Capturing Qubit Decoherence through Paraconsistent Transition Systems

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ABSTRACT

This position paper builds on the authors' previous work on paraconsistent transition systems to propose a modelling framework for quantum circuits with explicit representation of decoherence.

CCS CONCEPTS

- **Theory of computation → Logic and verification**.

KEYWORDS

quantum decoherence, paraconsistent logic

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1 THE PROBLEM

In quantum computation decoherence is a well-known phenomenon capturing the decay of a qubit in superposition to its ground state due to environmental interference of different sorts. Of course, a circuit is effective only if unitarians and measurements are performed to superposition states within a limited period of time after their preparation. When that time is exceeded there is an increasing probability that the circuit does not behave according to its design.

This position paper discusses a modelling framework for quantum circuits represented as a sort of transition systems which incorporate the effect of decoherence by means of a 'negative' transition relation. Our starting point is a recent research line pursued by the authors on what is called in reference [4] paraconsistent transition systems.

2 THE APPROACH

The original motivation comes from the domain of many-valued logics to deal with contexts in which the classical bivalent distinction is not enough, in particular when facing the need to capture vagueness or uncertainty. Residuated lattices, adding a commutative monoidal structure to a complete lattice such that the monoid composition has a right adjoint, the residue, provide the semantic universe for such logics. A suitable choice of the lattice carrier, which stands for the set of truth values, does the job — a typical example being the real $[0, 1]$ interval. Reference [2] explores, in a systematic way, the modal extensions of many-valued logics whose Kripke frames are defined over (variants of) residuated lattices.

We argue, however, that this is not yet the whole picture. In quantum computations, as in other modelling scenarios, there may also be a need to deal simultaneously with what could be called *positive* and *negative* accessibility relations, one weighting the possibility of a transition to be present, the other weighting the possibility of being absent. In the concrete contexts we are interested in, such weights are not complementary, and thus both relations should be explicitly incorporated in the Kripke frame.

A simplified case, taken from [4], assumes that weights for both transitions come from a residuated lattice over the real $[0, 1]$ interval. Then, the two accessibility relations jointly express a scenario of

- *inconsistency*, when the positive and negative weights are contradictory, i.e. they sum to some value greater than 1 (cf, the upper triangle in Fig. 1 filled in grey).
- *vagueness*, when the sum is less than 1 (cf, the lower, periwinkle triangle in Fig. 1);
- *strict consistency*, when the sum is exactly 1, which means that the measures of the factors enforcing or preventing a transition are complementary, corresponding to the red line in the figure.

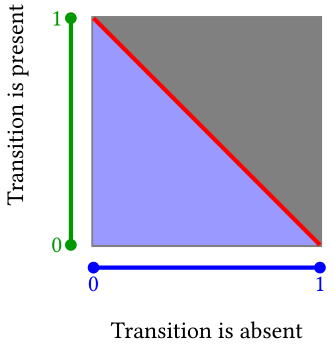

Figure 1: The vagueness-inconsistency square.

Exploring the three regions in the picture motivates an incursion into paraconsistent logic [3, 6], whose purpose is to regard inconsistent information as potentially informative. Such logics were originally developed in the decades of 1950 and 1960, mainly by F.

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Asenjo and Newton da Costa. Quickly, however, the topic attracted attention in the international community and the original scope of mathematical applications broadened out, as witnessed in a recent book on the engineering potential of paraconsistency [1].

Given a residuated lattice A to weight transitions, we have defined the notion of a *paraconsistent transition system* over A through a *positive* and a *negative* accessibility relation, taking weights from tha common universe. Equivalently, a transition between two states can be regarded as weighted by a pair of values. In a similar way, the valuation of a proposition in a state is a pair of (not necessarily complementary) values capturing, respectively, the degree upon which it may be considered to hold or to fail. A distance' between two such weights needs to be computed, namely to assess the level of vagueness or inconsistency in a given transition. As a smooth generalization of the example above, this is taken into account by a suitable metric over the carrier of A.

Not only the algebraic structure underlying such systems, but also the generated modal logic, were developed in [4]. Later [7] this was extended to the multi-modal case, thus giving rise to a *structured specification logic* [10] equipped with specific versions of the standard structured specification operators *à la CASL* [8]. This offers to the working software engineer the (formal) tools to specify such systems in a compositional way. Technically, the price to be paid to support this move consists of framing the logic as an institution [5].

3 THE QUESTION

Can this framework help to reason about quantum computation in the context of NISQ (*Noisy Intermediate-Scale Quantum*) technology [9] in which levels of decoherence of quantum memory need to be articulated with the length of the circuits to assess program quality? Such is the question we intend to address. This entails the need to model quantum circuits with an explicit representation of decoherence as a paraconsistent transition system. But it also requires a structured way to obtain new such systems from older', i.e. the study of a suitable algebra to compose (representations of) quantum circuits.

We argue that the actual possibility of a transition, approached simultaneously, from the affirmative and the negative perspectives, i.e. the possibility of the circuit remaining coherent during the execution of a program step, or becoming unstable, respectively, provides a basis to analyse the impact of qubit decoherence in quantum circuits optimization.

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