

# Numerical simulation of fracture in layered and sandwich structures: A systematic literature review

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## ARTICLE INFO

### Keywords:

Systematic literature review  
Moving mesh method  
Isogeometric analysis  
Crack propagation  
Layered structures

## ABSTRACT

A systematic literature review on numerical strategies suitable to simulate defects and discontinuities in layered and sandwich structures is presented here. Particular care is given to studies whose scope concerns the so-called Moving Mesh (MM) or Isogeometric Analysis (IGA) methods. A general overview of the peculiarities of each approach is also provided. A total of fifteen and twenty-six journal/conference articles were summarised and categorised for MM and IGA methods, respectively. Based on the available literature, it can be stated that MM approaches generally allow a lower computational burden due to the reduced number of re-meshing steps required when compared to standard finite element approaches. Conversely, IGA approaches bring strong advantages in the geometric description of curved shell structures and, due to a non-uniform rational b-spline interpolation function, ensure higher numerical accuracy in stress analysis problems.

## 1. Introduction

The continuous demand for higher mechanical performance materials in several engineering fields has determined the increasing application of composite laminates, sandwich panels, functionally graded materials, and fibre-based strengthening schemes. Under this scope, advanced numerical strategies are needed to predict the failure of laminated and layered structural systems, which tend to exhibit damage onset characterised by interlaminar and intralaminar crack propagation. The former is investigated through numerical strategies that account for the debonding between layers. The latter requires numerical strategies that are able to predict crack tracking and propagation.

In such a context, several strategies exist in the literature to predict the interfacial delamination in layered structures and can be applied at micro-, meso- or macro-scales [1,2]. Two formulations can be distinguished when simulating interfacial defects, i.e. an implicit or explicit crack representation. Implicit crack formulations are implemented within continuous models, in which proper constitutive relationships are introduced in the governing equations to predict local stiffness reduction [3]. However, implicit formulations do not provide any

information about the crack length scale, which is essential to describe fracture phenomena, and are unable to capture the formation of few dominant cracks. In this framework, an accurate choice of mesh discretisation is required and thus suggested to match the mesh size with the internal length involved in the material discontinuities.

In explicit representations, the internal discontinuities are considered to be geometrical entities that need to be updated and conditioned by the changes in their shape. Therefore, formulations based on Finite-Element (FEM) and Boundary-Element (BEM) Methods propose an explicit description of the micro-cracks in structures by updating the current mesh to the evolving cracked geometry [4]. The above-mentioned approaches require specific formulations and numerical tools to quantify the corresponding fracture parameters. For instance, crack evolution can be expressed as a function of classical Fracture Mechanics (FM) variables, such as the Stress Intensity Factor (SIF) or the Strain Energy Release Rate (SERR), whose definition requires the existence of an initial cracked length and a small region in which separation phenomena takes place [5]. The inaccuracy when reproducing crack localisation can be circumvented by proper crack growth criteria, which use coupled relationships in terms of energy and

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<https://doi.org/10.1016/j.jcomc.2022.100294>

stress variables that evaluate the applied loading, crack onset and evolution [6,7].

Alternatively, Cohesive Zone (CZ) models propose an explicit way to simulate debonding phenomena, including crack onset. The CZ approaches were first developed as an alternative to FM, by introducing the possibility of mitigating stress singularities and simulating large-scale decohesion phenomena. To this end, weak interfaces are distributed within the continuous media, and appropriate traction separation damage laws characterise their mechanical response [8,9]. Existing models in the literature are mainly classified as either non-potential-based or potential-based models [10]. Non-potential-based cohesive interaction models are relatively simple to develop because an asymmetric system is not required. However, these are unable to guarantee the consistency of the constitutive relationship for arbitrary mixed-mode conditions as different possible separation paths are not eligible. The traction-separation relationships across fracture surfaces are obtained from a potential function that characterises the fracture behaviour in potential-based models [11]. Note that the existence of a potential function for the cohesive constitutive relationship is addressed in conjunction with the non-negative work for closed processes. Due to the nature of the potential, the first derivative of the fracture energy potential provides the traction (cohesive interactions) over fracture surfaces, and its second derivative provides the constitutive relationship (material tangent modulus). Several potential-based models are available in the literature, such as models with specific polynomial orders, models with exponential expressions, and a model with general polynomials [8,12].

As reported in [10], CZ traction-separation relationships may be obtained by employing theoretical, experimental and computational techniques. For example, a traction-separation relation was obtained for double cantilever beam specimens [13]. Inverse analyses were employed to calibrate a traction-separation relationship to achieve the best predicted global load-displacement curve [14–16]. Based on a measured local displacement field, digital image correlation techniques and inverse analysis were employed to estimate fracture parameters and determine traction-separation relationships [17,18]. Additionally, macroscopic traction-separation relationships were also obtained by considering microstructure in conjunction with multi-scale analysis [19]. Several constitutive relationships of the CZ model have been developed based on effective displacement and effective traction, which easily define various CZ relations such as cubic polynomial [20], trapezoidal [21], smoothed trapezoidal [22], exponential [23], linear softening [24] and bilinear softening [25].

However, the CZ approach shows some numerical limitations, since its accuracy depends on the mesh discretisation, as the direction of crack propagation is restricted by the defined element size and orientation. Moreover, an initial finite stiffness may produce an excess of compliance in brittle solids; and spurious traction oscillations may appear for high stiffness values. Such problems may be partially circumvented by introducing a finer mesh refinement at the crack tip front to obtain a high resolution of the characteristic fracture length of the interface [26]. The resulting model is affected by computational complexities because of the large number of variables and nonlinearities involved at the interfaces.

From a practical standpoint, delamination may affect layered structures and several solutions were developed in industry to improve the debonding resistance [27]. Such solutions are often in the form of Through-The-Thickness (TTT) elements, such as rods or Z-pins. In particular, the Z-pinning approach is widely used in engineering applications since structural safety plays a relevant role, such as in naval and aerospace contexts. Z-pins might be adopted either as strengthening interventions after the localisation of damage or as appropriate details to be considered in the design phase to address stress concentrations [28–31]. It is worth mentioning how the improvement provided by the introduction of TTT reinforcements depends on the geometry and their interaction with the host structural element [32]. Referring to composite

laminates with TTT reinforcements, two computational issues emerge: (i) the adaption of an effective strategy to simulate the TTT reinforcements; and (ii) the modelling of the interfacial delamination. While the former was previously addressed, the modelling of the TTT reinforcements might be performed following different approaches that differ in terms of accuracy. In [33], the authors adopted distributed cohesive elements reproducing Z-pins behaviour with averaging bridging forces. Other approaches were developed to simulate the single Z-pin with spring elements, whose non-linear traction separation laws were formulated resorting to experimental campaigns and for both normal and tangential directions [34,35]. When failure is dominated by mixed-mode delamination, appropriate criteria must be adopted to capture the actual behaviour of the specimens.

