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Arbitrary Convergence Time Control for Aerial Manipulator with TSK Estimator

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***Abstract*—This paper investigates the stable control problem of unmanned aerial manipulator (UAM) in the presence of lumped disturbance, including modelling uncertainties and external inferences. These disturbances typically involve limited prior knowledge and change rapidly, presenting considerable challenges to real-time control accuracy. To address this issue, a Takagi-Sugeno-Kang estimator (TSKE) with *K*-closest fuzzy rules interpolation (K-FRI) is proposed to derive an approximation for the uncertain disturbances. The incorporation of K-FRI enhances the accuracy and convergence rate of the estimation under the conditions of a sparse fuzzy rule base with an incomplete fuzzy quantity space. Subsequently, a backstepping controller with arbitrary convergence time is introduced to guarantee the rapid and precise control of the UAM. The stability of both the TSKE and the controller with arbitrary convergence time is analysed through Lyapunov theory. The feasibility and performance of the proposed control strategy are validated via comparative experimental simulations, demonstrating its ability for robust estimation capability with stable control performance, at any convergence time of the UAM working under lumped disturbance.**

***Index Terms*—Unmanned aerial manipulator, fuzzy rule interpolation, arbitrary convergence time control, TSK estimator.**

I. Introduction

In recent years, unmanned aerial manipulators (UAM) have attracted lots of attention owing to their enhanced maneuvering capability and superior agility. As an innovative robotic system, UAM showcases excellent capabilities in various complex scenarios and challenge applications for unmanned operations, such as industrial inspection, agricultural monitoring, security observation, and search and rescue [1]. In these applications, the prevalence of uncertain disturbances is attributed to the harsh working environments of UAM, making it a focal point of shared concern in UAM research [2].

In general, the under-actuated, strong coupling, and nonlinear characteristics of the UAM pose substantial challenges for achieving rapid and accurate control performance [3]. Recently, a disturbance compensation robust H_∞ controller was designed to stabilise UAM flight while the manipulator is in operation [4]. A disturbance observer was adopted for robust control to enhance the position accuracy of the grasping operational flying robot [5]. Although the aforementioned controllers have achieved stable operation of aerial robots, the time required to reach a stable state varies with different tasks during the operational process. Consequently, achieving controllability of the convergence time when designing an exceptional controller is still recognised as a challenging issue. In [6], an adaptive finite-time control scheme with the prescribed performance was presented for an UAM to ensure finite-time convergence, implementing the specified transient and steady-state performance. A novel non-singular terminal sliding mode surface was proposed to ensure the global convergence of system states to the origin in finite time, even in the presence of disturbances in *n*-link robotic manipulators [7]. An admittance controller was developed for the *n*-link robotic manipulator to enable precise trajectory tracking within an arbitrary time frame [8].

Whilst existing techniques have explored control performances and achieved specified steady-state performance within controllable convergence time for an UAM or an associated manipulator under disturbances, a significant practical issue remains. That is, the lumped disturbance, consisting of modelling uncertainties and interferences from the environment, are essential factors in designing controllers which require careful consideration. A nonlinear disturbance observer (NDOB) was, therefore, employed to estimate uncertainties and external disturbances [9]. In [10], a robust integral error method was applied for disturbance rejection, and an invariance-based adaptive control method was proposed for an aerial robot to enhance the trajectory tracking ability. A disturbance-observer-based adaptive fuzzy tracking control scheme was introduced to achieve desirable and safe tracking performance for both position and attitude loops of a controlled unmanned helicopter [11]. In short, uncertain disturbances commonly exist in UAM applications and are expected to be compensated promptly and accurately. For this purpose, as indicated above, various attempts have been conducted to elevate the accuracy of uncertainty estimators.

The utilisation of fuzzy techniques for uncertainty estimation has become prominent due to its explicable and compre-

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hensible nature. Despite its popularity, challenges arise when dealing with a large rule base, leading to a dramatic increase in computational burden. Moreover, obtaining a complete knowledge base is extremely difficult, if not impossible, for any sizeable problems in practice, whether derived directly from domain experts or learned from historical data (or a combination of both). In response to these challenges, researchers are actively seeking favorable solutions for uncertainty estimation with limited fuzzy rule knowledge. Fuzzy rule interpolation (FRI) emerges as a valuable technique capable of generating new fuzzy rules based on existing sparse fuzzy knowledge on the fly [12]. As a benefit, feasible estimation and classification with high efficiency may be facilitated under limited initial fuzzy rule knowledge [13]. However, existing FRI methodologies are primarily designed for Mamdani inference models and not for Takagi-Sugeno-Kang (TSK) models. The Mamdani model requires prior information about the estimation range, whereas the TSK model can establish a polynomial to estimate the output. In addressing this limitation, a novel and effective FRI approach based on the TSK model while utilising only a small number of neighboring rules for unmatched observations has recently been developed [14]. From this, the significant potential of FRI based on the TSK model is emerging for high-efficiency disturbances and uncertainties estimation, particularly when dealing with limited prior knowledge.

