as more and more fundamental mechanisms are combined. The branching is stopped, either because the number of combined fundamental mechanisms which form a combined mechanism exceed a critical number N_{max}, or because the lowest β-index calculated in a ramification is greater than twice the β-index β_u for the mechanism which constitute the basis, i.e. $\beta \geq 2\beta_u$ or the time of computation exceeds the maximum time SECM assumed. With these stopping criteria a failure tree is determined. Each node symbolizes a mechanism.

The parameters $\epsilon 1$ and $\epsilon 2$ can be chosen as constants or as variables which depend partly on β_{min} and partly on the number of fundamental mechanisms used to obtain the actual mechanism. In the program package »MEKBETA» $\epsilon 1$ and $\epsilon 2$ are read in as constants.

Combination of a mechanism with the fundamental mechanisms can be made by the following procedure. Let Z_i in (6) be the mechanism considered and let Z_j, $j = 1, 2, \ldots, m$, be the fundamental mechanisms. $c = 1$ is chosen and $d = -A_{ik}/A_{jk}$ for each value $A_{jk} \neq 0$. $d = 0$ when $A_{jk} = 0$, i.e. d has a value different from zero when

$$A_{ik} + d\,A_{jk} = 0 \;\wedge\; A_{ik} \neq 0 \;\wedge\; A_{jk} \neq 0 \qquad (8)$$

One combined mechanism is obtained for each k where (8) is fulfilled. A combined mechanism is rejected if it already constitutes a node in the failure tree or if it has a β-index beyond a prescribed interval or if it is a fictitious mechanism.

In figure 3 an example of part of a failure tree is shown. The failure tree is determined using the program »MEKBETA».

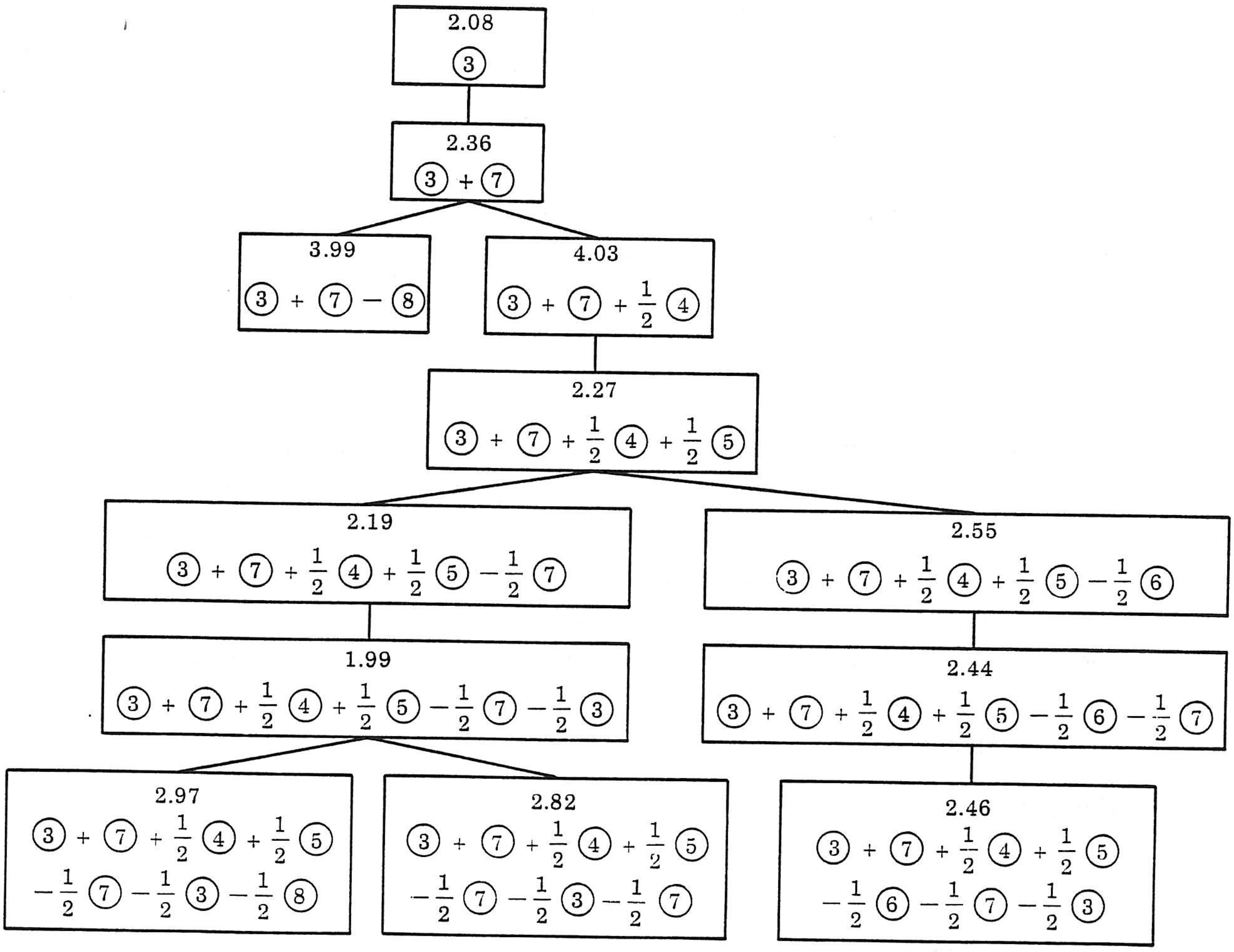


Figure 3. Failure tree [4.03 / (3) + (7) + $\frac{1}{2}$ (4)] symbolizes the mechanism made by combination of one times the fundamental mechanisms 3 and 7 and 1/2 times the fundamental mechanism no. 4. The combined mechanism has a β = 4.03. The failure tree is calculated in example 1, see section 5.

From figure 3 it appears that the β-index by combination of fundamental mechansims can increase somewhat and then decrease by additional combinations. This is the reason why $\epsilon 2$ is chosen > 0 (here 0.6).

During the formation of a failure tree some of the identified mechanisms are not significant to the failure probability (mechanisms with high β-indices). Evaluation of the reliability of the structure is made by modelling the significant mechanisms as elements in a series system. Only mechanisms with β-index in the interval $[\beta_{min}, \beta_{min} + \epsilon 3]$ are included. $\epsilon 3 \geqslant 0$ and β_{min} is the lowest β-index in the failure tree.

To evaluate the failure probability of a series system several methods are available. In the program »MEKBETA» the following methods are available:

- Ditlevsen bounds
- PNET approximation
- approximation based on the average correlation coefficient
- approximation based on the equivalent correlation coefficient
- bounds based on min. and max. correlation coefficient
- Hohenbichler's approximation

These methods are described in details in [5], section 3.2.1.

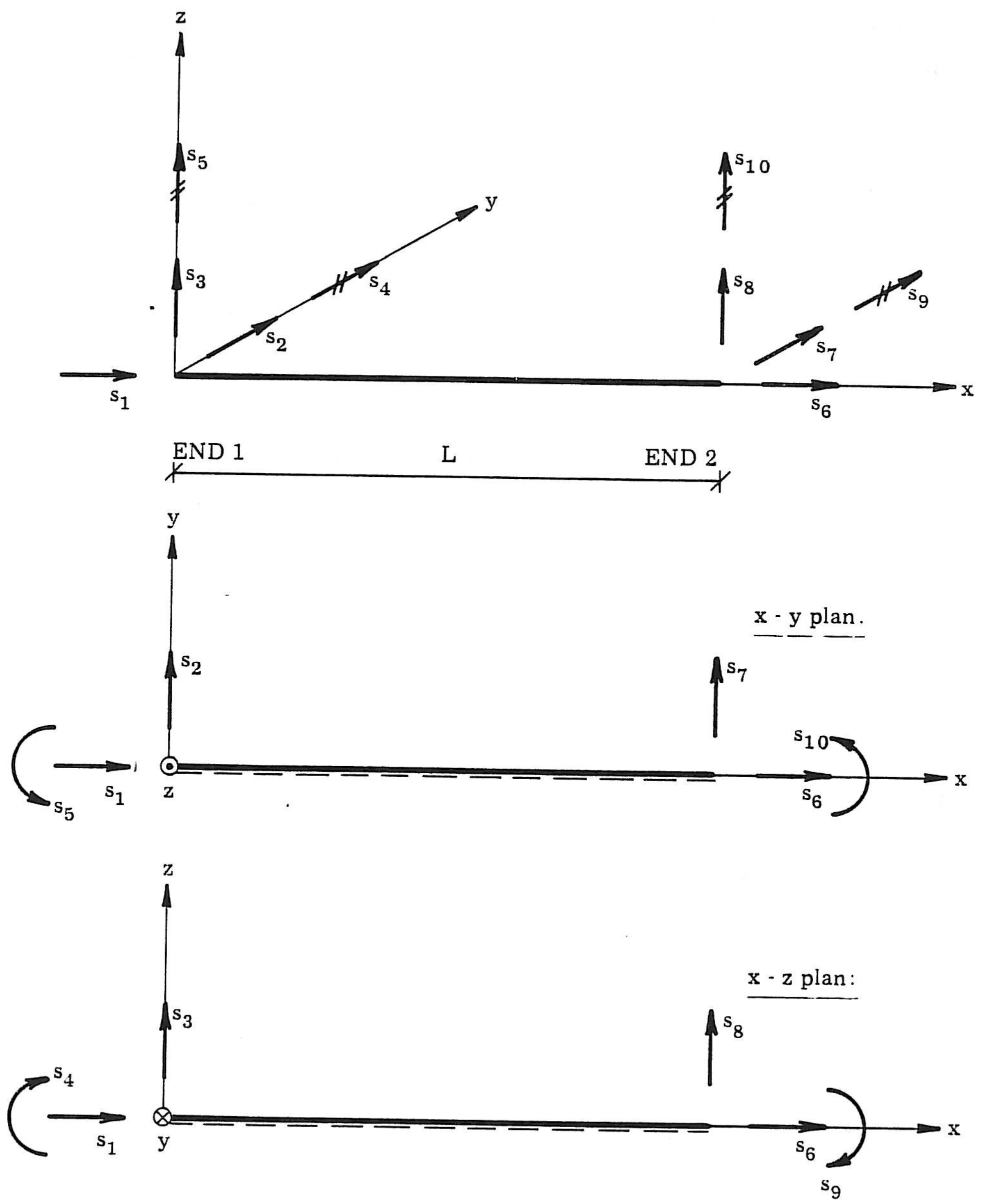
3. AUTOMATIC GENERATION OF FUNDAMENTAL MECHANISMS

As mentioned in section 2 all mechanisms can be written as linear combinations of the fundamental mechanisms. Therefore it is very important to be able to determine a set of fundamental mechanisms in an efficient way. Watwood [4] has described a method for automatic generation of the fundamental mechanisms. This method is used in the program package »MEKBETA» and will be described below.

