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Research article

q-Rung Orthopair fuzzy time series forecasting technique: Prediction based decision making

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Abstract: The literature frequently uses fuzzy inference methods for time series forecasting. In business and other situations, it is frequently necessary to forecast numerous time series. The q-Rung orthopair fuzzy set is a beneficial and competent tool to address ambiguity. In this research, a computational forecasting method based on q-Rung orthopair fuzzy time series has been created to deliver better prediction results to deal with situations containing higher uncertainty caused by large fluctuations in consecutive years' values in time series data and with no visualization of trend or periodicity. The main objective of this article is to handle time series forecasting with the usage of q-Rung orthopair fuzzy sets for things like floods, admission of students, number of patients, etc. After this, people can then manage issues that will arise in the future. Previously, there was a gap in determining the forecasting of data whose entire value of membership and non-membership exceeded 1. To fill this kind of gap, we used q-Rung orthopair fuzzy sets in time series forecasting. We also used numerous algebraic components for the q-Rung orthopair fuzzy time series, which has a union, max-min composition, cartesian product, and algorithm that are useful to calculate the method of data forecasting. Moreover, we also defined the algorithm and proposed MATLAB code that facilitates the execution of mathematical calculations, design, analysis, and optimization (structural and mathematical), and gives results with speed, correctness, and precision. At the end, we tested the model using historical student enrollment data and the annual peak discharge at Guddu Barrage. Furthermore, we calculated the error to get an idea of to what extent this method is suitable.

Keywords: fuzzy time series; intuitionistic fuzzy sets; q-Rung orthopair fuzzy sets; induced fuzzy set; nondeterminacy

1. Introduction

Making precise conclusions in decision analysis on the basis of unclear and confusing information is highly challenging. Information that is unclear or unsecured has long been a significant problem. Although reaching a final conclusion requires numerous actions and various criteria, the logical implications for decision-makers coping with such unclear, ambiguous, and erroneous information grow increasingly difficult. To deal with these sorts of decision-making problems, some research has been done in sectors such as kidney transplant [1], computer science, and environmental sciences, among others [2]. Before describing this paper we describe the brief history of research in dealing with uncertain data. In 1965, Zadeh defined the concept of a fuzzy set [3]. Several researchers have worked on the extension of fuzzy sets.

After this, Atanassov [4] gave the idea of intuitionistic fuzzy sets theory. Intuitionistic fuzzy sets (IFSs) have been found to be quite useful in dealing with ambiguity. There are circumstances when the evaluation of membership values is not achievable to our satisfaction owing to a lack of information (apart from the existence of ambiguity). Because evaluating non-membership values is not always possible, it is preferable to employ (IFS theory). The issues addressed by fuzzy set theories can also be addressed by IFS theory, although there are a few more problems that IFS theory is better suited to address than fuzzy set theory. Few of them work on a higher order of fuzzy sets [5]. Dubois [6] worked on the relation of fuzzy sets with other sets and also described some basic properties of the extension of fuzzy sets. The membership of an element in a fuzzy set is a single value between zero and one, according to intuitionistic fuzzy sets. The non-membership degree is immediately added to one of the degrees of membership. After this, intuitionistic fuzzy sets can be implicated in the soft set [7], which gives an idea of how we can use intuitionistic fuzzy sets in decision making [8]. As we know, fuzzy set theory is used for decision-making in real-life problems, so Lin [9] used it in the selection of the best air conditioner, and Wang [10] used it for an example related to the best car selection with the usage of IFSs for multicriteria decision-making, which is a type of fuzzy set.

Time series analysis and multiple regression models are two forecasting methodologies. The time series is used to deal with problems that change over time. Song and Chissom's implemented fuzzy time series [11] which describes the idea of fuzzy time series and its models. Song and Chissom forecasted data using fuzzy set theory [12, 13]. After this, Kumar and Gangwar [14] modified the technique of Song and Chissom. Non-determinism arises in the classical forecasting approaches due to noise (a greater component of time series) and is reduced using a minimal filter. Joshi and Kumar [15] and Gangwar and Kumar [16] used IFSs, utilizing the time series forecasting approach to integrate the degree of hesitation in fuzzy logical links and provide a few time series forecasting models [17]. Some researchers selected data from the University of Alabama's student enrollment and the market prices of SBI shares [18, 19] that could be solved using intuitionistic fuzzy time series forecasting. These methods represent distinct ways for computing the forecasting of IFS time series, after which they may be compare to each other using calculated errors.

We used the set q-Rung orthopair fuzzy sets (q-ROFSs), which are a modification of generalized fuzzy sets, in 2 dimensions. q-ROFS are used in this paper because sometimes we have data in 2 dimensions, so they tackle these types of data as well as generalize all types of fuzzy sets that are in 2 dimensions. Yager [20] investigated q-ROFSs as a continuation of IFSs and Pythagorean fuzzy sets (PFSs); after this, they described the generalized orthopair fuzzy set [21]. Also, Yager's Pythagorean fuzzy set [22] is an extension of IFSs that spreads the range of membership and nonmembership functions, allowing decision makers to describe their judgments more openly than the introduced IFSs and their subset [23]. This approach has been included in q-ROFSs. Many academics have focused on q-ROFSs because of their capacity to handle uncertainty more effectively. The sum of the q-th powers of the membership degree (μ) and non-membership degree (ν) should be less than 1 when representing fuzzy information using q-ROFSs ($0 \leq \mu^q + \nu^q \leq 1$, where $q \geq 1$). IFSs and PFSs are obviously special cases of q-ROFSs, where q is 1 or 2, respectively. Figure 1 shows the representation of q-ROFSs.

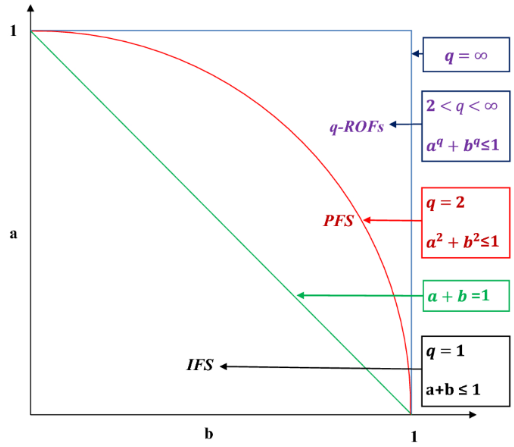

Figure 1. Generalization of the IFSs and PFS.

Khoshaim [24] linked q-ROFSs for decision-making in emergencies. Shaheen [25] explained why we need q-Rung orthopair fuzzy sets. Although the q-ROFS is a strong and valuable tool for dealing with uncertainty, decision-makers are frequently required to examine the real situation throughout the decision-making process. The preceding works mostly looked at q-ROFSs from a static standpoint. Decision-makers are usually needed to examine the real situation dynamically during the decision-making process. As a result, it is critical to include a time parameter in the q-Rung orthopair fuzzy number (q-ROFN) and then apply it to the time series q-ROFS [26]. Liu et al., [27] introduced the q-ROF preference relations based decision support methodology under q-ROFSs. The T-spherical fuzzy set (T-SFS) [28] is a modification of q-ROFS, but it is three-dimensional. Liu [29] gives spherical operators, which are based on decision making. In this paper, we used the concept of time series forecasting by using a 2-dimensional fuzzy set. In the future, researchers may use this technique and implicate our idea in T-SFS when the problem is in three dimensions by using the proposed algorithm.

Time series forecasting plays a crucial role in fuzzy analysis. It provides a framework for modeling and analyzing systems that involve vagueness or ambiguity. Time series forecasting models based on fuzzy logic can utilize linguistic terms (e.g., low, medium, and high) to represent different levels of variables and define fuzzy rules that describe system dynamics. Fuzzy time series forecasting models can better capture and predict nonlinear patterns in data. Time series forecasting is highly relevant and beneficial in daily life for various reasons. It helps individuals and businesses with financial planning and budgeting. It can predict future trends in income, expenses, sales, stock prices, exchange rates, and other financial variables. Investors and traders use time series forecasting to analyze stock market trends, predict stock prices, and make investment decisions.

- We implicate q-ROFSs in this paper for time series forecasting, which can help with future prediction problems.
- The new algorithm, which is applicable to time series forecasting by using the definition of q-ROFSs.
- Additionally, MATLAB code is provided to locate the values more quickly than Excel. You only need to enter the initial value, and the code will produce the final result.

