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Solving a harvest scheduling optimization problem with constraints on clearcut area and clearcut proximity

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Abstract

This study aims at solving a harvesting scheduling optimization problem with constraints on the clearcut area with additional constraints on clearcut proximity. The objective function is defined as the net present value generated by harvesting discounted by a penalty for each clearcut. This problem arises to reduce the negative environmental impact of excessive harvesting. We propose the connected-bucket model, the so-called bucket model with additional constraints on bucket connectivity and two definitions of stand adjacency, and a Dantzig–Wolfe decomposition. The decomposed model is solved by branch-and-price and the connected-bucket model by a general-purpose mixed integer programming solver (CPLEX). We compare the quality of the solutions obtained with both approaches for real instances. The branch-and-price approach found better solutions for the majority of the instances.

Keywords: integer programming; Dantzig–Wolfe decomposition; branch-and-price; bucket formulation; connectivity

1. Introduction

One of the most studied harvest scheduling problems by the operations research community aims at optimizing the net present value generated by harvesting subject to nonspatial constraints, for

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example, on volume or ending age, and spatial constraints. Volume constraints ensure a regular production of timber, mainly to guarantee that the industry is able to continue operating with similar levels of machine and labor utilization. The ending age constraint helps to prevent the model from overharvesting the forest. The most common spatial constraints are on the area of each clearcut. A clearcut is a continuous area of forestland from which all merchantable trees have recently been harvested, and it is surrounded by nonharvested areas, or in other words, a harvested cutblock that is not contained in other harvested cutblock (in mathematics, a maximal harvested cutblock with respect to set inclusion). Clearcut constraints preclude a clearcut from exceeding a maximum threshold area in any given time period or over a green-up time. Generally, green-up standards are applied when a proposed cutblock with an even-aged silvicultural system, such as clearcut or seed tree, is located adjacent to a previously harvested cutblock or cutblocks that are also under an even-aged management system, and these areas together will exceed the maximum threshold area. The green-up time usually refers to the amount of time it takes for a harvested cutblock to regenerate (e.g., for the trees to reach a specific height or age). These constraints have been used to decrease the environmental damage caused by harvesting, for example, on soil, water, biodiversity, and wildlife. See McDermott et al. (2010) for a detailed and systematic comparison of environmental forest policies and enforcement in 20 countries worldwide.

The most common type of harvesting/regeneration in the Czech Republic are clearcuts with replanting of new trees. According to the report produced by Fern in March 2008 (Fenton et al., 2008), about 15,000 ha of clearcuts are created in the country every year. Almost 84% of regeneration is planting or seeding. Law limits the maximum size of clearcuts to 1 ha (2 ha in floodplain and pine forests and on inaccessible mountain slopes). This practice has been repeatedly criticized for soil degradation and biodiversity impacts. Natural conditions are radically different in Central Europe: for example, average size of gaps created by fallen trees in natural beech forests in Slovakia is 250 m^2 and the gaps never exceed 0.4 ha. Concerning the situation of Czech forests, scientists stated in May 2006 that "consequences of clearcutting are serious," including soil damage, erosion, and radical impact on biodiversity, and "trees planted in the clearing tend to be more vulnerable to pests and wind," "legislation must significantly reduce or eliminate clearcutting. Small area shelterwood felling and selective harvesting methods should be preferred." Since then, two measures that have been progressively implemented in the Czech Republic to reduce the clearcutting impact are to reduce the size of the clearcuts and to prevent the creation of clearcuts within a given threshold distance from each other. Management systems addressing restrictions on the distance between clearcuts is most common in the case of *Picea abies* (spruce) and *Pinus sylvestris* (pine). The main goal of these restrictions is to make good stand borders of the exposed clearcuts to the wind. The aim of this work is to solve the harvest scheduling problem with constraints on the clearcut area and the additional restrictions on the clearcut proximity. To help reduce the number of clearcuts, the net present value generated by harvesting is discounted by a penalty for each clearcut. As far as we know, this problem has not yet had the attention of the operations research community.

Operation researchers have been discussing the so-called area restriction model, ARM (Murray, 1999), to model constraints on clearcut area. This approach enables one to harvest two or more adjacent stands in the same period unless the combined area is greater than the limit. Several definitions of adjacency have been used. The most common are the weak and strong adjacency (Goycoolea et al., 2005). Two stands are weak adjacent if they share at least a single point. Two stands are strong adjacent if they share a boundary with positive length. A harvested cutblock is a

Table 1
Models used to mostly solve the single-harvest ARM

Model	References
Path formulation	Martins et al. (1999); McDill et al. (2002); Murray and Weintraub (2002); Crowe et al. (2003)
Cluster formulation	Martins et al. (1999); McDill et al. (2002); Martins et al. (2005); Goycoolea et al. (2005); Vielma et al. (2007)
Bucket formulation	Constantino et al. (2008)
Full adjacent unit formulation	Gharbi et al. (2019)

Table 2
Exact approaches and heuristics based on mathematical programming enabling the user to overcome the size of the ARM models

Approach	Model on which the approach is based	References
Column generation and variable fixing heuristic	Cluster formulation	Martins et al. (2005)
Branch-and-price	Cluster formulation	Epstein et al. (2003); Martins et al. (2012)
Branch-and-cut	Path formulation	Tóth et al. (2013); Constantino and Martins (2018)
Dantzig–Wolfe decompositions and matheursitic	Bucket formulation	Martins et al. (2015)

set of harvested stands such that, between each pair of stands, there is a sequence of stands, each one adjacent to the next. Different adjacency definitions might be adopted in other spatial contexts as wildlife and habitat conservation. For example, two stands can be considered adjacent if they are within a certain distance from each other or if they share a boundary longer than a threshold value (Walters, 1996). See Kaya et al. (2016) and Yoshimoto and Asante (2018) for a broader view on exact integer programming approaches and heuristics for the ARM. A bibliography revision on ARM with green-up can be read in Borges et al. (2015).

In this study, we are interested in methods that are able to provide an optimal solution, at least in theory (exact methods), or in general, approaches that are able to provide a measure of the solution's quality. Exact approaches mostly consider a single harvest over the planning horizon for individual stands, that is, the minimum rotation in a stand is longer than the planning horizon. Table 1 summarizes some of the main models for the single-harvest ARM found in the literature.

Constraints on maximum clearcut size are modeled in the path formulation as cover constraints (also called path constraints). In the cluster and full adjacent unit formulations, those constraints are related directly with the variables. The linear programming relaxation of the cluster formulation is tighter than those of the bucket formulation (Martins et al., 2012) and the path formulation (Goycoolea et al., 2005), and there is no dominance relationship between the path and bucket formulations (Goycoolea et al., 2005). Solving these formulations via a general mixed integer programming solver (implementing branch-and-bound based approaches), with all variables and constraints, has limitations in terms of size. Table 2 summarizes some of the approaches found in the literature, which provide a measure of the solution's quality, enabling the user to overcome the size of the ARM models.

