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PAPER NO. 72

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STRESS-RESPONSE OF OFFSHORE STRUCTURES BY EQUIVALENT POLYNOMIAL EXPANSION TECHNIQUES

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ABSTRACT

This paper concerns an investigation of the effects of nonlinearity of drag loading on offshore structures excited by 2D wave fields, where the nonlinear term in the Morison equation is replaced by an equivalent cubic expansion. The equivalent cubic expansion coefficients for the equivalent drag model are obtained using the least mean square procedure. Numerical results are given. The displacement response and the stress response processes obtained using the above loading model are compared with simulation results and those obtained from equivalent linearization of the drag term.

1. Introduction

The loading imposed on structural members of an offshore structure subjected to wave action represents one of the major steps in design of deepwater bottom-supported structures. The wave loading is normally estimated using the well-known Morison equation for a member with dimensions such that the presence of the member does not significantly disturb the wave field.
This paper concerns an investigation of the effects of nonlinearity of drag loading on offshore structures excited by irregular 2D wave fields, where the nonlinear term in the Morison equation is replaced by an equivalent cubic expansion. The structural system is modelled by a linear system with a finite number of degrees of freedom. A system reduction based on an eigenmode expansion is applied, where the frequency response matrix of the system is expressed in two terms, corresponding to the quasi-static contribution and the dynamic contribution, respectively. Besides, the first order wave theory is applied. The influence of the velocity of the structure is ignored in the drag term. It is assumed that the sea surface can be considered as a realization of a stationary zero-mean Gaussian process, which is also homogeneous in the horizontal

space parameters. The response processes of the system are determined based on a spectral approach. The equivalent cubic expansion coefficients for the equivalent drag model are obtained using the least mean square procedure. The variance of the displacement response and the stress response processes obtained using the above loading model are compared with simulation results and with results obtained by using two different equivalent linearization methods of the drag term, namely by using the least mean square procedure and by the requirement that the variance of the original and the equivalent linear drag loading is alike.

2. Short-Term Model of the Sea States

The observed sea elevation, $\eta(\mathbf{x},t)$ at the fixed location $\mathbf{x} = (x, y)$ at a time t, can be considered as a realization of a non-stationary stochastic process, whose characteristic parameters vary slowly with time. Further, it is assumed that for short-term periods (a few hours) the sea surface $\eta(\mathbf{x},t)$ can be considered as a realisation of a stationary stochastic process, which is also homogenous in the horizontal space parameters. This process is assumed to be a zero-mean Gaussian process. A consequence of these simplifying assumptions is that within the short-term time scale the sea surface elevation is completely defined by the cross-covariance function $\kappa_{\eta\eta}(\Delta\mathbf{x},\tau)$, defined as

$$\kappa_{\eta\eta}(\Delta\mathbf{x},\tau) \;=\; E[\eta(\mathbf{x},t)\;\eta(\mathbf{x}+\Delta\mathbf{x},t+\tau)] \tag{1}$$

where $\Delta\mathbf{x} = (x_1 - x_2, y_1 - y_2)$, $\tau = t_1 - t_2$. (x_1, y_1) and (x_2, y_2) are the spatial coordinates of two points at the sea surface.

In structural analysis it may be more convenient to use spectral densities than correlation functions. Applying linear wave theory and assuming long crested waves the corresponding spectral density can be obtained as

$$S_{\eta\eta}(\Delta\mathbf{x},\omega) \;=\; \exp\big(-ik(\omega)(\Delta x\,\cos\theta + \Delta y\,\sin\theta)\big)\;S_{\eta\eta}(\omega) \tag{2}$$

where ω is the frequency (rad/sec), $\Delta x = (x_1 - x_2)$, $\Delta y = (y_1 - y_2)$. θ is the angle from the x-axis to the direction of wave propagation of the 2D sea state in counter-clockwise direction. $i = \sqrt{-1}$ and $k(\omega)$ is the wave number obtained as

$$\omega^2 \;=\; k\;g\;\tanh(kh) \qquad \omega \geq 0, \quad k \geq 0 \tag{3}$$

where g is the acceleration of gravity and h is the water depth.

$S_{\eta\eta}(\omega)$ is the double sided auto-spectral density function, and $S_{\eta\eta}(\Delta\mathbf{x},\omega)$ is the double sided cross-spectral density of the sea surface. For negative frequencies the wave number should be defined from the asymmetry condition

$$k(-\omega) = -k(\omega) \tag{4}$$

In most practical applications a standard formula involving a few sea state characteristics is used for $S_{\eta\eta}(\omega)$. Over the last 30 years many spectral expressions have been suggested. A common feature of most spectral models is that they are of a unimodal form and mainly meant to characterise a pure wind driven sea.