The geometry of the structure basically determines the fundamental mechanisms, but in addition it is necessary to make judgements regarding the locations of potential plastic hinges or axial collapse (elongation) mechanisms. It is well known that the problem of hinge location can be handled if the loading can be approximated in the form of concentrated loads and moments, since joints can be placed at all such load points as well as member intersections. The net result is that the structure is modelled as an assembly of beam elements; hinges will form only at the ends of these elements and in addition each element may elongate or shorten (axial collapse).

Unless a failure hinge is formed or axial collapse occurs within a member, the member must move as a rigid body within the mechanism.

Therefore, it is necessary to formulate equations that can enforce this requirement. For this purpose a coordinate transformation is introduced to the member-genralized coordinates that separates the rigid body motion from the member deformation.



- - - - : lower side of element

x, y, z : local coordinate system

Figure 4. Definition of generalized Coordinates for space beam element.

Consider the space beam element shown in figure 4. The rigid body modes (in terms of the generalized coordinates defined in figure 4) can be chosen as follows if positive rotation is defined so that it gives tension in the lower side.

s1	s2	s3	s4	s5	s6	s7	s8	s9	s10	
[1	0	0	0	0	1	0	0	0	0]	~ translation in x-direction
[0	1	0	0	0	0	1	0	0	0]	~ translation in y-direction
[0	0	1	0	0	0	0	1	0	0]	~ translation in z-direction
[0	0	0	−1	0	0	0	L	−1	0]	~ rotation about y-axis at end 1
[0	0	0	0	1	0	−L	0	0	1]	~ rotation about z-axis at end 1

The magnitude of each such motion will be the new generalized coordinate. Note that any rigid body notion can be represented by some combination of these coordinates. To complete the transformation, five additional independent coordinates are required. These are chosen as:

$$
\begin{array}{llllllllll l}
[0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0] & \sim \text{ end two displacing axially} \\
[0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0] & \sim \text{ end one rotating pos. about z-axi} \\
[0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1] & \sim \text{ end two rotating pos. about z-axi} \\
[0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0] & \sim \text{ end one rotating pos. about y-axi} \\
[0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0] & \sim \text{ end two rotating pos. about y-axi}
\end{array}
$$

Each of these latter five coordinates clearly involves deformations of the beams. The full transformation is thus assembled as:

$$\bar{s} = \bar{\bar{T}}\,\bar{s}' \tag{9}$$

in which the ten components of the column matrix s are defined in figure 4, and:

$$
\bar{s}' = \begin{bmatrix} s'_1 \\ s'_2 \\ s'_3 \\ s'_4 \\ s'_5 \\ \hline s'_6 \\ s'_7 \\ s'_8 \\ s'_9 \\ s'_{10} \end{bmatrix}
\begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \\ \\ \end{array}\right\} \leftarrow \text{deformations coordinates} \\ \left.\begin{array}{l} \\ \\ \\ \\ \\ \end{array}\right\} \leftarrow \text{rigid body coordinates} \end{array}
\tag{10}
$$

$$
\bar{\bar{T}} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -L \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & L & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\tag{11}
$$

Note that each column of the matrix $\bar{\bar{T}}$ is one of the »modes» previously defined.

Here the inverse relation is needed and is as follows:

$$\bar{s}' = \bar{\bar{T}}^{-1} s$$

$$\text{in which } \bar{\bar{T}}^{-1} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1/L & 0 & 0 & 1 & 0 & 1/L & 0 & 0 & 0 \\ 0 & 1/L & 0 & 0 & 0 & 0 & -1/L & 0 & 0 & -1 \\ 0 & 0 & -1/L & -1 & 0 & 0 & 0 & 1/L & 0 & 0 \\ 0 & 0 & 1/L & 0 & 0 & 0 & 0 & -1/L & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1/L & 0 & 0 & 0 & 0 & 1/L & 0 & 0 \\ 0 & 1/L & 0 & 0 & 0 & 0 & -1/L & 0 & 0 & 0 \end{bmatrix} \qquad (12)$$

Examination of the first five rows of $\bar{\bar{T}}^{-1}$ reveals some insight in the interpretation of the deformation coordinates $s_1', s_2', s_3', s_4', s_5'$. For example the first row gives:

$$s_1' = -s_1 + s_6 \qquad (13)$$

which is clearly the relative »stretching» of the element axially. The second row gives:

$$s_2' = s_5 + \frac{1}{L}(s_7 - s_2) \qquad (14)$$

which can be interpreted as the rotation of end one about the z-axis.

The 3rd, 4th, and 5th rows give:

$$s_3' = -s_{10} + \frac{1}{L}(s_2 - s_7)$$

$$s_4' = -s_4 + \frac{1}{L}(s_8 - s_3) \qquad (15)$$

$$s_5' = s_9 + \frac{1}{L}(s_3 - s_8)$$

For any mechanism, all members making up the structure move as rigid bodies. Deformation of each member must be prevented by enforcing the deformation coordinates $s_1', s_2', s_3', s_4', s_5'$ for each member to remain zero during the mechanism.

To enforce these constraints formally, we first introduce a constraint matrix $\bar{\bar{C}}^j$ for the j^{th} element which is made up of the first five rows of $\bar{\bar{T}}^{-1}$, i.e.:

$$\bar{\bar{C}}^j = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1/L & 0 & 0 & 1 & 0 & 1/L & 0 & 0 & 0 \\ 0 & 1/L & 0 & 0 & 0 & 0 & -1/L & 0 & 0 & -1 \\ 0 & 0 & -1/L & -1 & 0 & 0 & 0 & 1/L & 0 & 0 \\ 0 & 0 & 1/L & 0 & 0 & 0 & 0 & -1/L & 1 & 0 \end{bmatrix} \qquad (16)$$

The constraint matrix $\bar{\bar{C}}$ for the entire structure is constructed by »assembling» the $\bar{\bar{C}}^j$ for the members as follows:

$$\bar{\bar{C}} = \begin{bmatrix} \bar{\bar{C}}^1 & 0 & . & . & . & . & 0 \\ 0 & \bar{\bar{C}}^2 & 0 & . & . & . & 0 \\ 0 & 0 & \bar{\bar{C}}^3 & 0 & . & . & 0 \\ . & . & . & . & & & . \\ . & . & . & & . & & . \\ . & . & . & & & . & . \\ 0 & 0 & 0 & . & . & . & \bar{\bar{C}}^m \end{bmatrix} \tag{17}$$

in which m is the number of elements. This provides the relation:

$$\bar{\bar{C}}\,\bar{s} = s'_d \tag{18}$$

in which

$$\bar{s}'_d = \begin{bmatrix} s'_1 \\ . \\ . \\ . \\ s'_5 \\ \hline s'_1 \\ . \\ . \\ . \\ s'_5 \\ \hline . \\ . \\ . \\ . \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \\ \\ \end{matrix}\right\} \text{1. element} \\ \left.\begin{matrix} \\ \\ \\ \\ \\ \end{matrix}\right\} \text{2. element} \\ \\ \\ \\ \\ \end{matrix}$$

and $\bar{s}$ is redefined as:

$$\bar{s} = \begin{bmatrix} \bar{s} \\ \hline \bar{s} \\ \hline . \\ . \\ . \\ \hline \bar{s} \end{bmatrix} \begin{matrix} \sim \text{1. element} \\ \sim \text{2. element} \\ . \\ . \\ . \\ \sim \text{m. element} \end{matrix} \tag{19}$$

However, the components of $\bar{s}$ are not independent but restrained so that they move in such a way that the compatibility of the assembled structure is preserved. This can be enforced by introduction of the usual structural compatibility matrix:

$$\bar{s}^* = \bar{\bar{a}}\,\bar{r} \qquad (20)$$

where the components of $\bar{s}^*$ are the generalized coordinates of all elements expressed in the global coordinate system, $\bar{r}$ is the column matrix of the external degrees of freedom (in general three per joint in plane structures, and six per joint in spatial structures, unless constraints are imposed) expressed in the global coordinate system and finally $\bar{\bar{a}}$ is the compatibility matrix.

It now remains to introduce the coordinate transformations to link the element coordinates in the local systems, $\bar{s}$, to the element coordinates expressed in the global system, $\bar{s}^*$. This is also well known and can be expressed as:

$$\bar{s} = \bar{\bar{Q}}\,\bar{s}^* \qquad (21)$$

in which $\bar{\bar{Q}}$ is the transformation matrix.

Combining eqs. (18), (20) and (21) leads to:

$$\bar{\bar{C}}_1\,\bar{r} = s'_d \qquad (22)$$

in which

$$\bar{\bar{C}}_1 = \bar{\bar{C}}\,\bar{\bar{Q}}\,\bar{\bar{a}} \qquad (23)$$

To find a mechanism, one must find a solution to eq. (22) so that $\bar{s}'_d$ is zero:

$$\bar{\bar{C}}_1\,\bar{r} = 0 \qquad (24)$$

However, unless the structure is already a mechanism no such solution exists. In fact, $\bar{\bar{C}}_1$ is of dimension M by N in which M is three times the number of elements, for a plane frame, and five times the number of elements for a spatial frame (unless constraints are imposed), and N is the number of external degrees - of - freedom.

This is to be expected, since the assembled structure was not intended to be a mechanism without the formation of some plastic hinges or axial collapse.

At this point releases are introduced which will provide the possibility of mechanism formation. To keep the method as general as possible five releases per element will be inserted. The five releases inserted are two hinges at each end of each element (about the two local axes) plus detaching (axial only) the end one from its joint. This is shown for a simple space frame beam in figure 5.