Inspired by the above observations, an arbitrary convergence time control using TSK models is herein proposed to achieve stable control with high accuracy. Particularly, a TSK estimator (TSKE) incorporating K-closest fuzzy rules interpolation (K-FRI) is created for estimating lumped disturbance. Then, a backstepping control with arbitrary convergence time is introduced to guarantee stable control performances with high precision. The main contributions of this paper are listed below:

1) Stable control performance of an UAM is assured through the proposed backstepping control strategy, featuring arbitrary convergence time based on the TSKE. Notably, in contrast to asymptotic convergence and finite/fixed time convergence controllers, the convergence time of the proposed controller is arbitrary.
2) Nonlinear complex lumped disturbance estimation with high accuracy and robustness of the UAM can be achieved using the proposed TSKE with K-FRI. This approach offers the advantage of enhancing estimation precision and convergence rate by generating new rules through interpolation.
3) Disturbance bounds are not strictly required by the proposed fuzzy estimator, which is particularly advantageous in real-world dynamic scenarios where obtaining precise disturbance bounds may be challenging or impractical.

The remainder of this paper is organised as follows. Section II introduces the dynamics of an UAM under the lumped disturbance. Section III presents the novel control strategy with arbitrary convergence time. Simulations and results analysis are provided in Section IV. Finally, Section V concludes the paper and discusses future work.

II. UAM Dynamics Modelling

This section presents the dynamics of the UAM under investigation, encompassing the dynamics of the unmanned aerial vehicle (UAV) and the onboard manipulator.

The inertia frame and the UAV body-fixed frame are defined as Ω_w and Ω_b, respectively. The position and velocity of the UAV in the inertial frame are denoted by $\boldsymbol{p} = [x, y, z]^{\mathrm{T}}$ and $\boldsymbol{v} \in \mathbb{R}^3$, respectively. The attitude and angular velocity of the UAV in the body-fix frame are denoted by $\boldsymbol{\varphi} = [\phi, \theta, \psi]^{\mathrm{T}}$ and $\boldsymbol{\omega} \in \mathbb{R}^3$, respectively. Following the above specification, the dynamics model of the UAV can be established using Newton-Euler theory [15]:

$$\begin{cases} \dot{\boldsymbol{p}} = \boldsymbol{v} \\ \dot{\boldsymbol{v}} = -\dfrac{1}{m_u} R_b \boldsymbol{F}_t + \boldsymbol{G}_p + \boldsymbol{d}_p \\ \dot{\boldsymbol{\Phi}} = T\boldsymbol{\omega} \\ \boldsymbol{I}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega} \times \boldsymbol{I}\boldsymbol{\omega} + \boldsymbol{\tau}_a + \boldsymbol{d}_a \end{cases} \tag{1}$$

where $I \in \mathbb{R}^{3\times3}$ is the inertia matrix; $\boldsymbol{F}_t = [0, 0, f_t]^{\mathrm{T}}$ is the thrust with f_t in z-axis; the torque $\boldsymbol{\tau}_a \in \mathbb{R}^3$ is generated by the propellers; m_u is the mass of the UAV; $\boldsymbol{G}_p = [0, 0, g]^{\mathrm{T}}$ represents the gravity with g being the gravity acceleration; $\boldsymbol{d}_p \in \mathbb{R}^3$ and $\boldsymbol{d}_a \in \mathbb{R}^3$ respectively represent the lumped disturbance consisting of all relevant external disturbances, modelling uncertainties and unknown noise; $R_b \in \mathbb{R}^3$ is the rotation matrix and $T \in \mathbb{R}^{3\times3}$ is a transformation matrix.

For an n-joint onboard manipulator, the dynamics can be represented as:

$$\boldsymbol{M}_{\mathrm{m}}\ddot{\boldsymbol{q}} + \boldsymbol{C}_{\mathrm{m}}\dot{\boldsymbol{q}} + \boldsymbol{G}_{\mathrm{m}} = \boldsymbol{\tau}_{\mathrm{m}} + \boldsymbol{\tau}_{\mathrm{dq}} \tag{2}$$

where, $\boldsymbol{M}_{\mathrm{m}} \in \mathbb{R}^{n\times n}$, $\boldsymbol{C}_{\mathrm{m}} \in \mathbb{R}^{n\times n}$, $\boldsymbol{G}_{\mathrm{m}} \in \mathbb{R}^{n\times 1}$ are the positive inertia matrix, centrifugal and Coriolis matrix, and gravity matrix of the manipulator, respectively. The joint angles of the manipulator are defined as $\boldsymbol{q} = [q_1, ..., q_n]^{\mathrm{T}}$. The control input and the lumped disturbance are represented as $\boldsymbol{\tau}_{\mathrm{m}} \in \mathbb{R}^{n\times 1}$and $\boldsymbol{\tau}_{\mathrm{dq}} \in \mathbb{R}^{n\times 1}$, respectively.