Here, the JONSWAP spectrum is adopted as a model for wind sea. This spectrum can be written, Hasselmann et al. 1973

$$S_{\eta\eta}(\omega) = \alpha\, g^2 \omega^{-5} \exp\left(-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)^{-4}\right)\gamma^{\exp\left(-\frac{1}{2}\left(\left(\frac{\omega}{\omega_p}-1\right)/\sigma\right)^2\right)} \tag{5}$$

where

α equilibrium range parameter

ω_p spectral peak frequency ($= 2\pi/T_p$)

γ spectral peak parameter

σ spectral peak width parameter

The mean values from the JONSWAP measurements are usually adopted for σ, i.e. $\sigma = 0.07$ for $\omega \leq \omega_p$ and $\sigma = 0.09$ for $\omega > \omega_p$. Here σ is chosen as 0.08 for all frequencies and all sea states. For a sea state with a given value of the significant wave height H_s the remaining parameters (α, γ and ω_p) are related to each other through the following equation, Haver 1985

$$\gamma = \exp(3.484\,(1 - 0.1975\,\alpha\, T_p^4/H_s^2)) \tag{6}$$

Eq. (5) is expected to be a reasonable spectral model within the so-called JONSWAP range. In Haver 1985 the JONSWAP range is given by

$$3.6\sqrt{H_s} \leq T_p \leq 5\sqrt{H_s} \tag{7}$$

The lower bound corresponds to $\alpha = 0.016$ and $\gamma = 5.0$, and the upper bound (corresponding to fully developed sea) to $\alpha = 0.0081$ and $\gamma = 1.0$, where the JONSWAP wave spectrum equals the Pierson-Moskowitz wave spectrum. Within the JONSWAP range α is assumed to vary linearly with T_p and H_s as follows

$$\alpha = 0.036 - 0.0056\, T_p/\sqrt{H_s} \tag{8}$$

3. Stochastic Modelling of the Wave Loading

In this section we consider the loading on a structural element. It is well known that the force on a circular cylinder subjected to wave action consists of a drag as well as an inertia component. It is assumed that the total wave force per unit length on a fixed vertical cylinder of the diameter D at position $\mathbf{r} = (x_1, x_2, x_3)$ at the time t can be estimated by using Morison's equation as, Sarpkaya & Isaacson 1981

$$\mathbf{p}(\mathbf{r},t) = \begin{bmatrix} p_1(\mathbf{r},t) \\ p_2(\mathbf{r},t) \\ p_3(\mathbf{r},t) \end{bmatrix} = \mathbf{p}_D(\mathbf{r},t) + \mathbf{p}_I(\mathbf{r},t) \tag{9}$$

$$\mathbf{p}_D(\mathbf{r},t) = K_D(\mathbf{r})\ |\mathbf{u}_n(\mathbf{r},t)|\ \mathbf{u}_n(\mathbf{r},t) \tag{10}$$

$$\mathbf{p}_I(\mathbf{r},t) = K_I(\mathbf{r})\ \dot{\mathbf{u}}_n(\mathbf{r},t) \tag{11}$$

$$K_D(\mathbf{r}) = 1/2\, C_D(\mathbf{r})\ \rho D(\mathbf{r}) \tag{12}$$

$$K_I(\mathbf{r}) = 1/4\, C_M(\mathbf{r})\ \pi\ \rho\ D^2(\mathbf{r}) \tag{13}$$

where

$\mathbf{u}_n(\mathbf{r},t)$	Horizontal water particle velocity vector perpendicular to the cylinder at position $\mathbf{r}$ at the time t.
$\dot{\mathbf{u}}_n(\mathbf{r},t)$	Horizontal water particle acceleration vector perpendicular to the cylinder at position $\mathbf{r}$ at the time t.
$C_D(\mathbf{r})$	Drag coefficient.
$C_M(\mathbf{r})$	Coefficient of inertia.
ρ	Mass density of water.

The normal vectors $\mathbf{u}_n$ and $\dot{\mathbf{u}}_n$ can be expressed in terms of a unit vector with directional cosines $\mathbf{s} = (s_1, s_2, s_3)$ along the cylinder axis as follows

$$\mathbf{u}_n(\mathbf{r},t) = \begin{bmatrix} u_{n_1} \\ u_{n_2} \\ u_{n_3} \end{bmatrix} = \mathbf{S}(\mathbf{r})\ \mathbf{A}(\mathbf{r})\ \mathbf{u}(\mathbf{r},t)\ , \qquad \dot{\mathbf{u}}_n(\mathbf{r},t) = \begin{bmatrix} \dot{u}_{n_1} \\ \dot{u}_{n_2} \\ \dot{u}_{n_3} \end{bmatrix} = \mathbf{S}(\mathbf{r})\ \mathbf{A}(\mathbf{r})\ \dot{\mathbf{u}}(\mathbf{r},t) \tag{14}$$

where

$$\mathbf{S}(\mathbf{r}) = \begin{bmatrix} (1-s_1^2) & -s_1 s_2 & -s_1 s_3 \\ & (1-s_2^2) & -s_2 s_3 \\ \text{symm.} & & (1-s_3^2) \end{bmatrix} \tag{15}$$

$$\mathbf{A}(\mathbf{r}) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}\ , \qquad \mathbf{u}(\mathbf{r},t) = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\ , \qquad \dot{\mathbf{u}}(\mathbf{r},t) = \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} \tag{16}$$

$\mathbf{u}(\mathbf{r},t)$ and $\dot{\mathbf{u}}(\mathbf{r},t)$ signify the velocity and acceleration of a water particle in the 2D wave field at position $\mathbf{r}$ at the time t. u_1 is the horizontal water particle velocity in the direction of wave propagation and u_2 is the vertical water particle velocity (in the z_3-direction).