Furthermore, layered and sandwich structures might experience intralaminar crack propagation. This might generate a loss in the structural system stiffness and leads to failure modes that are hard to predict. In such cases, the arbitrariness of the crack path requires proper numerical procedures and criteria that can predict crack initiation, propagation, and direction. Most of the models available in the literature are based on the FEM because of its practicability when modelling complex structures, which ensures a good level of accuracy in predicting interfacial variables between dissimilar materials. FEM computational formulations require several re-meshing events to track arbitrary cracking. Although the first FEM approaches have simulated crack propagation using node-release or node-decoupling techniques for prescribed or constrained crack representations [36], re-meshing procedures are typically used nowadays. Nonetheless, a re-meshing procedure is cumbersome and requires high computational effort during the transition procedure [37]. BEM models could be used in FM, in which only the boundaries of the structure are represented by using a mesh discretisation. Such a hypothesis simplifies re-meshing procedures and reduces the computational costs required to generate new elements. However, complexities still remain in the definition of the singular integral [38]. As an alternative to both FEM and BEM approaches, meshfree methods were implemented to avoid the need for mesh discretisation but also to preclude the use of a re-meshing procedure since the current solution is expressed as a function of nodal quantities only [39,40]. The solution accuracy is determined by the influence function and its dependence on the reference nodes. However, such methodology is affected by intrinsic complexities in defining essential boundary conditions, especially when the Kronecker delta property is not verified, leading to high computational costs in the solving procedure.

In the last decades, mesh elements embedding the definition of the crack geometry through the direct modification of the strain-displacement relationship (e.g. the XFEM approach) were developed [41,42]. The basic idea of such formulations is that crack discontinuities are modelled by nonconforming elements with enriched shape functions with discontinuity properties. Non-linear problems require a further extension to predict fracture variables, especially in the presence of frictional effects. The methodology needs a different number of kinematic variables for each node, leading to a total number of mesh points that is dependent on the crack growth [43].

Alternatively, formulations have also been developed based on an implicit definition of the crack area, employing softening damage constitutive relationships or kinematic laws to predict strain localisation effects [44,45]. Known as smeared crack representations, these methods simulate the presence of material discontinuities by constitutive degradation models, which are supposed to affect mesh elements when damage activation conditions are satisfied. The implementation of damage-based constitutive laws has a solid background and makes smeared crack approaches particularly suitable for investigating many structural engineering problems [46–48]. However, such methodologies are typically affected by mesh-dependence phenomena and strain localisation problems of the solution defined in terms of internal material characteristic length.

Others methodologies based on the Discrete Element Method (DEM)

have been formulated in the last decade, and promising results have been obtained [49]. Likewise, the potential of methods based on Moving Mesh (MM) and Isogeometric Analysis (IGA) was also explored in the last decade, hence holding a prominent position within the framework of the element-based discretisation technique. Even though such approaches have different theoretical backgrounds, both allow high accuracy with promising computational costs.

Despite the huge potential shown by MM and IGA methods [50–58], a thorough and complete state-of-the-art literature review is still lacking. This work aims at covering this gap with a systematic review focusing on the use of MM and IGA based approaches for defect simulation in layered and sandwich structures. The authors have decided to follow a Systematic Literature Review (SLR) approach [59] rather than a classical expert review with ad-hoc literature selection. A SLR allows a methodologically rigorous review by formulating an initial set of specific research questions.

The paper is organised as follows: Section 2 briefly describes MM and IGA approaches. Section 3 presents a detailed description of the review methodology. Section 4 discusses the results found with the SLR. Finally, section 5 proposes an answer for each research question and underlines the main conclusions.

## 2. Brief frameworks description

FEM is recognised as one of the most powerful numerical methods to approximate the solution of partial differential equations (PDE). Crudely, it consists in the discretisation of the original geometry into a smaller and finite number of sub-geometry, i.e., finite elements, and transforming the governing PDEs that cannot be solved analytically into a system of algebraic equations. However, such a geometrical approximation, which is an intrinsic peculiarity of classic FEM, might affect the accuracy of the solution or dramatically increase the computational cost if a fine mesh is required. Such an issue is particularly emphasised in structural FM mechanics etc.

Several approaches have been proposed in the literature to minimise the discretisation bias and decrease the computational cost, e.g., local mesh refinement or adoption of high order shape functions. Among others, MM and IGA demonstrated their capability in simulating fracture phenomena in layered structures. The following subsections aim to provide a general overview of the fundamental concepts of MM [60] and IGA [61].

### 2.1. Moving mesh approach

MM approach addresses the capability of withstanding large mesh distortions to lessen the need for re-meshing events. In FM problems, high computational efficiency is especially desired in regions where strain localisation appears, i.e., around the crack tip. Therefore, MM approach guarantees local refinement that evolves as the crack propagates and ensures continuity of the solution. In this context, an important role is played by the kinematic description of the continuum that determines the relationship between the deforming continuum and the finite grid or mesh of computing zones [60]. The arbitrary Lagrangian-Eulerian (ALE) description in MM algorithms is typically adopted. In the ALE description, the computational nodes are moved in some arbitrarily specified way to give a continuous rezoning capability. Because of this freedom in moving the computational mesh offered by the ALE description, greater distortions of the continuum can be handled.

The MM approach was successfully implemented to simulate interlaminar and intralaminar crack propagation phenomena [62]. For more details concerning the theoretical framework and the numerical implementation, the reader can refer to [60,62].

### 2.2. Isogeometric based approaches

IGA has been developed to combine the advantages of advanced CAD systems with numerical approaches. To accomplish this, a non-uniform rational basis spline (NURBS) approach is adopted for the geometrical representation and employed in the analysis framework without making any geometrical approximation [61,63]. Consequently, the coupling of CAD and numerical analysis solvers into a unified framework reduces the associated burden of remeshing algorithms and thus minimises the computational cost to a great extent [64,65].