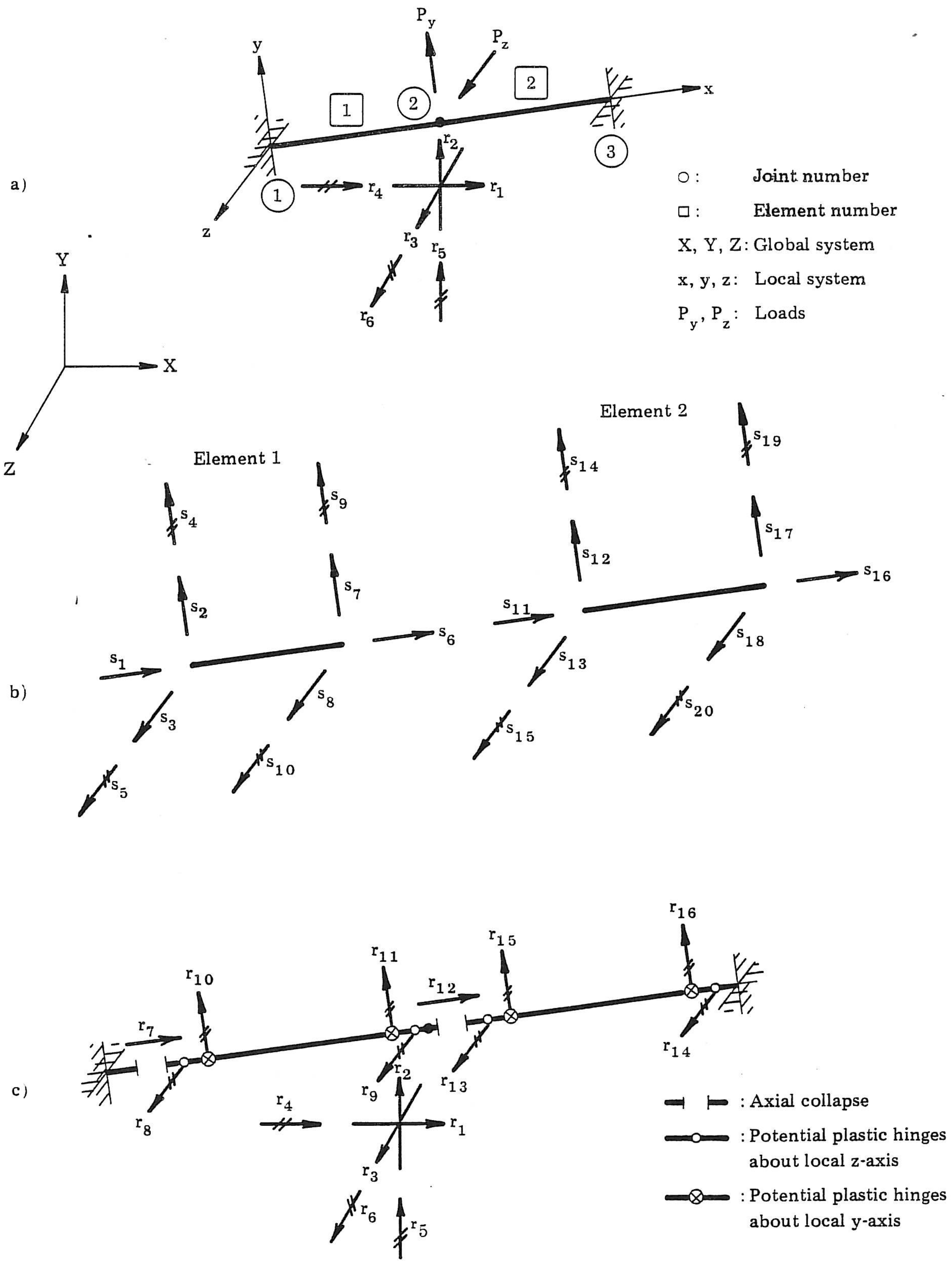


Figure 5. Simple spatial frame beam example.

Insertion of a release is equivalent to adding an external degree - of - freedom. By combining (20) and (21) we get

$$\bar{s} = \bar{\bar{Q}}\,\bar{\bar{a}}\,\bar{r} \tag{25}$$

The releases must be made with respect to the local coordinates s. In accordance with the definition of the coordinates shown in figure 5b and figure 5c. The rows of the matrix $\bar{\bar{Q}}\,\bar{\bar{a}}$ are replaced by zeros where they correspond to s_1, s_4, s_5, s_9, s_{10}, s_{11}, s_{14}, s_{15}, s_{19}, and s_{20}. This is equivalent to releasing the corresponding degree - of - freedom. Also, a column is added to $\bar{\bar{Q}}\bar{\bar{a}}$ in which the number one is placed in the row that was »zeroed out» earlier. Of course, $\bar{r}$ must be expanded by one component for each column added to $\bar{\bar{Q}}\,\bar{\bar{a}}$. This modified form of (25) is then inserted into (18) resulting in

$$C_2\, r_t = s'_d \tag{26}$$

in which $\bar{\bar{C}}_2$ is $\bar{\bar{C}}$ multiplied by the modified form of $\bar{\bar{Q}}\bar{\bar{a}}$, and $\bar{r}_t$ is the matrix of external degrees - of - freedom, $\bar{r}$, augmented by the member releases.

Now one may insist that s'_d is zero and will have the possibility of sulution, i.e., one seeks r_t with all components that are not zero so that:

$$C_2\, r_t = 0 \tag{27}$$

The dimension of C_2 are M by N_t in which M is the same as before and N_t is the number of external degrees - of - freedom, N, plus the number of possible potential plastic hinges and axial collapse. The value N_t will exceed M, the difference being the number of fundamental mechanisms.

4. DESCRIPTION OF THE COMPUTER PROGRAM PACKAGE »MEKBETA»

The program package »MEKBETA» can be used for automatic identification of fundamental mechanisms and for automatic identification of significant mechanisms in plane and spatial frames and lattice structures. Based on the significant mechanisms also a measure of the reliability of structure is determined.

The program is written in FORTRAN and contains 26 subroutines which can be partitioned into 3 main groups:

1) Program for identification of a set of fundamental mechanisms, i. e. formation and solution of (27).

2) Program for identification of the significant mechanisms (see section 2).

3) Program for estimating the reliability of the structure (see section 2).

In the construction of the program special consideration is given to minimize the consumption of computer memory and to minimize the time of computation.

The upper limit of the sizes of the structures which can be analysed is only dependent on the limits of the computer. The program is made so that the reservation of memory for all variables (vectors and matrices) is performed after the size of the problem (i.e. number of nodal points, number of elements, number of possible potential plastic hinges and axial collapse, type of problem (2 or 3 dimension) etc.) has been read in. This implies that only the amount of memory needed for the different variables is reserved. All information is stored in blank common areas. By changing the size of the common area (changing one number in the program text) the capacity of the program can easily be changed. In the output from the program it is shown how much room the run in question uses of the blank common areas. If the common area is too small the run is discontinued and in the output it is indicated how large the common area should be. It is also indicated in what line the change should be performed.

The most important assumptions are:

- the loads are assumed constant with time (static loads)
- the loads and the capacities of the cross-sections in the elements of the structure are modelled as normally distributed stochastic variables
- the yield function for combined axial force and bending moment is assumed to be cubic in normalized coordinates
- the simple classical »plastic method» (Hodge [2] and Neal [3]) can be used to analyse the structures against collapse.

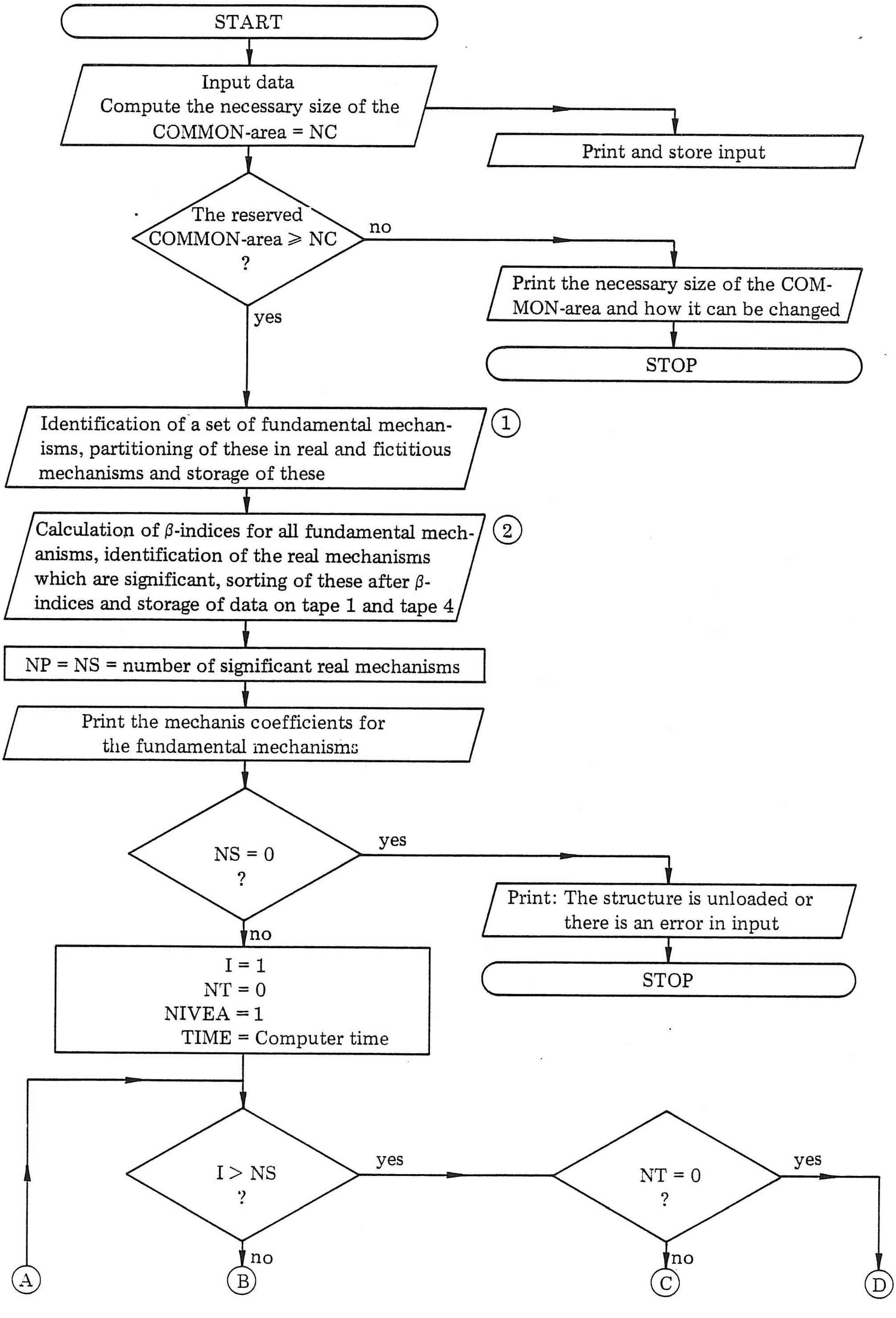
As input »MEKBETA» needs:

