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Research article

Efficient approximate analytical technique to solve nonlinear coupled Jaulent–Miodek system within a time-fractional order

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Abstract: In this article, we considered the nonlinear time-fractional Jaulent–Miodek model (FJMM), which is applied to modeling many applications in basic sciences and engineering, especially physical phenomena such as plasma physics, fluid dynamics, electromagnetic waves in nonlinear media, and many other applications. The Caputo fractional derivative (CFD) was applied to express the fractional operator in the mathematical formalism of the FJMM. We implemented the modified generalized Mittag-Leffler method (MGMLFM) to show the analytical approximate solution of FJMM, which is represented by a set of coupled nonlinear fractional partial differential equations (FPDEs) with suitable initial conditions. The suggested method produced convergent series solutions with easily computable components. To demonstrate the accuracy and efficiency of the MGMLFM, a comparison was made between the solutions obtained by MGMLFM and the known exact solutions in some tables. Also, the absolute error was compared with the absolute error provided by some of the other famous methods found in the literature. Our findings confirmed that the presented method is easy, simple, reliable, competitive, and did not require complex calculations. Thus, it can be extensively applied to solve more linear and nonlinear FPDEs that have applications in various areas such as mathematics, engineering, and physics.

Keywords: nonlinear coupled Jaulent–Miodek equation; fractional partial differential equations; Mittag-Leffler function; approximate solutions; nonlinear problems
Mathematics Subject Classification: 33E12, 35R11, 74H10

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1. Introduction

The Jaulent-Miodek model comprises coupled nonlinear partial differential equations (PDEs) that have many practical applications in engineering and science, especially physical phenomena. This model was proposed in 1988 by Jollint and Miodek [1] to theoretically represent a Josephson junction, a nonlinear superconducting device. After that, these equations have been studied and developed by many researchers to widely represent many physical applications, such as condensed matter physics, dynamics of semiconductor devices, plasma physics, diffusion of electromagnetic waves in nonlinear media, and the behavior of Bose-Einstein condensates (see e.g., [2–5]).

Fractional calculus (FC) is an arbitrary order of integrals and derivatives that are considered a generalization of integer calculus. Moreover, the differential equations are ordinary or partial that involve fractional order operators called fractional differential equations (FDEs). Recently, models containing the FC have attracted many researchers, which have prompted them to explain a few attractive several engineering and natural science problems in the form of fractional-order models (FOMs). These FOMs have many characteristics that distinguish them from classical models, as they have nonlocal operators, anomalous diffusion, nonlinearities, long-range interactions, and display memory effects (see e.g., [6–13]).

Through research in previous studies, we found many numerical and analytical methods for solving linear and non-linear PDEs. However, there is no unique method suitable for giving the best solutions for all mathematical models described by these PDEs where some methods work well with certain problems, but others do not work well with those problems. Each method has its advantages and shortcomings, which is due to the type of problem, surrounding circumstances, and the researcher's experience. We mention here some methods used to solve PDEs such as residual power series method [14–16], sine-Gordon expansion technique [17, 18], q-homotopy analysis transform method [19], finite difference method [20], Homotopy perturbation method [21], Elzaki transform decomposition method [22], Adams–Bashforth–Moulton method [23], Fourier spectral method [24], and many more approaches [25–30].

Our motivation of this article is to implement a new analytical-approximate method (i.e., MGMLFM) to obtain general solutions for nonlinear FPDEs and solve the following time-FJMM [31].

$$
\begin{aligned}
&{}^{C}_{0}D^{\alpha}_{t}U + \frac{\partial^3 U}{\partial x^3} + \frac{3}{2}V\frac{\partial^3 V}{\partial x^3} + \frac{9}{2}\frac{\partial V}{\partial x}\frac{\partial^2 V}{\partial x^2} - 6U\frac{\partial U}{\partial x} - 6UV\frac{\partial V}{\partial x} - \frac{3}{2}V^2\frac{\partial U}{\partial x} = 0,\\
&{}^{C}_{0}D^{\alpha}_{t}V + \frac{\partial^3 V}{\partial x^3} - 6V\frac{\partial U}{\partial x} - 6U\frac{\partial V}{\partial x} - \frac{15}{2}V^2\frac{\partial V}{\partial x} = 0,
\end{aligned}
\tag{1.1}
$$

with initial conditions (ICs) [31,32]

$$
\begin{aligned}
&U(x,0) = U_0 = \frac{1}{8}\lambda^2\left(1 - 4sech^2\left(\frac{\lambda x}{2}\right)\right),\\
&V(x,0) = V_0 = \lambda sech\left(\frac{\lambda x}{2}\right),
\end{aligned}
\tag{1.2}
$$

where U and V are anonymous functions dependent on the variables of space x and time t, ${}^{C}_{0}D^{\alpha}_{t}$ is CFD, $0 < \alpha \leq 1$ and λ is an arbitrary constant.