It is worth remarking that IGA is applied beyond the NUBRS basis functions. Other CAD tools were successfully investigated, such as T-spline [66], Polynomials splines over Hierarchical T-meshes [67], Locally Refined splines [68] and Hierarchical spline [69]. IGA peculiarities allowed its implementation in conjunction with other concepts arising from computational mechanics in order to successfully simulate both contacts and crack propagation phenomena [64], such as:

- 1 Explicit isogeometric enrichment for modelling material interfaces and cracks exactly [70].
- 2 Cohesive interlaminar debonding [57]. This method relies upon the ability to specify NURBS and T-splines' continuity through a knot insertion process.
- 3 Linear Elastic Fracture Mechanics [71], in which the partition of unity method is adopted to characterise strong dimensional discontinuities and crack tip singularities accurately.
- 4 As a variation of the eXtended Finite Element Method (XFEM), so that the method is usually referred to as XIGA (eXtended IGA) [72].

For a detailed description of IGA based approaches, readers are referred to [65], in which the authors provided a detailed overview of the method and underline the differences with respect to classical FEM.

## 3. Review methodology

This section provides an overview of the methodology adopted for the systematic literature review. Fig. 1 depicts a synoptic representation

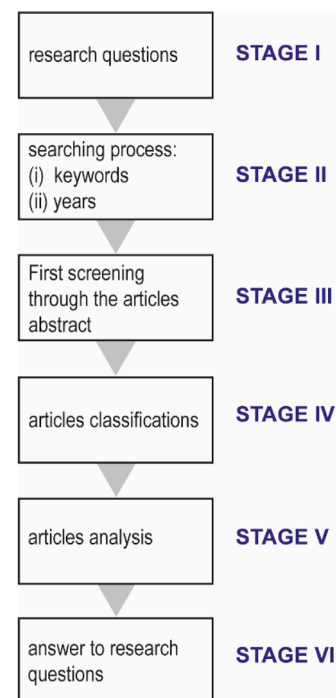


Fig. 1. Flowchart of the methodology adopted in the SLR.

**Table 1**

Search strings used in Web of Science.

Sub-category	Web of Science search string
MM	TS = (("Layered Structures" OR "Sandwich Structures" OR "Sandwich Panels" OR "Laminated" OR "COMPOSITES") AND ("Crack Propagation" OR "Delamination" OR "Debonding")) AND ("Moving Mesh" OR "Moving Finite Element" OR "ALE")
IGA	TS = (("Layered Structures" OR "Sandwich Structures" OR "Sandwich Panels" OR "Laminated" OR "COMPOSITES") AND ("Crack Propagation" OR "Delamination" OR "Debonding")) AND ("isogeometric")

**Table 2**

Search strings used in Scopus.

Sub-category	Scopus search string
MM	TITLE-ABS-KEY (("Layered Structures" OR "Sandwich Structures" OR "Sandwich Panels" OR "Laminated" OR "COMPOSITES") AND ("Crack Propagation" OR "Delamination" OR "Debonding")) AND ("Moving Mesh" OR "Moving Finite Element" OR "ALE")
IGA	TITLE-ABS-KEY (("Layered Structures" OR "Sandwich Structures" OR "Sandwich Panels" OR "Laminated" OR "COMPOSITES") AND ("Crack Propagation" OR "Delamination" OR "Debonding")) AND ("isogeometric")

of the workflow, in which all the followed stages are identified. Throughout the next section, both graphical representations and tables are presented to support the adopted methodology within the searching process and the definition of the inclusion/exclusion criteria concerning the research papers under analysis.

### 3.1. Research questions

The first stage (Fig. 1) aims to define relevant research questions that drive the SLR. In this regard, the following research questions were formulated for a better insight into the numerical methods based on MM and IGA:

- RQ 1 How is the research topic addressed?
- RQ 2 Which are the main advantages of MM and IGA approaches?
- RQ 3 Why are MM and IGA approaches used for the investigation of the crack propagation phenomena?
- RQ 4 Which are the main limitations of MM and IGA approaches?
- RQ 5 What are the opportunities that could trigger future research streamlines and innovative applications?

### 3.2. Searching process

The searching process was conducted in February 2022 through the abstract database from SCOPUS and Web of Science repositories. The authors limited the search to journal and conference articles published within the past ten years (Feb 2012–Feb 2022). The searching process has been performed using the Advanced Search tool available in SCOPUS and Web of Science and constraining the search to: (i) Document

Title; (ii) Abstract; (iii) Keywords. The first screening involved the selection of the structural systems under investigation for which the following keywords were used: "Layered Structures"; "Sandwich Structures"; "Sandwich Panels"; "Composites"; "Laminated". The second screening involved the journal articles or conference proceedings containing at least one of the subjects under investigation, in specific: "Crack Propagation"; "Delamination"; "Debonding". The third and last screening aims to cluster the research articles according to the two sub-categories under investigation, the MM and the IGA. This has been accomplished by filtering the articles whose keywords match at least one of the following: "Moving Mesh"; "Moving Finite Element"; "ALE"; "Arbitrary Lagrangian and Eulerian" for MM; and "Isogeometric" for IGA. For the sake of clarity, Table 1 and Table 2 report the search strings used for both the sub-categories.

### 3.3. Screening based on the article abstract reading

All the abstracts were read, and the papers not considered pertinent as well as the clones were removed. On the contrary, when the authors were aware of studies not selected in the search process but considered helpful for the discussion, they were included in the lists. Consequently, 42 articles were classified as eligible to conduct the SLR, of which 16 concerning MM and 26 for IGA.

### 3.4. Articles classifications

Articles can be clustered according to two research fields: MM and IGA based approaches. Table 3 and Table 4 present the classification adopted for the selected articles and for both the MM and IGA clusters,

**Table 3**

MM: articles classification.

