On Solving Multi-Attribute Decision Making Problem Using AHP

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Abstract. Multi-attribute Decision Making (MADM) is concerned with the elucidation of the levels of preference of decision alternatives, through judgments made over a number of criteria. Many complex MADM problems are characterized with both quantitative and qualitative-of preference of decision alternatives, through judgments made over a number of criteria. Many complex MADM problems are characterized with both quantitative and qualitative-attributes. In selection of its suppliers, an organization needs to take into account such attributes as quality, technical capability, supply chain management, financial soundness, environmental and so on. Many smart technologies are used in modern cities to improve society's well-being in many ways. The proposed research focuses on communication methods for data applications. Our main goal for this paper is to use little energy as possible while delivering as much data as possible by using multi-attribute decision-making.

1 Introduction

Multi-attribute decision-making (MADM) is extremely utilized in different domains, including society, economy, and management for invested in decision-making, plan estimation, standard appraisal and financial profit evaluation. [1, 4, 10] In actual life, getting a precise numerical value to be estimated the objects while decision-making is extremely difficult due to the complexity and ambiguity of intention topics and the person thought.

The Analytic Hierarchy Process (AHP) will be used for assess supply chain risks by ensuring that risk variables are classified consistently [9, 12]. Saaty created the AHP approach in the year 1970. The abbreviation AHP encapsulates its qualities, which are aimed at overcoming decision-makers cognitive constraints. The approach is distinguished to its simplicity and robustness, allowing it to be applied in a variety of fields, including manufacturing, logistics, bioenergy, and construction [2]. The AHP applications of determination issues is completed by two stage and it is classified as a support tool. AHP accepts decision-makers for instance complicated issues from the hierarchical framework,

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with the first phase including the structuring of the problem in levels and the second phase involving the evaluation [3, 8]. The actions below should be followed in order to make well-informed decisions and establish consistent priorities [5, 7].

1) Explain the issues and sort the information required;

2) *Problem decomposition*: conducting research, separating, and arranging the problem into a hierarchy.

3) Create a matrix table from verdict betwixt the couple of criteria and second matrix table is verdict to the assumed alternatives, allowing you to visualize the problem in terms of objectives, criteria, and alternatives. Every one element of the top quantity was employed the liken of the variables instantly beneath it;

4) The values for the priority for each criterion and alternative will be determined from the matrix of judgments.

The analysis begins with a determination of the criteria's relevance using the Saaty Scale of Fundamental which compares the much factor is mightily powerhouse than the another. The verdict creator have capable to compare options and build the decisions based on their preferences. The reciprocal requirement should be satisfied by the intensity of these preferences: If M is S times extra favoured than B. Then B's preference are $1/ss$ times that is M. The judgement matrix are normalised by dividing each element (M_{ij}) with the addition of the numbers to the appropriate column from the judgement matrix. Normalised eigenvector (w) was responsible for determining the relevance to each criterion and is evaluated by employing the line represents the normalised matrix variables. By multiplying the matrix of decisions, M with the vector of priority W, then dividing that new vector, Mw, with the original vector w, the greatest eigenvalue (max) is obtained [10, 12].

2 Preliminaries

2.1 Scalar Cardinality [6]

Scalar cardinality is defined by the summation of membership values of all elements in the set.

$$|A| = \sum x \qquad \textit{for all } x \in X \qquad (1)$$

2.2 Consistency Index (CI) [9]

The consistency index is evaluated to a value obtained by producing random inverse matrices with the same dimension, providing the consistency ratio (CR) that is intended to have the identical interpretation regardless of matrix size. The corresponding values from random matrices.

$$CI = \frac{\lambda_{max} - n}{n-1} \qquad (2)$$

where$\lambda_{max} = \frac{Average\ of\ sum\ of\ last\ column\ values}{Number\ of\ elements}$.

2.3 Consistency ratio [5, 9]

The consistency ratio (CR) will be determined as:

$$CR = \frac{CI}{RI} \qquad (3)$$

where RI reflects the average consistency index across a set of random elements with same inverse matrix. If CR is less than 0.10, it is acceptable. If it is more than 0.10, the evaluation matrix is judged contradictory.

