Modelling Forest Fire Spread Through Discrete Event Simulation

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Abstract Forest fires are becoming a more common occurrence in Portugal as well as worldwide. To extinguish or reduce them more quickly and effectively, it is crucial to understand how they spread. This paper presents a study and a model that shows how wildfires spread, assuming the forest can be represented by a graph, where the nodes correspond to forest stands and the arcs to the path between them. In order to do this, algorithms were developed in Python, using discrete event simulation, that allow modelling the progression of the fire on the graph. This fire propagation model takes into account several aspects of the forest, the wind being the most influential one. Some tests were performed, considering different ignition points, wind directions and wind speeds.

Keywords Fire spread · Discrete event simulation · Wind speed · Wind direction

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1 Introduction

In recent years, forest fires around the world have been affecting entire communities with the loss of human and animal lives, homes, public infrastructures, air quality, landscapes, among others. The existence of different ecosystems directly interferes with the size, frequency, and intensity of forest fires [1]. Therefore, in order to simulate fire spread, it is necessary to determine the best way to represent the forest and its characteristics. Some of the benefits of using a graph to represent a forest are its considerable small size and the inherent flexibility to model different levels of change, since nodes represent homogeneous sections of terrain and arcs represent the travel path between them [2].

Fuel, weather and topography are three of the factors that most influence how forest fire spreads and the Rothermel's model [3] uses these factors. The model computes the rate, R, which is the speed of the fire spread, through the following expression [3]:

$$R = \frac{I_R \xi (1 + \phi_w + \phi_s)}{\rho_b \epsilon Q_{ig}} \tag{1}$$

The components in the final equation for the Rothermel surface fire spread model are:

I_R: Intensity of the fire (Btu/ft2/min)–Rate of energy release per unit area of fire front

ξ: Propagating flux ratio–Proportion of the reaction intensity that heats adjacent fuel particles to ignition (no wind)

ϕ_w: Wind factor–Dimensionless multiplier that considers how wind affects the propagating flux ratio

ϕ_s: Slope factor–Dimensionless multiplier that considers how slope affects the propagating flux ratio

ρ_b: Bulk density (lb/ft3)–Amount of oven-dry fuel per cubic foot of fuel bed

ϵ: Effective heating number–Proportion of a fuel particle that reaches ignition temperature at the beginning of flame combustion

Q_{ig}: Heat of preignition (Btu/lb)–Amount of heat required to ignite one pound of fuel.

There are, however, other approaches to model the way fire can propagate across a specific landscape. Some papers simplify the calculation of the rate of fire spread and one of the simple forms is the 10% wind speed rule of thumb for estimating

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a wildfire's forward rate of spread in forests and shrub lands. The resulting rule of thumb is that the rate of fire spread in forests and shrub lands, under relatively dry conditions, is approximately equal to 10% of the average 10 m open wind speed (5-m average wind speed at a height of 10 m above the ground), where both values are expressed in the same units [4]. Other way to model fire spread is considering spread probabilities between cells. The probability of fire spreading from a burning cell to the neighbouring cells was addressed by [5] and can be described as:

$$P_{burn} = P_h(1 + P_{den})(1 + P_{veg})P_w \tag{2}$$

Where P_h denotes the constant probability that a cell adjacent to a burning cell containing a given type of vegetation and density will catch fire at the next time step; P_{den}, P_{veg} and P_w are the fire propagation parameters that depend on the density of vegetation, the type of vegetation and the wind speed respectively. P_w is described by:

$$P_w = e^{C_1 V} e^{V C_2 (cos\theta - 1)} \tag{3}$$

Where V denotes the wind speed, θ is the angle between the direction of the fire spread and the direction of the wind, and C_1 and C_2 are wind constants. These last two parameters were $0.045\,\mathrm{m}^{-1}\,\mathrm{s}$ and $0.131\,\mathrm{m}^{-1}\,\mathrm{s}$, respectively. They were determined and defined by [6] by encircling a simulator with a non-linear optimization technique, with the goal of minimizing the discrepancy between the number of burned cells predicted by the simulation and those that were actually burned (respected to a case study). The probability of fire spread increases as the angle between the direction of fire spread and the wind direction decreases because the wind will speed up the spread of the fire in the direction it is blowing in, but will slow it down in directions against the wind [2, 5].

The most common used probability distribution to represent the wind speed is the Weibull distribution since it has been found to fit a wide collection of recorded wind speed data [7]. Therefore, the wind speed probability density function is a two-parameter function (shape and scale) and can be calculated by the following equation [7]:

$$f(v) = \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{k-1} exp\left[-\left(\frac{v}{c}\right)^{k}\right], (k > 0, v > 0, c > 1) \tag{4}$$

where c is the Weibull scale parameter, with units equal to the wind speed units, k is the unitless Weibull shape parameter and v is the wind speed [7]. The two parameters of the distribution (shape and scale) can be obtained using known estimation methods such as the maximum likelihood method, the modified maximum likelihood method, and the graphical method. All of them require wind speed data: time-series wind data for the maximum likelihood method, wind speed data in frequency distribution format for the modified maximum likelihood method, and wind speed data in cumulative frequency distribution format for the graphical method [8]. To obtain these distribution parameters in a real situation, it is then necessary to have a sample of wind speeds in the specific geographic area, in order to be able to apply some estimation method.