Using tensor notation and index summation with the free indices i and j taking values 1,2 and 3, and $\alpha, \beta, \gamma, \delta, \lambda$ and μ taking values 1 and 2, the components of eq. (9) can be rewritten as

$$\begin{aligned} p_i(\mathbf{r},t) &= K_D u_{n_i} \sqrt{u_{n_j} u_{n_j}} + K_I \dot{u}_{n_i} \\ &= K_D A_{i\alpha} u_\alpha \sqrt{A_{j\beta} A_{j\gamma} u_\beta u_\gamma} + K_I A_{i\alpha} \dot{u}_\alpha \\ &= f_{i\alpha} u_\alpha \sqrt{e_{\beta\gamma} u_\beta u_\gamma} + c_{i\alpha} \dot{u}_\alpha \end{aligned} \tag{17}$$

where $A_{j\beta} = A_{j\beta}(\mathbf{r})$, $u_\alpha = u_\alpha(\mathbf{r},t)$ and $\dot{u}_\alpha = \dot{u}_\alpha(\mathbf{r},t)$ are components of $\mathbf{A}(\mathbf{r})$, $\mathbf{u}(\mathbf{r},t)$ and $\dot{\mathbf{u}}(\mathbf{r},t)$ defined by eq. (16). Further

$$f_{i\alpha}(\mathbf{r}) = K_D(\mathbf{r}) A_{i\alpha}(\mathbf{r}) \tag{18}$$

$$c_{i\alpha}(\mathbf{r}) = K_I(\mathbf{r}) A_{i\alpha}(\mathbf{r}) \tag{19}$$

$$e_{\alpha\beta}(\mathbf{r}) = A_{j\alpha}(\mathbf{r}) A_{j\beta}(\mathbf{r}) \tag{20}$$

As seen, the drag term of the loading depends non-linearly on the 2D velocity field $u_\alpha(\mathbf{r},t)$. Instead of eq. (17) an equivalent system is considered in which the drag term is given by a cubic expansion in the velocity field, i.e.

$$p_{i,eq}(\mathbf{r},t) = b_{i\alpha}(\mathbf{r}) u_\alpha + d_{i\alpha\beta\gamma}(\mathbf{r}) u_\alpha u_\beta u_\gamma + c_{i\alpha}(\mathbf{r}) \dot{u}_\alpha \tag{21}$$

In appendix A it is shown how the coefficients $b_{i\alpha}$ and $d_{i\alpha\beta\gamma}$ can be obtained using the least mean square procedure. In what follows the cross-covariance of the loading will then be analysed based on eq. (21).

Two points $\mathbf{r}_1$ and $\mathbf{r}_2$ of the structure are considered. The cross-covariance function for the equivalent wave loading components can be expressed as

$$\begin{aligned} \kappa_{p^{(1)}_{i,eq} p^{(2)}_{j,eq}}(\tau) &= b^{(1)}_{i\alpha} \Big(b^{(2)}_{j\beta}\ E[u^{(1)}_\alpha u^{(2)}_\beta] + d^{(2)}_{j\beta\gamma\delta}\ E[u^{(1)}_\alpha u^{(2)}_\beta u^{(2)}_\gamma u^{(2)}_\delta] \\ &+ c^{(2)}_{j\beta}\ E[u^{(1)}_\alpha \dot{u}^{(2)}_\beta] \Big) + d^{(1)}_{i\alpha\beta\gamma} \Big(b^{(2)}_{j\delta}\ E[u^{(1)}_\alpha u^{(1)}_\beta u^{(1)}_\gamma u^{(2)}_\delta] \\ &+ d^{(2)}_{j\delta\lambda\mu}\ E[u^{(1)}_\alpha u^{(1)}_\beta u^{(1)}_\gamma u^{(2)}_\delta u^{(2)}_\lambda u^{(2)}_\mu] + c^{(2)}_{j\delta}\ E[u^{(1)}_\alpha u^{(1)}_\beta u^{(1)}_\gamma \dot{u}^{(2)}_\delta] \Big) \\ &+ c^{(1)}_{i\alpha} \Big(b^{(2)}_{j\beta}\ E[\dot{u}^{(1)}_\alpha u^{(2)}_\beta] + d^{(2)}_{j\beta\gamma\delta}\ E[\dot{u}^{(1)}_\alpha u^{(2)}_\beta u^{(2)}_\gamma u^{(2)}_\delta] + c^{(2)}_{j\beta}\ E[\dot{u}^{(1)}_\alpha \dot{u}^{(2)}_\beta] \Big) \end{aligned} \tag{22}$$