Humans have worked hard to gain a deeper understanding of the developing world in which they inhabit. In this growing world, we encounter several problems and uncertainties caused by fuzziness in making the best decisions. That is why a large number of researchers have worked to handle this form of resistance to decision-making [30]. Previously, researchers made decisions based on the ranking and scoring of supplied data. However, they were not be helpful in predicting future values. Chen [31] first worked on forecasting enrollments depending on fuzzy time series. It is observed that although several authors are engaged in IFS time series forecasting [32]. Kumar [33] gave an idea of how we can deal with forecasting with the help of induced IFSs. Gautam [34] used this concept to deal with high-order IFS time series data. We choose the same data and compare the results. However, there is a difficulty in that there is no work on handling data in which the total of membership and non-membership is more than one. Thus, we aim to fill this gap. As a result, we use q-ROFSs to deal with this type of issue. This is a modification of all forecasting methods that can handle any sort of membership or non-membership value. We worked in this area to fulfill it by using q-Rung orthopair fuzzy time series (q-ROFTSs), which plays a critical function in dealing with time series forecasting in which the total of membership and non-membership is above one. In this paper, we describe a method for forecasting q-ROFTSs in which the chances of error are lower than in previous research work. We selected two types of data and calculated errors to determine whether each type of data is suited for our suggested strategy. This concept is useful for resolving a variety of issues that may arise in the future. The rest of the article is organized as follows:
(1) Fuzzy sets and q-ROFSs are defined, which is used throughout the rest of the article.
(2) Following that, we provide some definitions that will be used to implement my suggested technique, as well as create a triangle membership function that will be used in calculating the induced fuzzy set.
(3) We then define q-ROFTSs, and provide time variant and time invariant definitions.
(4) After that, we provide a flowchart in which we define step-by-step how we can forecast the data in our proposed method. We define the method of construction of q-ROFTSs, as well as how to construct q-ROFSs, their fuzzification, defuzzification, and also how to calculate the error.
(5) We give the MATLAB code for the example of the admission of students.
(6) We implemented the method on the admission of students by year and draw its results in tables.
(7) We take another example, the annual peak of the Guddu Barrage in Pakistan, to compare the results of both example.
(8) Finally, we give the conclusion of this paper.

Table 1 shows the list of abbreviations used in this paper.

Table 1. List of abbreviation.

Abbreviation	Definition
q-ROFSs	q-Rung Orthopair Fuzzy Sets
q-ROFN	q-Rung Orthopair Fuzzy Number
q-ROFR	q-Rung Orthopair Fuzzy Relation
q-ROFLR	q-Rung Orthopair Fuzzy Logic Relation
q-ROFLRGs	q-Rung Orthopair Fuzzy Logic Relation Groups
IFS	Intuitionistic Fuzzy Set
PFS	Pythagorean Fuzzy Set
q-ROFTSs	q-Rung Orthopair Fuzzy Time Series
AFE	Absolute Forecasting Error
MSE	Mean Square Error
MOM	Mean of Maximum

2. Preliminaries

In this section, we discuss some definitions related to q-ROFSs that were used previously before when q-ROFS was introduced.

Definition 1. *The fuzzy set concept introduced by Zadeh, is defined as: Let* Ł *be a set. A fuzzy set* Ł *in* S *is defined as:*

$$Ł = \{\langle s, \mu_{Ł}(s)\rangle \mid \forall\ s \in S\}$$

where $\mu_{Ł}(s)$ *is a membership function of the fuzzy set* Ł, $\mu_{Ł} : S \to [0,1]$, *and* $\mu_{Ł}(s)$ *provide the membership function s degree in* Ł.

Definition 2. *Assume a nonempty set S. In S, an intuitionistic fuzzy set* Å *is an attribute of the form:*

$$Å = \{\langle s, \mu_{Å}(s), \nu_{Å}(s)\rangle; s \in S\}$$

where the function, $\mu_{Å}(s), \nu_{Å}(s) :\to [0,1]$ *defines its membership and non-membership degrees, and for all aspects* $s \in S, 0 \leq \mu_{Å}(s) + \nu_{Å}(s) \leq 1$.

But, if the membership and non-membership values sum to greater than one, then PFS is used in this types of cases.

Definition 3. *A Pythagorean fuzzy set (PFS)* Å *in S is mathematically formed as:*

$$Å = \{\langle s, \mu_{Å}(s), \nu_{Å}(s)\rangle; s \in S\}$$

where the functions $\mu_{Å}(s) : S \to [0,1]$ *represents the degree of membership of s in* Å *and* $\nu_{Å}(s) : S \to [0,1]$ *represents the degree of nonmembership of s in* Å. *For every* $s \in S$, *the following circumstance should be satisfied:*

$$0 \leq \mu^2_{Å}(s_i) + \nu^2_{Å}(s_i) \leq 1.$$

In PFS, the power of membership and non-membership is 2, which also does not handle the condition of the total sum being less than 1, which is described in the fuzzy set main definition, so we used q-ROFS.

Definition 4. *Yager introduced the concept of q-Rung orthopair fuzzy sets q-ROFSs. A q-ROFSs* Å *in* S *is defined as an object of the following form:*

$$\text{Å} = \{\langle s, (\mu_{\text{Å}}(s), \nu_{\text{Å}}(s))\rangle \mid \forall\ s \in S\}$$

where $\mu_{\text{Å}}(s) : S \rightarrow [0,1]$ *and* $\nu_{\text{Å}}(s) : S \rightarrow [0,1]$ *indicate the degree of membership and nonmembership of* $s \in S$*, respectively. For each* $s \in S$*, the condition*

$$0 \leq \mu^q_{\text{Å}}(s_i) + \nu^q_{\text{Å}}(s_i) \leq 1$$

where the power $q \geq 1$ *satisfy.*
Where the sum of the q th power of the degree of membership and the q th power of the non-membership degree is limited to one.

The degree of indeterminacy [35] is given as

$$\pi^q_{\text{Å}}(s) = (\mu^q_{\text{Å}}(s_i) + \nu^q_{\text{Å}}(s_i) - \mu^q_{\text{Å}}(s_i)\nu^q_{\text{Å}}(s_i))^{1/q}.$$

For convenience, we call $\text{Å} = \langle \mu_{\text{Å}}(s), \nu_{\text{Å}}(s)\rangle$ *a q-ROFN represented by* $\text{Å} = \langle \mu_{\text{Å}}, \nu_{\text{Å}}\rangle$.

3. Cartesian product of q-ROFSs and its relation

Now we discuss some basic definitions that can be used to solve our proposed method.

Definition 5. *Let* $\text{Å} = \{\langle \mu_{\text{Å}}(z), \nu_{\text{Å}}(z)\rangle \mid \forall\ z \in S_1\}$ *and* $\text{Ç} = \{\langle \mu_{\text{Ç}}(t), \nu_{\text{Ç}}(t)\rangle \mid \forall\ t \in S_2\}$ *be the two q-ROFSs over* S_1 *and* S_2*, respectively. The statement that follows defines the cartesian products of q-ROFSs* Å *and* Ç.

$$\text{Å} \times \text{Ç} = \{\langle (z,t),\ \min(\mu_{\text{Å}}(z), \mu_{\text{Ç}}(t)),\ \max(\nu_{\text{Å}}(z), \nu_{\text{Ç}}(t))\rangle \mid z \in S_1, t \in S_2\}.$$

Definition 6. *A q-Rung orthopair fuzzy relation (q-ROFR) R which is a subset of* $(Z \times T)$ *between the two nonempty sets Z and T is a q-ROFS over* $Z \times T$*, which is defined as:*

$$R = \{\langle (z,t),\ \mu_R(z,t) + \nu_R(z,t)\rangle \mid z \in Z,\ t \in T\}$$

where $\mu_R : Z \times T \rightarrow [0,1]$ *and* $\nu_R : Z \times T \rightarrow [0,1]$ *are the degrees of membership and nonmembership, respectively, and meet the following criteria:*

$$0 \leq \mu_R(z,t) + \nu_R(z,t) \leq 1\ \forall\ (z,t) \in Z \times T.$$

Definition 7. *Let q-ROFSs(Z,T) be the set of all q-ROFSs over* $Z \times T$*. Let B, C* $\in$ *q-ROFSs(Z,T), and the union, compositions of relation B and C are defined as follows:*
$B \cup C = \{\langle (z,t),\ \max(\mu_B(z,t), \mu_C(z,t)),\ \min(\nu_B(z,t), \nu_C(z,t))\rangle \mid (z,t) \in Z \times T\}$.
$B\ o\ C = \{\langle (z,t),\ \max_{t \in T}(\min(\mu_B(z,t), \mu_C(z,t))),\ \min_{t \in T}(\max(\nu_B(z,t), \nu_C(z,t)))\rangle \mid (z,t) \in Z \times T\}$.

Definition 8. *[36] Triangle fuzzy sets are defined as q-ROFSs with membership functions that have a triangular shape. Membership function is what defines a triangular fuzzy set.*

$$\mu(z;[a,b,c]) = \begin{cases} \frac{z-a}{b-a} & , \text{ if } z \in [a,b] \\ \frac{c-z}{c-b} & , \text{ if } z \in [b,c] \\ 0 & , \text{ otherwise} \end{cases}.$$

4. q-Rung Orthopair fuzzy time series

The relevant knowledge measures for q-ROFTSs are mentioned in this part.

Definition 9. *Suppose $\Gamma(t)$, where (t=0,1,2,3,4,......) is the universe of discourse and $\Gamma(t) \subseteq R$. Assume that $Q_i(t)$ where $(i = 1, 2, 3...)$ are q-ROFSs stated in the universe of discourse $\Gamma(t)$. If $\Im(t)$ is the cluster of $Q_i(t)$, then $\Im(t)$ is a q-ROFT, $\Gamma(t)$ may differ based on the circumstances.*

Definition 10. *If $Q_j(t) \in \Im(t)$ is caused only by $Q_i(t-1) \in \Im(t-1)$ denoted by $Q_i(t-1) \rightarrow Q_j(t)$ or the same as $\Im(t-1) \rightarrow \Im(t)$, then the relationship fits into the following q-Rung orthopair fuzzy relational equation (q-ROFRE):*

$$\Im(t) = \Im(t-1) \; o \; R_i(t, t-1)$$

in which the symbol o is the max-min composition operator, defined in definition 7. The relation $R_i(t, t-1)$ is a q-ROFRs between $\Im(t)$ and $\Im(t-1)$ for which it the first-order model of $\Im(t)$.