The wind intensity can be defined using the Beaufort Scale that classifies the wind intensity in 13 levels [9]. For instance, level 2, described as a light breeze, corresponds to a wind speed within 6 to 11 km/h and level 8, described as a gale, corresponds to a wind speed within 62 to 74 km/h. The first can be associated with light winds and the second with strong winds.

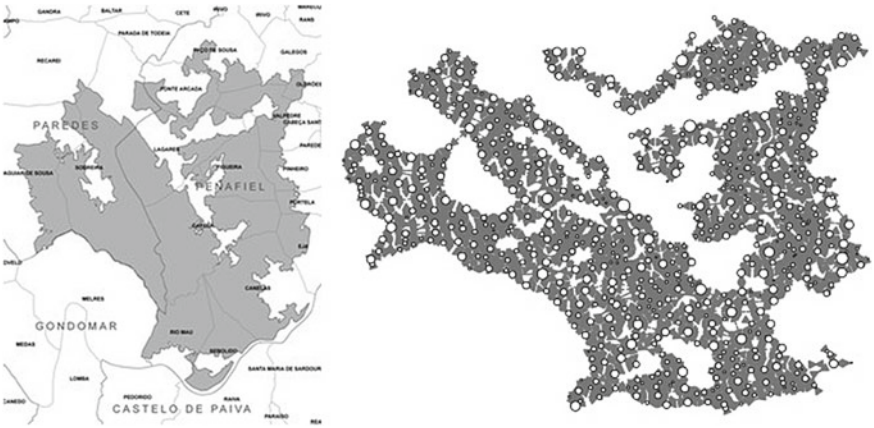
To model a forest fire, one of the first steps is to identify the starting point or points of the fire, i.e. it is necessary to locate the ignition of the fire and analyse whether the fire starts in one or multiple locations. It is possible to determine the probability of ignition in each cell or to model a distribution based on historical ignition locations in specific case studies [10]. Ignition probabilities tend to increase near roads and houses, and with human activity such as smoking, other negligent behaviours, intentional fire setting and other causes, and decrease when associated to land use [11].

Discrete event simulation is a stochastic modelling approach used to model real world systems as a sequence of events in time. It is widely used to address complex and dynamic systems, such as healthcare systems [12], military systems [13] or industrial systems [14]. There are not many studies that use discrete event simulation for modelling the spread of forest fires. Filippi et al. [15] present a novel method that allows the simulation of large-scale/high-resolution systems, focusing on the interface and fire spread. It is based on the discrete event simulation approach, which describes time progression in terms of increments of physical quantities rather than discrete time steps. However, the space representing the forest is not divided into nodes or cells, but into polygons with real coordinates.

The aim of this study is to develop a model to obtain a greater insight into how fire can spread through a specific landscape, and how the factors considered can affect both the behaviour of the fire and the time it takes to burn the entire area. It should thus contribute to the understanding of a problem that highly impacts the environment and the life of the communities.

2 Methodology

In this section, the approach applied to the problem at hand is explained. First by detailing how the forest can be represented, followed by the fire ignitions' topic, the fire spread approach and finally the simulation process, where it is possible to understand how the model works with the previously addressed specificities combined.



(a) (b)

Fig. 1 Forest Representation. (**a**) location of the study area [18]; (**b**) ZIF Graph derived from (**a**)

2.1 Forest Representation

The forest representation was made through the use of a graph where nodes correspond to stands (contiguous community of trees sufficiently uniform) and arcs to the adjacency between those stands. The nodes' coordinates were obtained by the coordinates of the Forest Intervention Zone (ZIF) of Paiva and Entre Douro e Sousa located in the north of Portugal. The network was then derived from the case study. The existing arcs in the graph were also generated from the information collected, representing the paths that exist between each of the stands. The referred ZIF contains several instances and, although it is possible to apply all these methodologies to any of them, only the northern part of the ZIF will be used as it is the one with the largest area. The graph of the Entre Douro e Sousa ZIF (b), and its real location (a), are represented in Fig. 1, where the white balls represent the stands, and the grey shading represents the arcs. The size of the nodes is proportional to the size of the corresponding stand, so a stand with a significant area becomes perceptible in larger nodes. It is possible to highlight the similarities of the real instance, shown in Fig. 1(a), to the one drawn in Fig. 1(b). This particular graph contains 687 stands which, in turn, covers a total area of approximately 6 611.85 ha.

It is important to have this graph as a representation of the forest since it effectively characterises a real land parcel which, with the specific information obtained from the area, can add value to the mitigation of possible fires and also show the applicability of the present work in real contexts.

Table 1 Fire ignitions possibilities

Fire ignitions	How to obtain the ignition points?	Simulation
–Multiple ignitions points	–Chosen	–Variable ignition point(s)
–One ignition point	–Generated by an ignition probability function	–Fixed ignition point(s)

2.2 Fire Ignitions

Once the network is built it is necessary to determine where and how the fire starts and subsequently spreads. For the fire ignitions the options presented in Table 1 can be assumed.