Our contribution is to exhibit the convenient analytical approximate solution of nonlinear FJMM that characterizes the behavior of several phenomena, onset from electrical circuits to biological processes, using a promising analytical method called MGMLFM. Furthermore, we offer a comparison between obtained solutions with familiar exact solutions and solutions gained by other methods in the literature, as well as evaluate the absolute error to prove the efficiency and eligibility of MGMLFM. Through this research and the presented results, we found several advantages of the proposed method that distinguish it from other conventional methods, including that it is easy, has simple computations, and does not require excessive effort. Also, the obtained solutions from this method are completely consistent with the exact solutions, and the value of absolute error is much lower compared to the methods available in the literature that solved this model under the same conditions.

This work is organized as follows. Section 2 supplies some basic concepts of FC that support this work. Section 3 illustrated the fundamental algorithm of the MGMLFM to solve the general nonlinear FPDEs. In Section 4, we implemented MGMLFM to determine the analytical approximate solutions for the FJMM. The numerical simulation of obtained results is investigated in Section 5 through some 2D and 3D plots and a comparison with the known exact solution and other different methods is presented in some tables to confirm the validation of our method. Section 6 shows the conclusion and discussion.

2. Preliminaries

We introduce in this part some essential concepts, properties and useful definitions of FC to advance this research [33–35].

Definition 2.1. *The fractional integral of order $\alpha > 0$ for function $\Phi(x,t)$ in $t \in [0,T]$ based on the Riemann-Liouville sense is specified by*

$$\begin{aligned} {}_0I_t^{\alpha}\Phi(x,t) &= \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}\Phi(x,\tau)d\tau, \quad t>0, \\ {}_0I_t^{0}\Phi(x,t) &= \Phi(x,t). \end{aligned}$$

Definition 2.2. *Let $\Phi(x,t)$ be an absolutely continuous function, then the CFD of order $n-1<\alpha\leq n\in\mathbb{N}$, $t\in[0,T]$ is given by*

$${}_0^C D_t^{\alpha}\Phi(x,t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1}\frac{\partial^n\Phi(x,\tau)}{\partial\tau^n}d\tau, \quad t>0,$$

when $0<\alpha<1$, *then we have*

$${}_0^C D_t^{\alpha}\Phi(x,t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t (t-\tau)^{-\alpha}\frac{\partial\Phi(x,\tau)}{\partial\tau}d\tau, \quad t>0.$$

Theorem 2.1. *let $\alpha\in(n-1,n]$, $t\in[0,T]$ and $\theta>-1$. Then, for a differentiable function $\Phi(x,t)$ we get*

$$\begin{aligned} {}_0^C D_t^{\alpha}\, {}_0I_t^{\alpha}\Phi(x,t) &= \Phi(x,t), \\ {}_0I_t^{\alpha}\, {}_0^C D_t^{\alpha}\Phi(x,t) &= \Phi(x,t) - \sum_{k=0}^{n-1}\frac{\partial^k\Phi(x,t)}{\partial t^k}|_{t=0}\frac{t^k}{k!}. \end{aligned}$$

Also, we have

$$
\begin{aligned}
{}^{C}_{0}D^{\alpha}_{t} t^{\theta} &= \frac{\Gamma(\theta+1)}{\Gamma(\theta-\alpha+1)} t^{\theta-\alpha}, \\
{}_{0}I^{\alpha}_{t} t^{\theta} &= \frac{\Gamma(\theta+1)}{\Gamma(\theta+\alpha+1)} t^{\theta+\alpha}.
\end{aligned}
$$

Definition 2.3. *The Mittag-Leffler function is defined as*

$$E_{\alpha}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\alpha+1)}, \quad \alpha > 0, \ \ z \in \mathbb{C}.$$

Lemma 2.1. *The CFD of generalized Mittag-Leffler function is given by*

$${}^{C}_{0}D^{\alpha}_{t} E_{\alpha}(\aleph t^{\alpha}) = {}^{C}_{0}D^{\alpha}_{t}\Big(\sum_{n=0}^{\infty} \frac{\aleph^n t^{n\alpha}}{\Gamma(n\alpha+1)}\Big) = \sum_{n=1}^{\infty} \frac{\aleph^n t^{(n-1)\alpha}}{\Gamma((n-1)\alpha+1)} = \sum_{n=0}^{\infty} \frac{\aleph^{n+1} t^{n\alpha}}{\Gamma(n\alpha+1)} = \aleph E_{\alpha}(\aleph t^{\alpha}).$$

Theorem 2.2. *[36] Suppose that $\Phi(x,t) = \sum_{k=0}^{\infty} \zeta^k \Phi_k(x,t)$, N is a nonlinear operator. Then, we have*

$$\frac{\partial^n}{\partial \zeta^n} N(\Phi)_{\zeta=0} = \frac{\partial^n}{\partial \zeta^n} N\left(\sum_{k=0}^{\infty} \zeta^k \Phi_k\right)_{\zeta=0} = \frac{\partial^n}{\partial \zeta^n} N\left(\sum_{k=0}^{n} \zeta^k \Phi_k\right)_{\zeta=0}.$$

3. Fundamental procedures for the MGMLFM

To demonstrate the procedure and algorithm of the MGMLFM, we consider the following general nonlinear FPDEs

$${}^{C}_{0}D^{\alpha}_{t} F(X,t) = L(F(X,t)) + N(F(X,t)), \tag{3.1}$$

subject to ICs

$$F(X,0) = \varpi(X), \tag{3.2}$$

where L linear operator and N nonlinear operator, $F = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_m \end{bmatrix}$, $X = [\, x_1 \ x_2 \ \cdots \ x_n \,]$, $n, m \in \mathbb{N}$, and $\varpi(X) = \begin{bmatrix} \varpi_1 \\ \varpi_2 \\ \vdots \\ \varpi_m \end{bmatrix}$.