Borges et al. (2015), John and Tóth (2015), McDill et al. (2016), Yoshimoto and Konoshima (2016), and Yoshimoto and Asante (2018, 2021) addressed the ARM with multiple harvests. In these third initial studies, constraints on the maximum clearcut size are modeled as cover constraints. In the last three references, the authors proposed optimization models that use common matrix algebra (Yoshimoto and Konoshima, 2016) and flow network constraints in maximum flow problems.

When a problem becomes too large or complicated to handle, a decomposition approach can be applied if the problem structure is suitable. The basic mechanism in all decomposition principles is to decompose the original problem into smaller subproblems, which are coordinated by a master problem. One of these techniques is the Dantzig–Wolfe decomposition. In Martins et al. (2015), the authors proposed four Dantzig–Wolfe decompositions for the bucket formulation: the $\mathcal{S}$-knapsack and the $\mathcal{S}$-knapsack-and-clique decompositions, and two similar decompositions of the bucket formulation with additional constraints on the connectivity of the buckets, the $\mathcal{R}$-knapsack and the $\mathcal{R}$-knapsack-and-clique decompositions. They establish theoretically that the linear programming bounds of the knapsack-and-clique decompositions are better than or equal to those of the knapsack decompositions, and that the linear programming bound of the $\mathcal{S}$-knapsack-and-clique decomposition is equal to that of the $\mathcal{R}$-knapsack-and-clique decomposition. According to these results and since the subproblem of the $\mathcal{S}$-knapsack-and-clique decomposition may be less difficult to solve than that of the $\mathcal{R}$-knapsack-and-clique decomposition, they proposed a solution approach based on the first decomposition. For solving the decomposed model, a matheuristic, a heuristic based on mathematical programming, was developed. In this work, neither the bucket connectivity constraints nor the $\mathcal{R}$-knapsack-and-clique decomposition was formulated.

We propose for the problem of this study the bucket formulation with additional constraints on bucket connectivity and two definitions of stand adjacency. One of these definitions is used to address bucket connectivity and the other to address constraints on bucket (clearcut) proximity. Then, we formulate the $\mathcal{R}$-knapsack-and-clique decomposition of the model. A branch-and-price is implemented to solve this decomposition. We compare both approaches, the Dantzig–Wolfe decomposition/branch-and-price and the connected-bucket formulation solved by a general mixed integer programming solver (CPLEX).

This paper is outlined as follows. In Sections 2 and 3, we describe the forest planning problem and the formulation, and in Section 4, the branch-and-price and the decomposition. In Section 5, we report computational experience as to the efficiency of the decomposition/branch-and-price. Our tests involved real instances. We compare the quality of the solutions obtained for these instances with the formulation/general MIP solver for the same time limit. In the last section, we present our conclusions.

2. The problem

In this study, we consider forests classified into stands. A stand is a grouping of vegetation sufficiently uniform in species composition, age, and condition to be managed as a single unit.

Table 3
Parameters and sets of the harvest scheduling problem

Parameter/set	Notation
I	Set of stands indexed by $i = 1, \ldots, \|I\|$
T	Planning horizon indexed by $t = 1, \ldots, \|T\|$
p_i^t	Net present value of removed timber from stand i in period t
v_i^t	Volume of removed timber from stand i in period t
a_i	Area of stand i
$cost$	Penalty assigned to a clearcut
Deviation allowed from target volume	Δ
Upper bound on the area of a clearcut	$A\max$

The harvest scheduling problem (referred to as problem P) deals with determining which stands should be harvested in each period during a given planning horizon in order to maximize the difference between the net present value generated by harvesting and the sum of costs assigned to clearcuts. Stand selection is subject to volume constraints (nonspatial constraints) and spatial constraints as follows.

1. *Volume constraints*: Lower and upper bounds on the volume of timber harvested in each period. These constraints ensure a regular production of timber.
2. *Clearcut size constraints*: An upper bound on the area of each clearcut in each period. These constraints limit the area of a clearcut in the period of intervention.
3. *Clearcut proximity constraints*: Stands that are within a certain distance of a clearcut cannot be harvested in the same period of intervention. These constraints prevent the creation of clearcuts within a threshold distance from each other. The green-up time is considered to be one period.

The spatial constraints together with the clearcut penalties in the objective function aim at reducing the negative environmental impact of excessive harvesting.

In this study, the following are assumed:

- The cost assigned to each clearcut is equal for all clearcuts.
- Each stand is harvested at the most once.
- A minimum age is required for harvesting.
- Harvesting occurs at the beginning of the periods, and all the periods are of the same length.

Table 3 displays the parameters and sets of the problem.

3. The connected-bucket formulation

The model that we propose for P is based on the bucket formulation. Two different definitions of adjacency are considered. The weak adjacency and what we will refer to very weak adjacency. Two stands are *very weak adjacent* if they are within a certain distance from each other.

Table 4
Graphs for the model

Graph	Description
$G = (I, E)$	The endpoints of each edge in E correspond to two adjacent stands according to the weak adjacency
$G = (I, A)$	The arc set A is obtained from E by setting two opposite directions associated with each edge in E
$G_k = (I_k, A_k)$	Subgraph of $G = (I, A)$ induced by set $I_k = \{i \in I : i \geq k\}$ (thus, $A_k = \{(i, j) \in A : i, j \in I_k\}$)
$GN = (I, EN)$	The endpoints of each edge in EN correspond to two adjacent stands according to the very-weak adjacency

The bucket formulation for the single-harvest ARM "fills" buckets, one at the most for each stand, in such a way that nonspatial constraints and constraints on clearcut area are satisfied. A nonempty bucket is a region that might be disconnected since there are no constraints to enforce its connectivity. Thus, each nonempty bucket represents a feasible clearcut or a set of feasible clearcuts. As for the problem of this study, flow constraints involving the weak adjacency are added to force buckets to be connected. The very weak adjacency is used to ensure at least a threshold distance between buckets.

Next, we will describe the model that we propose for P, the bucket model with additional constraints on bucket connectivity and two definitions of stand adjacency. We will refer to this model as a connected-bucket model. For the sake of simplicity, we will describe the model without addressing the minimum required age for harvesting. At the end, we will consider this aspect.

Four graphs are defined (Table 4).