In wildfires, especially when they are caused by humans, there may be one or more fire ignition points. In this work two ways to obtain these points in the network were assumed: in some situations the ignition points were selected through the analysis of the considered graph, by choosing a node that was in the border of the region, located in a specific area (for example, located in the south area). Alternatively, an ignition probability function was used to randomly choose one or two ignition points. Then, the simulation of the fire spread (that will be addressed in the following section) was implemented and during the run it can be assumed to keep the same ignition point(s) throughout the simulation or change this point(s) each time a new network is run. All this information is presented in the Table 1. Once the fire ignition point(s) are obtained the next step was to simulate how the fire spreads through the network.

2.3 Fire Spread

In order to include, in the network, the required information to simulate the fire spread it was decided to assign to each arc of the network two weights: the spread probability between the nodes connected by the arc (*weight1*) and the time of fire spread between those nodes (*weight2*). Assuming that there is fire in the network, the spread probability is the probability of fire spreading from one point to another. The wind direction was used as a way to adjust the probability estimation of fire spread. Each arc of the graph has a certain direction that corresponds to the possible direction of the fire spread and can be seen as a vector, *VecSpread*. Assuming that the wind direction is constant for the whole forest and can be represented as a vector, *VecWind*, it is possible to determine the angle between these two vectors and associate the result obtained in each arc with the respective probability of fire spread. The wind direction was defined considering eight possible scenarios: North, South, East, West, Northeast, Southeast, Northwest and Southwest. This way, the dot product of vectors was applied to each of the arcs of the graph with the wind direction vector. Once the angle between the wind direction and each of the arcs was obtained, the probability of fire spread was computed according to:

$$p = e^{cos\theta - 1} * p_h \tag{5}$$

Formula (5) is a simplified version of formula (2), where p_h denotes the constant probability that a node, adjacent to a burning node, containing a given type of vegetation and density, will catch fire at the next time step. Since there was no available data to determine the value of p_h for every arc, a constant value of 0.85 was assumed for the entire graph, after some initial tests, in order to obtain realistic values regarding the burnt area. Moreover, θ is the angle between the wind and the fire spread directions and it can be noted that the probability of propagation (p) is greater when the angle between the arc and the wind direction is small, and is equal to p_h when the angle between the wind direction and the arc is zero.

The fire spread probability is one of the weights associated with each arc but, in addition to this parameter, there is another weight: the time of fire spread, i.e., the time it takes for the fire to spread from one stand to another. It is possible to obtain this time if the distance travelled and the speed of fire spread are known. The distance between nodes is obtained through the Euclidean distance. The speed of fire spread is a more complicated measurement to obtain accurately. It was decided to use two different ways to calculate this measure: (i) the Rothermel model, and (ii) the 10% wind speed rule of thumb.

In these two models (Rhothermel and 10% wind speed) it is always necessary to consider the wind speed as an input. The wind speed was defined by the Weibull distribution with two parameters: scale and shape. Each of the parameters were obtained considering the results of the wind speed that was effectively being considered. Thus, two wind scenarios defined by the Beaufort scale were simulated: level 2 of the Beaufort scale, light breeze with winds between 6 and 11 km/h; and level 8 of the Beaufort scale, gale with winds between 62 and 74 km/h. For light breeze, the shape and scale parameters considered were 1.6 and 1, respectively. For gale, the shape and scale parameter considered were 4 and 6, respectively. These shape and scale parameters used were obtained by testing different values until the observations for the wind speed were compatible with the abovementioned intervals of speed, for light breeze and for gale (see in Fig. 2 the histograms of a thousand wind speed observations). The 95% confidence intervals for the mean of these observations were approximately [10, 11] km/h for light breeze, and [63, 65] km/h for gale. The Rothermel model was used in order to obtain the rate of fire spread that can be seen as the speed of fire spread itself. However, the only component that did not remain constant in the model was the wind speed. Therefore, as the other parameters remained constant, and the only direct influence on the speed of fire spread included was the wind, the values of all the other parameters of the Rothermel model were obtained from the literature [3], and the final equation was as follows:

$$R = \frac{I_R \xi (1 + \phi_w + \phi_s)}{\rho_b \epsilon Q_{ig}} \tag{6}$$

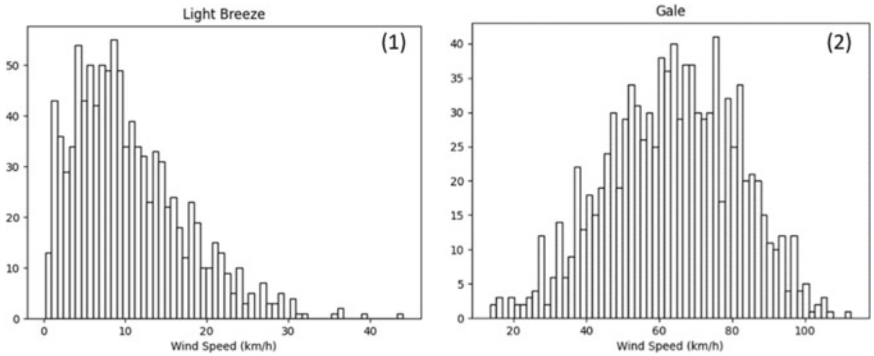


Fig. 2 Histograms of a thousand wind speed observations given by the Weibull distribution for (1) light breeze and (2) gale