Algorithm for MADM problem using AHP

Step 1: For MADM problem, let $B = (a_{ij})_{n\times m}$ where $a_{ij} > 0$. Then $M = \{m_1, m_2, \dots, m_n\}$. Collect data of attributes and analyse a AHP structure with goal of upper level, the attribute for second level and finally alternatives.

Step 2: Create different criteria for pair-wise comparison matrix. It will be developed with the help of scale of relative importance.

1	-	Same Weight
3	-	Average Weight
5	-	Robust Weight
7	-	More Robust Weight
9	-	Atmost Weight
2, 4, 6, 8	-	Middle Weight
$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$	-	Values for Inverse Comparison

Step 3: Compare and substitute the accurate values for each cell. x_{ij}, $i = j$ then values will be diagonal other row and column look like symmetric method.

Step 4: Add each column values. Hereafter every element from a column will be divided with the sum of the column.

Step 5: Normalized pairwise comparison matrix will be created and identify criteria weights (CW).

Step 6: From first table we multiply each value in the column with the CW values.

Step 7: Find weight sum value (WSV). Then divided with CW.

Step 8: Identify consistency ratio (CR). If $CR < 0.1$ then multiply the CW values with main table and identify the results.

3 Numerical Example

A person decides to purchase a new laptop. Now they chosen five various laptops x_i which is evaluated by using following five categories (attributes).

m_1	:	Design
m_2	:	Cost
m_3	:	Generation

m_4 : Reviews
m_5 : Weight

The evaluation over the laptop $x_i (i = 1,2,3,4,5)$ with respect to the indices u_j.

Moderate	**Good**	**Better**	**Excellent**	**Extra-ordinary**
1	2	3	4	5

Alternatives	m_1	m_2	m_3	m_4	m_5
Laptop 1	Excellent	51,990	10	05	1.85
Laptop 2	Average	50,990	11	04	1.65
Laptop 3	Good	49,990	04	04	1.40
Laptop 4	Better	52,190	08	03	2.04
Laptop 5	Good	44,000	10	02	1.50

Alternatives	m_1	m_2	m_3	m_4	m_5
Laptop 1	4	51,990	10	05	1.85
Laptop 2	1	50,990	11	04	1.65
Laptop 3	2	49,990	04	04	1.40
Laptop 4	3	52,190	08	03	2.04
Laptop 5	2	44,000	10	02	1.50

Alternatives	m_1	m_2	m_3	m_4	m_5
Laptop 1	4	51,990	10	05	1.85
Laptop 2	1	50,990	11	04	1.65
Laptop 3	2	49,990	04	04	1.40
Laptop 4	3	52,190	08	03	2.04
Laptop 5	2	44,000	10	02	1.50

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$\sum x$	12	249160	43	18	8.44

Alternatives	m_1	m_2	m_3	m_4	m_5
Laptop 1	0.33	0.21	0.23	0.28	0.22
Laptop 2	0.08	0.20	0.26	0.22	0.20
Laptop 3	0.17	0.20	0.09	0.22	0.17
Laptop 4	0.25	0.21	0.19	0.17	0.24
Laptop 5	0.17	0.18	0.23	0.11	0.15

	m_1	m_2	m_3	m_4	m_5
m_1	1	5	4	3	$\frac{1}{2}$
m_2	$\frac{1}{5}$	1	$\frac{1}{6}$	1	$\frac{1}{7}$
m_3	$\frac{1}{4}$	6	1	8	$\frac{1}{5}$
m_4	$\frac{1}{3}$	1	$\frac{1}{8}$	1	$\frac{1}{9}$
m_5	2	7	5	9	1

	m_1	m_2	m_3	m_4	m_5
m_1	1	5	4	3	0.5
m_2	0.2	1	0.17	1	0.14
m_3	0.25	6	1	8	0.2
m_4	0.33	1	0.13	1	0.11
m_5	2	7	5	9	1
Sum	3.78	20	10.3	22	1.95