Then, $\boldsymbol{\xi} = [\boldsymbol{p}^{\mathrm{T}}, \boldsymbol{\varphi}^{\mathrm{T}}, \boldsymbol{q}^{\mathrm{T}}]$ is denoted as the input and $\boldsymbol{\xi}_{\mathrm{d}} = [\boldsymbol{p}_{\mathrm{d}}^{\mathrm{T}}, \boldsymbol{\varphi}_{\mathrm{d}}^{\mathrm{T}}, \boldsymbol{q}_{\mathrm{d}}^{\mathrm{T}}]$ is the desired trajectory. The dynamics of the UAM under lumped disturbance can be derived by combining (1) and (2), as follows [16]:

$$\boldsymbol{M}(\boldsymbol{\xi})\ddot{\boldsymbol{\xi}} + \boldsymbol{C}(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}})\dot{\boldsymbol{\xi}} + \boldsymbol{G}(\boldsymbol{\xi}) = \boldsymbol{\tau} + \boldsymbol{d}_{\mathrm{ext}} \tag{3}$$

where $\boldsymbol{M} \in \mathbb{R}^{(6+n)\times(6+n)}$, $\boldsymbol{C} \in \mathbb{R}^{(6+n)\times(6+n)}$, $\boldsymbol{G} \in \mathbb{R}^{(6+n)\times 1}$, and the lumped disturbance $\boldsymbol{d}_{\mathrm{ext}} = [d_x, d_y, d_z, d_\phi, d_\theta, d_\psi, d_{q1}, d_{q2}]^{\mathrm{T}}$, with $\boldsymbol{\tau}$ being the input of the UAM.

III. Controller Design

This section describes an innovative control strategy with arbitrary convergence time of the UAM. The structure of the proposed control strategy is shown in Fig. 1. The stability analysis is also provided in this section.

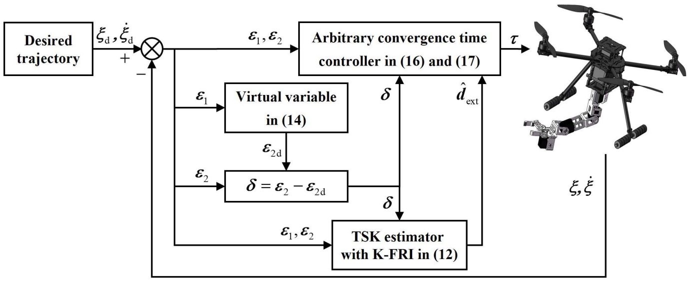


Fig. 1: Structure of proposed arbitrary convergence time control strategy for UAM.

A. TSK Estimator with K-FRI

A TSKE is herein devised to estimate lumped disturbance in the UAM. In particular, the K-closest fuzzy rules interpolation is introduced to enhance the capability of the estimator with limited prior knowledge and to improve the convergence rate. It is anticipated to perform a feasible and robust estimation with favourable efficiency.

Without losing generality, suppose that a TSK sparse fuzzy rule base consists of k_0 rules with each involving v antecedent input variables $O(b_1,,...,b_v)$ and being defined by:

$$\begin{aligned} R_i: \quad & \text{if } b_1 \text{ is } A_{i1},...,b_v \text{ is } A_{iv} \\ & \text{then } f_i(b_1,...,b_v) = a_{i0} + a_{i1}b_1 + ... + a_{iv}b_v \end{aligned} \tag{4}$$

where, $a_{i0}, a_{i1}, ..., a_{iv}$ are the parameters specifying the polynomials of a rule's consequent. The fuzzy sets $A_{i1},...,A_{iv}$ correspond to the antecedent input variables $O(b_1,,...,b_v)$ are divided into several input domains by fuzzy membership functions.

The task of modelling lumped disturbance for a given UAM is generally intricate and challenging. Leveraging on the fuzzy logic characteristics of the TSK estimator, unknown interferences can be estimated, making the TSKE a viable candidate for serving as the disturbance estimator for the UAM. However, challenges arise when dealing with unmatched inputs, as the sparse fuzzy rule base and incomplete input fuzzy quantity space can lead to invalidation and reduced estimation accuracy. To overcome this limitation, K-FRI is adopted to derive new fuzzy rules based on the closest K rules from the sparse fuzzy rule base that correspond to the unmatched input, ensuring the comprehensive utilisation of each input. This results in an improvement in estimation precision and response rate for the fuzzy estimator. Indeed, the tracking performance and stability of the UAM can be improved by combining TSKE and K-FRI.

Note that while executing the K-FRI, the judgment threshold $\mu_{\max} \in [0,1]$ plays a pivotal role during the rule interpolation process. If the threshold is surpassed by the maximum membership value of any elements within the support representing the fuzzy input, the input and its corresponding attribute value where the maximum element is located are considered matched. In such cases, the corresponding TSK fuzzy rule antecedent is triggered to construct the TSKE reasonably. Conversely, if the threshold is not exceeded, indicating that the input is not matched by any known fuzzy rule, a new fuzzy rule is then derived through K-FRI. This process also includes the determination of parameters for the consequent of the interpolated fuzzy rule.

For computational simplicity, as well as for the recognition of their popularity in the literature, triangular fuzzy membership functions are utilised to define the input fuzzy set. Furthermore, the representative value of a triangular membership function is calculated by $A_{\mathrm{r}} = (a_{\mathrm{r}1}, a_{\mathrm{r}2}, a_{\mathrm{r}3})$ [13]. Then, the Euclidean distances between the centroid of the triangular membership function and the input are computed as follows:

$$d_j(A_{\mathrm{r}}, b_j) = \frac{\sum_{i=1}^{3} |a_{\mathrm{r}i} - b_j|}{3}, \quad (j = 1,...,v) \tag{5}$$

The similarity metric based on the Euclidean distances measured by Eqn. (5) is commonly employed for evaluating the similarity between input and fuzzy sets [17]. It is computed as follows:

$$S_j(A_{\mathrm{r}}, b_j) = \frac{1}{1 + d_j(A_{\mathrm{r}}, b_j)}, \quad (j = 1,...,v) \tag{6}$$