$I_R = 4.4967e^{03} BTU/ft^2 min$
$\xi = 1.4782e^{-01}$
ϕ_w is not constant
$\phi_s = 3.8930e^{-01}$
$\rho_b = 1.8119e^{00} lb/ft^3$
$\epsilon = 9.2223e^{-01}$
$Q_{ig} = 3.9435e^{02} BTU/lb.$

The ϕ_w, which represents the wind factor, is the only parameter in the expression that varies since it depends on the wind speed, and it is determined according to the Rothermel model [3].

Alternatively, the 10% wind speed rule of thumb gives the speed of fire spread, under relatively dry conditions, and is approximately equal to 10% of the wind speed, where both values are expressed in the same units.

In the next section the necessary steps for simulating fire spread will be explained.

2.4 Discrete Event Simulation

The fire propagation along the network was defined based on discrete event simulation (DES). Given a graph with nodes and arcs that represents the forest, each arc has two distinct weights, as explained in the previous section: *weight1* represents the spread probability between the nodes, and *weight2* represents the time of fire spread between the respective nodes.

Before starting the simulation process, it is necessary to choose the ignition node, i.e., the place where the fire starts. It is also required to create a priority queue that will contain the active nodes along the propagation. The active nodes are defined as those that the fire can reach at a certain timestamp. The queue is arranged in ascending order according to the fire propagation times of the nodes and is processed by removing nodes from it as they change from the 'burning' state to the 'burned' state. Therefore, this priority queue contains the next nodes that the fire can reach and the time it takes to get there. The process starts with a fire ignition that corresponds to the initial node being added to the priority queue with a null propagation time. While the queue is not empty, the simulation process does not finish.

The initial step is to remove the node with the shortest propagation time from the priority queue, add it to the list of nodes that have already been visited and assume this node as the current node. If this node is still in the priority queue with a longer propagation time, it is deleted from the queue. Next, it is necessary to check the neighboring nodes of the current node and make them active or inactive using a '*Bernoulli Generator*'. This tool generates random binary numbers using a Bernoulli distribution with parameter textitp, that produces zero with probability *1-p* and one with probability *p*. The probability *p* used is determined according to equation (5) and corresponds to '*weight1*' (probability of the fire spread from one node to the other), and it is calculated a priori. For each of the neighboring nodes of the current node, it is thus essential to:

1. Check the probability ('*weight1*') concerning the arc that starts from the current node towards the neighboring node.
2. Apply the '*Bernoulli Generator*' to this probability.
3. Check the numbers obtained in 2 and define the neighboring nodes as active (if connected to the current node by an arc with '1' value) or inactive (if connected to the current node by an arc with '0' value).

Then, it is verified, according to step 3, which of the neighboring nodes are active. If the nodes that result from this step have already been visited, they are ignored. Otherwise, they remain active and, therefore, are added to the priority queue. The propagation time associated to each node in the queue is the sum of the previous node's time with the current new one ('*weight2*'). As long as the queue is not empty, the process is repeated. Note that with this approach, there can be isolated nodes eventually, i.e., that will never become active by the '*Bernoulli Generator*' and therefore the fire does not reach them. Figure 3 shows the flowchart corresponding to the discrete event simulation model used, making it easier to understand the algorithm. The DES model was implemented using Python and the output of its application can be visualized in the actual graph where the set of all paths can be identified.

3 Results

The DES method was tested with different wind directions using the data of the Entre Douro e Sousa ZIF. Applying the procedure to the graph shown in Fig. 1(b) gives a real insight into how the model can work, since the distances between stands and the area of each stand are real. The total area covered by the graph is approximately 6

