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Problem-solving in Arithmetic

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PROBLEM-SOLVING IN ARITHMETIC

By

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CHAPTER I

INTRODUCTION

For a century arithmetic has been a popular subject in the elementary schools, and has consumed more time than any other subject. The war had an effect in reducing this time on arithmetic, largely due to the war-time emphasis on health, food production, back-of-the-line morale, and kindred subjects. More recently, school activities and playground work have come in for a reasonable share of the school time. The Twenty-ninth Yearbook recognizes the child as the center of interest, and the final criterion of all values being the effect any technique of teaching or any content of instruction has on the child. It is the whole child, not a part of him, which is the reality to be kept in mind. A child's PRESENT self is but a part of himself, and an educational philosophy based on the assumption that only the present interests, needs, strengths, weaknesses, and whims of the child comprise the sole or dominating aspect of the child is disregarding that the child's future is a part of him. It is of importance to realize that this child of today, is destined to live in an environment, probably of the United States between 1935 and 1985. This total environment is not to be the actualities of which we may now be dreaming. This child must of course, be taught in methods utilizing CHILD psychology principles. Aims of this child's education must be influenced by his real nature and by the demands which life will place upon him. We should teach, then, those skills, informations, attitudes, judgments, habits, ideals, and ambitions which the child will find adequate and satisfying to his future adulthood and present childhood.

CHAPTER II

UNDERLYING THEMES OF ARITHMETIC INSTRUCTION

What, then, should be taught in the way of number knowledge and the application of number in order that the uses of society may best be served? A comprehensive study of the contributions number has made to the progress of civilization suggests a series of themes as Brueckner would call them that might well underlie the teaching of arithmetic. These should serve as one basis of guidance in directing the activities of pupils into worthwhile channels.

Number is a device invented by man which enables him to deal in an orderly way. He had to speak in vague generalities for his thoughts were chaotic and disorganized. Many systems of counting have been invented by man, the last being our decimal system which is the simplest of all, and the most efficient. No child could possibly be expected to invent such a system which was evolved by the human race over a period of a thousand years. He must be taught it and learn to use it. The number system is basic to all science. Our method of numbering years, the numbering of pages in books, house numbers, telephone numbers, the numbers on automobile licenses are other concrete illustrations of the ways in which number enables us to deal with various conditions in an orderly way.

Number has enabled man to deal with precision with quantitative aspects of the environment. Measurement is one of the most important applications of number. The various units of measurement, such as the inch, the degree, and the ounce have enabled man to express his concepts regarding objects in a meaningful, precise way. Expressions such as "quite tall" or "rather short" are almost meaningless because of lack of common standard among individuals as a basis of comparison. When we state that a person is five feet nine inches tall, we know exactly what is meant. Increased precision has been made possible by the invention of smaller units of measure or by reducing units to fractional parts. Inaccuracy of measurement is the consequence of lack of skill in applying the tools, of failure to describe by reference to a standard, of lack of insight as to procedure, and of inaccuracy of the measuring devices themselves. Without measuring devices, homes could not be built, automobiles could not be constructed, and mechanical progress would be impossible. Man is always at work devising new ways of measuring the forces of nature and aspects of the environment which he cannot now understand because they cannot be measured. Ordinary description merges gradually into measurements as standards and comparisons become more and more accurate.

Number enables man to deal systematically with quantitative aspects of his environment. The number system is at the basis of such plans of systematic arrangement as the Dewey decimal system used in many of our libraries. Statistical tables, charts, graphs, and similar diagrammatic devices are convenient ways of arranging large bodies of data in a systematic way so that they can be readily considered and interpreted. Organized tabular material, such as railroad time-tables, catalogues, are economical ways of systematizing much information. Budgets are systematic ways of allotting income and guiding expenditures. Our units of measurement enable us to collect systematic, accurate, descriptive data and to classify them on a logical basis, such as size, shape, weight, form, color, or height. The systematic methods of science have enabled us to verify facts. Science has added enormously to the truth that men may learn if they will.

Man has invented methods of reducing the labor of counting and computing. It is obvious that man first counted. The labor of counting or comparing large numbers of objects or things in different places was very great and likely to be incorrect. The invention of the process of addition and subtraction of at first small groups of numbers, then of larger groups, followed. Multiplication, invented still later, is a short-cut method of subtraction which has likewise greatly reduced the labor of computation. The decimal system is much more convenient in computation than the system involving common fractions. Increasingly the labor of computation is being done by mechanical devices, slide rules, etc. Tables containing information otherwise to be found by long laborious computations are available, such as interest tables, tables of logarithms, and tables of squares. To have a true appreciation of the convenience of such materials the pupils should have experience in performing some of the computations involved in some of them.

Present-day measuring devices for dealing with quantitative aspects of the environment are the more or less perfected end products of a long series of social institutions. Money was invented by man as a convenient basis of expressing value. It developed through a series of successive levels, beginning with barter, then the use of objects such as beads, shells, claws of animals, wampum, then crude metallic forms, to our present system of money, which includes metals, paper bills, and indirect means of indicating values, such as bonds, trade acceptances, and bills of exchange. Similarly the clock as we have it today advanced from such crude units of measurement as moons, days, candle clocks, water clocks, sundials, sand glasses to the really wonderful, marvellously accurate timepieces of the present time controlled by electricity which can measure time in fractional parts of a second. A similar evolution has taken place in the development of units for measuring length, area, volume, weight, and temperature. An appreciation of this evolutionary process should eventuate in the generalization that future improvements and refinements are likely and in many cases desirable. The pupil will thus be led to realize that we live in a world characterized by constant change.

Number has enabled man to gain increasing control over nature and to use the forces of nature to his advantage. Man invented the magnetic compass which enables him to determine direction. The woodsman, the mariner, and the traveler are safer because of this device. The aviator has measuring instruments which enable him to fly at night or through a fog with a minimum of risk. The thermometer enables man to take steps to regulate the temperature in foods in cold storage as a means of preventing their decay. The microscope enables the scientist to study minute germ cells and to determine their qualities. Medicine studies disease by determining its quantitative relationships and characteristics. The measurement of volume is involved in the control of ventilation and humidity.

Number has been an important element in developing co-operation among individuals, state, and nations of the earth. When each town and tribe had its own system of money there was so much confusion that trading was difficult. To obviate this, tribes, cities, even nations finally agreed to accept a uniform system of money. The necessity of eliminating confusion in regard to railroad timetables led to an agreement to adopt the standard time belts which are now commonly accepted. Confusion due to lack of uniformity in systems of measurement among the nations led to the invention of the metric system which is now the accepted standard in all countries except Great Britain and the United States. Insurance is a form of cooperative arrangement for the more efficient handling and use of money. Taxation may be considered to be the means whereby members of society contribute more or less freely and willingly to the general welfare.

The intelligent consideration of accurate quantitative information should be the basis on which decisions are made as to the problems involved in production, distribution, and consumption of the necessities of life. To be able to arrive at satisfactory solutions for such problems as the individual meets, he should have, or know how to secure, accurate and reliable information of a quantitative kind and the ability to interpret facts rationally and correctly.

Mathematics affords "an exact and easily workable symbolism for the expression of ideas" in precise way. Expressions such as average, median, mode, and standard convey simply and exactly ideas which it would be very difficult to express without these convenient symbols. Our magazines, newspapers, and books contain discussions in which appear many new terms such as index figures of bank failures, car loadings, prices, and so on, each of which is the attempt to express concisely a series of complicated relationships. The correct understanding of such quantitative terms as bonds, stocks, margins, involves not only a knowledge of their exact technical meanings, but also of the psychology they may suggest, such as the glamor of vivid advertisements, the methods of high-pressure salesmen.

CHAPTER III

TESTS

Standardized tests in arithmetic have been in general use for the past fifteen years and most of them have been of the survey type. The principle involved in the survey testing is that of securing a sampling of pupil achievement in a wide range of subject matter. When such samplings are tabulated and compared with norms representing many school systems, such a measure has value. General standards have been maintained through such comparisons. The value of the survey type is almost always general in nature. Probably once in four or five years is often enough for a school to apply survey tests, provided sufficient use is made in the meanwhile of diagnostic tests and of the preventive and remedial measures suggested by diagnostic steps. If given too often these survey tests tend to place pressure on teacher and pupils for securing better results without a clue or hint as to how such results should be obtained.

A more detailed treatment of results than that usually made of survey-test scores is necessary before difficulties can be diagnosed. To make possible an analysis of difficulties is the purpose of the diagnostic test. Until one knows what are the hard spots in learning arithmetic, and until he knows what difficulties are experienced in the various classes in a school, he is unable to address himself definitely to the problem of helping children learn in the most efficient manner. When a teacher first begins his work with a new class, he is comparatively ignorant of the abilities of the pupils. To be truly diagnostic, test results must first reveal weaknesses in all significant types of exercises in specific processes and operations measured. To complete the diagnosis one should go further and show why pupils have difficulties with such processes. Most measurement in the past has been for the purpose of checking what has been taught. Differences among children are great and should receive early attention from the teacher. A pre-instruction test will show at the beginning of a teaching period what remedial work needs to be done. A test may be given for diagnostic purposes at times other than at the beginning and close of a teaching period. This will make possible greater achievement and will prevent pupils from going on in an inefficient way until the end of the teaching, with no one knowing just what difficulties have been causing the trouble. Diagnostic testing is aimed not only at finding deficiencies and organizing remedial instruction. It is quite as valuable for furnishing better insight to the teacher of arithmetic by acquainting him with what is involved in the learning process in arithmetic and what types of processes and operations offer greatest difficulty. A good diagnostic test has two principal uses: the discovery of errors and the discovery of causes of the errors. Because of the danger of error in inferring from a test paper that a given answer resulted from a given cause, the individual method of diagnosis has been used. It is based upon the idea that improvements in arithmetic will result only from the removal of the causes of error and it is highly important that these causes be discovered with certainty. One can scarcely overemphasize the importance of discovering the mental processes which lie back of pupils' answers in arithmetic. In many cases pupils secure correct answers by methods which are very crude and laborious and which should be replaced by more efficient methods of thinking.

The selection of a test should depend largely on the subject matter it contains. No test is better than the problems it contains. The selection of materials in a test is of importance for its validity, for getting a real measure of the thing one sets out to measure. Since the survey test is limited in time and material, it is likely to have a better selection of material if it covers a relatively short range of subject matter. For the sake of validity and reliability, it is important the survey test contain problems that are frequently taught and used, and that are typical of others closely related in the skills involved.

The survey test must be given in a short period of time. It will probably cover the bare essentials so far as significant types of examples in an operation are concerned. All tests should be long enough to cover adequately the field to be measured, but short enough so that the administering is not burdensome. A test need not be longer for diagnostic purposes than for a survey, but the range of material used should be much less. A comparatively narrow unit must be used for diagnosis, so that all significant types in a process may be included.

Difficulties in an arithmetical process are chiefly due to three factors: the structure of a problem, the complexities and variations from type that develop in a process owing to increased length of a problem, and the combinations involved.

The reliability of a survey test may be judged fairly by analysis of its content and structural makeup. Unreliability or inaccuracy of measurement is caused by inadequate sampling, inadequate or ambiguous directions, unfamiliar forms of problems, and any other conditions distracting to pupils. If the test is made up of parts, each dealing with only one operation or a closely related group of skills, then the total score of each part will give some idea of the ability of a pupil in the operation so measured.

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CHAPTER IV

HOW ARITHMETIC DIFFICULTIES OCCUR

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Although great individual differences manifest themselves in arithmetical abilities, it should always be remembered that in an ordinary class, most of the pupils will have similar abilities in any given trait. This is true partly because they have similar abilities to begin with and partly because they have been under the same instruction in the subject. It is more economical in time to give group instruction than to give individual instruction. There are several reasons why groups as a whole, or a large part of a group, have the same difficulties in learning arithmetic. Failure of the teacher to know what is involved in the process may be caused by assuming some things are easily understood by a child which in fact are difficult to understand. Children may have been required to take long steps from an easy, known, and well-understood process to a difficult and distantly related process. The first requirement of instruction is that the teacher be familiar with the steps of learning involved.

Most new processes are easy for pupils to understand if the steps in learning have been observed. The new processes must be made understandable by relating them to previously learned processes and then being so drilled. In many cases, not enough drill is given for proper initial learning, but only enough to give an understanding of the process.

Teachers vary from year to year in the emphasis they place on different processes. Pupils may benefit or may suffer from these changes in emphasis. Teachers should be allowed some leeway in modifying methods and in placing emphasis.

It is necessary not only to drill a new process until a child understands it, but also to continue the drill until a habit is formed. After habit formation and the teacher finds it necessary to go on to a second process, the first process might be used in the second. Mechanical skills, regardless of how highly they may be developed, are soon lost unless practice in them is maintained. It is, therefore, important that a system of drill be maintained and that group measurement and diagnosis be continued in order to locate needs for remedial work, and to give a measure to show whether the drill is being maintained.

Only in rare instances is organized drill material furnished in an arithmetic test. The practice is likely to be poorly organized. The first exercises are often too involved. Intermediate steps leading up to the new operations may be necessary.

The touchstone which the man on the street applies to education is whether it will enable one to get on better. No argument is needed to establish the fact that the world is rapidly changing. It follows that the child must be endowed with an ability to make his own adjustments, and thus to meet unforeseen problems. Habits are essential for economy of action. They should be taught and learned. Confronted by unfamiliar situations one's habits no longer meet the need; one's number facts are insufficient. This means that the educated person is a thinking person, and that a fundamental problem of the school is to teach children to think. Greene tells us that as an ability it means the power to use methods of reasoning appropriate to the problems of life. Since one is not born with these characteristics, and since the circumstances of environment are not enough to compel their acquisition, it must be the office of education to cultivate them. Human beings cannot be taught to think unless placed in situations requiring thought. These situations are called problems, and arithmetic affords exercises which are likewise called problems. There is a distinction between problems of arithmetic, and problems of life. In arithmetic the problems do not 'arise'. The problem of arithmetic, however, may be made exceedingly valuable. They afford practice in thinking situations through. The problems of arithmetic are the most significant part of the subject. In problem-solving the pupil gets all the benefit that he can from doing abstract number work, plus a benefit peculiar to problem work - which is thinking.

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CHAPTER V

WHAT IS A PROBLEM?

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What is a problem? It is a situation that can only be reacted to intelligently and with the employment of insight. If the pupil does not react intelligently, even though he gets the right answer, he has not really solved the problem. The problematic character of the situation is such as has been experienced by somebody. What is a problem for one may be a routine situation for another. A problem needs must refer to a situation which is sufficiently familiar to the pupil for him to be able to realize the full significance of the conditions, and to see clearly what it is that he has to find out. Many so-called problems that appear in textbooks are not really problems for many pupils. Every problem is less of a problem to the extent to which it can be solved by memory of the methods by which similar problems have previously been solved. The expression drill in problem solving, might be a misnomer. It is, of course, desirable that pupils avail themselves of the economy of thinking which becomes possible when they recognize that an exercise is similar to one with which they have previously dealt intelligently. It is clear that at various points in a solution the pupil uses knowledge acquired in the solution of other problems, yet the reasoning employed as a whole has never been used before. Solution of problems by school pupils should be of this same insightful character. Teachers of arithmetic are coming to consider that their two most important teaching tasks are that of increasing the skill with which their pupils use the fundamentals, and that of increasing their knowledge of when to use them.

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CHAPTER VI

NATURE OF PROBLEM-SOLVING

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Problems in arithmetic may vary from, "How much do five pencils cost at two cents each?", to those in which a whole set of social relations is involved. Problems of the first type emphasize the computational function of arithmetic while those of the second type bring out the informational, sociological, and psychological functions. In our standard scales for measuring problem-solving ability, we find only problems of the first type. No method has been devised for measuring the ability to do the thinking or to perform the activities involved in solving problems of the second type. Bobbitt says, that in the community life, arithmetic is not primarily a matter of solving problems but rather of seeing things in quantitative ways and thinking in quantitative terms.

The greatest single factor which reduces the scores made by pupils on tests in problem solving is inaccuracy in the requited computations. Lutes has shown that if pupils are given practice beforehand on the computations involved in problems in tests, care being taken to make certain that the pupils will be unable subsequently to recognize the particular processes by embracing them in a mixed drill with others, there is a marked increase in the test score. This is due to the reduction in the number of errors in computations rather than to improvement in the thinking required to determine the method of solving the problem. If the purpose of measuring problem-solving ability is to measure the power of the pupil to think through the steps in reasoning out a solution rather than to measure ability to compute, it seems reasonable to propose that we reduce difficulty of computation to a minimum and check only to determine the ability of the pupil to do the thinking required.

CHAPTER VII

DIAGNOSING DIFFICULTIES OF PUPILS

The essential technical background of a person who wishes to make a diagnosis of difficulties in arithmetic includes:

(1) a knowledge of types of behavior which are evidences of underlying causes of inability not only in arithmetic, but in related subjects such as reading, and the analytical ability to observe and interpret these manifestations of difficulty;

(2) an adequate technique for bringing to the surface, facts concerning the nature of the pupil's disability or methods of work which otherwise would not be noted;

(3) a thorough knowledge of the factors underlying the development of arithmetical ability and the ways in which these factors operate;

(4) a knowledge of the specific skills and controls which constitute the list of habits needed for successful work in arithmetic;

(5) a knowledge of what remedial measures to apply when the diagnosis has been made.