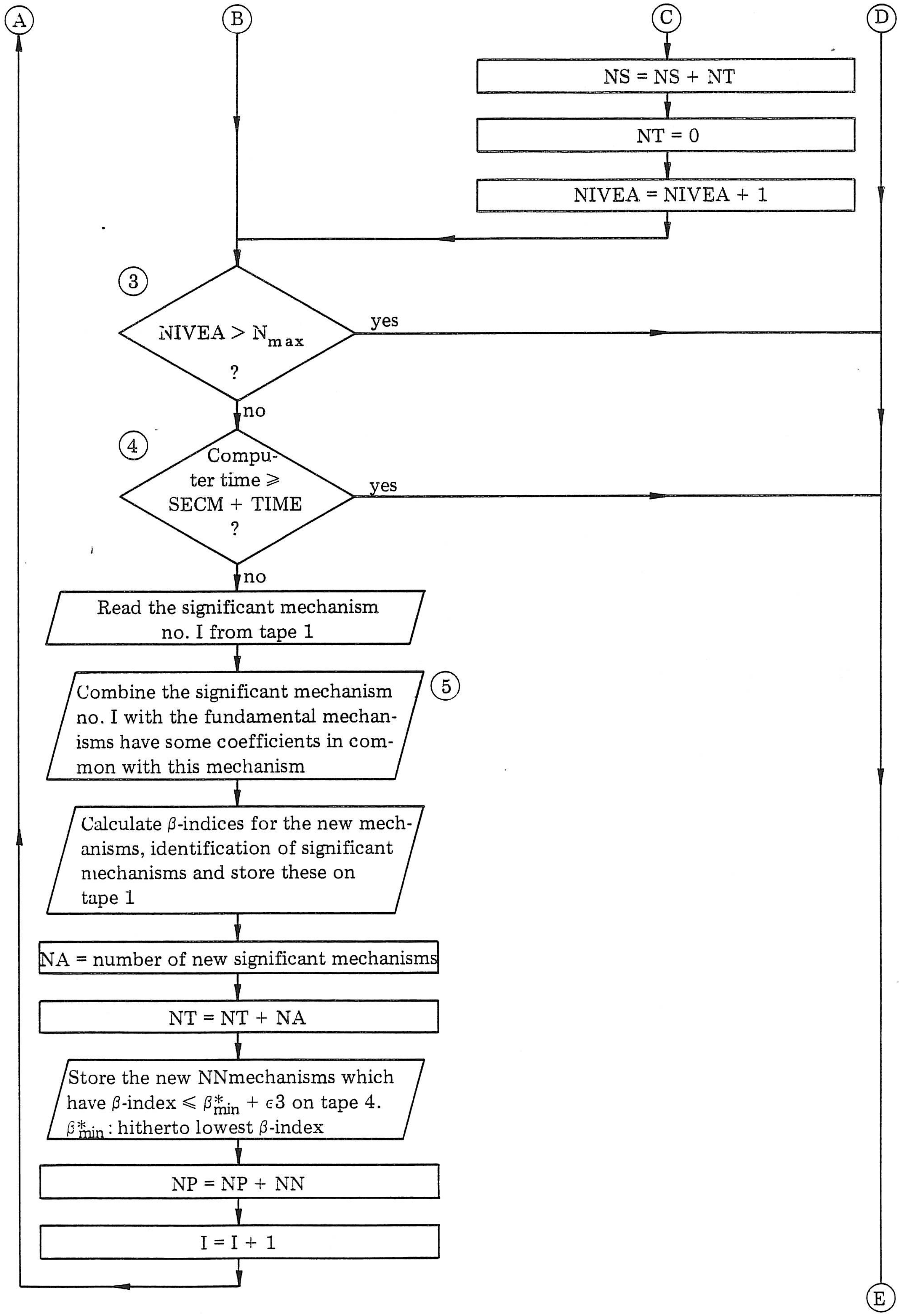
- the geometry of the structure
- placing of potential yield hinges and axial collapses
- expected values and covariances of stochastic variables (yield moments/yield forces and loads)
- ratio between tensile and compressive yield forces
- $\epsilon 1$, $\epsilon 2$, $\epsilon 3$, and N_{max}
- impossible combinations of yield hinges
- choice of methods to determine the reliability of the structure
- maximum time of computation for identification of significant mechanisms, SECM.