The MGMLFM assume the solution of Eq (3.1) as

$$
\begin{aligned}
F_1(X,t) &= \theta_1(X) E_{\alpha}(\aleph_1 t^{\alpha}) = \sum_{k=0}^{\infty} \theta_1(X) \aleph_1^k \frac{t^{k\alpha}}{\Gamma(k\alpha+1)}, \\
F_2(X,t) &= \theta_2(X) E_{\alpha}(\aleph_2 t^{\alpha}) = \sum_{k=0}^{\infty} \theta_2(X) \aleph_2^k \frac{t^{k\alpha}}{\Gamma(k\alpha+1)}, \qquad (3.3) \\
&\vdots
\end{aligned}
$$

$$F_m(X,t) = \theta_m(X)E_\alpha(\aleph_m t^\alpha) = \sum_{k=0}^{\infty}\theta_m(X)\aleph_m^k\frac{t^{k\alpha}}{\Gamma(k\alpha+1)},$$

where $\aleph_1, \aleph_2, \cdots, \aleph_m$ are anonymous coefficients. The auxiliary functions $\theta_1, \theta_2, \cdots, \theta_m$ satisfies $\theta_1 = \varpi_1, \theta_2 = \varpi_2, \cdots, \theta_m = \varpi_m$. Using assumptions (3.3) and Lemma 2.1 the FPDEs (3.1) satisfies

$$\sum_{k=0}^{\infty}\varpi(X)\aleph_m^{k+1}\frac{t^{k\alpha}}{\Gamma(k\alpha+1)} = L(\sum_{k=0}^{\infty}\varpi(X)\aleph_m^k\frac{t^{k\alpha}}{\Gamma(k\alpha+1)}) + N(\sum_{k=0}^{\infty}\varpi(X)\aleph_m^k\frac{t^{k\alpha}}{\Gamma(k\alpha+1)}), m = 1, 2, \cdots. \tag{3.4}$$

Consequently, we can write L as the following

$$\begin{aligned} L(F(X,t)) &= L(\sum_{k=0}^{\infty}\varpi(X)\aleph_m^k\frac{t^{k\alpha}}{\Gamma(k\alpha+1)}) = L(\varpi(X))\sum_{k=0}^{\infty}\aleph_m^k\frac{t^{k\alpha}}{\Gamma(k\alpha+1)} \\ &= \varepsilon\varpi(X)\sum_{k=0}^{\infty}\aleph_m^k\frac{t^{k\alpha}}{\Gamma(k\alpha+1)}, \end{aligned} \tag{3.5}$$

where ε is a constant. From the Theorem 2.2, N can be expanded as follows

$$\begin{aligned} N(F(X,t)) &= N(\sum_{k=0}^{\infty}\varpi(X)\aleph_m^k\frac{t^{k\alpha}}{\Gamma(k\alpha+1)}) = N(\sum_{k=0}^{\infty}\varpi(X)F_j(X,t)) \\ &= N(\varpi(X))\,(N(F_0(X,t)) + \sum_{k=1}^{\infty}(N(\sum_{i=0}^{k}F_i(X,t)) - N(\sum_{i=0}^{k-1}F_i(X,t)))). \end{aligned} \tag{3.6}$$

By decomposing Eqs (3.5) and (3.6) into Eq (3.4), we obtain a general recurrence relations to acquire $\aleph_m$. Consequently, we get a solution of Eq (3.1). For more details on the MGMLFM (see e.g., [37–40]). Regarding the error estimator and convergence of the given algorithm, we offer the following theorem.

Theorem 3.1. *Let's consider a Hilbert Space H defined as: $H = L^2((\epsilon,\eta)\times[0,T])$ with the associated norm $\left\|F^2\right\| = \int_{(\epsilon,\eta)\times[0,T]}F^2(x,\lambda)\,d\lambda\,d\tau < +\infty$ and the following two hypotheses $\mathcal{H}_1$, $\mathcal{H}_2$ are satisfied*

($\mathcal{H}_1$) $(\psi(F_1)-\psi(F_2), F_1-F_2) \geq K\|F_1-F_2\|^2$; $K > 0$, $F_1, F_2 \in H$,
($\mathcal{H}_2$) whenever a constant $\varrho > 0$, then there exist $M(\varrho) > 0$, since $\|F_1\| \leq \varrho$, $\|F_2\| \leq \varrho$, $\forall\, F_1,\, F_2 \in H$ and we have $(\psi(F_1)-\psi(F_2), \omega) \leq M(\varrho)\,\|F_1-F_2\|\,\|\omega\|$ for every $\omega \in H$,

where the operator $\psi(F_1)$ following Eq (3.1) is given by $\psi(F_1) = {}^{C}D_t^{\alpha}F_1(x,t) = L(F_1(x,t)) + N(F_1(x,t))$; L, N are linear and nonlinear differential operators in H. Then, the MGMLFM is convergence.