Some contributing factors to failure to master arithmetic processes are of a permanent kind; others are temporary in character and can be obviated. These contributing factors vary widely from pupil to pupil and may appear in various combinations.

Lack of mentality or native ability is undoubtedly the greatest single cause of failure to learn. In most schools teachers now have access to records which show the mental level of the pupil as determined by intelligence tests. Such information is invaluable in making a diagnosis.

Physical handicaps often seriously interfere with the development of power in arithmetic. The undernourished, anemic, fatigued child cannot make the sustained effort required. Defects of vision and of hearing are serious handicaps. Fortunately the careful examinations by the school doctors and the nurse can locate these faults and steps can be taken to remedy the conditions.

Emotional factors such as fear of the teacher, dislike for the subject may result in the development of a condition which will lead to unsatisfactory growth in arithmetic, for pupils with such emotional maladjustments may make little effort to master the subject by showing little or no initiative in their work. If the right attitude between teacher and pupil exists, the pupil will regard the teacher as a counselor and guide. Faulty attitudes of the pupil toward his school work are often due to unsatisfactory environment, such as strained relations in the home, indifference of parents toward his success in school, and broken homes. Dishonesty, untrustworthiness, and other undesirable moral traits often develop in such situations. It is the duty of the teacher to be familiar with the social background of the pupils in her class in order that she may intelligently analyze the needs of each individual, and make necessary adjustments of instruction.

Pedagogical factors over which the school has some control often contribute to the failure of pupils to make satisfactory progress in arithmetic. Instruction may be unskillfully done; materials of instruction may be inadequate and poorly constructed without consideration of the known difficulties the subject presents; instruction may be by mass methods and not individualized, a condition which results in the development of many faulty inefficient habits of work by the pupils; difficulty with some process may be due to weakness in some more basic skill which should have been mastered; due to excessive absence or to transfer from one school to another, there may be big gaps in the pupil's knowledge of processes. Almost all of the above can be obviated. An important and integral part of any diagnostic and remedial work is to make certain that pupils are using well-motivated and efficiently organized, and well-graded materials of instruction.

The approach to diagnosis of arithmetic difficulty may be either through measurement and general observation, or through the use of more refined and specialized clinic methods. Three kinds of diagnosis may be differentiated; general, analytical, and psychological.

By general diagnosis is meant any procedure which gives the teacher a general picture, not a detailed one, of the pupil's ability in arithmetic, as survey tests might do, yielding scores which can be interpreted to show the general level of the pupil's ability in arithmetic. Tests of this kind merely enable the teacher to select those pupils whose performances are below standard so that their work can be more carefully observed and systematically analyzed to determine specific deficiencies and faults and what steps should be taken in the remedial work.

Analytical diagnosis is to enable the teacher to determine the specific phases of arithmetic in which the pupil is deficient. The basis of an analytical diagnosis may be a pupil's performance on a series of carefully constructed diagnostic tests, each of which yields a measure of the pupil's ability in a single phase of arithmetic. Another kind of diagnostic test consists of a wide variety of types of examples in a particular process. Such exercises are very valuable to the teacher when a process is first being taught since the results enable the teacher to determine what types the pupils have not learned to solve. The method of analytical diagnosis reveals the processes in which a pupil is deficient, or the specific phases of a process which may be the source of difficulty. These results yield only a partial diagnosis, since the teacher must in addition determine the causes of the difficulty by more penetrating techniques that such tests provide, especially in the case of pupils whose work exhibits serious weaknesses.

By psychological diagnosis is meant any procedure the teacher may use to identify the more subtle causes or nature of the difficulties located by the analytical. To be able to make a satisfactory psychological analysis, the teacher must know the possible causes and symptoms of maladjustment, the most common faults that have been found to exist in the work of pupils, various methods of making the diagnosis, and the use that can be made of data in various school records. Through observation of the pupil at work or play, the teacher can learn much concerning the pupil's social behavior as well as his work habits. Faulty procedures can thus be readily discovered.

Through an analysis of the pupil's written work, the causes of many errors can be determined. An analysis of the written work does not reveal the mental processes of the pupil at each step of the solution. They may have been involved and uneconomical but this is not evident from what is written on the paper, so more intensive study must be made.

A type of diagnosis which may be used in such cases has been developed by Courtis and has since been employed in many of the investigations of causes of difficulty in arithmetical processes, that is, the analysis of the oral responses of the pupil as he works the problem aloud. The pupil is asked to state aloud the mental steps he may take in solving the problem and the language he may employ. This plan may be supplanted by sympathetic questioning to locate faults of which the pupil may not be conscious. This enables the teacher to note any roundabout, involved, or otherwise faulty habits of work which may be the basis of the difficulty. It is apparent that such faults cannot be detected by test scores or analysis of the written work. They must be searched out by techniques that are more clinical in nature. Such faults so discovered will impress on the teacher the necessity of seeing to it from the beginning that effective procedures and economical thought processes are mastered.

The use of such clinical methods in diagnosis brings to classroom instruction the technique that is found in other professions, such as medicine. Modern clinical medicine uses diagnostic devices which determine with precision what the cause of the difficulty is, and in the light of such information prescribes the remedial treatment.

The procedure to follow in making a clinical diagnosis may be summarized thus:

> "(1) The teacher should give a survey test to secure an initial picture as to the status of the class as a whole.
> (2) The teacher should make a careful analysis of the work on the test to locate the obvious deficiencies or the types of

exercises most frequently solved incorrectly by the class.

(3) The teacher should then select for careful study those pupils whose work was considerably below the standard in one or more of the processes. Usually not more than ten percent of the pupils in a class will need much study. The students should be given the ordinary assignment of work to be done at their seats, so that all may be profitably occupied.

(4) After the class has begun to work on the assignment, one pupil who has been selected for special study should be called to the teacher's desk. The pupil should be told that the purpose of the teacher is to help him to determine the cause and nature of his arithmetic difficulties and he should be encouraged to assume a cooperative attitude in the undertaking. The teacher should think of his part of the examination as being like that of a physician who is making a clinical diagnosis of the cause of the illness of an individual. The purpose of the diagnosis by the teacher should be the location of faulty methods of work, lack of knowledge on the part of the pupil, and other possible causes of inefficiency of work. At this step the teacher should not attempt to remedy the situation by teaching correct procedures.

(5) The teacher should next select a standardized diagnostic test in the process to be investigated, such as the Buswell-John or Brueckner tests, or, if they are not available, should use some similar set of examples prepared for the purpose. Usually only one process at a time should be studied, to avoid fatigue on the part of the pupil.

(6) The teacher should explain to the pupil that he will make it easier to diagnose his difficulties if he will do his work aloud, so that the teacher may observe his procedures. The teacher should illustrate the method by working one or two typical examples. Pupils readily respond to these directions and demonstrations, especially if the teacher has created the right attitude and if the examination is conducted in a friendly, helpful spirit.

(7) As the pupil works the teacher should make notes of the types of faults that are discovered. Such a record is facilitated by the use of the record blanks that are pre-

pared on certain of the standard diagnostic tests. It is obvious that the teacher must have a firsthand appreciation of the various kinds of errors that may be discovered and of their symptoms. Sometimes the pupil stops in the middle of an example and apparently is blocked by some difficulty. By careful questioning the teacher should make an effort to get the pupil to tell what his mental processes are during the period of apparent inactivity. Altho the method of securing the pupil's testimony as to his mental processes may not be a wholly reliable one, due to his inability to describe them accurately, an observing teacher with insight can usually secure quite a vivid picture of what mental activity takes place. The length of the time required for a diagnosis will, of course, vary according to the extent and nature of the faults discovered in the pupil's work. The average time required for a single process is between fifteen and thirty minutes.

(8) When the work of the test has been completed the teacher should carefully analyze the notes taken during the examination and summarize the findings of the diagnosis. These may be recorded on the standardized blank, on the pages of a notebook in which records of a diagnosis are kept, or may be filed in some other convenient form for reference.

(9) The necessary reteaching and remedial work should then be undertaken in the light of the findings of the diagnosis." 1

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1. Brueckner and Melby, Diagnosis and Remedial Teaching,

Leo J. Brueckner through diagnostic procedure found the chief causes of pupil difficulty in problem-solving to be:

"(1) Failure to comprehend the problem in whole or in part, due to inferior reading ability, inability to visualize the situation, lack of practise in solving problems, and similar conditions; (2) carelessness in reading, resulting in the omission of essential ideas or misreading; (3) inability to perform the computations involved, either thru forgetting of the procedure or failure to learn it; (4) confusion of process, resulting in the random trial of any process that may come to mind; (5) lack of knowledge of essential facts, rules, and formulas such as how many inches there are in a yard, or how to find the perimeter of a rectangle; (6) carelessness in arranging the written work, and general lack of neatness; (7) ignorance of quantitative relations due to lack of vocabulary or of understanding of principles, such as the relation between selling price, cost, profit, and margin; (8) lack of interest, due to repeated failure, to difficulty of problem material, its unattractiveness, and the like; (9) general lack of mental ability." 2

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2. Brueckner, Leo J., <u>Diagnostic and Remedial Teaching of Arithmetic</u>,

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Brueckner says the foregoing faults can be located by suitable diagnostic tests and devices, using the following steps:

"(1) Give the entire class survey tests in arithmetic processes and in problem solving to get measures of the pupils' ability to compute and to solve problems.

(2) Observe the work of pupils of inferior ability in problem solving located by this test especially during the reading period and when solving problems.

(3) Give analytical diagnostic tests in problem solving to determine the specific nature of the difficulty.

(4) Analyze written work in problem solving to discover, if possible, the nature of the difficulty.

(5) Supplement these data by a personal interview during which a clinical procedure should be followed and the pupil's methods of work analyzed in detail.

(6) Use any available data such as health records, social records, reading test records, intelligence test scores, and the like, that will help to arrive at a conclusion as to the source of difficulty.

(7) Scrutinize carefully and critically the quality of problem material that is presented to determine to what extent its inadequacies and deficiencies may be the contributing factor." 3

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3. Ibid.

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CHAPTER VIII

REMEDIAL WORK

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The remedial instruction is concerned with the removal of factors that have led to the difficulty and with the substitution of efficient methods of work for wasteful procedures that the pupil may have acquired. There appear to be at least three essential elements in a remedial program: developing a purposeful cooperative attitude of the pupil, correcting minor deficiencies due to temporary difficulties or specific gaps in training, and general reteaching in cases which exhibit such serious deficiencies that a restudy of essential fundamentals is necessary. The necessity for remedial teaching in the case of any pupil indicates the presence of an emergency situation which must be corrected. Helpful instruction can be done by removing the conditions that cause failure and building up in the pupil the feeling of satisfaction that results from successful efforts. This may be accomplished by pointing out efficient methods of attack, using simpler types of material, helping the pupil to measure the success of his efforts, using well-graded practise exercises, progress charts, and to show the utility of number, a variety of attractive, interesting activities. Some pupils have such a marked lack of ability to master arithmetic that remedial teaching in their cases is difficult. Brueckner submits the following general principles as basic in remedial instruction:

> "(1) Having the pupils solve many interesting, wellgraded problems during the arithmetic period will yield big returns.
>
> (2) Superior pupils apparently can devise technics of problem solving that are highly efficient and should be encouraged to do so.
>
> (3) Pupils of average or lower ability

must be taught systematic procedures to use in problem solving, since otherwise they may invent and acquire wasteful, uneconomical methods of work. There probably is no single best way for all pupils.

(4) Increasing accuracy of computation thru wellplanned use of systematically organized practise exercises will greatly increase scores on problem tests because of the elimination of the large number of errors in problems due to failure to compute correctly.

(5) Exercises in careful exact reading, such as following directions, are very helpful.

(6) Vocabulary exercises on important arithmetic terms and number concepts are essential.

(7) The use of original problems and concrete applications growing out of local situations and experiences of pupils is a valuable means of developing in the pupil the ability to sense number relations and to generalize his number concepts.

(8) In connection with work on various original problems, such specific reading skills as use of the index, table of contents, ability to summarize, and the like, are essential and should be taught as a part of the instruction in arithmetic.

(9) Neatness of work and orderly arrangement of solution should be emphasized.

(10) Standardized tests and other objective methods of showing the pupil his improvement in solving arithmetic problems at regular intervals during the year are an essential element in a remedial program." 1

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1. Brueckner, Leo J., op. cit.,

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CHAPTER IX

TYPE OF PRACTICE WORK

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The results of critical studies of sources of difficulties have enabled authors of instructional material to devise learning exercises for practice which will greatly reduce the difficulty of these steps.

Brueckner has summarized the principles that should underlie the organization of practice work in arithmetic:

> "(1) A background of meanings and appreciations of the uses and functions of number in the lives of children should form the basis of arithmetic work in the primary grades and should be associated with the formal teaching of number processes in all grades.
>
> (2) The organization of practise in arithmetic processes must be individualized to make provision for the wide range of individual differences in such factors as rate of learning, difficulties encountered, and rate of forgetting.
>
> (3) The assignment of practise should be made according to the needs of pupils revealed where possible by standard tests of ability and comprehensive diagnostic exercise.
>
> (4) Selfscoring of the work done, self-diagnosis of difficulties, and other procedures which will develop in the pupil a critical, constructive, cooperative attitude toward his work, should be used.
>
> (5) Instructional materials should be constructed on the basis of a careful analysis of the skills involved in each process and with due consideration of the known difficulties that experiments have shown the subject presents to the pupil.
>
> (6) In order to prevent loss of ability due to disuse, there must be a systematic program of practise to maintain the skills that are acquired.
>
> (7) In order to prevent the development of faulty, inefficient methods of work, the teacher must teach simple, economical procedures and methods of thinking, and from time to time check to make certain that pupils actually acquire them.

(8) Genuine and legitimate motivation should be a feature of all classwork." 1

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1. Brueckner, Leo J., *op. cit.*,

P. R. Stevenson uses as remedial exercises the following:

"I. Teach pupils to read problems and develop a technique for working them. When assigning the lesson have the pupils read the problem silently and teach them to pick out (1) what they are asked to find, (2) what is given to help answer the question, (3) what process or processes are to be used, and instruct them to estimate answers.

Problems which contain extra data should be so handled that the pupils will learn to pick out essential elements.

II. Teach the vocabulary used in problems. The pupils should state the problems in their own words, and in as many different ways as possible, while special attention should be given to technical words, such as *rate*, *salary*, *percent*, *dealer*, *commission*, etc.

III. Dramatize problems referring to measurements, e.g. pints, quarts, inches, feet, area, etc.

IV. Give a large variety of problems from life situations.

The pupils should be encouraged to make up problems dealing with their own or their parents' activities. An excellent device to increase the ease of understanding problems is to vary the language in which they are started. Problems centering around specific activities such as a field meet, a picnic, etc., arouse the interest of the pupils and serve their arithmetical purpose, too.

V. Give individual instruction. An excellent rule in arithmetic problems, as in other subjects, is to find out what the pupil needs and see that he gets it. For example, if he has vocabulary difficulties and cannot estimate answers,

give him special instruction in these
two phases." 2

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2. Stevenson, P. R., "Difficulties in Problem Solving", <u>Journal of Educational Research</u>, Feb., 1925, pp. 95-104.

CHAPTER X

INDIVIDUAL DIFFERENCES

In remedial work the organization of the practise on arithmetic processes must recognize the fact of individual differences. Pupils in the upper grades have progressed to widely varying levels of achievement. Differences in pupil interest, attitudes, emotional reactions, and ability to remember should be given due consideration. The matter of differences in maturity has led to the formation of Junior High Schools as one means of helping solve the difficulty. To plan for individual differences: give more difficult questions under the same topic to the more advanced pupils, let them help to direct and check the work of the backward ones, develop initiative, leadership in the less aggressive of the advanced pupils by appointing them as leaders, give maximum and minimum assignments.

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It is impossible for the borderline type ever to handle the same amount of subject matter that the normal child handles. It behooves the teacher to select for this child the subject matter within his capacity that is of highest social value and interest to him. The common core, or that which all children should acquire if possible, is vitally necessary to this child as it is the foundation of all quantitative thinking.

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CHAPTER XI

BORDERLINE PUPIL

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It is impossible for the borderline type ever to handle the same amount of subject matter that the normal child handles. It behooves the teacher to select for this child the subject matter within his capacity that is of highest social value and interest to him. The common core, or that which all children should acquire if possible, is vitally necessary to this child as it is the foundation of all quantitative thinking.

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CHAPTER XII

TRANSFER OF TRAINING

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The problem of transfer of training may be reduced to the specific questions:

(1) To what extent will specific training on certain topics carry over to other topics?

(2) On what level of mastery of a skill does transfer begin to operate?