From this, given a certain input, the K-closest known fuzzy rules of rule R_i can be determined according to the individual similarity measures across all relevant antecedent attributes concerned. Also, the interpolation weight based on similarity measures for each such rule R_i can be determined such that

$$\beta_i = \min(S_j(A_1, b_j), ..., S_j(A_K, b_j)) \quad (i = 1,...,K) \tag{7}$$

The consequent parameters of the interpolated fuzzy rules are obtained by the interpolation weight (7) and the consequent parameters of the selected K-closest rules:

$$\alpha_0 = \frac{\sum_{i=1}^{K} \beta_i a_{i0}}{\sum_{i=1}^{K} \beta_i}, ..., \alpha_v = \frac{\sum_{i=1}^{K} \beta_i a_{iv}}{\sum_{i=1}^{K} \beta_i} \tag{8}$$

Subsequently, the newly interpolated consequent of the unmatched input is constructed:

$$f_{\mathrm{new}}(b_1,...,b_v) = \alpha_0 + \alpha_1 b_1 + ... + \alpha_v b_v \tag{9}$$

Following the afore-described procedure, a new fuzzy rule is obtained and the total number of fuzzy rules in the sparse rule base is equivalently increased by one. The newly interpolated consequent can then be brought into the process of constructing TSKE, resolving the problem of encountering the unmatched input. In summary, the TSKE with K-FRI over an UAM is devised in terms of the following steps:

1) Let $\sigma_1 = \max(\mathrm{abs}(\boldsymbol{\varepsilon}_1))$ and $\sigma_2 = \max(\mathrm{abs}(\boldsymbol{\varepsilon}_2))$, where $\boldsymbol{\varepsilon}_1 = \boldsymbol{\xi}_{\mathrm{d}} - \boldsymbol{\xi}$ and $\boldsymbol{\varepsilon}_2 = \dot{\boldsymbol{\xi}}_{\mathrm{d}} - \dot{\boldsymbol{\xi}}$. Both inputs σ_1 and σ_2 are divided into y input fuzzy values in their respective quantity space and the fuzzy input membership values are set with pre-specified membership functions (which are implemented using triangular functions in this paper unless otherwise stated), and are denoted by
$$\boldsymbol{\mu}_{A_1} = [\mu_{A_1}^1, ..., \mu_{A_1}^y], \quad \boldsymbol{\mu}_{A_2} = [\mu_{A_2}^1, ..., \mu_{A_2}^y]$$
2) Construct TSKE with $y \times y$ fuzzy rules initially. Then, λ fuzzy rules are generated by K-FRI. After that, traditional fuzzy rule production-based reasoner, single

value fuzzyifier and centre mean defuzzifier are adopted in TSKE with K-FRI:

$$\boldsymbol{X}\boldsymbol{\Upsilon}\boldsymbol{\zeta}(\boldsymbol{x}_f)=\frac{\sum_{l_i=1}^{y\times y}W_{l_i}f_{l_i}(\sigma_1,\sigma_2)+\sum_{l_j=1}^{\lambda}W_{l_j}f_{l_j}(\sigma_1,\sigma_2)}{\sum_{l_i=1}^{y\times y}W_{l_i}+\sum_{l_j=1}^{\lambda}W_{l_j}} \tag{10}$$

where, $\boldsymbol{x}_f=[\sigma_1,\sigma_2]^{\mathrm{T}}$, $\boldsymbol{X}=[\sigma_1,\sigma_2,1]^{\mathrm{T}}$, $f(\sigma_1,\sigma_2)=a_0+a_1\sigma_1+a_2\sigma_2$ is the consequent polynomial, $W_{l_i}=\boldsymbol{\mu}_{A_1}\cdot\boldsymbol{\mu}_{A_2}$ represents the fuzzy rule strength of the l_i-th rule or that of the l_j-th new rule added by K-FRI, and $\boldsymbol{\zeta}(\boldsymbol{x}_f)$ is the underlying fuzzy membership function. The expression of the polynomial parameter matrix $\boldsymbol{\Upsilon}\in\mathbb{R}^{3\times(y\times y+\lambda)}$ is shown as follows:

$$\boldsymbol{\Upsilon}=\begin{bmatrix}a_{11} & \cdots & a_{(y\times y)1} & \cdots & a_{\lambda 1}\\ a_{12} & \cdots & a_{(y\times y)2} & \cdots & a_{\lambda 2}\\ a_{10} & \cdots & a_{(y\times y)0} & \cdots & a_{\lambda 0}\end{bmatrix} \tag{11}$$

3) Compute estimated lumped disturbance such that

$$\hat{\boldsymbol{d}}_{\mathrm{ext}}=[\boldsymbol{X}\hat{\boldsymbol{\Upsilon}}_1\boldsymbol{\zeta}(\boldsymbol{x}_f),...,\boldsymbol{X}\hat{\boldsymbol{\Upsilon}}_{6+n}\boldsymbol{\zeta}(\boldsymbol{x}_f)]^{\mathrm{T}} \tag{12}$$

where $\hat{d}_{\mathrm{ext}}(i)=\boldsymbol{X}\boldsymbol{\Upsilon}_i\boldsymbol{\zeta}(\boldsymbol{x}_f)$ is the i-th element of $\hat{\boldsymbol{d}}_{\mathrm{ext}}$, and $\hat{\boldsymbol{\Upsilon}}_i$ is the estimation of the polynomial parameter matrix $\boldsymbol{\Upsilon}_i$. Define $\tilde{\boldsymbol{\Upsilon}}_i=\boldsymbol{\Upsilon}_i-\hat{\boldsymbol{\Upsilon}}_i$. Then, the derivative of the estimation error is expressed by $\dot{\tilde{\boldsymbol{\Upsilon}}}_i=-\dot{\hat{\boldsymbol{\Upsilon}}}_i$.