	m_1	m_2	m_3	m_4	m_5
m_1	0.2646	0.25	0.3883	0.1364	0.2564
m_2	0.0529	0.05	0.0165	0.0455	0.0718
m_3	0.0661	0.30	0.0971	0.3636	0.1027
m_4	0.0873	0.05	0.0126	0.0455	0.0564

m_5	0.5291	0.35	0.4854	0.4091	0.5128

	m_1	m_2	m_3	m_4	m_5	CW
m_1	0.2646	0.25	0.3883	0.1364	0.2564	0.2591
m_2	0.0529	0.05	0.0165	0.0455	0.0718	0.0473
m_3	0.0661	0.30	0.0971	0.3636	0.1027	0.1859
m_4	0.0873	0.05	0.0126	0.0455	0.0564	0.0504
m_5	0.5291	0.35	0.4854	0.4091	0.5128	0.4573

CW	0.2591	0.0473	0.1859	0.0504	0.4573
	m_1	m_2	m_3	m_4	m_5
m_1	1	5	4	3	0.5
m_2	0.2	1	0.17	1	0.14
m_3	0.25	6	1	8	0.2
m_4	0.33	1	0.13	1	0.11
m_5	2	7	5	9	1

CW	0.2591	0.0473	0.1859	0.0504	0.4573
	m_1	m_2	m_3	m_4	m_5
m_1	0.2591	0.2365	0.7436	0.1512	0.2287
m_2	0.0518	0.0473	0.0316	0.0504	0.0640
m_3	0.0648	0.2838	0.1859	0.4032	0.0915
m_4	0.0855	0.0473	0.0242	0.0504	0.0503
m_5	0.5182	0.331	0.9295	0.4536	0.4573

		1			

	m_1	m_2	m_3	m_4	m_5	WSC
m_1	0.2591	0.2365	0.7436	0.1512	0.2287	1.6191
m_2	0.0518	0.0473	0.0316	0.0504	0.0640	0.2451
m_3	0.0648	0.2838	0.1859	0.4032	0.0915	1.0292
m_4	0.0855	0.0473	0.0242	0.0504	0.0503	0.2577
m_5	0.5182	0.3311	0.9295	0.4536	0.4573	2.6897

	m_1	m_2	m_3	m_4	m_5	Ratio
m_1	0.2591	0.2365	0.7436	0.1512	0.2287	6.2489
m_2	0.0518	0.0473	0.0316	0.0504	0.0640	5.1818
m_3	0.0648	0.2838	0.1859	0.4032	0.0915	5.5363
m_4	0.0855	0.0473	0.0242	0.0504	0.0503	5.1131
m_5	0.5182	0.3311	0.9295	0.4536	0.4573	5.8817

Using the definition 2.1,

$$\lambda_{max} = \frac{Average\ of\ sum\ of\ last\ column\ values}{Number\ of\ elements}$$

$= 5.5924$

$$Consistency\ Index = \frac{\lambda_{max} - n}{n - 1}$$

$= 0.1481$

$\frac{C.I.}{10} = 0.01481$

n	1	2	3	4	5	6	7	8	9	10
R.I.	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

$n = 5,$

$$C.R. = \frac{0.01481}{1.12}$$

$$= 0.0132 < 0.1$$

Alternatives	m_1	m_2	m_3	m_4	m_5	CW
Laptop 1	0.33	0.21	0.23	0.28	0.22	0.2591
Laptop 2	0.08	0.20	0.26	0.22	0.20	0.0473
Laptop 3	0.17	0.20	0.09	0.22	0.17	0.1859
Laptop 4	0.25	0.21	0.19	0.17	0.24	0.0504
Laptop 5	0.17	0.18	0.23	0.11	0.15	0.4573

- Laptop 1 score = 0.2529
- Laptop 2 score = 0.1811
- Laptop 3 score = 0.1591
- Laptop 4 score = 0.2283
- Laptop 5 score = 0.1832

Rank of all alternatives:
Laptop 1 > Laptop 4 > Laptop 5 > Laptop 2 > Laptop 3.