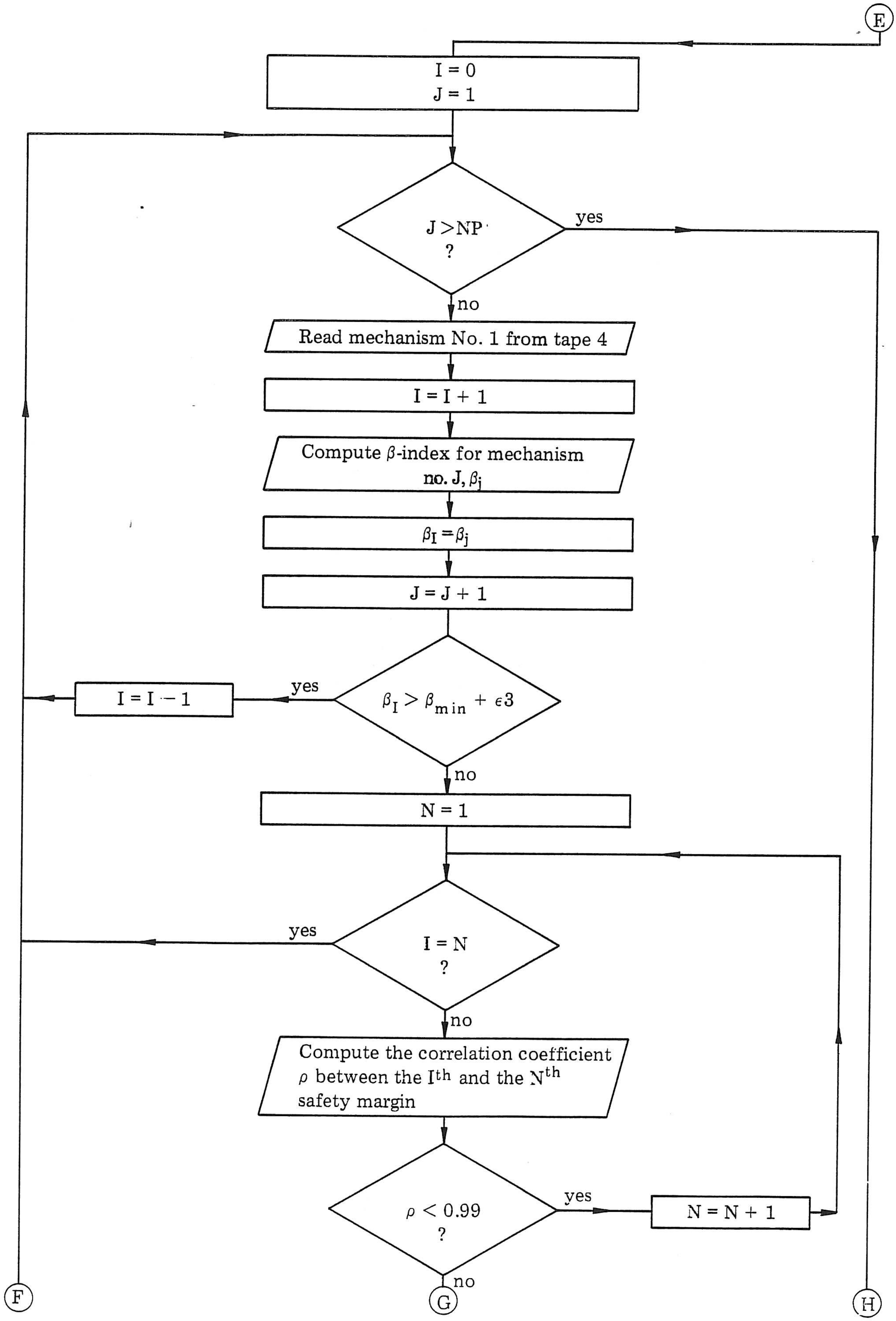
The output from the program is

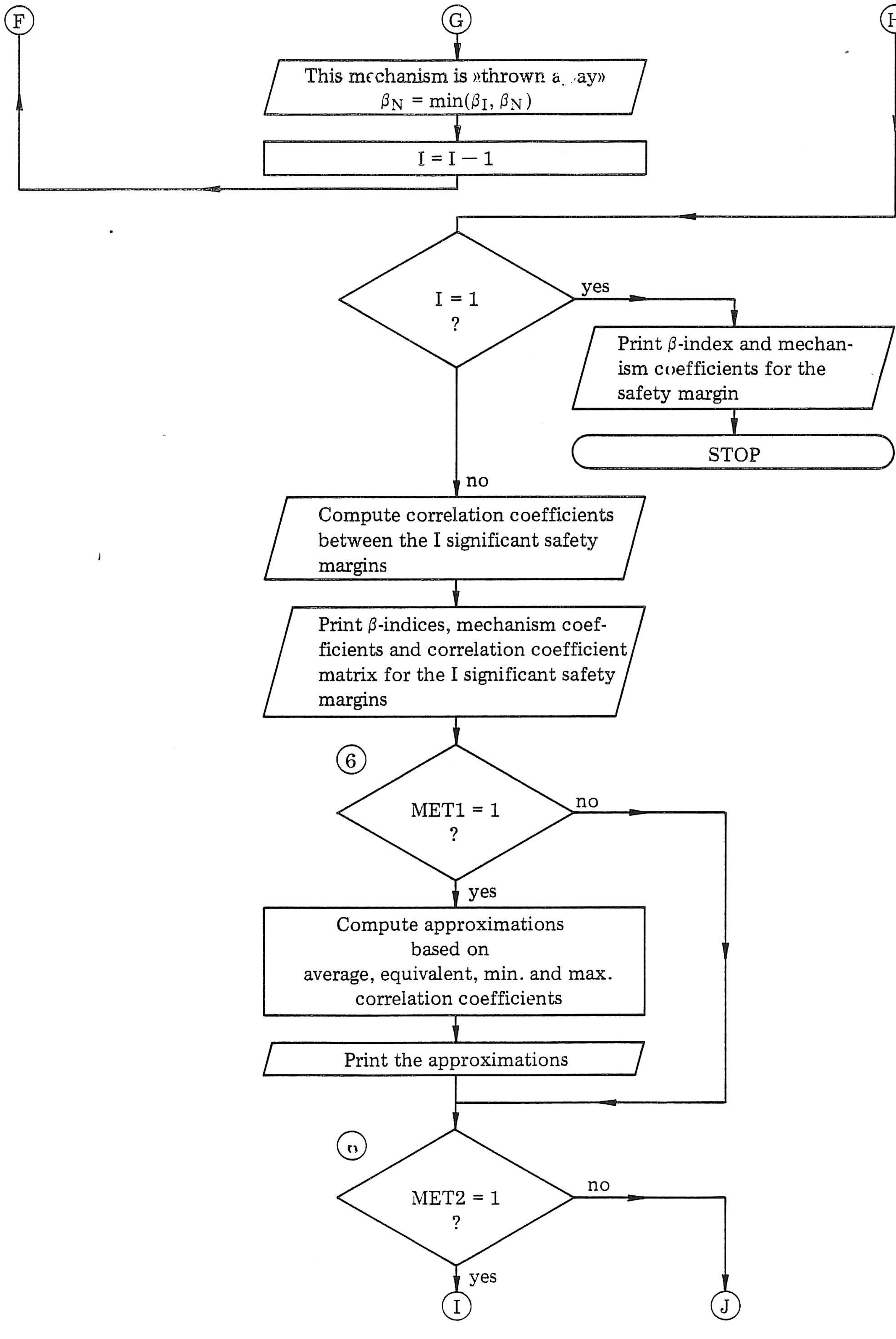
- printout of inputdata
- printout of fundamental mechanisms characterized by the coefficients A_{ij}, B_{ij}, and the β-indices of the mechanisms
- printout of the identified significant mechanisms characterized by the coefficients A_{ij}, B_{ij}, and the β-indices of the mechanisms
- the reliability of the structure (a generalized β-index) estimated by the chosen methods
- total time of computation, reserved and used computer memory in the run.

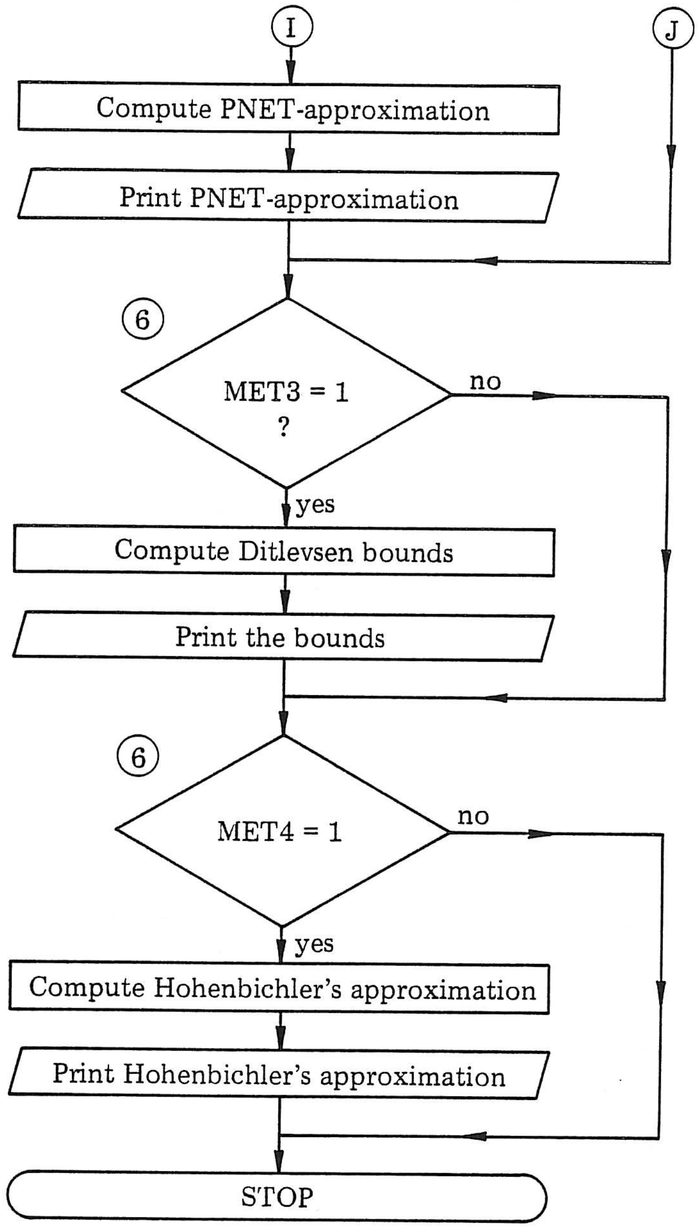
In the following a flow chart for »MEKBETA» is shown. The comments (marked by O in the flow chart) are shown on page 21.


START
Input data
Compute the necessary size of the COMMON-area = NC
Print and store input
The reserved COMMON-area ⩾ NC ?
no
Print the necessary size of the COMMON-area and how it can be changed
STOP
yes
Identification of a set of fundamental mechanisms, partitioning of these in real and fictitious mechanisms and storage of these
1
Calculation of β-indices for all fundamental mechanisms, identification of the real mechanisms which are significant, sorting of these after β-indices and storage of data on tape 1 and tape 4
2
NP = NS = number of significant real mechanisms
Print the mechanis coefficients for the fundamental mechanisms
NS = 0 ?
yes
Print: The structure is unloaded or there is an error in input
STOP
no
I = 1
NT = 0
NIVEA = 1
TIME = Computer time
I > NS ?
yes
NT = 0 ?
yes
no
no
A
B
C
D


A
B
C
D
NS = NS + NT
NT = 0
NIVEA = NIVEA + 1
3
NIVEA > N_{max}
?
yes
no
4
Computer time ≥ SECM + TIME
?
yes
no
Read the significant mechanism no. I from tape 1
5
Combine the significant mechanism no. I with the fundamental mechanisms have some coefficients in common with this mechanism
Calculate β-indices for the new mechanisms, identification of significant mechanisms and store these on tape 1
NA = number of new significant mechanisms
NT = NT + NA
Store the new NNmechanisms which have β-index ≤ β^*_{min} + ϵ3 on tape 4.
β^*_{min} : hitherto lowest β-index
NP = NP + NN
I = I + 1
E


E
I = 0
J = 1
J > NP
?
yes
no
Read mechanism No. 1 from tape 4
I = I + 1
Compute β-index for mechanism
no. J, βj
βI = βj
J = J + 1
yes
βI > βmin + ε3
I = I − 1
no
N = 1
yes
I = N
?
no
Compute the correlation coefficient
ρ between the Ith and the Nth
safety margin
ρ < 0.99
?
yes
N = N + 1
no
F
G
H


F
G
H
This mechanism is »thrown away»
$\beta_N = \min(\beta_I, \beta_N)$
I = I − 1
I = 1
?
yes
Print β-index and mechanism coefficients for the safety margin
STOP
no
Compute correlation coefficients between the I significant safety margins
Print β-indices, mechanism coefficients and correlation coefficient matrix for the I significant safety margins
6
MET1 = 1
?
no
yes
Compute approximations based on average, equivalent, min. and max. correlation coefficients
Print the approximations
MET2 = 1
?
no
yes
I
J


I
J
Compute PNET-approximation
Print PNET-approximation
6
MET3 = 1
?
no
yes
Compute Ditlevsen bounds
Print the bounds
6
MET4 = 1
no
yes
Compute Hohenbichler's approximation
Print Hohenbichler's approximation
STOP

The numbers in the flow chart on pages 16 - 20

(1) A real mechanism: a mechanism which has external work different from zero.

(2) A significant mechanism: a real fundamental mechanism which has β-index $\leqslant \beta^F_{min} + \epsilon 1$ where β^F_{min} is the β-index for the fundamental mechanism with the lowest β-index.

(3) N_{max}: max. number of combined fundamental mechanisms.

(4) SECM: max. computer time which can be used for identification of significant mechanisms.

(5) combining of mechanisms is shown in section 2.

(6) MET1, MET2, MET3, and MET4 are parameters managing which methods are used to estimate the reliability of the structure.

5. EXAMPLES

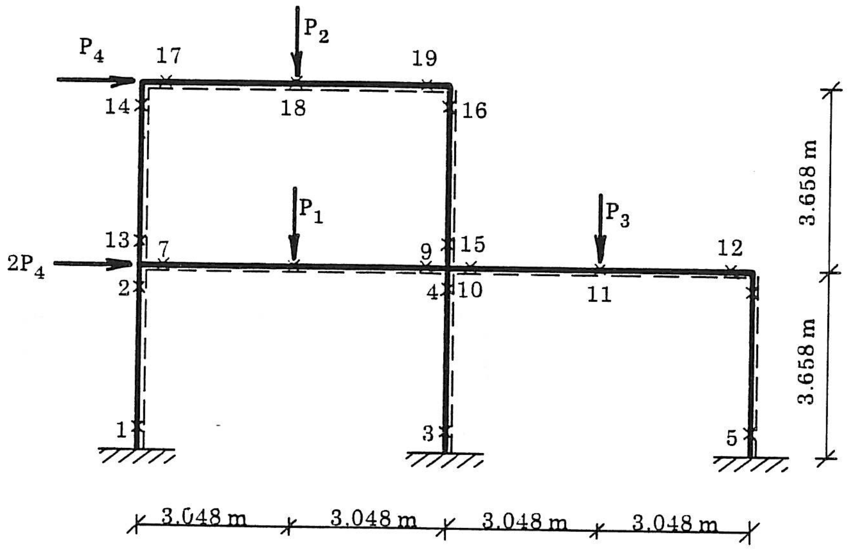
In order to test the computer program »MEKBETA», and to evaluate the method to estimate the reliability three examples are chosen. The problems chosen have all been analysed before. The two first examples have been analysed by P. Thoft-Christensen & J. D. Sørensen [6], but by another method. The last example has been analysed by P. Bjerager [7] and by G. Sigurdsson, J. D. Sørensen and P. Thoft-Christensen [1].