B. Backstepping Controller with Arbitrary Convergence Time

For effective trajectory tracking, the UAM is anticipated to rapidly converge to a stable state within a specified time. Based on the previously obtained estimation $\hat{\boldsymbol{d}}_{\mathrm{ext}}$, a backstepping controller can then be developed to drive the UAM towards convergent tracking error within an arbitrary time.

In particular, the state error of the UAM can be obtained from Eqn. (3), resulting in the following:

$$\begin{cases}\dot{\boldsymbol{\varepsilon}}_1=\boldsymbol{\varepsilon}_2\\ \dot{\boldsymbol{\varepsilon}}_2=\ddot{\boldsymbol{\xi}}_{\mathrm{d}}+\boldsymbol{M}^{-1}\boldsymbol{C}\dot{\boldsymbol{\xi}}+\boldsymbol{M}^{-1}\boldsymbol{G}-\boldsymbol{M}^{-1}\boldsymbol{\tau}-\boldsymbol{M}^{-1}\boldsymbol{d}_{\mathrm{ext}}\end{cases} \tag{13}$$

where $\boldsymbol{\varepsilon}_1=\boldsymbol{\xi}_{\mathrm{d}}-\boldsymbol{\xi}$ is the tracking error and $\boldsymbol{\varepsilon}_2=\dot{\boldsymbol{\xi}}_{\mathrm{d}}-\dot{\boldsymbol{\xi}}$.

The virtual variable is designed as:

$$\boldsymbol{\varepsilon}_{2\mathrm{d}}=\begin{cases}-\dfrac{\rho_1(\mathrm{e}^{\boldsymbol{\varepsilon}_1}-1)}{\mathrm{e}^{\boldsymbol{\varepsilon}_1}(t_{\mathrm{a}}-t)}, & t_0\le t<t_{\mathrm{a}}\\ 0, & \text{otherwise}\end{cases} \tag{14}$$

where $\rho_1\ge 1$, t_0 denotes the initial time and $t_{\mathrm{a}}>t_0$ the expected arbitrary time. Thus, $\Delta T_{\mathrm{a}}=[t_0,t_{\mathrm{a}}]$ indicates an arbitrary convergence time interval.

Lemma 1: Suppose that there exists a real-valued continuously differentiable function V and a real number $\rho\ge 1$. If the conditions $V(0)=0$, $V(x)>0,\forall x\ne 0$ are satisfied and the first derivative of V satisfies as follows:

$$\dot{V}\le\frac{-\rho(\mathrm{e}^{V}-1)}{\mathrm{e}^{V}(t_f-t)},\quad \forall t\in I_{\mathrm{t}} \tag{15}$$

Then, the equilibrium point $x=0$ is arbitrary time stable [7], where $I_{\mathrm{t}}\in[t_{\mathrm{s}},t_f]$ is a finite time interval with t_{s} being the initial time and t_f the convergence time.

Theorem 1: At any convergence time ΔT_{a}, the tracking errors $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ of an UAM and the estimation errors $\tilde{\boldsymbol{\Upsilon}}_i$ of TSKE all converge to 0. That is, the UAM under the proposed control strategy is stable, where the control input of the UAM is designed as:

$$\boldsymbol{\tau}=\boldsymbol{C}\dot{\boldsymbol{\xi}}+\boldsymbol{G}+\boldsymbol{M}(\ddot{\boldsymbol{\xi}}_{\mathrm{d}}+\boldsymbol{\varepsilon}_1)-\hat{\boldsymbol{d}}_{\mathrm{ext}}+\boldsymbol{M}\boldsymbol{\Xi} \tag{16}$$

with the estimator of the lumped disturbance $\hat{\boldsymbol{d}}_{\mathrm{ext}}$ being given as per Eqn. (12), and the term $\boldsymbol{\Xi}$ is designed as:

$$\boldsymbol{\Xi}=\begin{cases}\dfrac{\rho_1\mathrm{e}^{-\boldsymbol{\varepsilon}_1}\boldsymbol{\varepsilon}_2}{t_{\mathrm{a}}-t}+\dfrac{\rho_1(1-\mathrm{e}^{-\boldsymbol{\varepsilon}_1})}{(t_{\mathrm{a}}-t)^2}+\dfrac{\rho_2(\mathrm{e}^{\boldsymbol{\delta}}-1)}{\mathrm{e}^{\boldsymbol{\delta}}(t_{\mathrm{a}}-t)}, & t_0\le t<t_{\mathrm{a}}\\ 0, & \text{otherwise}\end{cases} \tag{17}$$

with $\boldsymbol{\delta}=\boldsymbol{\varepsilon}_2-\boldsymbol{\varepsilon}_{2d}$ and $\rho_1\ge 1$, $\rho_2\ge 1$.