where $E[\cdots]$ denotes the expected value, $p^{(i)}_{j,eq} = p_{j,eq}(\mathbf{r}_i, t_i)$, $u^{(i)}_\alpha = u_\alpha(\mathbf{r}_i, t_i)$, $b^{(i)}_{j\alpha} = b_{j\alpha}(\mathbf{r}_i)$, $d^{(i)}_{j\alpha\beta\gamma} = d_{j\alpha\beta\gamma}(\mathbf{r}_i)$, $i = 1, 2$ and $\tau = t_1 - t_2$. $u^{(1)}_\alpha$ and $u^{(2)}_\alpha$ are assumed to be simultaneously Gaussian distributed with zero mean. The cross-spectral densities between various components of the wave load can be found by deriving the Fourier transform of the cross-covariance function as follows

$$S_{p^{(1)}_{i,eq} p^{(2)}_{j,eq}}(\omega) = P_{ij\alpha\beta}\; S_{u^{(1)}_\alpha u^{(2)}_\beta}(\omega) + Q_{ij\alpha\beta\gamma\delta\lambda\mu}\; S_{u^{(1)}_\alpha u^{(2)}_\beta} * S_{u^{(1)}_\gamma u^{(2)}_\delta} * S_{u^{(1)}_\lambda u^{(2)}_\mu} \quad (23)$$

where

$$S_{u^{(1)}_\alpha u^{(2)}_\beta} * S_{u^{(1)}_\gamma u^{(2)}_\delta} * S_{u^{(1)}_\lambda u^{(2)}_\mu} =$$

$$\int_{-\infty}^{\infty} S_{u^{(1)}_\alpha u^{(2)}_\beta}(\omega - \omega_1) \int_{-\infty}^{\infty} S_{u^{(1)}_\gamma u^{(2)}_\delta}(\omega_1 - \omega_2)\; S_{u^{(1)}_\lambda u^{(2)}_\mu}(\omega_2)\; d\omega_2 d\omega_1 \quad (24)$$

$$\begin{aligned}
P_{ij\alpha\beta} = {} & b^{(1)}_{i\alpha} b^{(2)}_{j\beta} - \left(b^{(1)}_{i\alpha} c^{(2)}_{j\beta} - c^{(1)}_{i\alpha} b^{(2)}_{j\beta}\right) i\,\omega - c^{(1)}_{i\alpha} c^{(1)}_{j\beta} (i\,\omega)^2 \\
& + \left(d^{(2)}_{j\beta\gamma\delta} + d^{(2)}_{j\gamma\beta\delta} + d^{(2)}_{j\delta\gamma\beta}\right)\left(b^{(1)}_{i\alpha} + i\,\omega c^{(1)}_{i\alpha}\right) \kappa_{u^{(2)}_\gamma u^{(2)}_\delta}(0) \\
& + \left(d^{(1)}_{i\gamma\delta\alpha} + d^{(1)}_{i\delta\alpha\gamma} + d^{(1)}_{i\alpha\delta\gamma}\right)\left(b^{(2)}_{j\beta} - i\,\omega c^{(2)}_{j\beta}\right) \kappa_{u^{(1)}_\gamma u^{(1)}_\delta}(0) \\
& + \left(d^{(1)}_{i\mu\alpha\gamma} + d^{(1)}_{i\alpha\mu\gamma}\right) d^{(2)}_{j\delta\lambda\beta} \kappa_{u^{(1)}_\mu u^{(1)}_\gamma}(0) \kappa_{u^{(2)}_\delta u^{(2)}_\lambda}(0) + d^{(1)}_{i\gamma\lambda\alpha} d^{(2)}_{j\delta\beta\mu} \kappa_{u^{(1)}_\gamma u^{(1)}_\lambda}(0) \kappa_{u^{(2)}_\delta u^{(2)}_\mu}(0) \\
& + \left(d^{(1)}_{i\lambda\alpha\gamma} + d^{(1)}_{i\alpha\lambda\gamma}\right) d^{(2)}_{j\delta\beta\mu} \kappa_{u^{(1)}_\lambda u^{(1)}_\gamma}(0) \kappa_{u^{(2)}_\delta u^{(2)}_\mu}(0) + d^{(1)}_{i\gamma\mu\alpha} d^{(2)}_{j\delta\lambda\beta} \kappa_{u^{(1)}_\gamma u^{(1)}_\mu}(0) \kappa_{u^{(2)}_\delta u^{(2)}_\lambda}(0) \\
& + \left(d^{(1)}_{i\delta\alpha\gamma} + d^{(1)}_{i\alpha\delta\gamma}\right) d^{(2)}_{j\beta\lambda\mu} \kappa_{u^{(1)}_\delta u^{(1)}_\gamma}(0) \kappa_{u^{(2)}_\lambda u^{(2)}_\mu}(0) + d^{(1)}_{i\gamma\delta\alpha} d^{(2)}_{j\beta\lambda\mu} \kappa_{u^{(1)}_\gamma u^{(1)}_\delta}(0) \kappa_{u^{(2)}_\lambda u^{(2)}_\mu}(0)
\end{aligned} \quad (25)$$

$$Q_{ij\alpha\beta\gamma\delta\lambda\mu} = d^{(1)}_{i\alpha\gamma\lambda}\left(d^{(2)}_{j\beta\delta\mu} + d^{(2)}_{j\beta\mu\delta} + d^{(2)}_{j\delta\beta\mu} + d^{(2)}_{j\mu\beta\delta} + d^{(2)}_{j\delta\mu\beta} + d^{(2)}_{j\mu\delta\beta}\right) \quad (26)$$

$\kappa_{u^{(i)}_\gamma u^{(i)}_\delta}(0)$ signifies the cross-covariance of $u_\gamma(\mathbf{r}_i, t)$ and $u_\delta(\mathbf{r}_i, t)$.