Proof. The proof of this theorem can be proceed in the same manner in [7, 32]. □

4. Implementing MGMLFM on FJMM

In this section, we employ the above algorithm of the MGMLFM to solve the nonlinear coupled time-FJMM as stated in Eq (1.1) subject to ICs (1.2). Furthermore, the validity of the MGMLFM is proven by comparing acquired approximate solutions at $\alpha = 1$ with the next known exact solution [14,31].

$$\begin{aligned} U(x,t) &= \frac{\lambda^2}{8}\left[1 - 4sech^2\left(\frac{\lambda}{2}(x + \frac{1}{2}\lambda^2 t)\right)\right], \\ V(x,t) &= \lambda sech\left(\frac{\lambda}{2}(x + \frac{1}{2}\lambda^2 t)\right). \end{aligned} \tag{4.1}$$

To execute the MGMLFM on the FJMM model (1.1), the solution is assumed by the fractional power series as follows

$$\begin{aligned} U(x,t) &= \Upsilon(x)E_\alpha(At^\alpha) = \sum_{n=0}^{\infty}\Upsilon(x)A^n\frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \\ V(x,t) &= \Psi(x)E_\alpha(Bt^\alpha) = \sum_{n=0}^{\infty}\Psi(x)B^n\frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \end{aligned} \tag{4.2}$$

where A and B are anonymous coefficients. From Eq (1.2) the auxiliary functions yields $\Upsilon(x) = U_0$ and $\Psi(x) = V_0$. Using Lemma (2.1) and Eq (4.2), we have

$$\begin{aligned} &\sum_{n=0}^{\infty}\left[U_0A^{n+1} + \frac{\partial^3(U_0A^n)}{\partial x^3} + \left(\frac{3}{2}V_0C^n + \frac{9}{2}M^n - 6U_0L^n - 6U_0V_0E^n - \frac{3}{2}V_0^2P^n\right)\Gamma(n\alpha+1)\right]\frac{t^{n\alpha}}{\Gamma(n\alpha+1)} = 0, \\ &\sum_{n=0}^{\infty}\left[V_0B^{n+1} + \frac{\partial^3(V_0B^n)}{\partial x^3} - \left(6V_0Q^n + 6U_0H^n + \frac{15}{2}V_0^2R^n\right)\Gamma(n\alpha+1)\right]\frac{t^{n\alpha}}{\Gamma(n\alpha+1)} = 0, \end{aligned} \tag{4.3}$$

where

$$C^n = \sum_{k=0}^{n}\frac{B^k\frac{\partial^3(V_0B^{(n-k)})}{\partial x^3}}{\Gamma(k\alpha+1)\Gamma((n-k)\alpha+1)}, \qquad M^n = \sum_{k=0}^{n}\frac{\frac{\partial\left(V_0B^k\right)}{\partial x}\frac{\partial^2\left(V_0B^{(n-k)}\right)}{\partial x^2}}{\Gamma(k\alpha+1)\Gamma((n-k)\alpha+1)},$$

$$L^n = \sum_{k=0}^{n}\frac{A^k\frac{\partial(U_0A^{(n-k)})}{\partial x}}{\Gamma(k\alpha+1)\Gamma((n-k)\alpha+1)}, \qquad Q^n = \sum_{k=0}^{n}\frac{\frac{\partial(U_0A^k)}{\partial x}B^{(n-k)}}{\Gamma(k\alpha+1)\Gamma((n-k)\alpha+1)},$$

$$E^n = \sum_{k_1=0}^{n}\sum_{k_2=0}^{k_1}\frac{A^{k_2}B^{(n-k_1)}\frac{\partial(V_0B^{(k_1-k_2)})}{\partial x}}{\Gamma(\alpha k_2+1)\Gamma(\alpha(k_1-k_2)+1)\Gamma(\alpha(n-k_1)+1)}, \qquad H^n = \sum_{k=0}^{n}\frac{A^k\frac{\partial(V_0B^{(n-k)})}{\partial x}}{\Gamma(k\alpha+1)\Gamma((n-k)\alpha+1)},$$

$$P^n = \sum_{k_1=0}^{n}\sum_{k_2=0}^{k_1}\frac{\frac{\partial(U_0A^{k_2})}{\partial x}B^{(n-k_1)}B^{(k_1-k_2)}}{\Gamma(\alpha k_2+1)\Gamma(\alpha(k_1-k_2)+1)\Gamma(\alpha(n-k_1)+1)},$$

$$R^n = \sum_{k_1=0}^{n}\sum_{k_2=0}^{k_1} \frac{\frac{\partial(V_0 B^{k_2})}{\partial x} B^{(n-k_1)} B^{(k_1-k_2)}}{\Gamma(\alpha k_2+1)\Gamma(\alpha(k_1-k_2)+1)\Gamma(\alpha(n-k_1)+1)}.$$