5.1. Example 1

The structure shown in figure 6 is considered in this example. The loading and possible locations of potential yield hinges are also shown in figure 6. Yield capacities of yield hinges in the same line are considered fully correlated and yield capacities of yield hinges in different lines are mutually independent. The number of potential yield hinges is $n = 19$ and the degree of reduncance is $r = 9$. Therefore, the number of fundamental mechanisms is $m = n - r = 19 - 9 = 10$.

The expected values $E[\cdot]$ and the coefficients of variation ($V[\cdot]$) for the stochastic variables, P_i (loads) and R_i (yield-moment capacities) are shown in table 1.

In figure 7 one set of fundamental mechanisms is shown (identified by the program »MEKBETA»). The reliability indices β_i and the influence coefficients, A_{ij} and B_{ij} for the fundamental mechanisms are shown in table 2.



– – – –: lower side of elements

Figure 6. Geometry, loading and potential yield hinges (x).

variables	expected values	coefficients of variation
P_1	169 kN	0.15
P_2	89 kN	0.25
P_3	116 kN	0.25
P_4	31 kN	0.25
$R_1, R_2, R_3, R_4, R_5, R_6$	95 kNm	0.15
R_7, R_8, R_9	204 kNm	0.15
R_{10}, R_{11}, R_{12}	163 kNm	0.15
$R_{13}, R_{14}, R_{15}, R_{16}$	95 kNm	0.15
R_{17}, R_{18}, R_{19}	122 kNm	0.15

Table 1. Expected values and coefficients of variation for the stochastic variables.

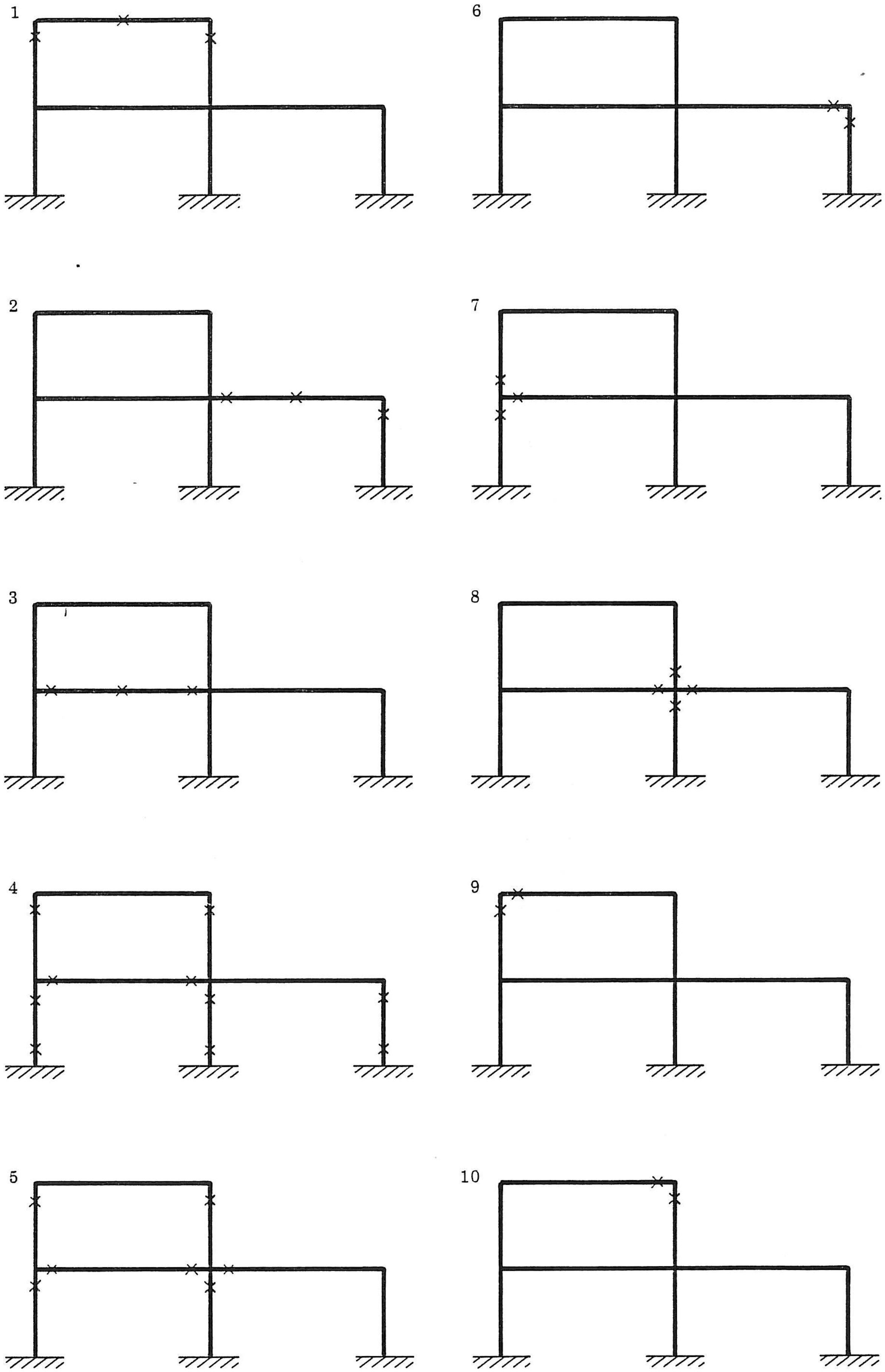


Figure 7. Set of fundamental mechanisms.

j\i	1	2	3	4	5	6	7	8	9	10
A_{ij}										
1				−1						
2				2	−1		−1			
3				−1						
4				2	−1			−1		
5				−1						
6		1		1		1				
7			−1	−1	1		1			
8			2							
9			−1	1	−1			−1		
10		−1		−1	1			1		
11		2								
12						1				
13							1			
14	−1			−1	1				−1	
15								1		
16	1			−1	1					1
17									1	
18	2									
19										1
B_{ij}										
1			3048							
2	3048									
3		3048								
4				7316						
β_i	1.98	1.99	2.08	8.85	10.18	9.12	10.75	12.65	9.36	9.36

Table 2. Fundamental mechanisms.

With N_{max} = 20, SECM = 25 seconds, $\epsilon 1$ = 10.0, $\epsilon 2$ = 0.6 and $\epsilon 3$ = 0.3, 12 not fully correlated significant mechanisms are identified. The reliability indices β_i and the influence coefficients, A_{ij} and B_{ij} are shown in table 3.

Estimates of the systems reliability index, β_s, are:

Ditlevsen bounds:

$$1.15 \leqslant \beta \leqslant 1.34$$

PNET approximation:

$$\beta_s \cong 1.17$$

The total computer time is 28.17 sec, which can be divided into:

Read input and identification of a set of fundamental mechanisms: 0.39 sec

Identification of significant mechanisms: 25.00 sec

Estimates of systems reliability:

with Ditlevsen bounds: 2.76 sec
with PNET approximation: 0.02 sec

j\i	1	2	3	4	5	6	7	8	9	10	11	12
A_{ij}												
1	−1/2	−1			−1/2	−3/4		−1/2	−1/2	−1/4		−1/2
2		1							−1/2	−3/4		
3	−1/2	−1			−1/2	−3/4		−1/2	−1/2	−1/4		−1/2
4	1/2	1			1/2	3/4		1/2		1/4		1/2
5	−1/2	−1			−1/2	−3/4		−1/2	−1/2	−1/4		−1/2
6	1/2	1		1	1/2	3/4		1/2	1	1/4		1/2
7						−1/4	−1					
8	1				1	2	2	1	2	2		1
9	−1/2					−1	−1	−1/2	−3/2	−1		−1/2
10				−1	−1/2							
11				2					1			
12												
13												
14			−1		1/2							1/2
15	−1/2					−3/4		−1/2	−1/2	−1		−1/2
16	1		1			3/2			2	2		1/2
17											−1	
18	1		2			3/2		1	2	2	2	
19								−1			−1	
B_{ij}												
1	1524				1524	3048	3048	1524	3048	3048		1524
2	1524		3048			2286		1524	3048	3048	3048	
3				3048					1524			
4	7316	10974			5487	10974		7316	9145	6401		7316
β_i	1.88*	1.91*	1.98*	1.99*	1.99*	2.05	2.08*	2.09*	2.15	2.17	2.17*	2.18*

Table 3. The 12 most significant mechanisms.

In [6] the β-unzipping method, based on finite element analysis, is used to determinate the significant mechanisms. From this it appears that with 2000 sec computer time 14 significant mechanisms are identified. Among these the mechanisms in table 3 marked with * are among the 10 most significant. Comparing the computer times (2000 sec and 30 sec) it is clear that the method which is used here in this example is the best one. By Monte-Carlo simulation Ma & Ang [8] have found the following estimate of the systems reliability:

$$\bar{\beta}_s = 1.20$$

This estimate is very close to the above-mentioned results.