Definition 11. *If $\Im(t)$ is caused by more q-ROFSs, $Q(t-n), Q(t-n+1), ...Q(t-1)$ then q-ROFRs is represented by $Q(t-n) = Q_{i_1}, Q(t-n+1) = Q_{i_2}, ..., Q(t-1) = Q_{i_n}$. The nth-order q-ROFTSs model is this one is defined above.*

Definition 12. *[37] Let $\Im(t)$ be a q-ROFTSs and $R_i(t, t-1)$ a first-order model of $\Im(t)$. If $R_i(t, t-1) = R_i(t-1, t-2)$ for any time t, then $\Im(t)$ is called a time-invariant fuzzy time series. But, if $R_i(t, t-1)$ relies on time, that is, $R_i(t, t-1)$ could be distinct from $R_i(t-1, t-2)$ for any time t, then $\Im(t)$ is called a time-variant q-ROFTSs.*

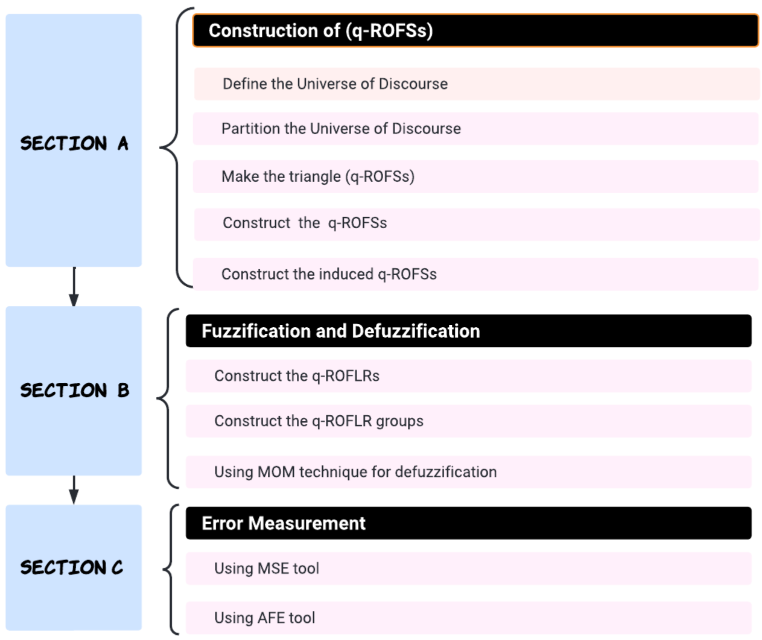

Figure 2. Flowchart of the proposed method.

5. q-Rung Orthopair fuzzy time series forecasting

Now we define our algorithm, which is divided into three parts: A, B, and C. In q-ROFTS, we define a solution to this sort of problem. Figure 2 gives the algorithm for handling q-ROFSs in time series analysis.

A. Construction of Fuzzy set and q-ROFSs

This section briefly describe the step-by-step method for creating fuzzy sets and q-ROFSs.

Step I. For the universe of discourse U depending upon the range of time series data that are provided, $U = [W_{\min} - W_1, W_{\max} + W_2]$ where $W_{\min}$ and $W_{\max}$ are the time series, lowest and highest data, respectively, and W_1 and W_2 are any suitable positive values to fully handle the time series data completely.

Step II. The universe of discourse, is divided into subsets of U, within intervals whose lengths are equal. The intervals will be set out in accordance to the number of fuzzy sets in $\Re$.

Step III. Make the triangle q-ROFSs $\Re_i$ according to the interval which was constructed in Step II, after which each interval in each triangle fuzzy set is applied to the triangular membership function.

Step IV. Construct Atanassov's q-ROFSs, which correspond to the the triangular fuzzy set $\Re$ using the construction approach of q-ROFSs which was given by Jurio at al [38].

Let $A_F \in FH_H(S)$, where $FH_H(S)$ shows the set of all fuzzy set in the Universal set S and Let $\partial, \hbar : S \to [0,1]$ are the two mappings. Then

$$Q = \{\langle s_i,\ f(\mu_{A_F}(s_i), \partial(s_i), \hbar(s_i))\rangle\ \forall\ s_i \in S\}$$

$$Q = \{\langle s_i,\ f_\mu(\mu_{A_F}(s_i), \partial(s_i), \hbar(s_i)),\ f_\nu(\mu_{A_F}(s_i), \partial(s_i), \hbar(s_i))\rangle\ \forall\ s_i \in S\}$$

where

$$f_\mu(z, t, \hbar) = z(1 - \hbar t)$$

$$f_\nu(z, t, \hbar) = 1 - z(1 - \hbar t) - \hbar t$$

where as the mapping $f : [0,1]^2 \times [0,1] \to L^*$ fulfills conditions:

(1) If $z_1 \leq z_2$, then $\partial(f(z, t_1, \hbar)) \leq \partial(f(z, t_2, \hbar)) \forall z, \hbar \in [0,1]$.
(2) c$f_\mu(z, t, \hbar) \leq z \leq 1 - f_\nu(z, t, \hbar) \forall z \in [0,1]$.
(3) $f(z, 0, \hbar) = (z, 1 - z)$.
(4) $f(0, t, \hbar) = (0, 1 - \hbar t)$.
(5) $f(z, t, 0) = (z, 1 - t)$.
(6) $\partial(f(z, t, \hbar)) = \hbar t$.

Example 1. *In this example we convert fuzzy set data into q-ROFS form by using the given formulas in Step IV.*

Let $Ł = \{(u_1, 0.3), (z_2, 0.8), (z_3, 0.1), (z_4, 0.5)\}$ *be the given fuzzy set.* z_i, ∂, *and* $\hbar$ *are picked in the following way.*

$\partial(z_i) = \min(\mu_Ł(z_1), \mu_Ł(z_2), \mu_Ł(z_3), \mu_Ł(z_4)) = 0.1$,
$\hbar(z_i) = \max(\mu_Ł(z_1), \mu_Ł(z_2), \mu_Ł(z_3), \mu_Ł(z_4)) = 0.8$.

Then we calculate the grade of membership and nonmembership for each z_1 is determined as follows:
$f_\mu(\mu(z_1), \partial(z_1), \hbar(z_1)) = f_\mu(0.3, 0.1, 0.8) = 0.27$,
$f_\nu(\mu(z_1), \partial(z_1), \hbar(z_1)) = f_\nu(0.3, 0.1, 0.8) = 0.64$.

We apply the same procedure for z_2, z_3 and z_4 and make the q-ROFSs, with Q corresponding to fuzzy set Ł.
$Q = \{(z_1, 0.27, 0.64), (z_2, 0.73, 0.18), (z_3, 0.09, 0.82), (z_4, 0.46, 0.46)\}$

Step V. Use the construction method and construct the induced q-ROFSs corresponding to the triangular fuzzy sets $\Re_i$ to add the hesitation degree in time series then the induced q-ROFSs are defined as follows:

Induced q-Rung Orthopair fuzzy sets

q-ROFSs have both membership and non-membership functions, making them an excellent tool for dealing with indeterminacy (hesitation). Because the acceptability in the achievement of fuzzy set theory are greater than that of q-ROFSs theory, and because q-Rung orthopair fuzzy values are frequently available as data, but the implementation requirements and methodologies are fuzzification of a q-ROFSs is needed to have induced fuzzy set.

Let $\mathring{A} = \{\langle s, (\mu_{\mathring{A}}(s), \nu_{\mathring{A}}(s))\rangle \mid \forall\ s \in S\}$ be a q-ROFSs then, the induced q-ROFSs are calculated in the following way:
(1) Calculate the indeterminacy degree which can be calculated using this formula $\ell(s_i) = 1 - \mu(s_i) + \nu(s_i)$.
(2) Choose maximum of $\mu(s_i)$ and $\nu(s_i)$.
(3) Add the indeterminacy degree $\ell(s_i)$ to the maximum of $\mu(s_i)$ and $\nu(s_i)$.

Example 2. $Ł = \{(u_1, 0.3), (z_2, 0.8), (z_3, 0.1), (z_4, 0.5)\}$ *is the fuzzy set presented in Example 1.*
$Q = \{(z_1, 0.27, 0.64), (z_2, 0.73, 0.18), (z_3, 0.09, 0.82), (z_4, 0.46, 0.46)\}$ *are q-ROFSs constructed in Example 1.*

Then, the induced q-ROFSs is

$$Q^* = \{(z_1, 0.73), (z_2, 0.82), (z_3, 0.91), (z_4, 0.54)\}.$$

B. q-Rung Orthopair fuzzification and q-Rung Orthopair defuzzification

In this section we explain the idea how to fuzzify and defuzzify our data.