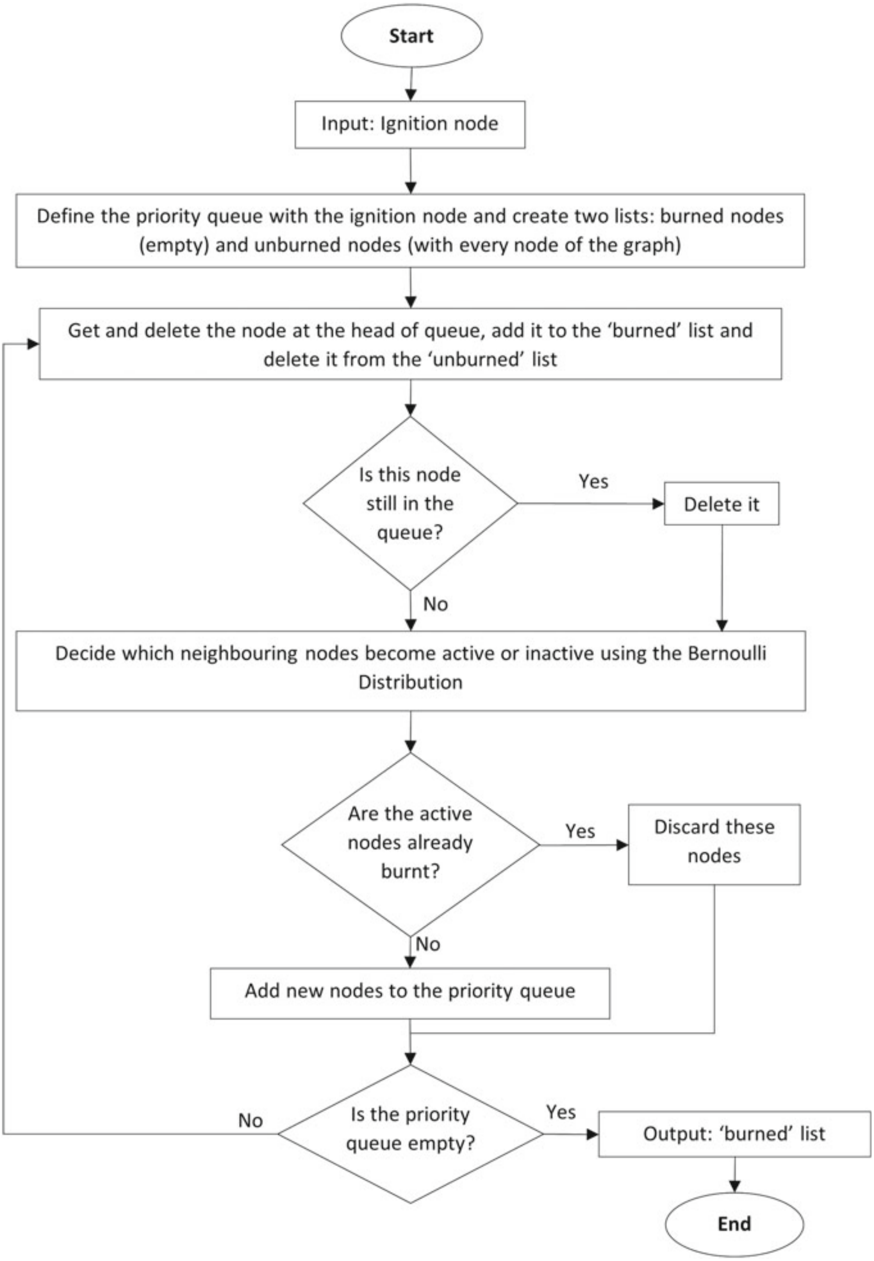


Fig. 3 Flowchart of the discrete event simulation model

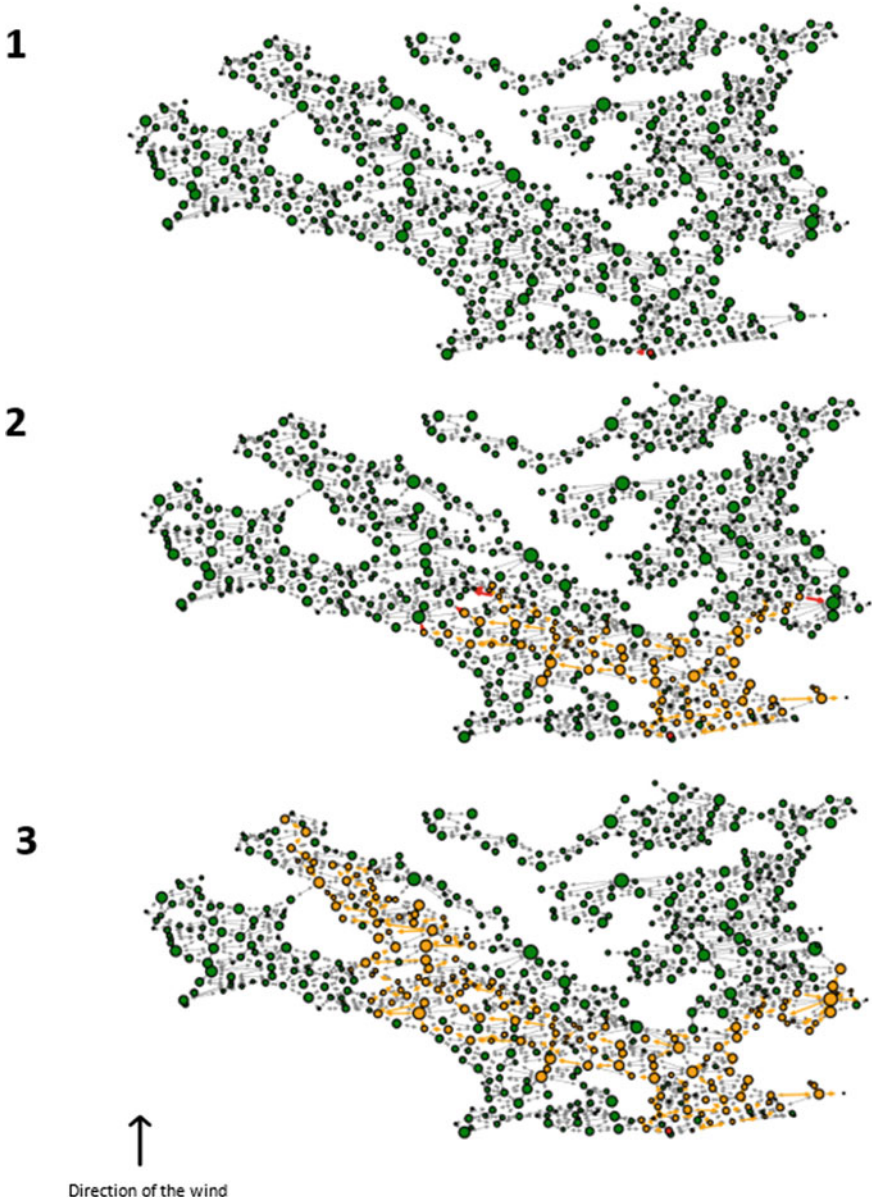


Fig. 4 Entre Douro e Sousa ZIF graph with DES applied: (1) the fire started but has not spread yet; (2) the fire spreading and (3) the fire extinguished

611.85 ha. The experiment described next assumes a fixed ignition point located in the south of the forest.

The fire spread in the forest with the wind direction to the north (winds come from the south and are directed north) can be seen in Fig. 4. It is noticeable that the direction of the fire spread effectively follows the wind direction. 30.65% of the stands in the forest were burnt, which corresponds to a burnt area of approximately 2 175.89 ha (32.9% of the forest area). Wind speed was 4.67 km/h and the speed of fire spread (given by the Rothermel model) was 0.12 km/h, which is a very slow speed, consequently it is expected that the fire spread in the forest will take a long time. In fact, the fire would take approximately 511 days to burn 2 175.89 ha with a weak wind speed. Clearly, under these conditions the result is unrealistic since there are firefighting means to extinguish the fire.

For strong winds the time of propagation in the forest is expected to decrease, since the speed of fire propagation increases. Thus, for the same conditions, but with a higher wind speed (gale), there is actually a shorter time: (i) the burnt area was 2 192.01 ha which means that 33.15% of the forest was burnt; (ii) the wind speed was 64.80 km/h and the fire spread speed was 1.39 km/h; and (iii) the time of fire spread in the forest was 42.5 days. Still, 42.5 days to burn an area of 2 000 ha is a long time, nonetheless there is an increase in the speed of fire spread, as expected. Further on, this unrealistic fire spread time will be addressed by applying another method (10% of the wind speed) to calculate the speed of fire spread.

Next, in order to gauge more realistic fire spread times, the 10% wind speed rule of thumb for the fire spread speed was applied, using the same conditions. Assuming a light wind a fire spread similar to the previous one is achieved. The wind speed was 8.54 km/h and the fire spread speed was 0.85 km/h. It can be seen that using this rule, instead of Rothermel model, the fire propagation is faster. The burnt area in this situation is 1 864.6 ha which corresponds to 28.2% of the forest and the fire propagation time was 67.3 days. Applying the same conditions but for strong wind, they clearly show a faster spread. With a wind speed of 75.1 km/h and a fire spread speed of 7.51 km/h, the burnt area was 2 029.74 ha (30.7% of the forest). The fire propagation time was almost 8 days. From an initial perspective, using the 10% wind speed rule of thumb appears to achieve more realistic results. In fact, the parameters used in the Rothermel model were constant, except for the wind speed, so the characteristics of this area may not match the values used for those parameters, which might explain these differences in results.