Reference	Problem simulated	Results validation	Rate dependent effects
[73]	Interlaminar crack propagation	Yes	Yes
[74]	Interlaminar crack propagation	Yes	Yes
[75]	Interlaminar crack propagation	Yes	No
[76]	Intralaminar crack propagation	Yes	Yes
[77]	Intralaminar crack propagation	Yes	Yes
[78]	Intralaminar crack propagation	Yes	No
[79]	Interlaminar crack propagation	Yes	Yes
[80]	Interlaminar crack propagation	Yes	Yes
[81]	Interlaminar crack propagation	Yes	No
[82]	Interlaminar crack propagation	Yes	Yes
[83]	Interlaminar crack propagation	Yes	Yes
[84]	Interlaminar crack propagation / Z-pin	Yes	Yes
[85]	Intralaminar crack propagation	Yes	No
[86]	Intralaminar crack propagation	Yes	Yes
[87]	Intralaminar crack propagation	Yes	No
[88]	Intralaminar crack propagation	Yes	No

**Table 4**  
IGA: articles classification.

Reference	Problem simulated	Results validation	Rate dependent effects
[89]	Simulation discontinuities	Yes	No
[90]	Simulation discontinuities	Yes	No
[91]	Interlaminar crack propagation	Yes	No
[92]	Simulation discontinuities	Yes	No
[93]	Intralaminar crack propagation	Yes	No
[94]	Intralaminar crack propagation	Yes	No
[95]	Intralaminar crack propagation	Yes	No
[96]	Intralaminar crack propagation	Yes	No
[97]	Intralaminar crack propagation	Yes	No
[98]	Intralaminar crack propagation	Yes	No
[99]	Intralaminar crack propagation	Yes	Yes
[100]	Intralaminar crack propagation	Yes	Yes
[101]	Simulation discontinuities	Yes	No
[102]	Intralaminar crack propagation	Yes	Yes
[103]	Simulation discontinuities	Yes	No
[104]	Intralaminar crack propagation	Yes	No
[105]	Interlaminar crack propagation	Yes	No
[106]	Interlaminar crack propagation	Yes	No
[107]	Interlaminar crack propagation	Yes	No
[108]	Interlaminar crack propagation	Yes	No
[109]	Interlaminar crack propagation	Yes	No
[110]	Interlaminar crack propagation	Yes	No
[111]	Interlaminar crack propagation	Yes	No
[112]	Interlaminar crack propagation	Yes	No
[113]	Interlaminar crack propagation	Yes	No
[114]	Intralaminar crack propagation	Yes	No

respectively. The selected articles have been chronologically sorted, from the oldest to the most recent, and categorised based on: (i) the object of simulation; (ii) the validation of the results; (iii) the capability to take into account rate-dependent effects.

#### 4. Articles analysis

MM approaches were successfully adopted for simulating interlaminar defects (interface delamination) as demonstrated in Table 3. Recent developments were also implemented for problems dealing with intralaminar defects, hence extending MM for the simulation of crack kinking phenomena in layered structures.

To what concerns IGA approaches, Table 4 suggests their potential lies in the geometrical representation of spatial layered shell structures. Some studies were devoted to investigating interlaminar discontinuities, whereas more recent works can also simulate crack evolution using enriched approaches and a CZ constitutive law. All the studies provided a complete validation of the results against experimental, numerical or analytical data extracted from the literature. In IGA based approaches, the general trend is to neglect the dynamic phenomena. In contrast, a percentage equal to 66% of the total number of selected articles can consider rate-dependent effects if one refers to MM approaches (Table 3).

Fig. 2(a) and Fig. 3(a) represent the distribution of the selected articles in terms of the phenomena considered (interlaminar crack propagation, interlaminar crack propagation / Z-pin and intralaminar crack propagation), and the numbers of published articles per year given in Fig. 2(b) and Fig. 3(b).

##### 4.1. Literature studies based on moving mesh (MM) approach

The first selected studies on a numerical model that couples ALE formulation with the FM concept has been proposed in [73,74]. Even though the general framework for simulating the dynamic crack propagation process using an ALE formulation can be applied to other problems apart from the delamination phenomenon, it is classified as *interlaminar* since it is unable to reproduce crack kinking. In this case, the computational point around the crack tip is rigidly moved following a straight crack propagation line. In the remaining parts, the smoothing

equation affects the motion of the computational points and reduces the need of re-meshing events when compared with standard FE approaches.

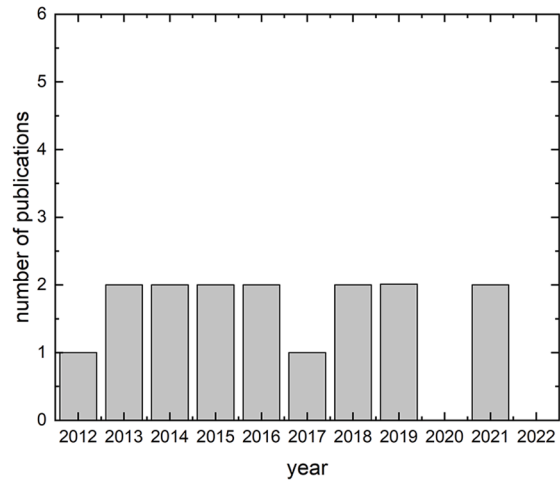
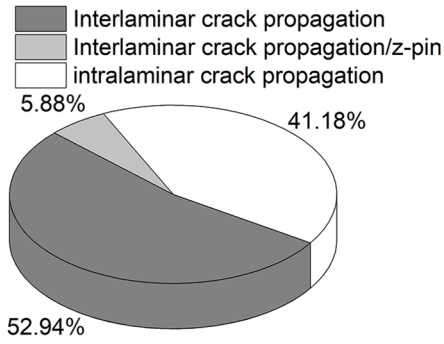
In [75], the authors developed a model to simulate dynamic interfacial debonding phenomena in FRP reinforced concrete beams. In this framework, ALE equations were considered in the concrete medium and within a plane-stress formulation, whereas the FRP strengthening was modelled using beam elements according to the Timoshenko beam theory. A pre-existing crack was considered, and the fracture variable was evaluated using the J-integral decomposition in the vicinity of the crack tip region, hence allowing for crack growth propagation on the basis of an explicit crack tip speed criterion. The same authors then extended this procedure in [79] for composite laminates, employing both a static and dynamic analysis. Both studies showed a good agreement with experimental tests or numerical data taken from the literature, and the authors highlighted that the accuracy for crack growth prediction is guaranteed by using a low number of elements compared to those required from the existing numerical solutions.

In the study given in [81], the same concept was developed to investigate composites made of unidirectional carbon fibre epoxy (T800/924C) tested under three-point-bending tests. In this case, the 3D discretisation allowed the description of the delamination front through the thickness of the sample.