Step I. For q-Rung orthopair fuzzification of time series, we select the q-ROFS Q_i whose degree of membership is maximums and the q-Rung orthopair fuzzy logic relations (q-ROFLRs) are made according to the specified rule: If Q_i is the q-ROFS given for year n and Q_j is the q-ROFS for the next of year n+1, then the q-ROFLRs is denoted by $Q_i \to Q_j$. Furthermore, Q_i is the current state, and Q_j is the next state.

Divide the derived q-ROFLRs into groups based on the current state of the production of q-ROFLRs, and the first-order q-ROFSs time-invariant relation R is solved as:

$$R = \bigcup_{i=1}^{n} Ri \tag{5.1}$$

where n denotes the number of q-ROFLRs groups.
Step II. The computation of the q-ROFSs output is done using the following equation:

$$\mathfrak{R}_i^* = \mathfrak{R}_{i-1}^* \; o \; R$$

where $\mathfrak{R}_{i-1}$ is the q-Rung orthopair fuzzified for year i-1 $\mathfrak{R}_i^*$ is the q-ROFSs forecasted for year i, and o is the max-min composition operator. The forecasted outputs are q-Rung orthopair defuzzifed with the help of Mean of Maximum(MOM) defuzzification method.

C. Measurement of error by using MSE and AFE

Mean square error (MSE) and Absolute Forecasting Error (AFE) are often used as tools to evaluate the accuracy of time series forecasting. The following definitions apply to all error measures.
MSE = $\frac{\sum_{i=1}^{n}(O_i-F_i)^2}{n}$,
Forecasting percentage error = $\frac{|F_i-O_i|}{O_i} \times 100$,
Percentage AFE = $\frac{sum\ of\ forecasting\ error}{n}$.
Here F_i and O_i are the forecasted and observed data of the time series at time t, and n is the number of time series. The smaller value of the MSE and AFE, the more accurate the result of forecasting is.

6. Linear programming using MATLAB

The MATLAB code to compute forecasting of students for the admission purpose is given below as:

```
clc; clear all ; close all
w1=67 w2=21                                  Suitable +ve values
y=[2006:1:2022]'                             Given Input data
P=50                                         step size
AD=[1858 1879 1557 1499 1631 1674 1498 1267 1577 1861 1691 1489 1390 1783 1497 1298 1671]'
Given Input data
```

A. Construction of Fuzzy Sets and (q-ROFSs)
Step 1.

```
wmin=min(AD); wmax=max(AD);                  Selecting interval
W_min=wmin-w1; W_max=wmax+w2;
U=[W_min,W_max]                              Universal set
```

Step 2.

```
h=(W_max-W_min)/P                            'no of intervals=14 in this case
```

Step 3.

for i=1:h
$\mathfrak{R}_i = [1200 + (i-1)p, 1200 + ip, 1200 + (i+1)p]$ triangular (q-ROFSs)

end
==========================for z1==============================

. if i==1
. z=[1298 1267];
for calculating of all z of triangular membership function using this formula

$$\mu(z;[a,b,c]) = \begin{cases} \frac{z-a}{b-a} & ,\ \text{if } z \in [a,b] \\ \frac{c-z}{c-b} & ,\ \text{if } z \in [b,c] \\ 0 & ,\ \text{otherwise} \end{cases}$$

Similarly calculating all z_1 to z_14.
. end
Step 4.
J(h,3)=W_max ; construction of (q-ROFSs)
end
Using Construction Method
for i=1:h
. if (i <= 2)
. T(i,2*i-(2*i-1))=min ([R(i,2*i-(2*i-1)) R(i,2*i-(2*i-1)+1)]);
. TT(i,2*i-(2*i-1))=max ([R(i,2*i-(2*i-1)) R(i,2*i-(2*i-1)+1)]);
. elseif (i >= 3) && (i <= 4)
. T(i,2*i-(2*i-1))=min ([R(i,2*i-(2*i-1))]);
. TT(i,2*i-(2*i-1))=max ([R(i,2*i-(2*i-1))]);
. end
end
Similarly calculated remaining for i=4 to i=14.
for i=1:h
f1(i)=(r1(i)*(1-T_max(i)*T_min(i))); for membership value
f2(i)=(1-r1(i)*(1-T_max(i)*T_min(i))-T_max(i)*T_min(i)); for non-membership value
Similarly find all other untill to f8(i)
end
Q=[f1 f2 f3 f4 f5 f6 f7 f8]
Step 5.
for i=1:h Induced (q-ROFSs)
$s1^t(i)$=max([f1(i) f2(i)]); $s2^t(i)$=max([f3(i) f4(i)]);
$s3^t(i)$=max([f5(i) f6(i)]); $s4^t(i)$=max([f7(i) f8(i)]);
end
q1=1-(f1+f2)+s1; q2=1-(f3+f4)+s2;
q3=1-(f5+f6)+s3; q4=1-(f7+f8)+s4;
SS=[q1 q2 q3 q4]

For Complete Code Contact the Corresponding Author

7. Case studies

We consider two examples to test to our method. The first one is related to the number of admissions of students in the future, and the second example is related to flood forecasting in Guddu Barrage's maximum flow of water, which may be helpful for many kinds of problems in the future.

7.1. Implementation of the proposed method of university admission

Analyzing the number of admissions in the mathematics department every year and developing an algorithm for predicting future admissions can provide valuable insights and benefits. We now discuss this topic further:

- Enrollment Trends and Planning:
  Understanding the trends and patterns in the number of admissions to the mathematics department can help with strategic planning and resource allocation. By analyzing historical data, you can identify fluctuations, growth rates, and other factors that influence enrollment. This information can guide decisions regarding faculty hiring, classroom capacity, course offerings, and overall departmental budgeting.
- Financial Planning:
  Accurate predictions of future admissions allow the mathematics department to plan its budget and allocate financial resources effectively. It helps in determining the funding required for scholarships, research grants, infrastructure development, and other expenses. Predictive algorithms can provide insights into future revenue streams, enabling better financial planning and management.
- Faculty Recruitment and Retention:
  Forecasting future admissions helps in faculty recruitment and retention strategies. By analyzing the expected number of students, the department can plan for the required number of faculty members and their areas of expertise. It allows the department to identify potential gaps and recruit faculty with specific research interests or teaching specializations, ensuring a well-rounded academic staff.
- Identifying Challenges and Opportunities:
  Predictive algorithms can help identify challenges and opportunities related to admissions. For example, if a decline in admissions is projected, the department can proactively address potential issues such as program attractiveness, curriculum relevance, or competition from other institutions. Conversely, an increase in admissions can prompt the exploration of expansion possibilities or the introduction of new programs.

Convert the q-ROFSs into induced q-ROFSs. Induced q-ROFSs are used in fuzzy logical relations in the proposed method. The MOM technique and composition operators max-min are also used for defuzzification.

We use our approach on data from the yearly admission of student and explains the findings step-by-step for easy understanding and verification of the proposed model.

Create a fuzzy set and q-ROFS. The fuzzy set and corresponding q-ROFSs for the time series data

of admission of student are constructed via the followings steps:
Step I. The universe of discourse for admission of student in the mathematics department is defined as $U = [1200, 1900]$ by taking $W_1 = 67$ and $W_2 = 21$. $W_{\min}$ and $W_{\max}$ are observed from Table 2.

Table 2. Admission of students in the mathematics department.

Years	Admission of student	q-Rung Orthopair induced fuzzified admission
2006	1858	$\mathfrak{R}^*_{13}$
2007	1879	$\mathfrak{R}^*_{13}$
2008	1557	$\mathfrak{R}^*_{7}$
2009	1499	$\mathfrak{R}^*_{6}$
2010	1631	$\mathfrak{R}^*_{9}$
2011	1674	$\mathfrak{R}^*_{10}$
2012	1498	$\mathfrak{R}^*_{6}$
2013	1267	$\mathfrak{R}^*_{2}$
2014	1577	$\mathfrak{R}^*_{7}$
2015	1861	$\mathfrak{R}^*_{13}$
2016	1691	$\mathfrak{R}^*_{10}$
2017	1489	$\mathfrak{R}^*_{6}$
2018	1390	$\mathfrak{R}^*_{4}$
2019	1783	$\mathfrak{R}^*_{12}$
2020	1497	$\mathfrak{R}^*_{6}$
2021	1298	$\mathfrak{R}^*_{2}$
2022	1671	$\mathfrak{R}^*_{10}$