From Eq (4.3) the term $t^{n\alpha} \neq 0$, but their coefficients $= 0$. Therefore, the recurrence relation is set as

$$\begin{aligned} A^{n+1} &= \frac{-\frac{\partial^3(U_0 A^n)}{\partial x^3} - \left(\frac{3}{2}V_0 C^n + \frac{9}{2}M^n - 6U_0 L^n - 6U_0 V_0 E^n - \frac{3}{2}V_0^2 P^n\right)\Gamma(n\alpha+1)}{U_0}, \\ B^{n+1} &= \frac{-\frac{\partial^3(V_0 B^n)}{\partial x^3} + \left(6V_0 Q^n + 6U_0 H^n + \frac{15}{2}V_0^2 R^n\right)\Gamma(n\alpha+1)}{V_0}. \end{aligned} \tag{4.4}$$

For $n = 0$, we have

$$\begin{aligned} A^1 &= \frac{-\frac{\partial^3(U_0 A^0)}{\partial x^3} - \frac{3V_0 C^0}{2} - \frac{9M^0}{2} + 6U_0 L^0 + 6U_0 V_0 E^0 + \frac{3}{2}V_0^2 P^0}{U_0} = \frac{4\lambda^3 \tanh\left(\frac{\lambda x}{2}\right)}{\cosh(\lambda x) - 7}, \\ B^1 &= \frac{-\frac{\partial^3(V_0 B^0)}{\partial x^3} + 6V_0 Q^0 + 6U_0 H^0 + \frac{15}{2}V_0^2 R^0}{V_0} = -\frac{1}{4}\lambda^3 \tanh\left(\frac{\lambda x}{2}\right), \end{aligned}$$

where $A^0 = 1$ and $B^0 = 1$. When $n = 1$ we have

$$\begin{aligned} A^2 &= \frac{-\frac{\partial^3(U_0 A^1)}{\partial x^3} - \frac{3V_0 C^1}{2} - \frac{9M^1}{2} + 6U_0 L^1 + 6U_0 V_0 E^1 + \frac{3}{2}V_0^2 P^1}{U_0}, \\ &= -\frac{\lambda^6(\cosh(\lambda x) - 2)\text{sech}^2(\frac{\lambda x}{2})}{\cosh(\lambda x) - 7}, \\ B^2 &= \frac{-\frac{\partial^3(V_0 B^1)}{\partial x^3} + 6V_0 Q^1 + 6U_0 H^1 + \frac{15}{2}V_0^2 R^1}{V_0}, \\ &= \frac{1}{32}\lambda^6(\cosh(\lambda x) - 3)\text{sech}^2(\frac{\lambda x}{2}). \end{aligned}$$

Similarly, by replacing diverse values for n we procure other coefficients of A and B. After that, we replace these gained coefficients in the next power series, leading to the approximation solutions of the nonlinear coupled FJMM that in Eq (4.2).

$$U(x,t) = U_0(A^0 + A^1\frac{t^\alpha}{\Gamma(\alpha+1)} + A^2\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + A^3\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \cdots),$$

$$V(x,t) = V_0(B^0 + B^1\frac{t^\alpha}{\Gamma(\alpha+1)} + B^2\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + B^3\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \cdots).$$

5. Numerical simulation and discussion

Here, we present graphical representations and tabled values of the obtained solutions through MGMLFM for nonlinear coupled FJMM Eq (1.1). A qualitative comparison between our analytical approximate solution with the given exact solution in Eq (4.1) is displayed in both 3D and 2D graphs. Also, we reported in some tables a quantitative comparison between our results and the known exact solutions. In addition, we evaluated the absolute error associated with the obtained solutions and compared it with some other methods found in the literature such as the Hermite wavelet method (HWM) [31], modified Laplace decomposition method (MLDM) [32], Laplace residual power series method (LRPSM) [16], and optimal auxiliary function method (OAFM) [1].

Figures 1 and 3 present the behavior of the obtained results from MGMLFM for U and V at $\alpha = 1$, respectively, compared with the given exact solution in Eq (4.1), in addition to displaying the absolute error between them. In Figures 2 and 4, we illustrate the influence of changing α on the behavior of solutions for the system dynamics.

In Tables 1 and 2, we show a comparison between the obtained numerical values of $U(x,t)$ and $V(x,t)$, respectively, with the given exact solution at $\alpha = 1$ and various values of t. Furthermore, the absolute errors associated with these approximate solutions are estimated and compared with other methods such as HWM and MLDM in Tables 1 and 2, as well as LRPSM in Tables 3 and 4. In addition, a comparison between absolute errors caused by MGMLFM with those resulting from the OAFM is presented in Tables 5 and 6.