Furthermore, a computational model based on MM methodology for debonding mechanisms in multilayered composite beams was given in [80] by using a shear deformable beam coupled with a moving-cohesive interface. Only the mesh nodes within the interface region were moved on the basis of the predicted fracture variables; a continuous rezoning procedure was applied in the interface region unaffected by the crack evolution to avoid highly distorted elements (Fig. 4). One can note how the use of beam elements for the modelling of structural layers allows a strong reduction of the computational cost compared to other works [75, 79].

In this framework, only the nodes of the computational mesh of the interface region were moved on the basis of the predicted fracture variables, reducing mesh distortions by using continuous rezoning procedures (Fig. 5).

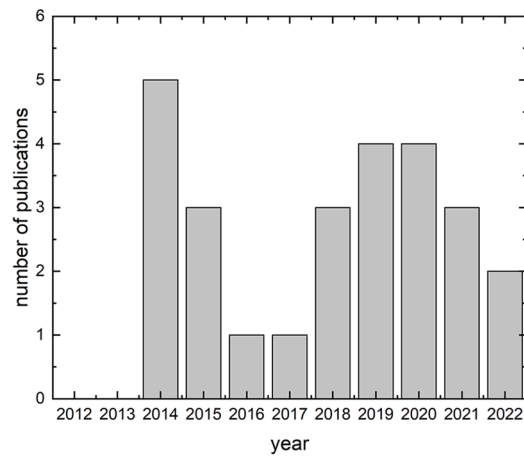
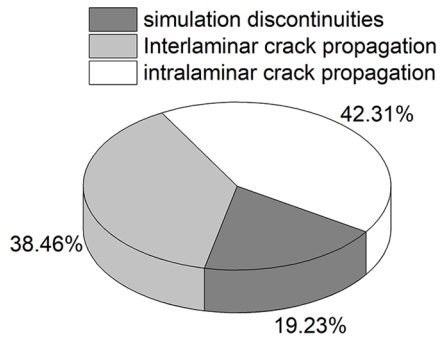
The use of moving mesh methodology in the proposed model is able to introduce nonlinear interface elements in a small region containing



(a)

(b)

Fig. 2. MM: (a) distribution of papers in terms of phenomena simulated, (b) number of publications in the period 2012–2022.



(a)

(b)

Fig. 3. IGA: (a) distribution of papers in terms of phenomena simulated, (b) number of publications in the period 2012–2022.

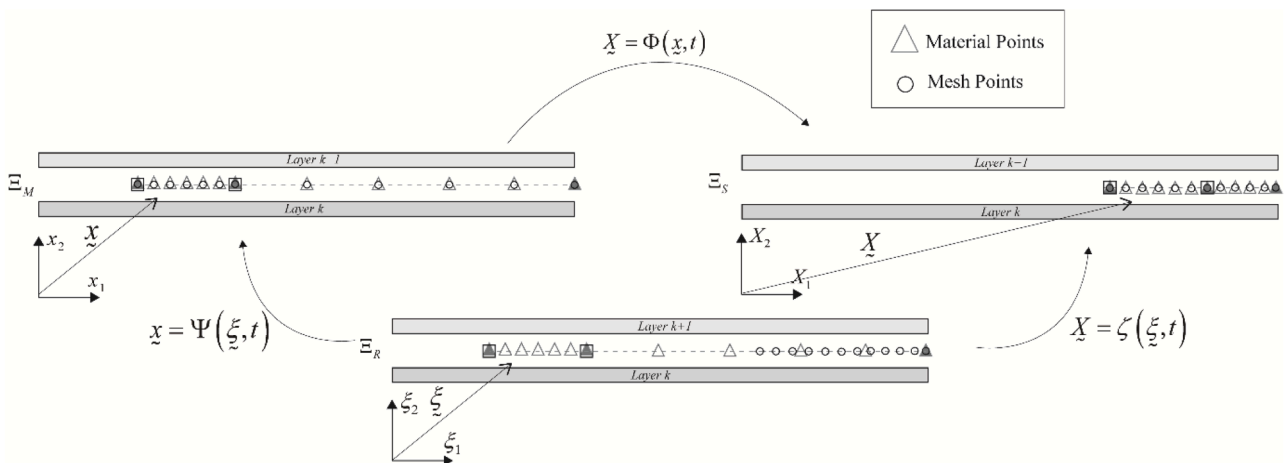


Fig. 4. ALE formulation: kinematic and referential configurations (adapted from [80]).

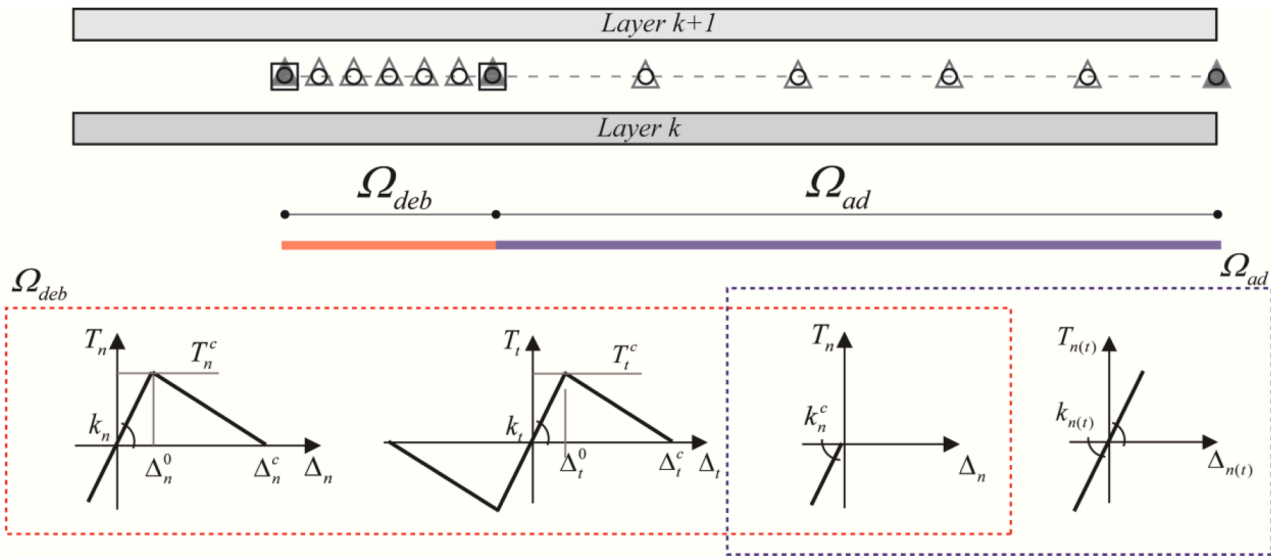


Fig. 5. Interface moving boundary: debonded and perfect adhesion regions (adapted from [80]).