Let $\mathcal{Q}$ be the set of maximal cliques of GN indexed by $P \in \mathcal{Q}$. A clique is the set of nodes of a complete subgraph of the graph, which has an edge between each pair of vertices, and it is maximal if it is not contained in any other clique. Cliques are used to prevent the stands adjacent to clearcuts, according to the very weak adjacency, from being harvested. Constraints on bucket connectivity guarantee that each nonempty bucket represents a single clearcut. We will refer to the stand with the smallest index in a nonempty bucket as a bucket label.

As each stand is harvested once at the most over the planning horizon, we can represent the set of buckets in the forest as $C = \{C_1, \ldots, C_{|I|}\}$.

For $i \in I_k$, $I_k^-(i) = \{(j, i) \in A_k\}$ and $I_k^+(i) = \{(i, j) \in A_k\}$ represent the set of arcs that end at i and the set of arcs that start at i, respectively, in G_k.

The decision variables of the connected-bucket model are the following:

1. $F_{lj}^{it} \geq 0$ is the flow that goes through arc (l, j) in direction to node i in period t; $\quad (l, j) \in A$; $i \in I \setminus \{l\}$; $t \in T$
2. $x_i^{kt} = \begin{cases} 1 & \text{if stand } i \text{ is selected to belong to bucket } C_k \text{ in period } t \\ 0 & \text{otherwise}; \; k \in I; \; t \in T; \; i = k, \ldots, |I| \end{cases}$
3. $w_P^{kt} = \begin{cases} 1 & \text{if at least one stand from clique } P \text{ is selected to belong to bucket } C_k \text{ in period } t \\ 0 & \text{otherwise}; \; k \in I; \; t \in T; \; P \in \mathcal{Q} : \max_{i \in P}\{i\} \geq k. \end{cases}$

The model is as follows.

Connected-bucket formulation

$$\max \sum_{t\in T}\sum_{k\in I}\sum_{i=k}^{|I|} p_i^t x_i^{kt} - \sum_{t\in T}\sum_{k\in I} cost\, x_k^{kt} \tag{1}$$

subject to

$$\sum_{t\in T}\sum_{k=1}^{i} x_i^{kt} \leq 1;\ \ i \in I \tag{2}$$

$$\sum_{i\in I} v_i^t \sum_{k=1}^{i} x_i^{kt} \geq (1-\Delta)\sum_{i\in I} v_i^{t-1}\sum_{k=1}^{i} x_i^{k,t-1};\ t = 2,\ldots,|T| \tag{3}$$

$$\sum_{i\in I} v_i^t \sum_{k=1}^{i} x_i^{kt} \leq (1+\Delta)\sum_{i\in I} v_i^{t-1}\sum_{k=1}^{i} x_i^{k,t-1};\ t = 2,\ldots,|T| \tag{4}$$

$$x_i^{kt} \leq w_P^{kt};\ k \in I;\ t \in T;\ i \geq k;\ P \in \mathcal{Q} : i \in P \tag{5}$$

$$\sum_{k \leq \max\limits_{i\in P}\{i\}} w_P^{kt} \leq 1;\ t \in T;\ P \in \mathcal{Q} \tag{6}$$

$$\sum_{i=k+1}^{|I|} a_i x_i^{kt} \leq (A\max - a_k)x_k^{kt};\ k \in I;\ t \in T \tag{7}$$

Connectivity constraints on bucket C_k; $k = 1,\ldots,|I|-1$; $t \in T$ (8)–(12)

$$x_i^{kt} \in \{0,1\};\ k \in I;\ t \in T;\ i = k,\ldots,|I| \tag{13}$$

$$w_P^{kt} \geq 0;\ k \in I;\ t \in T;\ P \in \mathcal{Q} : \max_{i\in P}\{i\} \geq k. \tag{14}$$

Connectivity constraints on bucket C_k; $k = 1,\ldots,|I|-1$; $t \in T$

$$\sum_{j\in I_k^+(k)} F_{kj}^{it} \geq x_i^{kt};\ k = 1,\ldots,|I|-1;\ i \in I_k \setminus \{k\};\ t \in T \tag{8}$$

$$\sum_{j\in I^+(l)} F_{lj}^{it} = \sum_{\substack{j\in I^-(l)\\ j\neq i}} F_{jl}^{it} + x_i^{lt};\ i \in I_k \setminus \{k\};\ l \in I_k \setminus \{i,k\};\ t \in T \tag{9}$$

$$\sum_{j\in I^-(i)} F_{ji}^{it} = \sum_{l=1}^{i-1} x_i^{lt};\ i \in I_k \setminus \{k\};\ t \in T \tag{10}$$

$$F_{lj}^{it} \leq F_{lj}^{jt};\ (l, j) \in A_k;\ i \in I_k \setminus \{l, j, k\};\ t \in T \tag{11}$$

$$F_{lj}^{it} \geq 0;\ (l, j) \in A_k;\ i \in I \setminus \{k, l\};\ t \in T \tag{12}$$

Function (1) states the management objective of maximizing the net present value of timber harvested discounting the cost of each clearcut. Constraints (2) state that each stand is harvested at the most once in the planning horizon. Constraints (3) and (4) allow the volume of timber harvested in each period to range from $1 - \Delta$ to $1 + \Delta$ times the volume of timber harvested in the previous period. Constraints (5) define the relationship between variables x and w: if stand i is assigned to bucket C_k ($x_i^{kt} = 1$), every clique P with node i is labeled with node k ($w_P^{kt} = 1$). Constraints (5) and (6) ensure that in each period every two adjacent stands (by the very weak adjacency) are in one bucket at the most (each clique cannot be labeled with two nodes). Constraints (7) guarantee that each bucket does not exceed the maximum allowed size. Constraints (7) also state that if C_k is nonempty then C_k contains stand k, the bucket label. Constraints (13) and (14) state binary and nonnegativity requirements on variables. The integrality of variables x, together with constraints (5), implies the integrality of variables w in at least one optimal solution.

Constraints (8)–(12) guarantee the connectivity of each nonempty bucket (with more than a single node) by identifying the bucket with a multicommodity network flow. For a single-commodity network flow: (i) the source is the bucket label (constraints (8)); (ii) there is flow conservation (constraints (9)); (iii) the sink is a selected stand (constraints (10)). Constraints (11) reduce the number of alternative solutions by ensuring that if a node receives flow in direction to other node then that node is also the sink of a single-commodity network flow.

We can also add the following constraints in order to eliminate alternative solutions:

$$F_{lj}^{it} \leq \sum_{k=1}^{i-1} x_i^{kt};\ i \in I_k \setminus \{k\};\ (l, j) \in A_k;\ t \in T. \tag{15}$$

These constraints ensure that there is no flow in direction to a node if this node is not assigned to a bucket.