Note that the fuzzy adaptive law of the polynomial parameter matrix is given by:

$$\dot{\hat{\boldsymbol{\Upsilon}}}_i=-\varsigma_i\Gamma_i\boldsymbol{X}\boldsymbol{\zeta}(\boldsymbol{x}_f),\quad (i=1,...,6+n) \tag{18}$$

where ς_i is the ith element of $\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{M}^{-1}$ and Γ_i is a positive real number.

Proof: The derivative of the error $\boldsymbol{\delta}$ is calculated by substituting (13) and (14), such that

$$\begin{aligned}\dot{\boldsymbol{\delta}}=&\ddot{\boldsymbol{\xi}}_{\mathrm{d}}+\boldsymbol{M}^{-1}\boldsymbol{C}\dot{\boldsymbol{\xi}}+\boldsymbol{M}^{-1}\boldsymbol{G}-\boldsymbol{M}^{-1}\boldsymbol{\tau}-\boldsymbol{M}^{-1}\boldsymbol{d}_{\mathrm{ext}}\\ &+\frac{\rho_1\mathrm{e}^{-\boldsymbol{\varepsilon}_1}\boldsymbol{\varepsilon}_2}{t_{\mathrm{a}}-t}+\frac{\rho_1(1-\mathrm{e}^{-\boldsymbol{\varepsilon}_1})}{(t_{\mathrm{a}}-t)^2}\end{aligned} \tag{19}$$

Consider the following Lyapunov function:

$$V_0=\boldsymbol{\varepsilon}_1^{\mathrm{T}}\boldsymbol{\varepsilon}_1+\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\delta}+\sum_{i=1}^{6+n}\mathrm{tr}(\tilde{\boldsymbol{\Upsilon}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\Upsilon}}_i) \tag{20}$$

The first derivative of V_0 can therefore be expressed as:

$$\begin{aligned}\dot{V}_0&=2\boldsymbol{\varepsilon}_1^{\mathrm{T}}\dot{\boldsymbol{\varepsilon}}_1+2\boldsymbol{\delta}^{\mathrm{T}}\dot{\boldsymbol{\delta}}+2\sum_{i=1}^{6+n}\mathrm{tr}(\tilde{\boldsymbol{\Upsilon}}_i^{\mathrm{T}}\Gamma_i^{-1}\dot{\tilde{\boldsymbol{\Upsilon}}}_i)\\ &=2\boldsymbol{\varepsilon}_1^{\mathrm{T}}(\boldsymbol{\delta}+\boldsymbol{\varepsilon}_{2d})+2\boldsymbol{\delta}^{\mathrm{T}}\dot{\boldsymbol{\delta}}-2\sum_{i=1}^{6+n}\mathrm{tr}(\tilde{\boldsymbol{\Upsilon}}_i^{\mathrm{T}}\Gamma_i^{-1}\dot{\hat{\boldsymbol{\Upsilon}}}_i)\end{aligned} \tag{21}$$

Then, by substituting Eqn. (16), (17), (18) and (19) into Eqn. (21), it can be obtained that

$$\begin{aligned}\dot{V}_0&=-\frac{2\rho_1\boldsymbol{\varepsilon}_1^{\mathrm{T}}(\mathrm{e}^{\boldsymbol{\varepsilon}_1}-1)}{\mathrm{e}^{\boldsymbol{\varepsilon}_1}(t_{\mathrm{a}}-t)}-\frac{2\rho_2\boldsymbol{\delta}^{\mathrm{T}}(\mathrm{e}^{\boldsymbol{\delta}}-1)}{\mathrm{e}^{\boldsymbol{\delta}}(t_{\mathrm{a}}-t)}\\ &\le-\frac{2\rho_1\|\boldsymbol{\varepsilon}_1\|(\mathrm{e}^{\|\boldsymbol{\varepsilon}_1\|}-1)}{\mathrm{e}^{\|\boldsymbol{\varepsilon}_1\|}(t_{\mathrm{a}}-t)}-\frac{2\rho_2\|\boldsymbol{\delta}\|(\mathrm{e}^{\|\boldsymbol{\delta}\|}-1)}{\mathrm{e}^{\|\boldsymbol{\delta}\|}(t_{\mathrm{a}}-t)}\\ &\le-\frac{2(\rho_1+\rho_2)\sqrt{V_0}(\mathrm{e}^{\sqrt{V_0}}-1)}{\mathrm{e}^{\sqrt{V_0}}(t_{\mathrm{a}}-t)}\end{aligned} \tag{22}$$

Denote $\eta=\sqrt{V_0}$ and substitute it into Eqn. (22). It then follows that

$$\dot{\eta}\le-\frac{(\rho_1+\rho_2)(\mathrm{e}^{\eta}-1)}{\mathrm{e}^{\eta}(t_{\mathrm{a}}-t)} \tag{23}$$

where $\rho_1+\rho_2\ge 1$. Consequently, $\eta=0$ at some time $t\le t_{\mathrm{a}}$ owing to the Lemma 1. Thus, the tracking errors $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ of the UAM and the estimation error $\tilde{\boldsymbol{\Upsilon}}_i$ of TSKE all converge to 0 at arbitrary convergence time ΔT_{a}.