Step II. Partition the universe of discourse U into 14 intervals:
$u_i = [1200 + (i - 1)p, 1200 + ip]$ where $i = 1, 2, 3, ...14$ and p = 50.
Step III. The following 14 triangular q-ROFSs $\mathfrak{R}_i$ ($i = 1, 2, 3, ...14$) according to the interval u_i, are defined on the universe of the discourse U:
$\mathfrak{R}_i = [1200 + (i - 1)p, 1200 + ip, 1200 + (i + 1)p]$ for $i = 1, 2, 3.......13$ where p = 50.
$\mathfrak{R}_i = [1200 + (i - 1)p, 1200 + ip, 1200 + ip]$ for $i = 14$ where p = 50.
The grade of membership to fuzzy sets defined in Step III are as follows:
$\mathfrak{R}_1 = 0.66/1267 + 0.04/1298.$
$\mathfrak{R}_2 = 0.34/1267 + 0.96/1298.$
$\mathfrak{R}_3 = 0.2/1390.$
$\mathfrak{R}_4 = 0.8/1390.$
$\mathfrak{R}_5 = 0.22/1489 + 0.06/1497 + 0.04/1498 + 0.02/1499.$
$\mathfrak{R}_6 = 0.78/1489 + 0.94/1497 + 0.96/1498 + 0.98/1499.$
$\mathfrak{R}_7 = 0.86/1557 + 0.46/1577.$
$\mathfrak{R}_8 = 0.14/1557 + 0.54/1577 + 0.38/1631.$
$\mathfrak{R}_9 = 0.62/1631 + 0.58/1671 + 0.52/1674 + 0.18/1691.$
$\mathfrak{R}_{10} = 0.42/1671 + 0.48/1674 + 0.82/1691.$
$\mathfrak{R}_{11} = 0.34/1783.$
$\mathfrak{R}_{12} = 0.66/1783.$

$\mathfrak{R}_{13} = 0.84/1858 + 0.78/1861 + 0.42/1879.$
$\mathfrak{R}_{14} = 0.16/1858 + 0.22/1861 + 0.58/1879.$
Step IV. Using construction method the following q-ROFSs $Q_i(i = 1, 2, 3, ...14)$ are created in the form of fuzzy sets. $\mathfrak{R}_i(i = 1, 2, 3, ...14)$:
$Q_1 = \{(1267, 0.642, 0.331), (1298, 0.038, 0.934)\}.$
$Q_2 = \{(1267, 0.229, 0.444), (1298, 0.646, 0.026)\}.$
$Q_3 = \{(1390, 0.192, 0.768)\}.$
$Q_4 = \{(1390, 0.288, 0.072)\}.$
$Q_5 = \{(1489, 0.219, 0.776), (1497, 0.059, 0.935), (1498, 0.039, 0.955), (1499, 0.019, 0.975)\}.$
$Q_6 = \{(1489, 0.183, 0.051), (1497, 0.221, 0.014), (1498, 0.226, 0.009), (1999, 0.230, 0.004)\}.$
$Q_7 = \{(1557, 0.519, 0.084), (1577, 0.278, 0.326)\}.$
$Q_8 = \{(1557, 0.129, 0.794), (1577, 0.499, 0.425), (1631, 0.35, 0.573)\}.$
$Q_9 = \{(1631, 0.550, 0.337), (1671, 0.515, 0.373), (1674, 0.461, 0.426), (1691, 0.159, 0.728)\}.$
$Q_{10} = \{(1671, 0.275, 0.380), (1674, 0.314, 0.340), (1691, 0.537, 0.118), \}.$
$Q_{11} = \{(1783, 0.300, 0.583)\}.$
$Q_{12} = \{(1783, 0.372, 0.191)\}.$
$Q_{13} = \{(1858, 0.543, 0.103), (1861, 0.504, 0.142), (1879, 0.271, 0.375)\}.$
$Q_{14} = \{(1858, 0.145, 0.762), (1861, 0.199, 0.707), (1879, 0.526, 0.381)\}.$
Step V. The induced q-ROFSs $\mathfrak{R}_i^*$ are calculated corresponding to Q_i.
$\mathfrak{R}_1^* = 0.669/1267 + 0.962/1298.$
$\mathfrak{R}_2^* = 0.771/1267 + 0.974/1298.$
$\mathfrak{R}_3^* = 0.808/1390.$
$\mathfrak{R}_4^* = 0.928/1390.$
$\mathfrak{R}_5^* = 0.781/1489 + 0.941/1497 + 0.961/1498 + 0.981/1499.$
$\mathfrak{R}_6^* = 0.949/1489 + 0.986/1497 + 0.991/1498 + 0.996/1499.$
$\mathfrak{R}_7^* = 0.916/1557 + 0.722/1577.$
$\mathfrak{R}_8^* = 0.871/1557 + 0.575/1577 + 0.650/1631.$
$\mathfrak{R}_9^* = 0.663/1631 + 0.627/1671 + 0.574/1674 + 0.841/1691.$
$\mathfrak{R}_{10}^* = 0.725/1671 + 0.686/1674 + 0.882/1691.$
$\mathfrak{R}_{11}^* = 0.700/1783.$
$\mathfrak{R}_{12}^* = 0.809/1783$
$\mathfrak{R}_{13}^* = 0.897/1858 + 0.858/1861 + 0.729/1879.$
$\mathfrak{R}_{14}^* = 0.855/1858 + 0.801/1861 + 0.619/1879.$

q-Rung Orthopair fuzzification and q-Rung Orthopair defuzzification

We now fuzzify and defuzzify our induced q-ROFSs.

Step I. Using the proposed algorithm for q-Rung orthopair fuzzification, time series data of admission of students are q-Rung orthopair fuzzified.

The admission in the year 2007 is 1879, which is present in the fuzzy set $\mathfrak{R}_{13}^*$ and $\mathfrak{R}_{14}^*$. Therefore, the following two q-ROFSs are constructed for the annual peak discharge in 2007.

$$\mathfrak{R}_{13}^* = (1879, 0.729),$$

$$\mathfrak{R}^*_{14} = (1879, 0.619).$$

The membership degree for the admission of 1879 is maximal in $\mathfrak{R}^*_{13}$. So, the q-Rung orthopair fuzzified value for the admission of 1879 is $\mathfrak{R}^*_{13}$.

Tables 3 and 4 show q-ROFLR and its group for the annual admission of students.

Table 3. q-Rung orthopair fuzzy logic relations.

$\mathfrak{R}^*_{13} \to \mathfrak{R}^*_{13}$	$\mathfrak{R}^*_{13} \to \mathfrak{R}^*_{7}$	$\mathfrak{R}^*_{7} \to \mathfrak{R}^*_{6}$	$\mathfrak{R}^*_{6} \to \mathfrak{R}^*_{9}$
$\mathfrak{R}^*_{9} \to \mathfrak{R}^*_{10}$	$\mathfrak{R}^*_{10} \to \mathfrak{R}^*_{6}$	$\mathfrak{R}^*_{6} \to \mathfrak{R}^*_{2}$	$\mathfrak{R}^*_{2} \to \mathfrak{R}^*_{7}$
$\mathfrak{R}^*_{7} \to \mathfrak{R}^*_{13}$	$\mathfrak{R}^*_{13} \to \mathfrak{R}^*_{10}$	$\mathfrak{R}^*_{10} \to \mathfrak{R}^*_{6}$	$\mathfrak{R}^*_{6} \to \mathfrak{R}^*_{4}$
$\mathfrak{R}^*_{4} \to \mathfrak{R}^*_{12}$	$\mathfrak{R}^*_{12} \to \mathfrak{R}^*_{6}$	$\mathfrak{R}^*_{6} \to \mathfrak{R}^*_{2}$	$\mathfrak{R}^*_{2} \to \mathfrak{R}^*_{10}$

Table 4. q-Rung orthopair fuzzy logic relationship groups.

$\mathfrak{R}^*_{2} \to \mathfrak{R}^*_{7}$	$\mathfrak{R}^*_{2} \to \mathfrak{R}^*_{10}$		
$\mathfrak{R}^*_{4} \to \mathfrak{R}^*_{12}$			
$\mathfrak{R}^*_{6} \to \mathfrak{R}^*_{2}$	$\mathfrak{R}^*_{6} \to \mathfrak{R}^*_{9}$	$\mathfrak{R}^*_{6} \to \mathfrak{R}^*_{4}$	$\mathfrak{R}^*_{6} \to \mathfrak{R}^*_{2}$
$\mathfrak{R}^*_{7} \to \mathfrak{R}^*_{6}$	$\mathfrak{R}^*_{7} \to \mathfrak{R}^*_{13}$		
$\mathfrak{R}^*_{9} \to \mathfrak{R}^*_{10}$			
$\mathfrak{R}^*_{10} \to \mathfrak{R}^*_{6}$	$\mathfrak{R}^*_{10} \to \mathfrak{R}^*_{6}$		
$\mathfrak{R}^*_{12} \to \mathfrak{R}^*_{6}$			
$\mathfrak{R}^*_{13} \to \mathfrak{R}^*_{13}$	$\mathfrak{R}^*_{13} \to \mathfrak{R}^*_{10}$	$\mathfrak{R}^*_{13} \to \mathfrak{R}^*_{7}$	

Using Table 4 we find all q-ROFLR groups which can be computed in this way:

$R_1 = \mathfrak{R}^{*t}_2 \times \mathfrak{R}^*_7$, $R_2 = \mathfrak{R}^{*t}_2 \times \mathfrak{R}^*_{10}$, $R_3 = \mathfrak{R}^{*t}_4 \times \mathfrak{R}^*_{12}$, and so on.

Similarly we find all the other. The union of q-ROFLRs is $R = \bigcup_{i=1}^{16} R_i$ which can give the matrix.

Constructed $\mathfrak{R}^*_1$ select those values of x in $\mathfrak{R}^*_1$ whose induced q-ROFSs membership is maximum then same choose x in others $\mathfrak{R}^*_i$. If the value x is not present in any $\mathfrak{R}^*_i$ then skip this place and write zero.