The number of variables and constraints is $\mathcal{O}(|I| \times |\mathcal{Q}| \times |T|$. Variables that have the value 0 in any feasible solution may not be included in the model. Observe that if a stand is too "far" from stand k then it is not worth assigning it to bucket C_k, because the area of any connected region with both stands would exceed the maximum. Thus, x_i^{kt} is defined when the shortest path between i and k is not greater than Amax, using graph G (Table 4) with weights assigned to nodes, where the weights are the areas of the respective stands. F_{ij}^{lt} is defined when the shortest path between i and l with arc (i, j) is not greater than Amax. w_p^{kt} is not defined when there is no x_i^{kt} for all $i \in P$. The elimination of these variables implies the reduction of the number of constraints (5), (7), and (8)–(11).

Addressing the minimum required age for harvesting forces the following variables to be 0 or, in other words, it is useless consider them: x_i^{kt} if stands i or k are not old enough to be harvested in period t; w_p^{kt} if there is no x_i^{kt} for all $i \in P$; F_{ij}^{lt} if stands i, j or l are not old enough to be harvested in period t. Again, the elimination of these variables implies the elimination of some constraints (5), (7), and (8)–(11).

4. Dantzig–Wolfe decomposition

In this section, we present a Dantzig–Wolfe decomposition of the connected-bucket model by reformulating the set $\mathcal{C}$ defined by constraints (5), (7), (13), (14), and the connectivity constraints (8)–(12). For the sake of simplicity, we will describe the decomposition without considering variable and constraint reductions and the minimum required age for harvesting. At the end, we will consider these aspects.

The representation of the convex hull of $\mathcal{C}$ by extreme points is used to strengthen the formulation. Each extreme point is represented by a variable in the master problem. As the convex hull has an exponential number of extreme points in general, the master problem is solved by branch-and-price, where each linear program throughout the enumeration tree of the branch-and-bound is solved by column generation. The pricing subproblems consider an objective function of reduced costs over set $\mathcal{C}$.

In this section, we first detail the master problem and the subproblem(s), then discuss the application of column generation and branch-and-price to solve the decomposed model. At last, we mention how branch-and-price can be combined with a general MIP solver to reduce the time to obtain high quality solutions.

4.1. The master problem

Let e be the number of variables w_P^{kt} in the connected-bucket model, $e = \sum_{k,t} e(k,t)$, where $e(k,t) = |\{P \in \mathcal{Q} : P \cap \{k, \ldots, |I|\} \neq \emptyset\}|$, and $f(k,t)$ the number of variables F_{lj}^{it} for $(l,j) \in A_k$; $i = k+1, \ldots, |I|$. For each period t, the set of cutblocks with stands $i \geq k$, containing k and with area not greater than $A\max$, will be called $\mathcal{R}^{kt}$. For each stand k, sets $\mathcal{R}^{kt}$ are identical.

$\mathcal{C}$ can be decomposed into $|I| \times |T|$ sets:
$\mathcal{C}^{kt} = \{(x, w, F) \in \{0,1\}^{(|I|-k+1)}[0, +\infty[^{e(k,t)+f(k,t)}$:

$$\left|\begin{array}{l}
\sum_{i=k+1}^{|I|} a_i x_i^{kt} \leq (A\max - a_k) x_k^{kt} \\
x_i^{kt} \leq w_P^{kt};\ i \geq k;\ P \in \mathcal{Q} : i \in P \\
\sum_{j \in I_k^+(k)} F_{kj}^{it} = x_i^{kt};\ i = k+1, \ldots, |I| \\
\sum_{j \in I_k^+(l)} F_{lj}^{it} = \sum_{\substack{j \in I_k^-(l) \\ j \neq i}} F_{jl}^{it};\ i, l = k+1, \ldots, |I|;\ l \neq i \\
\sum_{j \in I_k^-(i)} F_{ji}^{it} = x_i^{kt};\ i = k+1, \ldots, |I| \\
F_{lj}^{it} \leq F_{lj}^{jt};\ (l,j) \in A_k;\ i = k+1, \ldots, |I|;\ i \neq l, j \\
F_{lj}^{it} \geq 0;\ (l,j) \in A_k;\ i = k+1, \ldots, |I|;\ i \neq l \\
x_i^{kt} \in \{0,1\};\ i = k, \ldots, |I| \\
w_P^{kt} \geq 0;\ P \in \mathcal{Q} : \max_{i \in P}\{i\} \geq k
\end{array}\right\}.$$

For each stand k and period t, an element (x, w, F) of set $\mathcal{C}^{kt}$ is either the null vector or a vector $(\bar{x}^{kt}_S, \bar{w}^{kt}_S, F)$ where $S \in \mathcal{R}^{kt}$, and $\bar{x}^{kt}_S$, $\bar{w}^{kt}_S$ are defined as

- $\bar{x}^{kt}_{iS} = \begin{cases} 1 & \text{if stand } i \geq k \text{ belongs to region } S \text{ in period } t \\ 0 & \text{otherwise;} \end{cases}$
- $\bar{w}^{kt}_{PS} = \begin{cases} 1 & \text{if clique } P \in \mathcal{Q} \text{ is such that } P \cap S \neq \emptyset \text{ in period } t \\ 0 & \text{otherwise.} \end{cases}$

The decision variables of the master problem are the following:

- $y^{kt}_S = \begin{cases} 1 & \text{if region } S \in \mathcal{R}^{kt} \text{ is selected to be harvested in period } t \\ 0 & \text{otherwise; } k \in I;\ t \in T;\ S \in \mathcal{R}^{kt}. \end{cases}$

The master problem is as follows.

Master problem

$$\max \sum_{t\in T} \sum_{k\in I} \sum_{S\in\mathcal{R}^{kt}} \left(\sum_{i=k}^{|I|} p^t_i \bar{x}^k_{iS} - cost\, y^{kt}_S\right) \tag{16}$$

subject to

$$\sum_{S\in\mathcal{R}^{kt}} y^{kt}_S \leq 1;\ k \in I;\ t \in T \tag{17}$$

$$\sum_{k\leq \max\{i: i\in P\}} \sum_{S\in\mathcal{R}^{kt}} \bar{w}^{kt}_{PS} y^{kt}_S \leq 1;\ P \in \mathcal{Q};\ t \in T \tag{18}$$

$$\sum_{t\in T} \sum_{k=1}^{i} \sum_{S\in\mathcal{R}^{kt}} \bar{x}^{kt}_{iS} y^{kt}_S \leq 1;\ i \in I \tag{19}$$

$$\sum_{i\in I} v^t_i \sum_{k=1}^{i} \sum_{S\in\mathcal{R}^{kt}} \bar{x}^{kt}_{iS} y^{kt}_S \geq (1-\Delta) \sum_{i\in I} v^{t-1}_i \sum_{k=1}^{i} \sum_{S\in\mathcal{R}^{kt}} \bar{x}^{k,t-1}_{iS} y^{k,t-1}_S;\ t = 2, \cdots, |T| \tag{20}$$

$$\sum_{i\in I} v^t_i \sum_{k=1}^{i} \sum_{S\in\mathcal{R}^{kt}} \bar{x}^{kt}_{iS} y^{kt}_S \leq (1+\Delta) \sum_{i\in I} v^{t-1}_i \sum_{k=1}^{i} \sum_{S\in\mathcal{R}^{kt}} \bar{x}^{k,t-1}_{iS} y^{k,t-1}_S;\ t = 2, \ldots, |T| \tag{21}$$

$$y^{kt}_S \in \{0, 1\};\ k \in I;\ t \in T;\ S \in \mathcal{R}^{kt}. \tag{22}$$

4.2. The pricing subproblems

For the linear relaxation of the master problem, constraints (22) are simply replaced by $y^{kt}_S \geq 0$.