In order to make a more in-depth analysis of the results, the model that simulates the spread of fire in the Entre Douro e Sousa ZIF network was run a thousand times with all the previous assumptions, assuming the two alternatives for the calculation of the fire spread rate: the Rothermel model and the 10% wind speed rule of thumb. The results of the 95% confidence intervals (CI) for the means for the percentage of area burnt and the total spread time can be seen in Table 2. Assuming that the wind direction is always north, it can be noticed that the percentage of burnt area practically does not change, whether the rate of fire spread is determined using the Rothermel model or the 10% wind speed rule. It also does not change when the wind speed is stronger or weaker, because the way the fire spreads depends mostly on the

Table 2 Results obtained for the different scenarios of the wind speed regarding the time of fire spread in the ZIF graph and the percentage of burnt area

	Rothermel		10% wind speed rule	
Wind speed	Light breeze	Gale	Light breeze	Gale
95% CI for percentage of burnt area	[31.1, 33.5]	[31.1, 33.4]	[31.2, 33.4]	[30.0, 32.2]
95% CI for total time	[253.0, 264.2]	[40.0, 41.8]	[75.0, 78.1]	[7.3, 7.6]

probabilities of fire spread (*weight1*) that depend only on the wind direction that, in this case, was always the same (north). Regarding the total time of fire spread, there are noticeable differences, as it happened when the simulation was run only once, namely, when considering the 10% wind speed rule, there is a decrease in the time of propagation, both for strong and weak winds.

The units expressed for the total time of fire propagation are days. Therefore, for strong winds, and considering the speed of fire spread calculated by the Rothermel model, it is verified that the propagation time, on average, is between 40.0 to 41.8 days. Whereas, if the speed of fire spread is determined using 10% of the wind speed, this interval decreases to 7.3 to 7.6 days. Effectively, it is verified that the speed of fire spread is much lower for the Rothermel model and that can be explained by the disregard of the forest aspects. However, to burn 2 000 ha (30% of the total area) in a real situation would not take more than 40 days, so using the speed of fire spread as 10% of wind speed leads to more realistic results.