$\mathfrak{R}^*_1 = [0.962 \quad 0.974 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$,
$\mathfrak{R}^*_2 = [0.962 \quad 0.974 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$,
$\mathfrak{R}^*_3 = [0 \quad 0 \quad 0.808 \quad 0.928 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$,
$\mathfrak{R}^*_4 = [0 \quad 0 \quad 0.808 \quad 0.928 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$,
$\mathfrak{R}^*_5 = [0 \quad 0 \quad 0 \quad 0 \quad 0.981 \quad 0.996 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$,

$\mathfrak{R}_6^* = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0.981 \;\; 0.996 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0],$
$\mathfrak{R}_7^* = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0.916 \;\; 0.871 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0],$
$\mathfrak{R}_8^* = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0.916 \;\; 0.871 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0],$
$\mathfrak{R}_9^* = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0.841 \;\; 0.882 \;\; 0 \;\; 0 \;\; 0 \;\; 0],$
$\mathfrak{R}_{10}^* = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0.841 \;\; 0.882 \;\; 0 \;\; 0 \;\; 0 \;\; 0],$
$\mathfrak{R}_{11}^* = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0.700 \;\; 0.809 \;\; 0 \;\; 0],$
$\mathfrak{R}_{12}^* = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0.700 \;\; 0.809 \;\; 0 \;\; 0],$
$\mathfrak{R}_{13}^* = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0.897 \;\; 0.855],$
$\mathfrak{R}_{14}^* = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0.897 \;\; 0.855].$

Use the above $\mathfrak{R}_1^*$ to $\mathfrak{R}_{14}^*$ for calculating the Eq (5.1).

$$R = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0.916 & 0.871 & 0.841 & 0.882 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.916 & 0.871 & 0.841 & 0.882 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.700 & 0.8080 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.700 & 0.8081 & 0 & 0 \\
0.962 & 0.974 & 0.808 & 0.928 & 0 & 0 & 0 & 0 & 0.841 & 0.882 & 0 & 0 & 0 & 0 \\
0.962 & 0.974 & 0.808 & 0.928 & 0 & 0 & 0 & 0 & 0.841 & 0.882 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.916 & 0.916 & 0 & 0 & 0 & 0 & 0 & 0 & 0.897 & 0.855 \\
0 & 0 & 0 & 0 & 0.871 & 0.871 & 0 & 0 & & & 0 & 0 & 0.871 & 0.855 \\
0 & 0 & 0 & 0 & 0.841 & 0.841 & 0 & 0 & 0.841 & 0.841 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.882 & 0.882 & 0 & 0 & 0.841 & 0.882 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.700 & 0.700 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.809 & 0.809 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.897 & 0.871 & 0.841 & 0.882 & 0 & 0 & 0.897 & 0.855 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.855 & 0.855 & 0.841 & 0.855 & 0 & 0 & 0.855 & 0.855
\end{bmatrix}.$$

Step II. Using the max-min composition operator on the q-ROFLR group is applied to compute the q-Rung orthopair fuzzified admission, which can be defuzzified by using the MOM defuzzification method. Forecasted admissions of students obtained in the proposed method are in Table 6. We take the forecasted admission of the year 2007 is as:

Then q-ROFLR groups for the admission of students in 2007 is

$$\mathfrak{R}^*_{2007} = \mathfrak{R}_{2006}\, o\, R = \mathfrak{R}_{13}\, o\, R = \left|\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.897 & 0.871 & 0.841 & 0.882 & 0 & 0 & 0.897 & 0.855 \end{matrix}\right|,$$

q-ROFLR group for admission of student of other years are also computed in this way.

Table 5. q-Rung orthopair fuzzified prediction of students.

Year	Admission	$\mathfrak{R}^*_1$	$\mathfrak{R}^*_2$	$\mathfrak{R}^*_3$	$\mathfrak{R}^*_4$	$\mathfrak{R}^*_5$	$\mathfrak{R}^*_6$	$\mathfrak{R}^*_7$	$\mathfrak{R}^*_8$	$\mathfrak{R}^*_9$	$\mathfrak{R}^*_{10}$	$\mathfrak{R}^*_{11}$	$\mathfrak{R}^*_{12}$	$\mathfrak{R}^*_{13}$	$\mathfrak{R}^*_{14}$
2006	1858	–	–	–	–	–	–	–	–	–	–	–	–	–	–
2007	1879	0	0	0	0	0	0	0.897	0.871	0.841	0.882	0	0	0.897	0.855
2008	1557	0	0	0	0	0	0	0.897	0.871	0.841	0.882	0	0	0.897	0.855
2009	1499	0	0	0	0	0.916	0.916	0	0	0	0	0	0	0.897	0.855
2010	1631	0.962	0.974	0.808	0.928	0	0	0	0	0.841	0.882	0	0	0	0
2011	1674	0	0	0	0	0.882	0.882	0	0	0.841	0.882	0	0	0	0
2012	1498	0	0	0	0	0.882	0.882	0	0	0.841	0.882	0	0	0	0
2013	1267	0.962	0.974	0.808	0.928	0	0	0	0	0.841	0.882	0	0	0	0
2014	1577	0	0	0	0	0	0	0.916	0.871	0.841	0.882	0	0	0	0
2015	1861	0	0	0	0	0.916	0.916	0	0	0	0	0	0	0.897	0.855
2016	1691	0	0	0	0	0	0	0.897	0.871	0.841	0.882	0	0	0.897	0.855
2017	1489	0	0	0	0	0.882	0.882	0	0	0.841	0.882	0	0	0	0
2018	1390	0.962	0.974	0.808	0.928	0	0	0	0	0.841	0.882	0	0	0	0
2019	1783	0	0	0	0	0	0	0	0	0	0	0.700	0.808	0	0
2020	1497	0	0	0	0	0.809	0.809	0	0	0	0	0	0	0	0
2021	1298	0.962	0.974	0.808	0.928	0	0	0	0	0.841	0.882	0	0	0	0
2022	1671	0	0	0	0	0	0	0.916	0.871	0.841	0.882	0	0	0	0

The forecasted value of the proposed method and admission of students are given using the MOM defuzzification method.

Table 6. Forecasted value of the admission of students.

Year	Admission of students	Proposed method
2006	1858	–
2007	1879	1850
2008	1557	1850
2009	1499	1550
2010	1631	1300
2011	1674	1700
2012	1498	1700
2013	1267	1300
2014	1577	1550
2015	1861	1550
2016	1691	1850
2017	1489	1700
2018	1390	1300
2019	1783	1800
2020	1497	1800
2021	1298	1300
2022	1671	1550

Figure 3 shows the actual and observed value of the admission of the students.

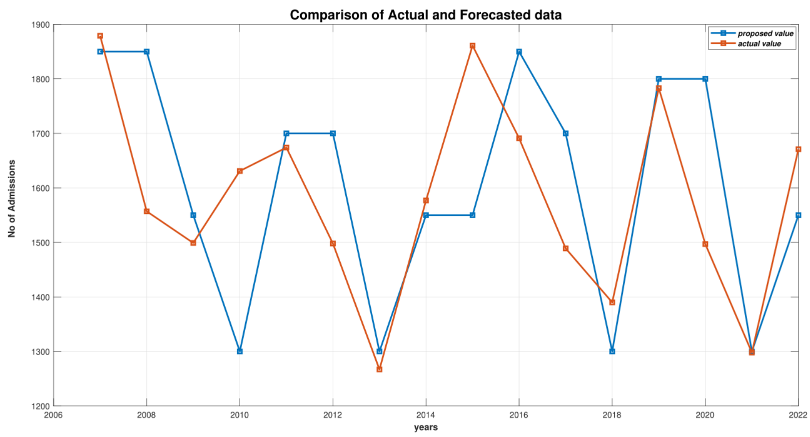

Figure 3. Graph of the admission of the students.

7.2. Implementation of the proposed method annual peak discharge at Guddu Barrage

The flood flow in Guddu Barrage, located in Pakistan, is a significant annual event that affects the region and its inhabitants. Predicting the future flow of water can be of immense value in terms

of disaster preparedness, response planning, and mitigation efforts. Developing an algorithm for predicting the flow of water can assist in managing and minimizing the impact of floods.
To discuss the example further, we explore the following points:

- Importance of predicting flood flow:
  Predicting the flow of water during floods is crucial for several reasons. It enables authorities and communities to prepare for potential disasters, evacuate vulnerable areas, and allocate resources effectively. Predictions can help in determining the required capacity of temporary shelters, emergency supplies, and medical aid. They can also aid in developing strategies for flood control and prevention.
- Factors affecting flood flow:
  Several factors contribute to the flow of floodwaters in Guddu Barrage and its surrounding areas. These include rainfall patterns, upstream water levels, snowmelt, topography, soil saturation levels, and the condition of reservoirs and dams. By analyzing historical data and monitoring these factors, it becomes possible to establish patterns and relationships that can inform predictions.
- Algorithm development for prediction:
  Developing an algorithm to predict the flow of water involves a combination of data analysis techniques, mathematical modeling, and machine learning. Historical flood data, including water levels and flow rates, is used to train the algorithm. The algorithm then learns the relationships between various factors and develops a predictive model. Continuous feedback and updating of the algorithm based on real-time data improve its accuracy over time.
- Benefits of predictive algorithms:
  Implementing an algorithm for future prediction of flood flow can bring several benefits. It enables authorities to issue early warnings, which can save lives and minimize property damage. Emergency response teams can be mobilized more effectively, and resources can be allocated efficiently based on projected flood severity. Additionally, infrastructure planning can be improved by considering potential flood scenarios, leading to better-designed structures and flood control measures.