Alternatives	m_1	m_2	m_3	m_4	m_5
Laptop 1	4	51,990	10	05	1.85
Laptop 2	1	50,990	11	04	1.65
Laptop 3	2	49,990	04	04	1.40
Laptop 4	3	52,190	08	03	2.04
Laptop 5	2	44,000	10	02	1.50
$\sqrt{\sum_{ij=1}^{n} (L_{ij})^2}$	5.83	111632.15	20.02	8.37	3.81

Alternatives	m_1	m_2	m_3	m_4	m_5
Laptop 1	0.69	0.47	0.50	0.60	0.49

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Laptop 2	0.17	0.46	0.55	0.48	0.43
Laptop 3	0.34	0.45	0.20	0.48	0.37
Laptop 4	0.51	0.47	0.40	0.36	0.54
Laptop 5	0.34	0.39	0.50	0.24	0.39

Weight	1.62	0.25	1.03	0.26	2.69
Alternatives	m_1	m_2	m_3	m_4	m_5
Laptop 1	1.12	0.12	0.52	0.16	1.32
Laptop 2	0.28	0.12	0.57	0.13	1.16
Laptop 3	0.55	0.11	0.21	0.13	0.99
Laptop 4	0.83	0.12	0.41	0.09	1.45
Laptop 5	0.55	0.10	0.52	0.06	1.05

Weight	1.62	0.25	1.03	0.26	2.69
Alternatives	m_1	m_2	m_3	m_4	m_5
Laptop 1	1.12	0.12	0.52	0.16	1.32
Laptop 2	0.28	0.12	0.57	0.13	1.16
Laptop 3	0.55	0.11	0.21	0.13	0.99
Laptop 4	0.83	0.12	0.41	0.09	1.45
Laptop 5	0.55	0.10	0.52	0.06	1.05
Ideal Best I_j^+	1.12	0.10	0.57	0.16	1.45
Ideal Worst I_j^-	0.55	0.12	0.21	0.06	0.99

Alternatives	m_1	m_2	m_3	m_4	m_5	M_j^+	M_j^-
Laptop 1	1.12	0.12	0.52	0.16	1.32	0.14	0.66
Laptop 2	0.28	0.12	0.57	0.13	1.16	0.89	0.48
Laptop 3	0.55	0.11	0.21	0.13	0.99	0.82	0.03

Laptop 4	0.83	0.12	0.41	0.09	1.45	0.35	0.57
Laptop 5	0.55	0.10	0.52	0.06	1.05	0.71	0.36

Alternatives	M_j^+	M_j^-	$M_j^+ + M_j^-$	R_i
Laptop 1	0.14	0.66	0.80	0.83
Laptop 2	0.89	0.48	1.37	0.35
Laptop 3	0.82	0.03	0.85	0.04
Laptop 4	0.35	0.57	0.92	0.62
Laptop 5	0.71	0.36	1.07	0.34

Alternatives	R_i	Rank
Laptop 1	0.83	1
Laptop 2	0.35	3
Laptop 3	0.04	5
Laptop 4	0.62	2
Laptop 5	0.34	4

4 Result

From the analyzation of AHP the C.R. will be 0.1. So, it is acceptable. Based on the above two evaluation laptop 1 should be best to purchase. In this work, an integrated approach has been proposed to show the effectiveness of the proposed model. The selection of AHP method provides various benefits. The numerical result shows that the proposed algorithm optimizes the selection of Laptop. The obtained result using proposed model is compared with the published result.

4.1 Comparison Analysis

The proposed method is compared to the standard model in Zhongsheng H, Bengang G and Xiaoyan X. [13] and the results are the same though they differ in the models proposed

5 Conclusion and future works

Different multi-attribute decision-making (MADM) methods often produce different outcomes for selecting or ranking a set of decision alternatives involving multiple attributes. In this paper we discuss about the AHP and TOPSIS method from MADM to compare some laptops to purchase with some attributes. Finally, identify the result that is laptop 1 will the better option to buy. Similarly, we can also analyse the problem based on mobile, financial, land, investment, etc. In future this work may be extended to the concepts as picture fuzzy graphs and planarity ideas can be explored. Furthermore, many real-life applications can be explored by extending this work to studies on the irregular and energy of graphs using AHP and TOPSIS method.

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