All the experiments done previously had only one ignition point and this node was always chosen strategically, that is, considering the wind direction, an ignition point was chosen that favoured the propagation of the fire in the forest. However, it is possible for a fire to have more than one ignition point and, as stated earlier, there are always areas where it is more likely for a fire to start. Next, it will be assumed there are two ignition points strategically placed, one to the south of the forest (the same node used previously) and the other to the east, and strong winds directed north (Fig. 5 (1)). The rate of fire spread considered is 10% of the wind speed. The fire spread for the two ignition points can be seen in Fig. 5 (2), where a clear connection of the two fires to a larger one can be noticed, however there may be cases where both fires do not merge. The total area burnt was 3 884 ha, which corresponds to approximately 58% of the forest and the total time of propagation was close to 10 days.

In order to evaluate the influence of multiple ignition points, the results with one ignition point (in a location prone to fire spread–south and another in a location unfavourable to fire spread - north) with those of two ignition points (south and east) are compared. To better analyse the results the model was run a thousand times for strong and weak winds, always with the same ignition points, and the results can be seen in Table 3. When applying the Z-test to the two samples of percentages of burnt

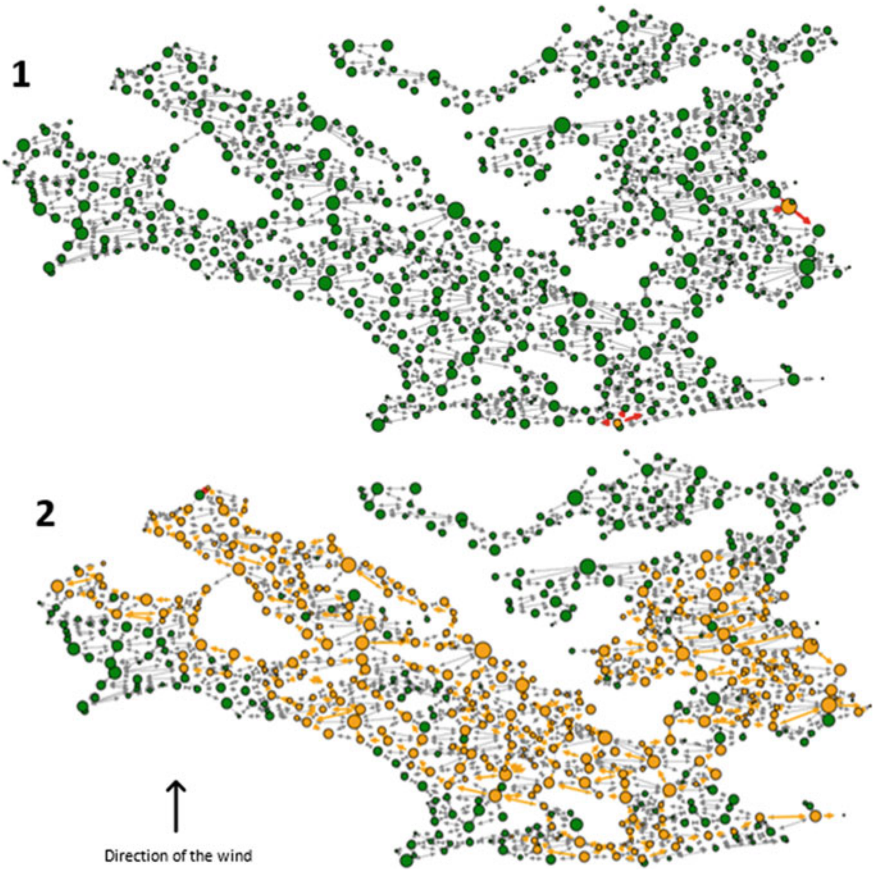


Fig. 5 Entre Douro e Sousa ZIF graph with DES applied: (1) the fire started but has not yet spread and (2) the fire extinguished

area for each of the three scenarios, a non-significant p-value (> 0.05) was obtained, indicating that there are no significant differences regarding the average percentage of burnt area, for light breeze versus gale.

The results presented in Table 3 for a fixed ignition point located south of the forest are the same as in Table 2, for the 10% wind speed rule. There is a slight decrease in the percentage of burnt area in the case of the two ignition points, when compared to one ignition point located in the south, and a strong decrease in the time of fire spread. There are two potential explanations for why the fire spread time decreased in the scenario of the two ignition points: (i) the decrease in the percentage of burnt area, since the fire propagation time decreases with the decrease in burnt area, and (ii) the existence of the two ignition points which causes the fire to propagate at the same time in two different places of the forest thus decreasing its propagation time.