Forecasting the annual peak of Guddu Barrage with the proposed method

Create a fuzzy set and q-ROFSs. The fuzzy set and corresponding q-ROFSs for the time series data of annual peak discharges of Guddu Barrage every year are constructed in the followings steps:
Step I. The universe of discourse for annual peak discharges of Guddu Barrage in Pakistan is defined as
$U = [2000, 26000]$ by taking $W_1 = 349$ and $W_2 = 556$. $W_{\min}$ and $W_{\max}$ are observed from Table 7.
Step II. Partition the universe of discourse U into the 12 intervals:
$u_i = [2000 + (i-1)p, 2000 + ip]$ where $i = 1, 2, 3, ...12$ and p = 2000.
Step III. The following 12 triangular q-ROFSs $\mathfrak{R}_i$ $(i = 1, 2, 3, ...12)$ according to the interval u_i, are defined on the universe of the discourse U:
$\mathfrak{R}_i = [2000 + (i-1)p, 2000 + ip, 2000 + (i+1)p]$ for $i = 1, 2, 3.......11$ where p = 2000,
$\mathfrak{R}_i = [2000 + (i-1)p, 2000 + ip, 2000 + ip]$ for $i = 12$ where p = 2000.

Table 7. Annual peak discharge of Guddu Barrage.

Years	Annual peak (m^3s)	q-Rung Orthopair induced fuzzified annual peak
1996	16954	$\Re_8^*$
1997	16118	$\Re_8^*$
1998	14130	$\Re_6^*$
1999	7438	$\Re_2^*$
2000	3204	$\Re_1^*$
2001	4408	$\Re_2^*$
2002	4999	$\Re_2^*$
2003	7479	$\Re_2^*$
2004	2349	$\Re_1^*$
2005	10740	$\Re_4^*$
2006	12928	$\Re_6^*$
2007	5426	$\Re_2^*$
2008	6199	$\Re_3^*$
2009	3838	$\Re_1^*$
2010	25444	$\Re_{12}^*$
2011	3803	$\Re_1^*$
2012	3512	$\Re_1^*$

The grade of membership of fuzzy sets are defined in Step III which as follows:
$\Re_1 = 0.17/2349{+}0.60/3204{+}0.75/3512{+}0.90/3803{+}0.91/3838{+}0.79/4408{+}0.50/4999{+}0.28/5426$,
$\Re_2 = 0.20/4408 + 0.49/4999 + 0.71/5426 + 0.90/6199 + 0.28/7438 + 0.26/7479$,
$\Re_3 = 0.09/6199 + 0.71/7438 + 0.73/7479$,
$\Re_4 = 0.63/10740$,
$\Re_5 = 0.37/10740 + 0.53/12928$,
$\Re_6 = 0.46/12928 + 0.93/14130$,
$\Re_7 = 0.06/14130 + 0.94/16118 + 0.52/16954$,
$\Re_8 = 0.05/16118 + 0.47/16954$,
$\Re_9 = \text{Null}$,
$\Re_{10} = \text{Null}$,
$\Re_{11} = 0.27/25444$,
$\Re_{12} = 0.72/25444$.

Step IV. Using construction method the following q-ROFSs $Q_i(i = 1, 2, 3, ...12)$ are constructed corresponding to the fuzzy sets $\Re_i(i = 1, 2, 3, ...12)$:
$Q_1 = \{(2349, 0.14, 0.70), (3204, 0.50, 0.33), (3512, 0.63, 0.21), (3803, 0.760, 0.08), (3838, 0.769, 0.07), (4408, 0.66, 0.17), (4999, 0.42, 0.42), (5426, 0.23, 0.60)\}$,
$Q_2 = \{(4408, 0.16, 0.65), (4999, 0.40, 0.41), (5426, 0.58, 0.23), (6199, 0.73, 0.08), (7438, 0.22, 0.59), (7479, 0.21, 0.60)\}$,
$Q_3 = \{(6199, 0.08, 0.85), (7438, 0.66, 0.27), (7479, 0.68, 0.25)\}$,
$Q_4 = \{(10740, 0.37, 0.22)\}$,
$Q_5 = \{(10740, 0.29, 0.50), (12928, 0.42, 0.37)\}$,

Q_6 = {(12928, 0.26, 0.30), (14130, 0.53, 0.04)},
Q_7 = {(14130, 0.05, 0.88), (16118, 0.88, 0.05), (16954, 0.49, 0.45)}
Q_8 = {(16118, 0.04, 0.92), (16954, 0.45, 0.51)},
Q_9 =Null,
Q_{10} =Null,
Q_{11} = {(25444, 0.25, 0.67)},
Q_{12} = {(25444, 0.34, 0.13)}.
Step V. The induced q-ROFSs $\mathfrak{R}_i^*$ are calculated corresponding to Q_i
$\mathfrak{R}_1^*$ = 0.86/2349+0.67/3204+0.79/3512+0.92/3803+0.93/3838+0.83/4408+0.58/4999+0.77/5426,
$\mathfrak{R}_2^*$ = 0.84/4408 + 0.60/4999 + 0.77/5426 + 0.92/6199 + 0.78/7438 + 0.79/7479,
$\mathfrak{R}_3^*$ = 0.92/6199 + 0.73/7438 + 0.75/7479,
$\mathfrak{R}_4^*$ = 0.78/10740,
$\mathfrak{R}_5^*$ = 0.71/10740 + 0.63/12928,
$\mathfrak{R}_6^*$ = 0.74/12928 + 0.96/14130,
$\mathfrak{R}_7^*$ = 0.95/14130 + 0.95/16118 + 0.55/16954,
$\mathfrak{R}_8^*$ = 0.96/16118 + 0.55/16954,
$\mathfrak{R}_9^*$ =Null,
$\mathfrak{R}_{10}^*$ =Null,
$\mathfrak{R}_{11}^*$ = 0.75/25444,
$\mathfrak{R}_{12}^*$ = 0.87/25444.

q-Rung Orthopair fuzzification and q-Rung Orthopair defuzzification

We now fuzzify and defuzzify of our induced q-ROFSs.
Step I. Using the proposed algorithm for q-Rung orthopair fuzzification, the time series data of annual peak discharges are q-Rung orthopair fuzzified.

The peak discharges for the year 1997 is 16, 118 which belongs to the fuzzy sets $\mathfrak{R}_7^*$ and $\mathfrak{R}_8^*$. So, the following two q-ROFSs are constructed for the peak discharges in 1997.

$$\mathfrak{R}_7^* = (16118, 0.95),$$
$$\mathfrak{R}_8^* = (16118, 0.96).$$

The degree of membership for the annual peak discharges 16,118 is maximal in $\mathfrak{R}_8^*$. So, the q-Rung orthopair fuzzified value for the annual peak discharges 16,118 is $\mathfrak{R}_8^*$.

Tables 8 and 9 show the q-ROFLR and its group for the annual peak discharge of Guddu Barrage.

Table 8. q-Rung orthopair fuzzy logic relations.

$\mathfrak{R}_8^* \to \mathfrak{R}_8^*$	$\mathfrak{R}_8^* \to \mathfrak{R}_6^*$	$\mathfrak{R}_6^* \to \mathfrak{R}_2^*$	$\mathfrak{R}_2^* \to \mathfrak{R}_1^*$
$\mathfrak{R}_1^* \to \mathfrak{R}_2^*$	$\mathfrak{R}_2^* \to \mathfrak{R}_2^*$	$\mathfrak{R}_2^* \to \mathfrak{R}_2^*$	$\mathfrak{R}_2^* \to \mathfrak{R}_1^*$
$\mathfrak{R}_1^* \to \mathfrak{R}_4^*$	$\mathfrak{R}_4^* \to \mathfrak{R}_6^*$	$\mathfrak{R}_6^* \to \mathfrak{R}_2^*$	$\mathfrak{R}_2^* \to \mathfrak{R}_3^*$
$\mathfrak{R}_3^* \to \mathfrak{R}_1^*$	$\mathfrak{R}_1^* \to \mathfrak{R}_{12}^*$	$\mathfrak{R}_{12}^* \to \mathfrak{R}_1^*$	$\mathfrak{R}_1^* \to \mathfrak{R}_1^*$

Table 9. q-Rung orthopair fuzzy logic relationship groups.