The cross-spectral densities of the water particle velocities $S_{u^{(1)}_\alpha u^{(2)}_\beta}(\omega)$ can be expressed in terms of the wave spectral density $S_{\eta\eta}(\omega)$ as

$$S_{u_\alpha^{(1)} u_\beta^{(2)}}(\omega) = U_\alpha(x_3^{(1)},\omega) U_\beta^*(x_3^{(2)},\omega) \exp\Big(-i\, k(\omega)(\Delta x_1 \ \cos\theta + \Delta x_2 \ \sin\theta)\Big) S_{\eta\eta}(\omega) \tag{27}$$

where $*$ denotes the complex conjugate and

$$U_1(x_3,\omega) = \omega \frac{\cosh(k(\omega)\ x_3)}{\sinh(k(\omega)\ h)} \tag{28}$$

$$U_2(x_3,\omega) = i\,\omega \frac{\sinh(k(\omega)\ x_3)}{\sinh(k(\omega)\ h)} \tag{29}$$

x_3-coordinates are measured form the bottom positive upwards.

4. Structural Response

It is assumed that the structure can be modelled as a space frame of three-dimensional beam elements connected by nodal points. If the structural system is assumed to be linear the dynamic equations may be written as

$$\mathbf{M}\ \ddot{\mathbf{x}} + \mathbf{C}\ \dot{\mathbf{x}} + \mathbf{K}\ \mathbf{x} = \mathbf{p}(t) \tag{30}$$

where

$\mathbf{x}$	Displacement vector
$\mathbf{M}$	Structural mass matrix
$\mathbf{C}$	Structural damping matrix
$\mathbf{K}$	Stiffness matrix
$\mathbf{p}$	Load vector

In what follows a system reduction is performed based on an expansion of the response $\mathbf{x}(t)$ on a basis made up of the undamped eigenmodes $\mathbf{\Phi}^{(m)}$

$$\mathbf{x}(t) = \sum_{m=1}^{N} q_m(t)\ \mathbf{\Phi}^{(m)} \tag{31}$$

N is the number of the degree of freedom in the system. It is assumed that only the lower-order modes contribute to the global dynamic response. However, the response of local elements exposed to wave loading may be dominated by high-frequency modes, and hence will be quasi-static in nature. If the number of dynamic modes considered is $M \leq N$, the modal coordinates q_m are then determined from the following equations

$$\ddot{q}_m + 2\omega_m\Big(\zeta_m\dot{q}_m + \sum_{\substack{n=1\\n\neq m}}^{M}\sqrt{\frac{\omega_n M_n}{\omega_m M_m}}\zeta_{mn}\dot{q}_n\Big) + \omega_m^2 q_m = \frac{P_m(t)}{M_m} \qquad m = 1,\ldots,M \tag{32}$$

$$q_m = \frac{P_m(t)}{M_m\omega_m^2} \qquad m = M+1,\ldots,N \tag{33}$$

ω_m is the undamped circular eigenfrequency corresponding to the mth eigenmode. The modal mass M_m, modal loading $P_m(t)$, damping ratio ζ_m and the coupling coefficents ζ_{mn} are defined by

$$M_m = \mathbf{\Phi}^{(m)\,T}\mathbf{M}\mathbf{\Phi}^{(m)} \tag{34}$$

$$P_m(t) = \mathbf{\Phi}^{(m)\,T}\mathbf{p}(t) \tag{35}$$

$$\zeta_m = \frac{\mathbf{\Phi}^{(m)\,T}\mathbf{C}\mathbf{\Phi}^{(m)}}{2\omega_m M_m} \tag{36}$$

$$\zeta_{mn} = \frac{\mathbf{\Phi}^{(m)\,T}\mathbf{C}\mathbf{\Phi}^{(n)}}{2\sqrt{\omega_m\omega_n M_m M_n}} \tag{37}$$

Notice that the summation convention has been abolished in the above equations.