(3) In what ways can transfer be facilitated and increased?

(4) To what extent will the teaching of arithmetic become generalized into quantitative methods of teaching?

The present status of the transfer problem seems to be - transfer exists to the extent that the same skills are used in the new situation. Transfer is often small because we fail to realize that the same skills could be used. If we want transfer or the applications of skills acquired in one situation to operate in another situation, we must train in the ability to look for uses of old skills. The old skills will not transfer by themselves except when the similarities are very close. The discovery of effective methods of developing quantitative modes of thinking by arithmetical or other instruction ranks among research problems of the highest order.

CHAPTER XIII

EXPERIMENT IN PROBLEM-SOLVING

The following is an experiment conducted to see if there could be an improvement in the conduct of problem-solving in the Eighth Grade.

As a survey test the Terman Group test - Form A - was administered to give a sampling of each pupil's ability, not showing in what respects any attainment would be lacking. The survey tests were scored on the basis of correct answers and from the pupil's score and his living age was determined his mental age. This mental age was equated into the I. Q. for each pupil. The following table shows the arrangement of the class according to scores received in the Terman Test, with the highest scores being given first. The upper quartile from and including 133 to the highest score 169 includes ten pupils which is just one-fourth of the class. The lowest quartile of the class, using twelve of the class includes the twelve scores falling below 100. The range of scores in the upper quartile was 36 points while the range of scores in the lower quartile was from 78 to 99 showing a difference of but 21 points. The pupils in the lower quartile were working a little closer together.

TABLE - I

Scores from TERMAN test FORM a in order of rank.

	Pupil	Score	Living Age	Mental Age
	23	169	13^{9}	17^{6}
	36	165	13^{5}	17^{5}
	33	161	12^{7}	17^{3}
	37	159	13^{10}	17_{1}
	35	154	13^{8}	16^{10}
Upper	18	152	13^{7}	16^{10}
quartile	5	152	13^{10}	16^{10}
	14	151	14^{1}	16^{9}
	32	136	15^{10}	16^{1}_{10}
	17	133	13^{10}	15_{11}
	3	132	14^{7}	15_{11}
	20	132	14^{3}	15_{11}
	9	132	13^{9}	15^{11}
	26	132	13^{11}	15^{7}
	31	126	14^{8}	15^{5}
	16	124	14^{3}	15_{4}
	11	123	14^{7}	15
	6	121	16^{2}	15^{3}
Class	27	120	13^{6}	15^{3}
Median 120	19	120	15^{5}	15^{3}
	22	117	14^{8}	15^{2}
	39	116	14^{7}	15^{2}
	24	116	15^{1}	15^{2}
	34	115	14^{10}	15^{1}
	28	112	13^{6}	14^{11}
	29	109	13^{6}	14^{9}
	40	107	14^{4}	14^{8}
	7	103	15^{5}	14^{5}
Lower	4	99	14^{11}	14^{3}
quartile	13	99	13^{8}_{1}	14^{3}_{2}
	1	97	14	14_{2}
	15	98	16_{4}	14
	30	88	15_{3}	13^{10}
	10	87	15_{11}	13^{9}
	12	85	15_{10}	13^{8}
	21	84	14	13^{7}
	8	81	14^{5}	13^{6}
	25	80	14^{11}	13^{5}
	38	79	14^{10}	13^{10}
	2	78	15^{7}	13^{4}

Every Pupil in lower quartile has a score of less than 100.

TABLE II

I. Q.'s from TERMAN test in order of rank.

	Pupil	I. Q. Score	Mental Age	Living Age
Upper quartile	33	137	17^{3}	12^{7}
	23	127	17^{6}	13^{9}
	36	125	17^{5}	13^{5}
	35	123	16^{10}	13^{8}
	18	123	16^{10}	13^{7}
	37	122	17^{1}	13^{10}
	5	121	16^{10}	13^{10}
	17	121	15^{10}	13^{10}
	14	118	16^{9}	14^{1}
	9	115	15^{11}	13^{9}
	26	114	15^{11}	13^{11}
	27	112	15^{3}	13^{6}
	20	111	15^{11}	14^{3}
	3	109	15^{11}	14^{7}
	16	108	15^{5}	14^{3}
	32	107	16^{1}	15^{10}
	22	107	15^{2}	14^{8}
	31	106	15^{7}	14^{8}
	13	105	14^{3}	13^{8}
Class Median 104	39	104	15^{2}	14^{7}
	11	104	15^{4}	14^{7}
	29	102	14^{9}	13^{6}
	40	102	14^{8}	14^{4}
	34	101	15^{1}	14^{10}
	28	101	14^{11}	13^{6}
	24	100	15^{2}	15^{1}
	1	100	14^{2}	14^{1}
Lowest quartile	19	98	15^{3}	15^{5}
	6	94	15^{3}	16^{2}
	4	94	14^{3}	14^{11}
	7	93	14^{5}	15^{5}
	8	93	13^{6}	14^{5}
	21	91	13^{7}	14^{10}
	30	90	13^{10}	15^{4}
	25	89	13^{5}	14^{11}
	15	87	14^{2}	16
	10	87	13^{9}	15^{3}
	38	87	13^{10}	14^{10}
	12	85	13^{8}	15^{11}
	2	85	13^{4}	15^{7}

Every pupil in lower quartile with mental age less than living age.

In the arrangement of the same class but according to the I. Q., the youngest boy in the class had 137, the highest I. Q. Number nine with an I. Q. of 115 has moved up into the upper quartile, with a living age of thirteen years, nine months.

The Compass Diagnostic Test No. XVIII, Advanced Form A in Problem Analysis was administered to the class. These diagnostic tests seem designed to enable each pupil to think by graduated steps into and through his particular difficulty. Failing in this, they enable the pupil to locate himself more precisely as to reasoning capability than is possible in the survey type by revealing specific sources of error. This test provides for five different items to be answered for each of the fifteen problems, for "Comprehension", for "What is Given", for "What is Called For", for "Probable Answer", and for "Correct Solution".

This test seemed to cover all of the separate arithmetic skills which have marked social utility and which receive emphasis in a well-balanced arithmetic course of study. The problems also seem to check with current business practice. The test samples an ability widely enough to furnish quite an accurate index to the ability of each pupil. The score as made by most of the pupils studied in the light of health, daily work, I.Q., scores in other tests, and the teacher's estimates seems about a normal measure.

The following page is a tabulation showing all errors resulting from the Compass Diagnostic Test, Form A, and showing the number of problems left unfinished. As all unfinished problems were the last problems of the test, it would seem to be through lack of time.

✓ means one error,

• counts as one-third error according to the key sent out by the publishers. The stated reason for that is that Part 2 of each problem being "What is Given" has on an average three correct answers thus each dot in Part 2 counts for but one-third.

means an error because the problem was unfinished. Thus, first vertical column reads one ✓, therefore one error. Second vertical column, i.e., Part 2, "What is Given", has forty-five dots (•), therefore fifteen errors because there were on an average three parts given in Part 2 in each problem.

1st TEST given DEC. 14, 1931

FORM "A."

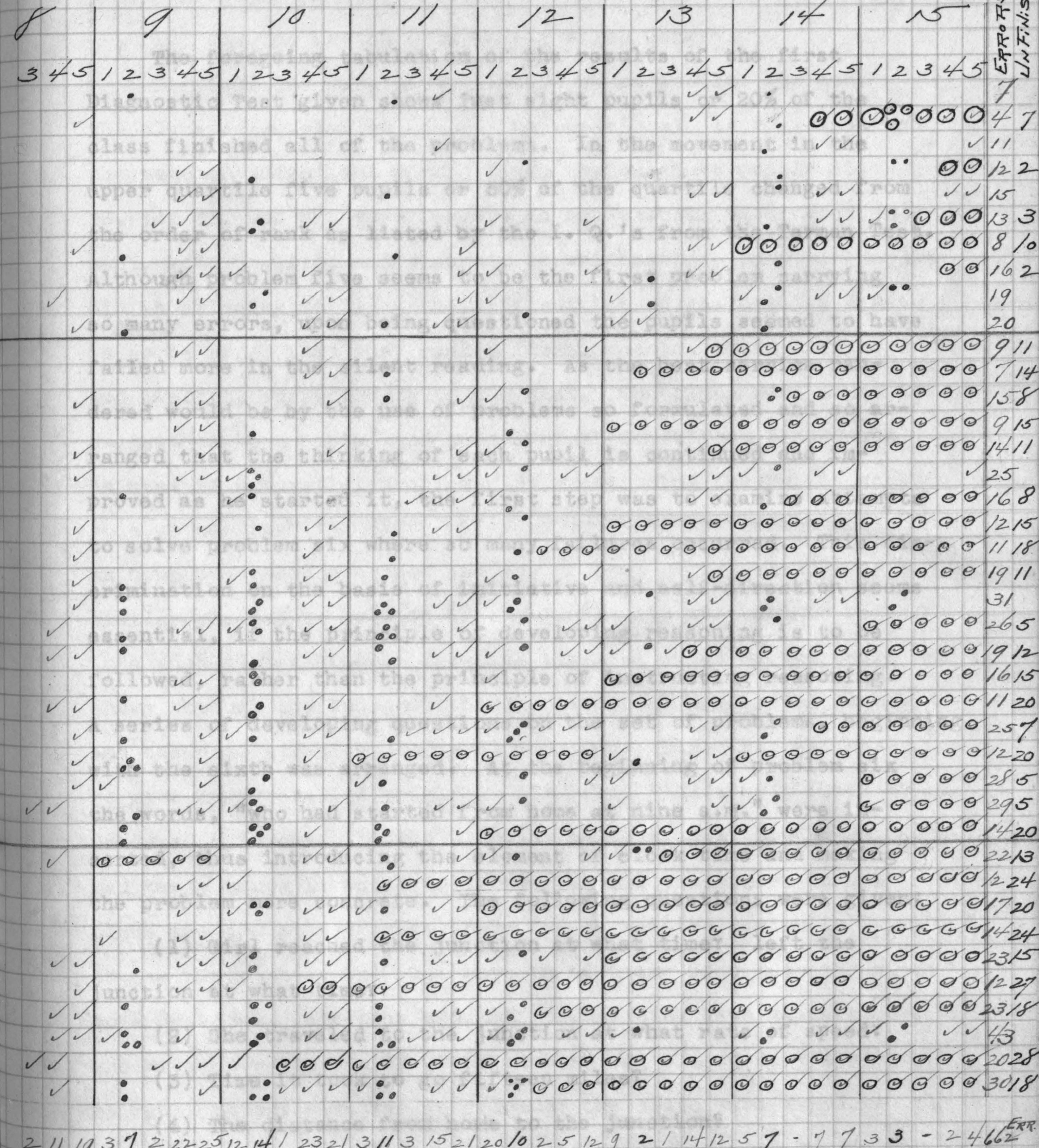
TABLE III. Compass Diagnostic

	1					2					3					4					5					6					7						
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2
		45÷3=					25÷3=					49÷3=																									
ERRORS	1	15	5	3	4	4	8	2	3	24	12	16	6	6	9	2	7	1	6	9	15	7	4	19	25	7	14	1	14	28	3	6	4	20	18	12	6
UNFINISHED	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
TOTAL ERRORS	28					41					49					25					70					64					31						

Pupils: CHAS.M. 23, ELIZ. 36, ELENOR 14, FRANK 19, MART. 33, ANTON 26, PEARL 3, JOE 18, CHEST. 40, WALSH 39, DAVID 5, DORIS 37, ANTON 31, PAVIK 29, ROB. D. 9, GEO. 2, ANNA 20, MARIE 17, EMMA 28, JACOB 35, ROB. F. 11, LEONA 12, HENRI 4, L.O 24, POPP 27, MAR. 32, JOE C. 7, LEE 22, BENO 1, MIKE 34, NORMA 13, ELVERA 16, HERB. 30, G.G. 15, YVON.. 8, DAN.. 10, A.O. 25, LUCIL. 38, BESS. 6, K.K. 21

IN PART 2 OF EACH PROBLEM THE NO. OF DOTS WERE ÷ 3, AS EACH • WAS BUT $\frac{1}{3}$ OF AN E[RROR]
FOR " " WAS MADE UP OF 3 SMALLER PARTS

Arithmetic Test — chart of Errors



2 11 19 3 7 2 22 25 12 14 1 23 21 3 11 3 15 21 20 10 2 5 12 9 2 1 14 12 5 7 - 7 7 3 3 - 2 4 662 ERR.

UNFIN. 1 1 1 1 1 1 2 2 3 5 5 5 5 5 8 8 11 11 11 14 15 16 17 20 22 22 24 26 26 29 29 30 32 32 - 440 UNFIN.

4 8 3 23 26 2 25 23 6 16 8 20 26 28 8 13 16 23 23 15 17 31 32 27 29 24 33 33 32 32 30 34 36

50 64 82 76 88 118 146 164

TOTAL ERRORS 1102

KEY: → ✓ means 1 ERROR
• counts as 1/3 ERROR
⊙ means UNFINISHED

The foregoing tabulation of the results of the first Diagnostic Test given shows just eight pupils or 20% of the class finished all of the problems. In the movement in the upper quartile five pupils or 50% of the quartile changed from the order of rank as listed by the I. Q.'s from the Terman Test. Although problem five seems to be the first problem carrying so many errors, upon being questioned the pupils seemed to have failed more in the silent reading. As the best service rendered would be by the use of problems so formulated and so arranged that the thinking of each pupil is continued and improved as he started it, the first step was to examine attempts to solve problem six where so many failures occurred. This discrimination on the basis of initiative and self-direction seems essential, if the principle of developing reasoning is to be followed, rather than the principle of instructing reasoning. A series of developing questions on the set of problems, beginning with the sixth was arranged. At the beginning of problem six the words, "who had started from home at nine a.m." were inserted, thus introducing the element of clock time and making the problem more concrete. The following questions were given:

(1) Girl reached the junction at what time? left the junction at what time?

(2) She traveled to the junction at what rate of speed?

(3) Time it took to go fifteen miles?

(4) The distance from home to the junction?

(5) The time the girl had at the junction in which to make the Express train?

(6) Did the girl travel two hours on the branch line?

(7) Was the branch line thirty miles long?

Difficulties were being cleared up on problem six and the class were asked to add to the questions. As the class and the teacher judged the work, it was obvious that pupils 23, 36, 14, 33, and 37, were leaders in the exercising of good judgment, good English, and accuracy in arithmetic. They were splendid in bringing to light the difficulties of the problem and it is to be noted that they ranked first, second, third, fifth, and twelfth, respectively in the test. The class were asked to suggest current social and geographical conditions that could be used in a parallel problem. By questioning the pupils were able to see that the fifty miles an hour and the thirty miles an hour possessed a common divisor of ten. This relation or a similar one was to be seen if the pupils did not want to be led into unusual fractions. The time the girl spent on the train was shown through the two conditions - length of her branch line, and the rate at which her train traveled. The class was asked to make up a parallel problem. With very little help Pupil 37 produced this one: "Going 80 miles an hour, Col. Lindbergh had three hours in which to carry his sick passengers in his plane the 180 mile distance from the devastated region in China to the river where the sick were to take the boat down the river to a hospital. How many minutes did the sick have to wait at the river for the boat?" This problem was placed on the board by Pupil 37 and with her as a leader the class were asked to give questions on this problem which Pupil 37 placed on the board. The other four leaders, Pupils 23, 36, 14, and 33, were asked to put on the board parallel problems for parts four and five for this problem six. These two parts carried the most errors in problem six, and obviously were the hardest from the arithmetic standpoint.

Problem five was taken up next and in the same way as above. Parts 1, 4, and 5 were most difficult. The social situation seemed not concrete enough for the fifteen pupils who failed on part one. Pupil 14 came forward with "Japan ordered 15 motorcycles from the Harley-Davidson Company in Milwaukee. Japan wished to use them in warfare so she has had machine guns put on each at a cost totaling $600.00 in all and armored plate cost $15.00 on each motorcycle. How much extra charge should the Harley-Davidson Company add to the original cost of $100.00 for each motorcycle?" Pupils 23, 36, 14, 33, and 37 were appointed as leaders with the following assignments. Pupil 23 had charge of problems 4, 7, and 13. Pupil 36 had 2, 8, and 15. Pupil 14 had 5, 12, and 14. Pupil 33 had problems 1, 10, and 11, but he was taken sick and Pupil 3, ranking seven in the results of the test was to "Carry On" and Pupil 37 had charge of 3, 6, and 9. Each pupil - leader had five class-mates in his group during their work-period from 8:30 to 9:00 while the teacher segregated the ten pupils of the lower quartile and worked with them at 8:30 and again from 3:30 to 4:00. At 9:00 o'clock until 9:40 the entire class assembled to judge the problems and questions.

The problems so devised portrayed situations in a school, in child's play, in a school-child's life, situations to be met by parents in the home, at the grocer's, at the coal dealer's. Situations of the real-estate dealer, the boy-scout, the newsboy, an older boy in a profit-making business, in gardening, in building a radio, in dress-making for girls, for adult workmen, for conditions on the farm, for the farmer. The current items of interest invaded and controlled his thought much. These problems had to be stated in terms of most frequently used fractions, of United States money, whole numbers, per cents, decimals, tables of square measure, linear measure, cubic measure, dry measure, liquid measure and area of a rectangle and of a triangle.