the process zone, reducing the numerical complexities and efforts, typically involved in a standard cohesive approach. Such a model was also adopted to investigate dynamic fracture phenomena for which a rate-dependent CZ law was implemented. Further development of the numerical model reported in [80] includes the capability to simulate crack initiation and crack coalescence phenomena in a multilayered composite beam [82]. Concerning the debonding phenomena, a procedure that allowed identifying the local where the onset of interfacial mechanisms occurs was still missing for the study of the cracked length evolution. In [82], two steps were considered in the analysis: (i) the evaluation of the crack initiation (ii) the evolution mechanism. In particular, a model with a coarser mesh is firstly assumed, yet with a proper mesh to consider crack initiation. Subsequently, since ALE cohesive elements were introduced in the model, a local enrichment is required to reproduce crack tip motion at the crack tip front. This procedure is implemented by means of a stop condition on the basis of the crack initiation criterion, and a restart procedure from the last converged step is achieved. In [82], the authors report a synoptic representation of the numerical procedure and the computational algorithm implemented in the FE environmental program.

In the context of TTT reinforcements in composite materials, the study given in [84] extends the governing equations and procedure described in [82] for Z-pinned composite laminates and sandwich panels [83]. A set of non-linear springs fixed to the material frame is introduced to model the effect of Z-pins. Furthermore, it is noteworthy recalling two CZs represent the model domain [84], the first defined in the moving or spatial domain, while the second fixed to the material domain. In order

to simulate the skin/core delamination of sandwich panels, the model adopted in [80,82] was generalised through the introduction of 2D plane-stress elements rather than shear-deformable beams for the skins for simulating the core region [83].

The first applications of MM to simulate crack evolution and direction in 2D media were developed in [76,77]. The authors investigated the dynamic crack propagation on different composite samples subjected to extreme loading conditions, e.g. impact loading. Several loading conditions were considered, including mixed-mode fractures achieved by an eccentric impact loading. Thus, the experiments allowed the calibration of the moving finite element model used to compute the time histories of the FM parameter during crack propagation. This work is the extension of the pioneering work developed in 1980 by the same authors [115].

A hybrid XFEM based method was proposed in [78]. Its hybrid nature resorts to the replacement of the XFEM interpolation method in the vicinity of the crack tip by one provided by a moving mesh patch. Such an approach is combined with a formed interaction integral adopted in conjunction with the maximum hoop stress criterion to compute the crack angle direction.

In [85], a numerical framework that combines concepts arising from structural mechanics and MM methodology was implemented in a unified workflow aiming to predict inclined crack growth on the basis of FM variables (Fig. 6). It was implemented for plane-stress problems and the moving computational nodes around the crack tip were modified, starting from a fixed referential coordinate system and based on a crack growth criterion, to predict the directionality and advancement of the

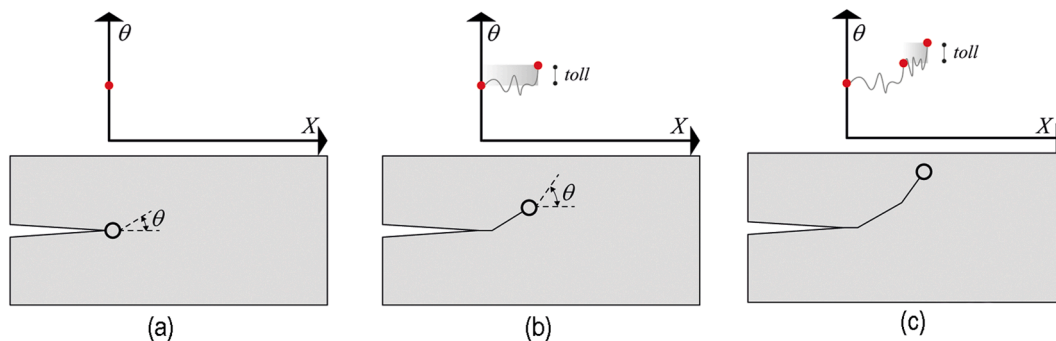


Fig. 6. Schematic representation of the proposed algorithm: (a) crack onset condition satisfied, evaluation of  $\theta$ ; (b) crack propagation in direction until the angle variation predicted is lower than  $tol\theta$ ; (c) tolerance condition is satisfied, new definition of the computational nodes (adapted from [85]).

crack. In addition, a smoothing and regularisation method based on Laplace’s equation was implemented in the remaining region. This ensured the consistency of mesh motion with small distortions and precluding a change in the mesh typology hence reducing the need for a re-meshing procedure. The model was validated using numerical and experimental results available from the literature. Furthermore, sensitivity and parametric analysis proved the low mesh dependency of the formulation. It is worth noting that such a procedure was formulated in a general way, which makes it suitable for simulating crack propagation in 2D media. In such a scope, the same authors implemented the proposed numerical scheme within the framework of sandwich panels for the simulation of the crack kinking phenomenon that typically involves the core region [116,117].

Recently, the numerical model proposed in [85] was generalised to account for dynamic crack growth phenomena [86]. In this case, a rate-dependent criterion was implemented and expressed in terms of crack angle and driven forces through the definition of the energy release rate. However, some concerns were found after the re-meshing events. In specific and when the tolerance criterion was satisfied, semi-automated re-meshing procedures were performed to transfer the nodal variables of the distorted as initial conditions of the new computational points due to the dynamic nature of the problem.

The concepts adopted in [85] were further applied in [87] to reproduce crack propagation phenomena in functionally graded materials. Lastly, the authors have generalised the approach developed in [85] to include thermo-mechanical loading conditions aiming to achieve a multi-physics character, see [88].