■

IV. Simulations and Results Analysis

In this section, comparative experimental simulations are conducted, contrasting the results between the application of the proposed TSKE with K-FRI and that of the same control strategy without K-FRI. The results analysis, including tracking performance of the UAM and statistical data, is provided to verify the effectiveness of the present approach.

A. Simulation Setup

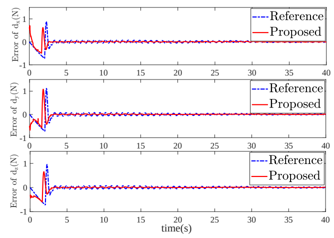
To verify the arbitrary time convergence performance of the proposed control strategy through simulations, a two-joint onboard manipulator is selected and the flight time of the UAM under investigation is set to 40s given continuous lumped disturbance. The mass of the UAV and the onboard manipulator are set as $m_u = 1.5$kg and $m_m = 0.7$kg, respectively. The inertia tensors of UAV are set as $I_x = 0.01$, $I_y = 0.02$ and $I_z = 0.03$. The length of the two rods of the onboard manipulator are set as $l_{\mathrm{m}1} = 0.2$m and $l_{\mathrm{m}2} = 0.2$m. The gravity acceleration is $g = 9.8\mathrm{m/s}^2$. The desired trajectory of the UAV is set to $\boldsymbol{p}_{\mathrm{d}} = [\sin\frac{\pi}{9.5}t,\ \sin\frac{\pi}{9.5}t,\ t]^{\mathrm{T}}$ and the expected joints of manipulator are set as $\boldsymbol{q}_{\mathrm{d}} = [1+0.5\sin\frac{\pi}{5}t, -0.5+0.5\sin\frac{\pi}{5}t]^{\mathrm{T}}$. The desired yaw angle is set as 0. The lumped disturbance is set to $\boldsymbol{d}_{\mathrm{ext}} = [\sin\frac{\pi}{8}t, \sin\frac{\pi}{8}t, \sin\frac{\pi}{8}t, 0.2\sin\frac{\pi}{8}t, 0.2\sin\frac{\pi}{8}t, 0.2\sin\frac{\pi}{8}t, 2+0.5\sin\frac{\pi}{10}t, 2+0.5\sin\frac{\pi}{10}t]^{\mathrm{T}}$. The convergence parameters of the controller are given as $\rho_1 = 3.5$ and $\rho_2 = 3$ and the arbitrary convergence time is set to $\Delta T_{\mathrm{a}} = 3$s. The parameters of the fuzzy adaptive law are set as: $\Gamma_1 = 200$, $\Gamma_2 = 160$, $\Gamma_3 = 160$, $\Gamma_4 = 0.5$, $\Gamma_5 = 0.8$, $\Gamma_6 = 1.2$, $\Gamma_7 = 470$, $\Gamma_8 = 470$.

To reflect on the situations where only limited prior knowledge is available, the sparse fuzzy rule base and incomplete input fuzzy quantity space are assumed in simulations, where triangular membership functions are chosen as membership functions for the input. Particularly, the input attributes are divided into five input fuzzy set values such that $A^1_{\sigma_1} = (-1.15, -1, -0.8)$, $A^2_{\sigma_1} = (-0.65, -0.5, -0.3)$, $A^3_{\sigma_1} = (-0.15, 0, 0.2)$, $A^4_{\sigma_1} = (0.35, 0.5, 0.7)$, $A^5_{\sigma_1} = (0.85, 1, 1.2)$ and $A^1_{\sigma_2} = (-5.5, -5, -4)$, $A^2_{\sigma_2} = (-3, -2.5, -1.5)$, $A^3_{\sigma_2} = (-0.5, 0, 1)$, $A^4_{\sigma_2} = (2, 2.5, 3.5)$, $A^5_{\sigma_2} = (4.5, 5, 6)$, respectively. The representative values of these membership functions can be easily computed and hence, are omitted to save space. The value of the interpolation parameter is set as $K = 3$.

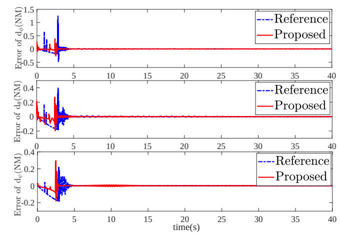
B. Results and Analysis

To enable comparative studies, as reference, the same controller and lumped disturbance are adopted while the K-FRI method is not utilised to construct TSKE. The fuzzy rules of the reference are given to be the same as those in the initial rule base of the proposed approach. Both the proposed strategy and the reference method guide the simulated UAM to track the same desired trajectory.

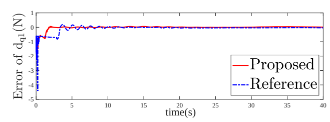
The results of the proposed method and those for the reference for lumped disturbance estimation errors are illustrated in Fig. 2. From this figure, it can be observed that both the estimation errors of the proposed and those of the reference converge rapidly to the zero-levels with the arbitrary convergence time $\Delta T_{\mathrm{a}} = 3$s. Specifically, the position and attitude lumped disturbance estimation errors are depicted in Fig. 2(a) and 2(b), respectively, where the estimated error convergence rate of the proposed approach is faster than that of the reference method. The lumped disturbance estimation errors of the tow manipulator joints are shown in Fig. 2(c) and 2(d), respectively, where the values of the estimation errors of the proposed method are smaller than those of the reference. That is, the proposed TSK fuzzy estimator with K-FRI exhibits faster convergence rates with excellent estimated performances.