The leader and his group had a choice of the following types of tests to be used as diagnostic, remedial or progress tests, true-false, yes-no, matching, completion, multiple-choice, analogy, same-opposite, or similar to any used in the Compass - Diagnostic Test.

Pupil 37 brought forward this problem as paralleling problem nine of the test. "I have a triangular corner of a piece of silk, having one straight edge 36" and the other straight edge is 16". How many modernistic designs can be painted on it, allowing one square foot for each design." The following questions were submitted and the leader tells us including questions that had heretofore puzzled the one who later gave it.

(1) Does length of Δ times its width equal the area?

(2) How much of the triangle is not used?

(3) 5 is nearer the correct answer than 2?

(4) Is the short dimension given, more or less than $\frac{1}{2}$ yd.?

(5) Is one design 12 inches?

(6) Write the part of the table of denominate numbers that you use.

Pupil 3 brought this problem from her group as a parallel to problem ten of the boy selling his newspapers. "After paying for chickens bought from a hatchery at $ .04 each, a boy had $ .80 left. His Mother had donated $1.40 and his Father had given him half a dollar. When the boy had shipped them to a poultry-raiser at $ .07 each, he found he had $18.10. How many chicks did the boy sell?"

(1) The answer is $2.20.

(2) When do you use the $ .04?

(3) What is the boy's entire income?

(4) Can you find his entire profit?

(5) Is the final process +, -, x , or ÷ ?

This problem was presented as a parallel to Problem eleven. "A sheep pasture is 120 feet wide and 180 feet long. An adjoining field where the lambs are kept, is 90 feet long by 60 feet wide. The owner wishes that these two pastures be reseeded with a peck of seed to about 100 square yards. How many bushels of grass seed will be necessary?"

(1) Write down the parts of the tables of denominate numbers used.

(2) Does amount of grass seed depend upon area?

(3) Give perimeter of smaller pasture.

(4) Give area covered by a bushel of grass seed.

(5) If the larger field is twice as long and twice as wide as the smaller one, is the area twice that of the smaller?

The following was submitted as a parallel to problem twelve. "If Chester, a newsboy in our class, receives 6% commission on his first ten dollars' worth of subscriptions to the "Liberty" and 3% on all above ten dollars, what should Chester remit to the office from $18.00 worth of subscriptions collected?"

(1) Give the number of dollars on which Chester gets 3%.

(2) "Remit to the owner" means what?

(3) Did Chester get 9% of $18.00?

(4) Does Chester get more or less than a dollar in commissions?

(5) What % of the first $10.00 goes to the office?

This problem was submitted to parallel problem 13. "Three mothers agreed to make our curtains for our Girls' Club-house for $12.00. Two of the mothers together were to furnish 5/6 of the money, while the third was to make the curtains and furnish the remainder of the money. How much money did the third mother save by doing the work?"

(1) If each mother shared alike in the $12.00, what would each pay?

(2) Is $1.50 or $2.50 nearest the correct answer?

(3) If each mother shared alike what difference would there be in amount of money paid by the third mother?

(4) Third mother's decreased amount of money paid by first mother.

(5) If there were 30 girls in that club, the second mother was paying for how many girls' shares?

Problem 14. "Three brothers were promised $5.20 to spade up and buy plants for, and fertilize their mother's garden. The garden was $5\frac{1}{2}$ wide by 20 ft. long and it took $\frac{5}{4}$ of a day, all three boys working together, to complete the work. They bought 1-2/3 in. yds. of fertilizer at 60¢ an cu. yd., and 15 doz. plants at 13¢ a doz. At what rate per day can you say that the mother paid the three boys together if they shared equally?"

(1) Do all 3 boys begin and finish at the same time?

(2) Question calls for what?

(3) Did the mother get but 3/4 day's work?

(4) Did the first boy receive a day's wages?

(5) Was a day's wages for the three boys together more or less than $2.25?

(6) What fact is given as to the money the boys receive?

(7) Consider a day as 9 hours if the boys began at 7:00 A.M. and had one hour off at noon, what time would it be when the boys had finished?

(8) $5.20 is for what?

Problem 15. "In the Spring a potato warehouse still had 1970 bus. of potatoes. One bin 12 ft. long, 6 ft. 9 in. wide and 9 ft. 4 in. deep was full and held 945 bus. A new bin 10 ft. 3 in. long and 10 ft. wide held the rest. How high up in this new bin did the potatoes come?"

(1) What fact helps you in finding the no. of bushels in the new bin?

(2) "How high up in this new bin" is equivalent to what question?

(3) Do you need to use "1970 bus."?

(4) Question calls for what?

(5) Draw and put dimensions on 2nd bin.

The following were among some of the very interesting problems presented:

(1) "The associated butchers of Cudahy met a demand of 315.7 lbs. of beef from the Steinmeyer Co. In the first load shipped there were 101.3 lbs. How many lbs. remained to be shipped?"

(2) The Curtiss-Wright airport is going to build a small triangular hangar in one corner of their landing field, in which to put their Ford Tri-motor airplane. If this hangar is to be 98' by 98' to the corner of the field and 30 Ft. high at $2.50 a cu. yd. what is the cost to cement its floor 9 inches deep, allowing 1 cu. yd. of cement for each 36 sp. ft. of ground?"

(3) "Capt. McCarthy had 3 hours in which to reach N. Y., 75 mi. away on his ship, the 'Clarabelle'. He had Pres. and Mrs. Hoover as passengers and they were to get the Overland Express train at N. Y. for Chicago. Their train was on time and traveled at the rate of 30 miles per hour. How long did they have to wait for their train?"

(4) "The first day in January, Canada shipped 25.6 tons of snow to the Gordon Park ski-jump. During the first week 6-4/5 tons were used, as it was quite warm. The following week it snowed and only 3.4 tons were used. How many tons still remained?"

These diagnostic and practice tests divided the class into smaller groups and thus arose the problems of individual differences and individualizing the instruction. What shall be done with the other groups while the teacher is working directly with one group? This is primarily a problem of management. It seems clear that a pupil should not be required to mark time in the work needed by the less capable. Also, the less capable should not be submerged in work beyond their ability. The best plan was to provide supplementary opportunities for the groups outside of those who are working on this arithmetic. These opportunities took various forms according to needs of pupils and their circumstances. Those pupils who on a previous day have successfully completed their diagnostic work were ready for the practice test. Those who had not been successful with the diagnostic work were helped and encouraged to try again, the ideal condition there being to encourage and establish the attitude and habit of self-diagnosis. Strong pupils at the outset were encouraged to state and work questions and problems that were diagnostic for difficulties encountered. In order to reduce confusion an individual systematic chart for keeping track of their progress was kept and checked by each pupil. A competent leadership was placed in charge of the diagnostic tests, and while the teacher took charge of a given group, a dependable leadership was assisting other pupils to get the diagnostic and practice work that they needed. There using of the diagnostic tests seemed promising, both from the standpoint of encouraging pupils to improve themselves independently and from the standpoint of remedial work directed by the teacher. It would seem that the way to improve reasoning ability of pupils is to start them at a level at which they are able to reason and enable them to think their way gradually to and into their original difficulty. The diagnostic tests were meant to be so arranged as to give definite approach to levels of difficulty in reasoning. For some pupils the diagnostic tests were all that were necessary. Some pupils who were found to be unable to do the problems of a given difficulty level, were able to work through the diagnostic tests and conquer the problems which previously they were unable to solve. An essential in securing ability to solve problems is to enable pupils to help diagnose their own difficulties and help provide their own remedial work. It would seem that the more direct work of the teacher should start where the pupils stop in the diagnostic work, and that the first step of remedial work was to go back in the diagnostic far enough to enable the pupil to resume his reasoning, then to approach the level at which he failed with steps so gradual that he is able to come up to his reasoning difficulty and pass through it. A further means of encouraging individual pupils to provide themselves with self-help is to have them make up problems with easy numbers but like the one on which they have failed. Any effort in this direction should be sympathetically encouraged.

The teacher undertook individual and remedial work with the lower quartile of the class to determine cause of errors. A peculiar type in this quartile was Pupil 21. In arithmetic he was a border-line pupil and was 14 yrs. 10 months of age. Practice work was presented to him and some of the findings recorded.

(1) Very poor in silent reading.

(2) Failed to make up essentials missed through absence and change of schools.

(3) Inability to clearly differentiate between linear, square, and cubic measure.

(4) Slight conception of quantitative relationships.

(5) Failure to understand arithmetic concepts with which he should be familiar.

(6) Very careless.

(7) A poor conception of fractional relationships.

(8) Lack of training in estimating answers.

(9) Lack of initiative.

(10) Short attention span.

After the work upon the parallel tests and their accompanying practise questions the same Compass Diagnostic test, Form A was again administered to the class as the publication of Form B had not been completed. The following tabulation shows the results of the second test. The upper quartile of the class shows 36 errors in the second test as apposed to 125 errors and twenty-four problems unfinished in the first test. The lower quartile shows 190 errors and thirty-eight problems unfinished in the second test as opposed to 216 errors and 187 problems unfinished in the first test.

TABLE IV

2nd TEST Compass Diagnostic

Jan. 29, 1932

Prob.	1					2					3					4				
Pupil	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
5							•					•								
35												•								
16												•								
19												•				✓				
36												•								
23					✓							•								
9												•								
14												•								
30												•								
29			✓									•								
3												•								
18												•								
33			✓							✓		•								
28							•					•								
27		•		✓			•													
34												•								
26										✓		•								
20										✓		•								
31		•								✓		•							✓	✓
24					✓							•								
7		•				✓				✓		•								
11					✓							•	✓							
37	✓											•							✓	
10											✓	•								
39		•	✓							✓										
6						✓				✓		••	✓							
2		•					•				✓			✓		✓				
8				✓	✓						✓					✓				✓
40												••								✓
15										✓		•								
12				✓	✓					✓		•								
4										✓	✓	•			✓				✓	
1						✓				✓		•				✓				
38											✓	••			✓	✓	••		✓	✓
17				✓	✓									✓						
13							✓			✓						✓			✓	
32		•	✓							✓		•		✓	✓	✓			✓	✓
25		••						✓			✓	•	✓	✓	✓					
21	✓	✓	✓				•			✓		••				✓	✓			✓
Errors	2	4	5	4	6	3	3	1	-	14	6	12	3	4	4	9	2	-	6	6

Prob.	5					6					7					8	
Pupil	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2
5					✓												
35							•										
16	✓																
19							•										
36														✓	✓		
23	✓						•										•
9					✓					✓							
14							•							✓	✓		
30					✓						✓						
29		•			✓	✓	••										
3		••					•										•
18							•			✓							
33		•					•										
28	✓						•				✓	•					•
27																	•
34		•								✓							
26	✓						•										
20																	
31		•			✓												
24					✓		•					•					
7											✓						
11	✓						•				✓			✓	✓	✓	
37	✓			✓			•							✓	✓		
10	✓						•••			✓				✓	✓	✓	
39					✓				✓	✓				✓	✓		
6						✓	••			✓							
2														✓	✓		
8			✓		✓		•	✓						✓	✓		
40	✓	•			✓		•		✓	✓	✓			✓	✓		
15	✓	•					•								✓		•
12	✓			✓	✓		•							✓	✓		
4		•		✓	✓				✓	✓						✓	
1	✓	••		✓	✓	✓	•		✓	✓			✓	✓		✓	
38	✓	••		✓	✓		•					••				✓	••
17			✓		✓		•		✓	✓							
13					✓		•										•
32	✓				✓		•							✓	✓		
25	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓
21	✓	✓		✓	✓	✓	✓			✓			✓			✓	✓
Errors	15	7	3	7	17	5	11	2	5	12	5	2	3	13	13	7	6

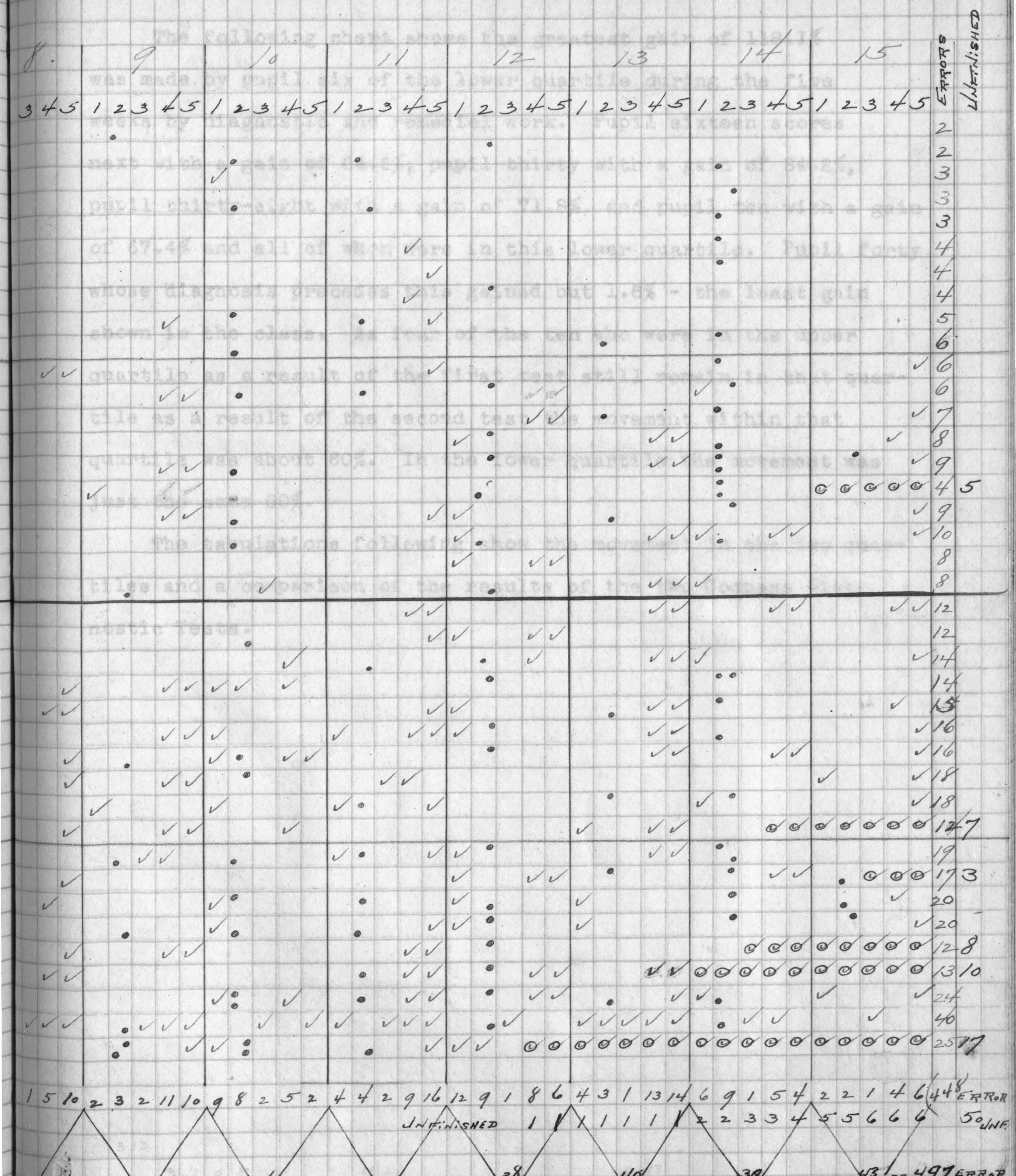
Upper 1/4 (line below pupil 29) — Class Median (line below pupil 24) — Lower 1/4 (line above pupil 12)

Prob.	1	2	3	4	5	6	7	8
Errors	21	21	29	23	49	35	36	29

IN PART 2 OF EACH PROB. the No. OF DOTS WAS ÷ 3, as EACH • WAS BUT 1/3 of AN ERROR, for " " WAS MADE UP of 3 SMALLER PARTS.

Arith. Test

CHART of ERRORS



KEY: → ✓ means 1 ERROR
• counts as $\frac{1}{3}$ ERROR

The following chart shows the greatest gain of 118.1% was made by pupil six of the lower quartile during the five weeks by diagnostic and remedial work. Pupil sixteen scores next with a gain of 84.6%, pupil thirty with a gain of 84.2%, pupil thirty-eight with a gain of 71.8%, and pupil ten with a gain of 67.4% and all of whom were in this lower quartile. Pupil forty whose diagnosis precedes this gained but 1.6% - the least gain shown in the class. As four of the ten who were in the upper quartile as a result of the first test still remain in that quartile as a result of the second test the movement within that quartile was about 60%. In the lower quartile the movement was just the same 60%.

The tabulations following show the movement in the two quartiles and a comparison of the results of the two Compass Diagnostic Tests.

Upper quartile on Pre-test

Table V.