Let Ω^{kt}, β_P^t, θ_i, μ^t, and ν^t denote the dual variables associated with constraints (17)–(21) of the linear relaxation of the master problem and Ω^{kt*}, β_P^{t*}, θ_i^*, μ^{t*}, and ν^{t*} assume an optimal dual solution of the linear relaxation of a restricted master problem. By definition of reduced cost, the objective function of the pricing subproblem kt is given by

$$\max_{S\in\mathcal{R}^{kt}} \left\{ \sum_{i\in S} \epsilon_i^{t*} + \sum_{P:S\cap P\neq\emptyset} \beta_P^{t*} \right\} + \Omega^{kt*},$$

where for all $i = k+1, \ldots, |I|$, $\epsilon_i^{1*} = p_i^1 + \theta_i^* - \mu^{2*}(1-\Delta)v_i^1 - \nu^{2*}(1+\Delta)v_i^1$
$\epsilon_i^{t*} = p_i^t + \theta_i^* + \mu^{t*}v_i^t + \nu^{t*}v_i^t - \mu^{t+1*}(1-\Delta)v_i^t - \nu^{t+1*}(1+\Delta)v_i^t$, $t = 2, \ldots, |T|-1$
$\epsilon_i^{|T|*} = p_i^{|T|} + \theta_i^* + \mu^{|T|*}v_i^{|T|} + \nu^{|T|*}v_i^{|T|}$ (with $\theta_i^* \leq 0$, $\mu^{t*} \geq 0$ and $\nu^{t*} \leq 0$), and for $i = k$, we have to discount the *cost* value in each expression.

The decision variables of the subproblem kt are the following:

- $F_{lj}^i \geq 0$ is the flow that goes through the arc (l, j) in direction to node i; $i \in I_k \setminus \{k\}$; $(l, j) \in A_k$;
- $x_i = \begin{cases} 1 & \text{if stand } i \text{ is selected to belong to region } S \in \mathcal{R}^{kt} \\ 0 & \text{otherwise; } i = k, \cdots, |I|; \end{cases}$
- $w_P = \begin{cases} 1 & \text{if at least one stand from clique } P \in \mathcal{Q} \text{ is selected to belong to region } S \in \mathcal{R}^{kt} \\ 0 & \text{otherwise; } P \in \mathcal{Q} : \max_{i\in P}\{i\} \geq k. \end{cases}$

The subproblem kt can be formulated by the following integer program.

Pricing subproblem kt

$$\max \sum_{i=k}^{|I|} \epsilon_i^{t*} x_i + \sum_{P:\max_{i\in P}\{i\}\geq k} \beta_P^{t*} w_P + \Omega^{kt*} \tag{23}$$

subject to

$$x_k = 1 \tag{24}$$

$$x_i \leq w_P;\ i \geq k;\ P \in \mathcal{Q} : i \in P \tag{25}$$

$$\sum_{i=k+1}^{|I|} a_i x_i \leq A\max - a_k \tag{26}$$

$$\sum_{j\in I_k^+(k)} F_{kj}^i = x_i;\ i = k+1, \ldots, |I| \tag{27}$$

$$\sum_{j\in I_k^+(l)} F_{lj}^i = \sum_{\substack{j\in I_k^-(l) \\ j\neq i}} F_{jl}^i;\ i, l = k+1, \ldots, |I|;\ l \neq i \tag{28}$$

$$\sum_{j \in I_k^-(i)} F_{ji}^i = x_i; i = k+1, \ldots, |I| \quad (29)$$

$$F_{lj}^i \leq F_{lj}^j; (l, j) \in A_k; i = k+1, \ldots, |I|; i \neq l, j \quad (30)$$

$$F_{lj}^i \geq 0; (l, j) \in A_k; i = k+1, \ldots, |I|; i \neq l \quad (31)$$

$$x_i \in \{0, 1\}; i = k+1, \ldots, |I| \quad (32)$$

$$w_P \geq 0; P \in \mathcal{Q} : \max_{i \in P}\{i\} \geq k. \quad (33)$$

Objective function (23) is to maximize the sum of the node weights ϵ_i^{t*} and the clique weights β_P^{t*} over the selected nodes and to add the value Ω^{kt*} to the optimal sum. Constraint (24) guarantees that region S contains stand k. Constraints (25) ensure that if a node is selected, then any maximal clique with this node is also selected. Constraints (26) ensure that the area of region S does not exceed Amax. Constraints (27)–(29) are the multicommodity flow constraints that guarantee the connectivity of region S. For each selected node, these constraints establish the flow network involving the nodes where the flow passes between node k and that node: (i) constraints (27) ensure that node k is the source; (ii) constraints (29) state the selected node as the sink; (iii) constraints (28) ensure the flow propagation. Constraints (30) eliminate alternative solutions. Constraints (31)–(33) state the variable types.

For each subproblem kt, stands that are too "far" from stand k do not belong to any region $S \in \mathcal{R}^{kt}$. Thus, the following variables may not be included in the model: x_i if the shortest path between i and k is greater than Amax; F_{lj}^i if the shortest path between l and i with arc (l, j) is greater than Amax; w_p when there is no x_i for all $i \in P$. The elimination of these variables implies the elimination of constraints (25) and (27)–(30).

Addressing the minimum required age for harvesting eliminates all subproblems kt such that stand k is not old enough to be harvested in period t. Furthermore, for each noneliminated subproblem, the following variables are set to 0: F_{lj}^i if stands i, l, or j are not old enough to be harvested in period t; x_i if stand i is not old enough to be harvested in period t; w_P if $x_i = 0$ for all i in clique P.

4.3. Column generation and branch-and-price

Column generation allows solving the linear relaxation of the master problem. Iteratively, a restricted master problem, where a small portion of variables is present, is enlarged by variables identified by the subproblems as having the most positive reduced costs. In this way, an optimal solution of the linear relaxation of the master problem is obtained without explicitly taking into account all variables.