Inserting eq. (33) and eq. (35) into eq. (31) the response can be written

$$\mathbf{x}(t) = \sum_{m=1}^{M} q_m(t)\mathbf{\Phi}^{(m)} + \left(\mathbf{K}^{-1} - \sum_{m=1}^{M}\frac{1}{M_m\omega_m^2}\mathbf{\Phi}^{(m)}\mathbf{\Phi}^{(m)\,T}\right)\mathbf{p}(t) \tag{38}$$

Deriving eq. (38) the following expansion of the inverse stiffnes matrix $\mathbf{K}^{-1}$ has been applied (Mercer's theorem)

$$\mathbf{K}^{-1} = \sum_{m=1}^{N}\frac{1}{M_m\omega_m^2}\mathbf{\Phi}^{(m)}\mathbf{\Phi}^{(m)\,T} \tag{39}$$

Using eq. (32) and eq. (38) the frequency response matrix of the system can then be written

$$\mathbf{H}(\omega) = \mathbf{H}_0(\omega) + \mathbf{K}^{-1} - \sum_{m=1}^{M}\frac{1}{M_m\omega_m^2}\mathbf{\Phi}^{(m)}\mathbf{\Phi}^{(m)\,T} \tag{40}$$

$$\mathbf{H}_0(\omega) = \mathbf{\Phi}\left(-\omega^2\mathbf{m}_0 + i\,\omega\mathbf{c}_0 + \mathbf{k}_0\right)^{-1}\mathbf{\Phi}^T \tag{41}$$

$$\mathbf{\Phi} = \left[\mathbf{\Phi}^{(1)} \ldots \mathbf{\Phi}^{(M)}\right] \tag{42}$$

$$\mathbf{m}_0 = \begin{bmatrix} M_1 & & \\ & \ddots & \\ & & M_M \end{bmatrix} \quad , \quad \mathbf{k}_0 = \begin{bmatrix} \omega_1^2 M_1 & & \\ & \ddots & \\ & & \omega_M^2 M_M \end{bmatrix} \tag{43}$$

$$\mathbf{c}_0 = \begin{bmatrix} 2\omega_1 M_1 \zeta_1 & 2\sqrt{\omega_1\omega_2 M_1 M_2}\zeta_{12} & \cdots & 2\sqrt{\omega_1\omega_M M_1 M_M}\zeta_{1M} \\ 2\sqrt{\omega_1\omega_2 M_1 M_2}\zeta_{21} & 2\omega_2 M_2 \zeta_2 & \cdots & 2\sqrt{\omega_2\omega_M M_2 M_M}\zeta_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ 2\sqrt{\omega_1\omega_M M_1 M_M}\zeta_{M1} & 2\sqrt{\omega_2\omega_M M_2 M_M}\zeta_{M2} & \cdots & 2\omega_M M_M \zeta_{MM} \end{bmatrix} \tag{44}$$

If the system in eq. (32) decouples in the damping term, i.e. if the coupling constants $\zeta_{mn} = 0,\ m \neq n,\ m,n = 1,\ldots,M$, eq. (40) reduces to

$$\mathbf{H}(\omega) = \mathbf{K}^{-1} + \sum_{m=1}^{M} \hat{H}_m(\omega)\mathbf{\Phi}^{(m)}\mathbf{\Phi}^{(m)\,T} \tag{45}$$

$$\hat{H}_m(\omega) = H_m(\omega) - \frac{1}{M_m\omega_m^2} \tag{46}$$

$$H_m(\omega) = \frac{1}{M_m(\omega_m^2 - \omega^2 + i\,2\zeta_m\omega_m\omega)} \tag{47}$$

$H_m(\omega)$ signifies the frequency response function in the mth mode.

The cross-spectral density of the components x_i and x_j of the displacement vector $\mathbf{x}(t)$ can be found as

$$S_{x_i x_j}(\omega) = \sum_{r=1}^{N}\sum_{s=1}^{N} H_{ir}^*(\omega) H_{js}(\omega) S_{p_r p_s}(\omega) \tag{48}$$

where the components H_{ir} indicate the (i,r) component of the matrix $\mathbf{H}$ and $S_{p_r p_s}(\omega)$ is the cross-spectral density of the components $p_i(t)$ and $p_j(t)$ of the loading vector $\mathbf{p}(t)$. The the variance of the displacement of a given degree of freedom i can be obtained from

$$\sigma_{x_i}^2 = 2\int_0^\infty S_{x_i x_i}(\omega)\, d\omega \tag{49}$$

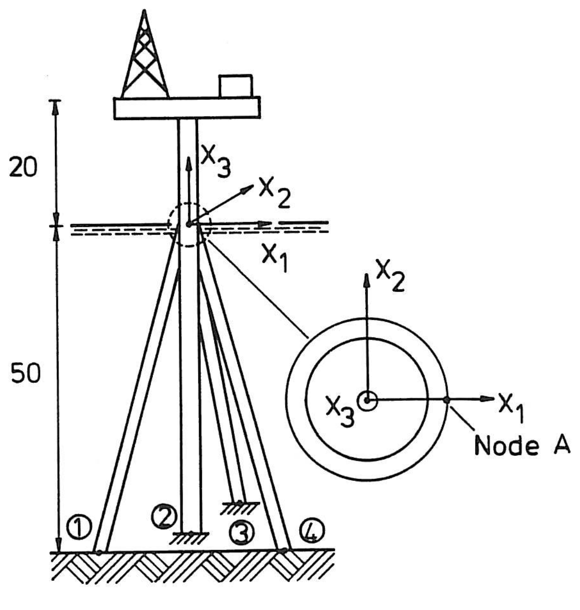
Finally the cross-spectral density of the stresses $s^{(1)} = s(\mathbf{r}_1, t)$ and $s^{(2)} = s(\mathbf{r}_2, t)$ in two points $\mathbf{r}_1$ and $\mathbf{r}_2$ of the structure can be found as

$$S_{s^{(1)}s^{(2)}}(\omega) = \sum_{i=1}^{N}\sum_{j=1}^{N} T_i^{(1)} T_j^{(2)} S_{x_i x_j}(\omega) \tag{50}$$

where $T_i^{(1)}$ is the stress in point $\mathbf{r}_1$ due to displacement $x_k = 1$ for $k = i$ and $x_k = 0$ for $k \neq i$.