Pupil	Pre-test	Final-test	Gain	% Gain	Unfinished	Living age	I.Q.	Mental age
23	68	72	4	5.	0	13^{9}	127	17^{6}
36	64	72	8	12½	7	13^{5}	125	17^{5}
14	64	71	7	10.	0	14^{1}	118	16^{9}
19	61	72	11	18.	2	15^{5}	98	15^{3}
33	60	68	8	13.	0	12^{7}	137	17^{3}
26	59	66	7	11.8	3	13^{11}	114	15^{11}
3	57	69	12	21.	10	14^{7}	109	15^{11}
18	57	69	12	21.	2	13^{7}	123	16^{10}
40	56	57	1	1.6	0	14^{4}	102	14^{8}
39	55	60	5	11.	0	14^{7}	104	15^{2}
5	55	73	18	32.	11	13^{10}	121	16^{10}
Av.	59.6	68	8.4	14	35	14^{1}	116.1	15^{11}

Upper quartile on Final-test

Pupil	Final test	Pre-test	Gain	%	Unfin.	Unfin. in Pre	Living age	I.Q.	Mental age	
5	73	55	18	32.	0	11	13^{10}	121	16^{10}	*
35	73	45	28	62.	0	11	13^{8}	123	16^{10}	
23	72	68	4	5	0	0	13^{9}	127	17^{6}	*
36	72	64	8	12½	0	7	13^{5}	125	17^{5}	*
16	72	39	33	84.6	0	24	14^{3}	108	15^{5}	✓
19	72	61	11	18.	0	2	15^{5}	98	15^{3}	*
9	71	50	21	42.	0	11	13^{9}	115	15^{11}	
14	71	64	7	10.	0	0	14^{1}	118	16^{9}	*
30	70	38	32	84.2	0	20	15^{4}	90	13^{10}	✓
18	69	57	12	21.	0	2	13^{7}	123	16^{10}	*
Aver.	71.5	54.1	17.4	37.1	0	8.8	14.1	114.8	16.2	

Tables showing movements in the quartiles as a result of {Diagnostic, Practice, Remedial} work

* were in 1st quartile as result of 1st test and remained in " " as result of 2nd test

✓ entered from lower quartile

Lower quartile — Pre-test

TABLE VI.

Pupil	Pre-test	Final-test	Gain	% Gain	Unfinished-Pre-test	Unfinished-Final-test	Living age	I.Q.	Mental age	
13	40	52	12	30	13	10	13^{8}	105	14^{3}	
16	39	72	33	84.6	24	0	14^{3}	108	15^{5}	✓
30	38	70	32	84.2	20	0	15^{4}	90	13^{10}	✓
15	37	56	19	51.3	24	7	16_{5}	87	14^{2}	
8	37	57	20	54	15	0	14	93	13^{6}	
10	36	61	25	69.4	27	0	15^{3}_{11}	87	13^{9}_{5}	
25	34	35	1	2.9	18	0	14_{10}	89	13_{10}	
38	32	55	23	71.8	0	0	14_{2}	87	13	
6	27	59	32	118.5	28	0	16_{10}	94	15^{3}_{7}	
21	27	33	6	22.2	18	17	14	91	13	
AV..	34.7	55	20.3	58.8	18.7	3.4	14.9	93.1	14.1	

Lower quartile — Final-test

Pupil	Pre-test	Final-test	Gain	% Gain	Unfinished-Pre-test	Unfinished-Final-test	Living age	I.Q.	Mental age	
~~15~~	56	37	19	51.3	24	7	16	87	14^{2}	*
12	56	44	12	27.2	5	0	15^{11}	85	13^{8}	
4	55	44	11	25.	12	3	14^{11}_{1}	94	14^{3}	
1	55	41	14	34.1	5	0	14_{10}	100	14^{2}	
~~38~~	55	32	23	71.8	0	0	14_{10}	87	13^{10}	*
17	55	48	7	14	15	8	13_{8}	121	15^{10}	
~~13~~	52	40	12	30	13	10	13_{10}	105	14^{3}_{1}	*
32	51	43	8	18	7	0	15_{11}	107	16	
~~25~~	35	34	1	2.9	18	0	14_{10}	89	13^{5}	*
~~21~~	33	27	6	22.2	18	17	14_{10}	91	13^{7}	*
Av..	50.3	39.	11.3	29.6	10.7	4.5	14	96.6	14^{5}	

Tables showing movements in the quartiles.

* were in Lower quartile as result of 1st test and
Remained " " quartile as result of 2nd test.

✓ moved into Upper quartile as result of 2nd test.

TABLE VII

Pupil	Age	Mental Age	Terman Score	I.Q.	1st Test Part ①	2nd Test Part ①	1st Test ②	2nd Test ②	1st Test ③	2nd Test ③	1st Test ④	2nd Test ④	1st Test ⑤	2nd Test ⑤	1st Test Total	2nd Test Total	Gain	% Gain	
23	13^{9}	17^{6}	169	127	14	15	13	14	15	15	13	14	13	14	68	72	4	5	
36	13^{5}	17^{5}	165	125	14	15	13	14	14	15	12	14	11	14	64	72	8	12½	
33	12^{7}	17^{3}	161	137	14	15	14	13	15	14	9	14	8	12	60	68	8	13	
37	13^{10}	17^{1}	159	122	13	12	11	13	11	15	10	9	9	12	54	61	7	12	
35	13^{8}	16^{10}	154	123	9	15	10	13	13	15	9	15	4	15	45	73	28	62	
18	13^{7}	16^{10}	152	123	12	14	14	13	14	15	10	14	7	13	57	69	12	21	
5	13^{10}	16^{10}	152	121	12	15	12	14	13	15	9	15	9	14	55	73	18	32	
14	14^{1}	16^{9}	151	118	14	15	14	14	15	15	11	13	10	14	64	71	7	10	
32	15^{10}	16^{1}	136	107	9	10	11	12	13	14	7	9	3	6	43	51	8	18	
17	13^{10}	15^{10}	133	121	12	14	10	13	12	12	9	9	5	7	48	55	7	14	
3	14^{7}	15^{11}	132	109	13	15	12	13	13	15	11	14	18	12	57	69	12	21	
20	14^{3}	15^{11}	132	111	11	13	11	13	12	15	9	13	8	11	51	65	16	32.6	
9	13^{9}	15^{11}	132	115	12	15	12	14	13	15	9	15	4	12	50	71	21	42.	
26	13^{11}	15^{11}	132	114	13	13	12	13	13	15	11	14	10	11	59	66	7	11.8	
31	14^{8}	15^{7}	126	106	13	14	11	14	12	15	10	13	6	11	52	65	13	25.	
16	14^{3}	15^{5}	124	108	9	13	8	14	10	15	7	15	5	15	39	72	33	84.6	
11	14^{7}	15^{4}	123	102	11	11	5	14	13	14	10	13	5	11	44	63	19	43.1	
6	16^{2}	15^{3}	121	94	8	11	7	13	1	14	6	12	5	9	27	59	32	118.1	GREATEST GAIN
27	13^{6}	15^{3}	120	112	11	15	7	12	11	15	7	12	8	12	44	66	22	50.	
19	15^{5}	15^{3}	120	98	13	14	12	13	15	15	11	15	10	15	61	72	11	18.	
22	14^{8}	15^{2}	117	107	8		9		12		9		4		42	ABSENT			
39	14^{7}	15^{2}	116	104	10	14	11	14	15	14	11	10	8	8	55	60	5	11.	
24	15^{1}	15^{2}	116	100	10	14	11	10	12	14	6	14	5	12	44	64	20	45.4	
34	14^{10}	15^{1}	115	101	10	13	2	12	11	14	10	14	8	13	41	66	25	60.9	
28	13^{6}	14^{11}	112	101	9	12	11	12	11	15	8	13	7	14	46	66	20	43.4	
29	13^{6}	14^{9}	109	102	9	14	10	12	12	14	10	15	10	14	51	69	18	35.2	
7	15^{5}	14^{5}	103	93	8	13	9	14	11	15	8	11	7	10	43	63	20	46.5	
4	14^{11}	14^{3}	99	94	11	12	10	13	12	14	6	9	5	7	44	55	11	25.	
13	13^{8}	14^{3}	99	105	9	11	9	12	10	13	7	8	5	8	40	52	12	30.	
1	14^{1}	14^{2}	97	100	8	7	10	12	11	14	7	10	5	12	41	55	14	34.1	
15	16	14^{2}	98	87	8	12	9	12	10	14	5	10	5	8	37	56	19	51.3	
30	15^{4}	13^{10}	88	90	5	14	8	14	11	15	8	14	6	13	38	70	32	84.2	
10	15^{3}	13^{9}	87	87	7	11	8	12	10	15	6	12	5	11	36	61	25	67.4	
12	15^{11}	13^{8}	85	85	9	11	10	12	14	14	7	10	4	9	44	56	12	27.2	
8	14^{5}	13^{6}	81	93	9	12	9	14	10	12	5	11	4	8	37	57	20	54.	
25	14^{11}	13^{5}	80	89	10	9	5	11	10	2	4	7	4	6	34	35	1	2.9	
38	14^{10}	13^{10}	79	87	8	8	3	9	11	15	4	13	6	10	32	55	23	71.8	
2	15^{7}	13^{4}	78	85	11	12	13	13	14	15	8	10	4	9	50	59	9	18.	
21	14^{10}	13^{7}	84	91	7	7	3	3	8	10	5	8	4	5	27	33	6	22.2	
40	14^{4}	14^{8}	107	102	10	8	12	12	15	15	9	13	10	9	56	57	1	1.6	LEAST GAIN

Chart of Errors

Table VIII

	Comprehension		What is Given		What is Called For		Probable Answer		Correct Solution	
	I		II		III		IV		V	
Errors	115		133		34		170		238	-- 1st Test
Errors	91		83		28		103		133	-- 2nd Test
Gain --	24		50		6		67		105	
% Gain	20%		37%		17%		33%		44%	Greatest gain in Part V

Chart showing Comparison of Errors in tests 1 and 2.

TABLE IX.

Frequency distribution showing No. of Pupils having Part I correct in each of the 15 problems in Tests 1 and 2

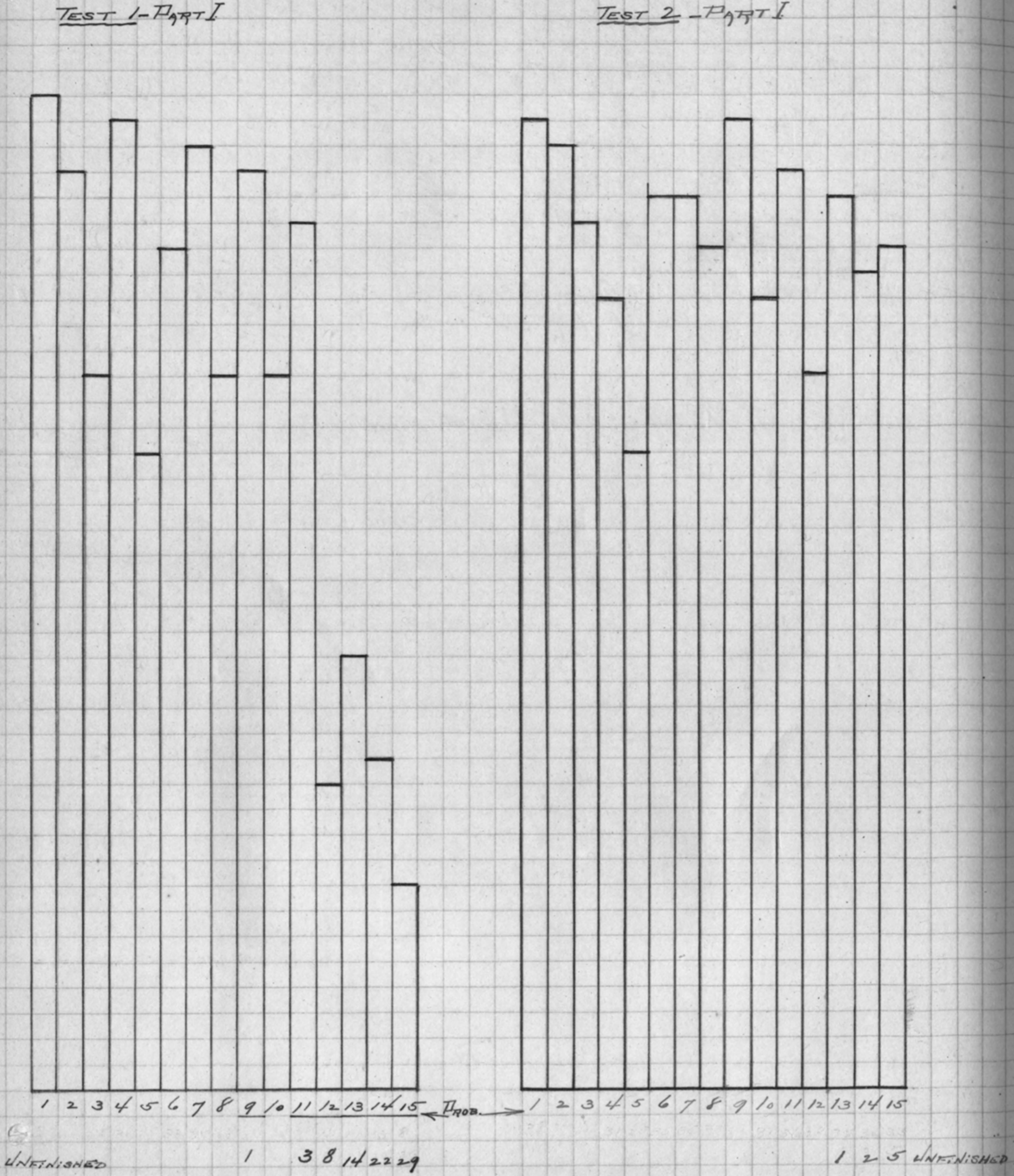
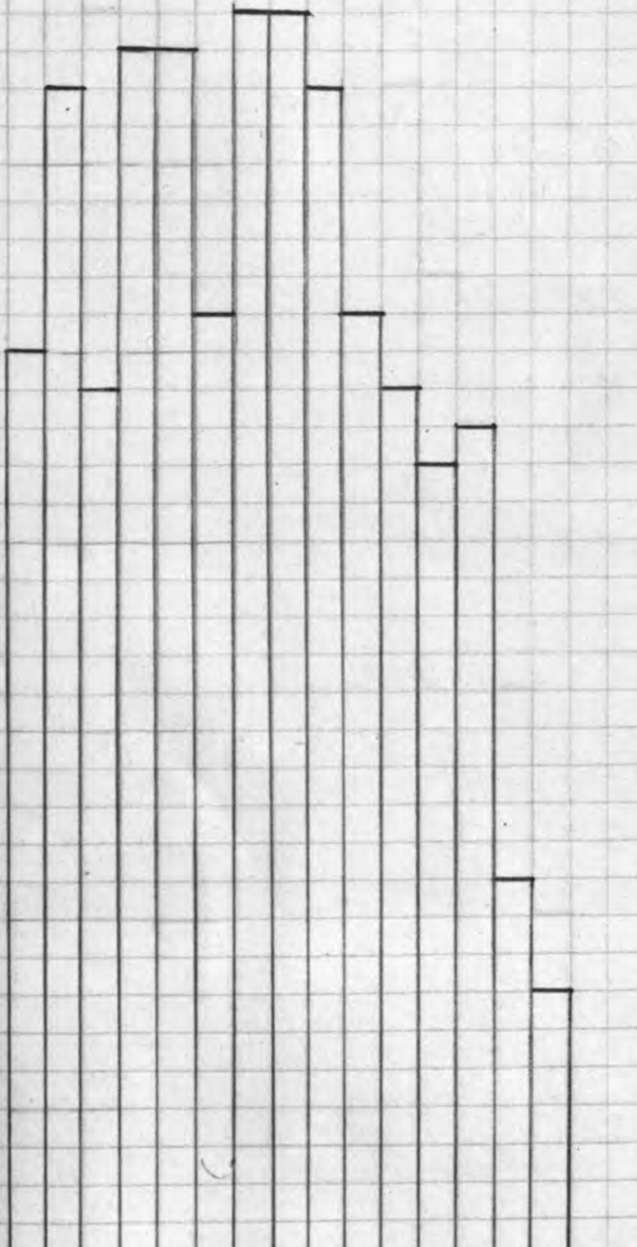


TABLE X

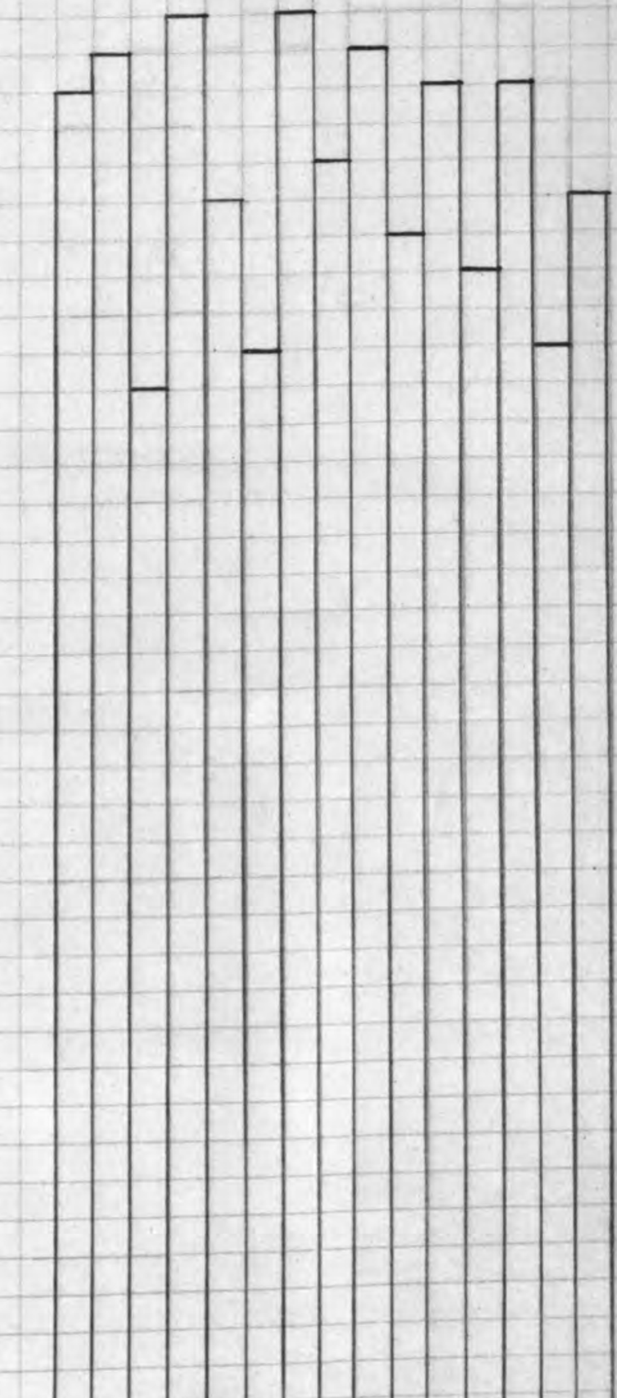
Frequency distribution showing No. of Pupils having Part II correct in each of the 15 problems in Tests 1 and 2

Test 1 - Part II.