5.2 Example 2

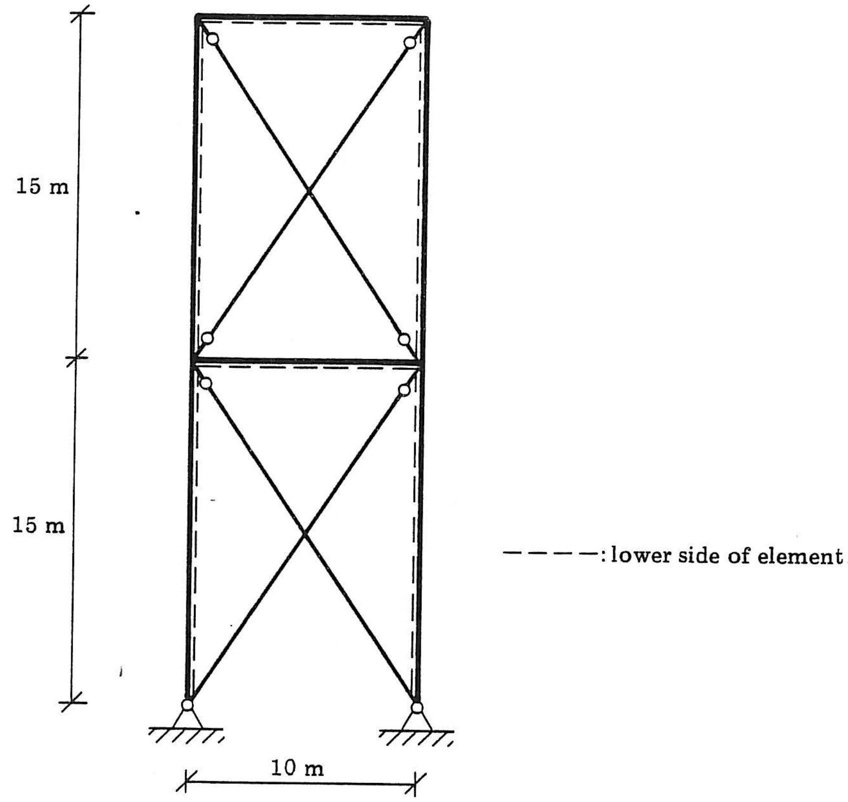
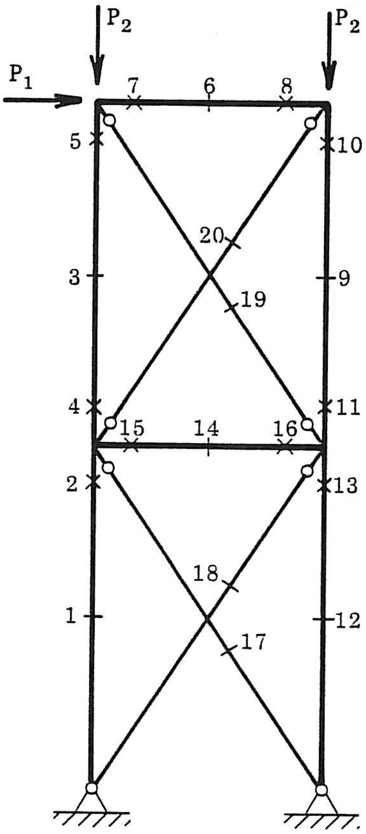


Figure 8. Geometry



$E[P_1] = 2.0$ kN	$V[P_1] = 0.1$	
$E[P_2] = 3.36$ kN	$V[P_1] = 0.1$	
$E[R_i] = 20$ kNm	$V[R_i] = 0.1$	i = 1, 2, 12, 13, 14, 15, 16
$E[R_i] = 6.04$ kNm	$V[R_i] = 0.1$	i = 4, 5, 7, 8, 10, 11
$E[R_i] = 9.0$ kN,	$V[R_i] = 0.1$	i = 3, 6, 9
$E[R_i] = 7.0$ kN	$V[R_i] = 0.1$	i = 19, 20
$E[R_i] = 8.0$ kN	$V[R_i] = 0.1$	i = 17, 18

Figure 9. Loading and possible locations of potential yield hinges (x) and axial collapses (—).

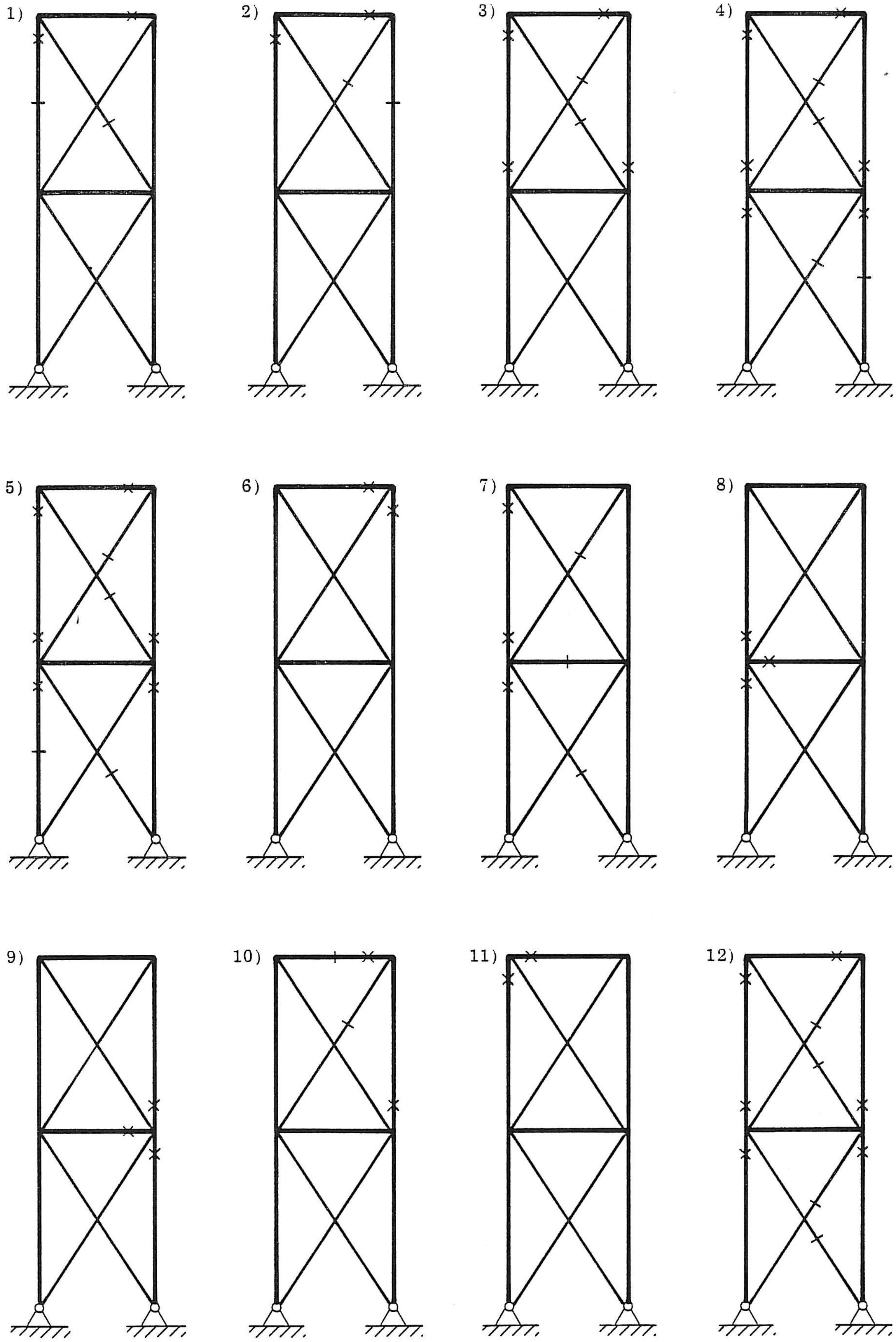


Figure 10. Set of fundamental mechanisms.

The structure shown in figure 8 is considered in this example. The loading and possible locations of potential yield hinges (x) and axial collapses (—) are shown in figure 9. Yield capacities of yield hinges in a column or a beam are considered fully correlated. Further it is assumed that the compression capacity of a brace is equal to 0.5 times the tensile capacity. The number of potential yield hinges and axial collapses is n = 20 and the degree of redundance is r = 8. Therefore, the number of fundamental mechanisms is m = n − r = 12. The expected values, (E[·]), and coefficients of variation (V[·]) for the stochastic variables, P_i, and R_i (yield capacities in tension for axial collapses), are shown in figure 9.

In figure 10 the set of fundamental mechanisms used is shown. The reliability indices β_i and the influence coefficients, A_{ij} and B_{ij} for the 12 fundamental mechanisms are shown in table 4.

With N_{max} = 20, SECM = 20 sec, $\epsilon 1$ = 3, $\epsilon 2$ = 5, and $\epsilon 3$ = 5, 8, not fully correlated, significant mechanisms are identified. The coefficents A_{ij} and B_{ij} and the reliability indices β_i are shown in table 5.

j\i	1	2	3	4	5	6	7	8	9	10	11	12
A_{ij}												
1					10							
2				−1	1		1	−1				1
3	10											
4			−1	1	−1		1	1				1
5	1	−1	1	−1	1		−1				−1	−1
6										15		
7											1	
8	−1	1	−1	1	−1	−1				1		1
9		10										
10						1						
11			1	−1	1				1	−1		−1
12				10								
13				1	−1				−1			−1
14							15					
15								1	1			
16												
17					8.32		8.32					8.32
18				8.32								−8.32
19	8.32		8.32	−8.32	8.32							−8.32
20		8.32	−8.32	8.32	−8.32		8.32			8.32		8.32
B_{ij}												
1			15									
2	10	10		10	10							
β_i	8.24	8.24	11.14	18.99	18.99	14.14	15.46	15.92	15.92	14.69	14.14	24.19

Table 4. Fundamental mechanisms.

j\i	1	2	3	4	5	6	7	8
A_{ij}								
1								
2						−1	−1	
3			10		10	10		
4		−1	−2	−1	−1	−1		−1
5			1	−1		1		
6								
7								
8			−1	1				
9		10	20	20	20	10	10	10
10						−1		
11		1	2	1	1	1	1	
12	10							
13								1
14								
15							1	
16								−1
17	8.32							
18								
19		8.32	24.96	8.32	16.64	16.64	8.32	8.32
20				8.32	8.32			
B_{ij}								
1	30	15	30	15	15	15	15	15
2	10	10	30	20	30	20	10	10
β_i	3.15	3.20	5.47	6.05	6.49	6.56	7.47	7.47

Table 5. The 8 most significant mechanisms.

The estimates of the systems reliability are:

Ditlevsens bounds:

$$2.97 \leqslant \beta_s \leqslant 2.97$$

PNET approximation:

$$\beta_s \cong 2.97$$

The total computer time is 21.23 sec, which can be divided into:

Read input and identification of one set of fundamental mechanisms: 0.29 sec

Identification of significant mechanisms: 20.00 sec

Estimates of systems reliability:

With PNET approximation: 0.03 sec

With Ditlevsen bounds: 0.91 sec

The results obtained in this example are very well in accordance with the results from [6], where the significant mechanisms are identified by the β-unzipping method based on finite element analyses. Compared with the results in [6] the computer time is of the same magnitude when the above method is used.