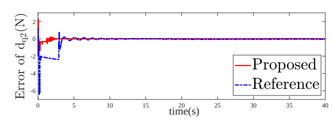
(a) Estimation errors of position lumped disturbance



(b) Estimation errors of attitude lumped disturbance



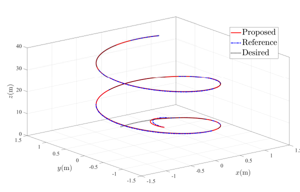
(c) Estimation errors of lumped disturbance for manipulator joint 1



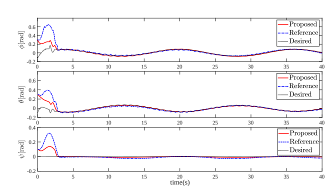
(d) Estimation errors of lumped disturbance for manipulator joint 2

Fig. 2: Simulation-based comparison on lumped disturbance estimation errors.

The comparative results of the position and attitude trajectory tracking performances for the UAM are shown in Fig. 3. The tracking curves of both the proposed method and the reference converge to the desired ones with the arbitrary convergence time $\Delta T_{\mathrm{a}} = 3$s. As K-FRI is adopted in the proposed approach, the trajectory tracking rates are faster for position and attitude trajectory tracking, and the convergence amplitudes are smaller for attitude tracking. That is, the proposed method based on TSKE with K-FRI exhibits faster convergence rates and outstanding trajectory tracking performances in the arbitrary convergence time.



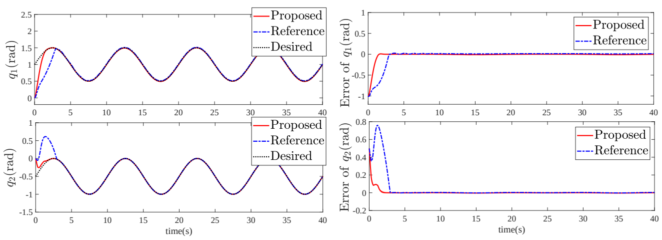
(a) Position tracking performances



(b) Attitude tracking performances

Fig. 3: Simulation-based comparison on UAM trajectory tracking performances.

The results of the manipulator trajectory tracking performances and those of tracking errors from both the proposed method and the reference are shown in Fig. 4(a) and 4(b), respectively. The arbitrary convergence time of tracking errors for both methods are $\Delta T_{\mathrm{a}} = 3$s. The proposed method utilises TSKE with K-FRI, achieving faster joint trajectory tracking convergence rates and smaller jitter of the tracking errors for onboard manipulator. This indicates once again, that the proposed method exhibits faster convergence rates with excellent error convergence performances in arbitrary convergence time.



(a) Tracking performance of manipulator joints (b) Tracking error of manipulator joints

Fig. 4: Simulation-based comparison on manipulator trajectory tracking performances.

To have a clear and intuitive examination of the performances regarding the two different approaches, statistic data of the results of utilising both methods are provided in Table I, where RMSE represents root-mean-square error and STD represents standard deviation. Comparing the RMSE results in Table I, the steady-state errors of the proposed approach based on TSKE with K-FRI are much smaller, exhibiting outstanding tracking performance.

TABLE I: Steady-State Error Statistics

State	Proposed method		Reference without K-FRI	
	RMSE	STD	RMSE	STD
x(cm)	5.14	6.98	5.53	7.01
y(cm)	5.99	7.12	6.40	7.13
z(cm)	3.81	5.54	3.97	5.68
ϕ(rad)	0.0493	0.0782	0.1155	0.1324
θ(rad)	0.0491	0.0651	0.0858	0.0930
ψ(rad)	0.0282	0.0287	0.0595	0.0624
q_1(rad)	0.1049	0.1628	0.1858	0.1870
q_2(rad)	0.0332	0.0357	0.0426	0.0409

Particularly, the number of the newly added fuzzy rules in TSKE with K-FRI reaches 1056 at the end, although the initial fuzzy rule base was sparse and the fuzzy quantity space incomplete. The unmatched input is not ignored and the information is not invalid by K-FRI. Consequently, TSKE with K-FRI possesses a remarkable estimation accuracy. Overall, the proposed strategy entails favourable tracking performance and convergence at arbitrary time, verifying the effectiveness of the proposed approach.

V. Conclusion

In this paper, an arbitrary convergence time UAM control strategy has been proposed. To construct a fuzzy estimator on lumped disturbance that works with an UAM exhibiting nonlinear characteristics while facing limited prior knowledge, a TSKE with K-FRI is developed that can perform with a sparse fuzzy rule base and incomplete fuzzy quantity space. Based on the proposed estimator, a backstepping controller with arbitrary convergence time is presented to guarantee the motion stability and accuracy of the UAM. The effectiveness of the scheme has been illustrated through comparative experimental simulations. However, this paper only explores the potential application of TSK estimator and fuzzy rule interpolation for disturbance estimation. Further research will focus on high-performance estimation with a rapid convergence rate, possibly involving higher-order uncertainties [18].

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