Unfinished 1 5 8 15 22 29

Test 2 - Part II.



Unfinished 1 2 5

TABLE XI.

Frequency distribution showing No. of Pupils having Part III. correct in each of the 15 problems in Tests 1 and 2

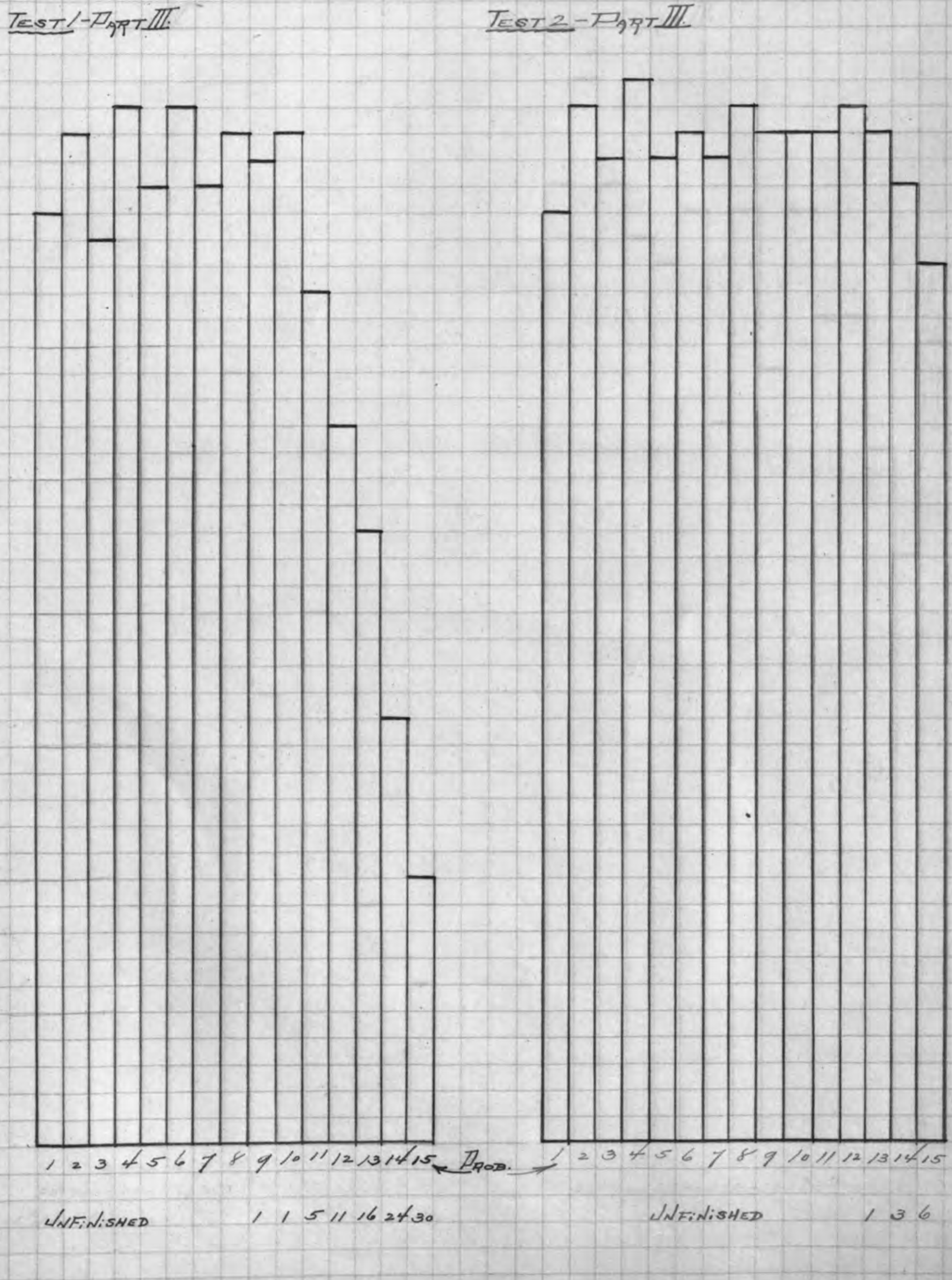


TABLE XII.

Frequency distribution showing No. of Pupils leaving Part IV. correct in each of the 15 prob. in Tests 1 and 2

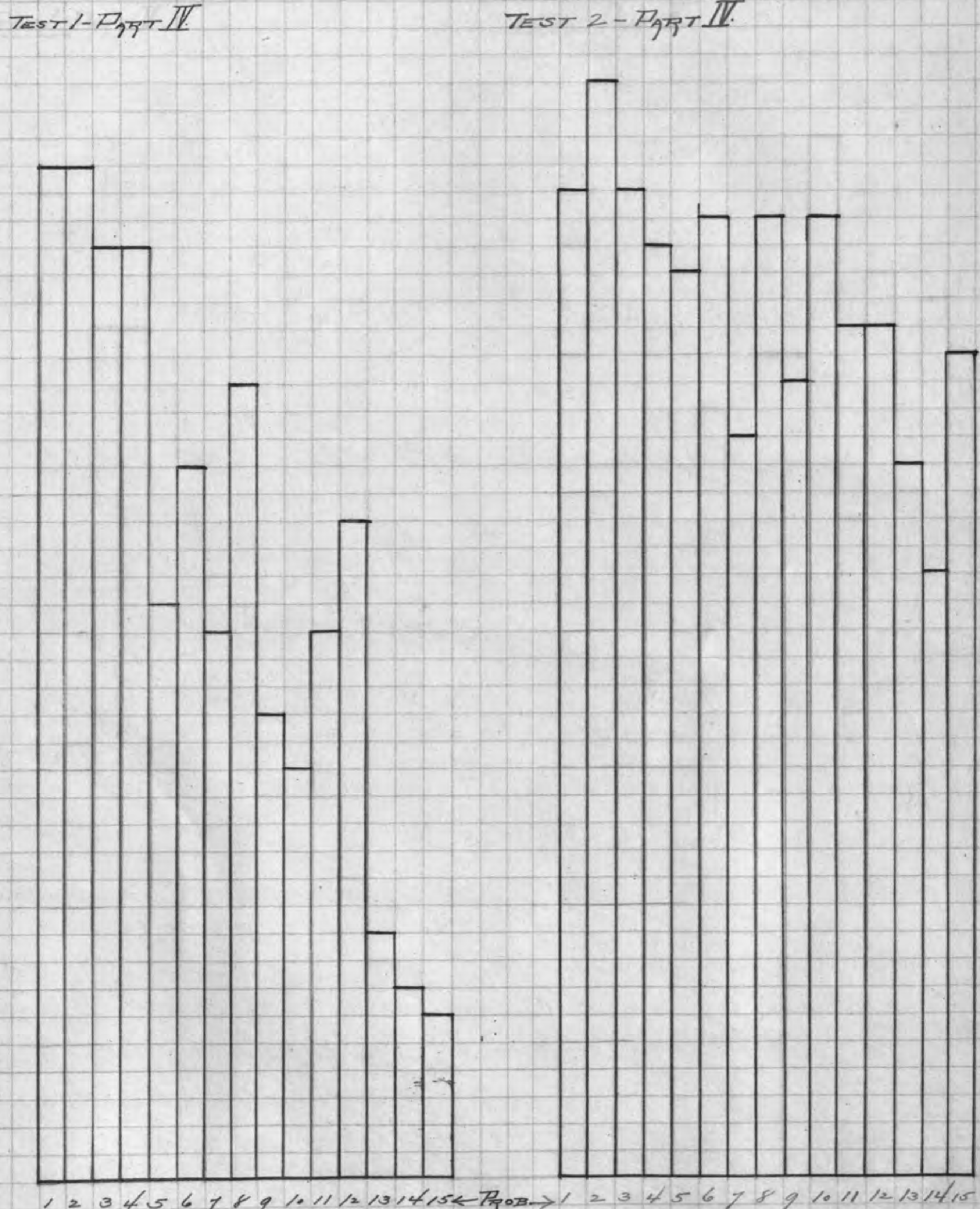
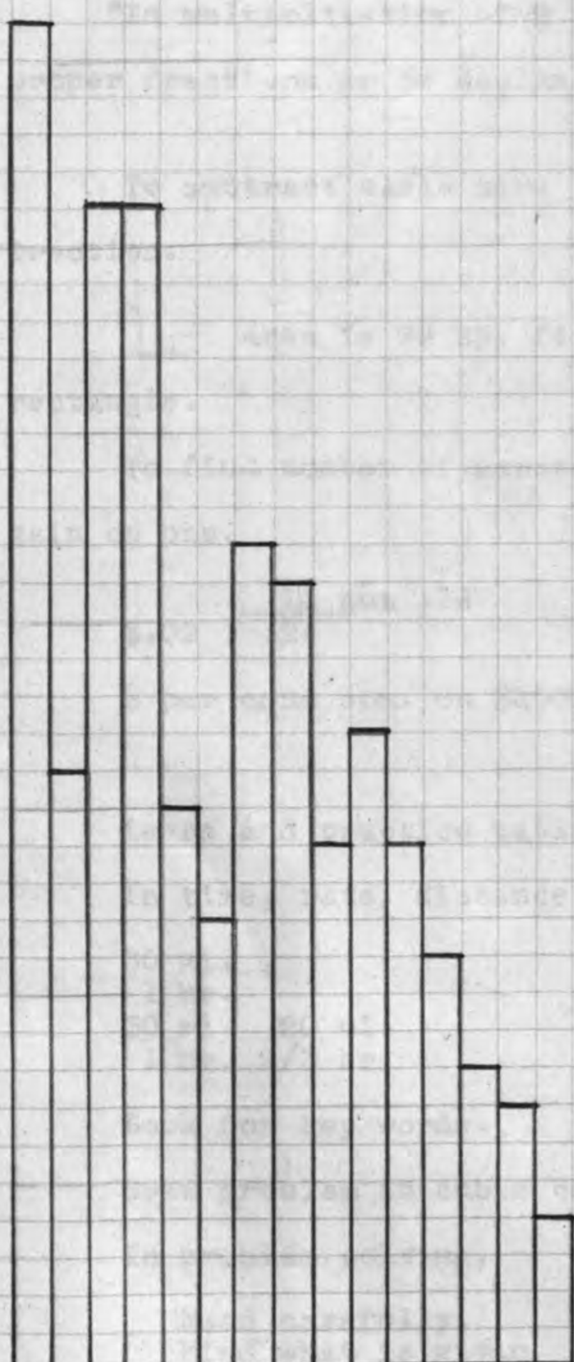


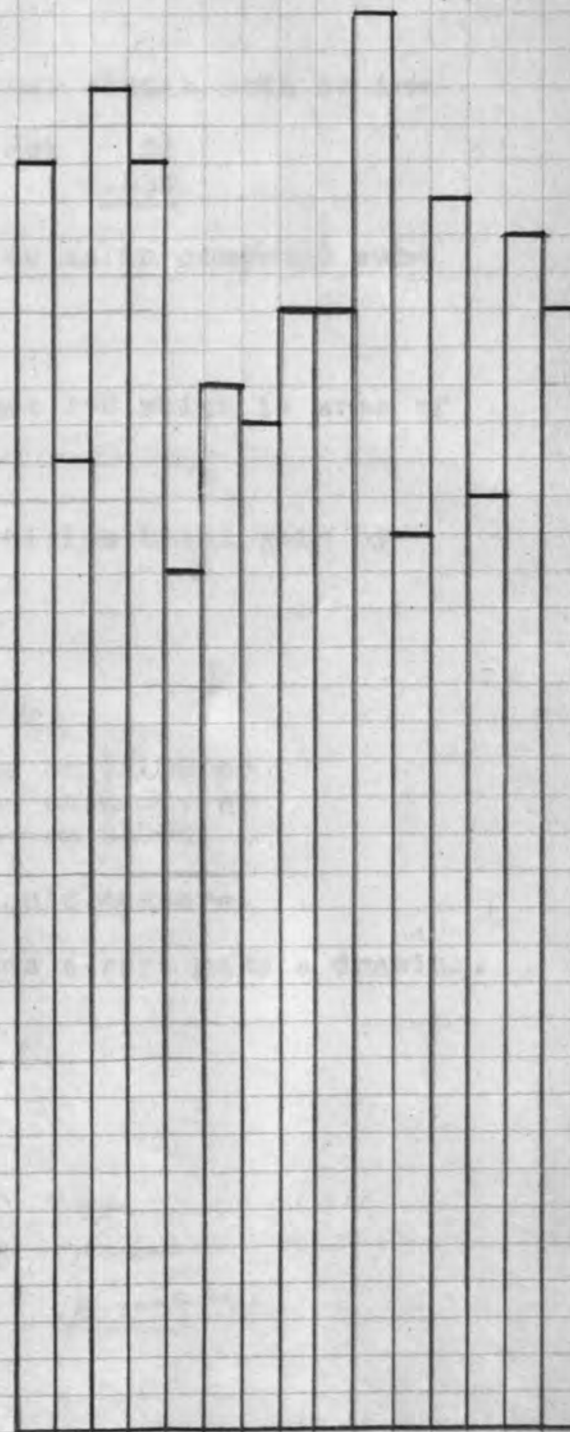
Table XIII.

Frequency distribution showing No. of Pupils having Part V. correct in each of the 15 problems, in Tests 1 and 2

Test 1 - Part V.

Test 2 - Part V.





1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 ← Prob. → 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Unfinished 1 2 5 11 20 26 30

Unfinished 1 1 4 6

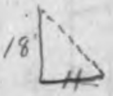
Each member of the class kept a note-book in which were recorded facts to be memorized, points of constructive self-criticism, and points of guidance. The following were gleaned from the note-books.

"In multiplication of 2 mixed nos. change both to improper fractions or to decimals and not $13\frac{1}{2}$
$\times .53\frac{1}{2}$

To subtract clock time

```
  8 60
  9:00
  8:40
  ----
    20
```

do as in compound subtraction.



area is 99 sp. ft. and not 198 which is area of rectangle.

To find number of newspapers, divide total gain by gain on one.

```
         12 not .12
$.02 ) .24
```

```
5 per cent com. on $1000. is 5¢ on $1.00 or
                            $5  on $100. or
                           $50  on $1000.
```

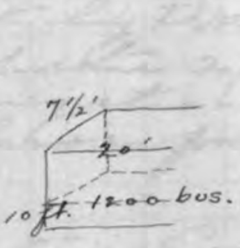
Learn and practice table of liquid measure.

In time, rate, distance problems always make a drawing.

```
|-- 30 mi. --|
    1 hr.
|-- 30 mi. --| 20 mi.
    1 hr.    2/3 hr.
```

Look for key words.

Draw problem in cubic contents



In problem solving,

Read carefully,
Find what is given,
Find what is called for,
Look for cues or key words,
Estimate a probable answer,
Check the solution.