5. Example

The considered model of a steel offshore platform consists of one central column and three inclined legs, see figure 1. It is assumed that the topside is 20 m above the still water level and the mean water-depth is 50 m. All structural elements are tubular beam elements made of steel with modulus of elasticity equal to $0.205 \cdot 10^9$ kN/m^2 and mass density equal to 7800 kg/m^3. The diameter of the column is 3 m and the diameter of the legs are 2 m, all with a wall thickness of 0.1 m. The total mass of the deck is assumed to be $3 \cdot 10^6$ kg.



	X_1	X_2	X_3
1	-5	-2.89	-50
2	0	0	-50
3	0	5.77	-50
4	5	-2.89	-50

Coordinates for the support nodes.

All dimensions in m.

Figure 1. Steel offshore platform.

The number of considered modes in the dynamic analysis is $M = 4$. Modal decoupling is assumed, and the damping ratios ζ_m are taken as 1% for all modes. The four lowest undamped circular eigenfrequencies are obtained as:

$$\omega_1 = 2.21 \text{ rad/sec.}$$
$$\omega_2 = 2.21 \text{ rad/sec.}$$
$$\omega_3 = 12.60 \text{ rad/sec.}$$
$$\omega_4 = 12.60 \text{ rad/sec.}$$

One sea state is considered, namely with significant wave height $H_s = 15$ m and spectral peak period $T_p = 20$ sec. with direction of wave propagation $\theta = 0^\circ$ (the x_1-direction). The drag coefficient C_D and the coefficient of inertia C_M in Morison's equation are taken as 1.3 and 2.0, respectively.

In table 1 the variance and the 4th order central moment of the horizontal displacement of the topside and the nominal stress in node A (see figure 1), obtained by four different methods. Notice that the 4th order central moment for all methods are obtained by simulation. For the different approaches the following designation are applied :

(a) Simulation, with the orginal drag loading.

(b) Equivalent cubic expansion of the drag loading (see section 3).

(c) Equivalent linear expansion of the drag loading using the least mean square procedure. Expansion coeffecients $b_{i\alpha}$ are obtained using eq. (A.8) where $d_{i\beta\lambda\mu}$ is set to zero.

(d) Equivalent linear expansion of the drag loading. Expansion coefficients $b_{i\alpha}$ are obtained by the requirement that the variance of the original and the equivalent linear drag loading are alike, see eq. (A.10).

	Displacement:		Stress:	
Method	Variance (m^2)	4th order moment (m^4)	Variance ($(N/m^2)^2$)	4th order moment ($(N/m^2)^2$)
(a)	$5.08 \cdot 10^{-4}$	$9.75 \cdot 10^{-7}$	$1.74 \cdot 10^{14}$	$9.75 \cdot 10^{28}$
(b)	$5.10 \cdot 10^{-4}$	$10.19 \cdot 10^{-7}$	$1.79 \cdot 10^{14}$	$10.13 \cdot 10^{28}$
(c)	$4.62 \cdot 10^{-4}$	$6.42 \cdot 10^{-7}$	$1.50 \cdot 10^{14}$	$6.75 \cdot 10^{28}$
(d)	$4.84 \cdot 10^{-4}$	$7.03 \cdot 10^{-7}$	$1.53 \cdot 10^{14}$	$7.02 \cdot 10^{28}$

Table 1. Variance and the 4th order central moment of the horizontal displacement of the topside and the nominal stress in node A, obtained by four different methods.

As seen, it is possible, using the equivalent cubic expansion of the drag term to estimate the variance as well as the 4th order moment for the displacement and the stress response with very little error. Both methods of equivalent linearization underestimates the variance as well as the 4th order moment of the response. The best linear approximation is obtained, when the equivalent linear expansion coefficients are calibrated to give the same variance as for the original drag loading. The 4th order moments of both methods of equivalent linearization are equal to 3 times the squared variances, as expected for Gaussian responses. Notice, that the considered example has been chosen to provide drag dominated loading. However, the considered wave climate and the dimensions of the structure are realistic. If the time needed for performing a calculation using equivalent linear expansion is set to 1, the calculation time for an equivalent cubic expansion is 7 and 112 for performing a simulation analysis. The convolution integrals in eq. (23) is calculated numerically by using Fast-Fourier-Transform (FFT) technique. These convolution integrals are the main reason for the increased calculation time of the cubic expansion technique.