Branch-and-price is required when the linear relaxation of the master problem has fractional variables. Branching on the master variables is usually avoid because it leads to unbalanced trees. The common branch strategy is based on the Dantzig–Wolfe core idea: to represent the decision variables of the initial formulation as convex combinations of the extreme points of the feasible region (in the limited case). Therefore,

$$x_i^{kt} = \sum_{S \in \mathcal{R}^{kt}} \bar{x}_{iS}^{kt} y_S^{kt}; k \in I; t \in T; i = k, \dots, |I| \quad (34)$$

$$w_P^{kt} = \sum_{S \in \mathcal{R}^{kt}} \bar{w}_{PS}^{kt} y_S^{kt}; k \in I; t \in T; P \in \mathcal{Q} : \max_{i \in P}\{i\} \geq k. \quad (35)$$

If all the y variables are integer in the linear relaxation, the x and w are also integers and the problem is solved. Otherwise, through (34) and (35), fixing the y variables in their optimal values, the optimal solution of the linear relaxation in the x and w is obtained. Two cases can occur, all variable x and w have integer values, or, at least one of them has a fractional value. In the former case, although some y variables are fractional, an optimal integer solution in the x and w was found. In the latter case, we branch on the x variables but as represented in the master problem (in the same manner as the initial model was transformed into the decomposed model). Given the example of a fractional variable $\bar{x}_{\bar{i}}^{\bar{k}t}$, representing the set of extreme points present in the restricted master problem by $\bar{\mathcal{R}}^{\bar{k}t}$, the branching constraints are

$$\sum_{S \in \bar{\mathcal{R}}^{\bar{k}t}} \bar{x}_{\bar{i}S}^{\bar{k}t} y_S^{\bar{k}t} = 0 \quad (36)$$

and

$$\sum_{S \in \bar{\mathcal{R}}^{\bar{k}t}} \bar{x}_{\bar{i}S}^{\bar{k}t} y_S^{\bar{k}t} = 1. \quad (37)$$

The presence of branching constraints of this type in the restricted master problem of any node of the search tree is taken care by column generation by including terms relative to their dual variables in the objective function of the subproblem(s).

It is well known that branch-and-price (and branch-and-bound) may not find an optimal solution in a reasonable amount of time. In order to reduce the time needed to obtain a (hopefully) good solution, we embedded a heuristic in branch-and-price that consists in using the general MIP solver not only to solve the linear relaxation of the restricted master problem but also the *integer* restricted master problem in some nodes of the search tree. After testing different settings, we decided to apply this heuristic after the linear relaxation is solved and in every node where new columns are generated.

Table 5
Characterization of the five datasets

Dataset	n	Feature	Min	Mean	Max	SD
R1	302	*area* (ha)	0.03	0.25	0.42	0.04
		age (year)	70	89.47	140	12.84
		vol (m^3/ha)	541	613.2	778	45.13
R2	69	*area* (ha)	0.07	0.24	0.38	0.04
		age (year)	61	93.71	127	18.01
		vol (m^3/ha)	478	620.35	734	81.48
R3	290	*area* (ha)	0.01	0.24	0.45	0.06
		age (year)	61	83.49	140	13.98
		vol (m^3/ha)	478	582.02	778	59.46
R4	134	*area* (ha)	0.02	0.23	0.42	0.06
		age (year)	60	86.49	153	21.53
		vol (m^3/ha)	478	593.32	796	78.21
R5	133	*area* (ha)	0.02	0.24	0.48	0.07
		age (year)	65	99.2	162	21.54
		vol (m^3/ha)	478	641.95	813	75.25

n, number of stands; *area*, stand area (ha); *age*, stand age (years); *vol*, stand volume (m^3/ha). Min, Mean, and Max are the minimum, average, and maximum values of each feature, respectively; SD is the standard deviation of the values of each feature.

5. Computational experience

5.1. Implementation

A straightforward implementation of the compact model was run with CPLEX 12.1. The branch-and-price used for solving the decomposed was implemented in the SearchCol++ framework (Alvelos et al., 2013). This C++ framework provides all the generic components of branch-and-price, being the user responsible to implement the problem specific part (through C++ derived classes)—essentially the subproblem solver. Within this framework, CPLEX 12.1 was used to solve the restricted master problems and the subproblems. All tests were conducted in a laptop equipped with a processor intel i7 at 2.4 GHz and with 12 GB of RAM.

5.2. Test instances

The test instances are relative to five datasets: R1, R2, R3, R4, and R5. The most relevant information about each dataset is displayed in Table 5. There are four features: n is the number of stands, *area* is the stand area (ha), *age* is the stand age (years), and *vol* is the stand volume (m^3/ha). For each of the last three features, there are four columns: Min, the minimum value; Mean, the average value; Max, the maximum value; and SD, the standard deviation. The discount rate used for the net present value calculation was 3%. The minimum age required for harvesting was set to 80 years and the threshold distance for the very weak adjacency to 25 m.

Table 6
Connected-bucket model, size, and performance

	Model size					Model performance		
Dataset	$\#Q$ / $w(GN)$	Amax (ha)	#constr.	#var.	Δ	Time (seconds)	Gap (%)	IP (€)
R1	276/5	0.5	457,738	8013	0.15	53	0.01	558,092
					0.20	27	0.01	559,259
		1.0	567,379	54,366	0.15	7200	*	*
					0.20	7201	*	*
R2	77/4	0.5	32,645	1859	0.15	1	0.01	112,756
					0.20	1	0.00	112,756
		1.0	55,161	11,540	0.15	7200	1.96	145,468
					0.20	7200	2.19	145,575
R3	272/5	0.5	443,595	6404	0.15	8	0.01	404,943
					0.20	9	0.00	406,464
		1.0	545,618	48,225	0.15	7200	11.60	508,134
					0.20	7200	10.65	513,125
R4	117/5	0.5	92,154	2386	0.15	0	0.01	189,929
					0.20	0	0.00	190,064
		1.0	117,215	12,642	0.15	7200	1.86	243,971
					0.20	7200	2.11	244,459
R5	103/5	0.5	69,573	2012	0.15	1	0.01	252,165
					0.20	2	0.01	254,937
		1.0	85,126	8764	0.15	7206	0.16	293,681
					0.20	7202	0.15	296,693

$\#Q$ and $w(GN)$, number of cliques and clique number of graph GN[a], respectively; Amax, maximum clearcut area (ha); #constr., number of constraints; #var., number of variables; Δ, parameter for harvested volume range; Time, total time (seconds); Gap, final relative gap (%); IP, optimal objective value or best feasible value (€).
[a] $w(GN)$ is the number of nodes of a largest clique of GN.
*No feasible solution was found.