TABLE XIV.

Chart of Pupil 35

Prob.	1st Test					⊗ Diag.					⊗ Prac.		⊗ Diag.				⊗ Prac.		⊗ Diag.				⊗ Prac.		Last test				
1	/	/	/	/	/																				/	/	/	/	/
2					✓																				/	/	/	/	/
3	✓	•		✓	✓						✓	✔														•			
4	/	/	/	/	/																				/	/	/	/	/
5					✓																				/	/	/	/	/
6		•			✓																					•			
7	✓			✓	✓						✓	✔													/	/	/	/	/
8					✓												✓	✔							/	/	/	/	/
9		•																								•			
10	✓	:		✓	✓			•						•						•						:			
11		•																											
12	✓	•			✓																				/	/	/	/	/
13				//	//																				/	/	/	/	/
14	//	//	//	//	//							✔													/	/	/	/	/
15	//	//	//	//	//					✓	✓	✔					✓	✔							/	/	/	/	/

Chart as kept by Pupil 35 with the exception of the 2 Diag. and Prac. tests preceding the Last test. They are not shown here for lack of space.

——— means correct in Compass Diag.
✓ means an error
• means 1 of the 5 parts is wrong
// means unfinished in test
X means test was given pupil because preceding test work finished
⊗ means that work completed

-*- -*- -*- -*- -*- -*- -*- -*- -*- -*-

-*-

CHAPTER XIV

CONCLUSION

-*-

-*- -*- -*- -*- -*- -*- -*- -*- -*- -*-

The problems in the diagnostic and practice work were designed different in content, in questions called for, but in reasoning similar in order to have a special bearing on transfer. Since the problems were different in content and slightly in procedure, there is substantial evidence of transfer. Transfer was also measured from problems on which there was no diagnostic nor practice work, and which were solved as supplementary work.

As to gains made, the relation between gains and degree of native ability can be studied by use of the I. Q. on the Terman Test and the scores made on the first and last Compass Diagnostic test. It was expected that there would be gains because of the familiarity with the material and the type of question and because the weaknesses revealed in the foregoing problems were the basis of the diagnostic and practice and remedial work. Retention is the final criterion of usefulness of any teaching materials. Permanency of gains was measured somewhat by giving some tests just before the graduation of the class. The diagnostic and practice material seems more potent than the regular arithmetic work in developing ability to solve problems on which there had been specific diagnostic and practice work. Such diagnostic and practice should afford more help in learning how to solve new problems. Following this page is a copy of the Compass Diagnostic Test which was used.

Dr. Kilpatrick in one of his lectures quoted some one as saying that we may as truly say that we have sold when no one has bought as that we have taught when no one has learned.

Standard Mathematical Service

COMPASS DIAGNOSTIC TESTS IN ARITHMETIC

RUCH—KNIGHT—GREENE—STUDEBAKER

EDITED BY G. W. MYERS

TEST XVIII: PROBLEM ANALYSIS: ADVANCED: FORM A

Name...Grade....................Boy or girl?..

Age..........When is your next birthday?...How old will you be then?................

School.. Date...
(Name) (City) (State)

SUMMARY OF PUPIL'S SCORE	PART 1	PART 2	PART 3	PART 4	PART 5	TOTAL
Scores on Parts of Test						
Educational Age Equivalent						
Grade Equivalent of Score						

Do Not Turn the Page until Told to Do So.

PROBLEMS	PART 1—COMPREHENSION	PART 2—WHAT IS GIVEN
Read each problem below. Then work across the two facing pages to the right, doing all Parts for one problem before going to the next. Do not go back and work on a Part after you have completed the one following. *Read the Sample below.*	*Put a cross (×) on the line before the* ***one*** *statement below which is true for each problem.*	*Put a cross (×) on the line be every statement below which tells a given in the problem.*
Sample *[Read the problem]* ***My reading book has 124 pages. I have read 72 pages. How many pages do I have left to read?*** →	Sample *[Check (×) true statement]* ____ I have read all my reader. ____ I have read less than half my book. ____ I have the most of my book to read. _×_ I have read a little more than half my book. ____ I should add to get the answer to this problem. →	Sample *[Check (×) what is given]* ____ Number of pages to rea _×_ Number pages in book. _×_ Number of pages I ha read. ____ Number of stories I ha read. ____ Number of pages with pi tures on them. →

Remember: **Work across the page to the right.**

[Read the problem] ***Problem 1*** ***A girl gave $\frac{1}{2}$ of her apple to her brother and $\frac{1}{4}$ of it to her pet rabbit. How much of the apple did she give away?*** →	*[Check true statement]* ____ The girl gave all of her apple away. ____ The girl gave away more of her apple than she kept. ____ The girl gave away exactly two-thirds of her apple. ____ The girl kept all of her apple. ____ The girl kept most of her apple. →	*[Check what is given]* ____ Part of apple girl ke ____ Part of apple girl gave rabbit. ____ Part of apple girl gave brother. ____ Part of apple given both brother and rabb ____ Part of apple the g threw away. →
[Read the problem] ***Problem 2*** ***A boy worked three evenings after school, working a total of $13\frac{1}{2}$ hours at $52\frac{1}{2}$ cents per hour. How much did he earn altogether?*** →	*[Check true statement]* ____ The boy worked for more than 24 hours. ____ The boy worked as many hours as he received dollars. ____ The amount the boy earned will be found by using multiplication. ____ The boy worked three whole days for me. ____ The boy received one dollar per hour. →	*[Check what is given]* ____ Number of hours b worked daily. ____ The days of the week which he worked. ____ Total amount the b earned. ____ Amount paid the boy p hour. ____ Total number of ho worked altogether. →

[PA]RT 3—WHAT IS CALLED FOR	PART 4—PROBABLE ANSWER	PART 5—CORRECT SOLUTION
[P]ut a cross (×) on the line before the [one] statement below which tells what is [call]ed for in the problem.	*Put a cross (×) on the line before the* ***one*** *statement below which gives the nearest probable answer to the problem. Do not take time to work the problem.*	*Put a cross (×) on the line before the* ***one*** *correct solution given for each problem. Figure in the margin if you want to.*
Sample *[Check (×) what is called for]* ___ Number of pages in book. ___ Number of pages yet to read. ___ Number of pages I have read. ___ Number of stories I have read. ___ Number of pages with pictures. →	**Sample** *[Check probable answer]* ___ One book. ___ About 124 pages. ___ About 72 stories. _×_ About 50 pages. ___ About 196 pages. →	**Sample** *[Check correct solution]* ___ $124+72=196$ ___ $124+72=196$; $196\div 2=98$ ___ $124-62=62$; $62+72=134$; $134\div 2=52$ ___ $124-72=72$ _×_ $124-72=52$

Remember: Work across the page to the right.

[Check what is called for] ___ Part of the apple given to brother ___ Part of apple given to rabbit. ___ Part of apple girl gave to both. ___ Part of apple girl kept. ___ Part of apple girl threw away. →	*[Check probable answer]* ___ One-half. ___ About two-thirds. ___ All. ___ About three-fourths. ___ One-third. →	*[Check correct solution]* ___ $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$ ___ $\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$ ___ $\frac{4}{4}-\frac{1}{2}=\frac{2}{4}$ or $\frac{1}{2}$ ___ $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}+\frac{1}{4}=\frac{4}{4}$ or 1 ___ $1-\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$ **Now Start Problem 2.**
[Check what is called for] ___ Number of hours boy worked on any one day. ___ Total amount earned by boy. ___ Number of days boy worked. ___ Amount paid boy per hour. ___ Number of hours boy worked altogether. →	*[Check probable answer]* ___ About 13 dollars. ___ 52 cents. ___ About 7 dollars. ___ 13 hours. ___ About $7\frac{1}{2}$ hours. →	*[Check correct solution]* ___ $13\frac{1}{2}\times .52\frac{1}{2}=\$6.76$ ___ $13\frac{1}{2}=\frac{27}{2}$ hrs. $52\frac{1}{2}=\frac{1.05}{2}$; $\frac{27}{2}\times\frac{105}{2}=\frac{28.35}{4}=7.09$ ___ $.52\frac{1}{2}¢\times 13\frac{1}{2}=\$6.76-.06\frac{1}{2}=\$6.82$ ___ $13\frac{1}{2}\times 3=40\frac{1}{2}$. $40\frac{1}{2}=81/2$; $52\frac{1}{2}=\$1.05/2$. $\frac{81}{2}$ hrs. $\times\frac{\$1.05}{2}=85.05/4=\$21.26$ ___ $13\frac{1}{2}+.52\frac{1}{2}=65+1=$; $66\times 3=1.98$

PROBLEMS	PART 1—COMPREHENSION	PART 2—WHAT IS GIVEN
[*Read the problem*] ***Problem 3*** ***A boy scout hiked to a camp 11.7 miles distant. He walked 6.3 miles in the forenoon. How far did he have to walk in the afternoon?***	[*Check true statement*] ____ Boy walked total distance to camp in morning. ____ Boy walked exactly one-half the distance to camp in the forenoon. ____ Boy walked all the way in the afternoon. ____ Boy walked most of the way in the forenoon. ____ Boy walked over half distance in the afternoon.	[*Check what is given*] ____ Distance camp was awa ____ Distance boy walked forenoon. ____ Distance boy walked afternoon. ____ Distance boy walked both forenoon and aft noon. ____ Distance the boy rode auto.
[*Read the problem*] ***Problem 4*** ***Two weeks ago I had 6.7 tons of coal in my coal bin. The first week was very cold and I burned 1.1 tons. The second week was quite warm and I burned only .8 tons. How much coal have I left?***	[*Check true statement*] ____ Burned entire supply of coal the first week. ____ Burned most of coal I had the first week. ____ Burned less coal the second week than the first. ____ Burned more coal the second week than the first. ____ Burned all the coal the second week.	[*Check what is given*] ____ Amount of coal in m bin at first. ____ Amount burned the fi week. ____ Amount burned the seco week. ____ Amount burned in t weeks. ____ Amount left in bin.
[*Read the problem*] ***Problem 5*** ***Mr. Day owns four 60 foot lots valued at $1000 each, located on a street which is now being paved. Curbing costs him $75 per lot and the total paving cost on the four lots will be $1500. How much should Mr. Day charge to each lot for curbing and paving costs?***	[*Check true statement*] ____ Value of lots affects the cost of paving per lot. ____ Curbing and paving cost less than value of lots. ____ Curbing costs more per lot than the paving. ____ It is necessary to use multiplication in solving this problem. ____ Mr. Day decided not to pave in front of his lots.	[*Check what is given*] ____ The value of each lot. ____ Width of street in front lots. ____ Cost of curbing per lot ____ Cost of curbing for the f lots. ____ Cost of paving for the f lots.
[*Read the problem*] ***Problem 6*** ***A girl had two hours in which to ride 50 miles on a branch line train to the junction where she was to take the Overland Express for Chicago. Her train was on time and traveled at the rate of 30 miles per hour. How many minutes did she have to wait at the junction until the Overland Express was due?***	[*Check true statement*] ____ Girl traveled two hours on Overland Express. ____ Overland Express was two hours late. ____ Train on branch line traveled 50 miles in less than two hours. ____ The girl waited two hours at the junction. ____ The branch line was only 30 miles long.	[*Check what is given*] ____ Distance to the junctio ____ Speed of branch line tr ____ Length of time girl had wait at junction. ____ Time girl had in which make train connect at junction. ____ Time required to m trip to Chicago.

PART 3—WHAT IS CALLED FOR	PART 4—PROBABLE ANSWER	PART 5—CORRECT SOLUTION
[Check what is called for] ____ Distance boy had to walk to reach camp. ____ Distance boy walked altogether. ____ Distance boy walked in forenoon. ____ Distance boy walked in afternoon. ____ Difference between what he walked in forenoon and afternoon.	*[Check probable answer]* ____ About 18 miles. ____ About 5.3 miles. ____ About 5.3 hours. ____ 1.4 hours. ____ 11 miles.	*[Check correct solution]* ____ $11.7 \times 6.3 = 73.71$ ____ $11.7 + 6.3 = 18.0$ ____ $11.7 - 6.3 = 5.3$ ____ $11.7 - 6.3 = 5.4$ ____ $11.7 + 6.3 = 17.0$
[Check what is called for] ____ Amount of coal in bin two weeks ago. ____ Amount of coal burned the first week. ____ Amount burned the second week. ____ Amount of coal burned in two weeks. ____ Amount of coal left in bin.	*[Check probable answer]* ____ About $4\frac{3}{4}$ tons. ____ About 8.6 tons. ____ 5.6 weeks. ____ About $4\frac{1}{2}$ weeks. ____ 6.7 tons.	*[Check correct solution]* ____ $6.7 + 1.1 - .8 = 7$ ____ $6.7 + 1.1 + .8 = 8.6$ ____ $\begin{cases} 1.1 + .8 = 1.9 \\ 6.7 - 1.9 = 4.8 \end{cases}$ ____ $\begin{cases} 1.1 + .8 = 1.9 \\ 6.7 + 2.9 = 9.6 \div 2 = 4.8 \end{cases}$ ____ $6.7 - .8 = 7.5 - 1.1 = 6.4$
[Check what is called for] ____ Number of lots to be paved. ____ Cost of curbing for each lot. ____ Cost of paving for all lots. ____ Cost of curbing and paving to charge to each lot. ____ Value of the four lots.	*[Check probable answer]* ____ 240 feet. ____ About 4.75 lots. ____ \$1425. ____ About 470 dollars. ____ \$1575.	*[Check correct solution]* ____ $\$1500 + 75 = 1575.00$ ____ $75 \times 4 = 1800.00$ ____ $\begin{cases} 1500 \div 4 = 375 \\ 375 + 75 = 450.00 \end{cases}$ ____ $\begin{cases} 1000 \times 4 = 4000 \\ 4000 \div 1500 = 2.67\% \\ 1500 \times 2.67\% = \$400 \\ 400 + 75 = 475 \end{cases}$ ____ $\begin{cases} 1000 \div 4 = 250 \\ 250 + 75 = 325.00 \end{cases}$
[Check what is called for] ____ Time required to go to Chicago. ____ Time girl had to wait at junction until Overland Limited was due. ____ Total distance girl rode on train. ____ Speed of branch line train. ____ Distance from girl's home to Chicago.	*[Check probable answer]* ____ 30 miles. ____ About 25 minutes. ____ About 50 minutes. ____ 50 miles. ____ About 48 minutes.	*[Check correct solution]* ____ $\begin{cases} 50 \times 2 = 100 \\ 100 + 30 = 130 \\ 130 \div 2 = 65; \quad 65 - 60 = 5 \end{cases}$ ____ $50 - 30 = 20$ ____ $\begin{cases} 50 \div 30 = 1\frac{2}{3} \\ 2 - 1\frac{2}{3} = \frac{1}{3} \\ \frac{1}{3} = 20 \end{cases}$ ____ $50 + 30 = 80; \quad 80 - 60 = 20$ ____ $\begin{cases} 30 \times 2 = 60; \quad 60 \div 50 = 1\frac{1}{5} \\ 2 - 1\frac{1}{5} = \frac{4}{5} = 48 \end{cases}$

PROBLEMS	PART 1—COMPREHENSION	PART 2—WHAT IS GIVEN
[*Read the problem*] ***Problem 7*** ***What was the total paid by our grocer for 6 dozen melons at 12½ cents apiece and 5 bushels of apples at $1.25 per bushel?***	[*Check true statement*] ____ Grocer bought six melons. ____ Grocer paid one dollar for five bushels of apples. ____ Grocer bought 12½ dozen apples. ____ Grocer bought six bushels of apples. ____ Grocer bought melons by dozen and apples by bushel.	[*Check what is given*] ____ Cost of five bushels apples. ____ Cost of melons apiece. ____ Number of dozens of m ons bought. ____ Number of bushels of a ples bought. ____ Cost of apples per bush
[*Read the problem*] ***Problem 8*** ***Irving sold 8⅓ dozen chicks for $8.00 and agreed to pay the express on them. The express bill was 56 cents. If the chicks cost him 5½ cents apiece to hatch, how much did he make on the deal?***	[*Check true statement*] ____ Irving sold one dozen chicks. ____ Irving sold the chicks at 5½ cents apiece. ____ Irving received 56 cents for the chicks sold. ____ Irving sold more than eight dozen chicks to an out of town buyer. ____ The chicks cost the buyer 8⅓ cents apiece.	[*Check what is given*] ____ Amount of express bill. ____ Cost of hatching per chic ____ Number dozen chicks sol ____ Cost per dozen chicks. ____ Selling price per chick.
[*Read the problem*] ***Problem 9*** ***John's garden is a triangular corner of the yard. One side is 11 feet and the other is 18 feet to the right angle corner. How many tomato plants can he set out in this space, allowing 1 square yard to each plant?***	[*Check true statement*] ____ John's garden is a circle in the center of the yard. ____ The garden is a small three-sided plot in the corner of the yard. ____ The garden is a square in the end of the yard. ____ John planted one tomato plant in his garden. ____ Addition is necessary to solve this problem.	[*Check what is given*] ____ The shape of the garde ____ The number of feet in o square yard. ____ Space required for o tomato plant. ____ The length of two sid of the triangular garde ____ The number of toma plants in the garden.
[*Read the problem*] ***Problem 10*** ***After buying his stock of papers for the day at 2 cents each, a newsboy had 35 cents change left. He earned 15 cents by running an errand for a man, and another man gave him a nickel. When all his papers were sold at 5 cents each, he had $1.30 in cash. How many papers did he sell?***	[*Check true statement*] ____ Newsboy made $1.30 by selling papers. ____ A man gave him 20 cents. ____ He earned more running errands than he did selling papers. ____ The papers cost the boy five cents each. ____ The newsboy made most of his money selling papers.	[*Check what is given*] ____ Amount made by selli papers. ____ Selling price of papers. ____ Amount given to boy. ____ Number of papers sold. ____ Amount boy had aft buying papers.