4.2. Literature studies based on Isogeometric Analysis (IGA) approach

From the IGA based studies, the study given in [89] presents a continuum shell formulation that is coupled with NURBS to generate the geometry of the surface. The behaviour of the shell in the thickness direction was computed by interpolating a high order B-spline. Delamination was modelled by introducing a strong discontinuity by means of a knot insertion in the thickness parametrisation, i.e., adding a new knot into the existing knot vector without changing the shape of the curve.

A similar study is presented in [90]. The authors investigated the IGA layerwise theory, in which the main idea relies on the use of different shape functions for in- and out-of-plane directions. Several numerical examples with pre-existing delamination were tested and compared with closed analytical solutions or classical FE literature approaches. The model is inspired by the observation that the continuity properties of a basis can be easily adjusted at the knots that represent physical interfaces within a laminate (Fig. 7). The method was applied in cross-ply laminates under cylindrical bending, and, again, the numerical accuracy was confirmed by an excellent agreement with analytical and classical FE solutions from the literature. The main outcome provided was that IGA layerwise approach performs better than Lagrange polynomial-based counterparts and for the same number of degrees of freedom, thus resulting in significant computational improvement. The

latter advantage justifies its further use for the implementation of delaminated patches, as demonstrated in [92].

Later works adopted IGA approaches since they use more accurate and efficient shape functions, particularly for describing the displacement field in the out-of-plane direction of shell elements. Nevertheless, one can point out other important research works that apply IGA for the simulation of the delamination [91]. In this study, an isogeometric cohesive element was formulated to simulate 2D and 3D delamination. The formulation was made fully automatic and needed minimal user intervention. The knot insertion algorithm was implemented directly from CAD data with an automatic generation of the cohesive weak interface, in which debonding may evolve. Validation was achieved against single and multiple delamination problems, and results were compared with numerical and analytical literature data. Other works, such as [97,98,106], proposed numerical models using 3D shell structures aiming to simulate the arbitrary delamination process, see Fig. 8. Such an algorithm allowed different discretisation refinement through the thickness on the basis of some criteria such as stress/deformation state or cracks initiation.

Similarly, one can find a flexible and efficient method in [113] for controlling the continuity of the out-of-plane approximation. A finer detail was only applied in areas of the structure where it is strictly required. Knot insertion was adopted to automatise an adaptive refinement of the shell model at arbitrary interfaces, thereby making it possible to predict multiple initiation and growth and coalescence delamination phenomena.

Following the same principles, an extended IGA approach coupled with cohesive methods to simulate weak interfaces is also reported in the literature [105,109,110]. In such cases, the main contribution of the IGA is in the description of the discontinuity field, whereas the approximation is locally enriched by additional degrees of freedom. The level set method is adopted to track discontinuity location with respect to existing mesh and to perform strong enrichment at the crack surface. It is noteworthy to mention that the same model was extended for the simulation of extremely cold environments [111] and to account for geometric nonlinearities [112]. This was achieved by using a reference configuration to reduce the difficulties associated with the computation of the crack opening, especially in the case of large rotations. The need that some problems pose, as the description of the stress-strain response at contact among different composite components like periodic materials, led to the development of hybrid isogeometric finite elements as given in [107].

Furthermore, IGAs have also been adopted to simulate intralaminar stationary cracks and their evolution in layered structures. In this context, some authors [93] proposed an IGA formulation within an orthotropic media by combining XFEM enriched functions and level set functions. This allowed an enrichment based on the collocation of the control points with respect to the crack tip points. In other words, the level set functions were used to distinguish the control points corresponding to the crack tips and crack faces. Similarly to what was addressed in [93], in [94,104], the authors adopted extended IGAs to simulate the functionally graded material’s crack propagation by

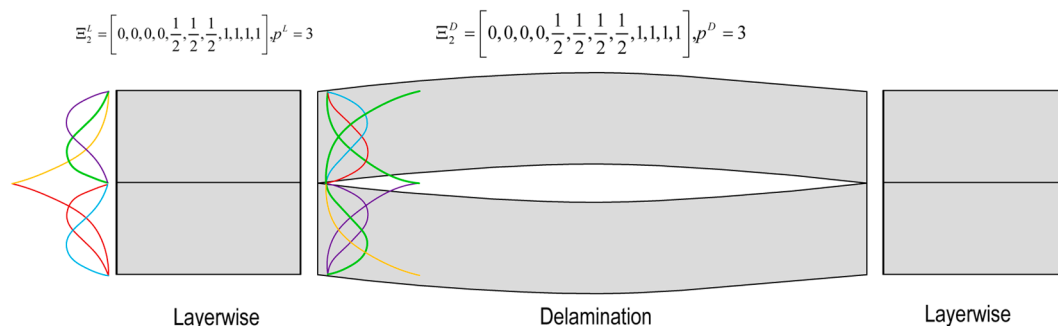


Fig. 7. Schematic representation of delamination modelling (adapted from [90]).



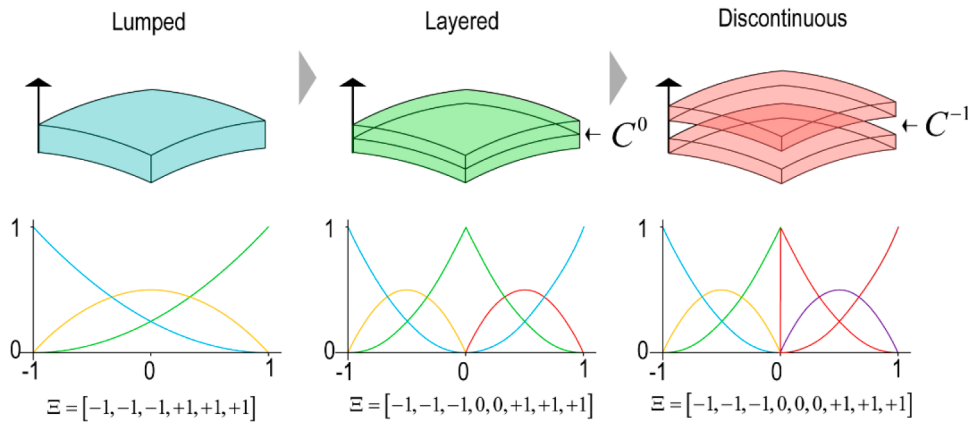


Fig. 8. Introducing weak and strong discontinuities in the thickness direction of the shell by knot insertion (adapted from [106]).