We conclude from the presented tables that the obtained approximate values when $\alpha = 1$ are closely aligned with the known exact solutions and the analysis errors are very small and better than those obtained in other presented methods.

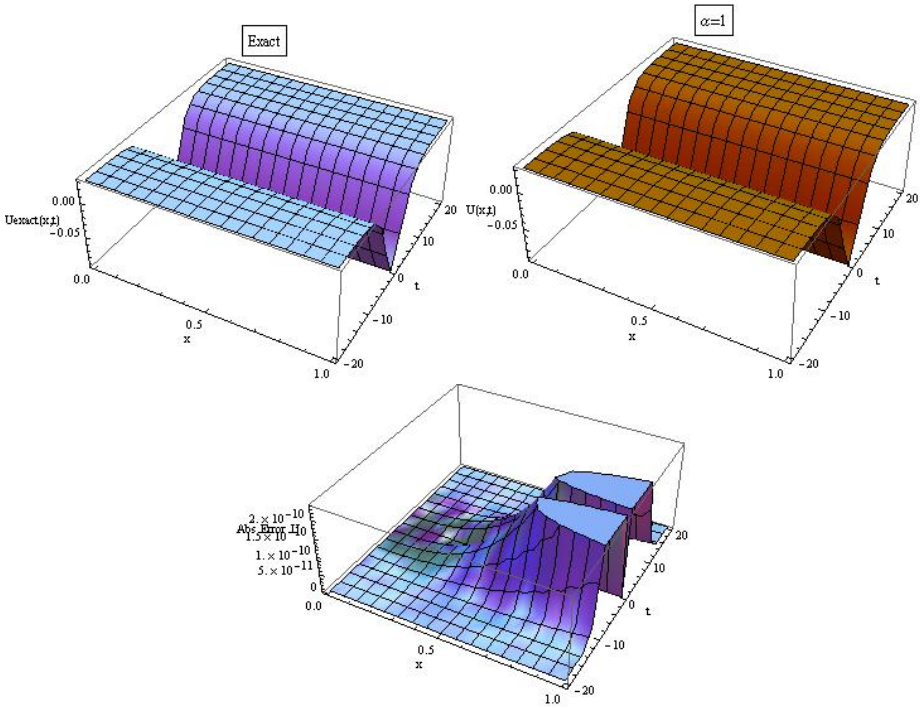
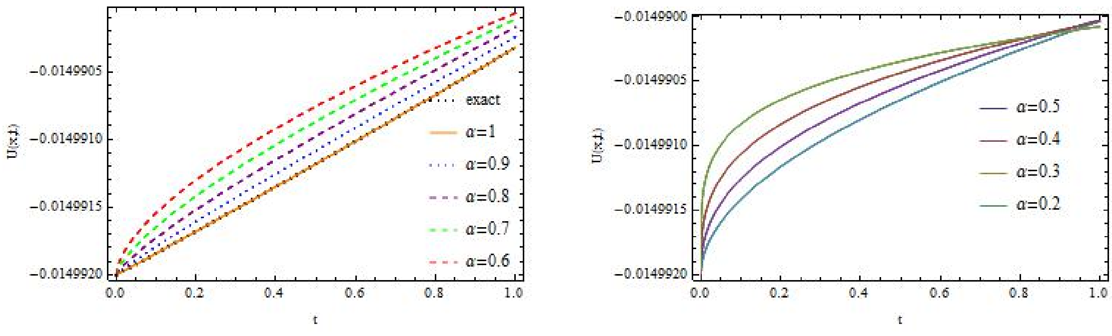


Figure 1. Comparison between the approximate solution for $U(x,t)$ at $\alpha = 1$ with exact solution for the FJMM.



(a) **(b)**

Figure 2. The influence of the fractional order on the MGMLFM approximate solution for $U(x,t)$ when $\lambda = 0.2$ and $x = 0.2$. (a) $\alpha = 1, 0.9, 0.8, 0.7, 0.6$. (b) $\alpha = 0.5, 0.4, 0.3, 0.2$.