6. Conclusions

A method to evaluate the variance response for displacements and stresses of offshore structures is described. The technique is based on an equivalent cubic expansion of the non-linear drag loadnig. One numerical example is given, where the loading is drag dominated. Analytical results obtained using this method are compared with those obtained by simulation where the original drag loding is used and with analytical results obtained by using two different equivalent linearization methods. It is shown that the equivalent cubic expansion method is able to estimate the variance as well as the 4th order moment for the displacement and the stress response with very little error. Both methods of equivalent linearization underestimates significantly the variance as well as the 4th order moment of the response. An equivalent linearization technique in which the equivalent linear expansion coeffecients of the drag term are obtained by the requirement that the variance of the orginal and the equivalent drag loading are alike, is shown to be superiour to the conventional least mean square procedure.

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Appendix A : Determination of equivalent polynomial expansion coefficients

A circular cylinder with directional cosines $\mathbf{s} = (s_1, s_2, s_3)$ along the cylinder axis as shown in figure A1 is considered.

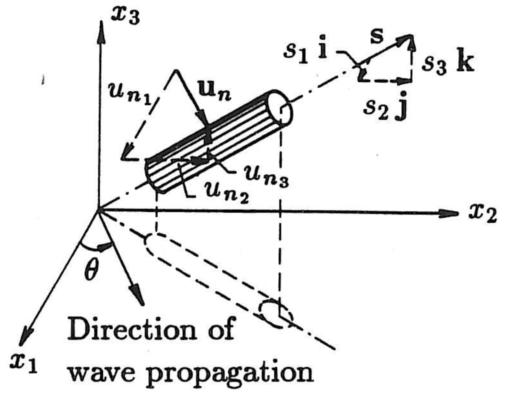

Figure A1. **i**, **j** and **k** represent the base vectors in the x_1, x_2, x_3-coordinate system.

The difference between the orginal loading (eq. (17)) and the equivalent loading (eq. (21)) is given by the error vector ϵ_i, i.e.

$$\begin{aligned} \epsilon_i &= p_i - p_{i,eq} \\ &= f_{i\alpha} u_\alpha \sqrt{e_{\beta\gamma} u_\beta u_\gamma} - b_{i\alpha}\, u_\alpha - d_{i\alpha\beta\gamma}\, u_\alpha u_\beta u_\gamma \end{aligned} \tag{A.1}$$

The least mean square criterion leads to the following conditions for the determination of the expansion coefficients

$$\frac{\partial}{\partial b_{i\alpha}} E[\epsilon_i \epsilon_i] = 0 \tag{A.2}$$

$$\frac{\partial}{\partial d_{i\alpha\beta\gamma}} E[\epsilon_i \epsilon_i] = 0 \tag{A.3}$$

resulting in the following system of two linear equations

$$b_{i\lambda}\ \mu_{\lambda\alpha} + d_{i\lambda\mu\nu}\ \mu_{\lambda\mu\nu\alpha} = f_{i\lambda}\ E[h_\lambda u_\alpha] \tag{A.4}$$

$$b_{i\lambda}\ \mu_{\lambda\alpha\beta\gamma} + d_{i\lambda\mu\nu}\ \mu_{\lambda\mu\nu\alpha\beta\gamma} = f_{i\lambda}\ E[h_\lambda u_\alpha u_\beta u_\gamma] \tag{A.5}$$

where

$$h_\alpha = h_\alpha(\mathbf{r},t) = u_\alpha(\mathbf{r},t)\sqrt{e_{\lambda\mu}(\mathbf{r})\, u_\lambda(\mathbf{r},t) u_\mu(\mathbf{r},t)} \tag{A.6}$$

$$\mu_{\lambda\alpha} = \mu_{\lambda\alpha}(\mathbf{r}) = E[u_\lambda(\mathbf{r},t)\, u_\alpha(\mathbf{r},t)] \tag{A.7}$$

The solution to eq. (A.4) and eq. (A.5) can be obtained as, Mørk 1989

$$b_{i\beta} = f_{i\lambda}\ E[h_\lambda u_\mu]\ \mu^{-1}_{\mu\beta} - 3\, d_{i\beta\lambda\mu}\, \mu_{\lambda\mu} \tag{A.8}$$

$$\begin{aligned} d_{i\beta\gamma\delta} &= \frac{1}{6} f_{i\lambda}\Big(E[h_\lambda u_\mu u_\nu u_\kappa]\, \mu^{-1}_{\mu\beta}\ \mu^{-1}_{\nu\gamma}\ \mu^{-1}_{\kappa\delta} \\ &\quad - E[h_\lambda u_\mu]\left(\mu^{-1}_{\mu\beta}\, \mu^{-1}_{\gamma\delta} + \mu^{-1}_{\mu\gamma}\, \mu^{-1}_{\beta\delta} + \mu^{-1}_{\mu\delta}\, \mu^{-1}_{\beta\gamma}\right)\Big) \end{aligned} \tag{A.9}$$

Equivalent linear expansion coefficients $b_{i\beta}$, requiring that the variance of the orignial and the equivalent linear drag loading are alike, can be obtained from the equations

$$b_{i\alpha}\, b_{j\beta}\ \mu_{\alpha\beta} = f_{i\alpha}\, f_{j\beta}\ E[h_\alpha\, h_\beta] \tag{A.10}$$

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