5.3. Results

In Table 6, size and performance of models (1)–(14) are presented. Table 7 displays branch-and-price performance. In Table 8, we compare both approaches.

Dantzig–Wolfe decomposition/branch-and-price outperforms the connected-bucket model/ branch-and-bound when the maximum clearcut area is 1.0 ha. When this limit is 0.5 ha, although the connected-bucket model runs much faster, Dantzig–Wolfe decomposition/branch-and-price obtained better solutions in the majority of the cases.

Although Amax equal to 1.5 ha is beyond the threshold used in practice, we also tested our approaches for that value. Branch-and-price was not able to find a feasible solution within two hours and "out-of-memory" error occurred during the process of solving the connected-bucket model.

For illustrative proposes, the solution obtained with branch-and-price for each dataset, where $A\text{max} = 1.0$ ha and $\Delta = 0.15$, is shown in Fig. 1.

Table 7
Branch-and-price performance

Dataset	*A*max (ha)	Δ	Total time (seconds)	Inc. time (seconds)	Feas. value (€)	Gap < 5% time (seconds)	Gap <1% time (seconds)	Final gap (%)
R1	0.5	0.15	7203	6618	558,093	110	110	0.37
		0.20	7202	1157	559,265	68	68	0.36
	1.0	0.15	7275	7275	660,976	7275	-	2.41
		0.20	7226	7226	668,160	7226	-	2.31
R2	0.5	0.15	464	5	112,756	5	117	0.01
		0.20	507	4	112,756	4	120	0.01
	1.0	0.15	7204	1089	145,676	89	-	1.38
		0.20	7205	263	145,566	85	-	2.10
R3	0.5	0.15	7202	1320	404,966	34	34	0.16
		0.20	7202	163	406,468	29	29	0.10
	1.0	0.15	7243	7243	532,608	7243	-	4.01
		0.20	7256	7256	536,405	7256	-	3.48
R4	0.5	0.15	19	6	189,929	6	6	0.01
		0.20	24	7	190,064	7	7	0.01
	1.0	0.15	7205	3779	244,748	73	665	0.62
		0.20	7200	2812	245,089	71	959	0.73
R5	0.5	0.15	55	53	252,174	8	8	0.00
		0.20	30	5	254,948	5	5	0.00
	1.0	0.15	7202	1572	293,685	56	56	0.27
		0.20	7202	3683	296,697	57	57	0.35

*A*max, maximum clearcut area (ha); Δ, parameter for harvested volume range; Total time, total time (seconds); Inc. time, time to find an incumbent solution (seconds); Feas. value, the objective value of the incumbent solution found (€); Gap < 5% time, time to get a relative gap < 5% (seconds); Gap < 1%, time to get a relative gap < 1% (seconds); Final gap, final relative gap (%).

Table 8
Comparing connected-bucket model/general MIP and Dantzig–Wolfe decomposition/branch-and-price

Amax	Connected-bucket model/general MIP and Dantzig–Wolfe decomposition/branch-and-price
0.5 ha	The connected-bucket model runs much faster (within the default CPLEX gap). Branch-and-price gets solutions within 1% of the optimum in reasonable time. The solution's quality is similar for both approaches within the same optimality parameters. In 6 of 10 instances, branch-and-price reached slightly better solutions. In the remaining instances, the solutions' value were the same.
1.0 ha	Both approaches reached the time limit for all instances. Branch-and-price obtained feasible solutions for all instances with gaps less than or equal to 4% and for four instances with gaps less than 1%. The connected-bucket model failed to find a feasible solution beyond the trivial solution (null solution) for two instances. Branch-and-price obtained better solutions than the connected-bucket model except for one instance.

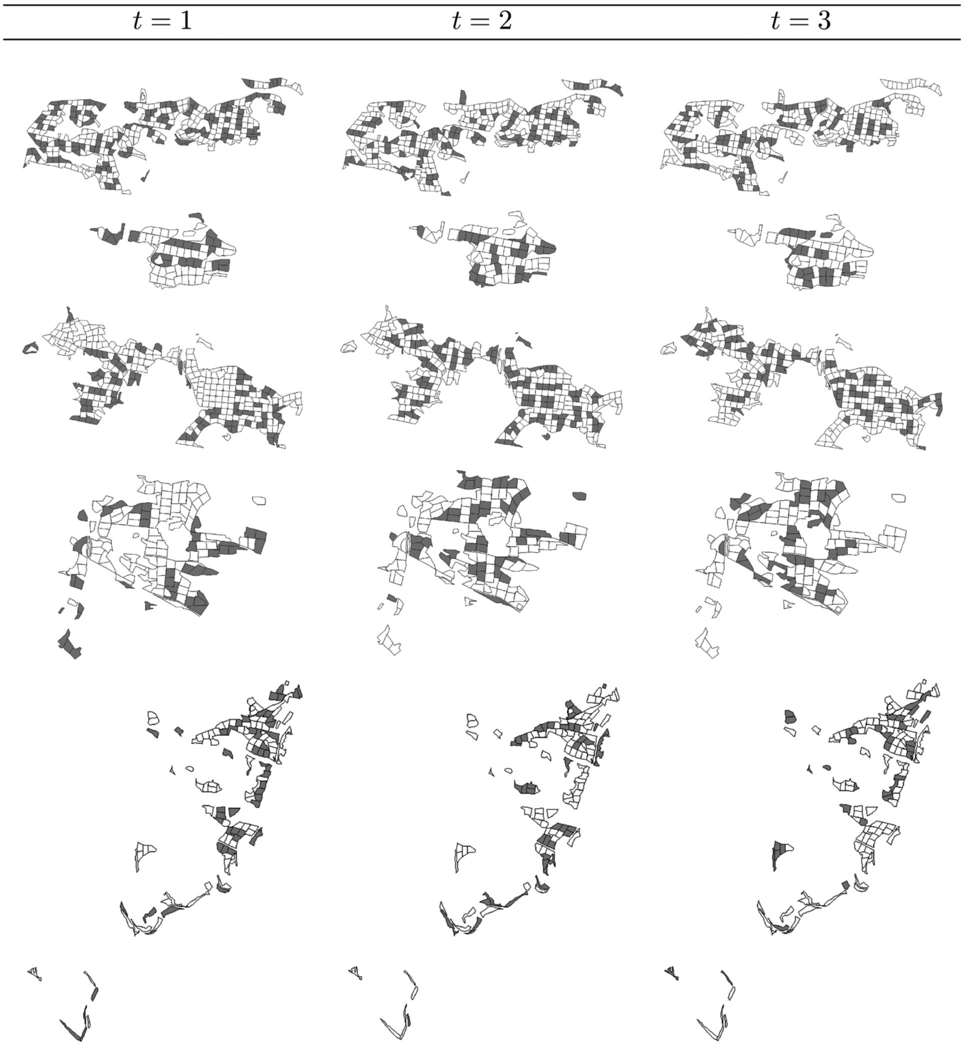

Fig. 1. Solutions obtained with branch-and-price for R1–R5 (top to bottom), where Amax = 1.0 ha and $\Delta = 0.15$. Stands harvested are in gray.