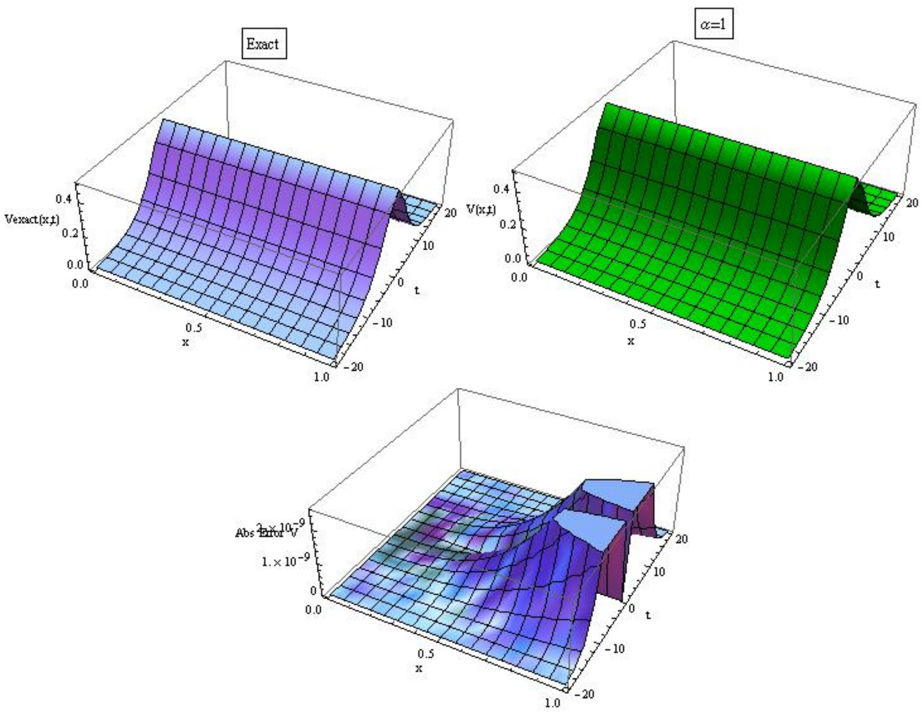
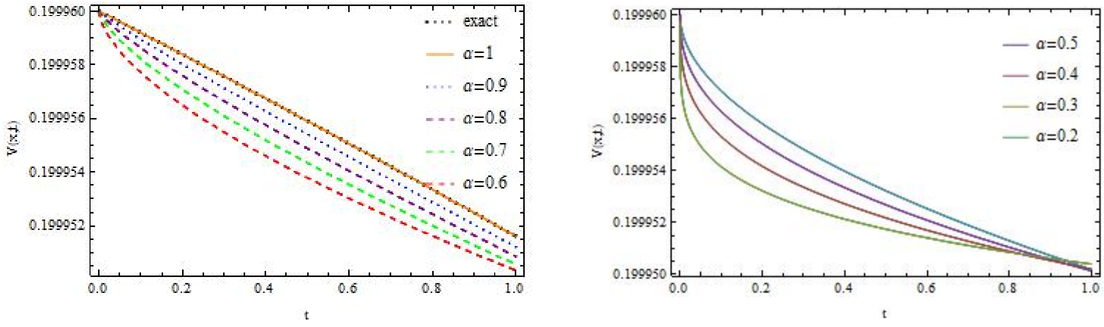


Figure 3. Comparison between the approximate solution for $V(x,t)$ at $\alpha = 1$ with exact solution for the FJMM.



(a) **(b)**

Figure 4. The influence of the fractional order on the MGMLFM approximate solution for $V(x, t)$ when $\lambda = 0.2$ and $x = 0.2$. (a) $\alpha = 1, 0.9, 0.8, 0.7, 0.6$. (b) $\alpha = 0.5, 0.4, 0.3, 0.2$.

Table 1. The MGMLFM and exact solutions for $U(x, t)$ with the absolute error provided by other methods in [31, 32] when $\lambda = 0.5$ and $\alpha = 1$.

x	t	Exact	MGMFM	Error MGMFM	Error HWM [31]	Error MLDM [32]
	0.2	-0.0933553	-0.0933553	4.16463×10^{-9}	1.3360×10^{-5}	4.16463×10^{-9}
0.2	0.4	-0.093263	-0.093263	3.43094×10^{-8}	6.3236×10^{-5}	3.43094×10^{-8}
	0.6	-0.093161	-0.0931609	1.19136×10^{-7}	1.6141×10^{-4}	1.19136×10^{-7}
	0.8	-0.0930495	-0.0930492	2.90296×10^{-7}	2.8892×10^{-4}	2.90296×10^{-7}

Table 2. The MGMLFM and exact solutions for $V(x, t)$ with the absolute error provided by other methods in [31, 32] when $\lambda = 0.5$ and $\alpha = 1$.

x	t	Exact	MGMFM	Error MGMFM	Error HWM [31]	Error MLDM [32]
	0.2	0.49921	0.49921	5.21689×10^{-9}	6.3948×10^{-5}	5.2168×10^{-9}
0.2	0.4	0.499025	0.499025	4.29844×10^{-8}	4.0998×10^{-5}	4.2984×10^{-8}
	0.6	0.498821	0.498821	1.49281×10^{-7}	1.4804×10^{-4}	1.4928×10^{-7}
	0.8	0.498597	0.498597	3.6381×10^{-7}	1.3136×10^{-4}	3.6381×10^{-7}

Table 3. The MGMLFM and exact solutions for $U(x, t)$ with the absolute error provided by LRPSM [16] when $\lambda = 0.02$ and $\alpha = 1$.