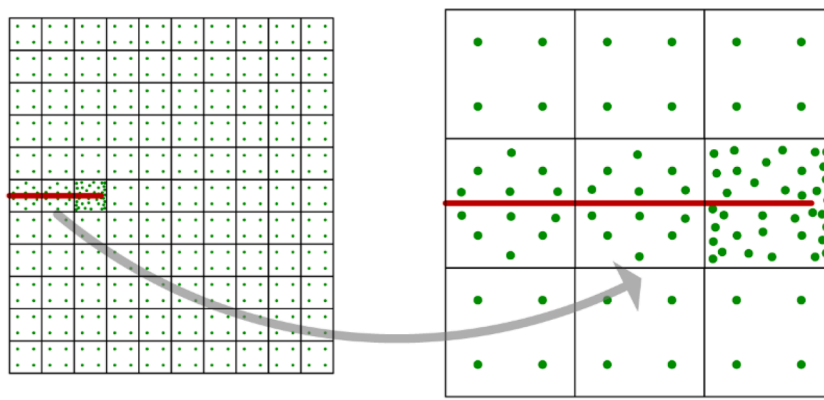


Fig. 9. Schematic representation of the generation of Gauss points in split and tip elements of crack (adapted from [94]).

adopting a first-order shear deformation theory for the investigation of out-of-plane loading conditions. Nonetheless and similarly to MM based methods, the region surrounding the crack tip needs higher computation accuracy in order to better compute the fracture variable that drives the crack propagation phenomenon. In [94], a standard Gaussian quadrature was employed in order to perform the numerical integration. A  $2 \times 2$  Gauss quadrature was adopted for elements with non-intersecting discontinuities, while the refinement of the Gauss quadrature was performed along the crack propagation path (Fig. 9). One can note how such an approach appears less efficient if compared with [85], in which the refined region moves along the crack propagation and the number of degrees of freedom is unchanged during the whole simulation.

The concepts expressed in [94] were also implemented in [96]. In addition, the authors developed a numerical model that can account simultaneously for inter and intra-laminar crack propagation. In [99, 100, 102], a refined IGA based multi-layer shell model was presented for the simulation of both intralaminar and interlaminar progressive damage. Specifically, the model accounts for intralaminar damage by coupling a smeared damage model in the direction of the material symmetry planes. Scalar-based damage variables (ranging from 0 to 1) reduced the initial elastic properties without affecting the orthotropic nature of the material. In addition, a zero-thickness CZM interface formulation was implemented to consider delamination among layers of laminated composite structures modelled with Kirchhoff–Love shells.

Recent developments were presented in [83]. The authors proposed a workflow in which the isogeometric collocation method is adopted to simplify the adaptive mesh refinement scheme for phase-field modelling. It represents a pioneering application of the coupled approach based on IGA and phase-field, particularly suitable for modelling fracture phenomena in fibre-reinforced composites.

## 5. Final remarks and answers to the research questions

A SLR on the potential of Moving Mesh (MM) or Isogeometric Analysis (IGA) based approaches for the simulation of defects in layered and sandwich structures was presented. From the analysis conducted on selected literature, an answer to the initial five research questions is reported as follows.

### RQ 1 How is the research topic addressed?

The numerical simulation of defects and crack propagation in layered and sandwich structures was addressed in the scope of MM and IGA methods. Different algorithms and numerical approaches were included in the analysis aiming to give a solid overview of how the main limitation of classical FEM based methods are overcome, either in terms of accuracy or computational cost. A total of forty-two articles were selected within the time period that includes the last decade, i.e. from 2012 to 2022.

### RQ 2 Which are the main advantages of MM and IGA-based approaches?

MM based models demonstrated their potential in simulating both interlaminar and intralaminar crack propagation in layered and sandwich structures. For simulating laminate delamination, MM tends to be adopted in conjunction with CZ, for which proper crack propagation criteria are implemented in order to prescribe the mesh motion along the interfaces. Such an approach seems to strongly reduce the degrees of freedom involved in the structural model, as a refined computational grid is considered only around the crack tip regions. Furthermore, MM

based models are also adopted to simulate intralaminar crack propagation. To this end, MM is being coupled with FM and solid mechanics concepts to find a unified framework for inclined crack propagation.

To what concerns IGA based approaches, it has been applied to (i) enrich the modelling to better represent material interfaces and cracks [66]; (ii) problems related to cohesive interlaminar debonding [55], in which the method takes advantage of the ability to specify NURBS and T-splines' continuity through a knot insertion process; (iii) Linear Elastic Fracture Mechanics [67], in which the partition of unity method is adopted to characterise strong dimensional discontinuities and crack tip singularities accurately; and (iv) as a variant of the eXtended Finite Element Method.

### RQ 3 Why are MM and IGA approaches used for the investigation of the crack propagation phenomena?

MM accurately simulates crack propagation by involving a limited number of computational points since only the region around the crack tip is typically enriched.

IGA eliminates the need for a bridge with CAD tool when mesh refinement is required because it is able to model exactly the geometry, even for a coarse mesh. Furthermore, the geometry is kept even when the mesh is refined [65].

### RQ 4 Which are the main limitations of MM and IGA approaches?

Concerning MM approaches, these seem to be unable to account for crack coalescence phenomena that typically involve layered structures.

Concerning IGA based approaches, the following limitations appear to exist: (i) lack of a suitable volume discretisation from given CAD boundary representations; (ii) efficient integration schemes and suitable error estimators; and (iii) few studies address dynamic crack propagation phenomena.

### RQ 5 What are the opportunities that could trigger future research streamlines and innovative applications?

MM full potential might be still explored for the simulation of 3D problems and by introducing a proper numerical scheme that accounts for fracture coalescence. Similarly, IGA is required to explore crack propagation simulation in 3D problems. The authors do not exclude future applications of MM, and IGA approaches in conjunction to study multidimensional problems involving layered and sandwich structures.

### Author contribution statement

**Marco Francesco Funari:** Conceptualisation, Methodology, Writing-Original draft preparation. **Luís C. M. da Silva:** Methodology, Writing-Original draft preparation. **Saverio Spadea:** Methodology, Writing-Original draft preparation. **Paolo Lonetti:** Methodology, Writing-Reviewing and Editing. **Paulo B. Lourenço:** Writing-Reviewing and Editing, Funding.

### Conflict of interest

The authors declare no conflict of interest

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