Table 3 Results obtained for the Entre Douro e Sousa ZIF graph with one and two ignition points for two different wind speeds

	Fixed ignition point (south of the forest)		
	Light breeze	Gale	Z test p-value
95% CI for percentage of burnt area	[31.2, 33.4]	[30.0, 32.2]	0.147
95% CI for total time	[75.0, 78.1]	[7.3, 7.6]	< 0.005
	Fixed ignition point (north of the forest)		
	Light breeze	Gale	Z test p-value
95% CI for percentage of burnt area	[5.5, 5.9]	[5.4, 5.8]	0.315
95% CI for total time	[26.3, 27.7]	[2.9, 3.1]	< 0.005
	Two fixed ignition points (south and east of the forest)		
	Light breeze	Gale	Z test p-value
95% CI for percentage of burnt area	[29.7, 31.6]	[28.9, 30.8]	0.231
95% CI for total time	[45.3, 47.4]	[5.3, 5.6]	< 0.005

The results were in line with what was expected since with two ignitions points of fire it would be expected that the fire spread is faster. Even so, the percentage of burnt area decreases slightly, contrary to what was expected.

The results presented are, as it was previously mentioned, for two fixed ignition points and one thousand runs, i.e., as a new network is formed, the ignition points remain unchanged, one to the south and the other to the east of the forest. The ignition point located to the south of the forest is favourable to fire spread since the wind direction is north, so there is a larger area through which the fire can effectively propagate. For the northern ignition point the opposite happens, making it unfavourable to the fire spread, which is corroborated by the low percentages of burnt area.

According to the literature ([11, 16, 17]), the existence of roads, the terrain elevation and the area, are some of the influential parameters in fire ignition and there is information regarding these three parameters for each of the stands. In order to visualise these three parameters, it was decided to assume an elevation above 290 m and an area greater than 9 ha. Therefore, in Fig. 6, the stands with these conditions, and with roads, are represented with a red border. It was considered that a stand without those conditions (i.e. without a red border) has a zero probability of ignition, while those identified with a red border have an equal probability of being selected as a point of ignition.

Since the elevation of the forest is higher to the east, the risk of fire ignition is also higher to the east where most of the stands have roads, higher elevation and larger area. Now, in order to analyse the results, the model was run a thousand times with variable ignition point within those that presented higher ignition risk, that is, in each

Fig. 6 Entre Douro e Sousa ZIF graph: fire ignition risk map

Table 4 Results obtained for the Entre Douro e Sousa ZIF graph with one variable ignition point for two different wind speeds and taking into account only the ignition points with higher risk

Wind direction: North	Light breeze	Gale	Z test p-value
95% CI for percentage of burnt area	[9.4, 10.7]	[9.1, 10.4]	0.486
95% CI for total time	[21.7, 23.4]	[2.8, 3.1]	< 0.05

run the ignition point changes to any node identified in Fig. 6. It was considered a wind direction to north and the results of these experiments can be seen in Table 4.

When comparing a fixed ignition point in the south of the forest (Table 3) with a variable ignition point (Table 4) there is a clear decrease in the percentage of burnt area as well as in the fire propagation time when the ignition points become variable, as expected since the points may not be favourable to the fire propagation, considering the direction of the wind.

4 Conclusions

The work described in this paper focused on the development of a Discrete Event Simulation method to model fire propagation in a graph, which proved to be effective and adequate. The model was developed in order to obtain a greater insight into how fire can spread through a specific landscape, and how the factors considered can affect the behaviour of the fire and the time it takes to burn the area. However, there are some limitations that can be pointed out. This method is greatly influenced by the probabilities of fire spread and therefore these should be defined as accurately as possible, but it should be noted that to obtain these values in a real situation is a complex process. The wind was chosen, as it is the parameter that most influences these probabilities, but as no other aspect of the forest was considered, the characterisation of the area should be improved.

Two other points that should be considered in further studies are the way the probabilities of fire ignition are defined and how to include the volatility of the wind in the model. The former was obtained in a very simple way, which, in future work, would require improvement since assigning a probability of zero to a certain number of nodes may be a too drastic approach. In the case of the wind, it was considered constant (both wind speed and direction) throughout the forest and during the entire time the fire was spreading. Clearly this assumption does not correspond to reality, so assigning greater volatility to the wind should be included in further analyses, based on, preferably, real data regarding wind behaviour in that area. If real data is used, the impact of wind speed and direction on the speed of fire spread can be included in the model in a more realistic manner, thus leading to more accurate results.

Furthermore, for future work fire projections could be incorporated, i.e., for certain conditions the fire can be projected in such a way that a new ignition in another place of the forest occurs, as well as the impact of fire fighting resources in the propagation of the fire, since it is expected that a forest fire is not left unattended until it extinguishes by itself.

The data obtained from the Paiva and Entre Douro e Sousa ZIF were very important for the verification and incorporation of the study in real situations. It is also important to use these data to improve the model (e.g., the calculation of probabilities) and to study the impact of other factors (e.g., slope, vegetation). The models applied showed results close to what would happen in reality, evidencing the importance and suitability of the study. Wind speed and direction, as well as the location of the fire ignition point, are factors that greatly affect fire behaviour, and this was verified in this study.

Forest fires unfortunately affect the planet more and more, therefore, their study and understanding are of great importance in order to help to reduce or eliminate them. The model applied should thus contribute to the understanding of this issue, as a problem that highly impacts the environment and the life of the communities.

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