x	t	Exact	MGMFM	Error MGMFM	Error LRPSM [16]
	0.1	-0.00014999999803187204	-0.00014999999803187202	$2.7105054312\times10^{-20}$	8.67132×10^{-8}
0.0099	0.3	-0.00014999999801596804	-0.00014999999801596802	$2.7105054312\times10^{-20}$	2.51868×10^{-7}
	0.5	-0.00014999999800000002	-0.00014999999800000000	$2.7105054300\times10^{-20}$	4.1395×10^{-7}
	0.7	-0.00014999999798396803	-0.00014999999798396800	$2.7105054312\times10^{-20}$	5.71027×10^{-7}
	0.9	-0. 00014999999979678720	-0. 00014999999979678720	$2.71050543120\times10^{-20}$	7.21256×10^{-7}
	1	-0.00014999999795980000	-0.00014999999795980000	$0.0000000000\times10^{00}$	8.62914×10^{-7}

Table 4. The MGMLFM and exact solutions for $V(x,t)$ with the absolute error provided by LRPSM [16] when $\lambda = 0.02$ and $\alpha = 1$.

x	t	Exact	MGMFM	Error MGMFM	Error LRPSM [16]
0.0099	0.1	0.019999999901593603	0.0199999999015936	$3.46944695195\times10^{-18}$	1.00061×10^{-8}
	0.3	0.019999999900798402	0.019999999900798402	$0.00000000000\times10^{00}$	9.00175×10^{-8}
	0.5	0.019999999900000003	0.0199999999	$3.46944695195\times10^{-18}$	2.50025×10^{-7}
	0.7	0.0199999998991984	0.0199999998991984	$0.00000000000\times10^{00}$	4.9002110×10^{-7}
	0.9	0.0199999998983936	0.0199999998983936	$0.00000000000\times10^{00}$	8.09995×10^{-7}
	1	0.01999999989799	0.019999999897990003	$3.46944695195\times10^{-18}$	9.99969×10^{-7}

Table 5. The MGMLFM and exact solutions for $U(x,t)$ with the absolute error provided by OAFM [1] when $\lambda = 0.2$ and $\alpha = 1$.

t	x	Exact	MGMFM	Error MGMFM	Error OAFM [1]
0.5	0.1	-0.014997580195199953	-0.014997580199784956	$4.585002516543923\times10^{-12}$	2.05729×10^{-7}
	0.2	-0.014991182592432142	-0.01499118260160548	$9.173337436885198\times10^{-12}$	2.252×10^{-7}
	0.3	-0.014980792306911109	-0.014980792320652778	$1.374166926082498\times10^{-11}$	2.44461×10^{-7}
	0.4	-0.014966417640955071	-0.014966417659235152	$1.828008057425112\times10^{-11}$	2.82237×10^{-7}
	0.5	-0.014948070069910207	-0.014948070092688998	$2.277879100620605\times10^{-11}$	7.21256×10^{-7}
	0.6	-0.014925764222701553	-0.014925764249929696	$2.722814335109902\times10^{-11}$	3.00695×10^{-7}
	0.7	-0.014899517857080208	-0.014899517888698881	$3.161867295764509\times10^{-11}$	3.18831×10^{-7}

Table 6. The MGMLFM and exact solutions for $V(x,t)$ with the absolute error provided by OAFM [1] when $\lambda = 0.2$ and $\alpha = 1$.

t	x	Exact	MGMFM	Error MGMFM	Error OAFM [1]
0.5	0.1	0.19998790061001168	0.1999879006399615	$2.994982040149807\times10^{-11}$	1.0×10^{-5}
	0.2	0.19995590810192201	0.19995590816181252	$5.98905092186186\times10^{-11}$	3.99×10^{-5}
	0.3	0.19990393846500928	0.19990393855473867	$8.9729390584381\times10^{-11}$	8.98×10^{-5}
	0.4	0.19983201765960865	0.19983201777902473	$1.194160881290429\times10^{-10}$	1.6×10^{-4}
	0.5	0.19974018158552978	0.19974018173443045	$1.489006695720718\times10^{-10}$	2.49×10^{-4}
	0.6	0.1996284760383726	0.1996284762165063	$1.781337022332962\times10^{-10}$	3.58×10^{-4}
	0.7	0.19949695665388084	0.1994969568609475	$2.070666693665401\times10^{-10}$	4.87×10^{-4}

6. Conclusions

In this study, we successfully approached the convenient approximate solution for the nonlinear time-FJMM by utilizing a new analytical technique called MGMLFM. The Caputo fractional operator is used to extend the proposed model into the FPDEs form. The fundamental analysis for the proposed method is investigated to acquire the analytical approximate solution of the general nonlinear FPDEs. We depicted a numerical simulation for gained results through two and three-dimensional plots and tabled data. We found excellent agreement between our results when compared with given exact solutions (4.1) as indicated in Figures 1 and 3. Also, we demonstrated the impact of fractional order α on the behavior of the approximate solutions in Figures 2 and 4. Moreover, a comparison between the absolute error resulting from the used method and some other methods in the literature is generated in the Tables 1–6 at various points of x and t when $\alpha = 1$. The findings of this research indicated the validity and efficiency of the MGMLFM for solving such models. Hence, MGMLFM is considered a promising method for handling real-life applications in various fields and an additive instrument for FC area and computational analysis methods.

Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence tools in the creation of this article.

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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