5.3 Example 3

In this example we consider the model of a steel jacket offshore platform in figure 11. The structure is a 12 times statically indeterminate spatial truss tower, having 48 structural elements. Each structural element is supposed to have one possible axial collapse. Therefore, the number of fundamental mechanisms is $m = 48 - 12 = 36$. The expected values of yield capacities in tension $E[R_i^+]$ are shown in figure 11.

The expected values of yield capacities in compression, $E[R_i^-]$, are

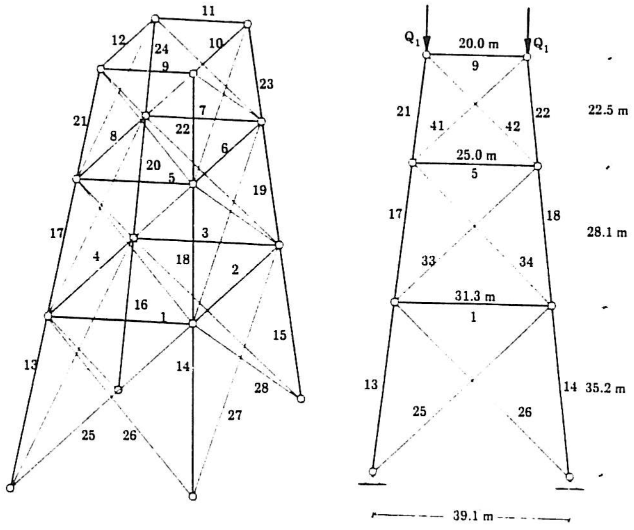
$$E[R_i^-] = 0.5\, E[R_i^+] \qquad i = 1, 2, \ldots, 48$$

All yield capacities R_i are assumed to have coefficients $\rho[R_i^+, R_j^+] = \rho[R_i^+, R_j^-] = \rho[R_i^-, R_j^-] = 0.5$ $(i \neq j)$ and coefficients of variation $V[R_i^+] = V[R_i^-] = 0.15$ for $i = 1, 2, \ldots, 48$.

The structure is subjected to 4 vertical dead loads each of the magnitude Q_1, see figure 11, and 12 horizontal wave loads all of the magnitude proportional to the quantity Q_2 and all having the same direction given by the angle θ, see figure 12.

Further, it is assumed that there is no correlation between resistance and load variables, and that the constants in the wave load model are:

$$(\theta, \gamma_1, \gamma_2, \gamma_3) = (30°, 1.000, 0.667, 0.126)$$



$E[R_i^+] = 67200$ kN $i = 1, \ldots, 4$

$E[R_i^+] = 37070$ kN $i = 5, \ldots, 8$

$E[R_i^+] = 16800$ kN $i = 9, \ldots, 12$

$E[R_i^+] = 103780$ kN $i = 13, \ldots, 24$

$E[R_i^+] = 37070$ kN $i = 25, \ldots, 32$

$E[R_i^+] = 23070$ kN $i = 33, \ldots, 40$

$E[R_i^+] = 13350$ kN $i = 41, \ldots, 48$

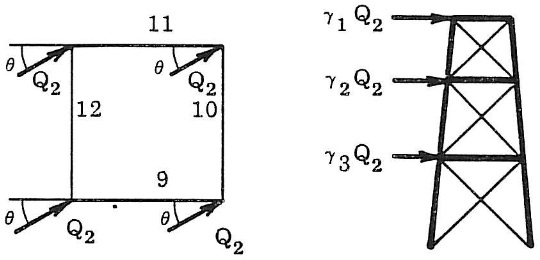
Figure 11. Spatial truss tover.

\i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
ij																		
1													12.80					
2															12.80			
3																12.80		
4														12.80				
5																	12.77	
6																		
7																		
8																		12.77
9					12.77													
0							12.77											
1								12.77										
2						12.77												
3													−1.40	−1.40				
4															−1.40			
5																−1.40		
6																		
7													1.42	1.42			−1.41	−1.41
8															1.42			
9																1.42		
0																		
1	1.40				−1.40	−1.40			−1.40		−1.40						1.40	1.40
2				1.40			−1.40		1.40			−1.40						
3			1.40					−1.40		1.40		1.40						
4		1.40								−1.40	1.40							
5															−1			
6													9.03	−1				
7																−1		
8															9.03			
9																		
0																9.03		
1													−1	9.03				
2																		
3													9.03	1.01				
4															1.01		9.01	−1.01
5															9.03			
6																1.01		
7																9.03		
3																		
9																	−1.01	9.01
0													1.01	9.03				
1				1			−1		−9			−1					9	1
2	1				9	−1			9		−1							
3			1					−1		1		−9						
4				1			9		1			9						
5		1								9	1							
6			1					9				1						
7	1				−1	9			−1	--9	9							
8		1								−1	−9						1	9
ij																		
	1.42	1.42	1.42	1.42														
					11.06	6.38	6.38	−11.06	22.11	22.11	12.77	12.77	1.40	0.81	0.81	−1.40	7.37	4.26
	2.38	2.38	2.38	2.38	8.06	8.09	8.09	8.21	8.29	8.29	8.34	8.34	8.37	8.37	8.38	8.38	8.47	8.48

Table 6. Fundamental mechanisms (to be continued).

j\i	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
A_{ij}																		
1																		
2																		
3																		
4																		
5																		
6	12.77																	
7		12.77																
8																		
9																		
10																		
11																		
12																		
13				−1.40			−1.40							1.40				
14				1.40				−1.40				1.40						
15			1.40					1.40									1.40	
16			−1.40				1.40											1.40
17				1.42		−1.41	1.42		−1.41		1.40			−1.40				
18	−1.41			−1.42		1.41		1.42		−1.41		−1.40	1.40					
19		−1.41	−1.42		1.41			−1.42		1.41					1.40		−1.40	
20			1.42		−1.41		−1.42		1.41							1.40		−1.40
21						1.40			1.40		−1.40							
22	1.40					−1.40				1.40			−1.40					
23		1.40			−1.40					−1.40					−1.40			
24					1.40				−1.40							-1.40		
25				−9.03				−1				1						
26				9.03			−1							1				
27			1					−9.03									1	
28				1				9.03				1						
29			9.03				1											1
30			−9.03					1									1	
31				−1			9.03							1				
32			−1				−9.03											1
33	−1.01			9.03		−9.01	1.01			-1.01			1	−1				
34				−9.03		9.01		1.01	−1.01		1	−1						
35		−1.01		−1.01	1.01			9.03		−9.01		−1			−1			
36	9.01		−1.01			1.01		−9.03		9.01			1				−1	
37			−9.03		9.01			−1.01	1.01							1	−1	
38		9.01	9.03		−9.01		−1.01			1.01					1			−1
39			1.01			−1.01	−9.03		9.01		1							−1
40				1.01	−1.01		9.03		−9.01					--1		1		
41						9			1		−1							
42	1					−9				1			−1					
43	9					−1				9			−1					
44		1			−1					−9					−1			
45		9			−9					−1					−1			
46					9				−1							−1		
47					1				−9							−1		
48						1			9		−1							
B_{ij}																		
1																		
2	4.26	−7.37	2.79	2.79	14.75	14.75	1.61	1.61	8.52	8.52								
β_i	8.48	8.52	8.83	8.83	8.83	8.83	8.83	8.83	8.85	8.85	8.03	8.24	8.03	8.24	8.03	8.03	8.24	8.24

Table 6. Fundamental mechanisms (continued).



$E[Q_1] = 40000$ kN $V[Q_1] = 0.05$

$E[Q_2] = 278$ kN $V[Q_2] = 0.30$

$\rho[Q_1, Q_2] = 0$

Figure 12. Illustration of the wave loading.

As mentioned earlier, each structural element has one possible axial collapse mode and the total number of possible axial collapses is 48. The axial collapses are numbered in the same order as the structural elements, see figure 11.

The reliability indices β_i and the coefficients of influence A_{ij} and B_{ij} for the fundamental mechanisms are shown in table 6.

With $N_{max} = 20$, SECM = 25.0 sec, $\epsilon 1 = 1.0$, $\epsilon 2 = 1.0$ and $\epsilon 3 = 2.0$ 12 not fully correlated significant mechanisms are identified. The coefficients A_{ij} and B_{ij} and the reliability indices β_i are shown in table 7.

Estimates of the systems reliability are:

Ditlevsen bounds:

$$1.85 \leqslant \beta_s \leqslant 1.92$$

PNET approximation:

$$\beta_s \cong 1.90$$

The total computer time is 29.91 sec, which can be divided into:

Read input and identification of one set of fundamental mechanisms: 1.68 sec

Identification of significant mechanisms: 25.00 sec

Estimate of systems reliability:

With PNET approximation: 0.03 sec

With Ditlevsen bounds: 3.20 sec

j\i	1	2	3	4	5	6	7	8	9	10	11	12
A_{ij}												
9						−1.42						
11					−1.42							
12							−1.42					
21				1.40		1.73	1.73		−0.16	0.17	0.17	
22		0.17									−0.02	−0.17
23	1.56				1.73			1.56				1.57
24			1.56					−0.16	1.56	−0.02		
41							1.23				0.11	−0.12
42		0.12		1		0.12			−0.11	0.12		
43	1.11							1.11				−0.11
44		0.12			1.23						−0.01	1.11
45	1.11		1.11					1	1.11	−0.01		
46					0.12							1.12
47			1.11	1			0.12		1		0.12	
48						1.23		−0.11		0.11		
B_{ij}												
1	1.58	0.18	1.58	1.42	1.59	1.59	1.59	1.42	1.42	0.16	0.16	1.42
2	2.46	0.27	1.42		2.80	−2.80	−3.44	2.46	1.42	−0.16	−0.27	1.58
β_i	2.33	2.33	2.35	2.38	3.49	3.59	3.60	3.65	3.67	3.72	3.74	3.87

Table 7. The 12 most significant mechanisms.

The results obtained in this example are also very well in accordance with results from [1] and [7]. The computer time in this example is much less than in [1], where the mechanisms are identified by the β-unzipping method based on finite element analyses.

6. REFERENCES

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