PART 1—COMPREHENSION	PART 2—WHAT IS GIVEN	PART 3—WHAT IS CALLED FOR	PART 4—PROBABLE ANSWER	PART 5—CORRECT SOLUTION
statement] ght six melons. l one dollar for els of apples. ght 12½ dozen ght six bushels ght melons by nd apples by	[*Check what is given*] ___ Cost of five bushels of apples. ___ Cost of melons apiece. ___ Number of dozens of mel ons bought. ___ Number of bushels of ap ples bought. ___ Cost of apples per bushel	[*Check what is called for*] ___ Cost of melons apiece. ___ Cost of melons per dozen. ___ Amount grocer paid for melons. ___ Amount grocer paid for apples. ___ Amount grocer paid for both melons and apples.	[*Check probable answer*] ___ 75 melons. ___ About $6.00. ___ About $15.00. ___ About 7 dozen. ___ 6.5 bushels.	[*Check correct solution*] ___ $.12\frac{1}{2}\times 6=.75;\ 5\times 1.25=6.25;\ .75+6.25=7.50$ ___ $12\times .12\frac{1}{2}=1.50;\ 1.25\times 5=6.25;\ 1.50+6.25=7.75$ ___ $.12\frac{1}{2}\times 6=.75$ $.75\times 12=9.00$ ___ $.12\frac{1}{2}\times 12=1.50;\ 1.50\times 6=9.00+1.25=10.25$ ___ $.12\frac{1}{2}\times 12=1.50;\ 1.50\times 6=9.00;\ 1.25\times 5=6.25$ $9.00+6.25=15.25$
statement] d one dozen the chicks at apiece. ived 56 cents hicks sold. more than eight icks to an out buyer. cost the buyer apiece.	[*Check what is given*] ___ Amount of express bill. ___ Cost of hatching per chick ___ Number dozen chicks sold ___ Cost per dozen chicks. ___ Selling price per chick.	[*Check what is called for*] ___ Profit on this sale. ___ Cost of all chicks. ___ Profit on each chick. ___ Cost of each chick. ___ Selling price per dozen chicks.	[*Check probable answer*] ___ About 2 dollars. ___ About 6⅔ dozens. ___ $67.22. ___ $8.56. ___ $6.08.	[*Check correct solution*] ___ $12\times 8\frac{1}{3}=100$ $100\times 5\frac{1}{2}=5.50$ ___ $12\times 8\frac{1}{3}=100;\ 8.00\div 100=.08;\ .08-.05\frac{1}{2}=.025$ $.025\times 100=2.50$ ___ $12\times 8\frac{1}{3}=100;\ 100\times .05\frac{1}{2}=5.50;\ 5.50+56=6.06$ $8.00-6.06=1.94$ ___ $8.00\div 8.33=\$.96;\ 12\times 5\frac{1}{2}=.66;\ .96-.66=.30$ $.30\div 12=.025;\ .025\times 100=2.50$
statement] len is a circle ter of the yard. n is a small d plot in the the yard. is a square in f the yard. ed one tomato his garden. necessary to problem.	[*Check what is given*] ___ The shape of the garden ___ The number of feet in one square yard. ___ Space required for one tomato plant. ___ The length of two sides of the triangular garden ___ The number of tomato plants in the garden.	[*Check what is called for*] ___ The area of our yard. ___ Number of tomato plants which can be set out in garden. ___ Number of square feet allowed plant. ___ Length of third side of garden. ___ Corner of yard in which garden was located.	[*Check probable answer*] ___ About 10 feet. ___ About 22 tomato plants ___ 198 square feet. ___ 99 square feet. ___ About 10 tomato plants.	[*Check correct solution*] ___ $18\times 11=198;\ 198\div 2=99$ $99\div 9=11$ ___ $18+11=29;\ 29\div 3=9\frac{2}{3}$ ___ $18\times 11=198;\ 198\div 2=99$ $99\div 3=33$ ___ $18\times 11=198;\ 198\div 9=22$ ___ $18+11=29;\ 29+9=37$
statement] nade $1.30 by pers. him 20 cents. more running han he did sell- s. cost the boy each. oy made most ney selling pa-	[*Check what is given*] ___ Amount made by selling papers. ___ Selling price of papers. ___ Amount given to boy. ___ Number of papers sold. ___ Amount boy had after buying papers.	[*Check what is called for*] ___ Total cost of newspapers. ___ Amount of money boy had before buying papers. ___ Amount of money boy received for papers. ___ Number of papers sold. ___ Profit made on each paper.	[*Check probable answer*] ___ About 75 cents. ___ 30 cents. ___ About 13 papers. ___ About 13 cents. ___ 20 papers.	[*Check correct solution*] ___ $.35+.15+.05=.50;$ $1.30-.50=.80;\ .80\div 2=40$ ___ $.35+2=.37;\ .15+.05=.20;$ $.37+.20=.57;$ $1.30+.57=1.87;\ 1.87\div 3=29$ ___ $.35\times 2=70;\ .15+.05=.20;$ $1.30+20=1.50;$ $1.50-.70=.80;\ .80\div 5=.15$ ___ $.35+.15+.05=.55;$ $1.30-.55=.75;\ .75\div .05=15$ ___ $.35+.02+.15+.05=.57$ $1.30+.57=1.87;$ $1.87\div 5=.36$

PROBLEMS	PART 1—COMPREHENSION	PART 2—WHAT IS GIVEN
[Read the problem] ***Problem 11*** *Our kitchen is 9 feet 6 inches by 12 feet. An adjoining hall is 4 feet by 4 feet. Mother wishes me to give both floors one coat of varnish. If a pint of varnish will cover 65 square feet, how many quarts of varnish will I need?*	*[Check true statement]* ____ The kitchen was the same size as the back hall. ____ The amount of varnish required depends upon the floor area to be varnished. ____ Mother wanted two coats of varnish on the floors. ____ The hall required more varnish than the kitchen. ____ Number of coats has nothing to do with number of quarts of varnish used.	*[Check what is given]* ____ Dimensions of kitchen. ____ Area of both floors gether. ____ Dimensions of hall. ____ Area covered by one of varnish. ____ Number of coats of nish to be applied.
[Read the problem] ***Problem 12*** *If a real estate agent receives 5 per cent commission on the first thousand and 2 per cent on all above that amount, what should he remit to the owner after selling a lot for $1850?*	*[Check true statement]* ____ Agent received 5% of sale price of lot. ____ Agent received 2% on the first thousand. ____ Agent should remit less than he sold the lot for. ____ Agent received 2% on first thousand and 5% on the balance. ____ Agent received 7% of the sale price of the lot.	*[Check what is given]* ____ Per cent agent receives first thousand. ____ Amount purchaser p agent for the lot. ____ Amount lot is sold for. ____ Amount agent should mit to owner. ____ Per cent agent receives amount above $1000.
[Read the problem] ***Problem 13*** *Four boys agreed to build a radio to cost $25.00. Three of the boys together were to provide $\frac{4}{5}$ of the money, while the fourth was to do all the work and furnish the remainder of the money. How much money did he save by doing the work?*	*[Check true statement]* ____ Boy's work on radio decreased amount of money he put in. ____ Boy who did the work put in as much money as the rest. ____ The three boys put in all the money. ____ Three of the boys divided the work on the radio. ____ One boy paid for his share wholly in work.	*[Check what is given]* ____ Total cost of radio set. ____ Part of money three b were to furnish. ____ Amount saved by boy did the work. ____ Portion of money furnis by fourth boy. ____ Share of radio set each owned.

PART 3—WHAT IS CALLED FOR	PART 4—PROBABLE ANSWER	PART 5—CORRECT SOLUTION
[*Check what is called for*] ____ Area of kitchen. ____ Area covered by one quart of varnish. ____ Number of quarts of varnish needed. ____ Area covered by one pint of varnish. ____ Number of coats of varnish needed for floor.	[*Check probable answer*] ____ About 130 square feet. ____ About 1 quart. ____ 1 square foot. ____ About 2 quarts. ____ 65 square feet.	[*Check correct solution*] ____ $9\frac{1}{2}+12=21\frac{1}{2}$; $21\frac{1}{2}\times 2=42$; $4\times 4=16$; $42+16=68$; $68\div 65=1$ ____ $12\times 9\frac{1}{2}=114$; $4\times 4=16$; $114+16=130$; $130\div 65=2$; $2\div 2=1$ ____ $12\times 9\frac{1}{2}=114$; $4\times 4=16$; $114-16=98$; $98-65=33$; $33\div 8=4$ ____ $9\frac{1}{2}\times 2=18$; $12\times 2=24$; $4\times 4=16$; $18+24+16=68$; $68\times 2=130$; $130\div 65=2$; $2\div 2=1$ ____ $9\frac{1}{2}\times 12=114$; $4\times 4=16$; $114+16=130$; $130-65=65$
[*Check what is called for*] ____ Selling price of lot. ____ Amount agent should remit to owner. ____ Agent's commission on lot. ____ Per cent owner pays agent for selling lot. ____ Per cent of sale price of lot agent's commission represents.	[*Check probable answer*] ____ About 65 dollars. ____ 100% of the sale price. ____ \$1500. ____ \$1817. ____ About \$1780.	[*Check correct solution*] ____ $1850\times .05=92.50$; $1850-92.50=1757.50$ ____ $1000\times .05=50$; $850.\times .02=17.00$; $50.00+17.00=67.00$; $1850-67.00=1783.00$ ____ $1000\times .05=50$; $850\times .02=17.00$; $50.00-17.00=33.00$; $1850-33.00=1817.00$ ____ $.05+.02=.07$; $1850\times .07=129.50$; $1850-129.50=1720.50$ ____ $1000.00\times .05=500.00$; $850\times .02=170.00$; $500+170.=670.00$; $1850-670=1180.00$
[*Check what is called for*] ____ Amount of money each boy was to furnish. ____ Total cost of radio set. ____ Amount of money three boys were to furnish. ____ Time required to build radio set. ____ Amount of money saved by boy who did the work.	[*Check probable answer*] ____ About 33 cents. ____ \$6.67. ____ About 1.7 hours. ____ About \$1.65. ____ \$1.25.	[*Check correct solution*] ____ $\frac{4}{5}$ of \$25.00 = 20.00; $20\div 3=6.67$; $25.-20.=5.$; $6.67-5.00=1.67$ ____ $25.-5.=20.00$; $20.\div 3=6.67$ ____ $\frac{5}{5}-\frac{4}{5}=\frac{1}{5}$; $\frac{1}{5}\times\$25.00=\$5.00$; $\frac{1}{4}\times\$25.00=\$6.25$; $\$6.25-\$5.00=\$1.25$ ____ $\frac{4}{5}$ of 25.00 = 20.00; $20.00\div 3=6.67$; $6.67\div 4=1.67$ ____ $25.00-5.00=20.00$; $\frac{4}{5}$ of 20.00 = 16; $16.\div 3=5.33$; $25.-20.=5.00$; $5.33-5.00=.33$

PROBLEMS	PART 1—COMPREHENSION	PART 2—WHAT IS GIVEN
[Read the problem] ***Problem 14*** ***Three workmen took the contract to build a cement sidewalk $3\frac{1}{2}$ feet wide and 50 feet long for \$39.60. It took all three $\frac{2}{3}$ of a day each to do the work. They had to buy $2\frac{1}{2}$ cu. yd. of sand at \$3.00 per cubic yard and 12 sacks of cement at \$1.10 per sack. At what rate per day were they paid for their labor if each shared equally?***	*[Check true statement]* ____ Workmen took the contract by the day. ____ Workmen each spent a day on the job. ____ Four workmen were employed on the job. ____ Three workmen worked $\frac{2}{3}$ of a day each. ____ Workmen worked more than three days on the job.	*[Check what is given]* ____ Money paid out for terials. ____ Number of square feet sidewalk to build. ____ Time taken to complete work. ____ Rate per day men w paid for their labor. ____ Number of workmen e ployed.
[Read the problem] ***Problem 15*** ***At harvest time I had 956 bushels of wheat. I had one bin 11′ 6″ long, 6′ wide, and 8′ 4″ deep which held 460 bushels when filled. I stored the rest of it in a bin 10′ long and $7\frac{3}{4}$′ wide. How high did it come in this bin when leveled off?***	*[Check true statement]* ____ One bin was large enough to store all the wheat. ____ Balance of wheat was hauled to market. ____ Space occupied by one bushel of wheat is given in the problem. ____ Wheat crop amounted to over one thousand bushels. ____ Another bin slightly larger than the first was required to store the wheat crop.	*[Check what is given]* ____ Number of bushels of wh raised. ____ Number of bushels c bin would hold. ____ All dimensions of one b ____ Space occupied by c bushel of wheat. ____ Height of wheat in seco bin after leveling.
	Score = No. right = __________ [Total possible score = 15 points]	*Score = No. right ÷ 3 =* ________ [Total possible score = 15 points]

PART 3—WHAT IS CALLED FOR	PART 4—PROBABLE ANSWER	PART 5—CORRECT SOLUTION
[Check what is called for] ___ Number of workmen employed. ___ Number of days required to do the work. ___ Rate per day workmen were paid for their labor. ___ Amount paid out for sand. ___ Amount paid out for cement.	*[Check probable answer]* ___ About 7.5 days. ___ \$6.90. ___ About \$9.50. ___ \$8.20. ___ 6.3 days.	*[Check correct solution]* ___ $39.60 \div 3 = 13.20$ ___ $3. \times 2\frac{1}{2} = 7.50$ $12 \times 1.10 = 13.20$ $7.50 + 13.20 = 20.70$ $39.60 - 20.70 = 18.90$ $18.90 \div 3 = 6.30$ ___ $3. \times 2\frac{1}{2} = 7.00$ $12 \times 1.10 = 12.10$ $7.00 + 12.10 = 19.10$ $39.60 - 19.10 = 20.50$ $20.50 \div 3 = 6.83$ ___ $12 \times 1.10 = 13.20$ $39.60 - 13.20 = 26.40$ $26.40 \div 3 = 8.80$ $8.80 \div 2 = 4.40$ $4.40 \times 3 = 13.20$ ___ $3.00 \times 2\frac{1}{2} = 7.50$ $12 \times 1.10 = 13.20$ $7.50 + 13.20 = 20.70$ $39.60 - 20.70 = 18.90$ $18.90 \div 3 = 6.30$ $6.30 \div 2 = 3.15$ $3.15 \times 3 = 9.45$
[Check what is called for] ___ Number of bushels of wheat left after one bin was full. ___ Total number of bushels of wheat raised. ___ Number bushels of wheat hauled to market. ___ Height to which second bin was filled when leveled off. ___ Number of bushels per cubic foot of bin space.	*[Check probable answer]* ___ About 8 cubic feet. ___ About 8 feet. ___ 7.4 feet. ___ 6.7 bushels. ___ 6.4 feet.	*[Check correct solution]* ___ $10 \times 7\frac{3}{4} = 77\frac{1}{2}$ $956 - 460 = 496$ $496 \div 77\frac{1}{2} = 6.4$ ___ $11\frac{1}{2} \times 6 \times 8\frac{1}{3} = 575$ $10 \times 7\frac{3}{4} = 77\frac{1}{2}$ sq. ft. $575 \div 77\frac{1}{2} = 7.4$ ___ $11\frac{1}{2} \times 6 \times 8\frac{1}{3} = 575$ $460 \div 575 = .8$ bu. $956 - 460 = 496$ $496 \div .8 = 620$ cu. ft. $10 \times 7\frac{3}{4} = 77\frac{1}{2}$ sq. ft. $620 \div 77\frac{1}{2} = 8$ ___ $11\frac{1}{2} \times 6 \times 8\frac{1}{3} = 575$ $575 \div 460 = 1.25$ cu. ft. per bu. $956 - 460 = 490$ $490 \times 1.25 = 613$ $7\frac{3}{4} \times 10 = 76.7$ $613 \div 76.7 = 8$ ft.
Score = No. right = ________ [Total possible score = 15 points]	*Score = No. right =* ________ [Total possible score = 15 points]	*Score = No. right =* ________ [Total possible score = 15 points]

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