6. Conclusion

This work studies a harvest scheduling optimization problem with constraints on the clearcut size with additional constraints on the clearcut proximity. The objective function is defined as the net present value generated by harvesting discounted by a penalty for each clearcut. This problem arises to reduce the excessive clear-cutting and to improve the quality of the stand borders of the exposed clearcuts.

We propose the connected-bucket model, the so-called bucket model with additional constraints on the connectivity of the buckets and two types of adjacency between stands. This model is solved by a commercial general MIP solver (based on branch-and-bound but with significant additional features, as cuts and primal heuristics). We also propose a Dantzig–Wolfe decomposition to strengthen the model. This decomposed model is solved by branch-and-price. The latter approach found better solutions for the majority of the instances than the connected-bucket model/branch-and-bound.

This study indicates that the Dantzig–Wolfe decomposition and branch-and-price might offer an interesting option for harvest scheduling optimization problems that prove to be difficult to solve with general mixed integer programming solvers. These solvers, typically more efficient for small instances, are more sensitive to the size of the problems, running out of memory quicker for large-scale cases.

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References

Alvelos, F., de Sousa, A., Santos, D., 2013. Combining column generation and metaheuristics. In *Hybrid Metaheuristics*. Springer, Berlin, pp. 285–334.

Borges, P., Martins, I., Bergseng, E., Eid, T., Gobakken, T., 2015. Effects of site productivity on forest harvest scheduling subject to green-up and maximum area restrictions. *Scandinavian Journal of Forest Research* 31, 5, 1–34.

Constantino, M., Martins, I., 2018. Branch-and-cut for the forest harvest scheduling subject to clearcut and core area constraints. *European Journal of Operational Research* 265, 2, 723–734.

Constantino, M., Martins, I., Borges, J.G., 2008. A new mixed integer programming model for harvest scheduling subject to maximum area restrictions. *Operations Research* 56, 3, 542–551.

Crowe, K., Nelson, J., Boyland, M., 2003. Solving the area-restricted harvest-scheduling model using the branch-and-bound algorithm. *Canadian Journal of Forest Research* 33, 9, 1804–1814.

Epstein, R., Goycoolea, M., Murray, A., Weintraub, A., 2003. An adjacency-modeling problem based on constructing harvesting areas. In Arthaud, G.J., Barrett, T.M. (eds) *Systems Analysis in Forest Resources*. Kluwer Academic, Dordrecht, pp. 279–289.

Fenton, E., Krul, L., Ozinga, S., Wainwright, R., 2008. Funding forests into the future? How the European Fund for rural development affects Europe's forests—the case of the Czech Republic. Technical Report, FERN.

Gharbi, C., Rönnqvist, M., Beaudoin, D., Carle, M.A., 2019. A new mixed-integer programming model for spatial forest planning. *Canadian Journal of Forest Research* 49, 12, 1493–1503.

Goycoolea, M., Murray, A.T., Barahona, F., Epstein, R., Weintraub, A., 2005. Harvest scheduling subject to maximum area restrictions: exploring exact approaches. *Operations Research* 53, 3, 490–500.

John, R.S., Tóth, S., 2015. Spatially explicit forest harvest scheduling with difference equations. *Annals of Operations Research* 232, 1, 235–257.

Kaya, A., Bettinger, P., Boston, K., Akbulut, R., Ucar, Z., Siry, J., Merry, K., Cieszewski, C., 2016. Optimisation in forest management. *Current Forestry Reports* 2, 1–17.

Martins, I., Alvelos, F., Constantino, M., 2012. A branch-and-price approach for harvest scheduling subject to maximum area restrictions. *Computational Optimization and Applications* 51, 1, 363–385.

Martins, I., Alvelos, F., Constantino, M., 2015. Decompositions and a matheuristic for a forest harvest scheduling problem. In Almeida, J., Oliveira, J., Pinto, A. (eds) *Operational Research*, Springer, Cham, pp. 237–260.

Martins, I., Constantino, M., Borges, J.G., 1999. Forest management models with spatial structure constraints. Working Paper No. 2/1999, C.I.O., University of Lisbon.

Martins, I., Constantino, M., Borges, J.G., 2005. A column generation approach for solving a non-temporal forest harvest model with spatial structure constraints. *European Journal of Operational Research* 161, 2, 478–498.

McDermott, C., Cashore, B.W., Kanowski, P., 2010. *Global Environmental Forest Policies: An International Comparison*. Earthscan, London.

McDill, M.E., Rebain, S.A., Braze, J., 2002. Harvest scheduling with area-based adjacency constraints. *Forest Science* 48, 4, 631–642.

McDill, M.E., Tóth, S.F., St John, R., Braze, J., Rebain, S.A., 2016. Comparing model I and model II formulations of spatially explicit harvest scheduling models with maximum area restrictions. *Forest Science* 62, 1, 28–37.

Murray, A., 1999. Spatial restrictions in harvest scheduling. *Forest Science* 45, 1, 45–52.

Murray, A., Weintraub, A., 2002. Scale and unit specification influences in harvest scheduling with maximum area restrictions. *Forest Science* 48, 4, 779–789.

Tóth, S.F., McDill, M.E., Könnyü, N., George, S., 2013. Testing the use of lazy constraints in solving area-based adjacency formulations of harvest scheduling models. *Forest Science* 59, 2, 157–176.

Vielma, J.P., Murray, A.T., Ryan, D.M., Weintraub, A., 2007. Improving computational capabilities for addressing volume constraints in forest harvest scheduling problems. *European Journal of Operational Research* 176, 2, 1246–1264.

Walters, K., 1996. Defining adjacency and proximity of forest stands for harvest blocking. GIS'96 Symposium, Canada.

Yoshimoto, A., Asante, P., 2018. A new optimization model for spatially constrained harvest scheduling under area restrictions through maximum ow problem. *Forest Science* 64, 4, 392–406.

Yoshimoto, A., Asante, P., 2021. Inter-temporal aggregation for spatially explicit optimal harvest scheduling under area restrictions. *Forest Science* 67, 5, 587–606.

Yoshimoto, A., Konoshima, M., 2016. Spatially constrained harvest scheduling for multiple harvests by exact formulation with common matrix algebra. *Journal of Forest Research* 21, 1, 15–22.