$\mathfrak{R}_1^* \to \mathfrak{R}_4^*$	$\mathfrak{R}_1^* \to \mathfrak{R}_2^*$	$\mathfrak{R}_1^* \to \mathfrak{R}_{11}^*$	$\mathfrak{R}_1^* \to \mathfrak{R}_1^*$	
$\mathfrak{R}_2^* \to \mathfrak{R}_2^*$	$\mathfrak{R}_2^* \to \mathfrak{R}_2^*$	$\mathfrak{R}_2^* \to \mathfrak{R}_1^*$	$\mathfrak{R}_2^* \to \mathfrak{R}_3^*$	$\mathfrak{R}_2^* \to \mathfrak{R}_1^*$
$\mathfrak{R}_3^* \to \mathfrak{R}_1^*$				
$\mathfrak{R}_4^* \to \mathfrak{R}_6^*$				
$\mathfrak{R}_6^* \to \mathfrak{R}_2^*$	$\mathfrak{R}_6^* \to \mathfrak{R}_2^*$			
$\mathfrak{R}_8^* \to \mathfrak{R}_8^*$	$\mathfrak{R}_8^* \to \mathfrak{R}_6^*$			
$\mathfrak{R}_{12}^* \to \mathfrak{R}_1^*$				

Using Table 9 we find all q-ROFLR groups which can be computed in this way:

$R_1 = \mathfrak{R}_1^{*t} \times \mathfrak{R}_4^*$, $R_2 = \mathfrak{R}_1^{*t} \times \mathfrak{R}_2^*$ and so on. Similarly we find all the others. The union of q-ROFLRs is $R = \bigcup_{i=1}^{16} R_i$ which can give in the following matrix.

$$R = \begin{bmatrix} 0.93 & 0.92 & 0.92 & 0.78 & 0.72 & 0 & 0 & 0 & 0 & 0 & 0.75 & 0.87 \\ 0.92 & 0.92 & 0.92 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.92 & 0.92 & 0.92 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.78 & 0.78 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.71 & 0.71 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.92 & 0.92 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.92 & 0.92 & 0 & 0 & 0.95 & 0.95 & 0.95 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.96 & 0.95 & 0.96 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.75 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.87 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Similarly annual peaks for other years are forecasted in this ways and are given Table 11. Forecasted annual peak of the proposed method and the actual annual peak are given using the MOM defuzzification method. The graph of the annual peak Guddu Barrage is shown in Figure 4, which shows the difference between the actual value and forecasted value.

Table 10. q-Rung orthopair fuzzified forecast of annual peak of Guddu Barrage.

Year	Annual peak (m^3s)	$\mathfrak{R}_1^*$	$\mathfrak{R}_2^*$	$\mathfrak{R}_3^*$	$\mathfrak{R}_4^*$	$\mathfrak{R}_5^*$	$\mathfrak{R}_6^*$	$\mathfrak{R}_7^*$	$\mathfrak{R}_8^*$	$\mathfrak{R}_9^*$	$\mathfrak{R}_{10}^*$	$\mathfrak{R}_{11}^*$	$\mathfrak{R}_{12}^*$
1996	16954	–	–	–	–	–	–	–	–	–	–	–	–
1997	16118	0	0.92	0.92	0	0	0.96	0.95	0.96	0	0	0	0
1998	14130	0	0.92	0.92	0	0	0.96	0.95	0.96	0	0	0	0
1999	7438	0	0.92	0.92	0	0	0.95	0.95	0.95	0	0	0	0
2000	3204	0.92	0.92	0.92	0	0	0	0	0	0	0	0	0
2001	4408	0.93	0.92	0.92	0.78	0.72	0	0	0	0	0	0.75	0.87
2002	4999	0.92	0.92	0.92	0	0	0	0	0	0	0	0	0
2003	7479	0.92	0.92	0.92	0	0	0	0	0	0	0	0	0
2004	2349	0.92	0.92	0.92	0	0	0	0	0	0	0	0	0
2005	10740	0.93	0.92	0.92	0.78	0.72	0	0	0	0	0	0.75	0.87
2006	12928	0	0	0	0	0	0.78	0.78	0	0	0	0	0
2007	5426	0	0.92	0.92	0	0	0.95	0.95	0.95	0	0	0	0
2008	6199	0.92	0.92	0.92	0	0	0	0	0	0	0	0	0
2009	3838	0.92	0.92	0.92	0	0	0	0	0	0	0	0	0
2010	25444	0.93	0.92	0.92	0.78	0.72	0	0	0	0	0	0.75	0.87
2011	3803	0.87	0	0	0	0	0	0	0	0	0	0	0
2012	3512	0.93	0.92	0.92	0.78	0.72	0	0	0	0	0	0.75	0.87

Table 11. Forecasted value of annual peak.

Year	Annual peak ($m^3 s$)	Proposed method
1996	16954	–
1997	16118	16000
1998	14130	16000
1999	7438	16000
2000	3204	6000
2001	4408	4000
2002	4999	6000
2003	7479	6000
2004	2349	6000
2005	10740	4000
2006	12928	3500
2007	5426	16000
2008	6199	6000
2009	3838	6000
2010	25444	4000
2011	3803	26000
2012	3512	4000

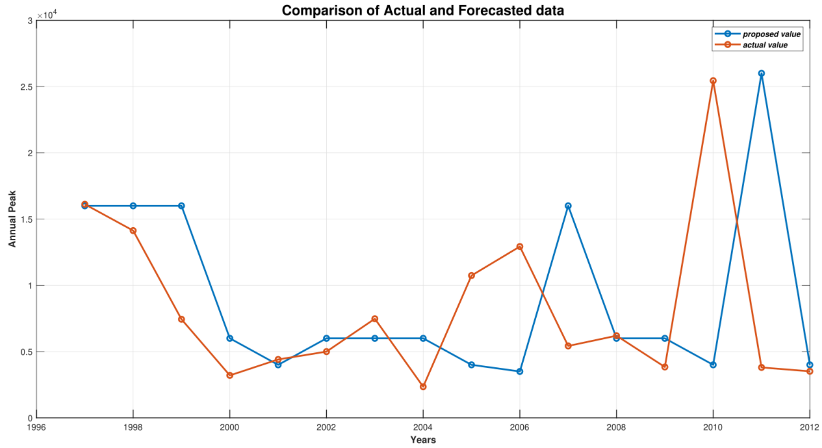

Figure 4. Graph of the annual peak of Guddu Barrage.

Measurement of error by using MSE and AFE

The MSE and AFE values are measured using the formula given in Section 5 Part C.

Table 12. MSE and AFE for proposed method.

Error	Admission	Guddu Barrage
MSE	32719.8	81559386.56
AFE	8.67259	93.29833446

8. Comparison analysis

First, I collected the information on each year's student admissions. The interval is between 1200 and 1900, and, after creating 14 subintervals using the proposed method, we observe that the inaccuracy is reasonable. Then, we take the data for the Guddu Barrage's maximum flow peak each year, whose interval falls between 2000 and 26000, and create a subinterval of 12 from it. It makes a great deal of mistakes. We get the conclusion that, if the interval's length is too large, then large amount of error occurs. We can create more subintervals since a smaller quantity of errors is acceptable. The number of subinterval is inversely proportional to error and therefore if the number of subintervals is small, there will be a lot of error. Additionally, if the actual data is poorly organised, such as with the Guddu Barrage, whose yearly peak is surprisingly low or high, there are many chances that an error may occur, and the graphs of the Guddu Barrage and admission of the student can also show the difference between the actual value and the forecasted value. Forecasting error results from authors on different algorithms applied to different fuzzy set definitions are given in Table 13.

Table 13. MSE and AFE from University of Albama's admissions.

Error	Cheng et al. [18]	Yolcu et al. [39]	Joshi and Kumar [32]
MSE	192086.6	648301.5	175559.6
AFE	2.0872	4.2886	2.06926

9. Discussion

These techniques are helpful such as the annual water flow in the Guddu Barrage and student admission. We can be ready for any type of disaster if we know what is expected such as student enrollment and water flow for the upcoming school year. These results are helpful for future predictions; similarly, more problems are also solved in this way, like stock change index problems, the number of patients with any kind of disease, etc. First, we convert raw data into a fuzzy set; after this, we convert it into q-ROFSs, and then the proposed method is applied to predict the future result.

10. Conclusions

Due to the fact that q-ROFS membership and non-membership sums are always smaller than one, they are becoming incredibly popular. Evidently, using IFSs cannot resolve this issue; thus, if the total of membership and non-membership exceeds 1, we use q-ROFSs to replace IFSs. In order to integrate indeterminacy in time series forecasting, we explain the concept of q-ROFTSs and offer a forecasting model based on it. The proposed method used a straightforward composition operator max-min for q-ROFSs, making it less complicated and more straightforward. The degree of nondeterminacy in q-ROFSs is taken and combined with the degree of membership in the suggested way to create the induced q-ROFSs. In order to verify the performance of the proposed method, was used on the admission of students. Following this proposed method, we may compute the forecasted value for the next years. If we want to know how many students will arrive in 2023, we use the same procedure and relate it to the year 2022 because of its previous initial value. In addition, we offered a MATLAB code that can complete our computation quicker than Microsoft Excel. We take another example in which data is not organized well according to our algorithm. After this, we give the idea that which type of data is most suitable for q-ROFTSs forecasting value. This method is not applicable when data is not converted into fuzzy set and q-ROFSs.

- In the future, researchers can impliment this idea for those data which cannot be handled, like the Guddu Barrage annual peak per year with the help of q-ROFTSs.
- The benefit of this article is that it may be used to forecast future difficulties. People already understand the concept; they have planned for the things that will be required in the future, such as investing in the stock exchange index market.
- They are also useful in a variety of enterprises.
- They can implement this idea in another algorithm in which the chances of error are less.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no conflicts of interest to report regarding the present study.

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